DA_exercises

October 11, 2017

Linear Model

$$z_n = 0.5z_{n-1} + \xi_{n-1} \tag{1}$$

with $\xi_{n-1} \sim N(0, B)$ and $z_0 \sim N(0, 0.4)$
Observations

$$y_n = z_n + \eta_n \tag{2}$$

$\eta_{n-1} \sim N(0, R)$
Kalman filter
Forecast formulas:

$$\hat{m}_{n+1} = Am_n \tag{3}$$
$$\hat{C}_{n+1} = AC_nA^\top + B \tag{4}$$

Analysis formulas

$$m_{n+1} = \hat{m}_{n+1} - K_{n+1}(H\hat{m}_{n+1} - y_{n+1}) \tag{5}$$
$$C_{n+1} = \hat{C}_{n+1} - K_{n+1}H\hat{C}_{n+1} \tag{6}$$

with Kalman gain

$$K_{n+1} = \hat{C}_{n+1}H^\top(R + H\hat{C}_{n+1}H^\top)^{-1} \tag{7}$$

Exercise: Please implement the Kalman filter for the example above
Lorenz equations

$$\dot{x} = \sigma(y - x) \tag{8}$$
$$\dot{y} = x(\rho - z) - y \tag{9}$$
$$\dot{z} = xy - \beta z \tag{10}$$

Ensemble Kalman Filter

$$z^i_{n+1} = \hat{z}^i_{n+1} - K_{n+1}(H\hat{z}^i_{n+1} - \tilde{y}^i_{n+1}) \tag{11}$$

$$m_n \approx \frac{1}{M}\sum_{i=1}^{M} z_n^i \tag{12}$$

$$C_n \approx \frac{1}{M}\sum_{i=1}^{M} (z_n^i - m_n)(z_n^i - m_n)^\top \tag{13}$$

Exercise: Please implement the Ensemble Kalman filter for the Lorenz equation
Particle filter
Exercise: Please implement the Particle filter with resampling for the Lorenz equation