Ch 1 Limits | Ch 2 Derivs | Ch 3 Curve analysis | Ch 4 Integrals | Ch 6 Exp/Log | Ch 5 Solids

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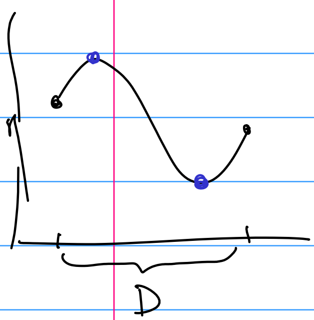
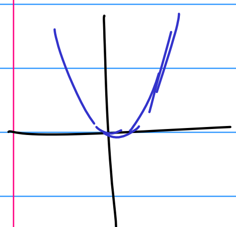
3.1 Find the local and absolute minimum/maximum of a function on an interval.

Interval/Domain



$f(c)$ is the absolute maximum of f on D if $f(c) \geq f(x)$ for all x in D.

$f(c)$ is the absolute minimum of f on D if $f(c) \leq f(x)$ for all x in D.



$f(x) = x^2$: $(0,0)$ is on this curve. (c, $f(c)$)

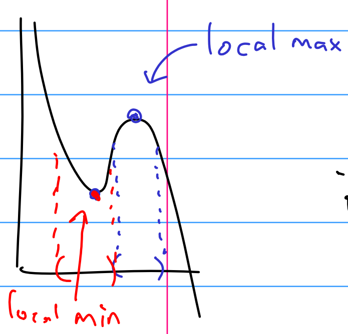
0 is the absolute minimum of $f(x)$ because $x^2 \geq 0$ for all x.

$0 \leq x^2$

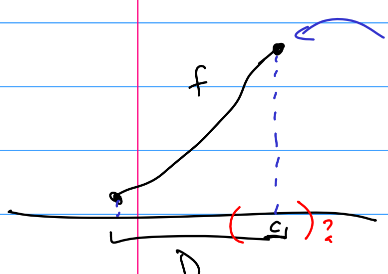
Pg 264

Local:

$f(c)$ is a local maximum of $f(x)$ on D if $f(c) \geq f(x)$ on some open interval in D, containing c.



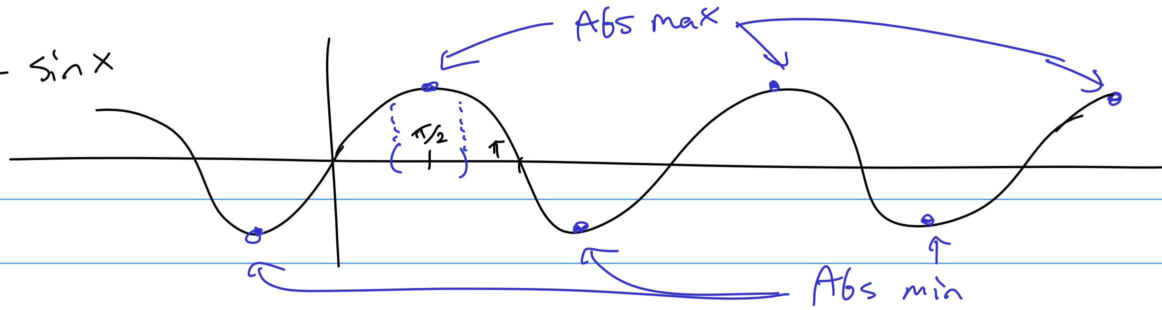
$f(c)$ is a local minimum of $f(x)$ on D if $f(c) \leq f(x)$ on some open interval in D, containing c.



Absolute max? Yes. $f(c) \geq f(x)$ for all x in D.

Local max? No. There is no open interval inside D that contains c.

$f(x) = \sin x$



Abs Max: 1.

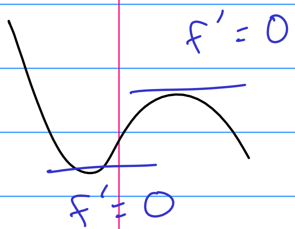
occurs in inf many places.

Abs max location

$\boxed{(\pi/2, 1)}$

Abs min: -1.

Each location of an abs min is also the location of a local min.



Extrema for $f(x)$ (Max/mins) tend to occur when $f'(x) = 0$.

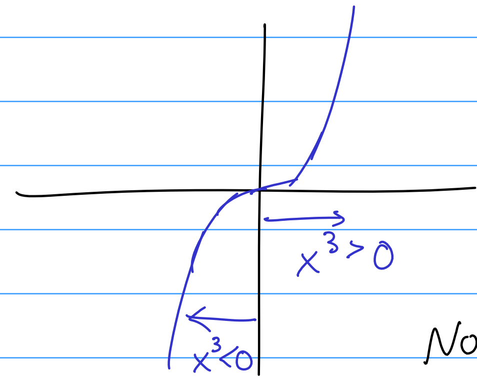
but not always!

$f(x) = x^3$

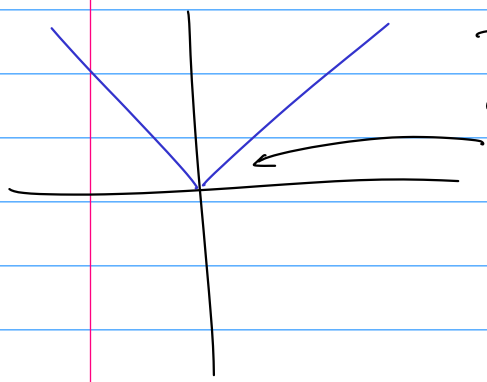
$f'(x) = 3x^2$ $\qquad 3x^2 = 0$

$x = 0.$

Extreme at $x = 0$?



No. :(



There is an abs/local minimum at 0

since $|x| \geq 0$ for all x.

But $|x|$ is not differentiable at 0.

"$f'(x) = 0$" would fail to catch this.

Pg 207

Fermat's Thm: If f has a local min/max at c,

AND if $f'(c)$ exists,

$\Longrightarrow$ then $f'(c) = 0$.

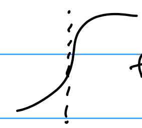
$\not\Leftarrow$. Counter example: $f(x) = x^3$

Def) A critical number of f is a number c in the domain of f such that $f'(c) = 0$ OR $f'(c)$ Does not exist.

Thm) If f has a local min/max at c, then c is a critical number of f.

$\Longrightarrow$

$\not\Longleftarrow$



$f'(c)$ DNE, but not a min/max.

Closed Interval Method — To find abs min/max on a continuous function f on a closed interval $[a,b]$:

- ☆ Find values of f at critical numbers of f in (a,b).
- ☆ Find values of f at the endpoints a and b.

The largest of the above values is the abs max.
" smallest " " " abs min.

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pg 212

\# 46 $\quad f(x) = 5 + 54x - 2x^3 \quad [0,4]$

$f'(x) = 0$? or DNE?

$f'(x) = 54 - 6x^2 = 0$ or ~~DNE~~ poly.

$= 6(9 - x^2) = 0$

$6(3-x)(3+x) = 0$

$x = 3 \quad x = \cancel{-3}$ ← Interval is only $[0,4]$

Critical numbers of f.

Endpoints:

$f(0) = 5$

$f(4) = 5 + 54(4) - 2(64)$

$= 5 + 216 - 128$

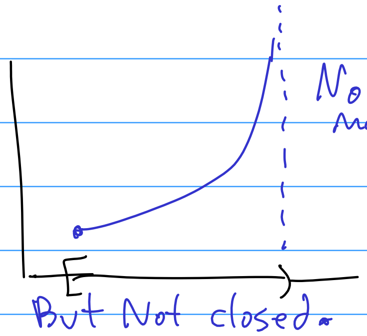
$=$

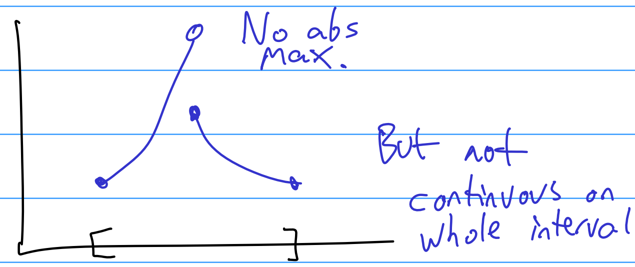
$f(4) = 93$ $\qquad$ $f(3) = 5 + 54(3) - 2(27)$

$= 113$

Abs max of 113 which occurs when $x = 3$.
Abs min of 5 which occurs when $x = 0$.

Closed Interval method works because of continuity!





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pg 212

#51 $\quad f(x) = x + \frac{1}{x} \quad [0.2, 4]$

$f'(x) = 1 - \frac{1}{x^2} = \underline{0}$ or $\underline{DNE}$

$1 - \frac{1}{x^2} = 0$

$1 = \frac{1}{x^2}$

$x^2 = 1$

$x = \cancel{\pm} 1$ $\qquad$ -1 is not in $[0.2, 4]$.

~~$f'(0)$ DNE but 0 is not in $[0.2, 4]$.~~

$\boxed{f(x) = x + \frac{1}{x}}$

end $\quad f(0.2) = 0.2 + 5 = 5.2$

crit $\quad f(1) = \quad = 2$

end $\quad f(4) = 4 + \frac{1}{4} = 4.25$

Abs max is 5.2, at $x = 0.2$
Abs min is 2, at $x = 1$.

3.2 Find numbers that satisfy the conclusions of Rolle's Thm and the MVT. (No need to memorize the theorems.)

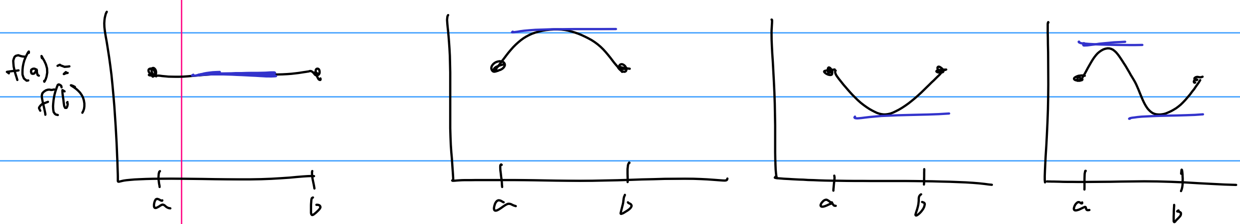
Rolle's Theorem

Let f be a function satisfying:

- ☆ f continuous on $[a,b]$
- ☆ f differentiable on (a,b)
- ☆ $f(a) = f(b)$

Then there exists a c in (a,b) such that $f'(c) = 0$.

pg 215 has proof



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pg 220

#7 $\quad f(x) = \sin\left(\frac{x}{2}\right) \qquad \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

☆ Continuous on $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$? Yes.

$\sin x$ cont everywhere, $\frac{x}{2}$ cont everywhere,

Compositions cont(cont) = cont.

☆ Diff'able on $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$?

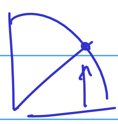
$f'(x) = \cos\left(\frac{x}{2}\right) \cdot \frac{1}{2}$ (chain rule)

$= \frac{1}{2}\cos\left(\frac{x}{2}\right)$ is defined everywhere

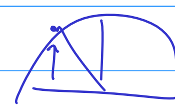
so $f(x)$ is diff'able everywhere.

☆ $f(a) = f(b)$?

$\sin\left(\frac{\pi/2}{2}\right) \qquad \sin\left(\frac{3\pi/2}{2}\right)$



$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \qquad \sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$



$=$

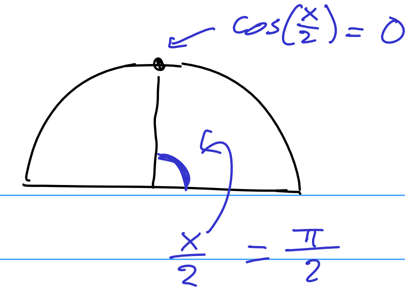
By Rolle's Thm, there exists c such that $f'(c) = 0$.

$$f'(x) = \frac{1}{2}\cos\left(\frac{x}{2}\right) = 0$$

$$\cos\left(\frac{x}{2}\right) = 0$$

$$\frac{x}{2} = \frac{\pi}{2}$$

$$\boxed{x = \pi} \qquad \pi \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$$



#9 $\quad f(x) = 1 - x^{2/3}$.

$f(-1) = f(1)$ but there is no c in $(-1, 1)$ such that $f'(c) = 0$.

$f(-1) = 1 - (-1)^{2/3} = 1 - 1 = 0$

$f(1) = 1 - (1)^{2/3} = 1 - 1 = 0$

$$f'(x) = -\frac{2}{3}x^{-1/3} = 0$$

$$-\frac{2}{3}\frac{1}{\sqrt[3]{x}} = 0$$

$$\frac{1}{\sqrt[3]{x}} = 0$$

$$1 = 0 \quad ??$$

$\Rightarrow$ There is no such c.

$f(x)$ not diff'able?

$f'(x) = -\frac{2}{3}\frac{1}{\sqrt[3]{x}}$ which does not exist when $x = 0$.

So this does not contradict Rolle's Thm.