

Priority Queue 優先佇列

* 有 "特例" 會被優先處理, 如: 看急診

應用: 找最近距離的城市...

因 selection sort, tree sort 有不同的時間複雜度

Heap 堆積

* 平衡的二元樹 (complete) 完整二元樹

上至下 [連續] 沒缺少任何一塊

\Rightarrow min-Heap, max-Heap $\text{Insert}(): O(\log n)$

Build heap?

ReheapDown() $\text{pq.Delete}(): O(\log n)$

ReheapUp() $\text{pq.Insert}(): O(\log n)$

heapInsert() efficiency: $O(\log n)$

* Double-ended Priority Queue (DEPQ)

Min-max Heap

Level 1

Level 2

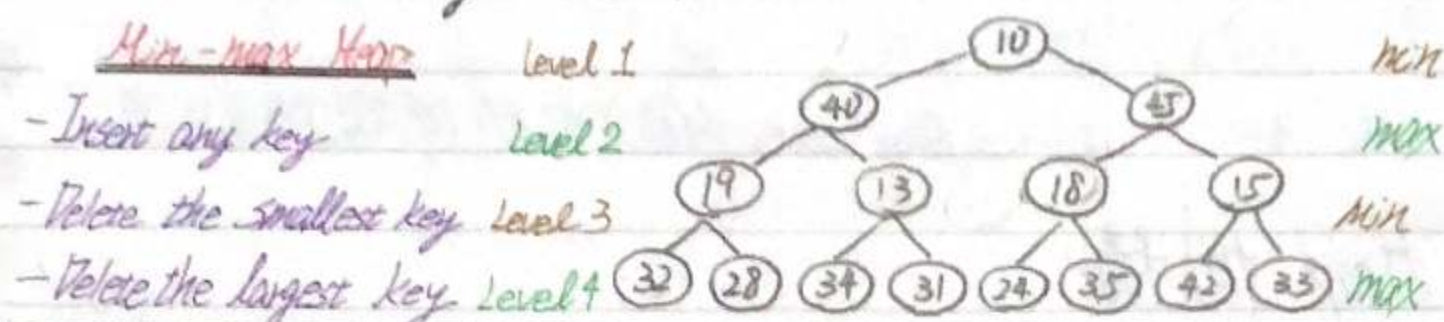
Level 3

Level 4

- Insert any key

- Delete the smallest key

- Delete the largest key



1. Decide which level \Rightarrow min or max

2. Check whether to swap with its parent

No \Rightarrow ReheapUp from the current node

Yes \Rightarrow ReheapUp from its parents

Min-max heap

Insert

1. Replace the root with the last element

2. Check whether to swap with its smaller child

No: ReheapDown from the root

Yes: ReheapDown from the root

Min-max heap

Delete the smallest

1. Replace the maximum with the last element

2. Check whether to swap with its larger child

No: ReheapDown from the current node

Yes: ReheapDown from the current node

Min-max heap

Delete the largest

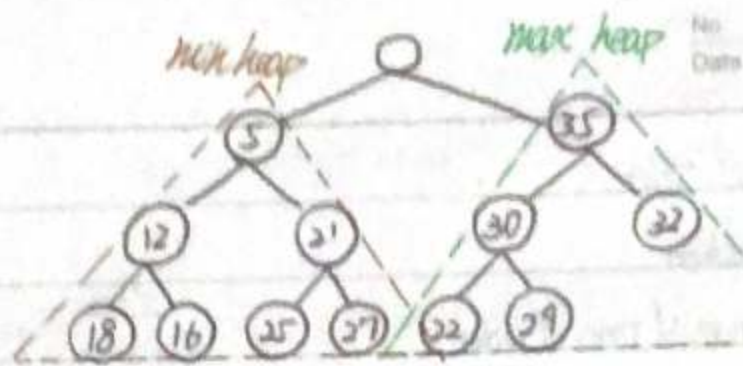
* Three 4-way trees \Rightarrow max heap + min heap + max heap.

Main idea

\rightarrow each node in max-heap has its parent in min heap

Heap 雙堆棧

- Insert any key
- Delete the smallest key
- Delete the largest key

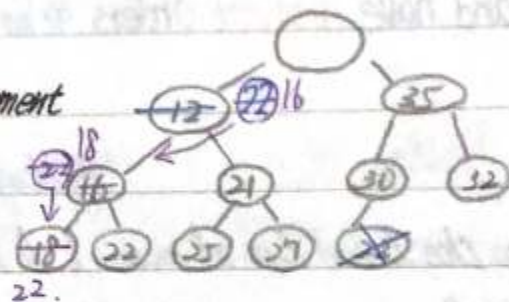


Heap: Insert

1. Examine the corresponding nodes: $left < right$
2. ReheapUp if necessary (recursion)

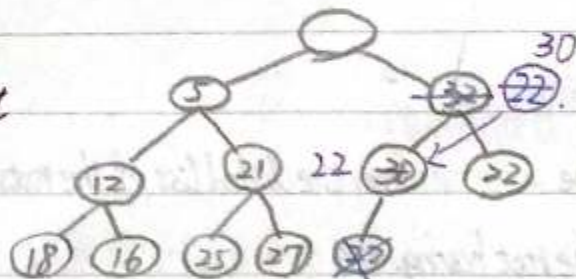
Heap: Delete the Smallest

1. Replace the root of min-heap with the last element
2. ReheapDown if necessary
3. Examine the corresponding nodes: $left < right$



Heap: Delete the largest

1. Replace the root of max-heap with the last element
2. ReheapDown if necessary
3. Examine the corresponding nodes: $left < right$



Main idea

- Two heap:

Pseudo root + min heap + max heap

Each node in max-heap corresponds to one in min-heap

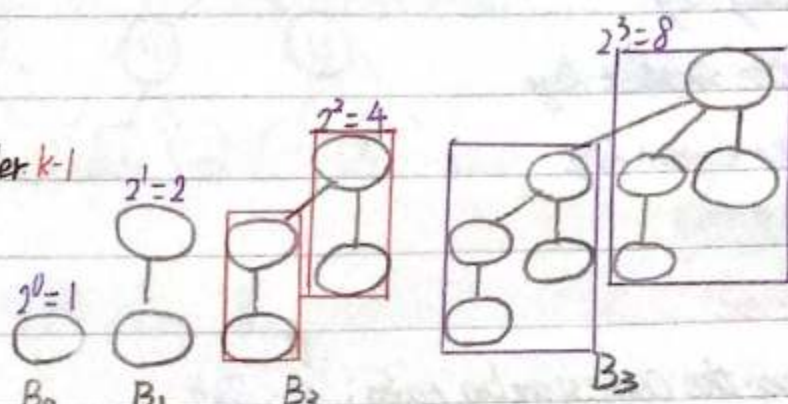
* Mergeable Priority Queues

Two cooks \rightarrow one cook // multiple servers: job queues

Binomial Tree

* Binomial tree of order k

- The root has k children
- Merged by two binomial trees of order $k-1$
- Number of nodes = 2^k
- Tree height = $k+1 \Rightarrow O(\log k)$
- C_i^k nodes at level i , for $i=0 \dots k$



* A binomial heap is a collection of binomial trees that satisfy the heap property and have distinct orders \Rightarrow Two binomial trees of the same order can be merged

* Insert:

1. Insert into the linked list of roots
2. Call merge function

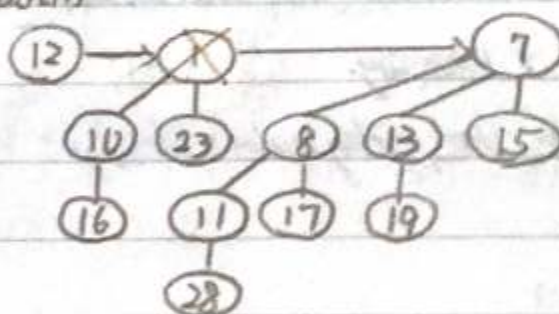
* Merge

1. A linked list sorted by the orders of binomial trees
2. Merge two binomial trees of the same orders [left to right]

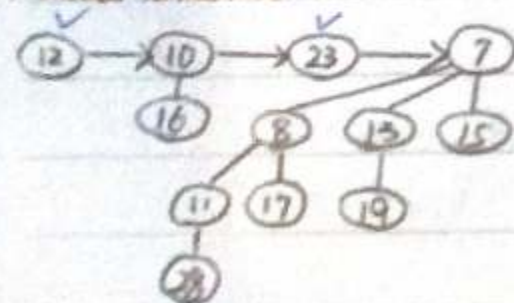
* Delete

1. Find the minimum from the linked list of the roots
2. Delete the root having the minimum
3. Add its children into the linked list
4. Call merge function

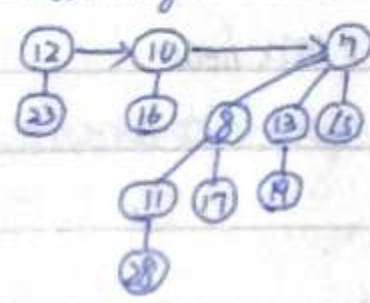
(a) head[H]



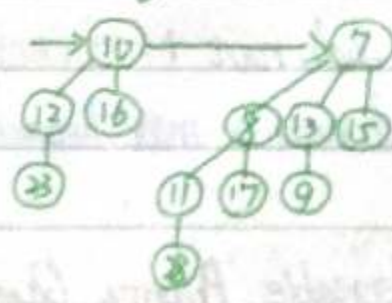
(b) Circular linked list



(c) Call merge



(d) $O(\log n)$



單元 3

No
Date

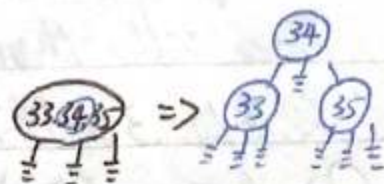
* 23 樹 - 2-node & 3-node [幾個小孩]

=> Search: $O(\log n)$

Insert: a leaf may contain either one or two

* 若 node 有兩個

=> 先做排序 => 中間值往上提 => 左右各自處理



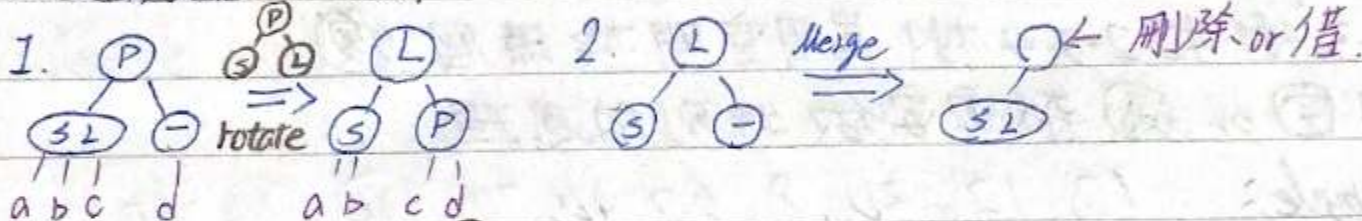
Step:

1. 找到樹葉

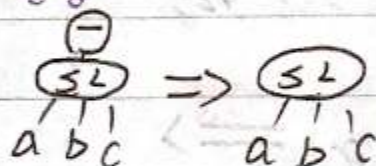
2. 放入 [若空放入] [滿 => 分裂] 樹根 -> 新增, else 左, 右, 或

Delete():

1. 重新分配 2. 合併



* 特殊刪到 root



Step:

1 找到樹葉

2. 刪除 [樹葉], 若非樹葉 => in-order successor 兄弟可借 1 item 借不到!

3 如果還有 item => done, else choose (a) Redistribute (b) Merge

Summary:

搜尋 2-3 tree 不會比 binary search tree 有效率, 就它有最小的樹高

∵ 2-3 tree ① node have two values

優點: 它保證平衡

2-3-4 樹

=> have 2-nodes, 3-nodes, 4-nodes.

* Are general tree, not binary trees

Never taller than 2-3 tree

效率比 2-3 樹好

Q: 4 item 不知道怎麼分 => 看到 3 個就先分

* 其它條件和 2-3 樹差不多

Delete: => 看到有 1 item 先幫它合併

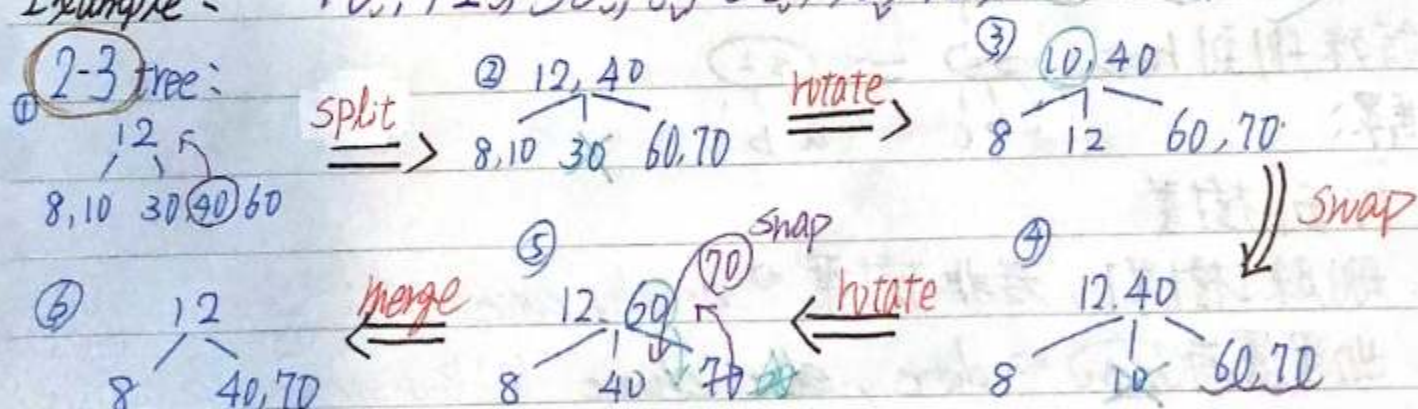
Summary:

2-3 樹和 2-3-4 樹是用空間換時間 (優)

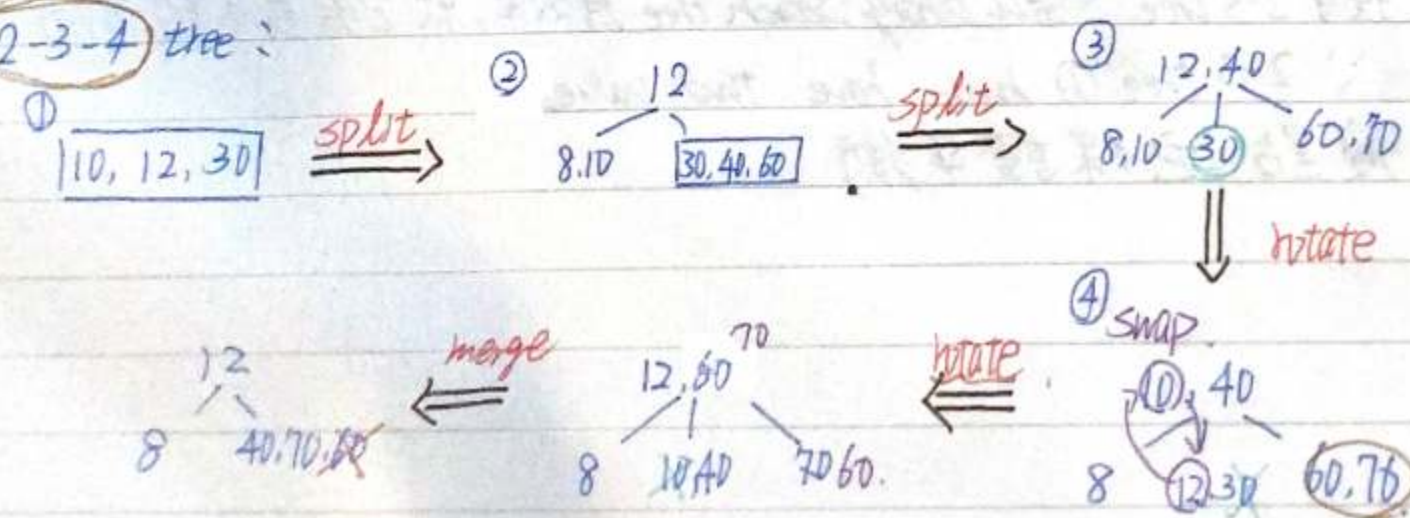
但 空 or 滿 都需要做特別的處理

Example: 10, 12, 30, 8, 60, 40, 70, 30, 10, 60

2-3 tree:



2-3-4 tree:



單元 4

AVL Tree:

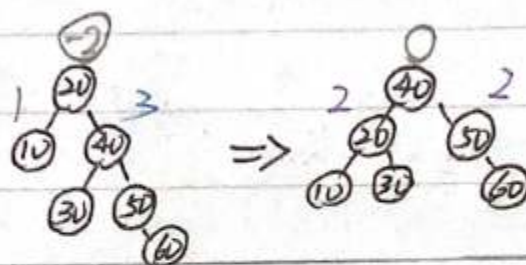
- A balanced binary search tree
- minimum-height

main:

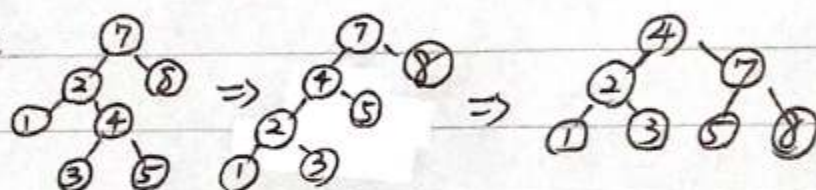
- 1 check balanced
- 2 if unbalanced \Rightarrow rotate

Rotate:

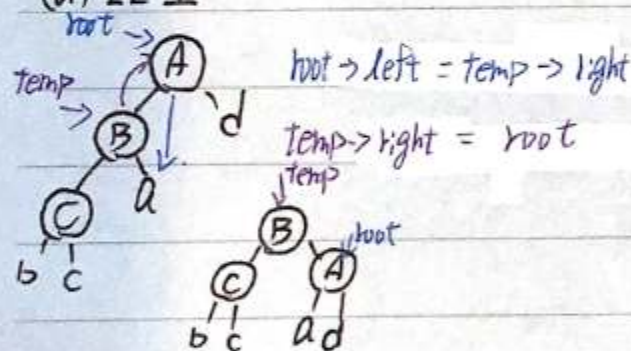
1 single rotation



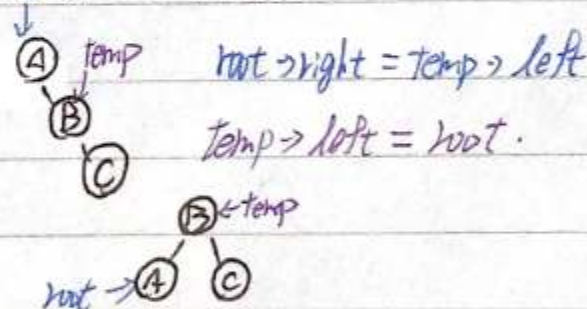
2. Double rotation



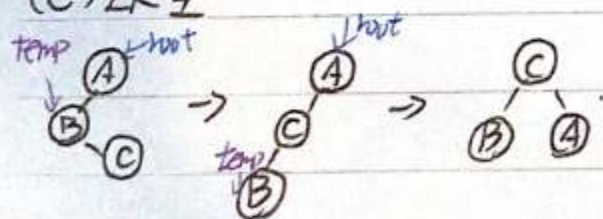
(a) LL 型



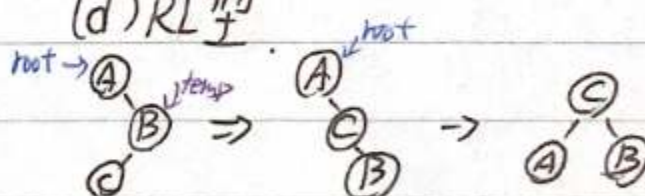
(b) RR 型



(c) LR 型



(d) RL 型



Balance?

