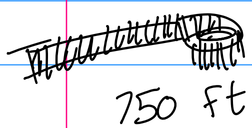
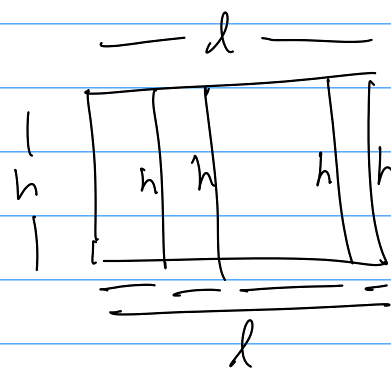
1. Write thing-to-optimize as a function of one variable.

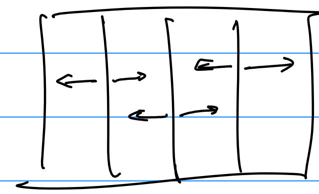
2. Pick a reasonable interval for the variable.

3. Do closed-interval method on function ① on interval ②.

Pg 265
#11







Constraint

$$750 = 5h + 2l$$

$$750 - 2l = 5h$$

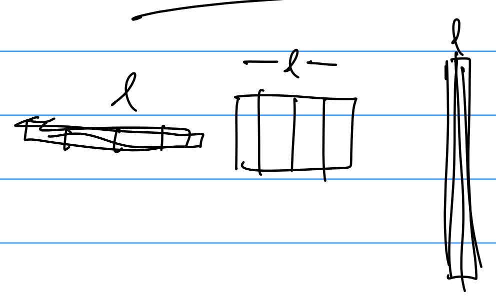
$$\frac{750 - 2l}{5} = h = 150 - \frac{2}{5}l$$

$$A = lh$$

$$A = l\left(150 - \frac{2}{5}l\right)$$

$$A = -\frac{2}{5}l^2 + 150l$$

①



$$0 < l < 375$$

②

$$A' = -\frac{4}{5}l + 150 = 0$$

$$150 = \frac{4}{5}l$$

$$\frac{5}{4}\cdot 150 = l = \frac{750}{4} = \frac{375}{2}$$

crit pt.

l

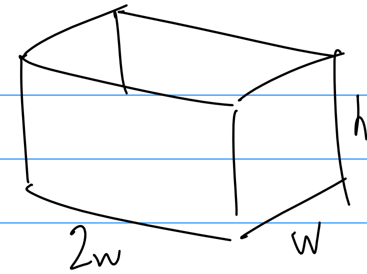
$$A(0) = 0$$

$$A\left(\frac{375}{2}\right) = \frac{375}{2}\left(150 - \frac{2}{5}\,\frac{375}{2}\right)$$

$$A(375) = 0$$

$$14062.5 \text{ ft}^2 \approx \frac{375}{2}\cdot 75$$

#16 $V = 10\ m^3$



Base: $\$10/m^2$
Sides: $\$6/m^2$

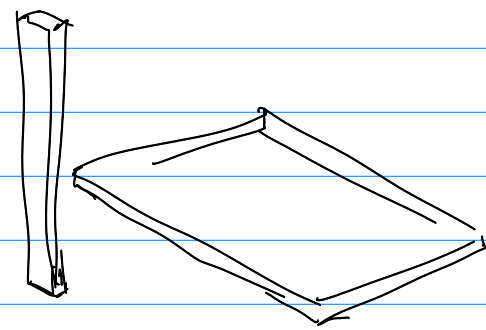
$(2w)w\cdot h = 10$
$2w^2h = 10$
$w^2h = 5$
$h = \frac{5}{w^2}$

minimize

$$\text{Cost} = \underbrace{2w^2}_{\text{Area base}} \cdot \underbrace{10}_{\text{cost base}} + \underbrace{\overset{wh+wh+2wh+2wh}{6wh}}_{\text{Area sides}} \cdot \underbrace{6}_{\text{cost sides}}$$

$$C = 20w^2 + 36w\left(\frac{5}{w^2}\right)$$

$$C = 20w^2 + 180w^{-1}$$



$$0 < w < \infty$$

$$C' = 40w - 180w^{-2} = 0 \text{ or DNE}$$

$w=0$

$$40w = 180w^{-2}$$

$$40w^3 = 180$$

$$w^3 = \frac{18}{4} = \frac{9}{2}$$

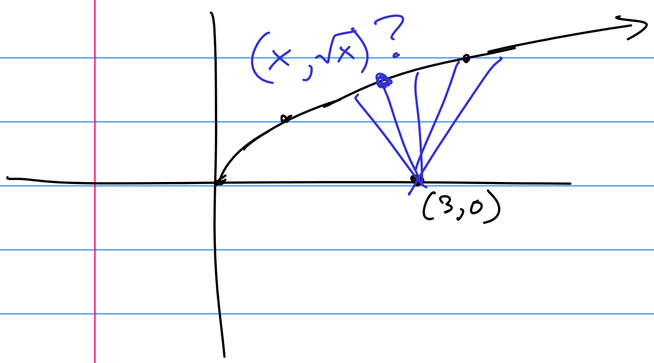
$$w = \sqrt[3]{\frac{9}{2}} \approx 1.651$$

crit pt.

$C(0) = \infty$

$C\left(\sqrt[3]{\frac{9}{2}}\right) \approx \$163.54$

#22 Point on $y=\sqrt{x}$ closest to $(3,0)$.



Minimize distance between $(x,\sqrt{x})$ and $(3,0)$

$$d=\sqrt{(3-x)^2+(0-\sqrt{x})^2}$$

$$d=\sqrt{x^2-6x+9+x}$$

$0 \le x \le 3$

$$d=\sqrt{x^2-5x+9}$$

$$d' = \frac{1}{2}(x^2-5x+9)^{-1/2}\cdot(2x-5) = \frac{2x-5}{2(x^2-5x+9)^{1/2}}$$

$$= \frac{2x-5}{2\sqrt{x^2-5x+9}}$$

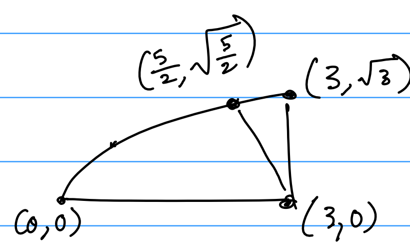
$d'=0$ → $2x-5=0$, $2x=5$, $x=5/2$

or

DNE → $x^2-5x+9=0$ or <0

$$x=\cancel{\frac{5\pm\sqrt{25-36}}{2}}$$ nonreal $\quad x^2-5x+9\neq 0$

Crit pt: $x=\frac{5}{2}$



$d(0)=3$

$d(\frac{5}{2})=\sqrt{11/4}\approx 1.6583$

$d(3)=\sqrt{3}\approx 1.732$

min.

$$\sqrt{\left(\frac{5}{2}-3\right)^2+\left(\sqrt{\frac{5}{2}}\right)^2} = \frac{1}{4}+\frac{5}{2} = \sqrt{\frac{11}{4}}$$

#50

$s = \frac{d}{t}$

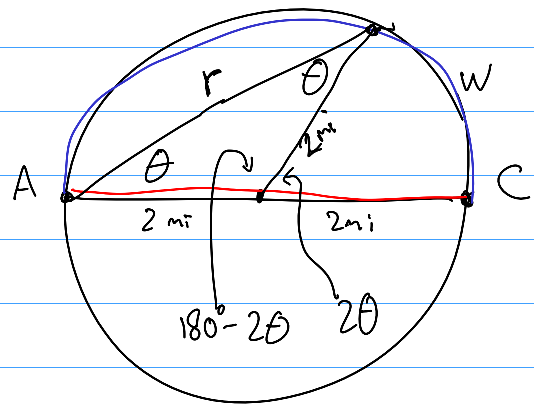
$t = \frac{d}{s}$

row $2\ \frac{mi}{hr}$ walk $4\ \frac{mi}{hr}$

minimize time.

$T = \frac{r}{2} + \frac{w}{4}$

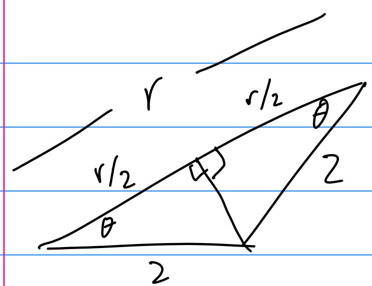
θ?



$0 < \theta < \frac{\pi}{2}$

$w = \frac{2\theta}{2\pi} \cdot 4\pi = \frac{8\pi\theta}{2\pi} = 4\theta$

$r = 4\cos\theta$



$\sin\theta =$

$\cos\theta = \frac{r/2}{2} = \frac{r}{4}$

$T = \frac{4\cos\theta}{2} + \frac{4\theta}{4}$

$T = 2\cos\theta + \theta$

$T' = -2\sin\theta + 1 = 0$

$1 = 2\sin\theta$

$\frac{1}{2} = \sin\theta$

$\theta = 30° = \frac{\pi}{6}$

crit pt

rowing $T(0) = 2$ hr

$T\left(\frac{\pi}{6}\right) = \sqrt{3} + \frac{\pi}{6} \approx 2.256$ hr

walking $T\left(\frac{\pi}{2}\right) = \frac{\pi}{2} \approx 1.57$ hr ← min time

ALWAYS CHECK ENDPOINTS!

#61

$\boxed{55{,}000}$

$\$19 \to 0$

$\$10 \to 27000$
$\$9 \to 30000$
$\$8 \to 33000$

$\$0 \to 57{,}000$

Maximize revenue

= price · attendance

$0 < A < 55{,}000$

$\boxed{0 < P < 19}$

$R = P \cdot A$

$A = 57000 - 3000p$

$R = p(57000 - 3000p)$

$R = 57000p - 3000p^2$

$R' = 57000 - 6000p = 0$

$570\cancel{00} = 60\cancel{00}p$

$\frac{57}{6} = p = \frac{19}{2} = \$9.50$

$R(0) = 0$

$R(9.50) = 9.5(28500) = \$270{,}750$

$R(19) = 0$

$R(10) = 270{,}000$