$$\lim_{x \to 8^+} \frac{1}{x-8} = \infty$$

Evaluate a limit?

- Graphs clue
- Tables clue
- Algebra proof

$$\lim_{x \to 5^+} \frac{1}{x-5} = \text{LN}$$

(joke)

1.6 Calculate limits using limit laws.

1, 2. $\lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) = \lim_{x \to a} \left(f(x) \pm g(x)\right)$ Why? Proof later.

3. $\lim_{x \to a} [c f(x)] = c \lim_{x \to a} f(x)$

4. $\lim_{x \to a} \left(f(x)\, g(x)\right) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

5. $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ if $\lim_{x \to a} g(x) \neq 0$.

Ex pg 70 #3

$$\lim_{x \to 3} (5x^3 - 3x^2 + x - 6)$$

$\lim_{x \to 3} x = 3$

$\lim_{x \to 3} 6 = 6$

$$\lim_{x \to 3} 5x^3 - \lim_{x \to 3} 3x^2 + \lim_{x \to 3} x - \lim_{x \to 3} 6$$

$$5 \lim_{x \to 3} x \lim x \lim x - 3 \lim x \lim x + \lim x - \lim 6$$

etc.

$$5(3)^3 - 3(3)^2 + 3 - 6$$

← evaluates at 3!

This works for polynomials.

Laws

n pos. int.

6. $\lim_{x\to a} \left[(f(x))^n \right] = \left[\lim_{x\to a} f(x) \right]^n$

7. $\lim_{x\to a} c = c$

8. $\lim_{x\to a} x = a$

9. $\lim_{x\to a} x^n = a^n$ (n pos int)

$\lim_{x\to -8} \sqrt{x} = ???$

10. $\lim_{x\to a} \sqrt[n]{x} = \sqrt[n]{a}$ (n pos int)

11. $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$ (n pos int)

If n is even, we can assume that $a > 0$ or $\lim_{x\to a} f(x) > 0$.

pg 70 #9

$$\lim_{x\to 2} \sqrt{\frac{2x^2+1}{3x-2}} \overset{\text{Law \#11}}{=} \sqrt{\lim_{x\to 2} \frac{2x^2+1}{3x-2}} \overset{\text{Law \#5}}{=} \sqrt{\frac{\lim(2x^2+1)}{\lim(3x-2)}}$$

Laws #1, 2, 3

$$= \sqrt{\frac{2\lim_{x\to 2} x^2 + \lim_{x\to 2} 1}{3\lim_{x\to 2} x - \lim_{x\to 2} 2}} = \underbrace{\sqrt{\frac{\overset{\#9}{2(2)^2} + \overset{\#7}{1}}{\underset{\#8}{3(2)} - \underset{\#7}{2}}}}_{\text{As if simply evaluating!}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

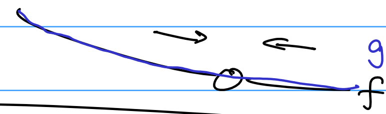
Pg 65 **Direct substitution property:**

If f is polynomial or rational function and $\underline{a}$ is in the domain of f...

$$\lim_{x\to a} f(x) = f(a)$$

$$\lim_{x\to 1}\frac{x^2-1}{x-1} = \ldots\ldots$$ Can't use DSP. but...

$$\frac{x^2-1}{x-1} = \frac{(x-1)(x+1)}{x-1} = x+1 \quad (x\neq 1)$$



$$\lim_{x\to 1}(x+1)$$

$$= 2$$

Thm

If $f(x) = g(x)$ when $x \neq a$,
then $\lim_{x\to a} f(x) = \lim_{x\to a} g(x)$
if these limits exist.

$$\lim_{x\to 0}\frac{|x|}{x}$$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

DSP not applicable here.

If $x \geq 0$... $\lim_{x\to 0^+}\frac{|x|}{x} = \lim_{x\to 0^+}\frac{x}{x} = \lim_{x\to 0^+} 1 = 1$

By above Thm

$\neq$

If $x < 0$... $\lim_{x\to 0^-}\frac{|x|}{x} = \lim_{x\to 0^-}\frac{-x}{x} = \lim_{x\to 0^-} -1 = -1$

$$\lim_{x\to 0}\frac{|x|}{x} = \text{DNE}. \quad \left(\lim_{x\to 0^+}\frac{|x|}{x} \neq \lim_{x\to 0^-}\frac{|x|}{x}\right)$$

If $f(x) \leq g(x)$ when $x \neq a$,

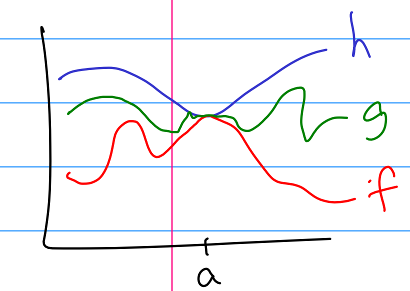
$$\lim_{x\to a} f(x) \leq \lim_{x\to a} g(x)$$

provided these limits exist.

Squeeze Thm If $f(x) \le g(x) \le h(x)$ when $x \neq a$,

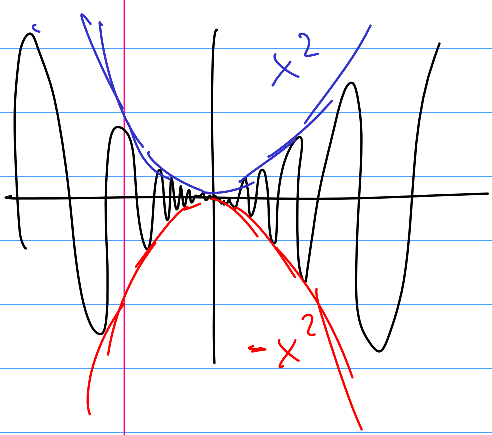
and $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$,

then $\lim_{x \to a} g(x) = L$.



$f(0)$ undef

$$\lim_{x \to 0} x^2 \sin \frac{1}{x} = \underline{\quad\quad} ?$$



$$g(x) = x^2 \sin \frac{1}{x}$$

Trig: $-1 \le \sin\left(\frac{1}{x}\right) \le 1$

mult by x^2

$$-x^2 \le x^2 \sin \frac{1}{x} \le x^2$$

DSP works here

$$\lim_{x \to 0} -x^2 = 0 = \lim_{x \to 0} x^2$$

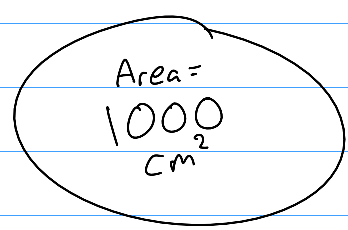
By Squeeze Thm, $\lim_{x \to 0} x^2 \sin \frac{1}{x} = \underline{0}$. □

1.7 Rigorously prove a limit or a limit law.

pg 81

#11

Needed:



$r = ?$

$$\pi r^2 = 1000$$

$$r^2 = \frac{1000}{\pi}$$

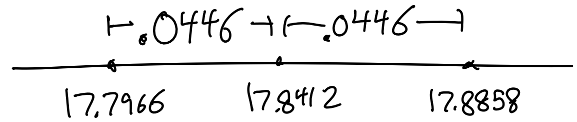
$$r = \sqrt{\frac{1000}{\pi}} \approx 17.8412$$

tolerance $\pm 5 \text{ cm}^2$

$$995 \le \text{Area} \le 1005$$

low $r = \sqrt{\frac{995}{\pi}} \approx 17.7966$

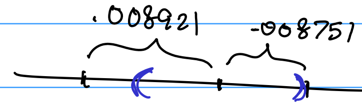
high $r = \sqrt{\frac{1005}{\pi}} \approx 17.8858$



$\pm .0446$ tolerance
in radius error

What if $\pm 1\ cm^2$ for area?

$999 < \text{Area} < 1001$



low $r = \sqrt{\frac{999}{\pi}} \approx 17.832318$

$\sqrt{1000/\pi} \approx 17.841241$

high $r = \sqrt{\frac{1001}{\pi}} \approx 17.850160$

$.008921$

$.008751$

$\pm .008751$ tolerance
in radius error

epsilon If Area error $= \varepsilon$	delta then radius error $= \delta$
$5\ cm^2$	$.0446\ cm$
$1\ cm^2$	$.008751\ cm$
ε	$\sqrt{\frac{1000+\varepsilon}{\pi}} - \sqrt{\frac{1000}{\pi}}$

formula!

$A_i = 1000$

$e_A = \varepsilon$

$A_\ell = 1000 - \varepsilon$

$A_h = 1000 + \varepsilon$

$r_\ell = \sqrt{\frac{1000-\varepsilon}{\pi}}$

$r_h = \sqrt{\frac{1000+\varepsilon}{\pi}}$

$r_i = \sqrt{\frac{1000}{\pi}}$

$r_h - r_i = \sqrt{\frac{1000+\varepsilon}{\pi}} - \sqrt{\frac{1000}{\pi}}$

$A = \pi r^2$

$\sqrt{\frac{A}{\pi}} = r$

I can get as close to $1000\ cm^2$ as I want, if I make r close enough to $\sqrt{\frac{1000}{\pi}}$.