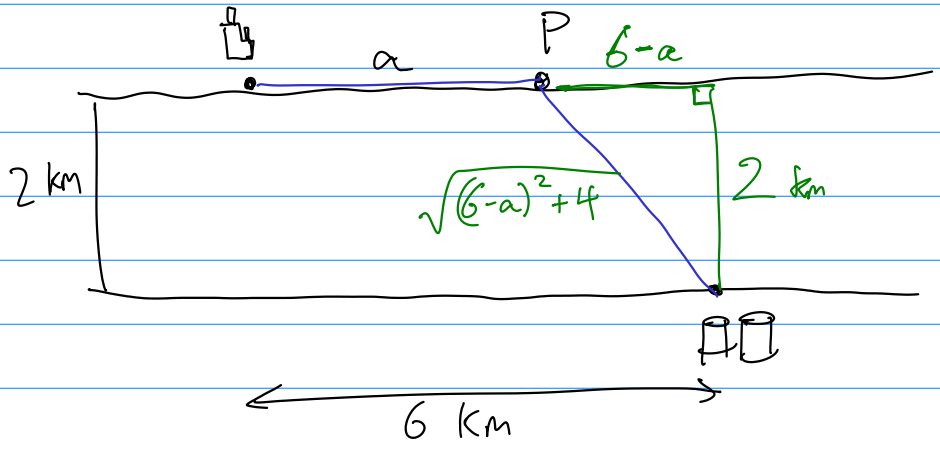
Pipe: $400,000/km over land
$800,000/km under river

Pg 267
#51



Minimize cost $C = 800000\sqrt{(6-a)^2+4} + 400000a$

$0 < a < 6$ $C = 400000\left(2\sqrt{a^2-12a+36+4} + a\right)$

$C = 400000\left(2\sqrt{a^2-12a+40} + a\right)$

$C' = 400000\left(2 \cdot \frac{1}{2}(a^2-12a+40)^{-1/2} \cdot (2a-12) + 1\right)$

$C' = 400000\left(\frac{2a-12}{\sqrt{a^2-12a+40}} + 1\right) = 0$ or ~~DNE~~ ~~$(6-a)^2+4 > 0$~~

$\frac{2a-12}{\sqrt{a^2-12a+40}} = -1$

$2a - 12 = -\sqrt{a^2-12a+40}$

$12 - 2a = \sqrt{a^2-12a+40}$

$144 - 48a + 4a^2 = a^2 - 12a + 40$

$$3a^2 - 36a + 104 = 0$$

$$a = \frac{36 \pm \sqrt{1296 - 1248}}{6}$$

$$a = \frac{36 \pm \sqrt{48}}{6}$$

$$a = \frac{36}{6} \pm \frac{\sqrt{48}}{6}$$

$$6 \pm \frac{4\sqrt{3}}{6}$$

$$a = 6 \pm \frac{2\sqrt{3}}{3}$$

$$0 < a < 6$$

$$a \neq 6 + \frac{2\sqrt{3}}{3}$$

$$a = 6 - \frac{2\sqrt{3}}{3} \quad \text{crit pt.}$$

$$\approx 4.8453$$

$$C = 800000\sqrt{(6-a)^2+4} + 400000a$$

$C(0) = \$5059644$

$C(4.8453) \approx \$3785640$

$C(6) = \$4,000,000$

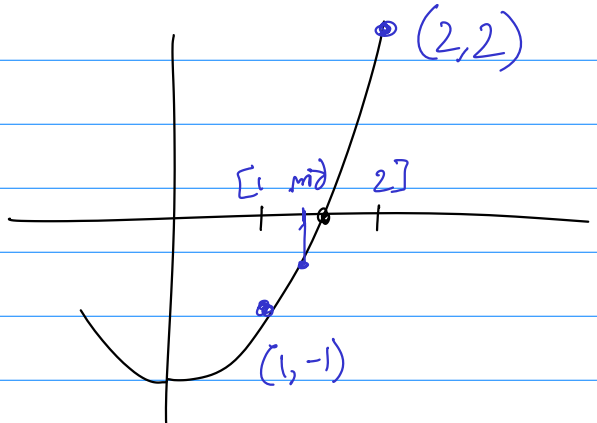
3.8 $\quad \sqrt{2} \approx ?$

$\sqrt{2} = x$ such that $x^2 = 2$

or

$f(x) = x^2 - 2 = 0$

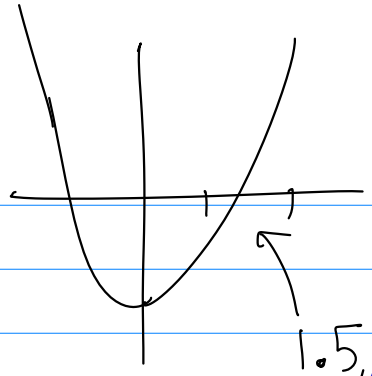
Bisection method



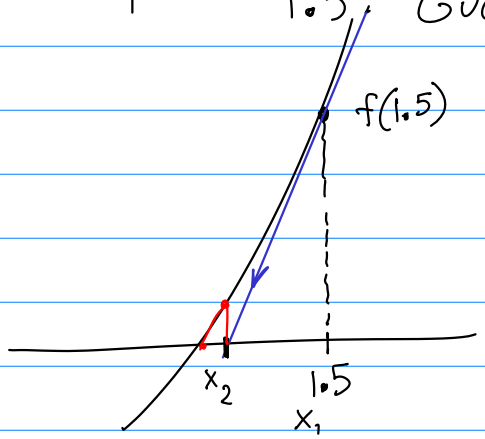
Newton's method

X_1 = initial guess.

$$X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)}$$



1.5? Guess.



$$y_2 - y_1 = f'(x_1)(x_2 - x_1)$$

$$0 - f(x_1) = f'(x_1)(x_2 - x_1)$$

$$\frac{-f(x_1)}{f'(x_1)} = x_2 - x_1$$

$$\boxed{x_1 - \frac{f(x_1)}{f'(x_1)} = x_2}$$

3.9 Find antiderivatives of functions. (General and specific)

$\frac{x^3}{3}$ $\quad \frac{x^3}{3} + 1 \quad \frac{x^3}{3} - 7$ $\quad \boxed{\frac{x^3}{3} + C}$ General antiderivative.

x^2

$2x$

$f(x)$: what derives to $f(x)$?

The antiderivative $F(x) + C$.

Arbitrary constant

$-\cos x + C$

$\sin x$

$f(x) = \sin x$

$F(x) = -\cos x + C$

etc.

$f(x) = 3x^2 - 9x + 1$

$F(x) = \underline{\quad ? \quad} + C$

$f(x) = 3$ $\quad$ $f(x) = -7$

$F(x) = 3x + C$ $\quad$ $F(x) = -7x + C$

	$f(x)$	$f'(x)$	$F(x)$
Constant	k	0	$kx + C$
Identity	x	1	$\frac{1}{2}x^2 + C$
Linear	$mx+b$	m	$\frac{1}{2}mx^2 + bx + C$
Power	x^n	nx^{n-1}	$\frac{1}{n+1}x^{n+1} + C$
Root	$\sqrt[n]{x} = x^{1/n}$	see above	see above
Recip	x^{-1}	$-\frac{1}{x^2}$	Ch 6
Const mult	$k \cdot g(x)$	$k \cdot g'(x)$	$k \cdot G(x) + C$
Sum/diff	$g(x) \pm h(x)$	$g'(x) \pm h'(x)$	$G(x) \pm H(x) + C$
Product	gh	$g'h + gh'$	Case-by-case ... or Calc II
Quotient	g/h	$\frac{hg' - gh'}{h^2}$	
Chain	$g(h(x))$	$g'(h(x)) \cdot h'(x)$	
Trig	$\sin x$	$\cos x$	$-\cos x + C$
	$\cos x$	$-\sin x$	$\sin x + C$

$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$
$\cot x$	$-\csc^2 x$

ch 6
or
Calc II