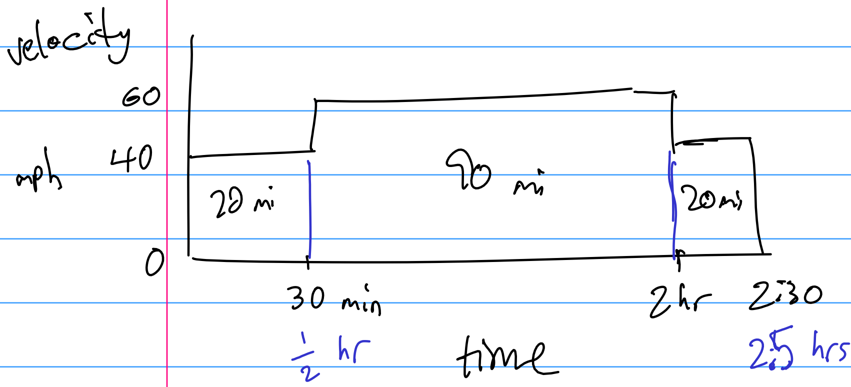
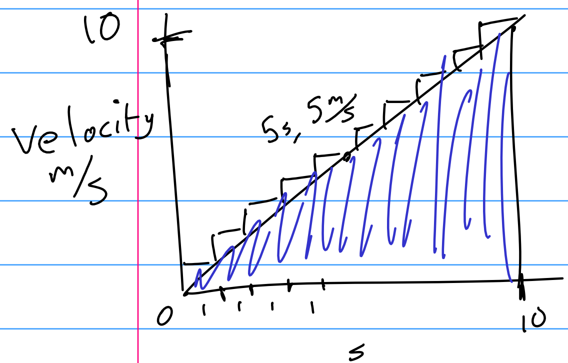
4.3,4 Evaluate indefinite integrals. (Results in functions)
4.3,4 Evaluate definite integrals. (Results in numbers)



Dist?

$\frac{1}{2}\cdot 40 = 20$ mi
$+\ 1.5 \cdot 60 = 90$ mi
$+\ \frac{1}{2}\cdot 40 = 20$ mi

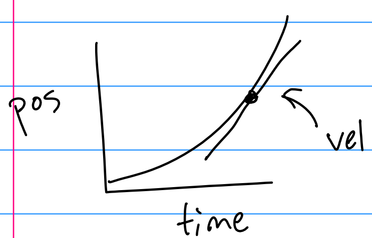
130 mi



Dist = estimate
$1+2+\ldots+10 = 55$ m

exact
Area: $10\cdot 10\cdot \frac{1}{2} = 50$ m

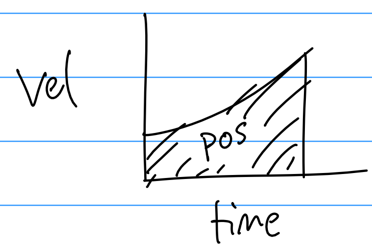
Ch 2, derivs



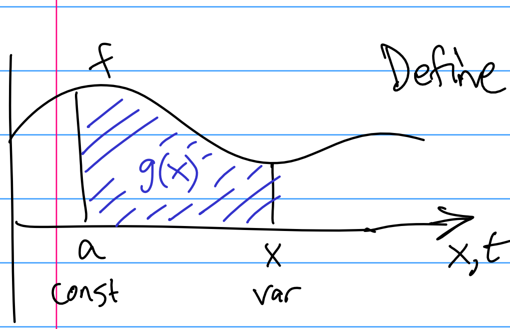
inverse?

antideriv?

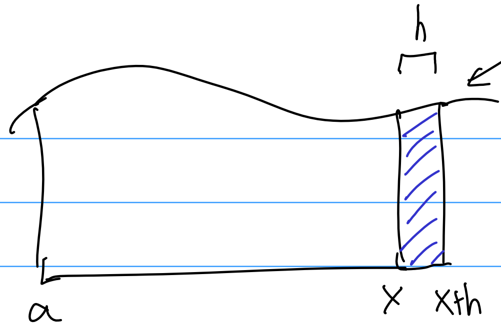
Ch 4, integrals:



Accumulation function



Define $g(x) = \int_a^x f(t)\,dt$



$$\underbrace{\int_a^x f(t)\,dt}_{g(x)} + \underbrace{\int_x^{x+h} f(t)\,dt}_{\approx\, h\cdot f(x)} = \underbrace{\int_a^{x+h} f(t)\,dt}_{g(x+h)}$$

$$g(x) + h\cdot f(x) \approx g(x+h)$$

$$h\,f(x) \approx g(x+h) - g(x)$$

$$f(x) \approx \frac{g(x+h)-g(x)}{h}$$

$$\lim_{h\to 0} f(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$$

As $h \to 0$, our approximation improves.

h not involved

$$f(x) = g'(x)$$

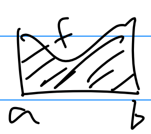
Antiderive

$$g(x) = F(x) + C$$

$$\text{and } g(x) = \int_a^x f(t)\,dt$$

Better way to find area?

But C is arbitrary — we need a unique area.



I want to find $\boxed{\int_a^b f(t)\,dt}$.

This equals $\int_a^b f(t)\,dt - \int_a^a f(t)\,dt$

$$= g(b) - g(a)$$

$$= \big(F(b) + C\big) - \big(F(a) + C\big)$$

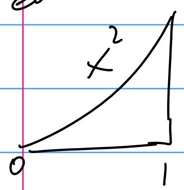
$$= F(b) - F(a)$$

C is the same in both, since $g(x)$ is defined consistently.

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Fundamental Theorem of Calculus

$\frac{1}{3}$ earlier...



$$F(x) = \frac{1}{3}x^3$$

$$\int_0^1 x^2\,dx = \frac{1}{3}(1)^3 - \frac{1}{3}(0)^3$$

$$= \boxed{\frac{1}{3}}$$

Why the dx?

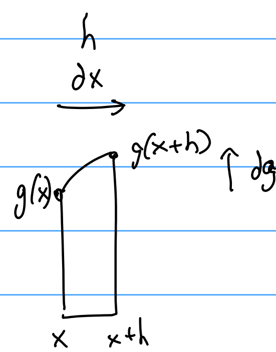
$$g'(x) = f(x)$$

$$\frac{dg}{dx} = f(x)$$

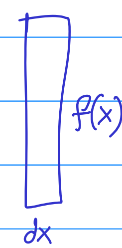
$$dg = f(x)\,dx$$

$$g(x+h) - g(x) = f(x)\,dx$$

$$= f(x)\,h$$



$$\int \underbrace{f(x)}_{\text{height}}\ \underbrace{dx}_{\text{width}}$$



Indefinite integral
no endpoints

(Antiderivative)

$$\int x^2\,dx = \frac{1}{3}x^3 + C$$

Gets a function

Definite integral
endpoints

$$\int_0^1 x^2\,dx = \frac{1}{3}$$

Gets a number

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#20 $\int_{-1}^{1} x^{100}\,dx = \frac{1}{101}x^{101}\Big|_{-1}^{1} = \frac{1}{101}(1)^{101} - \frac{1}{101}(-1)^{101}$

$\frac{1}{101} + \frac{1}{101}$

$= \boxed{\frac{2}{101}}$

#19 $\int_{1}^{3}(x^2+2x-4)\,dx$

$= \frac{1}{3}x^3 + x^2 - 4x\Big|_1^3 \qquad [F(3)] - [F(1)]$

$= \left(\frac{1}{3}(3)^3 + (3)^2 - 4(3)\right) - \left(\frac{1}{3}(1)^3 + (1)^2 - 4(1)\right)$

$= (9 + 9 - 12) - \left(\frac{1}{3} + 1 - 4\right)$

$6 - \left(-\frac{8}{3}\right)$

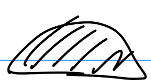
$6 + \frac{8}{3} = \frac{18}{3} + \frac{8}{3} = \boxed{\frac{26}{3}}$

#23 $\int_1^9 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_1^9 = \frac{2}{3}(9)^{3/2} - \frac{2}{3}(1)^{3/2}$

$x^{1/2}$

$\frac{2}{3}(27) - \frac{2}{3}(1)$

$= \frac{2}{3}(26) = \boxed{\frac{52}{3}}$



?

$\int_0^{\pi} \sin\theta\,d\theta = (-\cos\theta)\Big|_0^{\pi}$

$= (-\cos\pi) - (-\cos 0)$

$= (1) - (-1) = \boxed{2}$!

#29 $$\int_1^4 \frac{2+x^2}{\sqrt{x}}\,dx = \int_1^4 (2+x^2)x^{-1/2}\,dx$$

$$= \int_1^4 2x^{-1/2} + x^{3/2}\,dx$$ Pre-integration

$$4x^{1/2} + \frac{2}{5}x^{5/2}\Big|_1^4$$ Evaluation. no more $\int$ or dx!

$$\left(4(4)^{1/2} + \frac{2}{5}(4)^{5/2}\right) - \left(4(1)^{1/2} + \frac{2}{5}(1)^{5/2}\right)$$

$$= \left(4\cdot 2 + \frac{2}{5}\cdot 32\right) - \left(4 + \frac{2}{5}\right)$$

$$= 8 + \frac{64}{5} - 4 - \frac{2}{5}$$

$$= \frac{20}{5}\cancel{4} + \frac{62}{5} = \boxed{\frac{82}{5}}$$

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#32 $$\int_0^{\pi/4} \sec\theta \tan\theta\,d\theta = \sec\theta\Big|_0^{\pi/4}$$

$$= \sec\frac{\pi}{4} - \sec 0$$

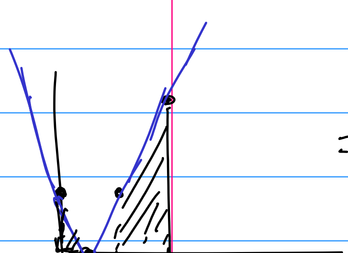
$$= \frac{1}{\cos\frac{\pi}{4}} - \frac{1}{\cos 0}$$

$$= \frac{1}{\sqrt{2}/2} - \frac{1}{1} = \frac{2}{\sqrt{2}} - 1$$

$$= \frac{2\sqrt{2}}{2} - 1$$

$$= \sqrt{2} - 1$$

#40 $$\int_0^2 |2x-1|\,dx$$



$$= \int_0^{1/2} -(2x-1)\,dx + \int_{1/2}^2 (2x-1)\,dx$$

$$= -\int_0^{1/2} (2x-1)\,dx + \int_{1/2}^{2} (2x-1)\,dx$$

$$-\left(x^2 - x\Big|_0^{1/2}\right) + \left(x^2 - x\Big|_{1/2}^{2}\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^2 - \tfrac{1}{2} \cancel{- (0^2 - 0)}\right) + \left((2^2 - 2) - \left(\left(\tfrac{1}{2}\right)^2 - \tfrac{1}{2}\right)\right)$$

$$-\left(\tfrac{1}{4} - \tfrac{1}{2}\right) + \left(2 - \left(\tfrac{1}{4} - \tfrac{1}{2}\right)\right)$$

$$-\left(-\tfrac{1}{4}\right) + \left(2 - \left(-\tfrac{1}{4}\right)\right)$$

$$\tfrac{1}{4} + \left(2 + \tfrac{1}{4}\right) = 2 + \tfrac{1}{2} = \boxed{\tfrac{5}{2}}$$

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#49 Oil leaks at $\overset{\text{rate}}{r(t)}$ gallons/min — "velocity"

$$\int_0^{120} r(t)\,dt = \text{~~total~~ oil leaked in first 120 min}$$

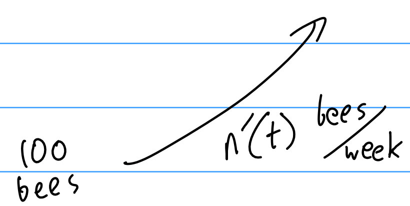
or 2 hrs. "position"

In general,

If $f'(x) = g(x)$,

$$f(x) = \int g(x)\,dx$$

#50



$n(t)$ = # of bees at time t

$$100 + \int_0^{15} n'(t)\,dt = \text{Total bees after 15 weeks}$$