SOEN 6011 : SOFTWARE ENGINEERING PROCESSES

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F2: Tangent Function, $tan(x)$

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https://www.overleaf.com/project/62cdb1d5b13422add2cceb12

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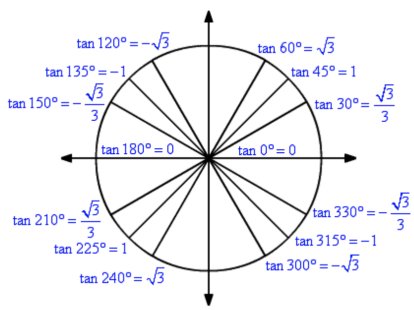
1)Problem1

a)Description of Function

[1] $tan(x)$ is a periodic function which is very important in trigonometry. The simplest way to understand the tangent function is to use the unit circle. For a given angle measure θ draw a unit circle on the coordinate plane and draw the angle centered at the origin, with one side as the positive x -axis. The x -coordinate of the point where the other side of the angle intersects the circle is $cos()$ and the y -coordinate is $sin()$. So, the tangent function is define as below:

$$tan(x) = \frac{sin(x)}{cos(x)}$$

The below graph shows values corresponding to different angles.



[1][2]The tangent function is undefined when $x = \pi \ / \ 2 + n\pi$ (where, n is integer) for which, $cos(x) = 0$. However, Tangent function does not have an amplitude. In addition, The graph intercept x-axis at $n\pi$ (where n is integer) and in y-axis at $(0, 0)$ point. The period of tangent function is π.

Range

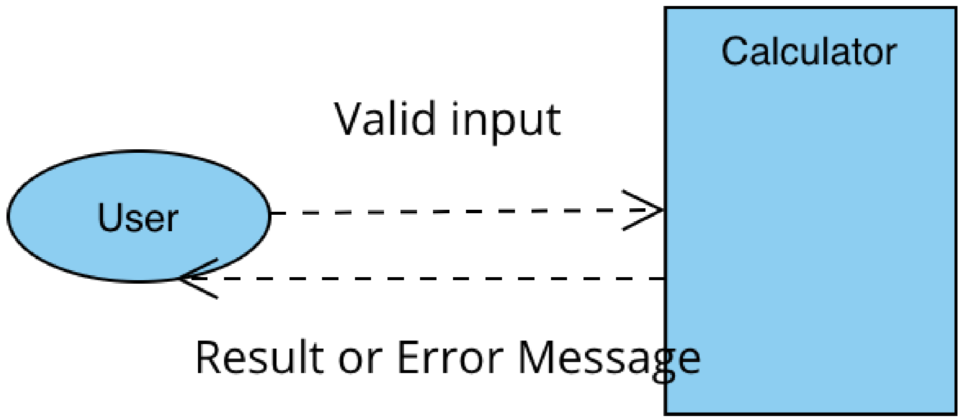
[1][2] The range of $tan(x)$ is all real number $\mathbb{R}$, $(-\infty, +\infty)$.

Domain and Co-domain

[1][2] The domain of tangent function is $x \in \mathbb{R}$, $x \neq \pi \ / \ 2 + n\pi$ where, n is an integer. The co-domain of $tan(x)$ is $(-\infty, +\infty)$.

b)Context of Use Model

Users can use the calculator to calculate the result of $sin()$, $cos()$ and $\frac{sin()}{cos()}$ which is $tan()$ of a degree. This degree shall be an integer or decimal, so the digits *0-9* and the decimal point must be available by the user. The user can select the appropriate function they want to use, and they shall be able to press a button to have the answer computed. The calculator should return the result or an error message that indicates why it was unable to do so.



2)Problem 2

Assumption:

For the given degree x, return the result of $tan(x)$. If the input value is invalid or cannot be calculated, return an error message.

Requirements:

Requirement Id	R1
Overview	$x = 0° + n\pi$
Description	For the given input $x = 0°$, the function may return 0 as output.
Priority	High
Type	Functional
Difficulty	Easy

Requirement Id	R2
Overview	x is Positive Degree
Description	For the given input x = any Positive Degree , the function may return corresponding $tan(x)$ value as output.
Priority	High
Type	Functional
Difficulty	Medium

Requirement Id	R3
Overview	x is Negative Degree
Description	For the given input x = any Negative Degree , the function may return corresponding $tan(x)$ value as output.
Priority	High
Type	Functional
Difficulty	Medium

Requirement Id	R4
Overview	$x = 90° + n\pi$
Description	For the given input x , the function may return ”Invalid” as output.
Priority	High
Type	Functional
Difficulty	Hard

3)Problem 3

a)Algorithm Selection

For this part, I will introduce two algorithms for implementing $tan(x)$ function. **Polynomial approximation and Maclaurin series.**

Algorithm1 : Polynomial approximation

[3]Polynomial approximation is an approximation of a curve with a polynomial. When we solve mathematical questions, we don't actually know how to calculate certain functions, such as the $sin()$ function. Therefore, to solve this kind of problems, mathematicians develop very good approximations to these functions - related functions which are very close to the function of interest, but much easier to calculate.

Advantages	Disadvantages
Easy to calculate	The approximation is only precise for small x, so some steps are needed when we calculate $tan(x)$

Algorithm2 : Maclaurin series

[4]A Maclaurin series is a Taylor series expansion of a function about 0,

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + ... + \frac{f^{(n)}(0)}{n!}x^n$$

The $tan(x)$ function's approximation is derived by the Maclaurin Series's explicit forms of $sin(x)$ and $cos(x)$.

$$sin(x) = x - x^3/3! + x^5/5! - x^7/7! + ... \quad (1)$$

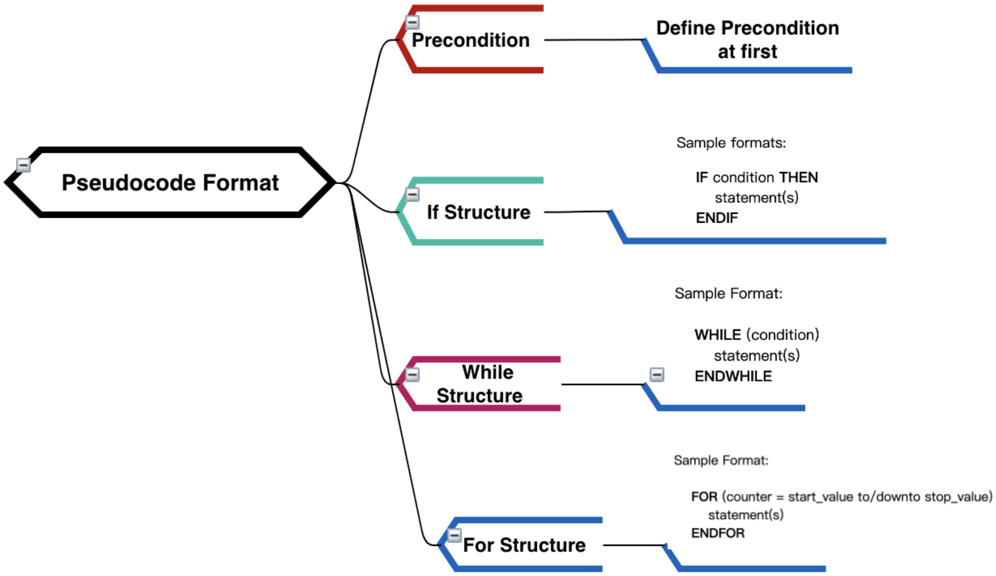
$$cos(x) = 1 - x^2/2! + x^4/4! - x^6/6! + ... \quad (2)$$

Then, we can use $\tan(x) = \frac{sin(x)}{cos(x)}$ to calculate.

Advantages	Disadvantages
The formula $\tan(x) = \frac{sin(x)}{cos(x)}$ is easy to understand.	Successive terms get very complex and hard to derive.

b)Mind Map For Pseudocode

In this part, I will use a mind map to decide a pseudocode format.



c) Pseudocode for each Algorithm

In this part, I will write pseudocode for each algorithm.

Algorithm 1 [5]Polynomial approximation

Require: $x \notin [0°, 180°]$
  function PERIODICITY(x)
    if $x > 180°$ **then**
      while $x > 180°$ **do**
        $x = x - 180°$ ▷ reduce x to the range $[0°, 180°]$
      end while
    else
      while $x < 0°$ **do**
        $x = x + 180°$ ▷ add x to the range $[0°, 180°]$
      end while
    end if
  return x ▷ get valid x
  end function

Require: $x \notin [0°, 90°]$
  function SYMMETRY(x)
    $tan(x) = -tan(180° - x)$ ▷ use the symmetry of $tan()$
  return $tan(x)$ ▷ get valid x
  end function

Require: $x \notin [0°, 45°]$
  function COFUNCTION(x)
    $tan(x) = -\frac{1}{tan(90° - x)}$ ▷ use the reciprocal of $tan()$
  return $tan(x)$ ▷ get valid x
  end function

Require: $x \notin [0°, 22.5°]$
  function TRIGONOMETRIC_IDENTITY(x)
    $tan(x) = -\frac{2tan(\frac{x}{2})}{1 - tan^2(\frac{x}{2})}$ ▷ use the trig identity of $tan()$
  return $tan(x)$ ▷ get valid x
  end function

Require: $x \in [0°, 22.5°]$
  function POLYNOMIAL(x)
    $\mathrm{x} = x * \frac{\pi}{180°}$ ▷ convert x to radians
    $tan(x) = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315}$ ▷ use the trig identity of $tan()$
  return $tan(x)$ ▷ get valid x
  end function

Algorithm 2 Maclaurin Series

Require: x in degrees
  function GETRAD(x)
    $val = x * \frac{\pi}{180°}$ ▷ Calculate x in radians
  return val ▷ return x in radians
  end function

Require: $n \neq 0$ and $length = 0$
  function CHECKDIGITS(n)
    while $n \leqq 1$ **do**
      $length+ = 1$ ▷ Calculate length
      $n* = 10$
    end while
  return $length$ ▷ return length
  end function

Require: $getrad(x) \neq NULL$ AND $n \neq 0$ AND x in radians AND $k = 1$ AND $m = 0$

```
function CalculateSin(getrad(x))
    sinres = x/k!
    while checkdigits(sinres) ≠ n do
        k = k + 2
        if m%2 == 0 then
            sinres −= x^k/k!
        else
            sinres += x^k/k!
        end if
        m += 1
    end while
return sinres                                  ▷ get value of sin(x)
end function
```

Require: $getrad(x) \neq NULL$ AND $n \neq 0$ AND x in radians AND $k = 2$ AND $m = 0$

```
function CalculateCos(getrad(x))
    cosres = 1
    while checkdigits(cosres) ≠ n do
        k = k + 2
        if m%2 == 0 then
            sinres −= x^k/k!
        else
            sinres += x^k/k!
        end if
        m += 1
    end while
return cosres                                  ▷ get value of cos(x)
end function
```

Require: $getrad(x) \neq NULL$ AND $calculatecos(x) \neq NULL$ AND $calculatesin(x) \neq NULL$

```
function CalculateTan(calculatecos(x),calculatesin(x))
    SinVal = calculatesin(x)
    CosVal = calculatecos(x)
return SinVal/CosVal                           ▷ calculation for tan(x)
end function
result ← tan(x)
```

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