4.5 Use the substitution rule to evaluate integrals.

$$\underbrace{\int 2x\sqrt{1+x^2}\,dx}$$?

$u = 1+x^2$

Derive u with respect to x.

$$\frac{du}{dx} = 2x$$

$$du = 2x\,dx$$

$$\int \sqrt{1+x^2}\,\underbrace{2x\,dx}$$

$$\int \sqrt{u}\,du$$

$$\int u^{1/2}\,du = \frac{2}{3}u^{3/2} + C$$

$$\boxed{\frac{2}{3}(1+x^2)^{3/2} + C}$$

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#8 $\int x^2 \sin(x^3)\,dx$

$u = x^3$

$\frac{du}{dx} = 3x^2$

$du = 3x^2\,dx$

Balancing the integral:

$$\frac{1}{3}\cdot 3\int x^2 \sin(x^3)\,dx$$

$$\frac{1}{3}\int 3x^2 \sin(x^3)\,dx = \frac{1}{3}\int \sin(x^3)\,3x^2\,dx$$

$$= \frac{1}{3}\int \sin(u)\,du$$

$$\frac{1}{3}(-\cos(u)) + C$$

$$\boxed{= -\frac{1}{3}\cos(x^3) + C}$$

#9 $\int (1-2x)^9 \, dx$

$u = 1-2x$

$du = -2 \, dx$

$= -\frac{1}{2} \int (1-2x)^9 (-2) \, dx$

$= -\frac{1}{2} \int u^9 \, du = -\frac{1}{2}\left(\frac{1}{10}u^{10}\right) + C = -\frac{1}{20}(1-2x)^{10} + C$

#20 $\int x\sqrt{x+2} \, dx$

$u = x+2$

$du = dx$

$u-2 = x$

$\int x\sqrt{u} \, du$

Can't integrate yet!

$\neq x \int \sqrt{u} \, du$

only constants can float through integrals.

$\int (u-2)\sqrt{u} \, du$

$\int u^{3/2} - 2u^{1/2} \, du$

$\frac{2}{5}u^{5/2} - 2 \cdot \frac{2}{3}u^{3/2} = \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$ ✓

$= (x+2)^{3/2}\left[\frac{2}{5}(x+2) - \frac{4}{3}\right]$

$= (x+2)^{3/2}\left[\frac{2}{5}x + \frac{\cancel{4}^{12}}{\cancel{5}_{15}} - \frac{\cancel{4}^{20}}{\cancel{3}_{15}}\right]$

$= (x+2)^{3/2}\left(\frac{2}{5}x - \frac{8}{15}\right)$

#17 $\int \sec^2\theta \tan^3\theta \, d\theta$

$u = \tan^3\theta$

$du = 3\tan^2\theta \sec^2\theta \, d\theta$

$$= \frac{1}{3}\int \tan\theta \quad 3\tan^2\theta \sec^2\theta \, d\theta$$

$$= \frac{1}{3}\int \sqrt[3]{u} \quad du \quad ?$$ maybe

Easier:

$$\int \sec^2\theta \tan^3\theta \, d\theta$$

$u = \tan\theta$
$du = \sec^2\theta \, d\theta$

$$\int \tan^3\theta \underbrace{\sec^2\theta \, d\theta}$$

$$\int u^3 \, du = \frac{1}{4}u^4 + C = \frac{1}{4}\tan^4\theta + C$$

#36 $\int_0^1 (3t-1)^{50} \, dt$

$u = 3t-1$
$du = 3\,dt$

$$\frac{1}{3}\int_{0=t}^{1=t} (3t-1)^{50} \, 3dt$$

$0 \le t \le 1$
$3(0)-1 \le u \le 3(1)-1$
$-1 \le u \le 2$

$$\frac{1}{3}\int_{\cancel{0}\ -1=u}^{\cancel{1}\ 2=u} u^{50} \, du$$

Method 1: Change the endpoints

$$= \frac{1}{3}\left(\frac{1}{51}u^{51}\right)\Big|_{-1}^{2}$$

$$= \frac{1}{3}\left(\frac{1}{51}2^{51} - \frac{1}{51}(-1)^{51}\right)$$

$$= \frac{1}{153}\left(2^{51}+1\right)$$ ✓

#38 $\int_0^{\sqrt{\pi}} x\cos(x^2) \, dx$

$$\boxed{\text{I want: } F(x)\Big|_0^{\sqrt{\pi}}}$$

$$u = x^2$$
$$du = 2x\,dx$$

Strategy 2: Keep endpoints, Do u-subst, back-subst, then evaluate.

"Back subst then evaluate"

$$\frac{1}{2}\int_{0=x}^{\sqrt{\pi}=x} \underbrace{2x}\ \underbrace{\cos(x^2)}\ \underbrace{dx}$$

$$= \frac{1}{2}\int_{0=x}^{\sqrt{\pi}=x} \cos(u)\ du$$

$$\frac{1}{2}\sin u\Big|_{0=x}^{\sqrt{\pi}=x} = \frac{1}{2}\sin(x^2)\Big|_0^{\sqrt{\pi}} = \frac{1}{2}\sin\left((\sqrt{\pi})^2\right) - \frac{1}{2}\sin 0$$

$$\frac{1}{2}\sin\pi - \frac{1}{2}\sin 0$$

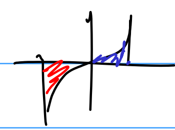
$$= 0$$

$$\int \sqrt{1+x^2}\,dx \ ? \qquad \begin{array}{l} u = 1+x^2 \\ du = 2x\,dx \end{array}$$

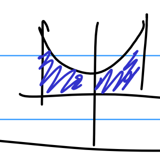
will learn in Calc II.

#41 $\displaystyle\int_{-\pi/4}^{\pi/4} \left(x^3 + x^4 \tan x\right) dx = 0\ ??$

pg 345: If f is odd, then $\displaystyle\int_{-a}^{a} f(x)\,dx = 0$



If f is even, then $\displaystyle\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx.$



#55 $\displaystyle\int_{-2}^{2} (x+3)\sqrt{4-x^2}\,dx$

Sum of two integrals. Interpret one as an area.

$$\int_{-2}^{2} x\sqrt{4-x^2} + 3\sqrt{4-x^2}\,dx$$

$$\int_{-2}^{2} x\sqrt{4-x^2}\,dx + \int_{-2}^{2} 3\sqrt{4-x^2}\,dx$$

Odd?

$f(x) = x\sqrt{4-x^2}$

$f(-x) = (-x)\sqrt{4-(-x)^2}$

$= -x\sqrt{4-x^2}$

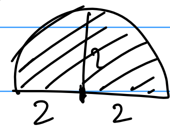
$f(x) = -f(-x)$

Odd! and endpoints are opposites.

$= 0$

$\cancel{u = 4-x^2}$

$\cancel{du = -2x\,dx}$

$3\Big[$  $\Big]$

$= 3\,\frac{1}{2}\pi 2^2$

$\boxed{= 6\pi}$

What does $\sqrt{4-x^2}$ look like?

$y = \sqrt{4-x^2}$

$y^2 = 4-x^2$

$x^2+y^2 = 4$

Circle with radius 2 centered at $(0,0)$.