Chapter 4: All objectives assessed by Exam 4

3.9 Find antiderivatives of functions. (General and specific)

3.9 MEMORIZE: Basic antiderivatives and antiderivative rules.

AppxE: Express sums of sequences in sigma notation.

AppxE: Calculate sums of sequences.

4.2 Evaluate definite integrals using integral properties.

4.3,4 Evaluate indefinite integrals. (Results in functions)

4.3,4 Evaluate definite integrals. (Results in numbers)

4.5 Use the substitution rule to evaluate integrals.

Memorize

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

Chapter 6 & 5: All objectives assessed by Exam 5

~~6.1 Find the derivative of an inverse of a function. (No need to memorize this formula.) als.~~

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

6.2,3,4 Find derivatives and integrals of exponential and logarithmic functions.

6.3 Evaluate logarithms. $\log_5 25$

6.3 Expand and collect logarithmic expressions. (Have the logarithm laws memorized!)

6.4 Use logarithmic differentiation to derive certain functions.

5.1 Compute the area between two curves.

5.2 Calculate the volume of solids using the disk/washer method. (No need to memorize the formu

5.3 Calculate the volume of solids using the shell method. (No need to memorize the formula)

5.5 Find the average value of a function over an interval.

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#38 Find f.

$$f''(t) = 4 - \frac{6}{t^4} \qquad t > 0$$

$$= 4 - 6t^{-4}$$

$f(1) = 6$

$f'(2) = 9$

$$f'(t) = 4t + 2t^{-3} + \cancel{C}\ \tfrac{3}{4}$$

$f'(2) = 9$

$$f'(2) = 4(2) + 2(2)^{-3} + C$$

$$8 + \frac{2}{2^3} + C$$

$$8 + \frac{1}{4} + C = 9$$

$$\frac{1}{4} + C = 1$$

$$C = \tfrac{3}{4}$$

$$f(t) = 2t^2 - t^{-2} + \frac{3}{4}t + \cancel{C}\ \frac{17}{4}$$

$$f(1) = 2 - 1^{-2} + \frac{3}{4} + C = 6$$

$$\frac{7}{4} + C = \frac{24}{4}$$

$$C = \boxed{\frac{17}{4}}$$

$$f(t) = 2t^2 - \frac{1}{t^2} + \frac{3}{4}t + \frac{17}{4}$$

$$\overset{8}{\frown}\ \overset{8}{\frown}\ \overset{8}{\frown}\ \overset{8}{\frown}\ \overset{8}{\frown}$$

$$83 + 91 + 99 + 107 + 115 + 123$$

$$\sum_{i=\boxed{1}}^{\boxed{6}} 75 + 8i$$

or $i = 0$ $\uparrow$ 5

Always increases by 1.

$$83 + 91 + 99 + 107 + \ldots\ldots + 8003$$

$$\sum_{i=1}^{991} 75 + 8i$$

$$75 + 8i = \cancel{8003}\ 7991 3$$
$$-\ 75$$
$$8i = 7928$$

$$8 \overline{)7928}\ 991$$
$$-72$$
$$72$$

$$\sum_{i=1}^{991} 75 + 8\sum_{i=1}^{991} i$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$75 \cdot 991 + 8\,\frac{991 \cdot 992}{2}$$

$$= \text{etc.}$$

Pg 505 Derive

#22 $g(t) = \dfrac{e^t}{1+e^t}$

$$g'(t) = \frac{(1+e^t)(e^t)' - e^t(1+e^t)'}{(1+e^t)^2} = \frac{(1+e^t)e^t - e^t e^t}{(1+e^t)^2}$$

$$= \frac{e^t + (e^t)^2 - (e^t)^2}{(1+e^t)^2} = \boxed{\frac{e^t}{(1+e^t)^2}}$$

$$f(x) = \ln(\log_3 x)$$

$$f'(x) = \frac{1}{\log_3 x} \cdot \frac{1}{x \ln 3} = \frac{1}{(\ln 3)\, x \log_3 x}$$

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$$\int \frac{\sin(\ln x)}{x}\, dx$$

$u = \ln x$

$du = \frac{1}{x}\, dx$

$$\int \sin(\ln x)\, \frac{1}{x}\, dx$$

$$\int \sin(u)\, du = -\cos u + C$$

$$-\cos(\ln x) + C$$

#101

$$\int \tan x \ln(\cos x)\, dx$$

$$-\int \frac{-\sin x}{\cos x} \ln(\cos x)\, dx$$

$u = \cos x$

$du = -\sin x\, dx$

$$-\int \frac{1}{u} \ln(u)\, du$$

$n \log a$

$= \log(a^n)$

$$-\int \frac{\ln(u)}{u}\, du$$

$v = \ln u$

$dv = \frac{1}{u}\, du$

$$-\int v\, dv = -\frac{1}{2}v^2 + C = -\frac{1}{2}(\ln u)^2 + C$$

$$= -\frac{1}{2}(\ln(\cos x))^2 + C$$

$$y = 2^{\boxed{x^2}} x^{\boxed{2^x}}$$

$$\ln y = \ln\left(2^{x^2} x^{2^x}\right)$$

$$\ln y = \ln\left(2^{x^2}\right) + \ln\left(x^{2^x}\right)$$

$$\ln y = x^2 \ln(2) + \underline{2^x}\ \underline{\ln x}$$

$$\frac{1}{y} y' = 2(\ln 2)x + (2^x)' \ln x + 2^x (\ln x)'$$

$$2(\ln 2)x + 2^x \ln 2 \ln x + 2^x \frac{1}{x}$$

$$y' = y\left(\underline{\qquad\qquad}\right)$$

$$y' = 2^{x^2} x^{2^x}\left(2(\ln 2)x + 2^x \ln 2 \ln x + 2^x \frac{1}{x}\right)$$

$$\int_0^1 \frac{3^x}{4^x}\,dx = \int_0^1 \left(\frac{3}{4}\right)^x dx = \left.\frac{\left(\frac{3}{4}\right)^x}{\ln\left(\frac{3}{4}\right)}\right|_0^1$$

$$\left(\frac{3}{4}\right)^x \xrightarrow{\text{Deriv}} \left(\frac{3}{4}\right)^x \ln\left(\frac{3}{4}\right)$$

$$= \frac{\left(\frac{3}{4}\right)^1}{\ln\left(\frac{3}{4}\right)} - \frac{\left(\frac{3}{4}\right)^0}{\ln\left(\frac{3}{4}\right)}$$

$$= \frac{3/4 - 1}{\ln\left(\frac{3}{4}\right)} = \frac{-1/4}{\ln\left(\frac{3}{4}\right)}$$

$$= \frac{-1}{4\ln\left(\frac{3}{4}\right)}$$

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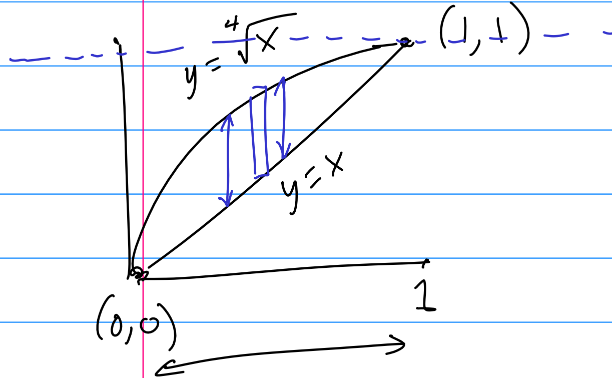
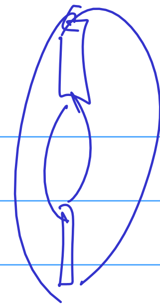
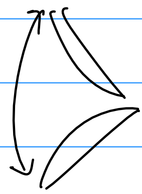
washer (disk) method.







How far is $y=x$ from $y=1$?

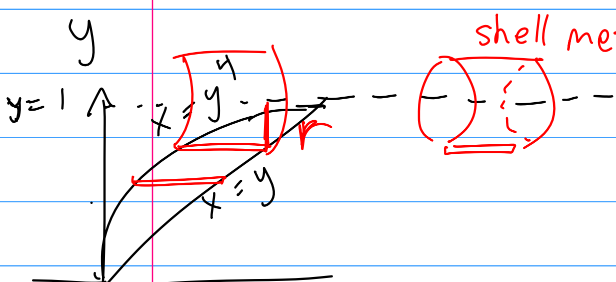
$f(x) = x-1$

How far is $y=\sqrt[4]{x}$ from $y=1$?

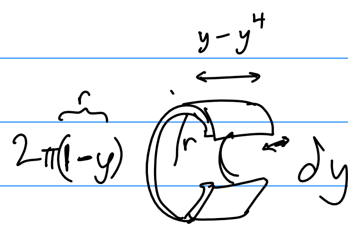
$g(x) = \sqrt[4]{x} - 1$

$$V = \int_0^1 \pi (x-1)^2\,dx - \int_0^1 \pi\left(\sqrt[4]{x}-1\right)^2 dx \quad \checkmark$$

OR



Volume of a shell:



$$\int_0^1 2\pi(1-y)\left(y-y^4\right) dy \quad \checkmark$$