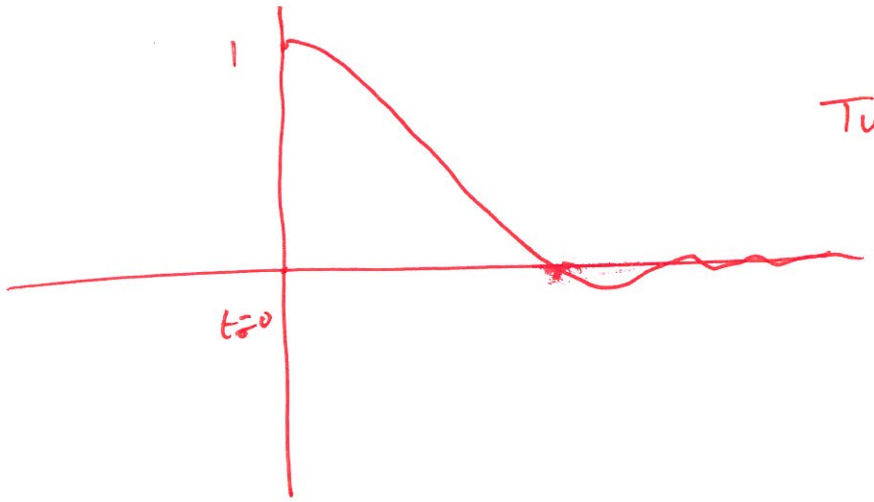
Q1)

i) (a)



Turbulent flow

2.5

(b)

Random noise signal

2.5

ii)

(a)

$$Re_{\tau_{av}} = 1000 \Rightarrow y^+ = 70 \Rightarrow$$

$$= 10{,}000 \Rightarrow y^+ = 200$$

$$= 100{,}000 \Rightarrow y^+ = 1000.$$

2.5

(b) Turbulence production $= \langle -uv \rangle^+ \dfrac{dU^+}{dy^+}$.

To get $\dfrac{dU^+}{dy^+}$, we need to know U^+.

$$U^+ = \frac{1}{\kappa}\ln y^+ + c \quad \text{for } y^+ > 50 \text{ and } y/\delta < 0.2$$

$$= -\frac{1}{\kappa}\ln(y/\delta) + B \quad \text{for } y/\delta > 0.2$$

We need to know if $y/\delta < 0.2$ or > 0.2 for the three peak locations.

For $Re_\tau = 1000$: $y^+ = 70$

$$\Rightarrow \frac{y}{\delta} = \frac{70}{1000} = 0.07$$

$Re = 10{,}000$: $y^+ = 200$

$$\Rightarrow y/\delta = \frac{200}{10000} = 0.02$$

$Re = 100{,}000$: $y^+ = 1000$

$$\Rightarrow y/\delta = \frac{1000}{100{,}000} = 0.01$$

(2.5)

For all 3 locations, $\frac{dU^+}{dy^+} = \frac{1}{\kappa y^+}$ (2.5)

$Re_\tau = 1000 \Rightarrow P = \frac{0.9}{\kappa y^+} = 0.032$

$= 10{,}000 = P = \frac{1}{\kappa y^+} = 0.0125$

$= 100{,}000 = P = \frac{1}{\kappa y^+} = 0.0025$

(2.5)

Q2. The evolution equation of K.E is,

$$\frac{dk}{dt} = P + T - \epsilon .$$

In a homogeneous isotropic decaying flow

$$T = 0 ; \quad P = 0$$

$$\therefore \quad \frac{dk}{dt} = -\epsilon .$$

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(i) The dissipation rate $\epsilon = A$

$$\frac{dk}{dt} = -A$$

Integrating,

$$k - k_0 = -A(t - t_0)$$

5

At $t = 0 ; \; k = k_0$

$$\Rightarrow k = k_0 - At .$$

(ii)

$$\frac{L}{\lambda} = \frac{k^{3/2}/\epsilon}{\sqrt{\frac{2k}{B\epsilon}15\nu}}$$

$$\frac{L}{\lambda} = \frac{k^{3/2}/\epsilon}{\sqrt{10\nu\, k/\epsilon}} \Rightarrow \frac{k^{3/2}}{\epsilon}\cdot\frac{\sqrt{\epsilon}}{\sqrt{k}}\,\frac{1}{\sqrt{10\nu}}$$

$$= \frac{k}{\sqrt{10\nu\epsilon}} \rightarrow 5$$

$$\frac{L}{\lambda} = \frac{k}{\sqrt{10\nu\epsilon}}$$

plugging in $k = k_0 - At$.

$$\frac{L}{\lambda} = \frac{k_0 - At}{\sqrt{10\nu A}} \quad \left[\text{Since } \epsilon = A\right]$$

$$= \sqrt{\frac{A}{10\nu}}\left(\frac{k_0}{A} - t\right) \rightarrow 2$$

(iii) Since the dissipation rate is a constant, the Kolmogorov scales of motion are all constants for all time.

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Q3 solution — 2014/2015

Methods ordered in terms of increasing cost / accuracy

1) Correlations (1) :
- + simple, fast (no extra transport eqn) (1/2)
- − only work for known cases / new flows problematic (1/2)

2) Integral methods (1) :
- + can be used in spatially varying flows, no extra eqns (1/2)
- − problematic for separating flows / complex fl (1/2)

3) One-point RANS (1) :
- + allows full field representation of turb (1/2)
- − depends on number of ad-hoc ~~eqns~~ assumptions (1/2)
- → reduced cost: averages over all scales, can be run in 2D, steady (time averaging) (1)

4) hybrid RANS/LES (1) :
- + combines advantages of RANS/LES (in theory) (1/2)
- − problems with APG in BL ~~part~~ separation due to RANS part (1/2)
- → 3D, unsteady simulation (more expensive than RANS) (1) but cheaper than LES due to RANS at wall

5) LES (1) :
- + geometry dependent large eddies resolved (1/2)
- − model uncertainty, errors near walls, high cost for wall-bounded flows (1/2)
- → 3D, unsteady (cheaper than DNS), by resolving energetic (anisotropic) structures more accurate than RANS in particular for separated flows, use of filtering (1)

6) DNS (1) :
- + Results 'exact' over all scales (1/2)
- − very expensive → limited use (1/2)
- no averaging / filtering, 3D unsteady (1)

Validation :
- − grid, domain size dependence checks
- − compare with reference, if possible
- − RANS: check first y^+
- − LES/DNS: check correlations, spectra

total of 2

total of 15

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Q4) i) Compact difference scheme with

$$\alpha\left(f'_{j+1}+f'_{j-1}\right)+f'_j=\frac{1}{\Delta x}\left[\frac{a}{2}\left(f_{j+1}-f_{j-1}\right)+\frac{b}{4}\left(f_{j+2}-f_{j-1}\right)\right]$$

use $f(x)=\sum_k \hat{f}(k)\,e^{ikx}$ and consider only a single mode

exact derivative: $f'=\frac{df}{dx}=ik\hat{f}e^{ikx}$ (1)

$f'_{j+1}=ik\hat{f}e^{ik(x+\Delta x)}$ (1) $\quad f'_{j-1}=ik\hat{f}e^{ik(x-\Delta x)}$ (1)

$f_{j+1}=\hat{f}e^{ik(x+\Delta x)}$ $\quad f_{j-1}=\hat{f}e^{ik(x-\Delta x)}$ (1) $\quad f_{j+2}=\hat{f}e^{ik(x+2\Delta x)}$ (1)

$$\Rightarrow \alpha ik\left(\hat{f}e^{ik(x+\Delta x)}+\hat{f}e^{ik(x-\Delta x)}\right)+ik\hat{f}e^{ikx}=\frac{1}{\Delta x}\left[\frac{a}{2}\left(\hat{f}e^{ik(x+\Delta x)}-\hat{f}e^{ik(x-\Delta x)}\right)+\frac{b}{4}\left(\hat{f}e^{ik(x+2\Delta x)}-\hat{f}e^{ik(x-2\Delta x)}\right)\right]$$ (1)

divide by $\hat{f}e^{ikx}$ (1)

$$\Rightarrow ik\Big[\alpha\underbrace{\left(e^{ik\Delta x}+e^{-ik\Delta x}\right)}_{2\cos(k\Delta x)\ (1)}+1\Big]=\frac{1}{\Delta x}\Big[\frac{a}{2}\overbrace{\left(e^{ik\Delta x}-e^{-ik\Delta x}\right)}^{2i\sin(k\Delta x)\ (1)}+\frac{b}{4}\overbrace{\left(e^{2ik\Delta x}-e^{-2ik\Delta x}\right)}^{2i\sin(2k\Delta x)\ (1)}\Big]$$

$$\cancel{i}k\Delta x=\frac{\cancel{i}\left(a\sin(k\Delta x)+\frac{b}{2}\sin(2k\Delta x)\right)}{1+2\cos(k\Delta x)}$$ (1)

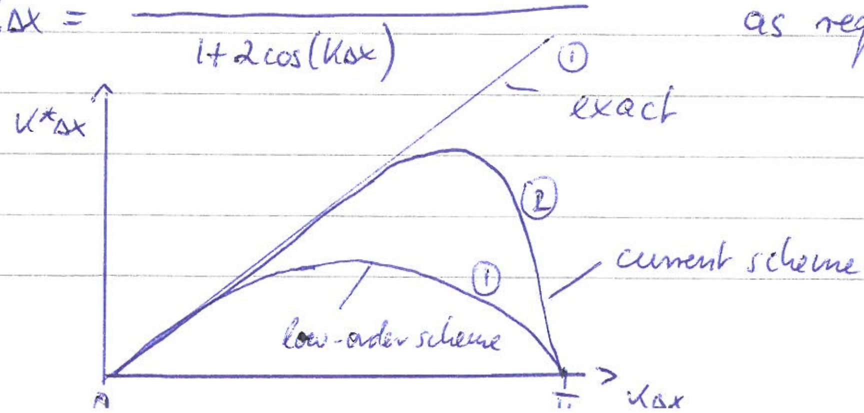
$$\Rightarrow k^*\Delta x=\frac{a\sin(k\Delta x)+\frac{b}{2}\sin(2k\Delta x)}{1+2\cos(k\Delta x)}$$ (1) as required

Sketch:

using $\alpha=1/2$

$a=14/9$

$b=1/9$



15

Q4

ii) 1) Damping of a RANS model (1), e.g FSM, where

$$\tau_{ij}^{hyb} = F \cdot \tau_{ij}^{RANS} \quad (1)$$

2) Blending of RANS / LES (1), e.g. $\nu_T^{hyb} = f \cdot \nu_T^{RANS} + (1-f)\, \nu_T^{LES}$ (1)

3) Changing length scale in transport eqn (DES) (1)

SA model: $A\left(\frac{\nu}{d}\right)^2$ replaced by $A\left(\frac{\nu}{\tilde{d}}\right)^2$ (1)

with $\tilde{d} = \min(d, C_{DES}\Delta)$

total of $\boxed{5}$