Coursework Project I

Konstantinos Drakopoulos

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Question 1

Investigate the stability and accuracy of the four schemes in terms of the Courant-Friedrichs-Lewy (CFL) numbers $\lambda = c\frac{dt}{dx}$ *and* $\mu = \nu\frac{dt}{dx^2}$*. In particular use numerical experimentation and/or von Neumann stability analysis to determine stability boundaries.*

Listing 1: Code used to find the maximum time step and the resulting CFL parameters

```
flg = 1;
dt = 2e-3;
step = 1e-3;
counter = 1;
while flg
    dt = dt + step;
    flg = wave(dt);
    if ~flg && counter <= 4
        dt = dt - step;
        step = step/(10*counter);
        flg = 1;
        counter = counter + 1;
    end
end
```

Table 1: CFL coefficients for a stable boundary

	Convection λ	Diffusion μ	dt
Euler - Central Diff 2	$2.7530e-03$	$4.0000e-06$	$2.0161e-05$
Euler - Central Diff 4	$2.7310e-03$	$4.0000e-06$	$2.0002e-05$
RK2 - Central Diff 2	$6.010206e-01$		
RK2 - Central Diff 4			

Question 2

Use the exact solution as an initial condition for your simulations and compare the numerical results against the exact ones at a later time. Consider whether the integral 'constants' $A = \int_{-\infty}^{\infty} f dx$ *and*

$E = \int_{-\infty}^{\infty} \frac{1}{2} f^2 dx$ *are conserved (or not) as appropriate.*

Question 3

Investigate the solutions when the viscosity is very small (in effect, zero) so that we reduce to the one-dimensional wave equation. Look the amplitude (growth/decay, stability) and dispersion (phase speed of wave components. One way to retain stability is to add an optimal small amount of artificial viscosity (e.g. the Lax-Wendroff method, Euler+CDiff2, using $\nu = 0.5c^2 dt$*).*

Question 4

Investigate the solutions when convection is negligible ($c = 0$*). Look at the accuracy and stability limit in terms of* $\mu = \nu \frac{dt}{dx^2}$ *for the different schemes.*

Question 5

Finally, discuss your results in terms of which schemes are best suited to be used for simulations of turbulent flows.