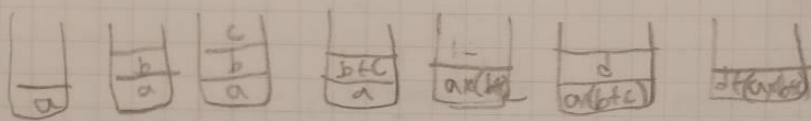


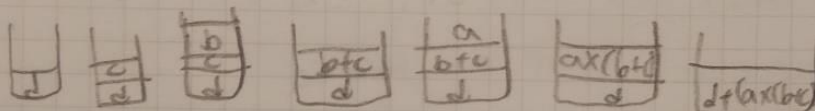
Application of Algebraic Expression

ex. postfix: pop the top 2 operand from the stack
while operator is entered
pushes the result back to the stack

ex. $abc + xcd +$: postfix



Prefix $+x a+bcd$



Infix to Postfix

first save all the operands and operators to each two stacks
and compare the two operator's precedence, ~~is~~ execute
the operator who has the higher precedence first
or if it have braces then no need to compare the precedence

$$((a \times (b+c)) + d)$$

+

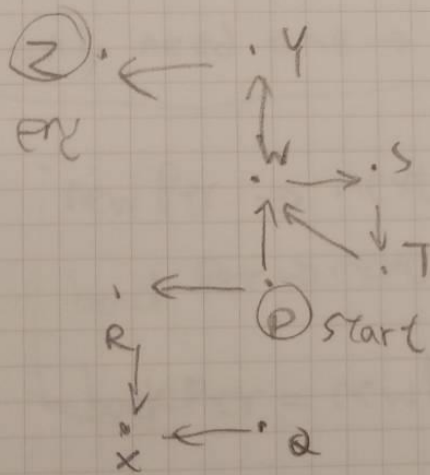
+

*

Search a path

find the next city and marked the origin as visited
if the next city is visited then go back to the city where it starts and do it again

and if there is no way to go ~~and also we can't~~
and it didn't reach to the destiny then go back to the first location



push P
R → next unvisited
X
pop X ∈ No unvisited
R
push W
S
T
pop T
S
push Y
Z

Queue

FIFO

Application

Reading a string

Recognize Palindrome

simulation

→ anything about lining

is Empty

enqueue

dequeue

get Front

~~de~~

o palindromes
the string

→ save to queue and stack

while(queue.isempty() queue.pop == stack.pop)

stack.push as same as Queue.enqueue

~ .pop(top)

~ .dequeue(front)

if (top != front)

count ~~on~~ it isn't palindrome

Implementations of the ADT Queue

A linear linked list with two external references

one front \rightarrow

one back

A circular --- one

only need one \rightarrow back

```
newPtr  $\rightarrow$  next = NULL;  
backPtr  $\rightarrow$  next = newPtr;  
backPtr = newPtr;
```

enqueue =

```
QueueNode *newPtr = new QueueNode  
newPtr  $\rightarrow$  item = newItem  
newPtr  $\rightarrow$  next = NULL  
backPtr  $\rightarrow$  next = newPtr  
backPtr = newPtr;  
if (isEmpty())  
    frontPtr = newPtr;  
else  
    backPtr  $\rightarrow$  next = newPtr  
backPtr = newPtr
```

dequeue

tempPtr = frontPtr

frontPtr = frontPtr->next

tempPtr->next = NULL

delete tempPtr

TempPtr = frontPtr

frontPtr = NULL

backPtr = NULL

tempPtr->next = NULL;

delete tempPtr

Simulation

→ modeling the behavior

event-driven simulation

26 5 25 0

22 4

arrival event & departure event

calculate statics

- total waiting time
- average waiting time

△ Make Decision

- should we add teller

Big (O) Notation

Time efficiency

space efficiency

Three factors that will effect (without algorithm)

specific implementation

computer

data

execution time related to the number of operations

```
for (a=1; a<=n; a++)  
  for (b=1; b<=a; b++)  
    for (c=1; c<=k; c++)
```

loop take $(n^2 + n) \times k$ time

if Algorithm A requires time proportional to $n^2 \rightarrow O(n^2)$
B $n \rightarrow O(n)$

if runtime sometimes be n or n^2 or n^3 runtime
although n^2 or n^3 may be, also be the of the algorithm
but we will select the best answer as the time proportion
that is $O(n)$

$O(1)$ $O(\log_2 n)$ $O(n)$ $O(n \log_2 n)$ $O(n^2)$ $O(n^3)$ $O(2^n)$
← good bad →

$$O(5n^3 + 3n)$$

ignore the lower order term

and ignore the higher order term's constant

$$O(f(n)) + O(g(n))$$

$$= O(f(n) + g(n))$$

Sequential search

Strategy \rightarrow stop when the desired item is found

worst case

best case

Average case

Efficiency of sorting Algorithm

internal sort

external sort

stable sort

stable

unstable

bubble
insertion
merge
radix

quick
heap
selection

5 2 5a 5b 6 8a 8b 19 27

in stable it always be the same sequence

but in unstable

5a & 5b may switch

or
8a & 8b

bubble sort

find the small one ~~and~~ if the next one is bigger than swap
selection sort

swap the smallest one to the first
and swap the second smallest one to the second ... etc.

Insertion sort

put the next value to sorted list and insert as it's value
region

may swap the front letter to the very back end \rightarrow will
usually mess the sort

$$n + \sum_{pass=1}^{n-1} (n-pass+1) + \sum_{pass=1}^{n-1} (n-pass)$$

$$= n + 2 \left[n^2(n-1) - n^2(n-1)/2 \right] + (n-1) = n^2 + n - 1 \Rightarrow O(n^2) \leftarrow \text{loop}$$

$$4 \times n(n-1)/2 = 2n^2 - 2n \Rightarrow O(n^2) \leftarrow \text{comparison}$$

Data exchange $O(n)$

compare $O(n^2)$ \rightarrow still $O(n^2)$

although no earlier termination even if it have been sorted

because it will only move the data $O(n)$, so when we

met a list with single large data than it's appropriate

bestcase $O(n)$

worst case $O(n^2)$

when the data is not complete

Shell Sort \rightarrow stronger version insertion sort

insert but don't sort perfectly

Merge Sort

divide and conquer and compose together

if there is n number

separate so $n-1$ space swap $\approx 2n$

→ worst $n-1 + 2n = 3n-1 \rightarrow O(n)$

levels: $\log_2 n$

average: $n \times \log_2 n$

→ the second array will take as origin array

Quick Sort

also divide and conquer

Choose a pivot

Partitions the array about the pivot

→ to ~~split~~ split items ~~to~~ to two parts

→ left of the split

right of the split

do it again

Radix Sort

sort as the based Radix

compare the number each by each from high to small

but also need to separate to group

Binary Tree

Binary search Tree

position oriented ADT

insert t to n^{th} position

delete ~~from~~

ex.

list stack queue binary tree

Value oriented ADT

Insert according to its value

Delete only knowing \checkmark

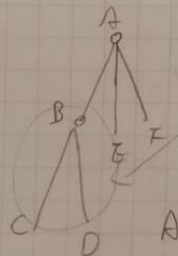
ex.

sorted list, binary search tree

Trees : are composed of nodes and edges

↓
hierarchical \rightarrow parent-child (two nodes)

Ancestor - descendant (among nodes)



subtree of a tree

\Rightarrow any node and its descendant Λ

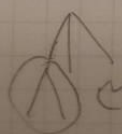
A single node r , the root

sets

Parent of node B = A

child \sim = C, D

Root: the only node in the tree with no parent



Subtree of node B contains child of node B and its descendant's

Leaf

- A node with no child

Siblings

- node with common parent

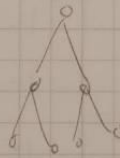
Ancestor from node to root

Descendant

from node to leaf

Binary Tree

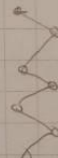
→ ~~can~~ one can have one siblings or non



Height=3



H=5

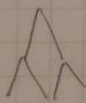


H=n

the max of from root to leaf is Height

Full Binary tree

every node levels $< h$ has two children



complete Binary Trees



→ is full to level $h-1$
fill from left to right

Balanced Binary Trees



The ADT Binary tree

tree1.setRootData(F)

tree1.attachLeft(G)

tree2.setRootData(D)

tree2.attachLeftSubtree(tree1)

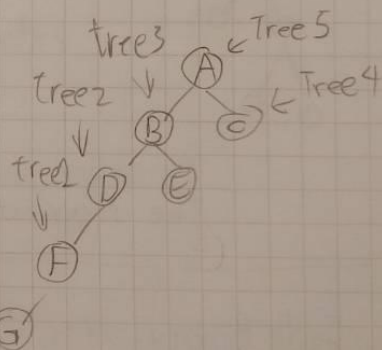
tree3.setRootData(B)

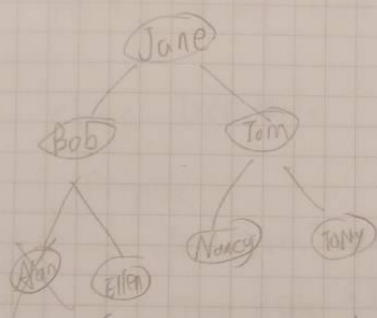
tree3.attachLeftSubtree(tree2)

tree3.attachRight(E)

tree4.setRootData(C)

tree5.createBinaryTree(A, tree3, tree4)

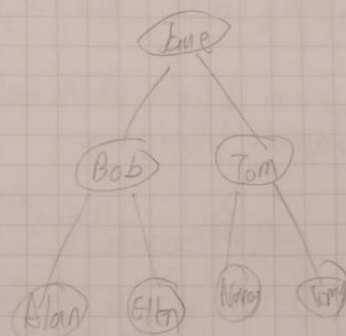




		leftChild	right
0	Jane	1	2
1	Bob	-1	4
2	Tom	5	6
3	Coke	-1	-1
4	Ellen	3	-1
5	Nancy	-1	-1
6	Tony	-1	-1
7	?	-1	8
8	?	-1	9
9	?	-1	-1

root
 0
 free
 16

Free list

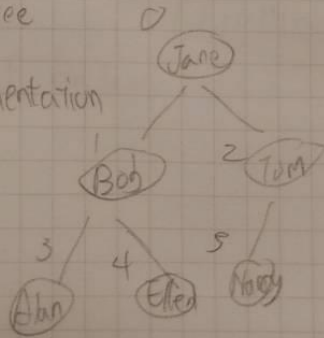


Jane	1	2
Bob	3	4
Tom	5	6
Coke	-1	-1
Ellen	-1	-1
Nancy	-1	-1
Tony	-1	-1
?	-1	8
?	-1	9

root
 0
 free
 17

a complete binary tree with array-based implementation

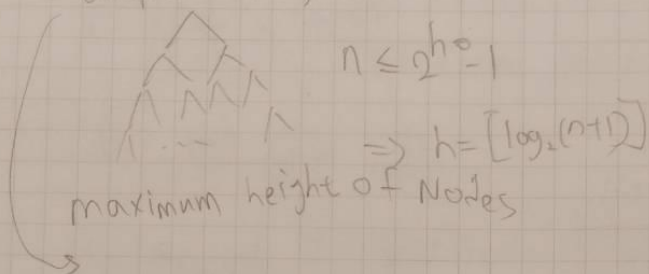
$leftChild = 2 * parent + 1$
 $rightChild = 2 * parent + 2$
 $parent = (child - 1) / 2$



- 0 Jane
- 1 Bob
- 2 Tom
- 3 Alan
- 4 Ellen
- 5 Nancy

Height and No. of Nodes

minimum height of N nodes
- complete binary tree



maximum height of Nodes

$$(2^{h-1} - 1) + 1 \leq n$$

$$h = \log_2(n) + 1$$

properties

N_2 : 3 nodes with 2 children

N_0 : 4 Leaves

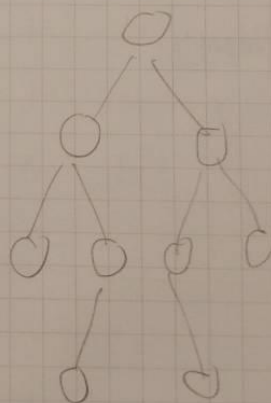
$$N_0 = N_2 + 1$$

B : 8 Branches

$$B = |E| = 2 * N_2 + 1 * N_1$$

$$N_0 + N_2 + N_1 = |V| = |E| + 1$$

$$= 2 * N_2 + N_1 + 1$$



Pointer Based ADT Binary Tree

class Tree Node

{ Private

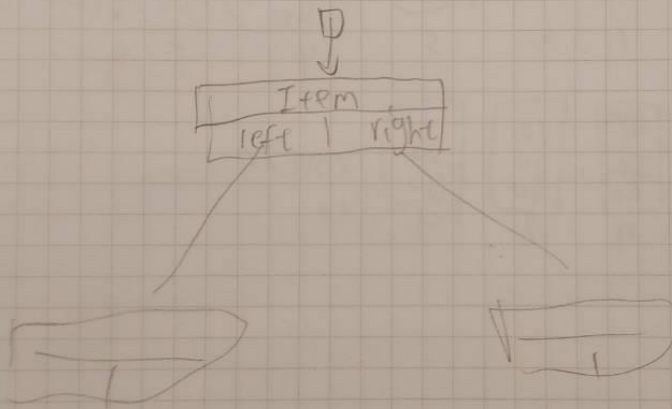
TreeItemType item // data portion

Tree Node *leftChildPtr // pointer to left child

Tree Node *rightChildPtr // pointer to right child

}

Tree Node *root // pointer to the root



recursive traversal algorithm

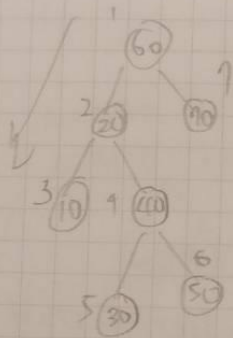
traverse (in binTree: BinaryTree)

If (binTree is not empty)

preorder → traverse (left subtree of binTree's root)

inorder → (right)

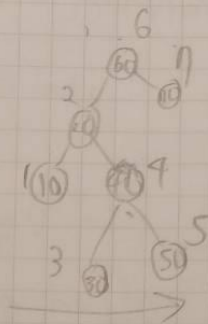
postorder →



pre order



遇見則輸出



in order

↓

回去過試輸出

回去過試輸出

60

1

20
60

2

10
20
60

3

NUL
10
20
60

4

10
20
60

5 6

visit 10

NUL
10
20
60

7

10
20
60

8

20
60

9

visit 20

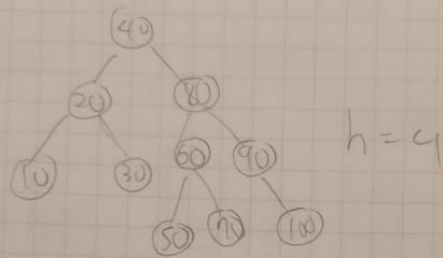
10

Binary search tree

left < right

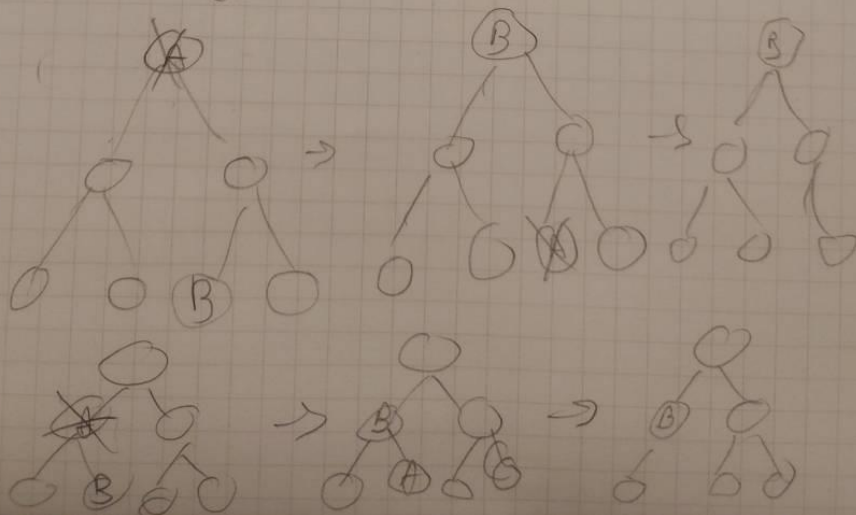
40 20 10 80 90 100 30 60 50 70

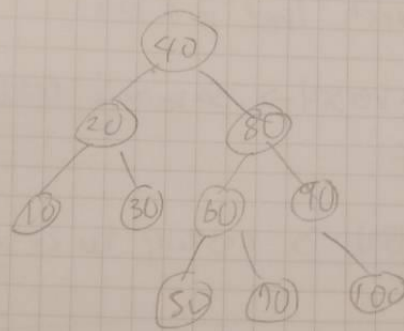
↓



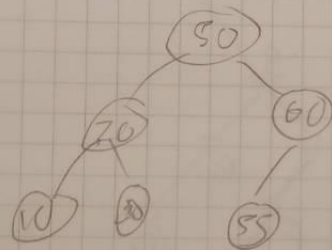
Delete \Rightarrow when we delete a node
we still need to connect the
last, when the node will connect
more than 2 node then we have special
moves

ex.





+55 - 90 - 10 - 100 - 40 - 10



Efficiency of Binary Search Tree

Operation	Average	Worst case
Retrieval	$O(\log n)$	$O(n)$
Insertion	$O(\log n)$	$O(n)$
Deletion	$O(\log n)$	$O(n)$
Traversal	$O(n)$	$O(n)$

→ Tree sort

Build a binary search tree by n insertions

Average $O(n \log n)$
Worst $O(n \cdot n)$

Saving a Binary Search Tree in a File

~~we can~~ pre order traversal \rightarrow save and restore
to original tree

inorder traversal \rightarrow restore to a balanced
tree

Left child \rightarrow the leftmost child
Right child : right next siblings

