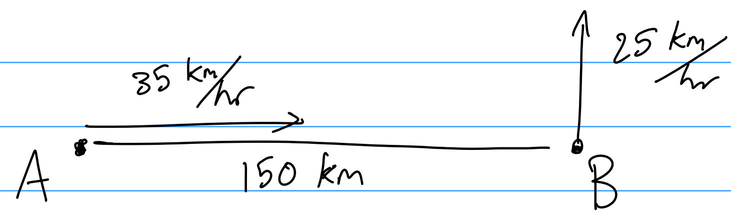
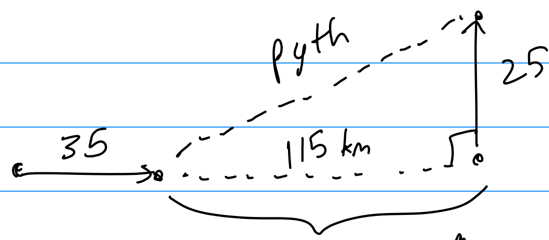
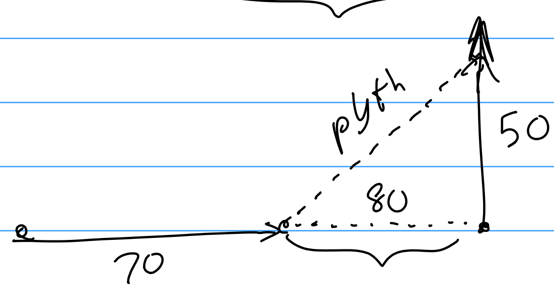
Pg 186 #16



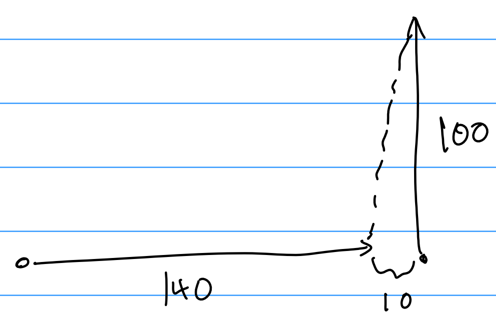
1 hr later

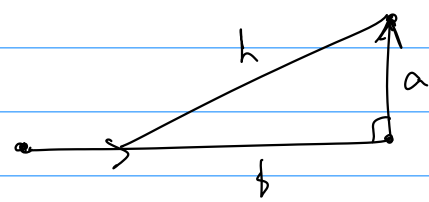


2 hr later



4 hr





$$a^2 + b^2 = h^2$$

a, b, and h are functions of time.

$a = a(t)$
$b = b(t)$
$h = h(t)$

$[a(t)]^2$

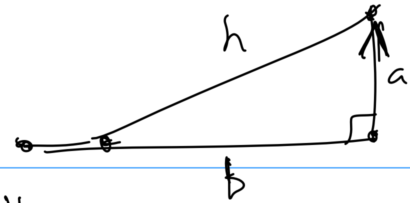
$2[a] \cdot \frac{da}{dt}$

Chain rule

$$\frac{d}{dt}\left(a^2 + b^2\right) = \frac{d}{dt}\left(h^2\right)$$

$$= 2a\frac{da}{dt} + 2b\frac{db}{dt} = 2h\frac{dh}{dt}$$

MOST GENERAL: $a\frac{da}{dt} + b\frac{db}{dt} = h\frac{dh}{dt}$



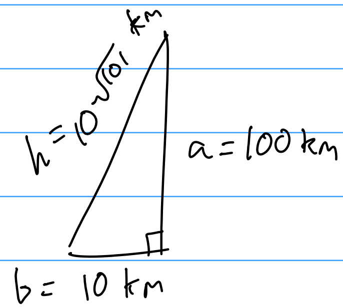
$25\ \text{km/hr}$

General, dynamic, but more specific to this problem $a \cdot \left(25\frac{km}{hr}\right) + b\left(-35\frac{km}{hr}\right) = h\frac{dh}{dt}$

a is increasing   b is decreasing

Even more specific, at one given time, static

4 PM



$100^2 + 10^2 = h^2$

$10100 = h^2$

$h = \sqrt{10100}$

$h = 10\sqrt{101}$

$$(100\ km)\left(25\frac{km}{hr}\right) - (10\ km)\left(35\frac{km}{hr}\right) = \left(10\sqrt{101}\ km\right)\frac{dh}{dt}$$

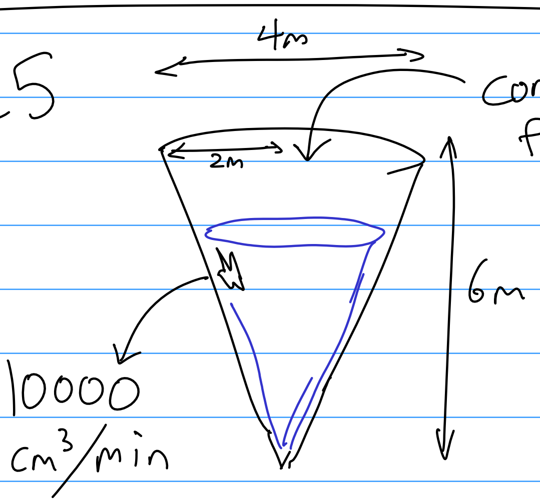
$$(2500 - 350)\frac{km^2}{hr} = \left(10\sqrt{101}\ km\right)\frac{dh}{dt}$$

$$\frac{2150\ \frac{km^2}{hr}}{10\sqrt{101}\ km} = \frac{dh}{dt}$$

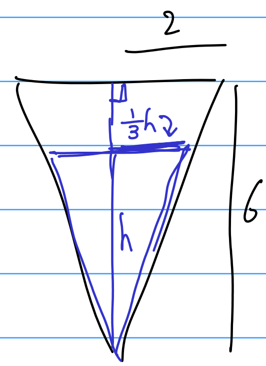
$$\frac{dh}{dt} = \frac{215}{\sqrt{101}}\ \frac{km}{hr}$$

$$\approx 21.3933\ \frac{km}{hr}$$

#25

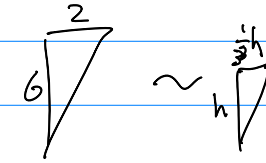


$V = \frac{1}{3}\pi r^2 h$



$r = \frac{1}{3}h$

similar triangles



h is a good variable in this problem.

$V = \frac{1}{3}\pi r^2 h$ and $r = \frac{1}{3}h$

$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$

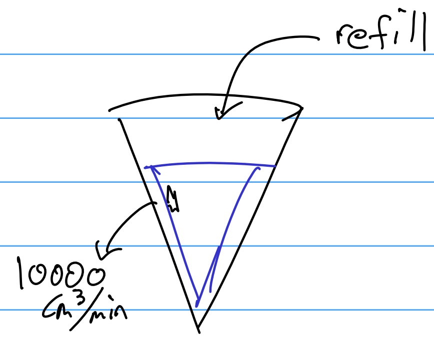
$V = \frac{\pi h^3}{27}$

$V = V(t)$

$h = h(t)$

$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{\pi h^3}{27}\right)$

$\underline{\frac{dV}{dt}} = \frac{\pi}{27}\frac{d}{dt}(h^3) = \underline{\frac{\pi}{27}3h^2\frac{dh}{dt}}$



$\frac{dV}{dt} = \frac{\pi}{9}h^2\frac{dh}{dt}$

Water level rising $20 \ cm/min$

Units!

when $h = 2 \ m = 200 \ cm$

$\frac{dV}{dt} = \frac{\pi}{9}(200 \ cm)^2\left(20 \ cm/min\right)$

$= \frac{\pi}{9} 40000 \ cm^2 \ 20 \ cm/min$

$\frac{dV}{dt} = \frac{800000\pi}{9} \ cm^3/min$

OVERALL rate of change of volume

(Leak)

$\frac{dV}{dt} = \text{Refill} \ cm^3/min - 10000 \ cm^3/min$

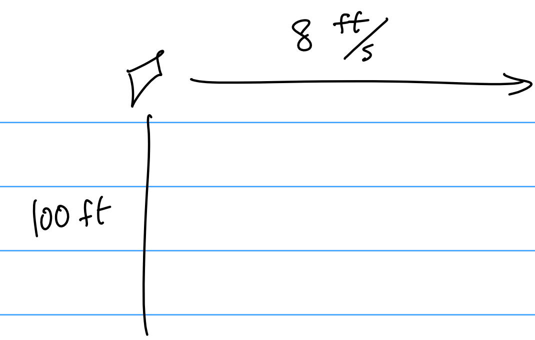
$\underline{\left(10000 + \frac{800000\pi}{9}\right) cm^3/min} = \text{Refill} = \boxed{289252 \ cm^3/min}$

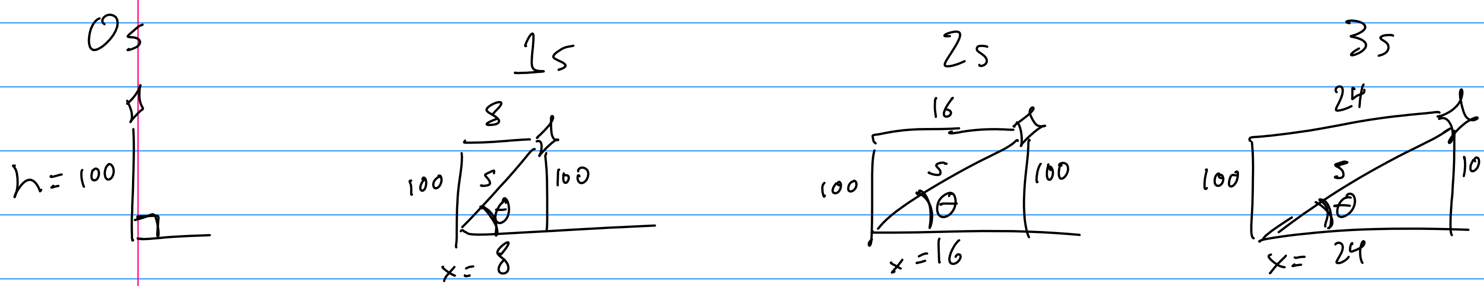
m^3/min ?

$\left(\frac{1 \ m}{100 \ cm}\right)^3 = \frac{1 \ m^3}{1000000 \ cm^3}$

$289252 \frac{cm^3}{min} \cdot \frac{1 \ m^3}{1000000 \ cm^3} = .289252 \ m^3/min$

#30





$\tan\theta = \frac{100}{x}$ ← x increases at a constant rate

$\cancel{\cos\theta = \frac{x}{s}}$ Too many variables

$\sin\theta = \frac{100}{s}$

$x \tan\theta = 100$

$x = x(t)$
$\theta = \theta(t)$

$$\frac{d}{dt}\left(x \tan\theta\right) = \frac{d}{dt}(100)$$

product rule

$$\frac{d}{dt}(x)\tan\theta + x\frac{d}{dt}(\tan(\theta)) = 0$$

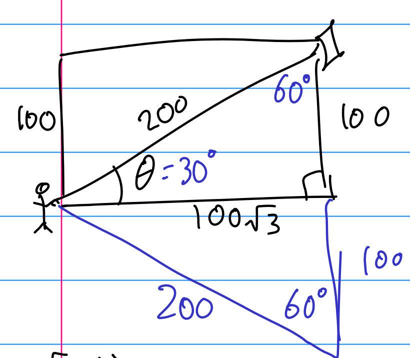
chain rule

$$\frac{dx}{dt}\tan\theta + x\sec^2\theta\frac{d\theta}{dt} = 0$$

Dynamic $\left(8\ \text{ft/sec}\right)\tan\theta + x\sec^2\theta\frac{d\theta}{dt} = 0$

$$8\ \frac{\text{ft}}{\text{sec}}\frac{1}{\sqrt{3}} + 100\sqrt{3}\ \frac{4}{3}\frac{d\theta}{dt} = 0$$

Static



$\sin\theta = \frac{100}{200}$

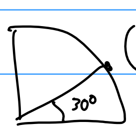
$\sin\theta = \frac{1}{2}$

$100^2 + x^2 = 200^2$

$10000 + x^2 = 40000$

$x^2 = 30000$

$x = \sqrt{30000} = 100\sqrt{3}$



$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\tan 30° = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\sec 30° = \frac{2}{\sqrt{3}}$ $\quad\sec^2 30° = \frac{4}{3}$

$$8\,\frac{ft}{sec}\,\frac{1}{\sqrt{3}} + 100\sqrt{3}\overset{ft}{}\,\frac{4}{3}\,\frac{\partial\theta}{\partial t} = 0$$

$$\frac{8}{\sqrt{3}}\,\frac{ft}{sec} + \frac{400\sqrt{3}}{3}\,ft\,\frac{\partial\theta}{\partial t} = 0$$

$$\sqrt{3}\left(\frac{400\sqrt{3}}{3}\,ft\,\frac{\partial\theta}{\partial t}\right) = \left(-\frac{8}{\sqrt{3}}\,\frac{ft}{sec}\right)\sqrt{3}$$

$$400\,ft\,\frac{\partial\theta}{\partial t} = -8\,ft/sec$$

$$\frac{\partial\theta}{\partial t} = \frac{-8\;ft/sec}{400\;ft} = -\frac{1}{50}\,\frac{rad}{sec}$$

#37

$$PV = C \quad \text{const}$$

$\leftarrow$ The fact!

Product rule

$$\frac{\partial}{\partial t}(PV) = \frac{\partial}{\partial t}(C) \qquad \begin{matrix} P(t) \\ V(t) \end{matrix}$$

$$\frac{\partial P}{\partial t}V + P\frac{\partial V}{\partial t} = 0$$

$$\left(20\,\frac{kPa}{min}\right)\left(600\,cm^3\right) + (150\,kPa)\frac{\partial V}{\partial t} = 0$$

$$12000\,\frac{kPa\,cm^3}{min} + (150\,kPa)\frac{\partial V}{\partial t} = 0$$

$$(150\,kPa)\frac{\partial V}{\partial t} = -12000\,\frac{kPa\,cm^3}{min}$$

$$\frac{\partial V}{\partial t} = \frac{-120\cancel{0}0\,\cancel{kPa}\,cm^3/min}{15\cancel{0}\,\cancel{kPa}} \qquad 1200 \div 15 = 80$$

$$\frac{\partial V}{\partial t} = -80\,\frac{cm^3}{min}$$

2.9 Find the linearization of a function at a point.

$\uparrow$

Equation of the tangent line.

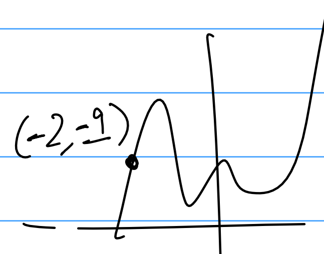
Formula on pg 188 — optional to know.
Just the point-slope formula.

Pg 192

linear equation of the tangent line at $x=a$.

\# 1 Find the linearization $L(x)$ of f at a.

$f(x) = x^3 - x^2 + 3$ $\qquad a = -2$



$f'(x) = 3x^2 - 2x$

$f'(-2) = 3(-2)^2 - 2(-2)$ $\qquad y = (-2)^3 - (-2)^2 + 3$

$12 + 4 = 16$ $\qquad -8 - 4 + 3 = -9$

$L(x) = f(a) + f'(a)(x-a)$

$L(x) = -9 + 16(x-(-2))$

$-9 + 16(x+2)$