Coursework Project I

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Question 1

Plot the Probability Density Function (PDF) of the streamwise velocity from both datasets and calculate its first four moments. Using the moment information, comment on type of distribution exhibited by the two sets of data.

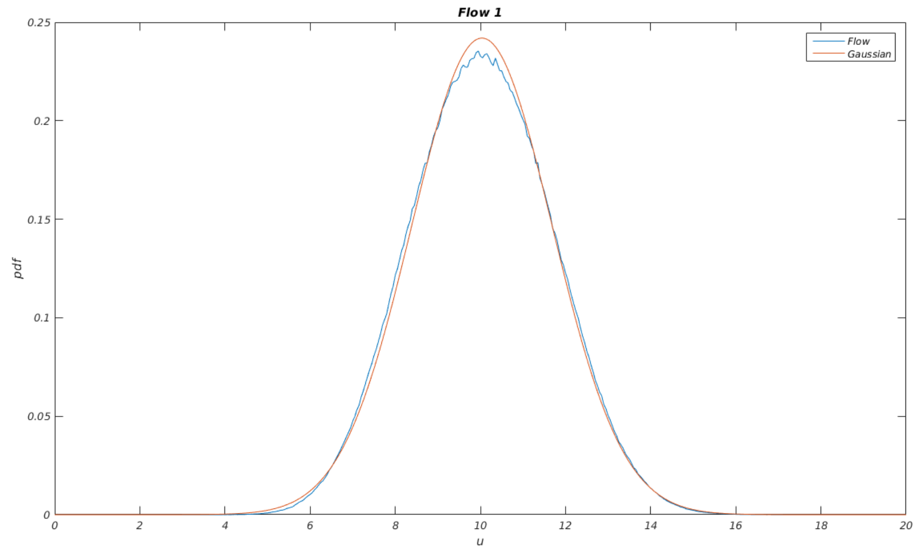

Figure 1: The Probability Density function for *flow 1*

Figures 1 and 2 show the probability density functions for the two flows with bin sizes of $0.05m/s$. From these figures it can be seen that they respect a *Gaussian* or *Normal* distribution. Table 1 shows that the statistical data for the two cases is not different between the two cases.

Table 1: Comparison of the first four moments for the two flows.

	Flow 1	Flow 2
1st Moment: Mean	10.04	10.04
2nd Moment:Variance	2.72	2.72
3rd Moment:Skewness	0.05	0.05
4th Moment:Kurtosis	2.78	2.78

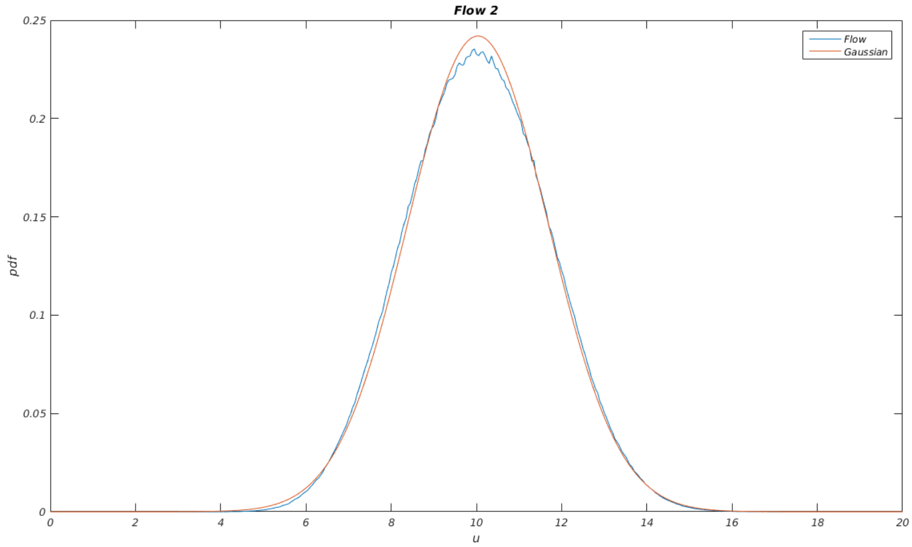

Figure 2: The Probability Density function for *flow 2*

Question 2

Plot the Probability Density Function (PDF) of the streamwise velocity gradient (i.e. ${}^{\partial u_1}/_{\partial x_1}$*) from both datasets and calculate its first four moments. Using the moment information, comment on type of distribution exhibited by gradients in the two sets of data. Use Taylor's hypothesis to convert temporal gradient to spatial gradient.*

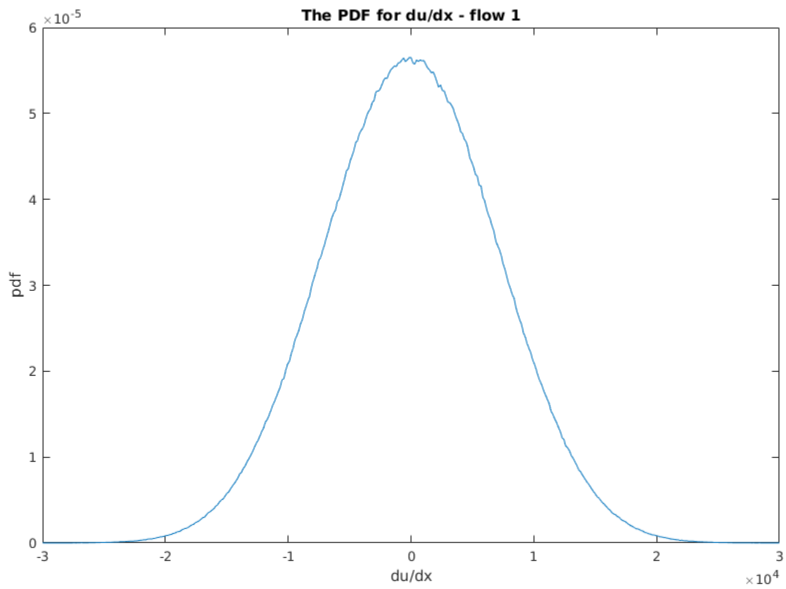

Figure 3: The normalised Probability Density function of ${}^{\partial u_1}/_{\partial x_1}$ for *flow 1*

Figure 3 shows the probability distribution of $\partial u_1/\partial x_1$ for *flow 1*. The curve shows that the velocity derivative to be more evenly distributed for this flow than the second flow case (fig. 4).

Kurtosis is a measure of how 'spread out' a distribution is and it is often compared to the number 3, given by the *Gaussian* distribution. A kurtosis lower than this is said to be *platykurtic* and a higher value is said to be *leptokurtic*. *Skewness* shows if the data has more outliers towards the right (positive) or left end of the tails.

Table 2 shows that the high variance and low kurtosis suggest a more 'spread out' distribution for *flow 1* when compared to the second. The second flow case is highly leptokurtic and the first is platykurtic

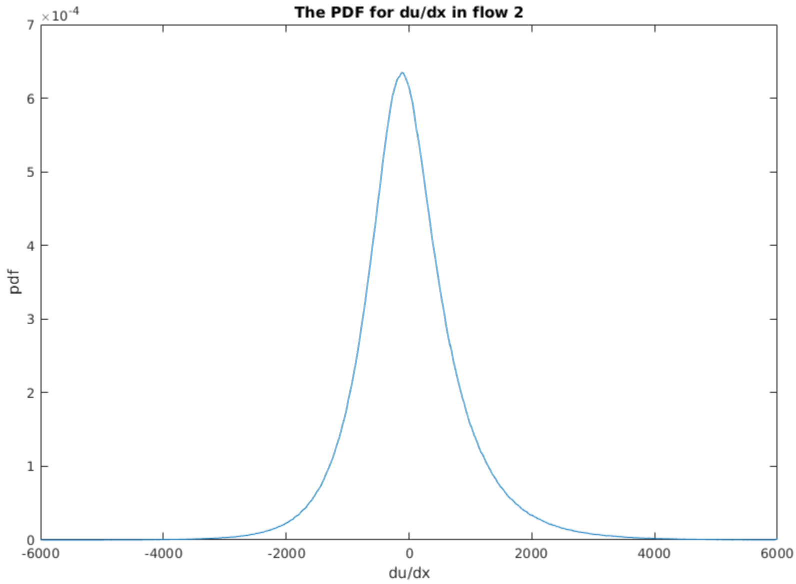

Figure 4: The normalised Probability Density function of ${}^{\partial u_1}/_{\partial x_1}$ for *flow 2*

but with a kurtosis close to that of a gaussian. The second case has more positive outliers due to the skewness of 0.55 whereas in the first flow case the datapoints are well centered ($S \approx 0$).

Table 2: Comparison of the first four moments for the ${}^{\partial u_1}/_{\partial x_1}$ of the two flows.

	Flow 1	Flow 2
1^{st} Moment: Mean	-0.002	-0.225
2^{nd} Moment:Variance	4.86×10^7	7.4×10^5
3^{rd} Moment:Skewness	-1.22×10^{-4}	0.55
4^{th} Moment:Kurtosis	2.89	5.92

Question 3

Using the homogenous isotropic assumptions and Taylor's hypothesis, calculate the dissipation rate from the dataset. Assume kinematic viscosity, $\nu = 1.5 \times 10^{-5} m^2/s$.

Dissipation is given by:

$$\epsilon = 15\nu \left\langle \left[\frac{\delta u}{\delta x}\right]^2 \right\rangle \tag{1}$$

Which gives $\epsilon_{flow1} = 1.09 \times 10^4\ {}^{J}/_{kg \cdot s}$ and $\epsilon_{flow2} = 166.4{}^{J}/_{kg \cdot s}$. This shows that a larger amount of energy is transformed from turbulent flow energy to heat in the first flow case which means that higer amounts of energy are lost in the first flow case.

Question 4

Using the dissipation and kinematic viscosity, calculate the Kolmogorov scales of both flows.

Table 3: Kolmogorov microscales for flows 1 and 2

Flow	η
1	6.758×10^{-7}
2	2.727×10^{-6}

Question 5

Calculate the autocorrelation function for both datasets. Compare the two curves and comment on the results.

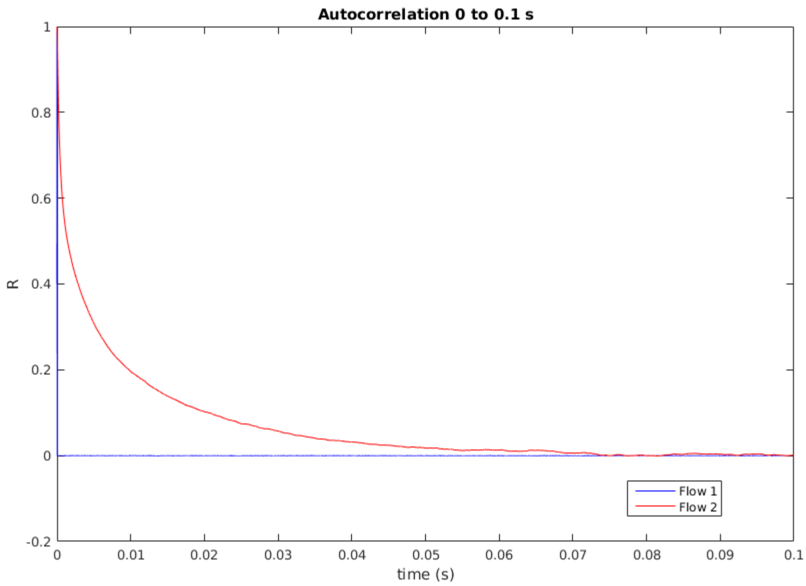

Figure 5: The comparison between the autocorrelation behaviours of the two flows from the origin to $0.1s$ with a window size of $1s$.

Figure 5 shows how sharply autocorrelation drops for the first flow case and in figure 6 it can be seen how it oscilates around 0, which is most probably the result of some signal or floating point operation error than an actual description of a phyisical property. Autocorrelation is also small for the second flow case but more pronounced at the start. Figure 6 shows how there is a small amount of autocorrelation occuring at the start.

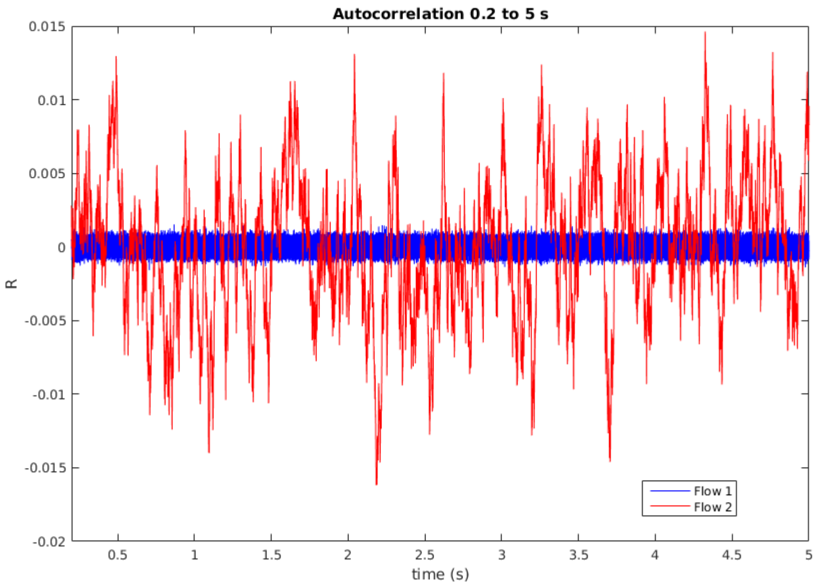

Figure 6: The comparison between the autocorrelation behaviours of the two flows between 0.2 and 5 s.

Question 6

Using the autocorrelation function, calculate the Integral length scale and the Taylor micro scale for both datasets. Comment on the results obtained.

The values in table 4 are obtained using eq. (2) and (3), where ρ is the autocorrelation coefficient and τ the time. To obtain eq. (3) a second degree central difference scheme with a fourth order of accuracy was used at $\tau = 0$. The chosen window size is of $1s$ and the integral scale is given in meters (i.e. the acquisition signal mutiplied by eq. (2) and the mean velocity $\langle u \rangle$).

$$T_{int} = \int_0^\infty \rho(\tau) d\tau \tag{2}$$

$$\lambda = \sqrt{-\frac{2}{\rho''(0)}} \tag{3}$$

Table 4: Integral scale(T_{int}) and Taylor microscale(λ) for flows 1 and 2

Flow	T_{int}	λ
1	$9.44 \times 10^{-5} m$	1.49×10^{-5}
2	$6.53 \times 10^{-2} m$	1.51×10^{-4}

Table 4 shows that in the second flow case the energy carying eddies are two orders of magnitude larger than in the first case. The Taylor microscale is an indicator of the eddy scale at which te turbulent energy is dissipated trough viscosity.

Question 7

Using the information that the window length necessary to calculate the energy spectrum should be at least 50 integral time scales, calculate the energy spectra for both datasets.

Figure 7 shows how the spectral energy varies with frequency. *Flow 2* exhibits high energy for low frequencies of the order of 10^3, whereas *flow 1* has a constant energy of order 1 for all frequencies. The spikes in present in the second flow case graph are due to measurement error.

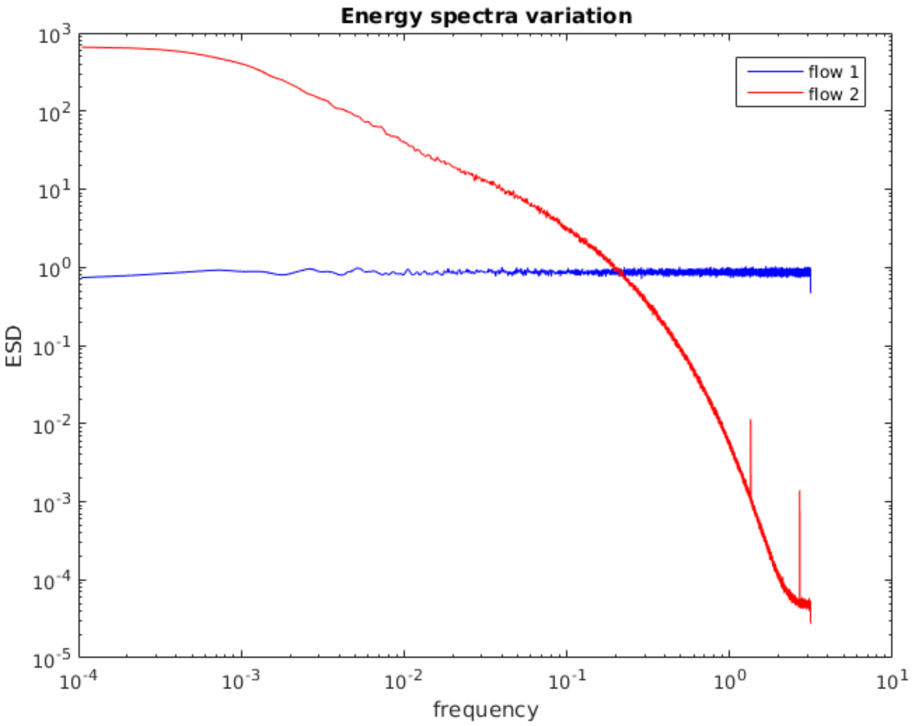

Figure 7: Comparison between the energy spectral densities of both flow cases as they change with frequency for a window size of $20'000$ data points

Question 8

Plot the pre-multiplied spectrum for both datasets and calculate the dominant time-scale (and hence length scale using Taylor's hypothesis) using this data by locating the peak in this pre-multiplied spectrum.

The dominant time scale is given by finding the index of the maximum value of the pre-multiplied spectrum and using it to choose the corresponding wave-number. The length scales for flows 1 and 2 are $\sim 2m$ and $4.58 \times 10^{-3}m$ respectively. When compared to the T_{int} values in table 4 they differ to a large degree especially for *flow 1*. In the second case the difference is not very large given the different approaches taken and they seem to be reasonable if they are thought of as describing a range for the sizes of the eddies that carry the largest part of the kinetic energy.

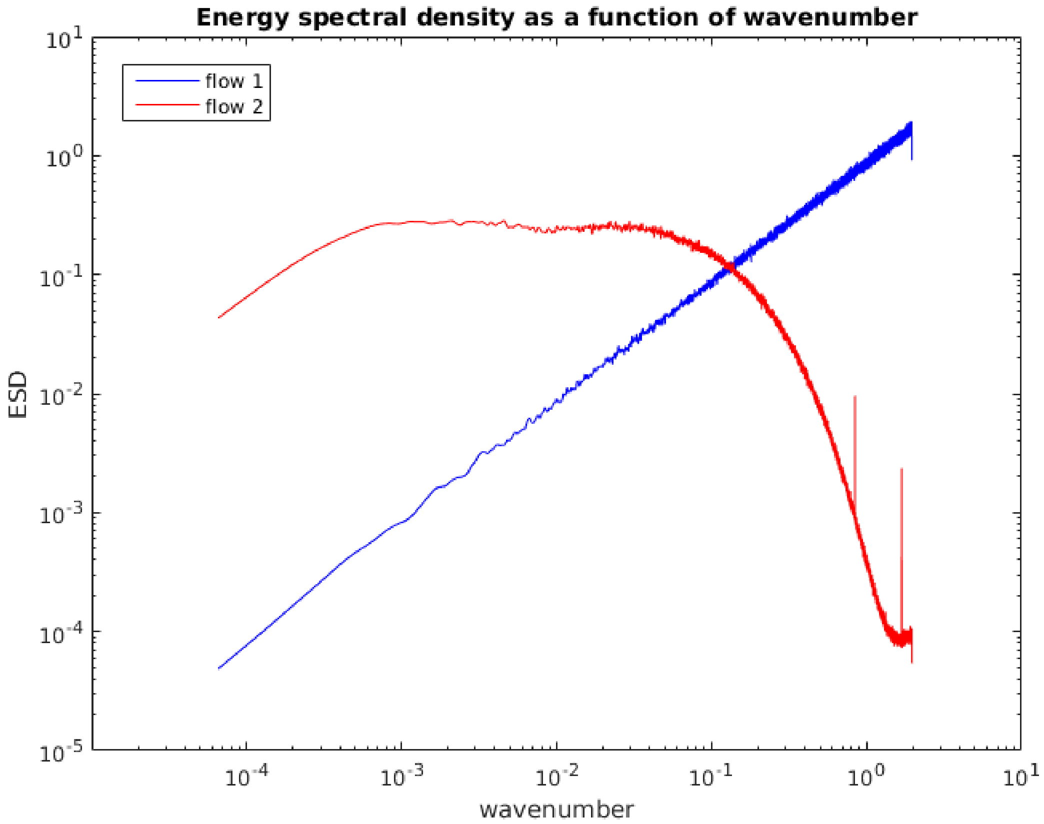

Figure 8: Comparison between the pre-multiplied energy spectral densities in the spatial domain of both flow cases as they change with wave-number for a window size of $20'000$ data points

Question 9

Plot the dissipation spectrum and verify if the dissipation computed from the spectrum matches the dissipation calculated using the gradients. Comment on the match between the two methods for the two datasets.

The dissipations for flow cases 1 and 2 are 4.886×10^{-4} and 1.269×10^{-6} respectively. These values are very different from those seen in section 3 by several orders of magnitude.

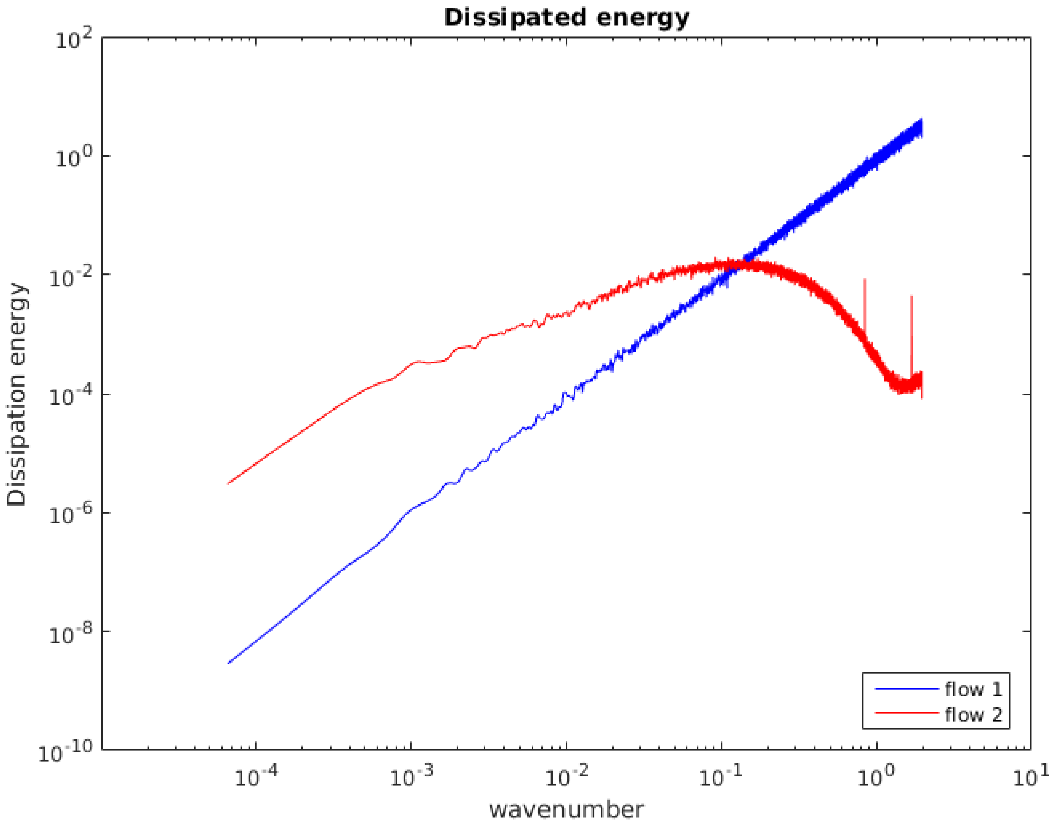

Figure 9: Comparison between the energy spectral densities of both flow cases as they change with frequency for a window size of $20'000$ data points

Question 10

Based on everything you have seen from the results, what can you say about the nature of flows in "flow1" and "flow2".

Q1 shows that statistical data alone is not sufficient to describe the nature of a flow since both have the same values (see 1). *Q2* starts showing the difference between the two flows: the du/dx PDF of the first flow has a very wide with a kurtosis similar to that of a normal distribution. This would suggest that the flow is not fully turbulent as in the second case, where the high amount of mixing forces the derivative PDF to be highly leptokurtic and thus have a higher agglomeration around the mean.

Q3 shows the energy dissipation computed through statistical methods so that it can be compared to that obtained through spectral methods in *Q9*. This was, however, not achieved with the differences being by several orders of magnitude higher in *Q3*. The reason for this is unknown and even after numerous attempts at solving this discrepancy, no solution was found neither in the lecture slides nor in other sources[1][2][3].

Q5 examines the autocorrelation for the two flows and as is expected the first flow displays 0 autocorrelation as it would be expected of a laminar or transitioning flow, where structures do not exist to move with the flow. In *Q6* the scales related to turbulent flow are computed statistically and then compared to spectral methods in *Q8*, where a short discussion is made around the differences.

The conclusion of this exercise is that the statistics of a laminar and turbulent flow can look very similar or they can be identical but when examined around using other physical parameters the distinctions become clear. Spectral methods seem to be a more reliable for differentiating between flows and they make the computation of other physical parameters easier and computationally more efficient through the use of Fourier transforms.

References

[1] H Burchard. Unknown. `http://www.io-warnemuende.de/tl_files/staff/burchard/pdf/Turb_Chap4_WS08.pdf`, 2001. Could not find title and author of book but it is on Prof. Burchard's personal page.

[2] William K George. *Lectures in Turbulence for the 21st Century*. Chalmers University of Technology, 2009.

[3] David C Wilcox et al. *Turbulence modeling for CFD*, volume 2. DCW industries La Canada, CA, 1998.