****

Part1) Central Limit Theorem

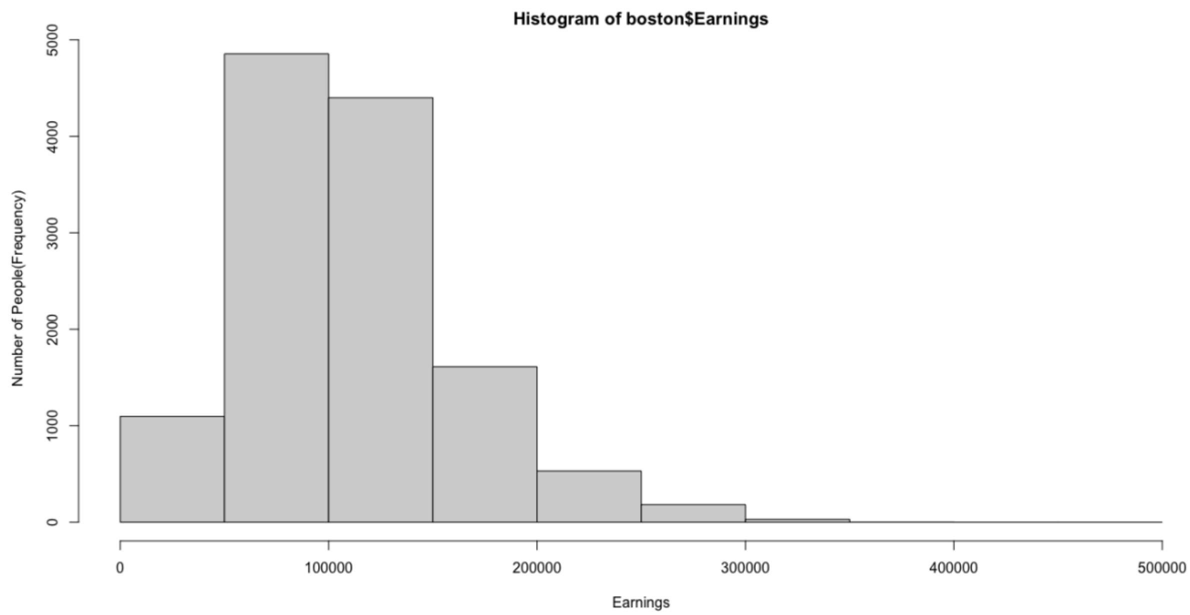
Code Section 1.a:

```
#Dataset
boston <- read.csv(
  "https://people.bu.edu/kalathur/datasets/bostonCityEarnings.csv",
  colClasses = c("character", "character", "character", "integer", "character"))
#---------------------------------------------------------------------------
#a)histogram of earnings
breaks_hist <- seq(0,500000,by= 50000)
options(scipen = 4)
par(mar=c(5,5,2,2))
hist_plot <- hist(boston$Earnings, breaks = breaks_hist,xlab = "Earnings",
     ylab ="Number of People(Frequency)")
#Mean Earnings
mean(boston$Earnings)
#Standard deviation of Earnings
sd(boston$Earnings)
```

Console Section 1.a:

```
#Dataset
> boston <- read.csv(
+   "https://people.bu.edu/kalathur/datasets/bostonCityEarnings.csv",
+   colClasses = c("character", "character", "character", "integer", "character"))
> #---------------------------------------------------------------------------
> #a)histogram of earnings
> breaks_hist <- seq(0,500000,by= 50000)
> options(scipen = 4)
> par(mar=c(5,5,2,2))
> hist_plot <- hist(boston$Earnings, breaks = breaks_hist,xlab = "Earnings",
+      ylab ="Number of People(Frequency)")
> #Mean Earnings
> mean(boston$Earnings)
[1] 108680.9
> #Standard deviation of Earnings
> sd(boston$Earnings)
[1] 50474.7
```

Plot section 1.a:



Inferences:
1. The data set looks skewed (not normal).
2. Most of the people belongs to 50k to 150k earnings.
3. there is small number of people that has income beyond 300k.
4. when we do box plot, we can find there are outlines towards the upper end.

Code section 1.b:

```
library(sampling)

#b)
set.seed(7356)

sample.1000 <- 1000
sample_size <- 10
xbar.10 <- numeric(sample.1000)
for (i in 1:sample.1000) {
  s10_rows <- sample(nrow(boston),10,replace = FALSE) #using sample() method
  s10_sample <- boston[s10_rows,] #mapping selected rows to Boston dataset

  xbar.10[i] <-mean(s10_sample$Earnings) # making 1000 samples of size 10

}

mean.1000_10 <- mean(xbar.10)
sd.1000_10 <- sd(xbar.10)
```

```
par(mar=(c(2,2,2,2)))
hist(xbar.10,xlab = "Earnings", ylab = "Frequency", main = "Histogram of sample means of size
10",
     ylim = c(0,250))
```

Console section 1.b:

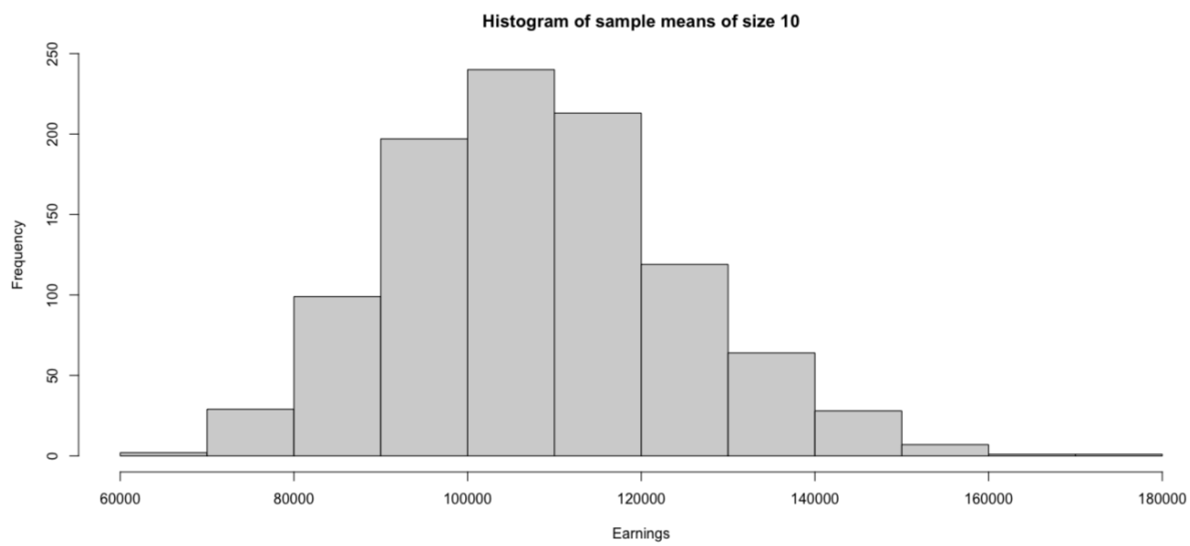
```
> library(sampling)
#b)
set.seed(7356)

sample.1000 <- 1000
sample_size <- 10
xbar.10 <- numeric(sample.1000)
for (i in 1:sample.1000) {
  s10_rows <- sample(nrow(boston),10,replace = FALSE) #using sample() method
  s10_sample <- boston[s10_rows,] #mapping selected rows to Boston dataset

  xbar.10[i] <-mean(s10_sample$Earnings) # making 1000 samples of size 10

}

mean.1000_10 <- mean(xbar.10)
#mean of sample size 10 of 1000 samples
mean.1000_10
sd.1000_10 <- sd(xbar.10)
#Standard deviation of sample size 10 of 1000 samples.
sd.1000_10
par(mar=(c(5,5,2,2)))
hist(xbar.10,xlab = "Earnings", ylab = "Frequency", main = "Histogram of sample means of size
10",
     ylim = c(0,250))
```

Console section 1.b:

```
> set.seed(7356)
> sample.1000 <- 1000
> sample_size <- 10
> xbar.10 <- numeric(sample.1000)
> for (i in 1:sample.1000) {
+   s10_rows <- sample(nrow(boston),10,replace = FALSE) #using sample() method
+   s10_sample <- boston[s10_rows,] #mapping selected rows to Boston dataset
+
+   xbar.10[i] <-mean(s10_sample$Earnings) # making 1000 samples of size 10
+
+ }
```

> mean.1000_10 <- mean(xbar.10)
> #mean of sample size 10 of 1000 samples
> mean.1000_10
[1] 108216.2
> sd.1000_10 <- sd(xbar.10)
> #Standard deviation of sample size 10 of 1000 samples.
> sd.1000_10
[1] 16297.48
> par(mar=(c(5,5,2,2)))
> hist(xbar.10,xlab = "Earnings", ylab = "Frequency", main = "Histogram of sample means of size 10",
+ ylim = c(0,250))

Plot section 1.b:



Code section 1.c:
set.seed(7356)
sample_size_40 <- 40 #for sample size 40
xbar.40 <- numeric(sample.1000) #initializing list of 1000 0s
for (i in 1:sample.1000) {
 s40_rows <- sample(nrow(boston),40,replace = FALSE) #getting random 40 rows out of original dataset
 s40_sample <- boston[s40_rows,] # mapping those sample data to original data

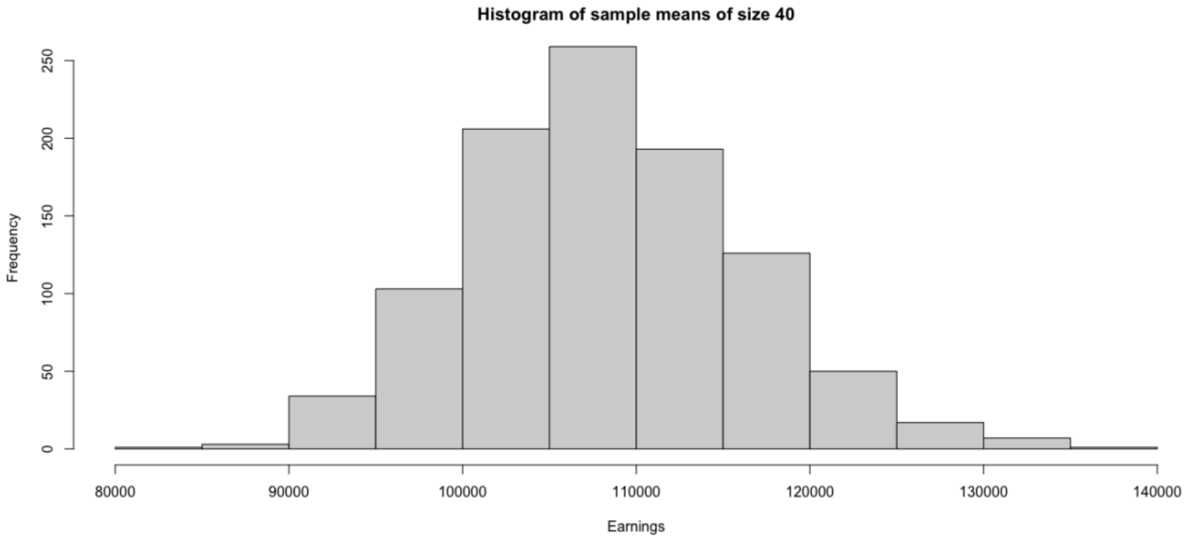
 xbar.40[i] <-mean(s40_sample$Earnings) # replacing those

```
}
#mean of the sample size of 40 of 1000 samples
mean.1000_40 <- mean(xbar.40)
mean.1000_40
#SD of the sample size of 40 of 1000 samples
sd.1000_40 <- sd(xbar.40)
sd.1000_40
#histogram of sample means
par(mar=c(5,5,2,2))
hist(xbar.40,main = "Histogram of sample means of size 40",xlab = "Earnings",ylab =
"Frequency")
```

Console section 1.c:

```
> #c)
> set.seed(7356)
> sample_size_40 <- 40 #for sample size 40
> xbar.40 <- numeric(sample.1000) #initializing list of 1000 0s
> for (i in 1:sample.1000) {
+   s40_rows <- sample(nrow(boston),40,replace = FALSE) #getting random 40 rows out of
original dataset
+   s40_sample <- boston[s40_rows,] # mapping those sample data to original data
+
+   xbar.40[i] <-mean(s40_sample$Earnings) # replacing those
+
+ }

> #mean of the sample size of 40 of 1000 samples
> mean.1000_40 <- mean(xbar.40)
> mean.1000_40
[1] 108335.2
> #SD of the sample size of 40 of 1000 samples
> sd.1000_40 <- sd(xbar.40)
> sd.1000_40
[1] 8013.736
> #histogram of sample means
> par(mar=c(5,5,2,2))
> hist(xbar.40,main = "Histogram of sample means of size 40",xlab = "Earnings",ylab =
"Frequency")
```

Plot section 1.c:



Code section 1.d:

```
#d)
# means of three type of distribution
mean_combine <- c(Original=mean(boston$Earnings),Sample_10=mean.1000_10,
                  Sample_40=mean.1000_40)

mean_combine

# it can be seen that mean of the all three type of data are almost same
sd_combine <- c(Orignal=sd(boston$Earnings),Sample_10=sd.1000_10,
                Sample_40=sd.1000_40)
sd_combine
# however standard deviation of all three type of data are different. sd of the
# 1000  mean sample of size 10 is less diverse than original. while data with sample
# size 40 has SD way less than that of sample size of 10.

#theoretical values of sd can be calculated by using formula where sd of origincal
# sd divided by square root of sample sizes

theoritical_sd <- sd_combine[1]/c(sqrt(10),sqrt(40))
theoritical_sd
```

Console section 1.d:

```
> #d)
> # means of three type of distribution
> mean_combine <- c(Original=mean(boston$Earnings),Sample_10=mean.1000_10,
+                 Sample_40=mean.1000_40)
> mean_combine
 Original Sample_10 Sample_40
 108680.9  108216.2  108335.2
>
> # it can be seen that mean of the all three type of data are almost same
> sd_combine <- c(Orignal=sd(boston$Earnings),Sample_10=sd.1000_10,
+               Sample_40=sd.1000_40)
> sd_combine
  Orignal Sample_10 Sample_40
50474.701 16297.481  8013.736
> # however standard deviation of all three type of data are different. sd of the
> # 1000  mean sample of size 10 is less diverse than original. while data with sample
> # size 40 has SD way less than that of sample size of 10.
>
> #theoretical values of sd can be calculated by using formula where sd of origincal
> # sd divided by square root of sample sizes
>
> theoritical_sd <- sd_combine[1]/c(sqrt(10),sqrt(40))
> theoritical_sd
[1] 15961.502  7980.751
```

Inferences:

1.From above original and theoretical values of population data and samples data followed the Central Limit Theorem.

2. as the sample sizes increases, samples will have small SD which means sample means are less distributed.

3. Mean of the original data set as well as means sample of size 10 and 40 remains similar. Theoretically, this should have same values when all possible samples are drawn but we took 1000 samples only.

Part2) Central Limit Theorem – Negative Binomial distribution

Code section 2.a:

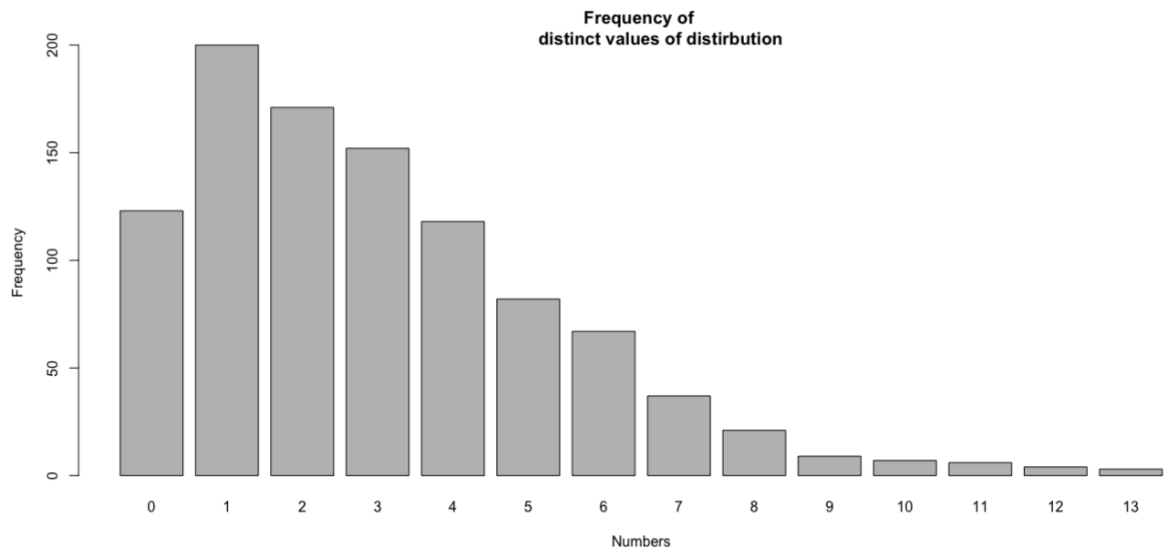
```
set.seed(7356)
```

```
#a)

random.1000 <-rnbinom(1000,3,0.5)

#checking frequency of the number

Freq_1000 <- table(random.1000)

barplot(Freq_1000,xlab = "Numbers",ylab = "Frequency",main = "Frequency of
        distinct values of distirbution")
```

Console section 2.a:

```
> #Part 2

> set.seed(7356)

> #a)

> random.1000 <-rnbinom(1000,3,0.5)

> #checking frequency of the number

> Freq_1000 <- table(random.1000)

> barplot(Freq_1000,xlab = "Numbers",ylab = "Frequency",main = "Frequency of
+        distinct values of distirbution")
```

Plot section 2.a:



Code section 2.b:

```
#b)

# four sample sizes

sample_sizes <- c(10,20,30,40)

# 5000 samples of each of sample size types

xbar.5000 <- numeric(5000)

list_mean <- c() # To store list of all four means

list_SD <- c() # to store list of all four SD

par(mfrow = c(2,2))


for (size in sample_sizes) {

  for (i in 1:5000) {

    xbar.5000[i] <- mean(sample(random.1000,size,replace = FALSE))
```

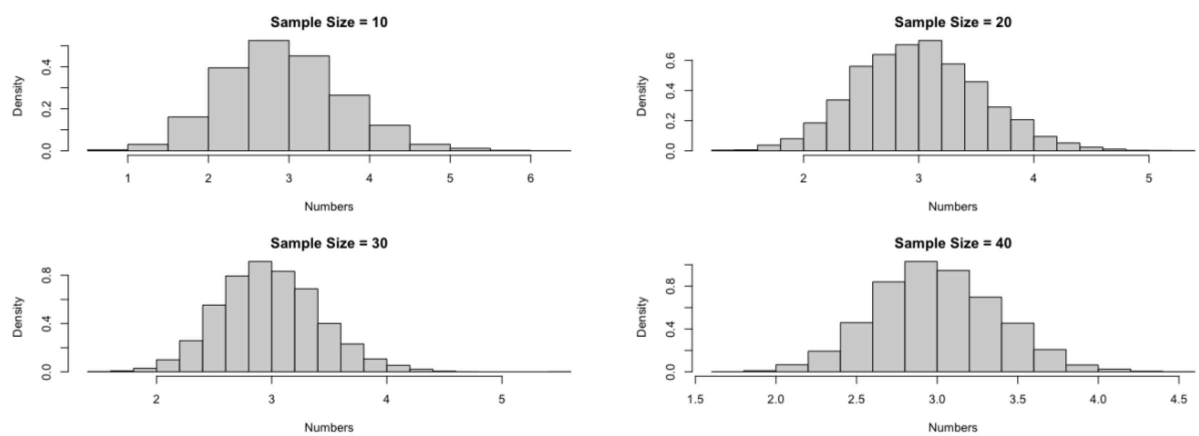
```
  }
  hist(t(xbar.5000),prob=TRUE,
       breaks = 15,main = paste("Sample Size =", size),xlab = "Numbers")

  cat("Sample Size = ",size, " Mean = ", mean(xbar.5000),
      " SD = ", sd(xbar.5000), "\n")
  list_mean <- c(list_mean,mean(xbar.5000))
  list_SD <- c(list_SD,sd(xbar.5000))
}
```

Console section 2.b:

```
> # four sample sizes
> sample_sizes <- c(10,20,30,40)
> # 5000 samples of each of sample size types
> xbar.5000 <- numeric(5000)
> list_mean <- c() # To store list of all four means
> list_SD <- c() # to store list of all four SD
> par(mfrow = c(2,2))
> for (size in sample_sizes) {
+   for (i in 1:5000) {
+     xbar.5000[i] <- mean(sample(random.1000,size,replace = FALSE))
+
+   }
```

```
+  hist(t(xbar.5000),prob=TRUE,
+       breaks = 15,main = paste("Sample Size =", size),xlab = "Numbers")
+
+  cat("Sample Size = ",size, " Mean = ", mean(xbar.5000),
+      " SD = ", sd(xbar.5000), "\n")
+  list_mean <- c(list_mean,mean(xbar.5000))
+  list_SD <- c(list_SD,sd(xbar.5000))
+ }
Sample Size =  10  Mean =  2.98154  SD =  0.7476477
Sample Size =  20  Mean =  3.02617  SD =  0.5456392
Sample Size =  30  Mean =  3.001313  SD =  0.435892
Sample Size =  40  Mean =  3.007995  SD =  0.3800001
```

Plot section 2.b:



Code section 2.c:

```
#c)
#from above calculation
#mean from a
```

```
mean(random.1000)
#SD from a
sd(random.1000)

#means from b
list_mean
#SDs from b
list_SD


# from above, we can conclude that means of population and sample are almost similar
# While SD of sample means is lower than population. also, In sample means, the
# data variability decreases as the size of the sample increases.

#Theoretical sample sd calculation can also be done

samples.SD <- sd(random.1000)/sqrt(sample_sizes)
samples.SD

# sample SD for different sizes is almost same as the theoretical SD we get from
```

Console section 2.c:

```
#c)
> #from above calculation
> #mean from a
> mean(random.1000)
[1] 3.013
> #SD from a
> sd(random.1000)
[1] 2.434082
> #means from b
> list_mean
[1] 2.981540 3.026170 3.001313 3.007995
> #SDs from b
> list_SD
[1] 0.7476477 0.5456392 0.4358920 0.3800001

> # from above, we can conclude that means of population and sample are almost similar
> # while SD of sample means is lower than population. also, In sample means, the
> # data variability decreases as the size of the sample increases.

> #Theoretical sample sd calculation can also be done
> samples.SD <- sd(random.1000)/sqrt(sample_sizes)
> samples.SD
```

[1] 0.7697244 0.5442773 0.4444006 0.3848622

Inferences:
1.From above, we can conclude that means of population and sample are almost similar
2. While SD of sample means is lower than population. also, In sample means, the
3.Data variability decreases as the size of the sample increases and graphs looks more like Normal distribution plot as the sample sizes increases.
4.sample SD for different sizes are almost same as the theoretical SD we get from formula.

Part3) Sampling

Code part 3.a:

#number of employees working in each department can be done by using table

table.name <- table(boston$Department)

#now sorting the value and selecting top5 department.

top5_depart <- sort(table.name,decreasing = TRUE)[1:5]

top5_depart

#mapping with original data set

subset_top5 <- subset(boston,boston$Department %in% names(top5_depart))

#a)

library(sampling)

set.seed(7356)

sample.with.replace <- srswr(50,nrow(subset_top5)) # using R function

row.number <- (1:nrow(subset_top5))[sample.with.replace!=0] # mapping with top 5 dataset's rows

subset.with.replace <- subset_top5[row.number,] # getting subset from top5 data set

```
#frequencies

table(subset.with.replace$Department)

# percentage with respect to sample size

table(subset.with.replace$Department)/50

#Alternatively

prop.depart.a <- prop.table(table(subset.with.replace$Department))

prop.depart.a


for (m in 1:length(prop.depart.a)) {


  cat(names(prop.depart.a)[m],"will have", prop.depart.a[m]*100,"%", "\n")

}
```

Console Part 3.a:

```
> #part 3

> #number of employees working in each department can be done by using table

> table.name <- table(boston$Department)

> #now sorting the value and selecting top5 department.

> top5_depart <- sort(table.name,decreasing = TRUE)[1:5]

> top5_depart


Boston Police Department   Boston Fire Department    BPS Special Education  BPS Facility
Management    Boston Public Library

                   2732                   1672                    611                    415                    384

>
```

```
> #mapping with original data set

> subset_top5 <- subset(boston,boston$Department %in% names(top5_depart))

> #a)

> library(sampling)

> set.seed(7356)

> sample.with.replace <- srswr(50,nrow(subset_top5)) # using R function

> row.number <- (1:nrow(subset_top5))[sample.with.replace!=0] # mapping with top 5 dataset's
rows

> subset.with.replace <- subset_top5[row.number,] # getting subset from top5 data set

> #frequencies

> table(subset.with.replace$Department)


  Boston Fire Department Boston Police Department    Boston Public Library  BPS Facility
Management    BPS Special Education

                    12                       27                        3                       2                       6

> # percentage with respect to sample size

> table(subset.with.replace$Department)/50


  Boston Fire Department Boston Police Department    Boston Public Library  BPS Facility
Management    BPS Special Education

                  0.24                     0.54                     0.06                     0.04                     0.12

> #Alternatively

> prop.depart.a <- prop.table(table(subset.with.replace$Department))

> prop.depart.a
```

```
  Boston Fire Department Boston Police Department    Boston Public Library  BPS Facility
Management    BPS Special Education

                 0.24                    0.54                    0.06                    0.04                    0.12

>

> for (m in 1:length(prop.depart.a)) {

+

+   cat(names(prop.depart.a)[m],"will have", prop.depart.a[m]*100,"%", "\n")

+ }

Boston Fire Department will have 24 %

Boston Police Department will have 54 %

Boston Public Library will have 6 %

BPS Facility Management will have 4 %

BPS Special Education will have 12 %
```

Code section 2.b:

```
#b)
set.seed(7356)
inclusion.prob <- inclusionprobabilities(subset_top5$Earnings,50)
length(inclusion.prob)
unequal.probab <- UPsystematic(inclusion.prob)
head(unequal.probab)
new.samples.50 <- getdata(subset_top5,unequal.probab)
head(new.samples.50)
#alternatively we can map the selected rows with subset_top5 as follows
new.samples <- (subset_top5)[unequal.probab !=0,]
#frequency of employee in each department can be calculated by
frequency_depart <- table(new.samples.50$Department)
#calculating proportion
prop.depart.b <-prop.table(frequency_depart)
```

```
for (i in 1:length(prop.depart.b)) {

  cat(names(prop.depart.b)[i],"will have", prop.depart.b[i]*100,"%", "\n")
}
```

Console section2.b:

```
#b)
> set.seed(7356)
> inclusion.prob <- inclusionprobabilities(subset_top5$Earnings,50)
> length(inclusion.prob)
[1] 5814
> unequal.probab <- UPsystematic(inclusion.prob)
> head(unequal.probab)
[1] 0 0 0 0 0 0
> new.samples.50 <- getdata(subset_top5,unequal.probab)
> head(new.samples.50)
     ID_unit                    NAME              Department         Title Earnings ZipCode
112       44 Alessandro,Dennis Charles   Boston Fire Department   Fire Fighter   145968   02132
412      164   Aylward,Michael Anthony   Boston Fire Department   Fire Fighter   137181   02118
720      290              Bent,Thomas Boston Police Department Police Officer   134682   02132
965      405          Bowen,Raymond A Boston Police Department Police Officer   176469   02136
1185     508          Brown,Nytisha D Boston Police Department Police Officer   176367   02021
1424     623   Caggiano,Joseph Albert Boston Police Department Police Officer   141363   02128
> #alternatively we can map the selected rows with subset_top5 as follows
> new.samples <- (subset_top5)[unequal.probab !=0,]
> #frequency of employee in each department can be calculated by
> frequency_depart <- table(new.samples.50$Department)
> #calculating proportion
> prop.depart.b <-prop.table(frequency_depart)
>
> for (i in 1:length(prop.depart.b)) {
+
+   cat(names(prop.depart.b)[i],"will have", prop.depart.b[i]*100,"%", "\n")
+ }
Boston Fire Department will have 44 %
Boston Police Department will have 48 %
Boston Public Library will have 2 %
BPS Facility Management will have 2 %
BPS Special Education will have 4 %
```

Code section 3.c:

```
#c)
```

```
set.seed(7356)
#ordering the data using Department variable
order.department <- order(subset_top5$Department)
#mapping to dataset according to ordered rows
ordered.top5 <- subset_top5[order.department,]
#finding relative frequency of employee in each deparment
frequency.top5 <- table(ordered.top5$Department)

#finding proportions in each department based on their employee numbers.
prop.50 <- round(50*frequency.top5/sum(frequency.top5))
sum(prop.50)
50*frequency.top5/sum(frequency.top5)
# while using sum, it adds up to 49 only but we are supposed to have sample of size
# 50.So I need to find the department that can be added one more. here in proportion
# table Boston police department have 23.495 which would have 24 if it had 0.005 more value.
# so this is the closest  department  that can be used to add one more values and make it 24
instead 23.
#changing second value to 24 from 23.
prop.50[2] <- 24
#now total number of sample becomes 50
sum(prop.50)

st.d <-strata(ordered.top5,stratanames = "Department",size = prop.50,
                    method = "srswor",description =TRUE )
#now retrieving those 50 samples as we get from strata() method using getdata() method.
sample.d <- getdata(subset_top5,st.d)
#checking the frequency
table(sample.d$Department)

#finding proportion of each department according to number of employee in each department
prop.table.c <- round(prop.table(prop.50),2)
prop.table.c

for (n in 1:length(prop.50)) {

  cat(names(prop.50)[n],"will have", prop.table.c[n]*100,"%", "\n")
}
```

Console section 3.c:

```
set.seed(7356)
> #ordering the data using Department variable
> order.department <- order(subset_top5$Department)
> #mapping to dataset according to ordered rows
> ordered.top5 <- subset_top5[order.department,]
```

```
> #finding relative frequency of employee in each deparment
> frequency.top5 <- table(ordered.top5$Department)
>
> #finding proportions in each department based on their employee numbers.
> prop.50 <- round(50*frequency.top5/sum(frequency.top5))
> sum(prop.50)
[1] 49
> 50*frequency.top5/sum(frequency.top5)

  Boston Fire Department Boston Police Department    Boston Public Library  BPS Facility
Management    BPS Special Education
           14.379085               23.495012               3.302374               3.568971               5.254558
> # while using sum, it adds up to 49 only but we are supposed to have sample of size
> # 50.So I need to find the department that can be added one more. here in proportion
> # table Boston police department have 23.495 which would have 24 if it had 0.005 more
value.
> # so this the closest  department  that can be used to add one more values and make it 24
instead 23.
> #changing second value to 24 from 23.
> prop.50[2] <- 24
> #now total number of sample becomes 50
> sum(prop.50)
[1] 50
>
> st.d <-strata(ordered.top5,stratanames = "Department",size = prop.50,
+                      method = "srswor",description =TRUE )
Stratum 1

Population total and number of selected units: 1672 14
Stratum 2

Population total and number of selected units: 2732 24
Stratum 3

Population total and number of selected units: 384 3
Stratum 4

Population total and number of selected units: 415 4
Stratum 5

Population total and number of selected units: 611 5
Number of strata  5
Total number of selected units 50
> #now retrieving those 50 samples as we get from strata() method using getdata() method.
```

```
> sample.d <- getdata(subset_top5,st.d)
> #checking the frequency
> table(sample.d$Department)

  Boston Fire Department Boston Police Department    Boston Public Library  BPS Facility
Management    BPS Special Education
                  14                       24                       3                       4                       5
>
> #finding proportion of each department according to number of employee in each
department
> prop.table.c <- round(prop.table(prop.50),2)
> prop.table.c

  Boston Fire Department Boston Police Department    Boston Public Library  BPS Facility
Management    BPS Special Education
                0.28                    0.48                    0.06                    0.08                    0.10
>
> for (n in 1:length(prop.50)) {
+
+   cat(names(prop.50)[n],"will have", prop.table.c[n]*100,"%", "\n")
+ }
Boston Fire Department will have 28 %
Boston Police Department will have 48 %
Boston Public Library will have 6 %
BPS Facility Management will have 8 %
BPS Special Education will have 10 %
```

Code section 3.d:

```
#d)
#list of mean of four samples
list.mean.4 <- c(mean(subset_top5$Earnings),mean(subset.with.replace$Earnings),
            mean(new.samples$Earnings),mean(sample.d$Earnings))
list.mean.4
#mean of original
mean(boston$Earnings)
```

Console section 3.d:

```
#list of mean of four samples
> list.mean.4 <- c(mean(subset_top5$Earnings),mean(subset.with.replace$Earnings),
+              mean(new.samples$Earnings),mean(sample.d$Earnings))
```

```
> list.mean.4
[1] 133921.4 127668.4 158944.3 142998.9
> #mean of original
> mean(boston$Earnings)
[1] 108680.9
```

Comparison/inferences:

Here, mean of boston dataset is lower than means of fours, in which one dataset is subset of the original dataset where only top5 departments are included based on their number of employees. While other 3 are samples drawn from 3 different sampling methods. From this, it can be concluded that the employees in top5 department, which is based on number of employees working, also have higher earnings than other department that is why mean of the earnings from top 5 department higher than means of the main dataset. Here, mean of the samples drawn using simple random sampling with replacement are closer to the mean of the earnings of original boston dataset. However, if we compare among three types of sampling techniques. Stratified sampling using proportional sizes is closer to the top 5 department subset as mean earning of top 5 subset is closer than with other.

The End:

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