$e^{\ln x} = x$

$\ln(e^x) = x$

$f(x) = e^x$

$f^{-1}(x) = \ln x = \text{"}\log_e x\text{"}$

$f(x) = 2^x$

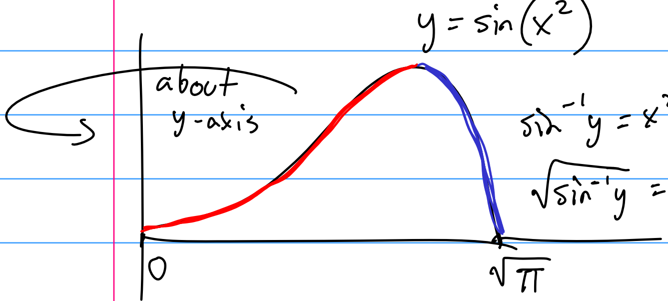
$f^{-1}(x) = \log_2 x$

$2^{\log_2 x} = x$

$\log_2(2^x) = x$

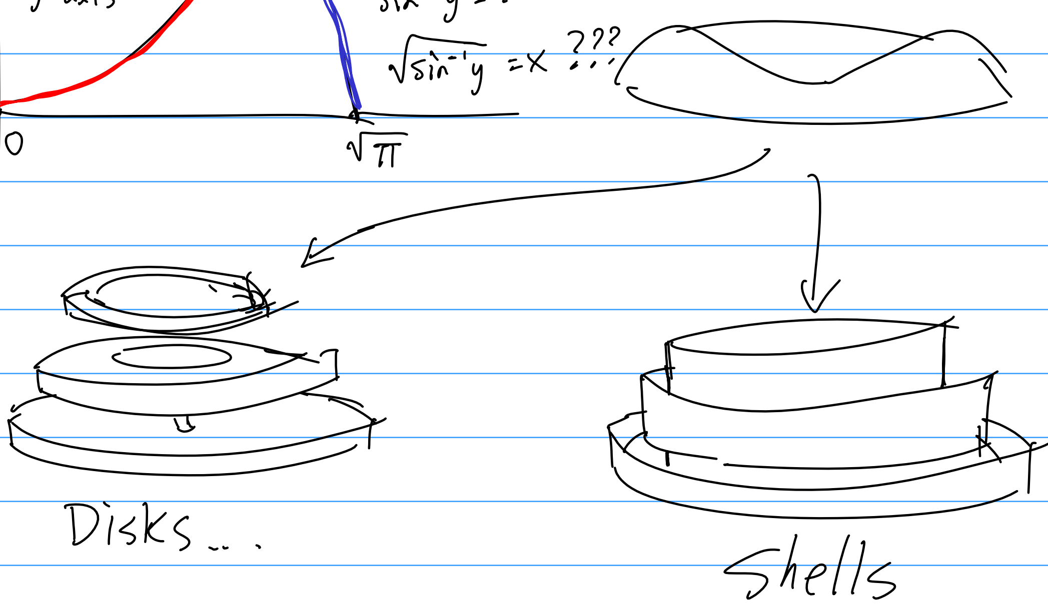
Pg 381

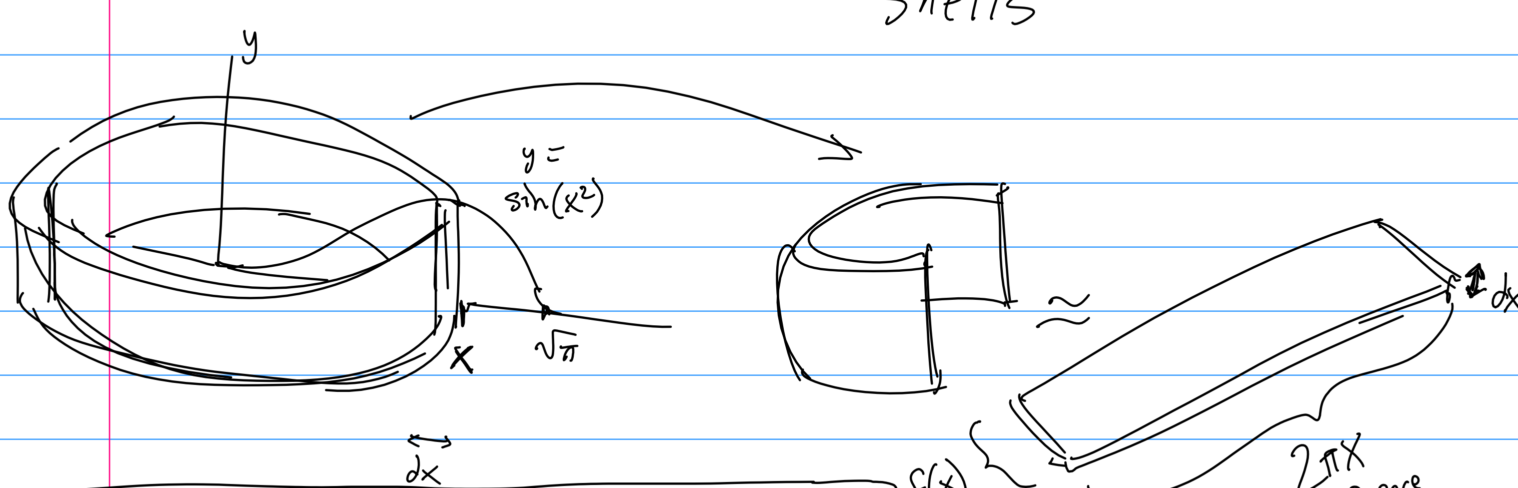
#2 2 disk methods?



$\sin^{-1} y = x^2$

$\sqrt{\sin^{-1} y} = x$???





Region under f(x) from a to b is revolved about the y-axis

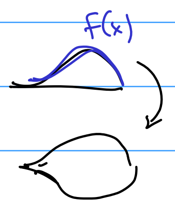
$$V = \int_a^b 2\pi x f(x)\, dx$$

"Shell method"

Disk/Washer:

$y = f(x)$

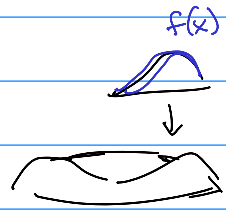
x-axis

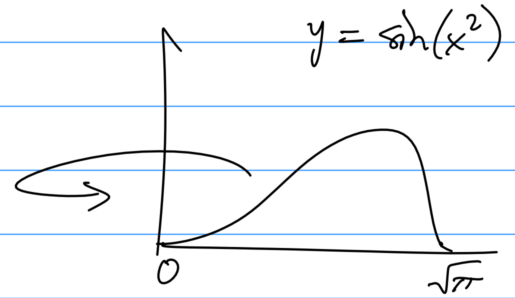


Shell:

$y = f(x)$

y-axis





Endpoints still along x-axis!

$$\int_0^{\sqrt{\pi}} 2\pi x \sin(x^2)\,dx$$

$$\pi \int_0^{\sqrt{\pi}} \sin(x^2)\, 2x\, dx \qquad u = x^2 \quad du = 2x\,dx$$

$$\pi \int_{0=x}^{\sqrt{\pi}=x} \sin u\, du = \pi(-\cos u)\Big|_{0=x}^{\sqrt{\pi}=x} = \pi(-\cos(x^2))\Big|_0^{\sqrt{\pi}} =$$

$$\pi(-\cos(\pi) - -\cos 0)$$

$$\pi(-(-1) - -(1))$$

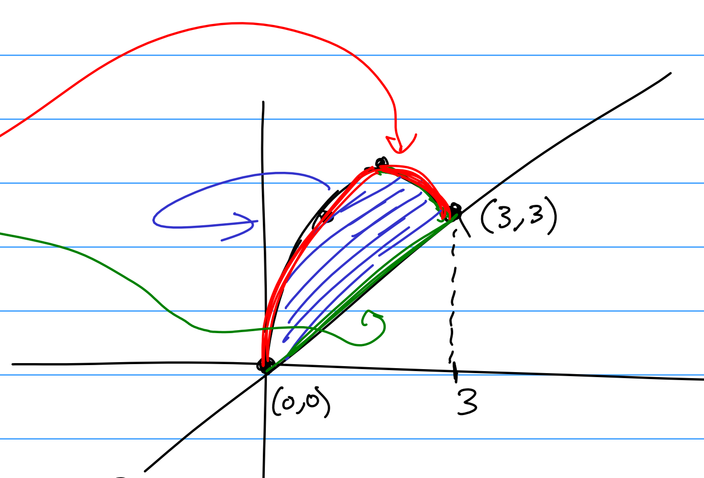
$$V = \boxed{2\pi}$$

#6

$y = 4x - x^2$

$y = x$

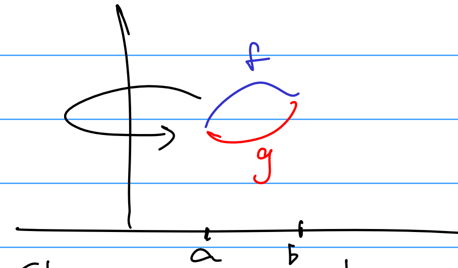
Difference of 2 shell methods.



$$\int_0^3 2\pi x(4x - x^2)\,dx - \int_0^3 2\pi x(x)\,dx$$

$$\pi\int_0^3 8x^2-2x^3\,dx \;-\; \pi\int_0^3 2x^2\,dx$$

$$\pi\left(\frac{8}{3}x^3-\frac{2}{4}x^4\right)\Big|_{\cancel{0}}^{3} - \pi\left(\frac{2}{3}x^3\right)\Big|_{\cancel{0}}^{3}$$



$$\int_a^b 2\pi x f(x)\,dx - \int_a^b 2\pi x\, g(x)\,dx$$

$$\pi\left(\frac{8}{3}3^3-\frac{1}{2}3^4\right)-\pi\left(\frac{2}{3}3^3\right)$$

$$\pi\left(72-\frac{81}{2}\right)-\pi(18)$$

$$\pi\left(72-\frac{81}{2}-18\right) \qquad 72-18=54=\frac{108}{2}$$

$$\pi\left(\frac{108}{2}-\frac{81}{2}\right)=\frac{27\pi}{2}$$

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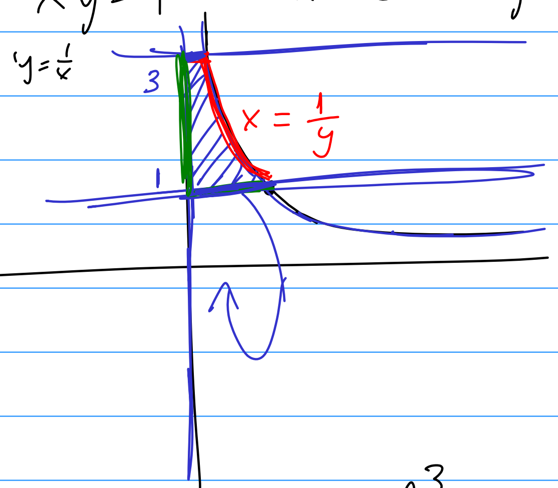
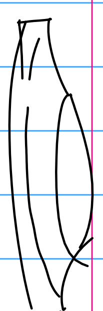
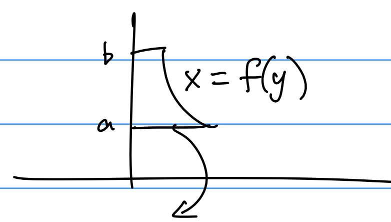
$xy=1 \qquad x=0 \qquad y=1 \qquad y=3$





Shell method sideways:



$$V=\int_a^b 2\pi y\, f(y)\,dy$$

$$V=\int_1^3 2\pi y\,\frac{1}{y}\,dy$$

$$V=\int_1^3 2\pi\,dy = 2\pi y\Big|_1^3 = 2\pi(3)-2\pi(1)$$

$$=\boxed{4\pi}$$

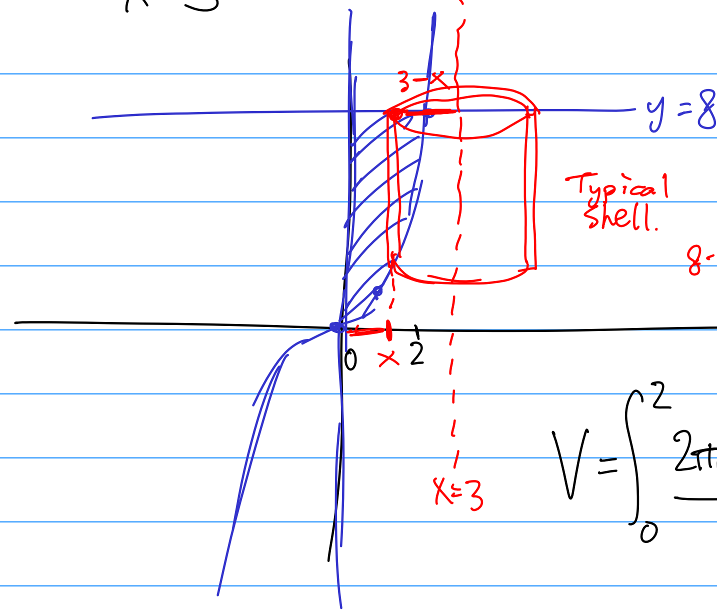
#15

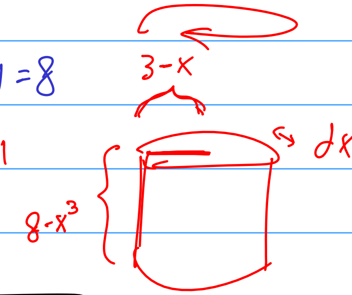
$y = x^3$

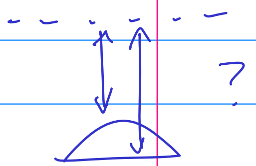
$y = 8$

$x = 0$

$x = 3$







$$V = \int_0^2 \underline{2\pi(3-x)}\,\underline{(8-x^3)}\,\underline{dx}$$

etc.

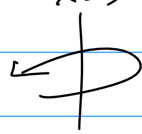
#18

$y = \sqrt{x}$

$x = 2y$

$y = \frac{x}{2}$

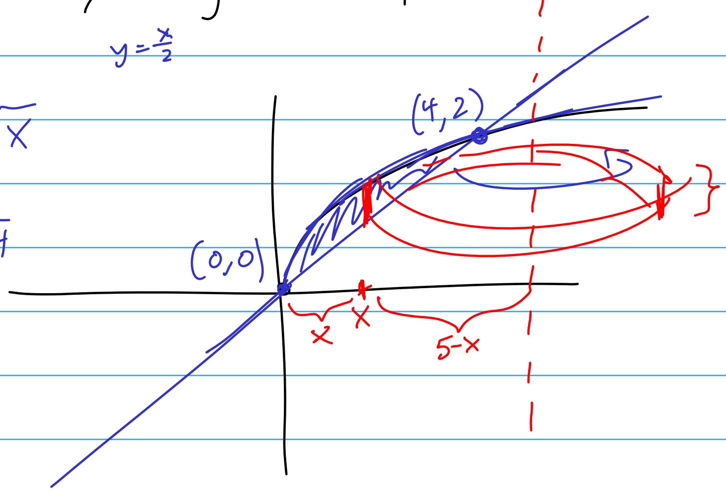
$x=5$



$\frac{x}{2} = \sqrt{x}$

$\frac{4}{2} = \sqrt{4}$

$2 = 2$



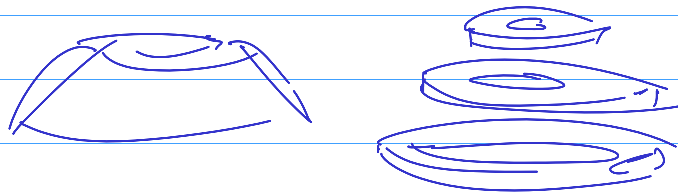
$r = 5-x$

$Circ = 2\pi(5-x)$

$\sqrt{x} - \frac{x}{2}$

$$\int_0^4 2\pi(5-x)\left(\sqrt{x} - \frac{x}{2}\right) dx$$

etc.



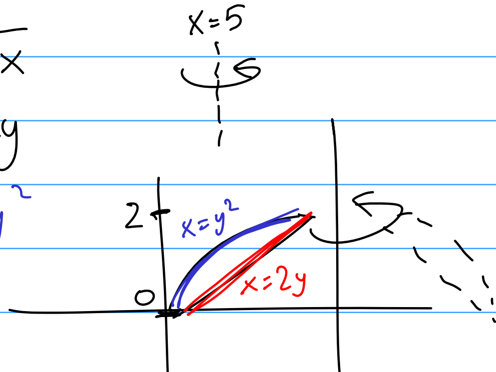
OR... Convert to disk method!

$y = \sqrt{x}$

$x = 2y$

$\hookrightarrow x = y^2$

$x=5$



How far is $x=y^2$ from $x=5$?

$5-y^2$

How far is $x=2y$ from $x=5$?

$5-2y$

$$\int_0^2 \pi(5-y^2)^2\,dy - \int_0^2 \pi(5-2y)^2\,dy.$$

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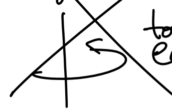
Circle

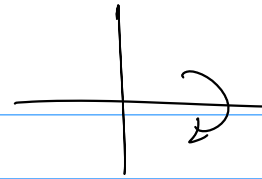
$x^2 + (y-1)^2 = 1$

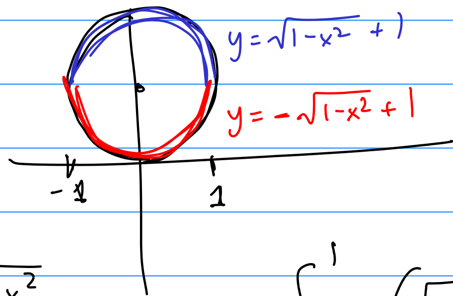
center: $(0,1)$ $\quad r = 1$

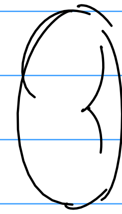
~~y-axis~~ too easy.



x-axis?







Disks?

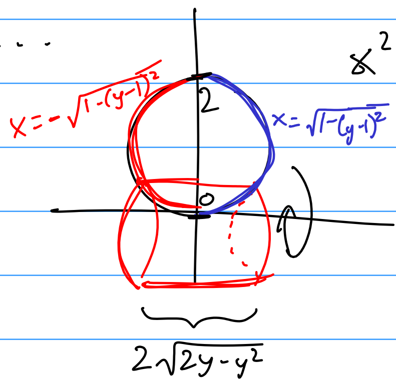
$(y-1)^2 = 1 - x^2$

$y - 1 = \pm\sqrt{1-x^2}$

$y = \pm\sqrt{1-x^2} + 1$

$$\int_{-1}^{1} \pi\left(\sqrt{1-x^2}+1\right)^2 dx - \int_{-1}^{1} \pi\left(-\sqrt{1-x^2}+1\right)^2 dx$$

OR...



$r = y$

Circ $= 2\pi y$

$x^2 + (y-1)^2 = 1$

$x^2 = 1 - (y-1)^2$

$x = \pm\sqrt{1-(y-1)^2}$

$\sqrt{1-(y^2-2y+1)}$

$\sqrt{1-y^2+2y-1}$

$\sqrt{2y-y^2}$

$$\int_0^2 2\pi y\left(2\sqrt{2y-y^2}\right)dy$$

???

Probably not possible

$$\int_{-1}^{1} \pi\left(\sqrt{1-x^2}+1\right)^2 dx - \int_{-1}^{1} \pi\left(-\sqrt{1-x^2}+1\right)^2 dx$$

$$\pi\int_{-1}^{1} \left(\sqrt{1-x^2}+1\right)^2 - \left(-\sqrt{1-x^2}+1\right)^2 \, dx$$

$$\pi\int_{-1}^{1} \left(\left(1-x^2\right) + 2\sqrt{1-x^2} + 1\right) - \left(\left(1-x^2\right) - 2\sqrt{1-x^2} + 1\right) dx$$

$$\pi\int_{-1}^{1} 4\sqrt{1-x^2}\, dx$$

not with our current methods.