# A Comparison of Parametric and Nonparametric Estimation of Multiple-Input Transfer Function Models

**Angelita P. Tobias**
University of the Philippines Diliman

## Introduction

Autoregressive Moving Average (ARMA) is a traditional approach to univariate time series analysis. Suppose that the time series $y_t$ is related to an input time series $x_t$, then we could use current and past values of $x_t$ in addition to the historical values of $y_t$ to understand the structure of $y_t$. Wei (2006) discussed a function describing the dynamics that exists between the output series $y_t$ and the input series $x_t$ through a linear filter as follows:

$$y_t = v(B)x_t + n_t$$

The term $v(B) = \sum_{j=0}^{\infty} v_j B^j$ is referred to as the transfer function of the filter and $n_t$ as the noise series. When $x_t$ and $n_t$ follows some ARMA process, then the function is also known as **ARMAX (**Autoregressive Moving Average with Exogenous variable) model.

Parametric transfer function have been widely used but it is limited to linear, stationary and cointegrated series. Moreover, fitting single input transfer function is simple but the process becomes tedious for multiple-input case

## Objectives

This study aimed to

- Propose Generalized Additive Model (GAM) as a robust nonparametric approach in estimating multi-input transfer function models.
- Compare the predictive ability of parametric transfer function (ARMAX) with the proposed nonparametric approach (GAM).

## Proposed Model

The proposed model is denoted by

$$y_t = \sum_{i=1}^{p} f_i(x_{i,t}) + \varepsilon_t$$

where

$\varepsilon_t \sim N(0,1)$
$y_t$ - output series
$x_{i,t}$ - $i^{th}$ input series
$p$ - number of input series
$t$ - length of time series
$f(x_{i,t})$ - smooth function of $y_t$ on $x_{i,t}$

The proposed model was estimated through **Generalized Additive Model (GAM**).

## Methodology

The general flow of the study is


**Simulate**

A total of 40 scenarios were considered with 100 replicates each for a total of 4,000 simulated data sets.

| Simulation parameters and levels | |
|---|---|
| Form | • Finite<br>• Exponentially decaying |
| Time points | • 60 time points<br>• 360 time points |
| Number of inputs | • 2 inputs<br>• 3 inputs |
| Level of correlation | • All inputs correlated<br>• All inputs uncorrelated<br>• Two inputs correlated |
| Misspecification error | • With<br>• Without |

**Estimate and Assess**

For each data set,

- GAM and ARMAX model were fitted.
- Mean Absoluter Percent Error (MAPE) was computed from each model for comparison.

## Results

Overall, the nonparametric approach performed better than the parametric approach specially when there is misspecification error in the data.


**Figure 1.** Overall MAPE by method.

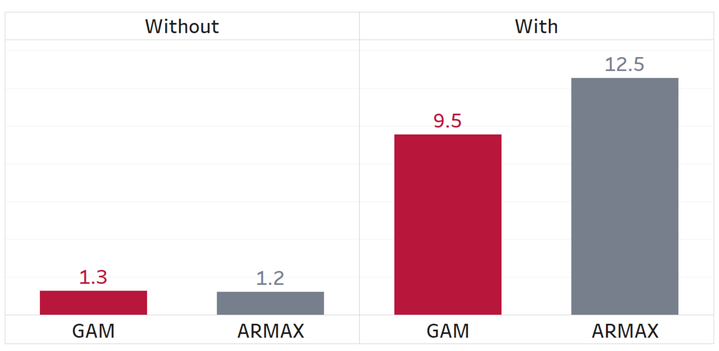


**Figure 2.** Comparison of MAPE by presence of misspecification error.

In terms of the form of transfer function, the two methods are comparable under the finite form.

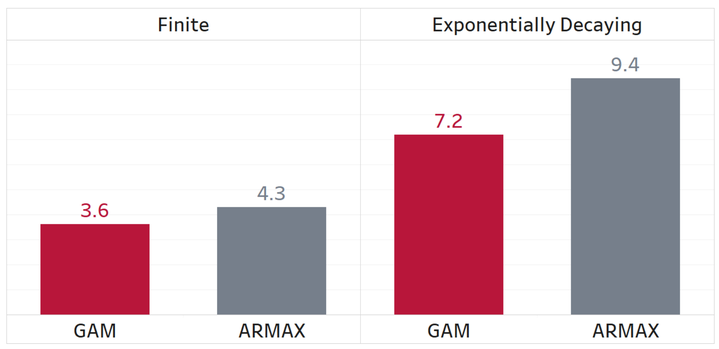


**Figure 3.** Comparison of MAPE by form of transfer function.

## Results (continuation)

Meanwhile, GAM showed superiority under exponentially decaying form of transfer function (Figure 3).

The Generalized Additive Model have also addressed the issue on short time series and performed better compared to ARMAX on both short and sufficiently long time series. (Figure 4).

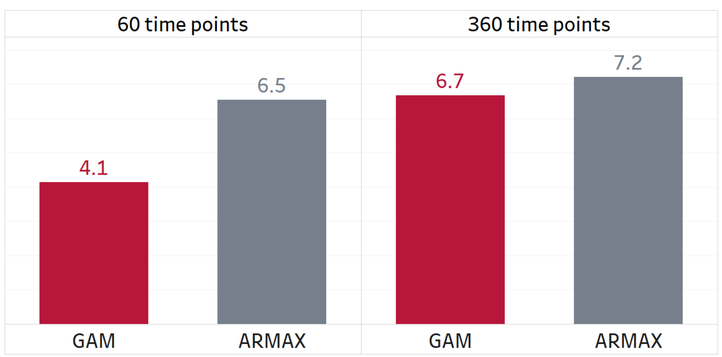


**Figure 4.** Comparison of MAPE by length of time points.

Moreover, the nonparametric approach had addressed the complexity of fitting multi - input transfer function and proved that it is robust even when correlations exist between input series as shown in Figure 5 and 6.

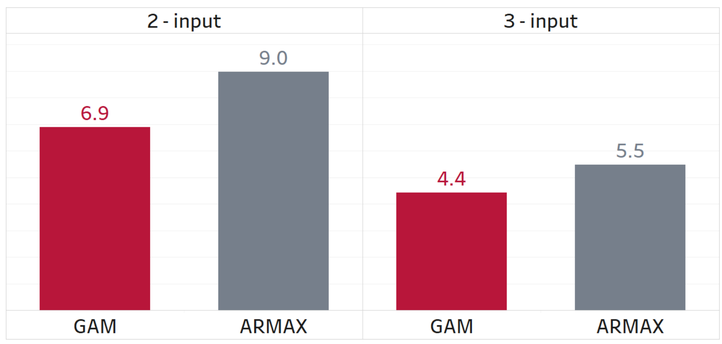


**Figure 5.** Comparison of MAPE by number of input series

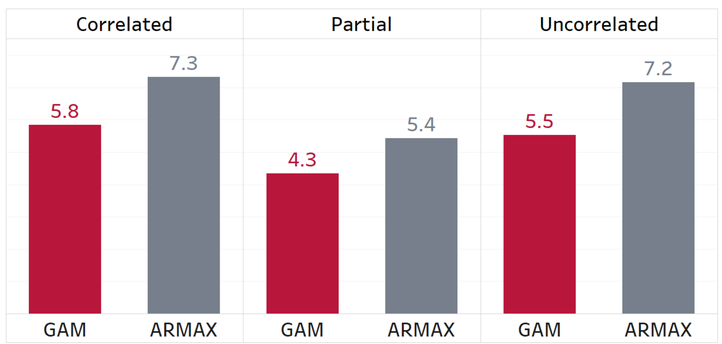


**Figure 6.** Comparison of MAPE by level of correlation

## Conclusion

The nonparametric procedure (GAM) produced better estimates than the parametric approach.

- It addressed the presence of misspecification error in the data.
- It is more convenient in fitting multi-input transfer function regardless of the level of correlation that exists between input series.
- It can handle short time series data which is very common on real data sets.

## Recommendations

To further test and prove robustness of the proposed approach,

- Add more input series.
- Induce structural change in the series.