$$\ln(x) = \log_e(x)$$

$\frac{d}{dx}(\ln x)$? Implicit diff on pg 428 —

OR

Deriv of an inverse

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$f(x) = e^x$ $\quad$ $f'(x) = e^x$

$f^{-1}(x) = \ln x$ $\quad$ $(f^{-1})'(x) = \dfrac{1}{f'(\ln x)} = \dfrac{1}{e^{\ln x}} = \boxed{\dfrac{1}{x}}$

	$f(x)$	$f'(x)$	$F(x)$
	$\ln x$	$\frac{1}{x}$	$x \ln x - x$
Earlier...	$\frac{1}{x}$	$-\frac{1}{x^2}$	$\ln\lvert x\rvert + C$

no need to memorize

$f(x) = \underline{x}\ \underline{\ln(x)} - x$ $\quad$ Find $f'(x)$.

$f'(x) = 1 \cdot \ln x + x \cdot \frac{1}{x} - 1$

$= \ln x + 1 - 1 = \underline{\ln x}$

#4 $\quad f(x) = \ln(\sin^2 x)$

$$f'(x) = \frac{1}{\sin^2 x} \cdot (\sin^2 x)'$$

$$\frac{1}{\sin^2 x} \cdot 2\sin x \cdot \cos x$$

$$= \frac{2\sin x \cos x}{\sin^2 x} = 2\frac{\cos x}{\sin x} = 2\cot x$$

$f(x)$	$f'(x)$	$F(x)$
$\ln(g(x))$	$\frac{g'(x)}{g(x)}$	(n/a)

$$f(x) = \ln(8x^5 - 4x^2 + 5)$$

$$f'(x) = \frac{40x^4 - 8x}{8x^5 - 4x^2 + 5}$$

#9 $g(x) = \ln(xe^{-2x})$

$= \ln(x) + \ln(e^{-2x})$ Log laws, no deriv yet

$= \ln x - 2x$

Now Derive

$= \frac{1}{x} - 2$

#20

$$y = \ln(\csc x - \cot x)$$

$$= \frac{1}{\csc x - \cot x} \cdot (-\csc x \cot x - (-\csc^2 x))$$

$+ \csc^2 x$

$$= \frac{\csc^2 x - \csc x \cot x}{\csc x - \cot x} = \frac{\csc x \cancel{(\csc x - \cot x)}}{\cancel{(\csc x - \cot x)}} = \underline{\csc x}$$

$[\log_b x]'$?

$$\log_b x = \frac{\ln x}{\ln b} = \ln x \cdot \frac{1}{\ln b}$$

change of base law

$$[\log_b x]' = \frac{1}{x} \cdot \frac{1}{\ln b} = \frac{1}{x \ln b}$$

$f(x)$	$f'(x)$	$F(x)$
$\log_b x$	$\frac{1}{x \ln b}$	$\boxed{\frac{1}{\ln b}(x \ln x - x)}$ No need to memorize

---

$[b^x]'$? pg 433

OR

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$f(x) = \log_b x \qquad f'(x) = \frac{1}{x \ln b}$

$$f^{-1}(x) = b^x \qquad (f^{-1})'(x) = \frac{1}{f'(b^x)} = \frac{1}{\frac{1}{b^x \ln b}} = \underline{b^x \ln b}$$

$f(x)$	$f'(x)$	$F(x)$
b^x	$b^x \ln b$	$b^x / \ln b$

#8 $f(x) = \log_{10}(\sqrt{x})$

$f(x) = \frac{1}{2}\log_{10}(x)$ Log law, not deriv

$f'(x) = \frac{1}{2}\frac{1}{x\ln 10} = \frac{1}{x\,2\ln(10)}$

#17

$f(x) = x^5 + 5^x$

$f'(x) = 5x^4 + 5^x \ln 5$

#18 $g(x) = \underbrace{x \sin(2^x)}_{\text{product}}$

$1\cdot\sin(2^x) + x\cos(2^x)\cdot 2^x \ln 2$

$\sin(2^x) + (\ln 2)\, x\, 2^x \cos(2^x)$

#26

$F(t) = 3^{\cos 2t}$ chain

$F'(t) = \underbrace{3^{\cos 2t} \ln 3}\,(\cos 2t)'$

$3^{\cos 2t} \ln 3\,(-\sin(2t))\,2$

$= -2\ln 3 \sin(2t)\, 3^{\cos 2t}$

Pg 438

#71 $\int_2^4 \frac{3}{x}\,dx$

$3\int_2^4 \frac{1}{x}\,dx$

$3\Big[\ln|x|\Big]_2^4$

$3(\ln 4 - \ln 2)$

$\ln(2^2)$

$3(2\ln 2 - \ln 2)$

$\boxed{3\ln 2}$

#72 $\int_0^3 \frac{dx}{5x+1} = \frac{1}{5x+1}\,dx$

$u = 5x+1$

$du = 5\,dx$

$\frac{1}{5}\int_{0=x}^{3=x} \frac{5\,dx}{5x+1} = \frac{1}{5}\int_{0=x}^{3=x} \frac{1}{u}\,du$

$\frac{1}{5}\ln|u|\Big|_{0=x}^{3=x}$

$\frac{1}{5}\ln|5x+1|\Big|_0^3 = \frac{1}{5}(\ln 16 - \ln 1)$

$= \frac{1}{5}(\ln 16 - 0)$

$= \frac{1}{5}\ln 16$

$= \frac{1}{5}\ln(2^4)$

$\boxed{= \frac{4}{5}\ln 2}$

#81 $\int_0^4 2^s\,ds$

$\frac{2^s}{\ln 2}\Big|_0^4$

$= \frac{2^4}{\ln 2} - \frac{2^0}{\ln 2} = \frac{16-1}{\ln 2} = \boxed{\frac{15}{\ln 2}}$

6.4 Use logarithmic differentiation to derive certain functions.

$f(x) = \frac{(3x+5)^9\sqrt[3]{x^2+1}}{4^x x^7 (x+1)}$

$f(x) = x^x$

Derive:

#45 $y = \sqrt{\dfrac{x-1}{x^4+1}}$

$\ln y = \ln\sqrt{\dfrac{x-1}{x^4+1}}$ Log rules

$\ln y = \frac{1}{2}\ln\left(\dfrac{x-1}{x^4+1}\right)$

$\ln y = \frac{1}{2}\left(\ln(x-1) - \ln(x^4+1)\right)$

$\ln(y) = \frac{1}{2}\ln(x-1) - \frac{1}{2}\ln(x^4+1)$ Implicit diff

$\frac{1}{y}y' = \frac{1}{2}\frac{1}{x-1} - \frac{1}{2}\frac{4x^3}{x^4+1}$

$\frac{1}{y}y' = \dfrac{1}{2(x-1)} - \dfrac{2x^3}{x^4+1}$

$y' = y\left(\dfrac{1}{2(x-1)} - \dfrac{2x^3}{x^4+1}\right)$

$y' = \sqrt{\dfrac{x-1}{x^4+1}}\left(\dfrac{1}{2(x-1)} - \dfrac{2x^3}{x^4+1}\right)$

#46

$y = \sqrt{x}\, e^{x^2-x}(x+1)^{2/3}$

$\ln y = \ln\left(\sqrt{x}\, e^{x^2-x}(x+1)^{2/3}\right)$ Log laws

$= \ln(\sqrt{x}) + \ln(e^{x^2-x}) + \ln((x+1)^{2/3})$

$\ln y = \frac{1}{2}\ln x + x^2 - x + \frac{2}{3}\ln(x+1)$ Diff!

$\frac{1}{y}y' = \frac{1}{2}\frac{1}{x} + 2x - 1 + \frac{2}{3}\frac{1}{x+1}$

$$\frac{1}{y}y' = \frac{1}{2x} + \frac{2}{3(x+1)} + 2x - 1$$

$$y' = y\left(\frac{1}{2x} \sim\!\sim\!\sim \right)$$

$$y' = \sqrt{x}\, e^{x^2-x}(x+1)^{2/3}\left(\frac{1}{2x} + \frac{2}{3(x+1)} + 2x - 1\right)$$

$f(x) = a^b$ const. $f'(x) = 0$

$f(x) = x^b$ power rule. $f'(x) = bx^{b-1}$

$f(x) = a^x$ exp rule. $f'(x) = a^x \ln a$

$f(x) = x^x$ doesn't fit rules, but log diff can help.

$$y = x^x$$

$$\ln y = \ln(x^x)$$

$$\ln y = x \ln x$$

product rule

$$\frac{1}{y}y' = 1 \cdot \ln x + x\frac{1}{x}$$

$$\frac{1}{y}y' = \ln x + 1$$

$$y' = y(\ln x + 1)$$

$$y' = x^x(\ln x + 1)$$

Grows very fast!

vs e^x $f(x) = e^x$

- - -

Any $f(x) = g(x)^{h(x)}$ can be derived using Logarithmic Differentiation.

#22

$$H(z) = \ln\sqrt{\frac{a^2 - z^2}{a^2 + z^2}}$$

Do NOT do log diff!

There's already an ln!

$$H(z) = \frac{1}{2}\ln\left(\frac{a^2 - z^2}{a^2 + z^2}\right)$$

$$H(z) = \frac{1}{2}\left(\ln\left(a^2 - z^2\right) - \ln\left(a^2 + z^2\right)\right)$$

No rule allows us to expand $\log_b(a+c)$ or $\log_b(a-c)$.

Derive, continue as usual.

Proof of the power rule.

$$y = x^n$$

$$\ln y = \ln(x^n) \qquad \text{Log diff}$$

$$\ln y = n \ln x$$

$$\frac{1}{y} y' = n \frac{1}{x}$$

$$y' = y\, n \frac{1}{x}$$

$$y' = x^n n \frac{1}{x} = \underline{n x^{n-1}}$$