SEMESTER 1 EXAMINATION 2014-2015

Turbulence: Physics & Modelling

Duration: 120 mins.

Attempt to answer **all four** questions.
Questions 1 and 3 carry 15 points each while questions 2 and 4 carry 20 points each out of a total 70.

Numerical questions:
Marks shown are for guidance only.
Marks will not be awarded unless full working is shown.

Only University approved calculators may be used.
A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

- The turbulent kinetic energy equation for an incompressible flow in the absence of body forces can be written as,

$$\frac{dk}{dt} = P + T - \epsilon$$

where,

$$k = \tfrac{1}{2}\langle u_i u_i \rangle = \tfrac{1}{2}\langle q^2 \rangle$$
$$P = -\langle u_i u_j \rangle \tfrac{\partial U_i}{\partial x_j}$$
$$\epsilon = 2\nu \langle s_{ij} s_{ij} \rangle$$
$$T = \tfrac{\partial}{\partial x_j}\left[-\tfrac{1}{\rho}\langle p u_i \rangle \delta_{ij} - \tfrac{1}{2}\langle q^2 u_j \rangle + 2\nu \langle s_{ij} u_i \rangle \right]$$

- The mean velocity profile in a turbulent boundary layer can be written as,

$$\begin{aligned} U^+ &= \frac{1}{\kappa} ln\, y^+ + C, \quad \text{for } y^+ \geq 50 \text{ and } y/\delta \leq 0.2 \\ &= -\frac{1}{\kappa} ln\, \frac{y}{\delta} + B, \quad \text{for } y/\delta > 0.2 \end{aligned}$$

where, κ = 0.4, C = 5.0 and B is a constant that varies from flow to flow.

Q.1

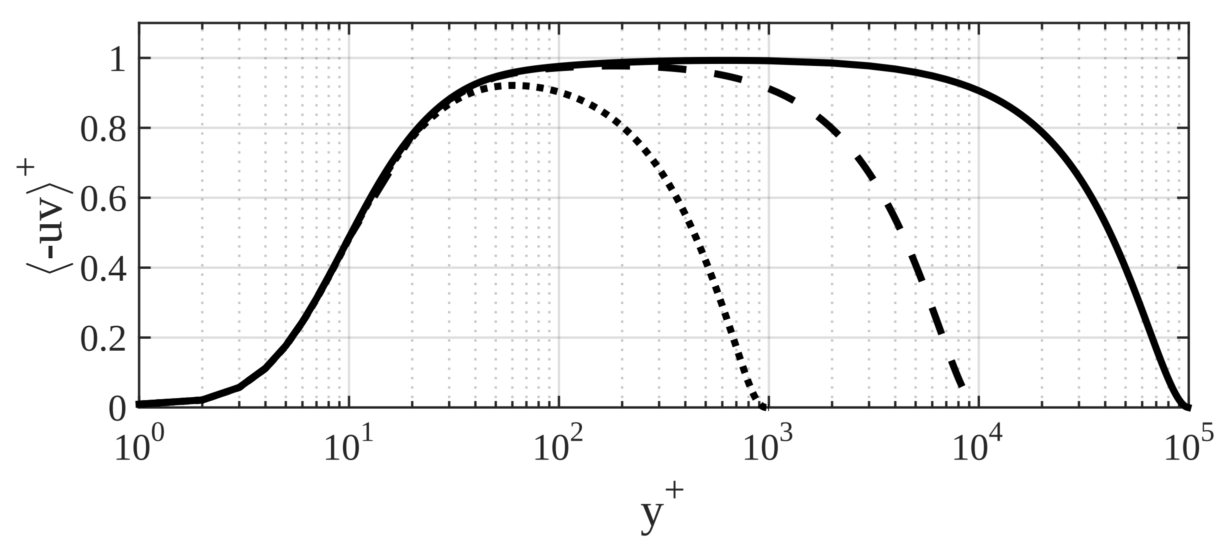
i) Sketch the auto-correlation function of the streamwise velocity for a turbulent flow. Also, sketch the auto-correlation function of a random noise signal.

[5 marks]

ii) Figure 1 shows the inner-scaled (i.e. scaled with skin-friction velocity and viscosity) Reynolds shear stress profile of a turbulent boundary layer for three different Reynolds numbers. The solid line is for Re_τ = 100000, the dashed line is for Re_τ = 10000 and the dotted line is for Re_τ = 1000.

a) Find the location of the peak Reynolds shear stress in inner-units (or wall-units).

b) Calculate the inner-scaled turbulence production at the location of the peak value of the Reynolds shear stress for all three Reynolds numbers.



[10 marks]

Q.2
Consider a decaying homogeneous isotropic turbulent flow with kinetic energy, $k = \frac{3}{2}\sigma_u^2$ (where σ_u^2 is the variance of the streamwise velocity) and dissipation, $\epsilon = 2\nu\langle s_{ij}s_{ij}\rangle = 15\nu\langle(\partial u/\partial x)^2\rangle$ (where ν is the kinematic viscosity of the fluid). The integral length scale (L) and the Taylor micro scale (λ) for this flow are defined as

$$L = \frac{k^{3/2}}{\epsilon} \quad \text{and} \quad \lambda = \sqrt{\frac{\sigma_u^2}{\langle(\partial u/\partial x)^2\rangle}} \; .$$

Experiments carried out in this decaying turbulent flow show that the dissipation rate remains a constant (= A) for all times.

i) Show that the kinetic energy at some time $t > 0$ can be written as

$$k = k_0 - A\,t,$$

where $k = k_0 \text{ at } t = 0$.

[8 marks]

ii) Show that the ratio of the integral length scale to the Taylor microscale decreases linearly with time, i.e.,

$$\frac{L}{\lambda} = \sqrt{\frac{A}{10\nu}}\left(\frac{k_0}{A} - t\right) \; .$$

[7 marks]

iii) How do the Kolmogorov scales of motion (length scale, time scale and the velocity scale) change with time?

[5 marks]

Q.3
Complete a list of the general computational strategies available to engineers for obtaining information about turbulent flows, ordered according to cost and complexity. Your answer should also include:

- Comment on the advantages and disadvantages of each strategy, with regard to cost, accuracy, computer resources and efficiency. Also state the reasons for certain methods being more costly than others and why they might be more accurate.
- A description of the role and meaning of *averaging* or *filtering* as used by each strategy.
- The steps that can be taken to validate the accuracy of solutions obtained using a turbulence closure.

[15 marks]

Q.4

i) Let the first derivative of a function $f' = df/dx$ be discretized by a sixth-order compact central-difference approximation

$$\alpha\left(f'_{j+1} + f'_{j-1}\right) + f'_j$$
$$= \frac{1}{\Delta x}\left[\frac{a}{2}\left(f_{j+1} - f_{j-1}\right) + \frac{b}{4}\left(f_{j+2} - f_{j-2}\right)\right]$$

(where f and x are respectively the dependent and independent variables, Δx is the constant grid spacing and j is the grid-point index, with $x_j = j\Delta x$). Show that the modified (effective) wavenumber $k^*\Delta x$ of this spatial discretization scheme is

$$k^*\Delta x = \frac{a\sin(k\Delta x) + b/2\sin(2k\Delta x)}{1 + 2\alpha\cos(k\Delta x)}$$

by applying the approximation to the single complex Fourier mode, $\widehat{f}(k)\exp(\mathrm{i}kx)$, at wavenumber k.
Using the coefficients $\alpha = 1/2$, $a = 14/9$ and $b = 1/9$, sketch the variation of the real part of $k^*\Delta x$ versus $k\Delta x$ for $0 \le k\Delta x \le \pi$. Include the exact solution and a lower-order method in your sketch.

[15 marks]

State three conceptually different unified hybrid RANS/LES methods and briefly explain, using the relevant equations, how two of those approaches employ RANS and LES within the same overall framework.

[5 marks]

END OF PAPER