Coursework Project I

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Question 1

Plot the Probability Density Function (PDF) of the streamwise velocity from both datasets and calculate its first four moments. Using the moment information, comment on type of distribution exhibited by the two sets of data.

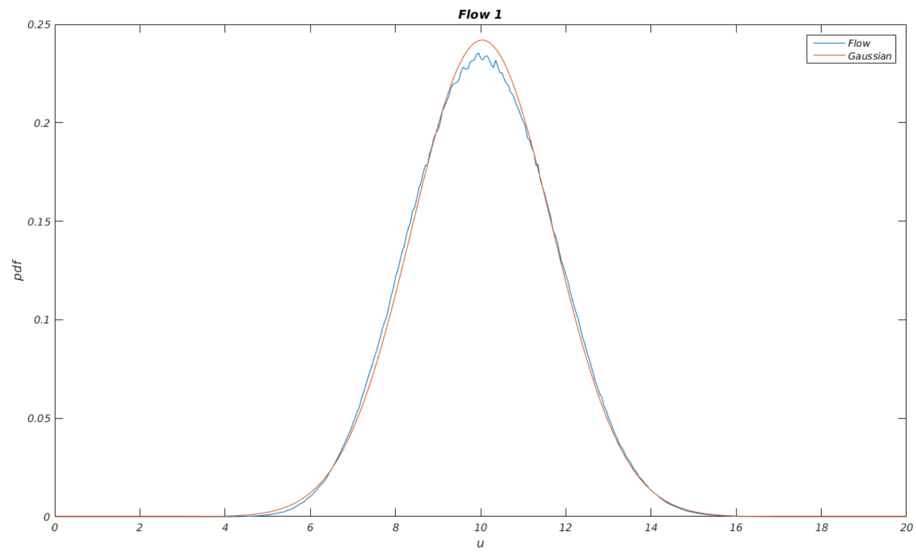

Figure 1: The Probability Density function for *flow 1*

Figures 1 and 2 show the probability density functions for the two flows with bin sizes of $0.05 m/s$. From these figures it can be seen that they respect a *Gaussian* or *Normal* distribution. Table 1 shows that the statistical data for the two cases is not different between the two cases.

Table 1: Comparison of the first four moments for the two flows.

	Flow 1	Flow 2
1^{st} Moment: Mean	10.04	10.04
2^{nd} Moment:Variance	2.72	2.72
3^{rd} Moment:Skewness	0.05	0.05
4^{th} Moment:Kurtosis	2.78	2.78

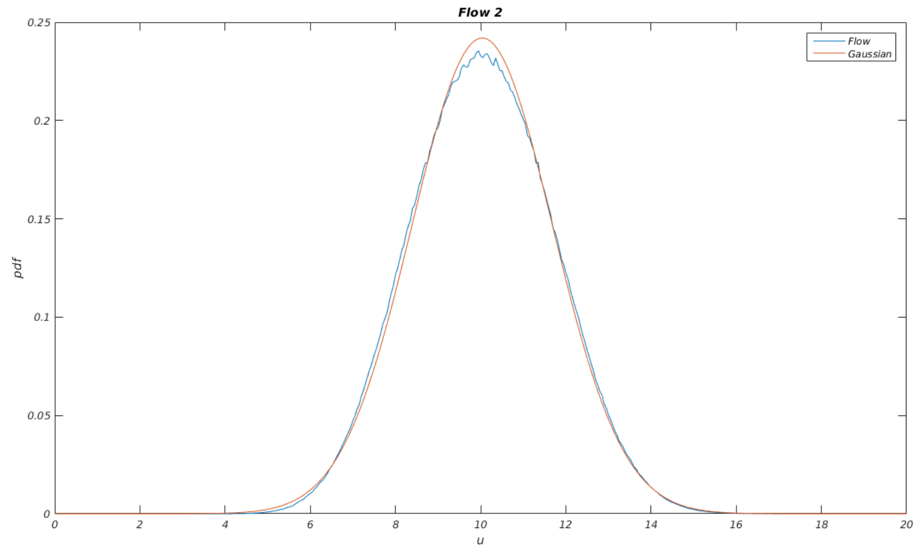

Figure 2: The Probability Density function for *flow 2*

Question 2

Plot the Probability Density Function (PDF) of the streamwise velocity gradient (i.e. $^{\partial u_1}/_{\partial x_1}$) from both datasets and calculate its first four moments. Using the moment information, comment on type of distribution exhibited by gradients in the two sets of data. Use Taylor's hypothesis to convert temporal gradient to spatial gradient.

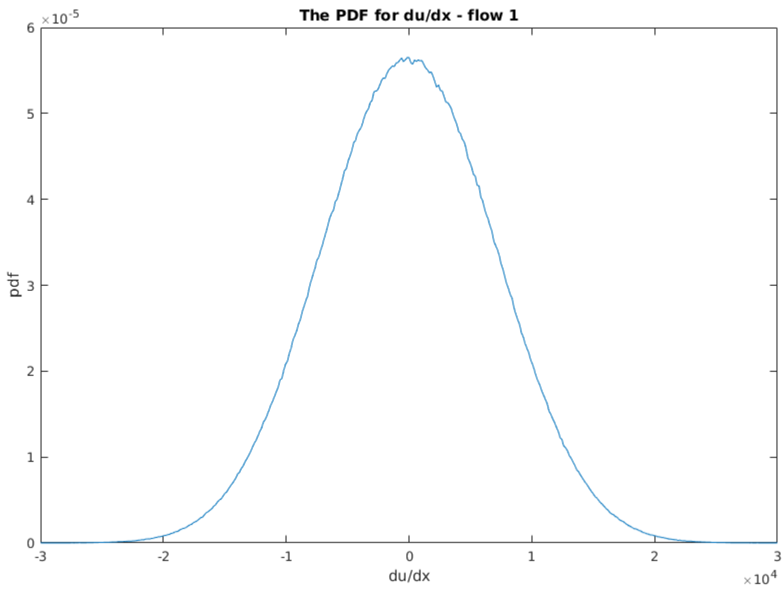

Figure 3: The normalised Probability Density function of $^{\partial u_1}/_{\partial x_1}$ for *flow 1*

Figure 3 shows the probability distribution of $\partial u_1/\partial x_1$ for *flow 1*. The curve shows that the velocity derivative to be more evenly distributed for this flow than the second flow case (fig. 4).

Kurtosis is a measure of how 'spread out' a distribution is and it is often compared to the number 3, given by the *Gaussian* distribution. A kurtosis lower than this is said to be *platykurtic* and a higher value is said to be *leptokurtic*. *Skewness* shows if the data has more outliers towards the right (positive) or left end of the tails.

Table 2 shows that the high variance and low kurtosis suggest a more 'spread out' distribution for *flow 1* when compared to the second. The second flow case is highly leptokurtic and the first is platykurtic

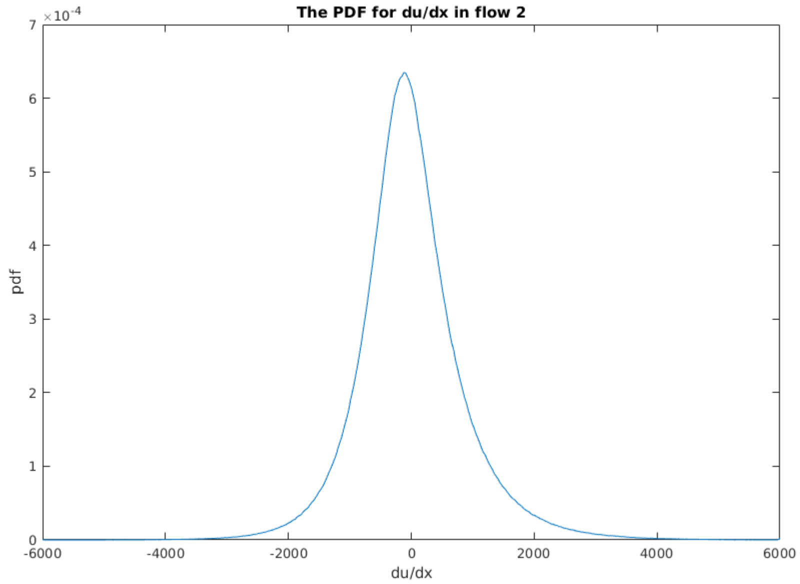

Figure 4: The normalised Probability Density function of ${}^{\partial u_1}/_{\partial x_1}$ for *flow 2*

but with a kurtosis close to that of a gaussian. The second case has more positive outliers due to the skewness of 0.55 whereas in the first flow case the datapoints are well centered ($S \approx 0$).

Table 2: Comparison of the first four moments for the ${}^{\partial u_1}/_{\partial x_1}$ of the two flows.

	Flow 1	Flow 2
1$^{\text{st}}$ Moment: Mean	-0.002	-0.225
2$^{\text{nd}}$ Moment:Variance	4.86×10^7	7.4×10^5
3$^{\text{rd}}$ Moment:Skewness	-1.22×10^{-4}	0.55
4$^{\text{th}}$ Moment:Kurtosis	2.89	5.92

Question 3

Using the homogenous isotropic assumptions and Taylor's hypothesis, calculate the dissipation rate from the dataset. Assume kinematic viscosity, $\nu = 1.5 \times 10^{-5} m^2/s$.

Dissipation is given by:

$$\epsilon = 15\nu \left\langle \left[\frac{\delta u}{\delta x} \right]^2 \right\rangle \tag{1}$$

Which gives $\epsilon_{flow1} = 1.09 \times 10^4 \; {}^{J}/_{kg \cdot s}$ and $\epsilon_{flow2} = 166.4 {}^{J}/_{kg \cdot s}$. This shows that a larger amount of energy is transformed from turbulent flow energy to heat in the first flow case which means that higer amounts of energy are lost in the first flow case.

Question 4

Using the dissipation and kinematic viscosity, calculate the Kolmogorov scales of both flows.

Table 3: Kolmogorov microscales for flows 1 and 2

Flow	η
1	6.758×10^{-7}
2	2.727×10^{-6}

Question 5

Calculate the autocorrelation function for both datasets. Compare the two curves and comment on the results.

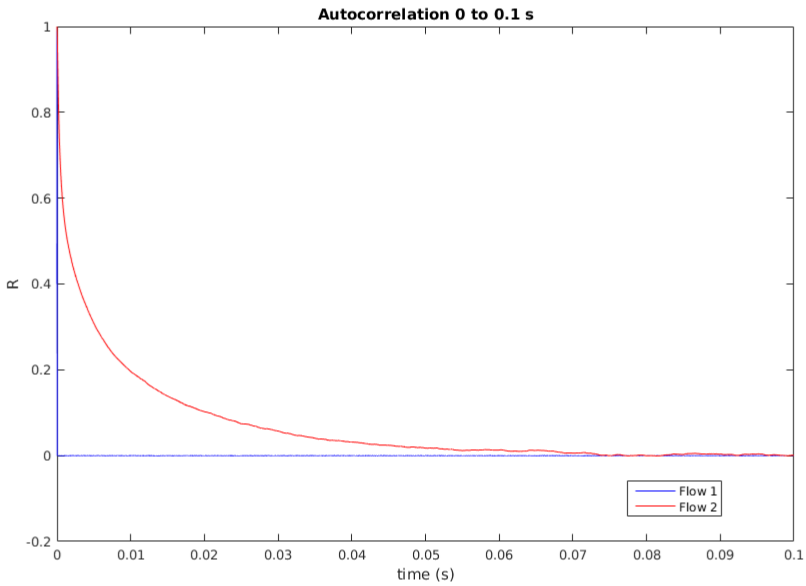

Figure 5: The comparison between the autocorrelation behaviours of the two flows from the origin to 0.1 s.

Figure 5 shows how sharply autocorrelation drops for the first flow case and in figure 6 it can be seen how it oscilates around 0, which is most probably the result of some signal or floating point operation error than an actual description of a phyisical property. Autocorrelation is also small for the second flow case but more pronounced at the start. Figure 6 shows how there is a small amount of autocorrelation occuring at the start.

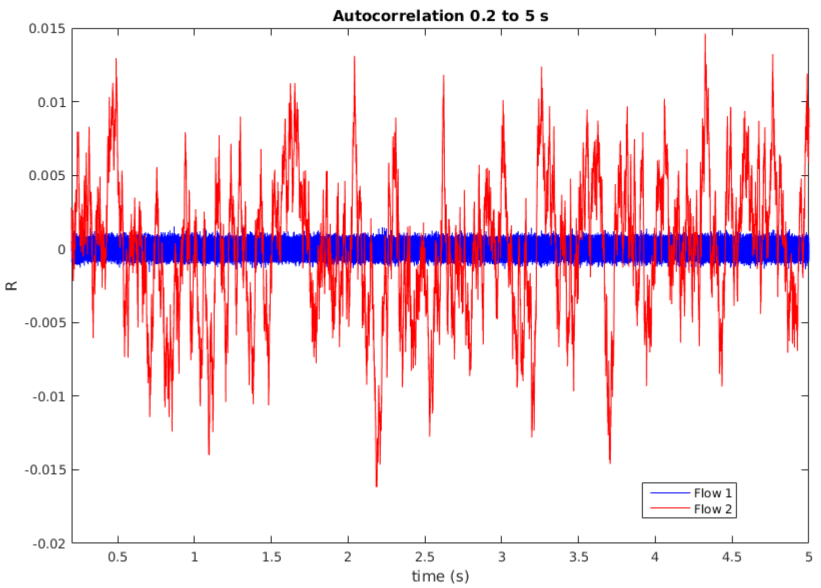

Figure 6: The comparison between the autocorrelation behaviours of the two flows between 0.2 and 5 s.

Question 6

Using the autocorrelation function, calculate the Integral length scale and the Taylor micro scale for both datasets. Comment on the results obtained.

The values in table 4 are obtained using eq. (2) and (3), where ρ is the autocorrelation coefficient and τ the time. To obtain eq. (3) a second degree central difference scheme with a fourth order of accuracy was used at $\tau = 0$. The chosen window size is of $1s$ and the integral scale is given in seconds (i.e. the acquisition signal mutiplied by eq. (2)).

$$T_{int} = \int_0^\infty \rho(\tau) d\tau \tag{2}$$

$$\lambda = \sqrt{-\frac{2}{\rho''(\tau)}} \tag{3}$$

Table 4: Integral scale(T_{int}) and Taylor microscale(λ) for flows 1 and 2

Flow	T_{int}	λ
1	$7.46 \times 10^{-6}s$	1.49×10^{-5}
2	$6.62 \times 10^{-3}s$	1.51×10^{-4}

Table 4 shows that in the second flowcase the velocity is correlated with itself by three orders of magnitude longer than in the first. The Taylor microscale is a description of how often a signal passes 0.

Question 7

Using the information that the window length necessary to calculate the energy spectrum should be at least 50 integral time scales, calculate the energy spectra for both datasets.

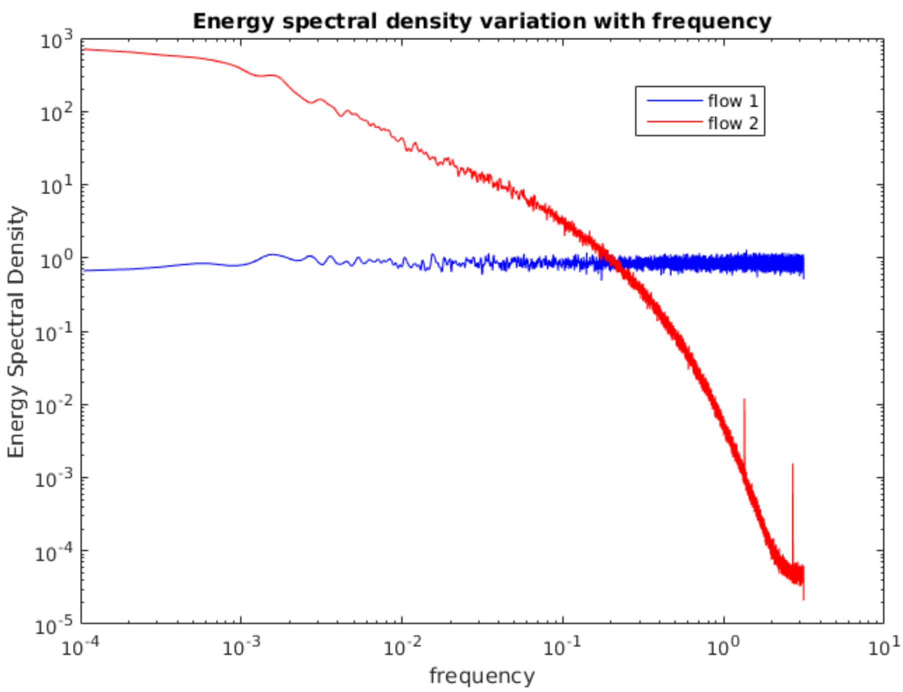

Figure 7: Comparison between the energy spectral densities of both flow cases as they change with frequency for a window size of $20'000$ data points

Figure 9 shows how the spectral energy varies with frequency. Flow 2 exhibits high energy for low frequencies of the order of 10^3, whereas flow 1 has a constant energy of order 1 for all frequencies.

Question 8

Plot the pre-multiplied spectrum for both datasets and calculate the dominant time-scale (and hence length scale using Taylor's hypothesis) using this data by locating the peak in this pre-mulitplied spectrum.

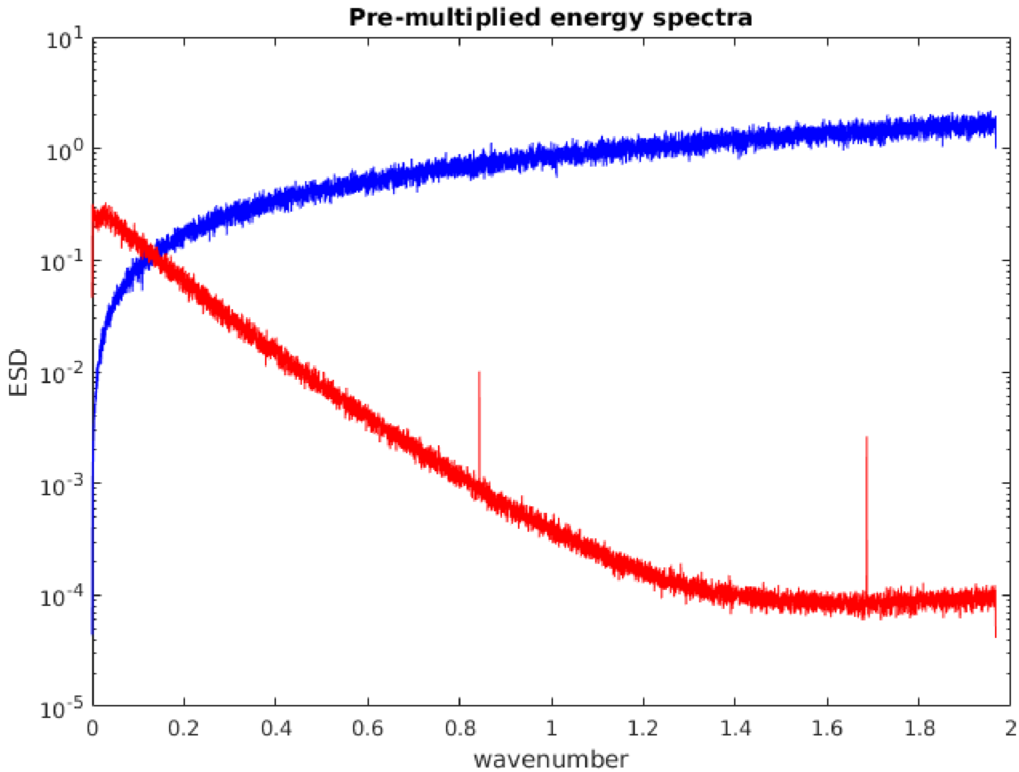

Figure 8: Comparison between the energy spectral densities in the spatial domain of both flow cases as they change with wavenumber for a window size of $20'000$ data points

Question 9

Plot the dissipation spectrum and verify if the dissipation computed from the spectrum matches the dissipation calculated using the gradients. Comment on the match between the two methods for the two datasets.

Question 10

Based on everything you have seen from the results, what can you say about the nature of flows in "flow1" and "flow2".

This can be interpreted as large range of eddy sizes passing by the hot wire probe which would suggest that it is placed near a body in a high *Re* flow.

In contrast, figure 4 shows that most values are near 0 which suggests that the flow has mostly eddies of a similar size.

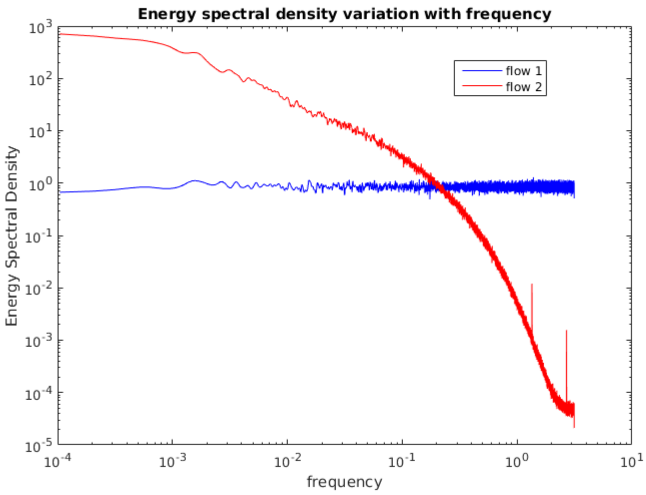

Figure 9: Comparison between the energy spectral densities of both flow cases as they change with frequency for a window size of $20'000$ data points