3.2 Find numbers that satisfy the conclusions of Rolle's Thm and the MVT. (No need to memorize the theorems.)

Pg 216:

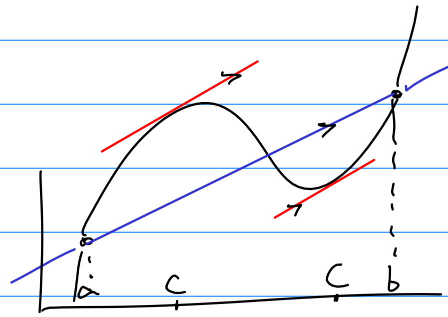
MVT: Let f be a function satisfying:

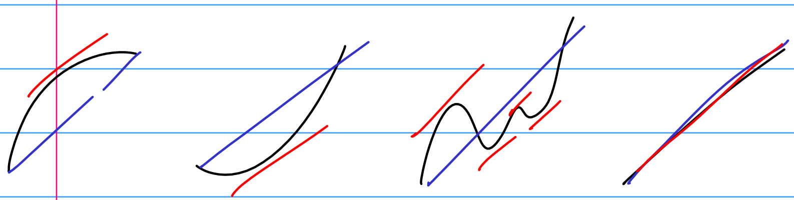
- ☆ f cont on $[a, b]$
- ☆ f diff'able (a, b)

Then there is a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} = \text{Slope}$$

of the line through $(a, f(a))$, $(b, f(b))$





Pg 220

#14 $f(x) = \frac{1}{x}$ $[1, 3]$

$$\frac{2x^2-3}{5x^9-1}$$

$\frac{1}{x}$: cont on $[1, 3]$?

Yes, because rational functions are continuous on their domains and $[1, 3]$ is within the domain of $\frac{1}{x}$.

diff'able on $(1, 3)$?

$f'(x) = -\frac{1}{x^2}$ defined when $x \neq 0$.

So $f(x)$ is diff'able on $(1, 3)$.

$\therefore$ there exists a c in $(\overset{a}{1}, \overset{b}{3})$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

$$= \frac{\frac{1}{3} - \frac{1}{1}}{3 - 1} = \frac{\frac{1}{3} - 1}{2} = \frac{3}{3} = \frac{1-3}{6} = \frac{-2}{6}$$

There is a c such that $f'(c) = \boxed{-1/3}$

$$f'(c) = -\frac{1}{c^2} = -\frac{1}{3}$$

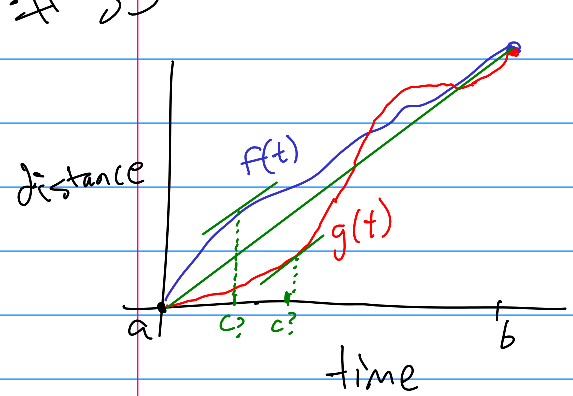
$$c^2 = 3$$

$$c = \cancel{\pm}\sqrt{3}$$

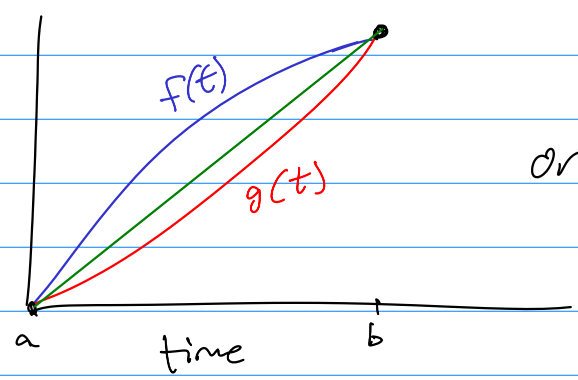
Only $\sqrt{3} \in (1, 3)$.
"is in"

$$\boxed{c = \sqrt{3}}$$

#35



or

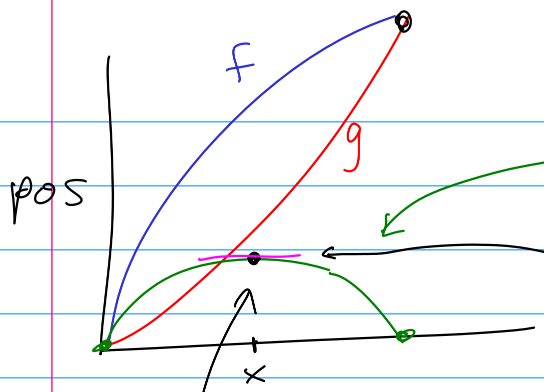


or etc.

Prove there was some time at which the runners had the same speed.

slope of secant line = avg speed of each runner.

same speed?



$(f-g)(x)$

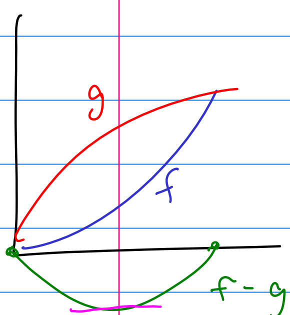
$(f-g)'(x) = 0$ here

$f'(x) - g'(x) = 0$

$f'(x) = g'(x)$

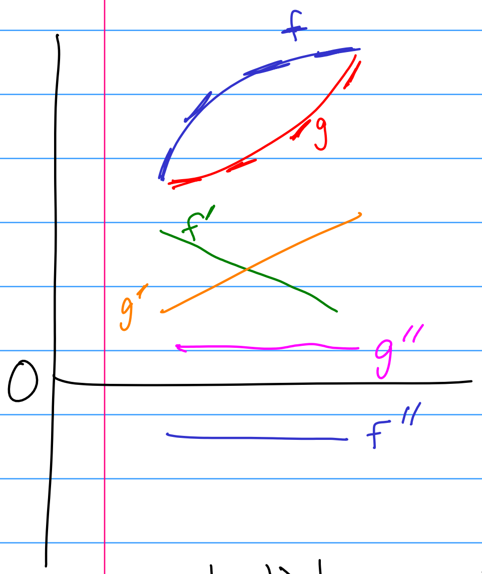
Maximum;
$(f-g)'(x) = 0$

At some x, the runners have the same speed.



By Rolle's Thm, such an x always exists.

3.3 Based on f'(x) and f''(x), determine properties (concavity, inc/dec) of f(x).



Concavity

potential min/max. → critical point

Only place these can occur.

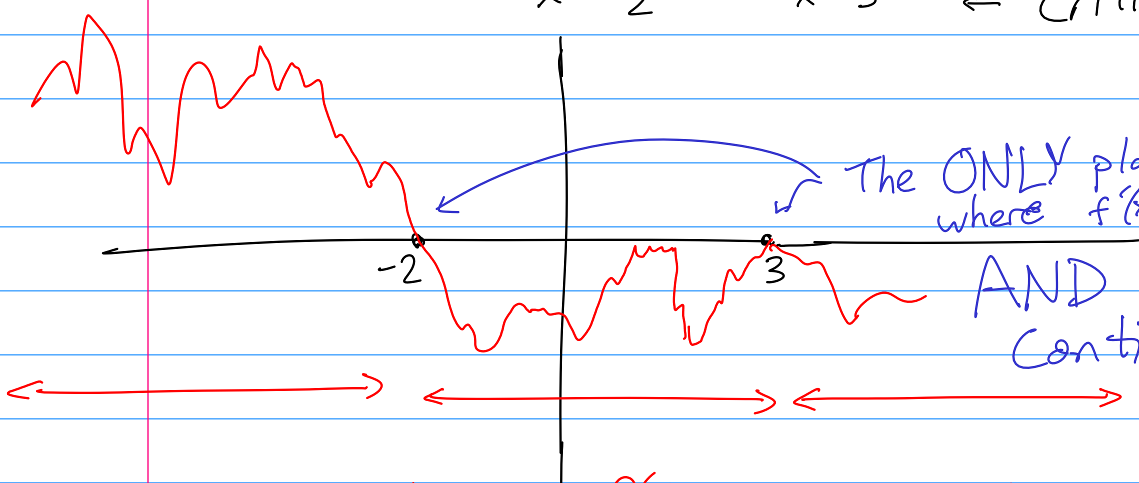
f(x)	f'(x)	f''(x)
increasing	positive	—
decreasing	negative	—
critical point	0 or DNE	—
Concave up	increasing	positive
concave down	decreasing	negative
possible inflection point		0 or DNE

Pg 229

\# 34 $f(x) = 36x + 3x^2 - 2x^3$ where is $f \nearrow$? $\searrow$?

$$f'(x) = 36 + 6x - 6x^2$$
$$= -6x^2 + 6x + 36$$
$$= -6(x^2 - x - 6)$$
$$= -6(x+2)(x-3) = 0$$

$x = -2$ $x = 3$ ← Critical numbers.



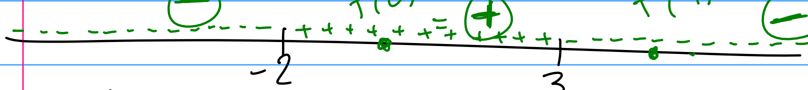
The ONLY places where $f'(x)$ meets 0.

AND $f'(x)$ is continuous!

On each interval, $f'(x)$ is either all positive or all negative!

$-6(x+2)(x-3)$ Test points.

$f'(x)$ $f'(-3) = -6(-1)(-6)$ − $f'(0) = -6(2)(-3)$ + $f'(4) = -6(6)(1)$ −



$f(x)$ $\searrow$ $\nearrow$ $\searrow$

$f(x)$ is increasing on $(-2, 3)$

decreasing on $(-\infty, -2)$ and $(3, \infty)$.

b) Crit numbers: $x=-2$ and $x=3$

min ↘↗ max ↗↘

$\begin{array}{r} 6\;12 \\ \cancel{7}2 \\ -28 \\ \hline 44 \end{array}$ $\begin{array}{r} 108 \\ -27 \\ \hline 81 \end{array}$

$f(-2) = 36(-2) + 3(-2)^2 - 2(-2)^3 = -72 + 12 + 16 = -44$ min

$f(3) = 36(3) + 3(3)^2 - 2(3)^3 = 108 + 27 - 54 = 81$ max

c) ∪? ∩?

$f'(x) = 36 + 6x - 6x^2$

$f''(x) = 6 - 12x$

possible inflection point:

$f''(x) = 0 \qquad 6 - 12x = 0$

$6 = 12x$

$\frac{1}{2} = x$

Test points

$f''(x)$ $\quad f''(0) = 6 \qquad\qquad f''(1) = -6$

+ + + + + + + + + + 0 + + $\frac{1}{2}$ - - - - 0 - - - - - - - - - - - - -

$f(x)$

concave up $\qquad\qquad$ concave down

$x = \frac{1}{2}$ is an inflection point.

[point at which the concavity changes and at which $f(x)$ is continuous.

$f(\frac{1}{2}) = 36(\frac{1}{2}) + 3(\frac{1}{4}) - 2(\frac{1}{8})$

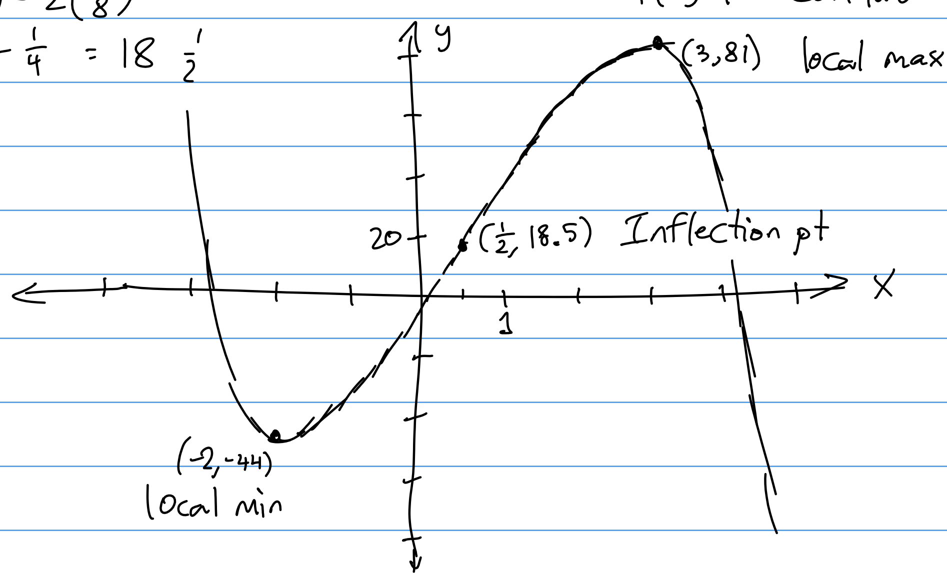
$18 + \frac{3}{4} - \frac{1}{4} = 18\frac{1}{2}$

min: $(-2, -44)$

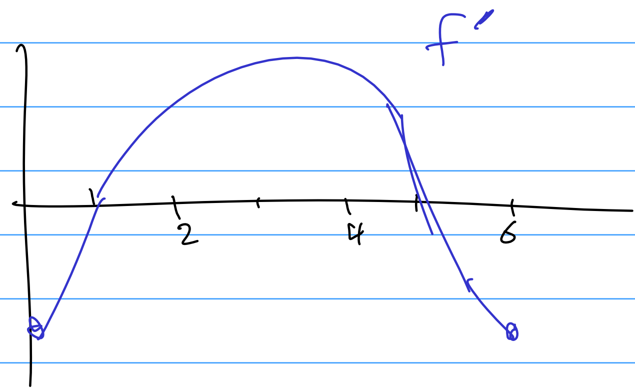
max $(3, 81)$

↘ -2 ↗ 3 ↘

∪ $\frac{1}{2}$ ∩



Pg 227 #5



$f \nearrow$? $(1, 5)$

$f \searrow$? $(0, 1), (1, 6)$

f max or min? 1 and 5

$f'(x) = 0$

$f'(x) = 0 \quad f''(x) > 0 \Rightarrow f(x)$ is a local min

$f'(x) = 0 \quad f''(x) < 0 \Rightarrow f(x)$ is a local max.