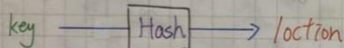


* Basics

1. Hash table: 以陣列型態來存放資料
2. Hash function: 透過 key 值計算出該放在 Hash table 哪個位置。



* Collision 碰撞

- Definition: 同一個 Hash table 的位置放了 2 次。
(放進去前發現裡面已經有資料了)

$$\text{Prob} \{N \text{ different}\} = \frac{(\text{table size} - 1) \times \dots \times (\text{table size} + 1 - N)}{(\text{table size})^{N-1}}$$

$$\text{Prob} \{ \text{collision} \} = 1 - \text{Prob} \{ N \text{ different} \}$$

1. Digit selection:

10927104 → 放進 [4]

2. Folding:

key: 10927104, table size = 10

① $1+0+9+2+7+1+0+4 = 24$
 $2+4 = 6 \rightarrow \text{放進} [6]$

② $109+27+10+4 = 150$
 $1+5+0 = 6 \rightarrow \text{放進} [6]$

3. Modulo arithmetic

* table size should be prime

• 當數值很大的時候會使用 @

key = 10927104, table size = 13

① $109+27+10+4 = 150$

$150 \% 13 = 7 \rightarrow \text{放進} [7]$

or
② $(109 \% 13) + (27 \% 13) + (10 \% 13) + (4 \% 13) = 7 \rightarrow \text{放進} [7]$

My Opinions
Thoughts, inspirations, and suggestions

4. Converting character strings

key: 10027104, table size = 13

① $49+48+48+50+55+49+48+52 = 399$

$399 \% 13 = 9 \rightarrow \text{放進} [9]$

② $(\dots (49 \times 2^8 + 48) + \dots + 48) \times 2^8 + 52 = 22260$

$22260 \% 13 = 4 \rightarrow \text{放進} [4]$

char	ASCII
'0'	48
'1'	49
'2'	50
'3'	51
'4'	52
'5'	53
'6'	54
'7'	55
'8'	56
'9'	57

凡是值得做的事，
都是上坡路。
——約翰·麥克斯威爾

a. Linear probing 線性探索

- 找到位置後假如有資料的話就放進下一個。
- 缺點: 搜尋上可能有困難。

b. Quadratic probing 平方探索

- 和 a 類似，但不是 +1，是 +1²，+2²，+3²，直到放進去。

優點: 較 a 分散

缺點: 可能會一直放到重覆的位置。

ex:

key: 10027207 → Hash → [5] Hash: sum % 7

$5+1^2 = 6 \% 7 = [6]$ $5+7^2 = 54 \% 7 = [5]$

$5+2^2 = 9 \% 7 = [2]$ $5+8^2 = 69 \% 7 = [6]$

$5+3^2 = 14 \% 7 = [0]$ $5+9^2 = 86 \% 7 = [2]$

$5+4^2 = 21 \% 7 = [0]$ $5+10^2 = 105 \% 7 = [0]$

$5+5^2 = 30 \% 7 = [2]$

$5+6^2 = 41 \% 7 = [6]$

[0]	10027210
[1]	
[2]	10027216
[3]	
[4]	
[5]	10027135
[6]	10027234

c. Double hashing 双重碰撞

- 使用 2 個 hash functions. h_1 是初使位置. h_2 是 step size.
- * $h_1 \neq h_2$ 且 $h_2(\text{key}) > 0$
- * table size 最好是質數且和 step size 互質就能探訪每個位置。

ex:

function: sum % 7

$h_2 = 5 - (36 \% 5)$

step size: $5 - (\text{sum} \% 5)$

$= 5 - 1$

key: 9927144

$= 4$

$9+9+2+7+1+4+4 = 36$

final location:

$1+4 - 5 \% 7 = 5$

$5+4 = 9 \% 7 = 2$

[0]	
[1]	9827145
[2]	9927144
[3]	
[4]	
[5]	9927135
[6]	10027203

My Opinions
Thoughts, inspirations, and suggestions

Ex:

Hash function: sum % 7, step size: sum % 5 + 1

	sum	h_1	h_2	location
9927104	32	[4]	x	[4]
10027101	12	[5]	x	[5]
10027161	18	[4]	4	[1]
10027249	25	[4]	1	[6]
9827166	39	[4]	5	[2]
10027262	20	[6]	1	[0]

Number of probes:
 $1+1+2+3+2 = 11$

習慣就是習慣，
誰也不能將其扔出窗外，
只能一步一步地引下樓。
——馬克·吐溫

My Notes

Important Concepts worth keeping

Collision Resolution (進階)

a. Buckets: (多容器)

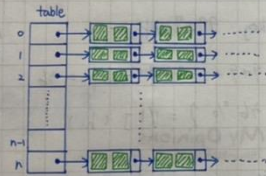
* 每個 hash table 的位置都是 array, 讓一個位置能存放很多資料。

b. Separate chaining: (分開鏈結)

* 每個 hash table 的位置都是一個 linked list。

* Bucket + Separate chaining:

- Memory management
- Worst case: $O(n)$



Efficiency:

* Average-case Analysis

1. Load factor α

- Current number of items in the table size.
- Measure how full a hash table is.

2. Depends on whether the search is successful.

- Unsuccessful searches generally require more time than successful searches.

You do not rise to the level of your goals. You

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My Questions

Problems & Difficulties needing exploration

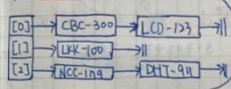
雜湊的应用

My learning weather report

1. Hash Join

Accidents	Maintenance	Nested-loop join:
Item 10 MKT-612 LCD-123 KFA-214 NCC-179 MKT-500 LKK-100	Item 10 CBC-300 LCD-123 NCC-919 LKK-100 DHT-911	6×5 $= 30 \text{ comparisons.}$

先 build



後 probe

MKT-612	[0]	5 builds
LCD-123	[0]	6 probes
KFA-214	[1]	$2+2+1+2+2+1$
NCC-179	[2]	$= 10 \text{ comparisons}$
MKT-500	[2]	
LKK-100	[1]	

My Opinions

Thoughts, inspirations, and suggestions



2. Count-Min Sketch

H1	H2	CAB
[0] 0	[0] 0	3 1 2
[1] 0	[1] 0	A B E
[2] 1	[2] 0	1 > 5
[3] 1	[3] 1	F G
[4] 2	[4] 3	6 7
[5] 1	[5] 1	

(key * 2) % 7

* 選小的

* 可能不正確

ex. 再加工 3 個 I

的話 B=5,

但 B 只有 2.

Counting:

A = 2

B = 2

C = 1

D = 0

E = 1

F = 1

G = 1

Inefficient Operations on Hashing:

* Traversal

- Visit all the data in sorted order.

* NN Search

- Find the item that has the smallest or the largest search key.

* Range Query

- Find the items between two searches keys.

決定你成功或失敗的，
不是你的目標，
而是你的系統。
《原子習慣》 21

My Notes

Important Concepts worth keeping

Ch6. 圖形概論

Today: / /

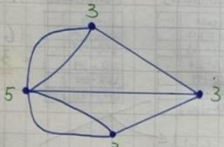
* 七橋問題

• $G = \{V, E\}$

- $V(G)$: 點的數量

- $E(G)$: 邊的數量

- degree: 點上邊的數量



$$\sum \text{degree}(V_i) = |E(G)| \times 2$$

$$5 \times 3 + 3 \times 3 = 7 \times 2$$

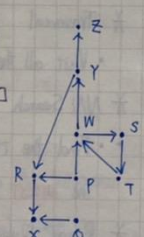
* 一筆畫

- degree 皆為偶數

- 0 或 2 個點的 degree 為奇數

* Adjacency Matrix:

	P	Q	R	S	T	W	X	Y	Z
P	0	0	1	0	0	1	0	0	0
Q	0	0	0	0	0	0	1	0	0
R	0	0	0	0	0	0	0	0	1
S	0	0	0	0	1	0	0	0	0
T	0	0	0	0	0	1	0	0	0
W	0	0	0	0	0	0	1	0	0
X	0	0	0	0	0	0	0	1	0
Y	0	0	0	0	0	0	0	0	1
Z	0	0	0	0	0	0	0	0	0



- R 的 In-degree: 2

- R 的 Out-degree: 1

- R ← P

successor predecessor

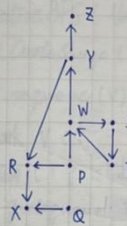
- traverse $(G): O(|V|^2)$

My Questions

Problems & Difficulties needing exploration

Adjacency List

My learning weather report



P	→	R	→	W
Q	→	X		
R	→	X		
S	→	T		
T	→	W		
W	→	S	→	Y
X				
Y	→	R	→	Z
Z				

* 只能求 Out-degree

* traverse $(G): O(|V| + |E|)$

My Opinions

Thoughts, inspirations, and suggestions

Other Graph Representations

* Mapping from vertex labels to array indices

P	Q	R	S	T	W	X	Y	Z
0	1	2	3	4	5	6	7	8

* Sequential representation

- nodes + edges

[0]	[1]	[2]	[3]	...	[7]	[8]	[9]	[10]	[11]	[12]	...
10	12	13	14	...	19	20	21	22	23	24	...

執行習慣的時間長短，
習慣的次數多寡重要。

《原子習慣》

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The amount of time you have been performing a habit is not as important as the number of the times you have performed it. «Atomic Habits»

* DFS: 利用 stack

Iterative DFS (Vertex v) {

s.createStack();

s.push(v);

Mark v as visited;

while (!s.isEmpty()) {

u = s.getTop();

if (unvisited w is adjacent to u) {

s.push(w);

Mark w as visited;

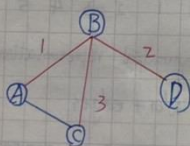
} // if

else s.pop();

} // while

} // DFS

DFS traversal sequence:
AB, BC, BD



暫停吸口氣，
靜思問自己，
閱讀 10 秒鐘，
持之以年功。
【且慢功】

Iterative BFS (Vertex v) {

q.CreateQueue();

q.enqueue(v);

Mark v as visited;

while (!q.isEmpty()) {

q.dequeue(u);

while (each unvisited w is adjacent to u) {

Mark w as visited;

q.enqueue(w);

} // while

} // while

My Opinions

Thoughts, inspirations, and suggestions

P → R → W

Q → X

R → X

S → T

T → W

W → S → Y

X → Y

Y → R → Z

Z

DFS: PR, RX, PW.

WS, ST, WY, YZ

PRXWSTYZ

BFS: PR, PW, RX.

WS, WY, ST, YZ

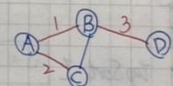
PRWXSTYZ

BFS: 利用 queue

My learning

weather report

BFS traversal sequence:
AB, AC, BD



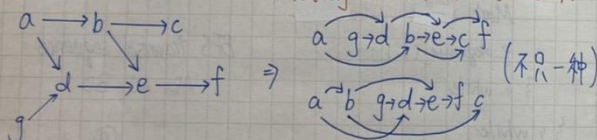
* Topological Sort

• Definition:

必須是 Directed Acyclic Graph (DAG)

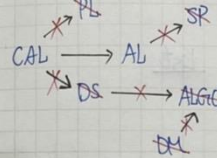
具有方向性且沒有 cycles

• Example:



• topSort I:

1. Find a vertex that has no successor (out-degree = 0).
2. Add the vertex to the beginning of a list.
3. Remove that vertex from the graph, as well as all edges that lead to it.
4. Repeat the previous steps until the graph is empty.



out-degree	successor	predecessor
0	AL	CAL
0	ALGO	DS, DM
0	CAL	SP
0	DM	ALGO
0	DS	AL
0	PL	CAL
0	SP	DM

① ALGO

② DM, ALGO

③ DS, DM, ALGO

④ PL, DS, DM, ALGO

⑤ SP, PL, DS, DM, ALGO

⑥ AL, SP, PL, DS, DM, ALGO

⑦ CAL, AL, SP, PL, DS, DM, ALGO

in-degree	predecessor	successor
0	AL	SP
0	ALGO	DM
0	CAL	AL, DS, DM
0	DM	ALGO
0	DS	ALGO
0	PL	CAL
0	SP	DM

in-degree = 0

① CAL

② CAL, DM

③ CAL, DM, DS

④ CAL, DM, DS, PL

⑤ CAL, DM, DS, PL, AL

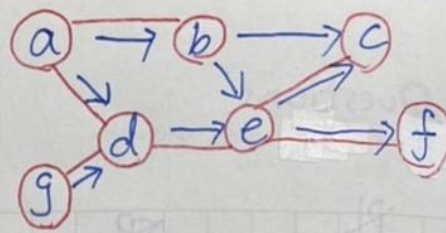
⑥ CAL, DM, DS, PL, AL, ALGO

⑦ CAL, DM, DS, PL, AL, ALGO, SP

My Notes

Important Concepts worth keeping

• topSort 2 (利用 DFS)



My Problems

My Thoughts,

Action	Stack s	List
★ push a	a	
★ push g	ag	
push d	agd	
push e	agde	
push c	agdec	
pop c, add c to List	agde	c
push f	agdef	
pop f, add f to List	agde	fc
pop e, add e to List	agd	efc
pop d, add d to List	ag	defc
pop g, add g to List	a	gdefc
push b	ab	
pop b, add b to List	a	bgdefc
pop a, add a to List (empty)		abgdefc

人生不應像孤島，
有時轉身分享，
有時求助，
有時轉身打氣，
有時互相提攜。

【轉身功】

(從左 → 右，所以先將 in-degree = 0 的放入 stack。)