Computer Exercice:
Classical Loop-Shaping

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***Abstract* – Blablabla !**

4 Exercises

4.1 Basics

We consider a system which can be modeled by the transfer function:

$$G(s) = \frac{3(-s+1)}{(5s+1)(10s+1)}$$

Exercise 4.1.1

We want to design a lead-lag controller which eliminates the static control error for a step response in the reference signal.

The controller transfer function is the following:

$$F(s) = K\frac{\tau_D s+1}{\beta\tau_D s+1}\frac{\tau_I s+1}{\tau_I s+\gamma}$$

We want to fulfill the following criteria:

- Phase margin of 30° at the cross-over frequency $\omega_c = 0.4$ rad/s.
- No static control error for a step response

Phase-lag action

Setting the parameters $\tau_I = 1$s and $\gamma = 0$ leads to an integral part in the system. This guarantee no static control error. ($\lim_{t\to\infty} e(t) = 0$).

Phase-lead action

The lag-action of the controller leads to the following values in the frequency domain:

- Phase margin: $\phi_0 = -50°$
- Gain: $|G(\omega = 0.4 \text{ rad/s}| = 0.9436$

Thus:

- $\beta = \frac{1-\sin(30°-\phi_0)}{1+\sin(30°-\phi_0)} = 0.0086$
- $\tau_D = \frac{1}{\omega_c\sqrt{a}} = 26.9459$
- $K = \frac{\sqrt{a}}{m} = 0.0983$

Controller

The two previous paragraph lead to the design of the controller as:

$$F(s) = K\frac{\tau_D s+1}{\beta\tau_D s+1}\frac{\tau_I s+1}{\tau_I s+\gamma} = 0.0983\frac{26.9459s+1}{0.2319s+1}\frac{s+1}{s}$$

Figure 1 & 2 show the bode diagrams of the different function of the system and the step response.

The two criteria (phase-margin and static error null) are fulfilled.

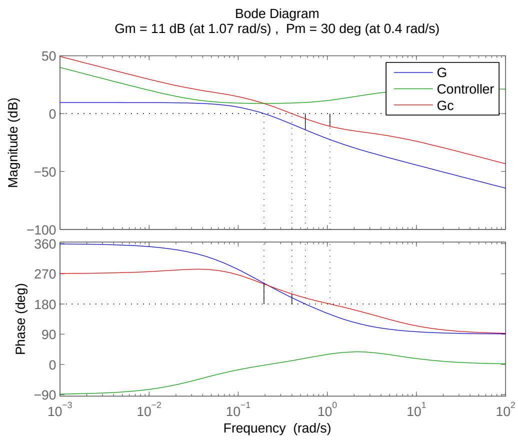

Figure 1: Bode diagram of the system's functions
Phase-margin: 30°

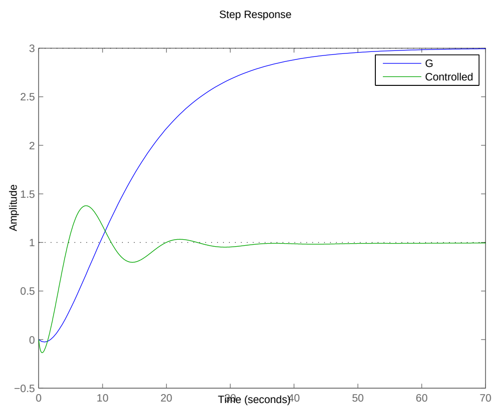

Figure 2: Step response of the system with and without the controller
Phase-margin: 30°

Exercise 4.1.2

Adding the lead lag controller to the system leads to the caracteristics repported in the following table

Bandwith $(-3dB)$	$[0, 0.75]$(rad/s)
Resonance peak M_T	5.8dB at 0.4(rad/s)
Rise time t_r	2.46 s
Overshoot $D\%$	37.8%

Exercise 4.1.3

Using the method depicted in Exercice 4.1.1 with $\tau_I =$ 1s places the integral action too close to the cross-over frequency: The lag-action and the lead-action of the controller are overlapping.

Therefore, we increase the value of τ_I to $\tau_I = 10$s and apply the same method as in Exercice 4.1.1 to get the values of τ_D and β.

The resulting controller fulfill all the criteria. (see Figure 3 and 4).

The lead lag controller with a phase-margin of 50° has the following caracteristics:

Bandwith $(-3dB)$	$[0, 0.99]$(rad/s)
Resonance peak M_T	2.06dB at 0.56(rad/s)
Rise time t_r	2.22 s
Overshoot $D\%$	15.7%

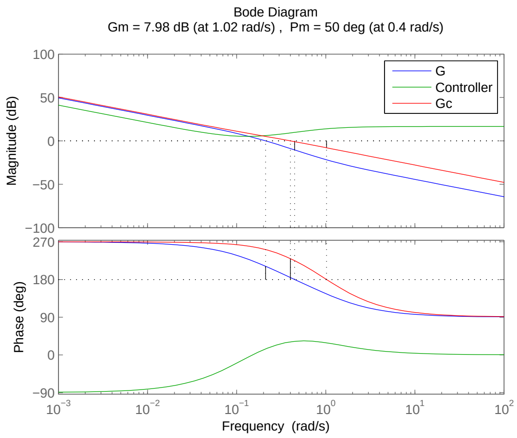

Figure 3: Bode diagram of the system's functions
Phase-margin: 50°

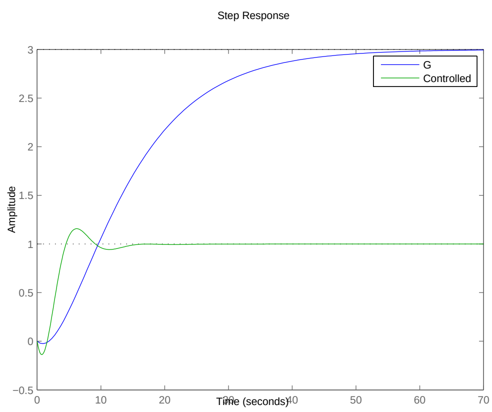

Figure 4: Step response of the system with and without the controller
Phase-margin: 50°

4.2 Disturbance attenuation

In this subsection, we need to design a controller which both tracks the reference signal and attenuates the disturbances.

The transfer function of the system is:

$$G(s) = \frac{20}{(s+1)((\frac{s}{20})^2 + \frac{s}{20} + 1)}$$

The disturbance transfer function is:

$$G_d(s) = \frac{10}{s+1}$$

The purpose of this subsection is to designed the pre-filter function and the feedback function (F_r and F_y) with the following specifications:

- Rise time $t_r \leq 0.2$ s
- Overshoot $D(\%) \leq 10\%$
- Step in the disturbance:

$$|y(t)| \leq 1, \forall t \text{ and } |y(t)| \leq 0.1, \forall t \geq 0.5 \text{ s}$$

- Control signal obeys:

$$|u(t)| \leq 1, \forall t$$

Exercise 4.2.1

Since $G_d(s) = \frac{10}{s+1}$, the cross-over frequency is $w_c = 10$ rad/s.

We have the following result:

$$\forall \omega < \omega_c, |Gd(j\omega)| > 1$$

Control action is needed at least for $\omega \in [0, \omega_c]$.

Here, we will try to design F_y in such a way that:

$$L(s) = F_y G \approx \frac{\omega_c}{s}$$

Unproper feedback

Let:

$$F_y = G^{-1}\frac{\omega_c}{s} = \frac{200s^3 + 4200s^2 + 84000s + 80000}{160000s}$$

This controller has 3 zero and 1 pole. Thus, it is not proper. However – using `Matlab` – we can plot the step response of the system and the step response to a step perturbation (see Figure 5).

We can see that even if the performance are poor (Error static $\leq 5\%$ for $t \geq t_d = 7$ s) we still have an attenuation of the perturbation and a step response of good quality.

Proper feedback

Assuming that this controller is the one we want to implement, we need to design it in a proper way. For now, it has 3 zeros and 1 pole. Therefore it is needed to add two more poles.

Let:

$$F_y(s) = G^{-1}\frac{\omega_c}{s(s+p_1)(s+p_2)}$$

We need to choose $p_{1,2}$ in order to not change the system performance two much.

We use the following criteria:

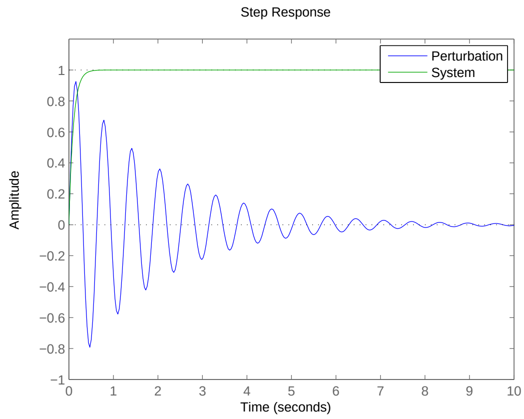

Figure 5: Non-proper step response of the system

- $p_1 = p_2$
- p_1 and p_2 take action after ω_c

Let's pick $p_1 = p_2 = p = 100\omega_c$. In order to have the same bode diagram for $\omega < \omega_c$, we need to translate the gain curve with a gain of p^2. Figure 6 shows the bode diagram of $L(s) = F_y(s)G(s)$ (we can see that $\forall\omega < \omega_c, |L_{proper}|_{dB} \approx |L_{unproper}|_{dB}$). Figure 7 shows that the step response to a perturbation with L_{proper} is almost the same than with $L_{unproper}$.

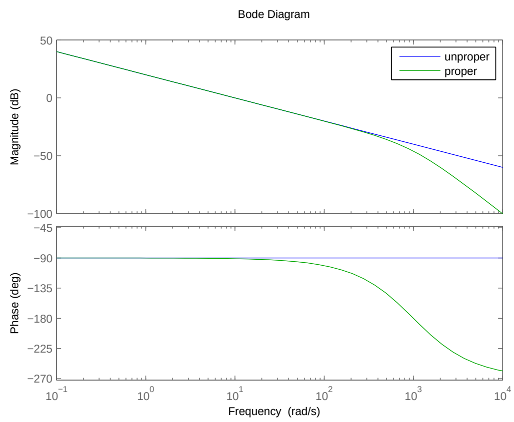

Figure 6: Bode diagram of $L(s)$ with the proper & unproper F_y

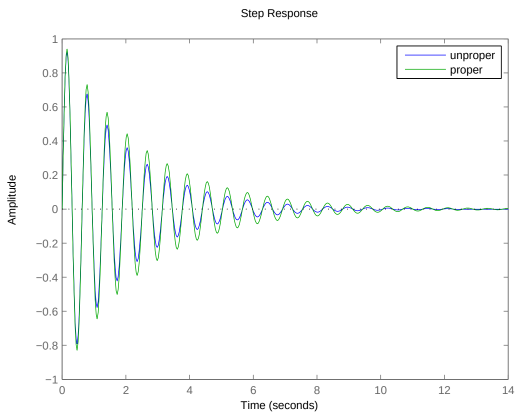

Figure 7: Step response of the system with the proper & unproper F_y