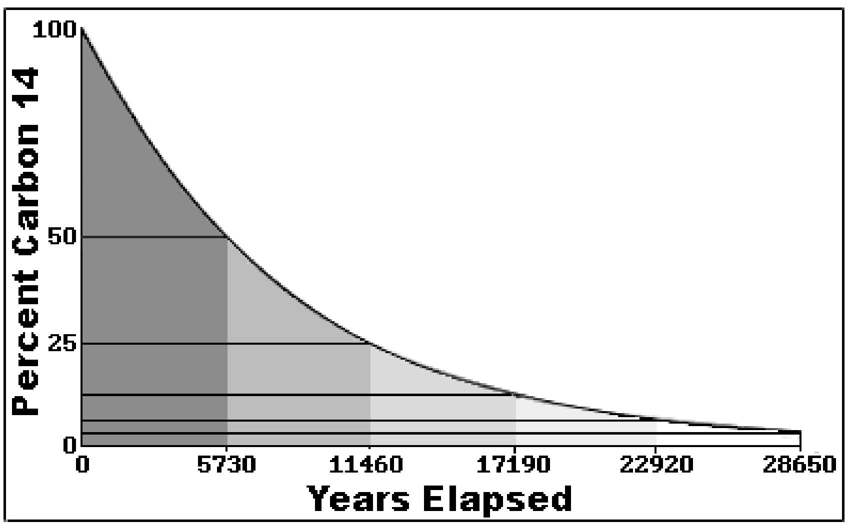
Program: RadioCarbon Dating

1 Background

In the 1940's Dr. Willard F. Libby invented carbon dating for which he received the Nobel Prize in chemistry in 1960.

Carbon dating is an accurate method by which archaeologists can determine the age of ancient artifacts. The stable form of carbon is carbon-12 and the radioactive isotope carbon-14 decays over time into carbon-12 and other particles. Carbon is naturally in all living organisms and is replenished in the tissues by eating other organisms or by breathing air that contains carbon. At any particular time all living organisms have approximately the same ratio of carbon-12 to carbon-14 in their tissues. When an organism dies it ceases to replenish carbon in its tissues and the decay of carbon-14 to carbon-12 changes the ratio of carbon-12 to carbon-14. Experts can compare the ratio of carbon-12 to carbon-14 in dead material to the ratio when the organism was alive to estimate the date of its death. Radiocarbon dating can be used on samples of bone, cloth, wood and plant fibers.

The half-life of carbon-14 is 5730 ± 30 years, and the method of dating lies in trying to determine how much carbon-14 (the radioactive isotope of carbon) is present in the artifact and comparing it to levels currently present in the atmosphere.



The decay of Carbon-14

Figure 1 illustrates the relationship between how much Carbon 14 is left in a sample and how old it is.

The following equation gives the quantitative relationship between the original number of nuclei present at time zero (N_0) and the number (N) at a later time t:

$N = N_0 e^{-\lambda t}$

where $e = 2.71828\ldots$ is the base of the natural logarithm, and λ is the decay constant for the nuclide.

The shorter the half-life, the larger is the value of λ, and the faster the exponential $e^{-\lambda t}$ decreases with time. The relationship between the decay constant λ and the half-life $t_{\frac{1}{2}}$ is

$\lambda = \frac{\ln(2)}{t_{\frac{1}{2}}} \approx \frac{0.693}{t_{\frac{1}{2}}}$ where $t_{\frac{1}{2}} = 5730$ for carbon-14.

Note that at any given time after the organism has died, $\frac{N}{N_0}$ is the percent of carbon-14 remaining in the sample compared to when it was alive. Knowing this percentage, we can calculate the age of the object.

The half-life of a radioactive isotope describes the amount of time that it takes half of the isotope in a sample to decay. This half-life is a relatively small number, which means that carbon-14 dating is not particularly helpful for very recent deaths and deaths more than 50,000 years ago. After 5,730 years, the amount of carbon 14 left in the body is half of the original amount. If the amount of carbon 14 is halved every 5,730 years, it will not take very long to reach an amount that is too small to analyze. When finding the age of an organic organism we need to consider the half-life of carbon-14 as well as the rate of decay, which is –0.693.

To determine the age of an object (from time of death), we can use the following equation:

$$t = \left[\frac{\ln\left(\frac{N}{N_0}\right)}{(-0.693)} \right] \times t_{\frac{1}{2}}$$

Example 1

Given a relic that has 92% carbon-14. How old is the relic?

Knowing that 92% of the C^{14} remains means that $\frac{N}{N_0} = 0.92$. Therefore, we can substitute into the above equation:

$$t = \left[\frac{\ln(0.92)}{(-0.693)} \right] \times 5730$$

Solving for t we get $t \approx 689$ years.

Example 2

Given a fossil with 35% carbon-14 compared to a living sample. How old is the fossil?

$$t = \left[\frac{\ln(0.35)}{(-0.693)} \right] \times 5730$$

$t \approx 8680$ years.

2 Your Task

You will develop methods to solve various aspect of radiocarbon dating. Then reflect on your solution and answer the reflection prompts.

2.1 Implement your solution

Modify the supplied Java source file named `Week3Program.java`. You should complete the following three methods:

int radiocarbonDate (double p)

Given the percentage of carbon-14 remaining in an artifact, return the year that the artifact was created or the organism died. The date is the current year minus the age of the artifact. Your result should be truncated, not rounded.

int radiocarbonAge (double p)

Given the percentage of carbon-14 remaining in an artifact, return the age of the artifact in years. Here a percentage is a double in the range [0.0, 1.0]. Your result should be truncated, not rounded.

double radiocarbonPercent (int age)

Given the age of an artifact in years, return the percent of carbon-14 left in it. Here a percentage is a double in the range [0.0, 1.0]. Remember that $\frac{N}{N_0}$ is the percent of carbon-14 remaining in the object.

2.2 Program Reflection

Record your answers to the following reflection prompts in the `Week3ProgramReflection.docx` document.

Conceptual Understanding

1. Discuss the decay constant (λ) and its role in the formulas used in this assignment. How is it derived?
2. Why is the base of the natural logarithm (e) crucial in the mathematical formulation of radiocarbon dating?
3. How can radiocarbon dating be potentially flawed? What are some factors that could affect the accuracy of the dating method?

Code Comprehension

1. Explain the logic behind your `radiocarbonAge( double p )` How did you ensure that the age is truncated and not rounded as per the assignment requirements?
2. Walkthrough the `radiocarbonPercent( int age )` How did you calculate the percentage of Carbon-14 remaining in an artifact given its age?
3. Describe any libraries or built-in Java methods you used for mathematical calculations (e.g., `Math.log( )`, `Math.E`)? If so, explain their functionalities and why you chose to use them.

Test Comprehension

1. Explain how the test case for `radiocarbonPercent( int age )` What is it checking for, and how does it determine success or failure?
2. Analyze the test case for `radiocarbonDate( double p )`. Why is a tolerance of 5 years allowed for the calculated year?
3. Did you encounter any edge cases while testing your methods? How did you handle them?

Insights

1. How has this assignment deepened your understanding of the interplay between computer science and other scientific disciplines such as archaeology or geology?
2. What ethical considerations should be taken into account when using radiocarbon dating on culturally or religiously significant artifacts?
3. Can the computational methods you've learned in this assignment be adapted for other forms of radioactive dating or scientific calculations? If so, how?

3 Hints

The following may prove useful.

- `Math.E`, `https://docs.oracle.com/en/java/javase/8/docs/api/java.base/java/lang/Math.html#E`
- `Math.log( )`, `https://docs.oracle.com/javase/8/docs/api/java/lang/Math.html#log-double-`
- `Math.pow( )`, `https://docs.oracle.com/javase/8/docs/api/java/lang/Math.html#pow-double-double-`
- `java.util.Calendar`, `https://docs.oracle.com/javase/8/docs/api/index.html?java/util/Calendar.html`

4 Provided Files

- `Week3Program.java`
- `Week3ProgramReflection.docx`

5 Critiquing

Your program shoulds:

- Compile with no errors or warnings.
- Your `radiocarbonDate( )` method should pass our tests.
- Your `radiocarbonAge( )` method should pass our tests.
- Your `radiocarbonPercent( )` method should pass our tests.
- You must follow the *Programming Style Guidelines*.
- You must complete the Program Reflection.

6 Submitting

Submit your assignment to Canvas. Your submission must include the following files:

- Week3Program.java
- Week3ProgramReflection.pdf