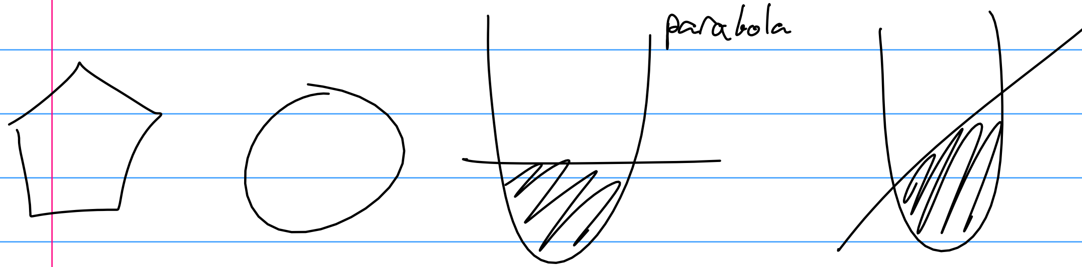
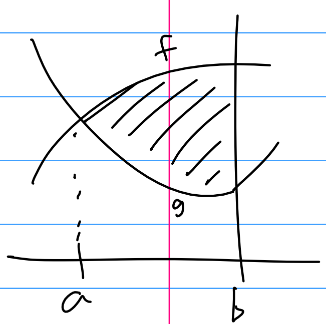
5.1 Compute the area between two curves.



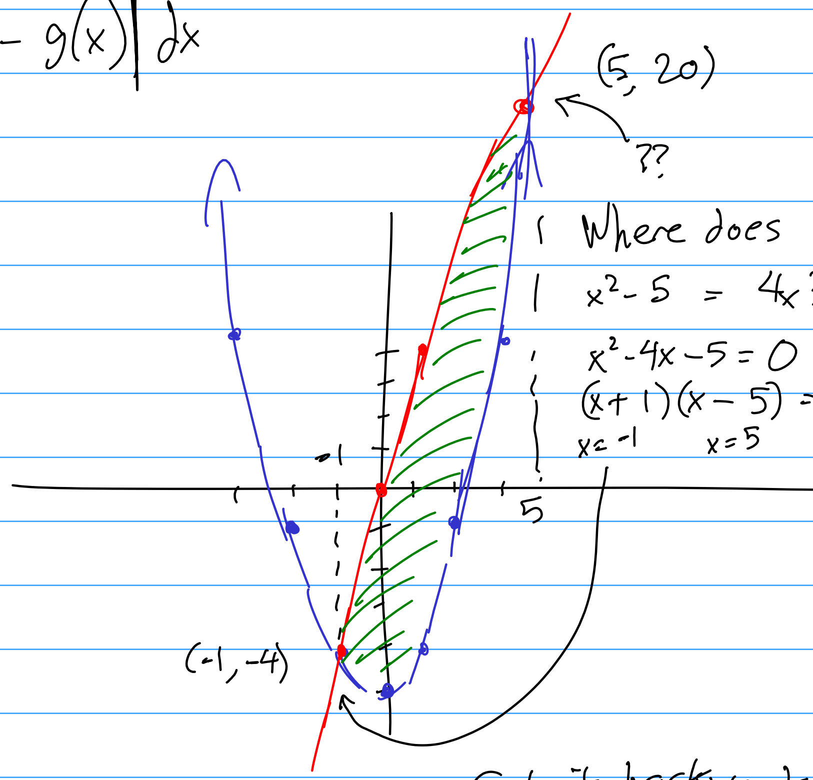
Area between $f(x)$ and $g(x)$ from a to b



$$\text{Area} = \int_a^b f(x) - \int_a^b g(x)$$

$$\text{Area} = \int_a^b \left| f(x) - g(x) \right| dx$$

Ex: $f(x) = x^2 - 5$
$g(x) = 4x$



Where does $x^2 - 5 = 4x$?
$x^2 - 4x - 5 = 0$
$(x+1)(x-5) = 0$
$x = -1 \qquad x = 5$

$$A = \int_{-1}^{5} (x^2 - 5) - 4x \, dx$$

$$= \int_{-1}^{5} x^2 - 4x - 5 \, dx$$

$$\frac{1}{3}x^3 - 2x^2 - 5x \Big|_{-1}^{5}$$

$$= \left(\frac{1}{3}125 - 2\cdot 25 - 25\right) - \left(-\frac{1}{3} \overset{+3}{\cancel{- 2 + 5}}\right)$$

$$\frac{125}{3} - 75 + \frac{1}{3} - 3$$

$$\frac{126}{3} - 78 = 42 - 78 = -36 \quad ?$$

Got it backwards?

$$\int_a^b g(x) - f(x) \, dx$$

is just

$$-\int_a^b f(x) - g(x) \, dx.$$

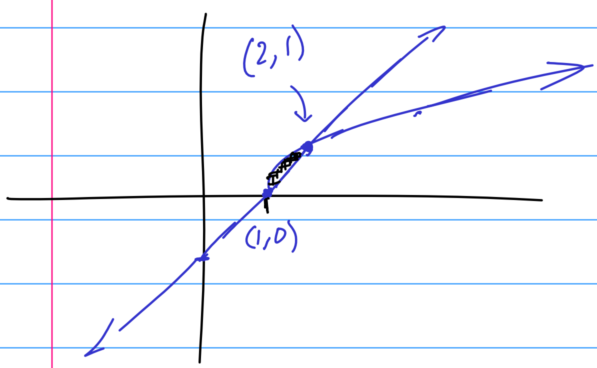
Area: 36.

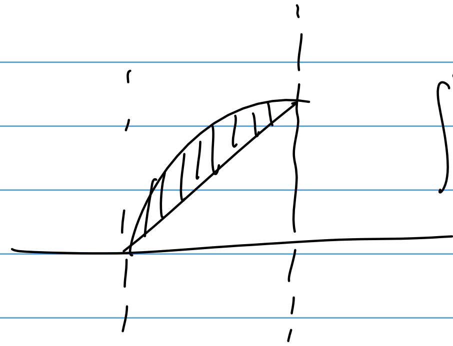
Pg 362

#18 $y = \sqrt{x-1}$ $\quad x - y = 1$

$x = 1 + y$

$y = \sqrt{x-1}$ $\quad y = x - 1$





$$\int_1^2 \sqrt{x-1} - (x-1)\,dx$$

u subst?

$u = x - 1$
$du = dx$

$$\int_{\cancel{1}=x}^{\cancel{2}=x} \sqrt{u} - u\,du \qquad u=1,\ u=0$$

$$\int_0^1 \sqrt{u} - u\,du \qquad \frac{2}{3}u^{3/2} - \frac{1}{2}u^2 \Big|_0^1$$

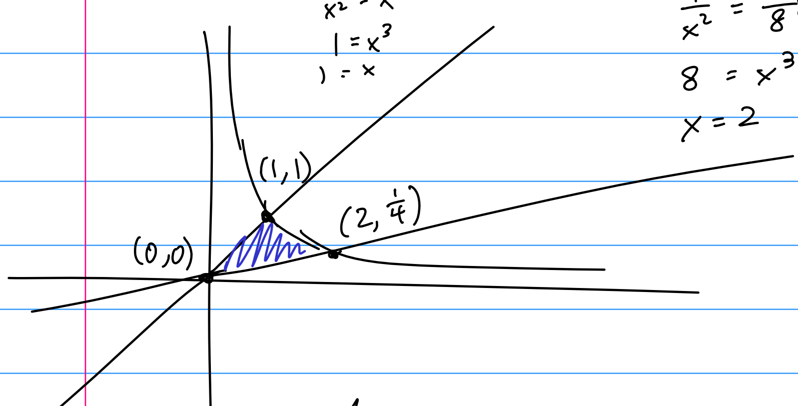
$$\left(\frac{2}{3} - \frac{1}{2}\right) - (0 - 0)$$

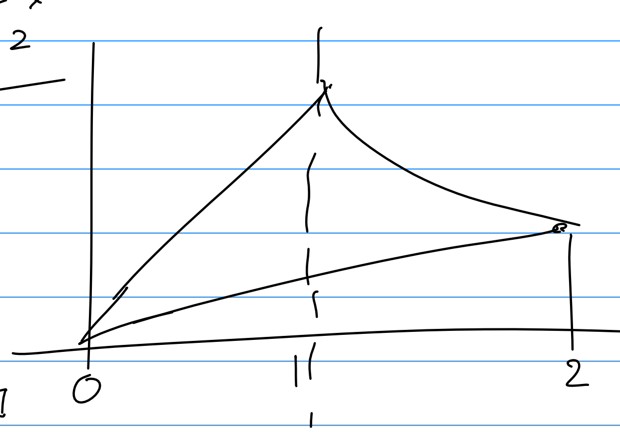
$$= \frac{4}{6} - \frac{3}{6} = \boxed{\frac{1}{6}}$$

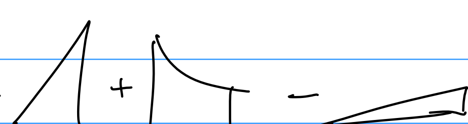
#27 $y = \frac{1}{x^2}$ $\quad y = x$ $\quad y = \frac{1}{8}x$

$\frac{1}{x^2} = x$
$1 = x^3$
$1 = x$

$\frac{1}{x^2} = \frac{1}{8}x$
$8 = x^3$
$x = 2$





Area = 

$$\int_0^1 x\,dx + \int_1^2 \frac{1}{x^2}\,dx - \int_0^2 \frac{1}{8}x$$

(x^{-2})

$$= \frac{1}{2}x^2\Big|_0^1 + -x^{-1}\Big|_1^2 - \frac{1}{8}\,\frac{1}{2}x^2\Big|_0^2$$

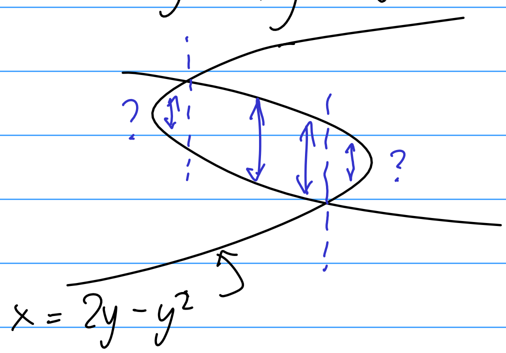
$$\frac{1}{2}(1) + \left(-(2^{-1}) - -(1^{-1})\right) - \frac{1}{8}\frac{1}{2}(2^2)$$

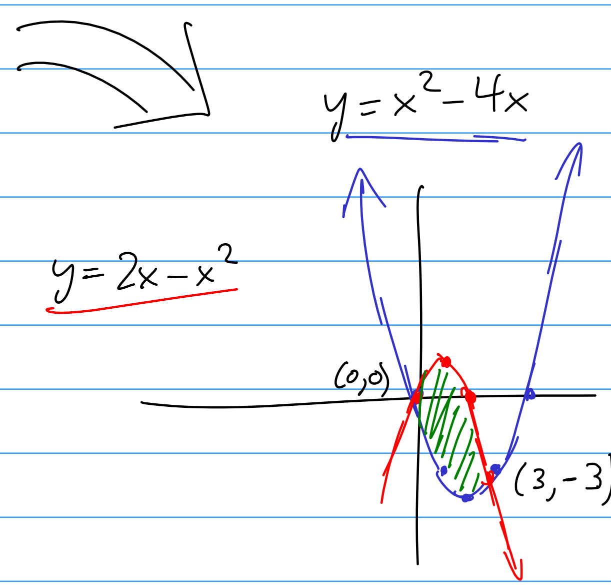
$$\cancel{\frac{1}{2} + -\frac{1}{2}} + 1 - \frac{1}{16}4$$

$$1 - \frac{1}{4} = \boxed{\frac{3}{4}}$$

#4 $x = y^2 - 4y$

Can swap x & y.



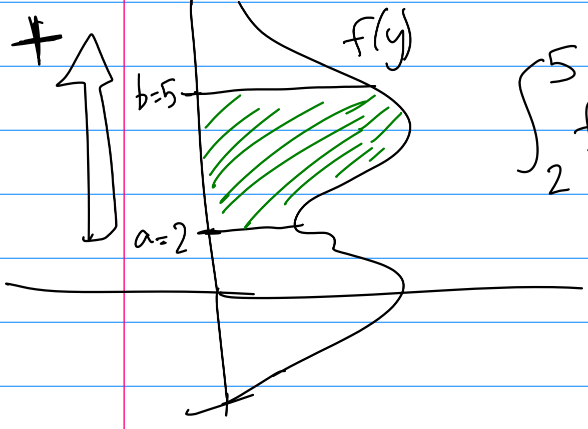


$$\int_0^3 (2x - x^2) - (x^2 - 4x)\,dx$$

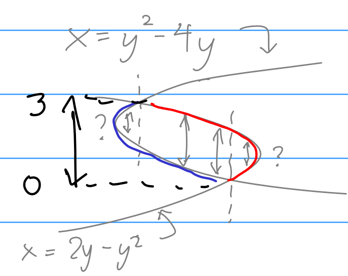
$$\int_0^3 6x - 2x^2\,dx$$

etc.

We can integrate a function of y with respect to y.



$$\int_2^5 f(y)\,dy$$



$$\int_0^3 (2y - y^2) - (y^2 - 4y)\,dy$$

Same result.

#20

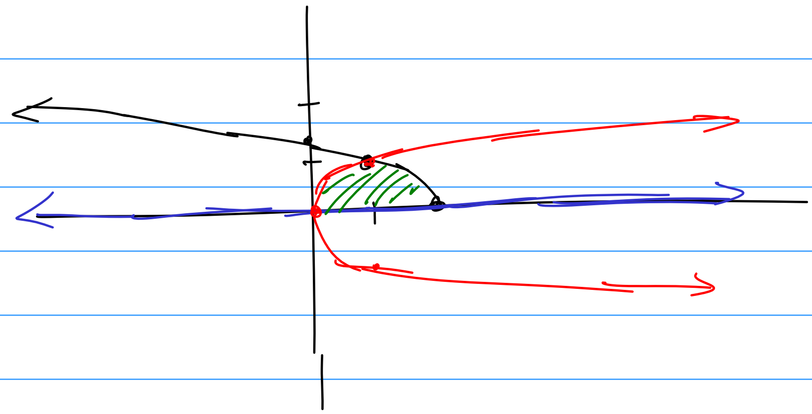
$x = y^4$ $\qquad$ $y = \sqrt{2-x}$ $\qquad$ $y = 0$

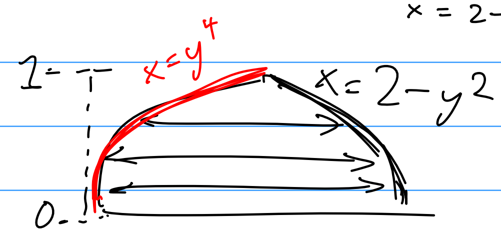
$y = \sqrt{2-x}$

$y^2 = 2-x$

$y^2 + x = 2$

$x = 2-y^2$

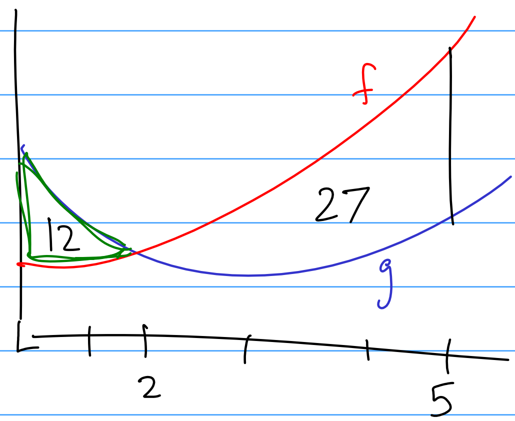




$$\int_0^1 (2-y^2) - y^4 \, dy$$

etc.

#29



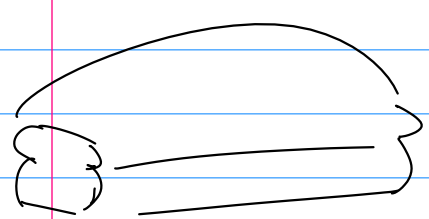
$f - g \geq 0$ everywhere?

Total area = 39

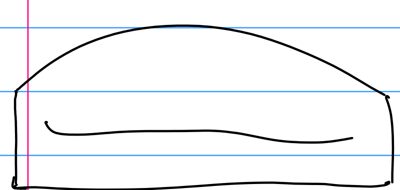
but

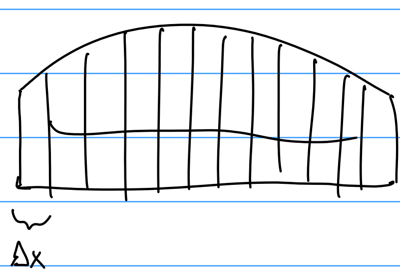
$$\int_0^5 f-g \, dx = 15.$$

5.2 Calculate the volume of solids using the disk/washer method. (No need to memorize the formula)

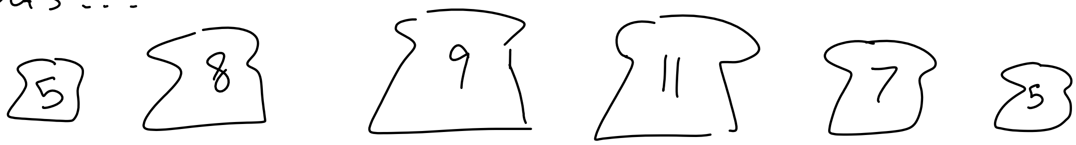


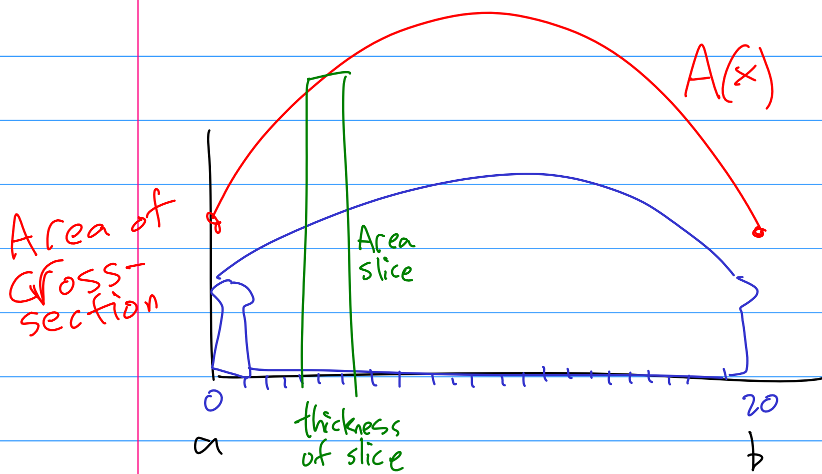
slices





Areas . . .

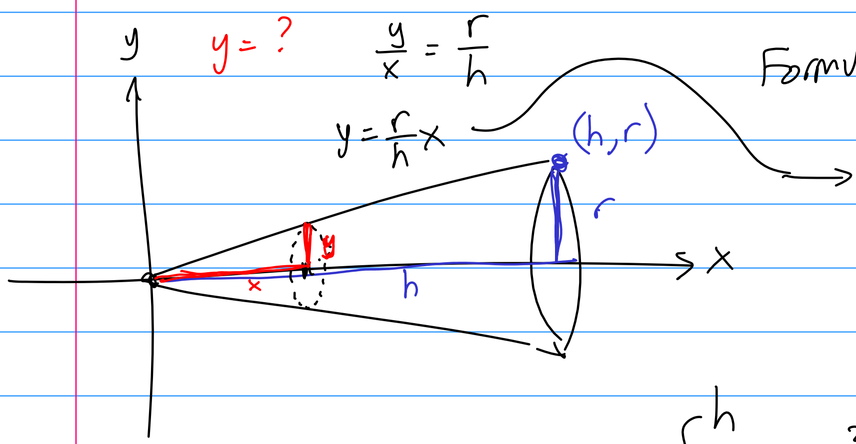




$$\text{Volume} = \int_a^b A(x)\,dx \;!$$

pg 368 Volume of a sphere

Volume of a cone $= \frac{1}{3}\pi r^2 h$?



Formula for cross-section area?

$$A(x) = \pi\left(\frac{r}{h}x\right)^2$$

$$= \pi\frac{r^2}{h^2}x^2$$

$$V = \int_0^h \pi\frac{r^2}{h^2}x^2\,dx$$

$$\pi\frac{r^2}{h^2}\frac{1}{3}x^3\Big|_0^h$$

$$\pi\frac{r^2}{h^2}\frac{1}{3}h^3 = \boxed{\frac{1}{3}\pi r^2 h}$$