$f(x) = \sqrt{x^2 + 1}$?

$f'(x) =$ ______ ?

$\cancel{f(x) = (x^2+1)^2}$

$\cancel{\downarrow}$

$\cancel{2(2x)}$

$\cancel{4x}$?

Name	$f(x)$	$f'(x)$
Chain	$g(h(x))$	$g'(h(x)) \cdot h'(x)$

In Liebniz,

if $y = f(u)$

and $u = g(x)$,

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} \ !$$

$(x^2)'$

$\frac{d}{dx}(x^2)$ professional!!

$f(x) = \sqrt{x^2+1}$

$= (g \circ h)(x) = g(h(x))$

$g(x) = x^{1/2}$

$g(x) = \sqrt{x} \rightarrow g'(x) = \frac{1}{2}x^{-1/2}$

$h(x) = x^2+1 \rightarrow h'(x) = 2x$

$f'(x) =$

$g'(h(x)) \cdot h'(x)$

$g'(x^2+1)$

$\frac{1}{2}(x^2+1)^{-1/2} \cdot 2x$

$x(x^2+1)^{-1/2}$

$$\frac{x}{(x^2+1)^{1/2}}$$

$$= \frac{x}{\sqrt{x^2+1}}$$

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#8 $F(x) = (1+x+x^2)^{99}$

$= g(h(x))$

$g(x) = x^{99} \longrightarrow g'(x) = 99x^{98}$

$h(x) = 1+x+x^2 \longrightarrow h'(x) = 2x+1$

$F'(x) = 99(1+x+x^2)^{98} \cdot (2x+1)$

#13 $f(\theta) = \cos(\theta^2) = g(h(\theta))$

$g(\theta) = \cos\theta \qquad g'(\theta) = -\sin\theta$

$h(\theta) = \theta^2 \qquad h'(\theta) = 2\theta$

$f'(\theta) = g'(h(\theta)) \cdot h'(\theta)$

$-\sin(\theta^2) \cdot 2\theta$

#39

$f(t) = \underline{\tan}(\underline{\sec(\cos t)})$

$\boxed{[f(g(h(x)))]' = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)}$

Outline:

$\tan'(\sec(\cos t)) \cdot [\sec(\cos t)]'$

$\underline{\tan}'(\sec(\cos t)) \cdot \underline{\sec}'(\cos t) \cdot \underline{\cos}'(t)$

$$\sec^2(\sec(\cos t)) \cdot \sec(\cos t)\tan(\cos t) \cdot -\sin t$$

#21

$$g(u) = \left(\frac{u^3-1}{u^3+1}\right)^8$$

$$= p(q(u))$$

$$p(u) = u^8 \longrightarrow p'(u) = 8u^7$$

$$q(u) = \frac{u^3-1}{u^3+1} \longrightarrow q'(u) = \frac{(u^3+1)(3u^2)-(u^3-1)(3u^2)}{(u^3+1)^2}$$

$$= \frac{3u^5+3u^2-3u^5+3u^2}{(u^3+1)^2} = \frac{6u^2}{(u^3+1)^2}$$

$$p'(q(u)) \qquad q'(u)$$

$$g'(u) = 8\left(\frac{u^3-1}{u^3+1}\right)^7 \cdot \frac{6u^2}{(u^3+1)^2}$$

Wkst answers

1. 0
2. 3
3. -1
4. $2x+10$
5. $0.1x^{-0.9}$
6. $f(x) = x^{1/6}$ $\qquad f'(x) = \frac{1}{6}x^{-5/6}$

7. $f(x) = 9\frac{1}{x} + x\frac{1}{9}$

$f'(x) = 9\left(-\frac{1}{x^2}\right) + \frac{1}{9} = \frac{1}{9} - \frac{9}{x^2}$

const mult   recip   const mult

8. $\frac{x\cos x - \sin x}{x^2}$

9. $3x^{-1/4} + 2x^{-2/4} + x^{-3/4}$

10. $\frac{x-4}{\sqrt{x^2-8x}}$

11. $f(x) = \frac{1}{\tan x} = \cot x$

$f'(x) = -\csc^2 x$

12. $-\sin(\sqrt{x}) \cdot \frac{1}{2}x^{-1/2}$

13. $f(x) = (x-3)^{-2}$ $f'(x) = -2(x-3)^{-3}(1)$

$= \frac{-2}{(x-3)^3}$

14. $\sec(\sec x)\tan(\sec x)\sec x \tan x$

15. $\frac{11}{(2x+1)^2}$

16. $-8\cos^7 x \sin x$

17. $f(x) = \frac{x-2}{x+5}$ $f'(x) = \frac{7}{(x+5)^2}$

$(x \neq -2)$ $(x \neq -2)$

18. $2(\cos^2 x - \sin^2 x)$

19. $10(x+1)^9(2x-1)^{11} + (x+1)^{10}\, 11(2x-1)^{10}(2)$

20. $f(x) = \overset{x^{6/2}}{x^3}\sin(x)\overset{x^{1/2}}{\sqrt{x}}$

$f(x) = x^{7/2}\sin(x) \qquad f'(x) = \underline{\frac{7}{2}x^{5/2}\sin x + x^{7/2}\cos x}$

$(x+1)^{10}(2x-1)^{11}$ product rule

$[(x+1)^{10}]'(2x-1)^{11} + (x+1)^{10}[(2x-1)^{11}]'$

chain $[10(x+1)^9(1)](2x-1)^{11} + (x+1)^{10}[11(2x-1)^{10}(2)]$ chain

$= 10(x+1)^9(2x-1)^{11} + 22(x+1)^{10}(2x-1)^{10}$

$2(\sin x)(\cos x)$ $\qquad\qquad$ $\sin(2x)$

const mult $\qquad$ ↓ product

$2[\cos x \cos x + \sin x(-\sin x)]$

$2(\cos^2 x - \sin^2 x)$ $\qquad\qquad$ $2\cos(2x)$