Math 6141 - Numerical methods

Coursework 2

Due: 9 December 2015

You must submit electronically (via Blackboard) a copy of the Python scripts required by the tasks below. The deadline is 12:00 Wednesday, December 9th 2015.

Penalties for late submission*: 10% of the total marks for the assignment per working day after the assignment is due (where a working day is a 24 hour period starting form original hand in deadline, skipping days on which the University is closed), for up to 5 working days. No marks will be obtained for submissions that are later than 5 working days.*

Assessment Criteria*: Assessment will primarily be based on the successful solution of the problems and your method of solution, which should be justified. You will also be assessed on the clarity of your code. Please ensure that all of it is written in good style, is properly commented and is robust; e.g., all input arguments should be explained and checked. Figures produced should be clearly presented with labels, titles, etc.*

Plagiarism*: Your are encouraged to discuss your work with your colleagues in general terms, but your submitted work* **MUST** *be your own.*

1 Modelling hair

A major use of numerical methods is in the creation of computer graphics in films, games, and other media. The generation of "realistic" animations of, for example, smoke, or water, or explosions, are typical problems. One case that remains difficult is the simulation of hair, particularly under external forces such as wind and rain.

2 Model

The first reasonable model of human hair used the beam equation applied to each hair individually. This coursework follows that model. The model we use here makes a number of simplifying assumptions:

1. The human head is a sphere with radius $10\,\mathrm{cm}$.
2. Each hair meets the head normal to the sphere.
3. The forces acting on each hair are gravity (in the $-z$ direction) and a constant wind (in the $+x$ direction).

We want to contruct the location of each hair. We will use the parameter s along the length of the hair: $s = 0$ at the free end of the hair, and $s = L$, where $L = 4\,\mathrm{cm}$ is the constant hair length, where the hair meets the head.

The model works separately for each hair. It uses a mixture of coordinate systems. The location of the hair will be displayed in the (x, y, z) coordinate system. The location of the hair will depend on the parameter s along its length,

$$x(s) = R\cos(\theta(s))\cos(\phi), \tag{1}$$
$$y(s) = -R\cos(\theta(s))\sin(\phi), \tag{2}$$
$$z(s) = R\sin(\theta(s)). \tag{3}$$

Here θ is the angle from the x axis, measured anti-clockwise. ϕ is the rotational angle. θ varies with the parameter s; the angle ϕ is assumed fixed for each hair, but may vary between hairs. The value of θ is assumed known at $s = L$, where it meets the head.

The equations for an individual hair are

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}s^2} = s f_g \cos(\theta) + s f_x \cos(\phi)\sin(\theta). \tag{4}$$

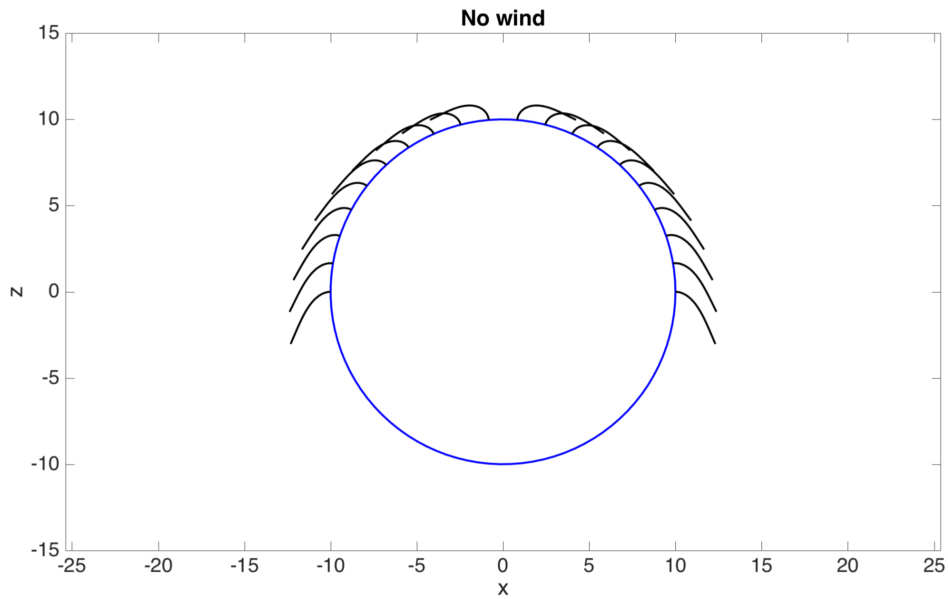

Figure 1: Locations of 20 hairs in the x, z plane under the force of gravity alone. The boundary value problem for each hair is solved, fixing the location where the hair meets the head.

f_g is proportional to the force from gravity. After including appropriate values for human hair stiffness, $f_g = 0.2$ is the value used here. f_x is proportional to the force from the wind.

$$\left.\frac{\mathrm{d}\theta}{\mathrm{d}s}\right|_{s=0} = 0, \quad \theta(L) = \theta_L. \tag{5}$$

Once $\theta(s)$ is known the (x, y, z) coordinates can be found as

$$\frac{\mathrm{d}x}{\mathrm{d}s} = \cos(\theta)\cos(\phi) + f_x \sin(\phi), \tag{6a}$$

$$\frac{\mathrm{d}y}{\mathrm{d}s} = -\cos(\theta)\sin(\phi) + f_x \cos(\phi), \tag{6b}$$

$$\frac{\mathrm{d}z}{\mathrm{d}s} = \sin(\theta). \tag{6c}$$

Note: if, after solving for θ in $s \in [0, L]$, you choose to solve for (x, y, z) by integrating for $s \in [L, 0]$ (as the boundary conditions are given at L), you will need to change the sign in all equations (6). Alternatively you can change parameters to use $\hat{s} = L - s$.

3 Task

1. Write a function that given L, R, f_x, and a list of values of $\theta(L)$ and $\phi(L)$ which specify where the hairs meet the head, returns the (x, y, z) coordinates of the hairs.

2. Fix $\phi = 0$ (work within the (x, z) plane) and $f_x = 0$ (no force due to the wind). Compute and plot the location of 20 hairs with $L = 4$ where the locations where the hairs meet the head are given by $\theta(L)$ being evenly spaced on $[0, \pi]$. Plot the locations in the (x, z) plane.

3. Repeat the previous task with $f_x = 0.2$.

4. Compute the locations of 100 hairs, with the force due to the wind set to $f_x = 0.1$. These should meet the head at a 10×10 grid of θ, ϕ locations, where $\theta(L)$ is evenly spaced on $[0, 0.49\,\pi]$ and ϕ is evenly spaced on $[0, \pi]$. Plot the results in the (x, z) plane, the (y, z) plane, and in 3 dimensions.

5. Fix $\phi = 0$ again. Fix to 10 hairs with $\theta(L)$ evenly spaced on $[0, \pi]$. Find the critical value of f_x, $F > 0$, such that for $f_x < F$ no hair intersects the head, whilst for $f_x > F$ some hair intersects the head. By "intersecting the head" we mean that the radius $R_{hair}(s) = \sqrt{x(s)^2 + y(s)^2 + z(s)^2}$ is less than R for some $s < L$.

3.1 Summary and assessment criteria

You should submit the code electronically as noted above.

The primary assessment criteria will be a general code that solves the hair-modelling problem. In top-level code comments the reasons for the choice of algorithm should be clearly given, stating the (dis)advantages of this and alternative methods.

The secondary assessment criteria will be the clarity and robustness of your code. Your functions and scripts must be well documented with appropriate internal comments describing inputs, outputs, what the function does and how it does it. The code should also be well structured to make it easy to follow. Input must be checked and sensible error messages given when problems are encountered. The clarity of the output (such as plots or results printed to the command window) is also important.

Code efficiency is not important unless the algorithm takes an exceptional amount of time to run.

The original paper that uses a "physical" approach to hair modelling is by Anjyo et al. from 1992. A survey of hair modelling approaches is given by Bartails et al. (2006). The particular model used here follows Howison's book on Practical Applied Mathematics. For a broader range of modelling techniques applied to computer graphics, Ron Fedkiw's webpage is a good place to start.