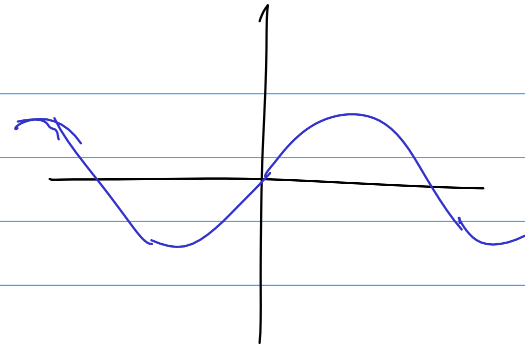
$\sin(-x) = -\sin(x)$?

This is true because $\sin(x)$ is odd.



In general, $f(-x) = -f(x)$ ONLY if f is odd.

1.3 Decompose a function into two or more functions.

Pg 44

#43 $F(x) = (2x+x^2)^4 = (f \circ g)(x) = f(g(x))$

$f(\underline{x}) = (\underline{x})^4$

$g(x) = 2x + x^2$

#45 $F(x) = \dfrac{\sqrt[3]{x}}{1+\sqrt[3]{x}} = (f \circ g)(x)$

$f(g(x))$

$f(x) = \dfrac{x}{1+x}$

$g(x) = \sqrt[3]{x}$

$f(g(x)) = f(\sqrt[3]{x}) = \dfrac{\sqrt[3]{x}}{1+\sqrt[3]{x}}$

$\text{trig}^2(x) = [\text{trig}(x)]^2$

#51 $f(g(h(t))) = \sin^2(\cos t) = S(t)$

$[\sin(\cos t)]^2$

$f(t) = t^2$

$g(t) = \sin t$

$h(t) = \cos t$

1.5 Determine the (one-, two-sided) limit of a function, if it exists.

$$f(x) = \frac{x-3}{x^2-9} \qquad f(3) = \text{undefined.} \quad \frac{0}{0}$$

Cannot divide by zero.

Simplify ?

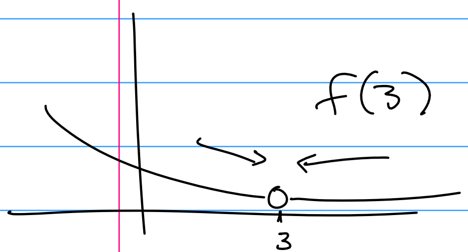
$$f(x) = \frac{x-3}{(x-3)(x+3)} = \frac{1}{x+3} \qquad f(3) = \frac{1}{6}$$

$$\frac{x-3}{x^2-9} = \frac{1}{x+3} ?$$ Only if this is true for every x.

But $f(3)$ here $\neq$ $f(3)$ here.

We can only say

$$\frac{x-3}{x^2-9} = \frac{1}{x+3}, \quad x \neq 3.$$



f(3) looks like it should be $\frac{1}{6}$! But is undefined.

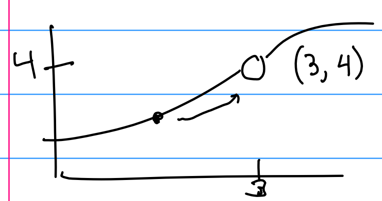
lim( ) is sort of like a function

☆ $\lim\limits_{x \to a} f(x) = L$

☆ The limit (as x approaches a) of f(x) is L.

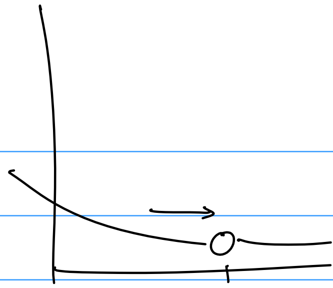
☆ As x gets closer to a, f(x) gets closer to L.

☆ We can make f(x) as close to L as we want by making x close enough to a.



$$\lim_{x \to 3} f(x) = 4$$

$f(x) = \dfrac{x-3}{x^2-9}$



$f(3) =$ undefined

but

$$\boxed{\lim_{x\to 3} f(x) = \frac{1}{6}.}$$

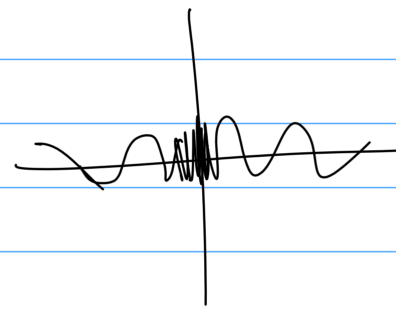
Based on graph,
table,
algebra.

This matters most.

Limits are concerned with values NEAR a number, not AT a number.

pg 53
Calculators have limited memory!

$f(x) = \sin\left(\dfrac{2\pi}{x}\right)$



$f(0)$ undef

$\lim_{x\to 0} f(x) = ?$

x	$\sin\left(\frac{2\pi}{x}\right)$
1	0
$1/2$	0
$1/3$	0
$1/4$	0
⋮	

As $x \to 0$,
$\sin\left(\frac{2\pi}{x}\right) \to 0.$

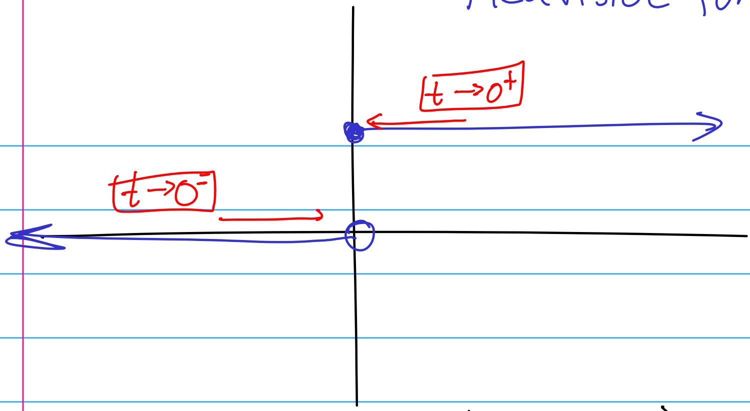
x	$\sin\left(\frac{2\pi}{x}\right)$
4	1
$4/5$	1
$4/9$	1
⋮	
↓	↓
0	1

As $x \to 0$,
$\sin\left(\frac{2\pi}{x}\right) \to 1$?

If there are multiple possibilities for L, the limit does not exist.

$$\lim_{x\to 0} \sin\left(\frac{2\pi}{x}\right) = \text{DNE}$$

Heaviside function



$$H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0 \end{cases}$$

$$H(0) = 1 \qquad \lim_{t \to 0} H(t) = \text{DNE}$$

from the left $\boxed{\lim_{t \to 0^-} H(t) = 0}$

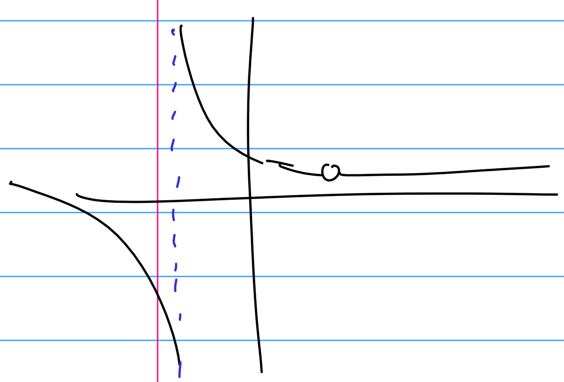
$t \to 0^-$, $t < 0$

from the right $\boxed{\lim_{t \to 0^+} H(t) = 1}$

when $t \to 0^+$, $t > 0$.

1.5 Describe behavior of functions near asymptotes, using limits.

$$f(x) = \frac{x-3}{x^2-9}$$



$f(-3) = \text{undefined}$

$$\lim_{x \to -3} f(x) = \text{DNE}$$

$$\lim_{x \to -3^+} f(x) = \text{DNE}$$ No one real number is being approached.

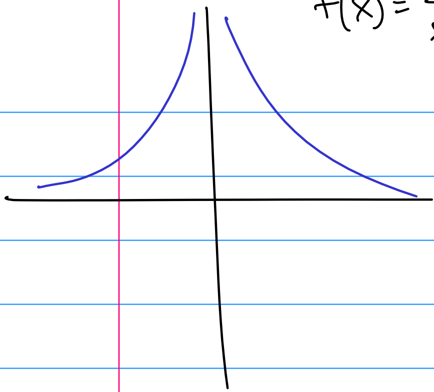
$$\lim_{x \to -3^-} f(x) = \text{DNE}$$

But this notation is not very descriptive.

$$\lim_{x \to -3^+} f(x) = \infty$$ ← Not a real number. Limit still DNE.

$$\lim_{x \to -3^-} f(x) = -\infty$$

$$\lim_{x \to -3} f(x) = \text{DNE}$$ ← Occurs if either one-sided limit fails to exist.



$f(x) = \frac{1}{x^2}$

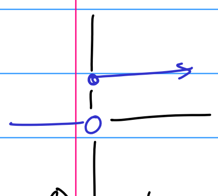
$$\underbrace{\lim_{x \to 0^+} f(x) = \infty}$$

$f(x)$ can be made as large as we want by taking x close enough to 0.

$$\lim_{x \to 0^-} f(x) = \infty$$

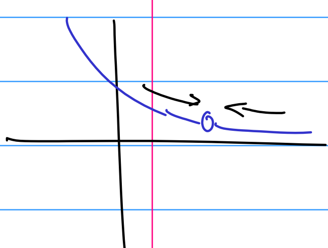
so $\lim_{x \to 0} f(x) = \infty$

$$\boxed{\text{If } \lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x), \text{ then } \lim_{x \to a} = L.}$$



$\lim_{t \to 0} H(t) = DNE$

$0 \neq 1$



$$\left.\begin{array}{l} \lim_{x \to 3^+} f(x) = \frac{1}{6} \\ \lim_{x \to 3^-} f(x) = \frac{1}{6} \end{array}\right\} \Rightarrow \lim_{x \to 3} f(x) = \frac{1}{6}$$

Pg 60

#7

$\lim_{t \to 0^-} g(t) = -1$

$\lim_{t \to 0^+} g(t) = -2$

$\lim_{t \to 0} g(t) = DNE$

$\lim_{t \to 4} g(t) = 3$

$\lim_{t \to 2^-} g(t) = 2$

$\lim_{t \to 2^+} g(t) = 0$

$\lim_{t \to 2} g(t) = DNE$

$g(2) = 1$