Make your own Aerological Diagram

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1 Introduction

This is a how-to guide for creating an aerological diagram. The purpose of this report is to give a blueprint to the reader to create their own aerological diagram, by providing in adequate detail the equations involved in producing such a plot. So far, the document assumes a background in coding should the reader wish to produce their own diagram, as well as a familiarity with basic atmospheric thermodynamics. **A revised document will include either Python code or pseudo code so that the reader can use the extra guidance as they desire.**

To produce a basic F160 diagram, one requires code that (a) Produces a blank set of axes with representative isotherms, dry adiabats, saturated adiabats and isohumes, and (b) takes temperature, dewpoint and pressure data and plots these on the blank diagram. We turn to understanding the procedures that going into transform temperatures and pressures to the coordinate system of the aerological diagram, and understanding how the various isopleths are determined.

1.1 The basic transformation

We start off with the transformations that comprise the axes of the skew T diagram. We denote the pressure by P, the temperature by T and the dewpoint by T_d. On the vertical (Y-) axis, we have the negative of the natural logarithm of the pressure, and on the horizontal (X-) axis we transform the temperature by scaling and translating the transformed pressure and original temperature,

$$Y = \ln{(P_0/P)} \tag{1}$$
$$X = T + a.\ln{(P_0/P)} \tag{2}$$

where P_0 is a reference pressure, which we usually take to be $P_0 = 1050$ mb but can be changed, and a is a 'rotation' factor. This rotation factor can be tuned for choice, but a $= 30\,C^{\circ}$ is usually quite good.

1.2 The isotherms

These are the most straightforward to calculate. At a given pressure P, an isotherm of a fixed and specified temperature T_0 is calculated as

$$X = T_0 + a.\ln(P_0/P).$$

An isotherm therefore appears on the skew-T digram (where the vertical coordinate is equation (1)) as a straight line with slope a. [**insert pseudo-code**]

1.3 Dry adiabats

To calculate the dry adiabats, start off with the surface temperature and pressures, T_0 and P_0. The equation[1] for how the temperature and pressure of a parcel change as it ascends/descends dry-adiabatically is

$$T = T_0 \left(\frac{P}{P_0} \right)^{C_p/R_d} \tag{3}$$

Where $C_p = 1003.5\, Jkg^{-1}K^{-1}$ is the specific heat capacity of dry air at constant pressure, and $R_d = 287.058\, Jkg^{-1}K^{-1}$ is the specific gas cosntant of dry air. Their ratio is input as 3.496. Given a set of pressures, the corresponds temperatures can be calculated using (3). The corresponding transformed temperatures are then calculated using (1) & (2). [**insert pseudo-code**]

1.4 Isohumes

The isohumes are lines of constant water content, which can be stated in terms of the mixing ratio r in g/kg. Bear in mind that the isohumes are curves of *dewpoint* as a function of pressure. The mixing ratio can be calculated given the temperature T, pressure P and dewpoint T_d. Firstly, calculate the saturation vapour pressure (the vapour pressure of water when the gaseous phase is in equilibrium with a plane surface of liquid water). We are not aware of an exact formula here. The formula we use is the August-Roche-Magnus formula,

$$e_s(T) = 6.1094 \exp\left(\frac{17.625.T}{T + 243.04} \right). \tag{4}$$

This formula assumes the temperature is supplied in **degrees centigrade**. The vapour pressure of water in a parcel of dewpoint T_d is

$$e(T_d) = 6.1094 \exp\left(\frac{17.625.T_d}{T_d + 243.04} \right). \tag{5}$$

The mixing ratio associated with a parcel of dewpoint T_d at pressure P is

$$r(T_d) = \frac{\epsilon.e(T_d)}{P - e(T_d)} \tag{6}$$

where $\epsilon = 0.622$ is the ratio of dry air's specific gas constant to saturated air's specific gas constant. A step-by-step guide for the isohume of fixed and specified mixing ratio r_0 is:

- Rearrange (6) in terms of $e(T_d)$
$$e(r_0, P) = \frac{r_0.P}{r_0 + \epsilon}$$
- Subsitute the expression in step 1 into an inversion of equation (5) in terms of T_d
$$T_d = 243.04 \frac{\ln e(r_0, P)}{17.625 - \ln e(r_0, P)}$$

We changed the notation of $e(T_d)$ to $e(r_0, P)$ because we are treating the pressure and mixing ratio as known variables and using these to calculate the vapour pressure, which in turn is used to calculate the dewpoint. The corresponding transformed dewpoints are then calculated using (1) & (2).[**Insert pseudo-code here**]

[1] In this equation, T_0 is actually the potential temperature given we are assuming P_0 is the surface pressure. It is usually framed as an equation for the potential temperature and called *Poisson's equation*. We think better to primarily frame this as the equation of a parcel undergoing dry adiabatic ascent or descent. This invites one to call on the equation in different contexts that do not involve surface pressure and temperature.

1.5 Saturated adiabats

The saturated adiabats are the most complex to calculate, as they require the numerical solution of a differential equation, albeit the simplest case of a differential equation. The saturated adiabatic lapse rate is the rate at which a parcel saturated with respect to water will cool as it continues to rise. Mathematically, this can be written as

$$\frac{dT}{dP} = \frac{\frac{R_d.T}{C_p} + r\frac{L_v}{C_p}}{P\left(1 + \frac{L_v^2.r.\epsilon}{C_p.R_d.T^2}\right)} \tag{7}$$

Where $L_v = 2.5 \times 10^6 J/kg$ is the latent heat of vaporisation of water, $C_p = C_{p,\,dry}(1 + 1.84r)$, $C_{p,dry} =$ is the specific heat of moist air of mixing ratio r. One can choose to approximate $C_p \approx C_{p,dry}$ or use the exact expression, since[we assume] the approximation is not a bad one. As mentioned above, to solve for the saturated adiabats requires the numerical solution of (7). This is because the moist lapse rate is itself quite a complicated function of temperature and pressure, making (7) an equation that cannot be integrated exactly to determine the temperature curves. To determine the saturated adiabats, We step out the calculation of a saturated adiabat:

- Specify the temperature and Pressure at the surface, T_0 and P_0.
- Specify a set of pressures at which you wish to calculate the temperature: P_0, P_1, ..., P_i, ... P_N. On a typical aerological diagram, one might take $P_0 = 1050$ mb and $P_N = 100$ mb. The set of pressures must have increments $\Delta P = P_{i+1} - P_i$ small enough so that the numerical method is accurate enough.
- Apply the numerical method as detailed in the appendix
- with the calculated set of temperatures T_0, $T_1, \ldots,$ T_i, ..., T_N corresponding to the pressures specified in step 2, transform both variables to the skew T coordinate system using (1) & (2).
- Repeat the above steps for a variety of surface temperatures T_0 to get a variety of saturated adiabats **[insert pseudo code]**

2 Conclusion

The theory behind the basic curves that constitute the aerological diagram has been reviewed as briefly as required to allow the reader to produce an aerological diagram given a set of balloon sounding data (precisely, a set of temperatures, dewpoints and pressures). An example of Python code which gives a basic working example of a customisable aerological diagram[2] can be found at `https://github.com/KDMpehle/Meteorology`. Figure 1 shows an example of some sounding data plotted using this code.

In concluding the report, we justify its writing. There are advantages to being able to code one's own aerological diagram. Namely, the ability build on the code, adding features that perform and plot the results of various thermodynamic or dynamic procedures which may not be available in pre-packaged plotters, as we have done with the numerically calculated wet-bulb trace in figure 1. Also (and no less importantly, in the author's opinion), scientific programming in established programming languages such as Python is a valuable up-skilling opportunity for meteorologists, especially if more and more the direction of many meteorological companies is to provide customised and specialised weather services—we hope this document is a helpful educational resource to this end.

[2]We understand that there are better and more elegant ways to produce a code. For example, going object-oriented and defining an aerological diagram as a class, instead of as a collection of functions as the linked code does.

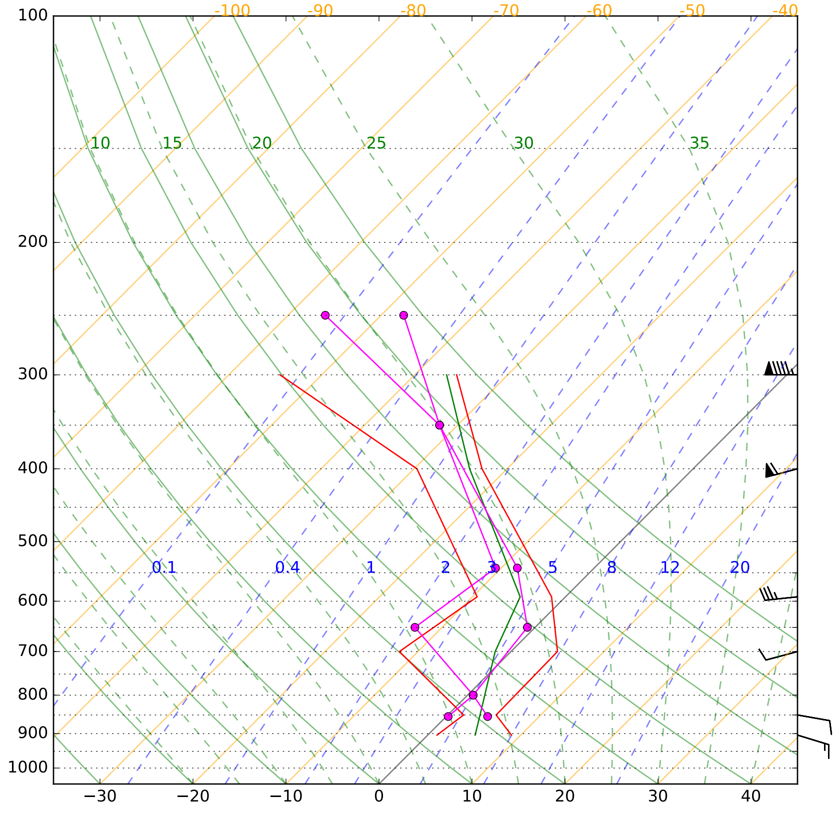

Figure 1: A sample aerological diagram with plotted data. The red curves are the familiar temperature and dewpoint soundings. The green curve is a calculated wet-bulb temperature trace corresponding to a portion of the sounding data, and the pink curves are the traces lifted after lifting by 50hPa. Wind barbs can be seen on the right of the diagram.

Appendix: The Euler method

Here we outline the Euler method algorithm for solving (7) and finding the saturated adiabats. To solve an equation like (7) computationally, one specifies a surface conditions T_0 and P_0 "grid" of pressures P_0, P_1, …, P_i, … P_N. We can then use the (7) to calculate approximations for the temperature at each pressure. Since (7) is really a statement of slope, one approximates a change in temperature when a small change in pressure occurs by

$$\Delta T = \left(\frac{\frac{R_d.T}{C_p} + r\frac{L_v}{C_p}}{P\left(1 + \frac{L_v^2.r.\epsilon}{C_p.R_d.T^2}\right)} \right) \Delta P$$

Therefore, when the pressure changes from P_i to P_{i+1}, with $DeltaP = P_{i+1} - P_i$, the temperature will change as

$$T_{i+1} - T_i = \left(\frac{\frac{R_d.T}{C_p} + r\frac{L_v}{C_p}}{P\left(1 + \frac{L_v^2.r.\epsilon}{C_p.R_d.T^2}\right)} \right) \Delta P$$

or

$$T_{i+1} = T_i + \left(\frac{\frac{R_d.T}{C_p} + r\frac{L_v}{C_p}}{P\left(1 + \frac{L_v^2.r.\epsilon}{C_p.R_d.T^2}\right)} \right) \Delta P$$

In the Euler approximation, one substitutes T_i and P_i into the slope (moist adiabatic lapse rate in this context),

$$T_{i+1} = T_i + \left(\frac{\frac{R_d.T_i}{C_{p,i}} + r_i\frac{L_v}{C_{p,i}}}{P_i\left(1 + \frac{L_v^2.r_i.\epsilon}{C_{p,i}.R_d.T_i^2}\right)} \right) \Delta P$$

where we recall that specific heat capacity of moist air C_p and the mixing ratio r vary with pressure. In the Euler method, the specific approximation that is made is to substitute the currently known values, the values at step i, to calculate the temperature at the next step $i+1$. This makes the procedure simple as, at step $i+1$, the temperature and pressure at previous step i are known. The numerical method is prone to numerical instabilities if a small enough grid spacing (pressure increment) ΔP is not employed. Other methods exist for solving this equation numerically, methods that are more accurate or more numerically stable. The use and discussion of these is academic however, as we found that the Euler method does well and is stable in this context, with a pressure increment of $\Delta P = 2$ mb. Connecting 1050 mb to 100 mb gives around 500 grid points with this grid spacing. A modern computer can perform the method on this number of grid points without any noticeable wait.