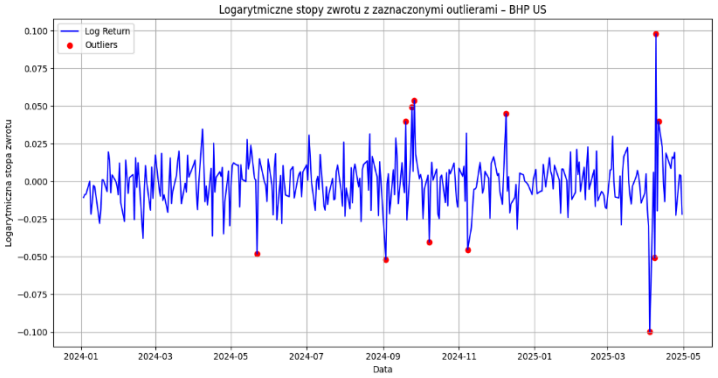
Brief company descriptions:

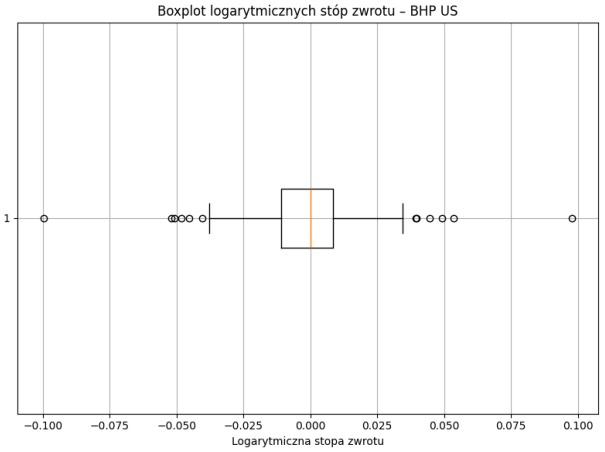
BHP Group Limited (BHP.AX) - A global **mining company specializing in the extraction of natural resources** such as iron ore, copper, nickel, and coal. It operates mainly in **Australia**, **North America**, and **South America**.

Pfizer Inc. - An **American** pharmaceutical company known for producing **drugs, vaccines, and biotechnology**. The company operates worldwide and is active in the areas of research, development, and sales of healthcare products.

LVMH Moët Hennessy Louis Vuitton - LVMH is the world's largest **luxury goods conglomerate**, comprising brands in the fashion, perfume, spirits, and watch sectors. It operates globally and is listed on the Euronext exchange in **Paris**.

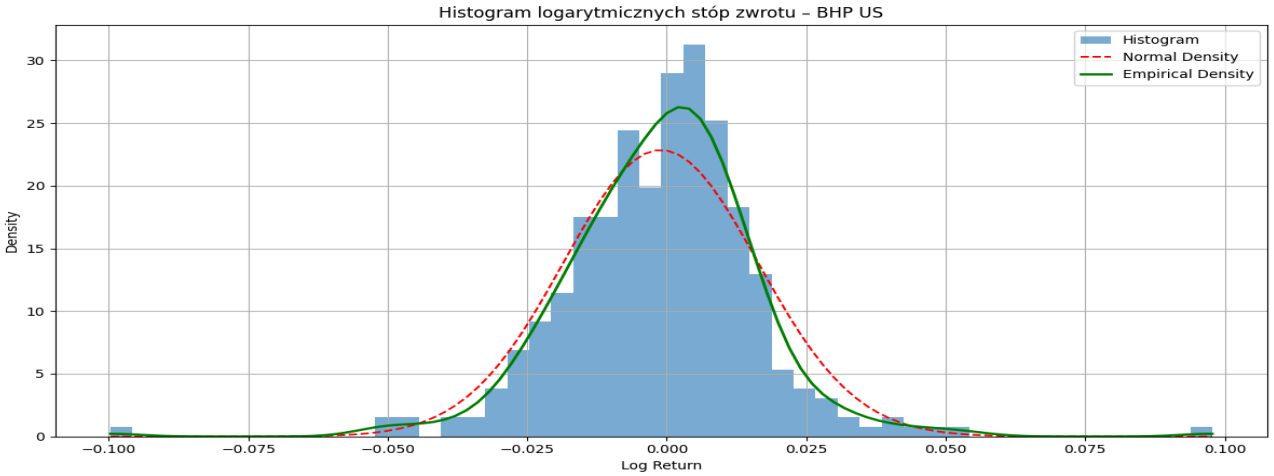
BHP Group Limited (BHP.AX)





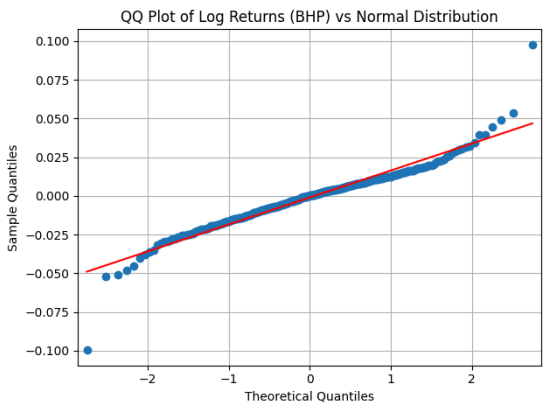
1. Characteristics of the return distribution:

The charts above allow us to notice the presence of outliers on both sides of the distribution, especially on the side of negative returns. The red dots show when the most extreme return changes occurred -- e.g., in March 2025.



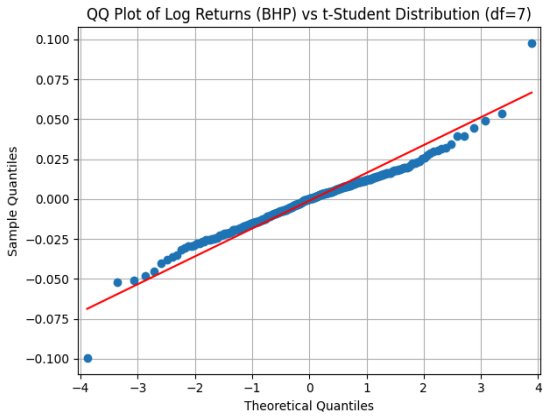
The **histogram** shows that the distribution of logarithmic returns is left-skewed (skewness ≈ -0.84) and leptokurtic (kurtosis ≈ 5.91), meaning it has fat tails and a high concentration of values around the mean. It deviates from the normal distribution, which is confirmed by the difference between the dashed line and the empirical density.

2. Assessment of distribution fit:



QQ plot vs normal

The quantile-quantile plot shows deviations from a straight-line pattern in the tails, suggesting that the distribution is not normal.



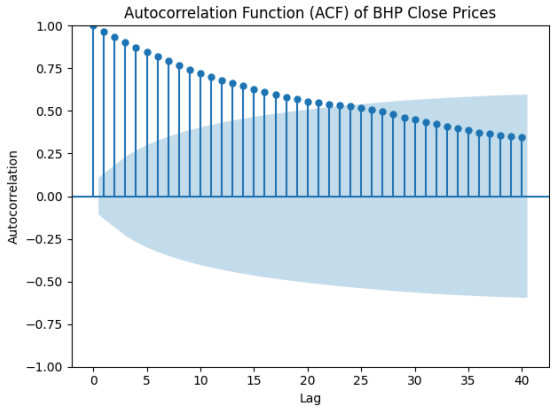
QQ plot vs Student's t-distribution df=7
The data fits the Student's t-distribution better -- the line runs closer to the points, especially at the extremes. This confirms the existence of fat tails.

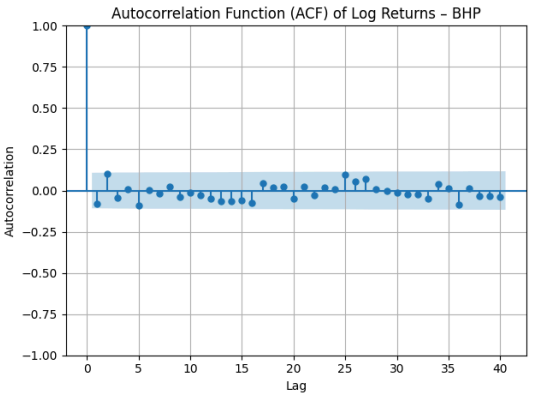
3. Statistical Tests

Jarque-Bera**: statistic = 467.39, p-value ≈ 0** ⟶ we reject normality

Kolmogorov-Smirnov**: statistic = 0.47, p-value ≈ 0** ⟶ we reject normality

4. Autocorrelation:





Strong autocorrelation is present in the price series - the bars are significantly above the significance limits.

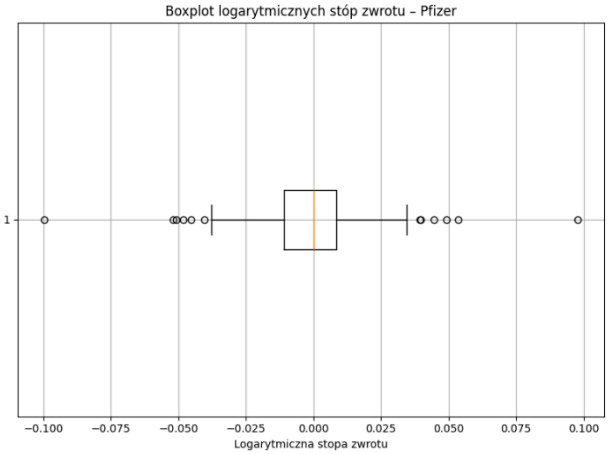
The autocorrelation of returns is small - all points are within the significance area.

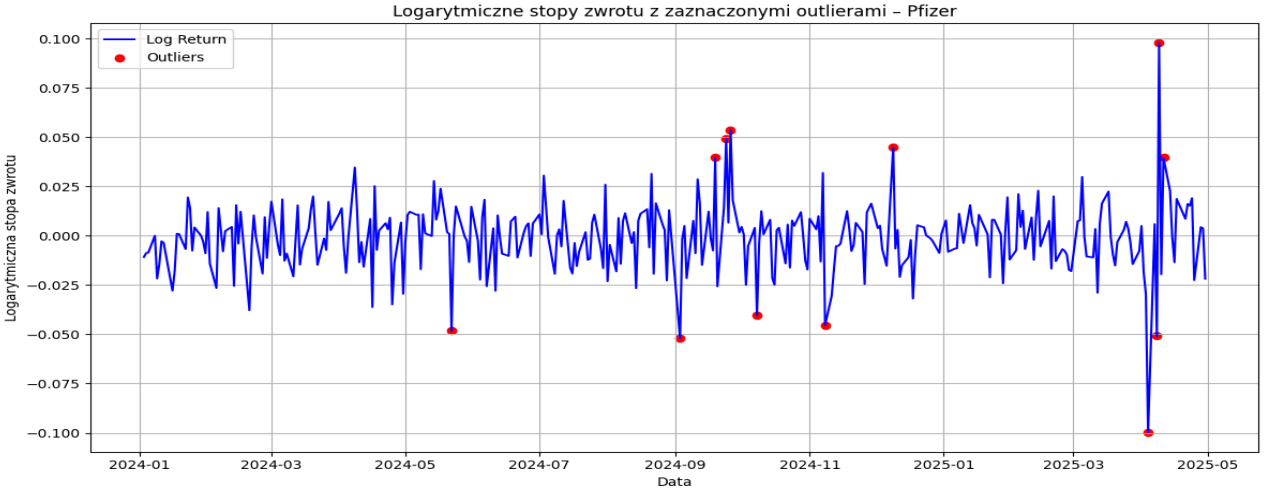
For closing prices: p-value ≈ 0 ⟶ we reject the hypothesis of no autocorrelation

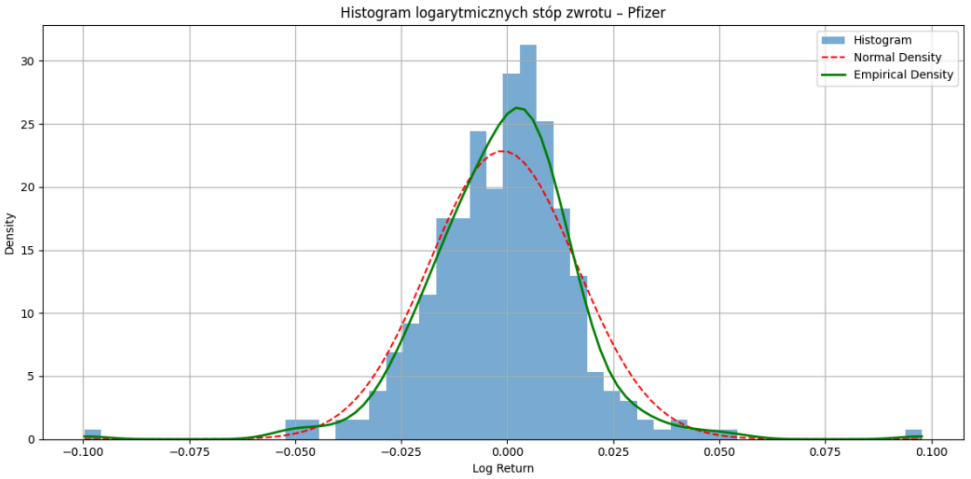
For returns: p-value = 0.7987 ⟶ **no grounds to reject the hypothesis of independence**
This means that **returns are independent**, which is consistent with the efficient market hypothesis.

Pfizer

1. **Characteristics of the return distribution**
   The time series data indicate relative stability of the logarithmic returns, with a few clear deviations -- mainly in March and April 2025. The boxplot and the chart with marked outliers confirm the presence of extreme observations on both sides of the distribution, especially negative ones.





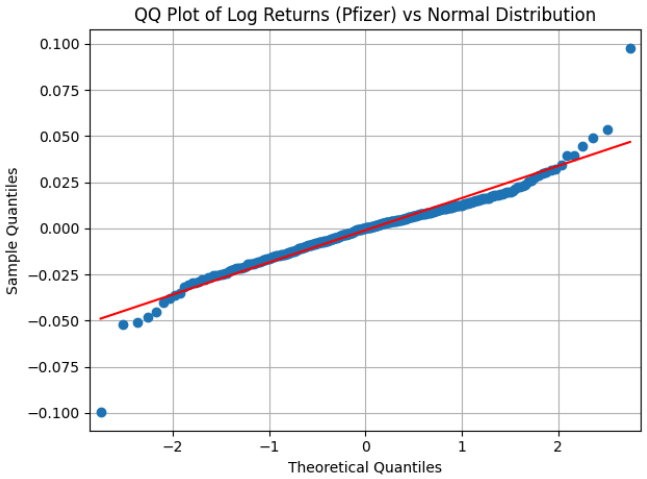


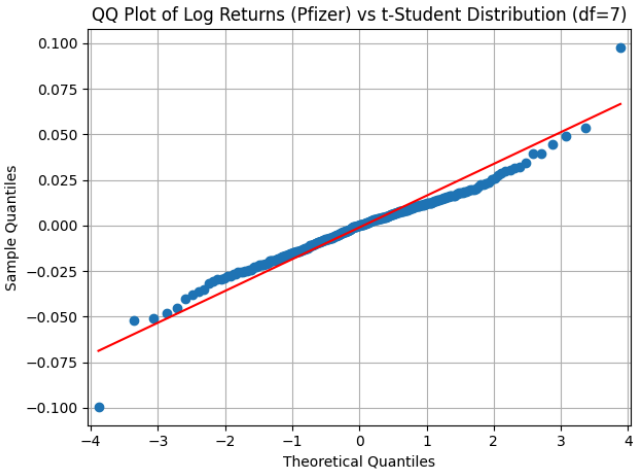
The **histogram** indicates that the return distribution is left-skewed *(skewness = -0.84)* and leptokurtic *(kurtosis = 5.91)*, which means the presence of fat tails and a high concentration of values around the mean.

2. Assessment of distribution fit:

QQ plot vs normal shows clear deviations in the tails, confirming a lack of conformity with the normal distribution.

The Student's t-distribution better represents the data - the line runs closer to the points at the extremes, confirming the presence of fat tails.

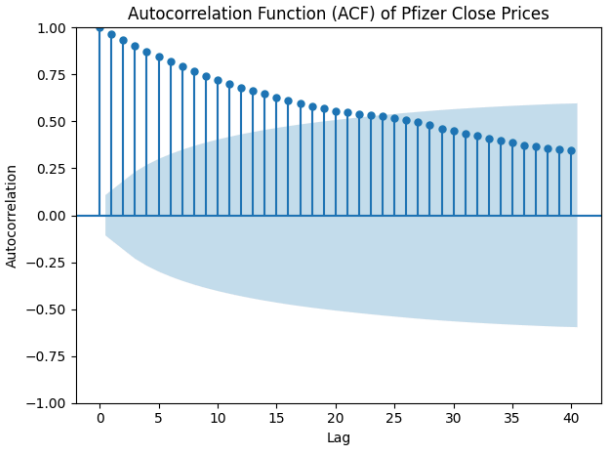


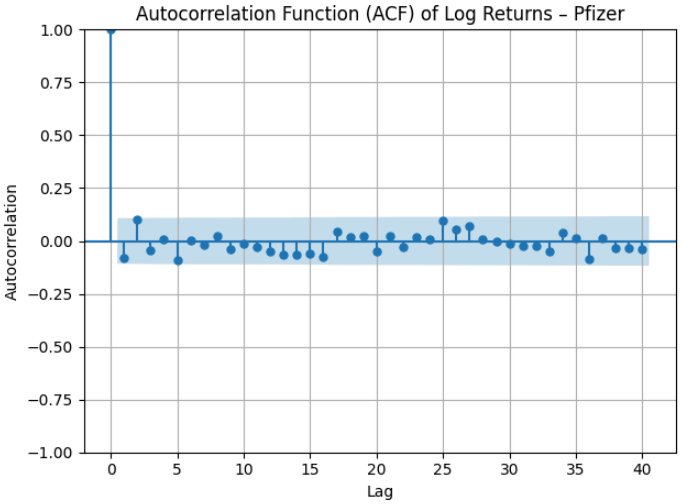


3. Autocorrelation

ACF of closing prices - There is strong autocorrelation - the price series is not random.

ACF logarytmic returns - The values fall within the significance limits ⟶ no autocorrelation in the returns.



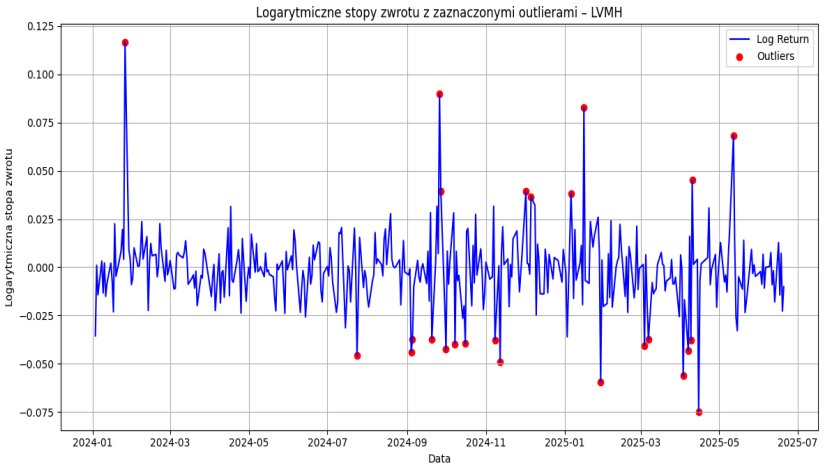


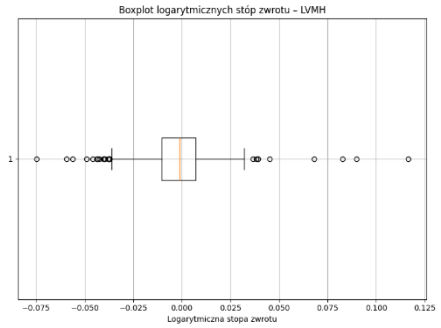
Ljung-Box Test

- Closing prices: p-value ≈ 0 ⟶ we reject the hypothesis of independence
- Returns: p-value ≈ 0.41 ⟶ no grounds to reject the hypothesis of independence

LVMH:

1. **Characteristics of the return distribution:**





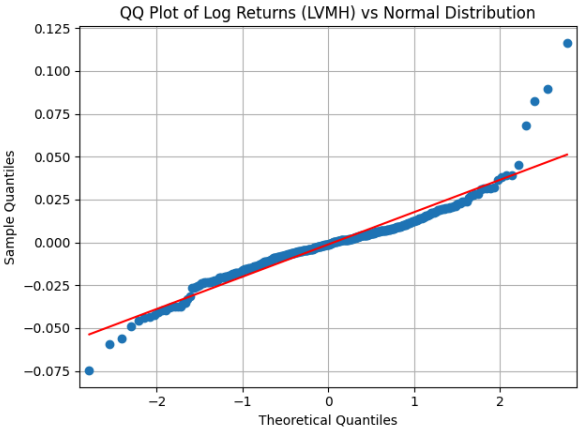
The time series chart shows moderate fluctuations in returns with single outliers, mainly on the positive side. The boxplot confirms the presence of extreme values, although their number is limited.

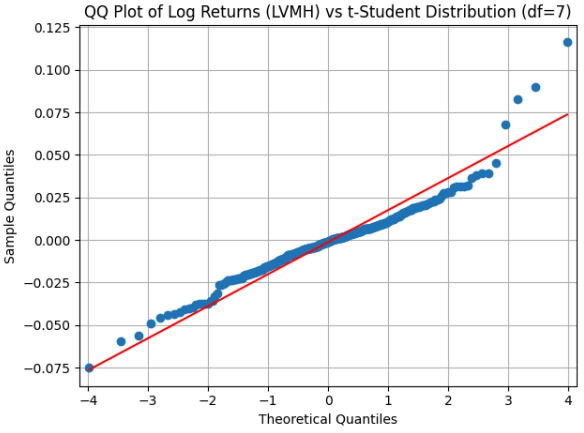
The histogram indicates that the return distribution has positive skewness (≈ 0.87) and is leptokurtic (kurtosis ≈ 6.89). This indicates the presence of fat tails and a higher probability of [Image: media/image16.png] extreme values occurring than in the case of a normal distribution. Despite a small mean (≈ -0.0012), high variability (standard deviation ≈ 0.0188) confirms the risky nature of the instrument.

2. **Assessment of distribution fit:**

QQplot vs normal - the line deviates from a straight-line pattern, suggesting a lack of conformity with the normal distribution, especially in the tails.

QQplot vs Student's t-distribution (df=7) - the fit is clearly better; the points align closer to the reference line, especially at the extremes of the distribution.





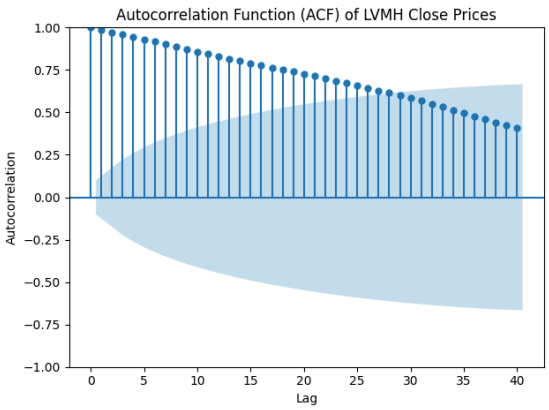
3. **Autocorrelation:**

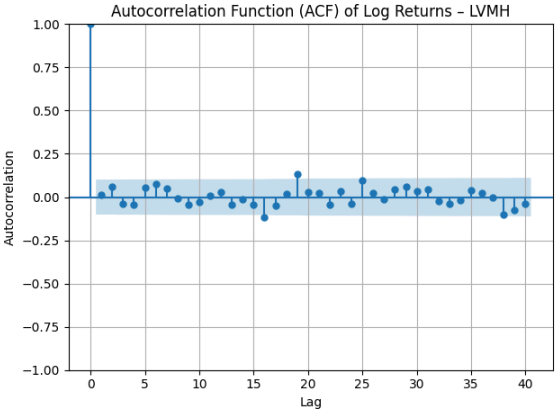
Autocorrelation of closing prices: strong - the bars significantly exceed the significance limits ⟶ observations are dependent over time.

Autocorrelation of returns: none - all bars fall within the significance limits ⟶ no autocorrelation.

Ljung-Box Test:

- **Closing prices:** p-value ≈ 0 ⟶ **we reject the hypothesis of independence** - significant autocorrelation is present.
- **Returns:** p-value ≈ 0.41 ⟶ **no grounds to reject the hypothesis of independence.**

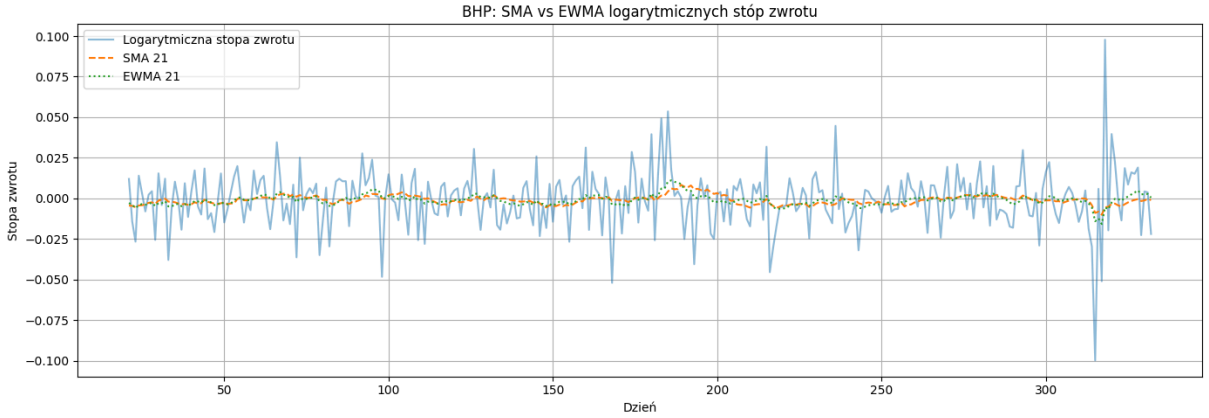


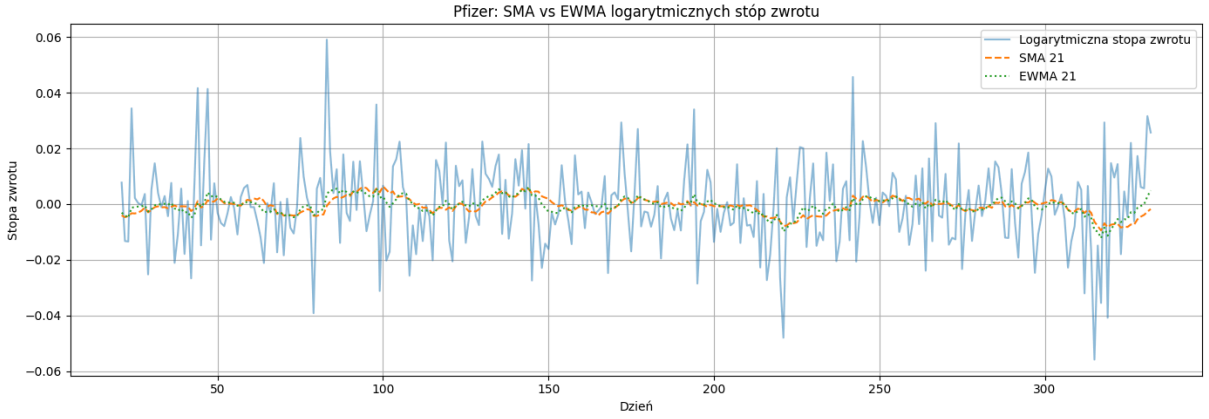


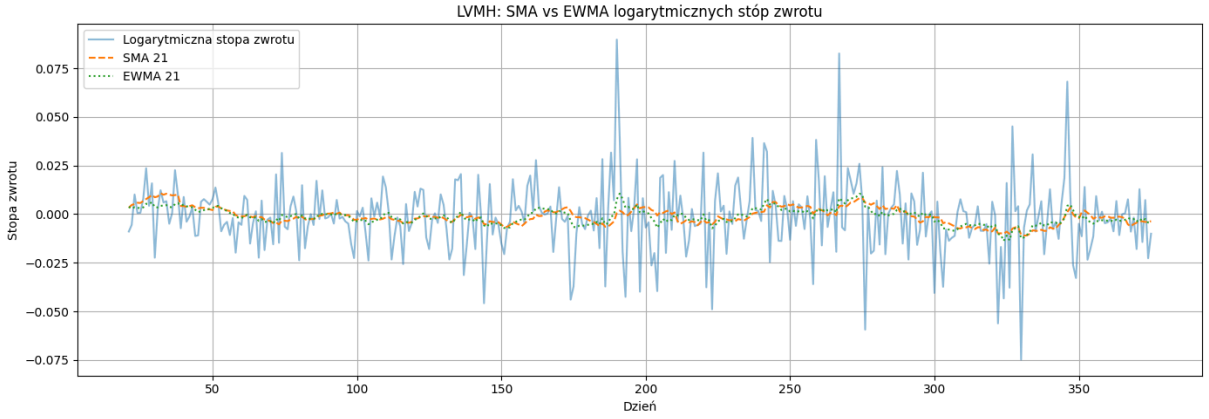
Comparison and general conclusions

All three companies have **non-normal return distributions**, but **LVMH is the closest to a normal distribution**. **BHP** is characterized by the largest deviation from normality and the highest kurtosis, indicating the **highest risk of extreme changes**.

Volatility measures - analysis and comparison of company risk







LVMH exhibits the highest volatility - its chart shows numerous jumps in returns and a greater spread relative to the moving averages (SMA and EWMA) - making it the most risky.

Value at Risk (VaR) and Conditional Value at Risk (CVaR)

Parametric method (VaR_param, CVaR_param) and non-parametric method results breakdown:

Spółka	VaR (param)	CVaR (param)	VaR (nonparam)	CVaR (nonparam)
BHP	–0.020973	–0.030706	–0.028680	–0.039004
Pfizer	–0.025170	–0.031413	–0.024279	–0.032389
LVMH	–0.032208	–0.040081	–0.031793	–0.043500

The non-parametric method is generally more justified - it accounts for the fat tails and asymmetry that we observe in the charts and normality tests (e.g., QQ plots and JB, KS tests).

The most useful measure in practice seems to me to be the non-parametric Value at Risk (VaR) -- especially when talking about financial data, as it does not rely on the assumption of a normal distribution. Furthermore, it is easy to interpret for people who are not analysts but hold decision-making positions in a company - it informs about the maximum expected loss at a given confidence level (e.g., 5%) over a specified time horizon. Its advantage is also that it is widely used in financial institutions.