```
restart;
```

 # Modified Wavenumber of Finite Difference Schemes

```
f := x -> exp(I*k*x);
```

$$x \to e^{I k x} \tag{1}$$

 # second order

```
df2 := (f(dx) - f(-dx))/(2*dx);
Kdx2 := convert(df2, trig)*dx/I;
series(Kdx2, dx = 0, 6);
```

$$\frac{1}{2}\,\frac{e^{I k dx} - e^{-I k dx}}{dx}$$

$$\sin(k\,dx)$$

$$k\,dx - \frac{1}{6}\,k^3\,dx^3 + \frac{1}{120}\,k^5\,dx^5 + O(dx^7) \tag{2}$$

 # fourth order

```
df4 := (f(-2*dx) - 8*f(-dx) + 8*f(dx) - f(2*dx))/(12*dx);
Kdx4 := simplify(convert(df4, trig)*dx/I);
series(Kdx4, dx = 0, 8);
```

$$\frac{1}{12}\,\frac{e^{-2 I k dx} - 8\,e^{-I k dx} + 8\,e^{I k dx} - e^{2 I k dx}}{dx}$$

$$-\frac{1}{6}\sin(2\,k\,dx) + \frac{4}{3}\sin(k\,dx)$$

$$k\,dx - \frac{1}{30}\,k^5\,dx^5 + \frac{1}{252}\,k^7\,dx^7 + O(dx^9) \tag{3}$$

 # sixth-order

```
df6 := (-f(-3*dx) + 9*f(-2*dx) - 45*f(-dx) + 45*f(dx) - 9*f(2*dx) + f(3*dx))/(60*dx);
Kdx6 := simplify(convert(df6, trig)*dx/I);
series(Kdx6, dx = 0, 10);
```

$$\frac{1}{60}\,\frac{-e^{-3 I k dx} + 9\,e^{-2 I k dx} - 45\,e^{-I k dx} + 45\,e^{I k dx} - 9\,e^{2 I k dx} + e^{3 I k dx}}{dx}$$

$$\frac{1}{30}\sin(3\,k\,dx) - \frac{3}{10}\sin(2\,k\,dx) + \frac{3}{2}\sin(k\,dx)$$

$$k\,dx - \frac{1}{140}\,k^7\,dx^7 + \frac{1}{720}\,k^9\,dx^9 + O(dx^{11}) \tag{4}$$

 # Compact Scheme, 6th order

```
dfc := a/(2*dx)*(f(dx) - f(-dx)) + b/(4*dx)*(f(2*dx) - f(-2*dx));
left := f(0) + alpha*(f(dx) + f(-dx));
```

$$dfc6 := simplify\left(\frac{convert\left(\frac{dfc}{left}, trig \right) \cdot dx}{I} \right);$$

$$Kdxc6 := simplify\left(subs\left(\left\{ \text{alpha} = \frac{1}{3}, a = \frac{14}{9}, b = \frac{1}{9} \right\}, dfc6 \right) \right);$$

$$series(Kdxc6, dx = 0, 8);$$

$$\frac{1}{2}\ \frac{a\left(e^{I k dx} - e^{-I k dx}\right)}{dx} + \frac{1}{4}\ \frac{b\left(e^{2 I k dx} - e^{-2 I k dx}\right)}{dx}$$

$$1 + \alpha\left(e^{I k dx} + e^{-I k dx}\right)$$

$$\frac{1}{2}\ \frac{2\,a\sin(k\,dx) + b\sin(2\,k\,dx)}{1 + 2\,\alpha\cos(k\,dx)}$$

$$\frac{1}{6}\ \frac{28\sin(k\,dx) + \sin(2\,k\,dx)}{3 + 2\cos(k\,dx)}$$

$$k\,dx - \frac{1}{2100}\ k^7\,dx^7 + O\left(dx^9\right) \qquad \textbf{(5)}$$

 # Plot Modified Wavenumber

$$plot(subs(dx = 1, [k \cdot dx, Kdx2, Kdx4, Kdx6, Kdxc6]), k = 0..Pi);$$