$$\text{defined} \underset{\Leftarrow}{\not\Rightarrow} \text{continuous} \underset{\Leftarrow}{\not\Rightarrow} \text{diff'able}$$

implies,

$$A \Rightarrow B$$

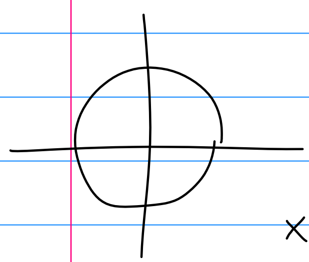
(if A, then B)

Contrapositive

$$(A \Rightarrow B) \iff (\sim B \Rightarrow \sim A)$$

not B   not A

2.6 Find dy/dx of an equation using implicit differentiation.

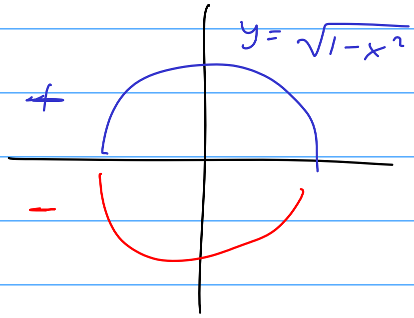


unit circle.

$$x^2 + y^2 = 1$$

$$y^2 = 1 - x^2$$

$$y = \pm\sqrt{1-x^2}$$



If $y = \sqrt{1-x^2}$,

chain rule

$$y' = \boxed{\frac{dy}{dx}} = \boxed{\frac{d}{dx}}\left(\sqrt{1-x^2}\right) = \frac{1}{2}(1-x^2)^{-1/2} \cdot (-2x)$$

object, function

operator "function"

$$g(x) = \sqrt{x} \longrightarrow g'(x) = \frac{1}{2}x^{-1/2}$$

$$h(x) = 1 - x^2 \longrightarrow h'(x) = -2x$$

$$\frac{-2x}{2}\,\frac{1}{\sqrt{1-x^2}} = \frac{-x}{\sqrt{1-x^2}}$$

pg 165 Crazy implicit curves!

$$y = 3x^2 - 1 + \sqrt{x}$$

Explicit.

$$y^3 + xy + x^3 = \sin\left(\frac{x}{y}\right)$$

Implicit

$y = ???$

Implicit Diff: Assume $y = \boxed{\text{a func of } x.}$

$\frac{d}{dx}(1) = 0$ $\frac{d}{dx}(x) = 1$ $\frac{d}{dx}(x^2) = 2x$

$\frac{d}{dx}(y) = \frac{dy}{dx}$

$\frac{d}{dx}(\underset{\text{func}}{x}\,\underset{\text{func}}{y}) = \frac{d}{dx}(x)\,y + x\frac{d}{dx}(y)$

product rule.

$= y + x\frac{dy}{dx}$

$\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$

Chain rule.

$(\text{func})^2$

OR

$\frac{d}{dx}(y^2) = \frac{d}{dx}(y \cdot y) = y\frac{dy}{dx} + \frac{dy}{dx}y$

product rule

$= 2y\frac{dy}{dx}$

Unit circle

$x^2 + y^2 = 1$

$\downarrow$ Derive both sides.

$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(1)$

$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = 0$

$\underline{2x + 2y\frac{dy}{dx} = 0}$

An equation in terms of x, y, and $\frac{dy}{dx}$.

Solve for $\frac{dy}{dx}$!

$2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$

Earlier: $y = \sqrt{1-x^2}$

$\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}}$

$\frac{dy}{dx} = \frac{-x}{y}$!

Find dy/dx

#5 $x^2 - 4xy + y^2 = 4$

$$\frac{d}{dx}(x^2) - \frac{d}{dx}(4xy) + \frac{d}{dx}(y^2) = \frac{d}{dx}(4)$$

$$2x - 4\frac{d}{dx}(xy) + 2y\frac{dy}{dx} = 0$$

$$2x - 4\left(\frac{d}{dx}(x)\cdot y + x\frac{d}{dx}(y)\right) + 2y\frac{dy}{dx} = 0$$

$$2x - 4\left(y + x\frac{dy}{dx}\right) + 2y\frac{dy}{dx} = 0$$

$$2x - 4y - 4x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} - 4x\frac{dy}{dx} = 4y - 2x$$

Explicit

$y = x^2$

$y' = 2x$

Implicit

$x^2 + y^2 = 1$

$y' = \frac{-x}{y}$

$$\frac{dy}{dx}(2y - 4x) = 4y - 2x$$

$$\frac{dy}{dx} = \frac{4y - 2x}{2y - 4x} = \frac{2y - x}{y - 2x}$$

Slope at $(8,2)$? $\frac{dy}{dx}(\overset{x}{8},\overset{y}{2}) = \frac{2(2) - 8}{2 - 2(8)} = \frac{4-8}{2-16} = \frac{-4}{-14} = \boxed{\frac{2}{7}}$

Line?

$y = \frac{2}{7}x + b$

$2 = \frac{2}{7}(8) + b$

$\frac{14}{7}\ \cancel{2} = \frac{16}{7} + b$

$b = -\frac{2}{7}$

$$y = \frac{2}{7}x - \frac{2}{7}$$

$$y \sin(x^2) = x \sin(y^2) \qquad \frac{dy}{dx} = ?$$

$$\frac{d}{dx}\left(y \sin(x^2)\right) = \frac{d}{dx}\left(x \sin(y^2)\right) \qquad \frac{d}{dx}(y^2) = 2yy'$$

$$\frac{d}{dx}(y)\sin(x^2) + y\frac{d}{dx}\left(\sin(x^2)\right) = \frac{d}{dx}(x)\sin(y^2) + x\frac{d}{dx}\left(\sin(y^2)\right)$$

$$y'\sin(x^2) + y\left(\cos(x^2)\,2x\right) = 1\cdot\sin(y^2) + x\left[\cos(y^2)\cdot 2yy'\right]$$

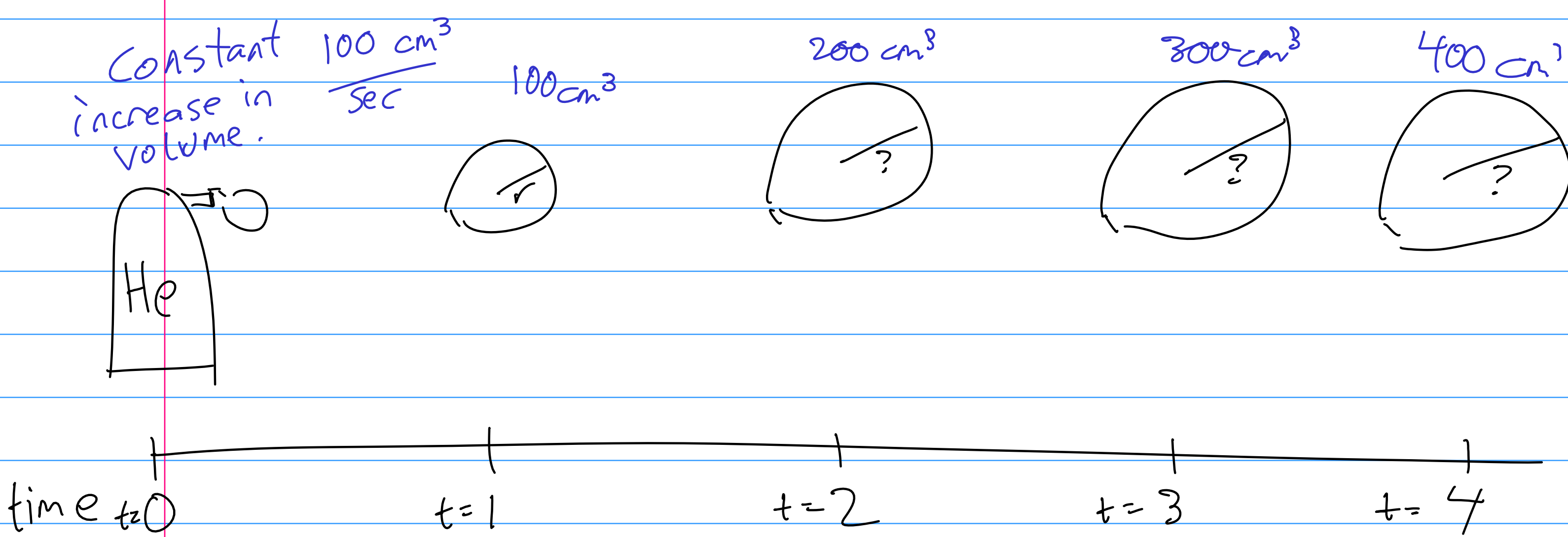
$$y'\sin(x^2) + 2xy\cos(x^2) = \sin(y^2) + 2xy\cos(y^2)\,y'$$

$$y'\sin(x^2) - 2xy\cos(y^2)\,y' = \sin(y^2) - 2xy\cos(x^2)$$

$$y'\left(\sin(x^2) - 2xy\cos(y^2)\right) = \sin(y^2) - 2xy\cos(x^2)$$

$$y' = \frac{\sin(y^2) - 2xy\cos(x^2)}{\sin(x^2) - 2xy\cos(y^2)}$$

2.8 Solve related-rates problems.



If $V = V_0$, $r = \ldots$ can be easily calculated at any given moment (value of t)

But at a moment, what is the rate of increase of r?

rate of change of volume $\rightarrow \frac{dV}{dt}$ and rate of change of radius $\rightarrow \frac{dr}{dt}$ are related!

sphere volume

$$V = \frac{4\pi}{3} r^3$$

ANY variable can be (and is) a function of time.

Start with a fact relating the variables.
Derive it with respect to t (time).

$$\frac{d}{dt}(V) = \frac{dV}{dt}$$

$$\frac{d}{dt}(r) = \frac{dr}{dt}$$

$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4\pi}{3} r^3\right)$$

$$\frac{dV}{dt} = \frac{4\pi}{3} \frac{d}{dt}(r^3)$$

$(r \text{ func})^3$

chain rule.

$$\frac{dV}{dt} = \frac{4\pi}{3}\left(3r^2 \frac{dr}{dt}\right)$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

A related-rates equation relates the rates of change of the variables and the variables themselves.

Above, we were given $\frac{dV}{dt} = 100 \frac{cm^3}{sec}$.

When $r = 10$ cm, how fast is r increasing? $\left(\frac{dr}{dt}\right)$?

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$100 \frac{cm^3}{sec} = 4\pi (10 \text{ cm})^2 \frac{dr}{dt}$$

$$100\ \frac{cm^3}{sec} = \left(400\pi\ cm^2\right)\frac{dr}{dt}$$

$$\frac{100\ cm^3/sec}{400\pi\ cm^2} = \underline{\frac{dr}{dt} = \frac{1}{4\pi}\ cm/sec}$$

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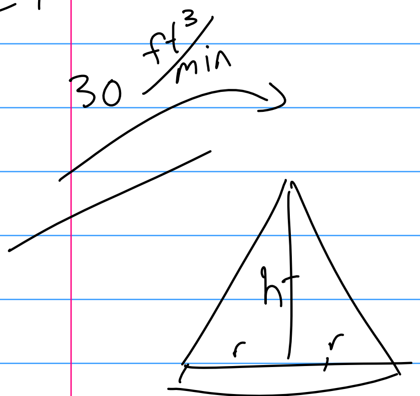

$h = 2r$

$r = \frac{h}{2}$

$$V = \frac{1}{3} h \pi r^2$$

$$\cancel{V = \frac{1}{3}(2r)\pi r^2}$$

but h and $\frac{dh}{dt}$ ~~are~~ what work easier in this problem.

$$V = \frac{1}{3} h \pi \left(\frac{h}{2}\right)^2$$

$$V = \frac{\pi}{12} h^3$$

$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{\pi}{12} h^3\right)$$

$$\frac{dV}{dt} = \frac{\pi}{12}\frac{d}{dt}\left(h^3\right)$$

$$\frac{dV}{dt} = \frac{\pi}{12} 3h^2 \frac{dh}{dt}$$

$$30\ ft^3/min = \frac{\pi}{12} 3 (10\ ft)^2 \frac{dh}{dt}$$

$$30\ \frac{ft^3}{min} = \frac{\pi}{4} 100\ ft^2 \frac{dh}{dt}$$

$$30\ ft^3/min = 25\pi\ ft^2 \frac{dh}{dt}$$

$$\frac{30\ ft^3/min}{25\pi\ ft^2} = \frac{dh}{dt} = \boxed{\frac{6}{5\pi}\ ft/min}$$