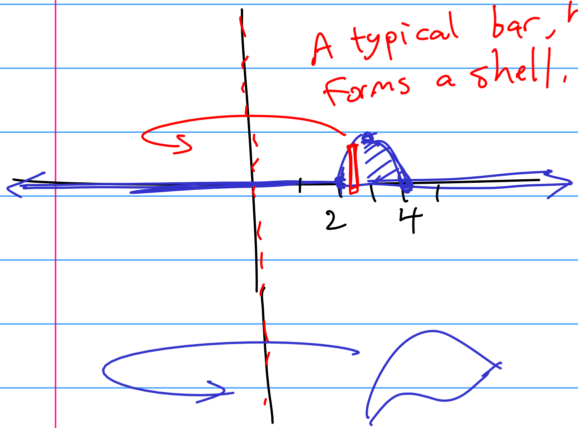
Pg 383 #37

$y = -x^2 + 6x - 8$ about the y-axis

$y = 0$

Disk	Shell ?
3	10



A typical bar, here, forms a shell.

x	y
-2	-24
-1	
0	-8
1	-3
2	0
3	1
4	0
5	-3

shell method

$$\boxed{V = \int_a^b 2\pi x f(x)\,dx}$$

$$V = \int_2^4 2\pi x\left(-x^2+6x-8\right)dx.$$

$$= \pi\int_2^4 -2x^3 + 12x^2 - 16x\,dx$$

$$\pi\left[-\frac{\overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}}}x^4 + 4x^3 - 8x^2\right]_2^4$$

$$\pi\left(\left[-\tfrac{1}{2}256 + 256 - 128\right] - \left[-\tfrac{1}{2}16 + \cancel{4\cdot 8} - \cancel{8\cdot 4}\right]\right)$$

$$\pi\left(\left[\cancel{-128 + 256} - 128\right] - \left[-8\right]\right)$$

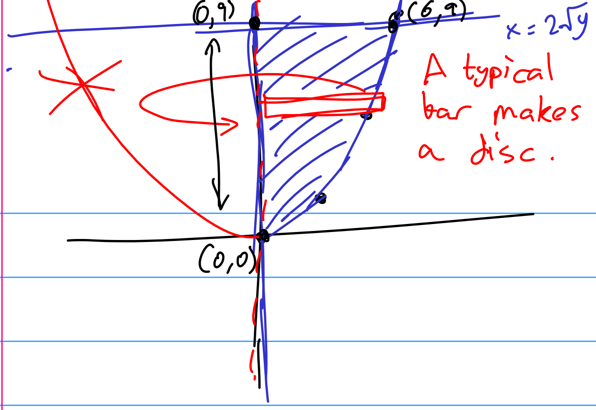
$$= 8\pi$$

$x = 2\sqrt{y}$ $\quad x = 0$ $\quad y = 9$ about y-axis

$\frac{x}{2} = \sqrt{y}$

$\frac{x^2}{4} = y$

Disk?	Shell?
7	5



A typical bar makes a disc.

x	y
0	0
2	1
4	4
6	9

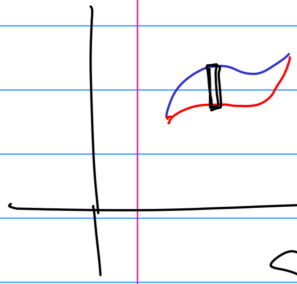
$$\int_0^9 \pi\left[2\sqrt{y}\right]^2 dy$$

$$= \int_0^9 4\pi y\, dy$$

$$= 2\pi y^2\Big|_{\cancel{0}}^{9}$$

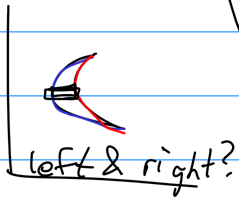
$$= 2\cdot 81\,\pi \quad = \boxed{162\pi}$$

Is the function better bounded

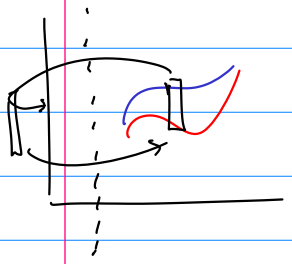


above & below

or



Sketch the typical bar.
(like a Riemann sum)
Revolve it about the axis.

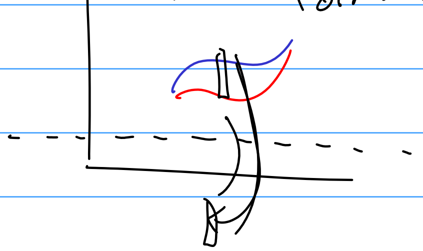


parallel?

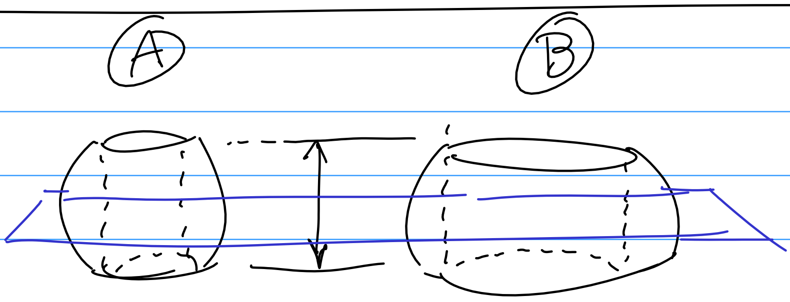
Forms a shell.

perp?

Forms a disc(s).



Pg 383
#48



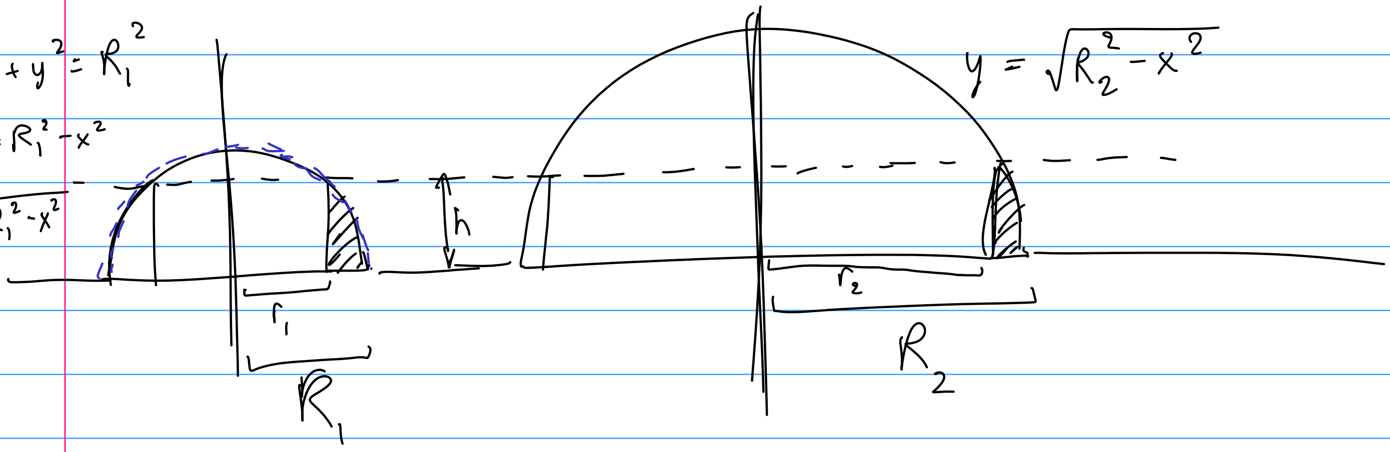
more wood?

A	B	Same
4	6	4

$x^2 + y^2 = R_1^2$

$y^2 = R_1^2 - x^2$

$y = \sqrt{R_1^2 - x^2}$



$y = \sqrt{R_2^2 - x^2}$

shell method

$$\int_{r_1}^{R_1} 2\pi x\sqrt{R_1^2-x^2}\,dx$$

$$u = R_1^2 - x^2$$
$$du = -2x\,dx$$

$$-\pi\int_{r_1}^{R_1} -2x\sqrt{R_1^2-x^2}\,dx$$

$$-\pi\int_{r_1=x}^{R_1=x} \sqrt{u}\,du$$

$$-\pi\left[\frac{2}{3}u^{3/2}\right]_{r_1=x}^{R_1=x}$$

$$-\pi\left[\frac{2}{3}\left(R_1^2-x^2\right)^{3/2}\right]_{r_1}^{R_1}$$

$$-\pi\left[0-\frac{2}{3}\left(R_1^2-r_1^2\right)^{3/2}\right]$$

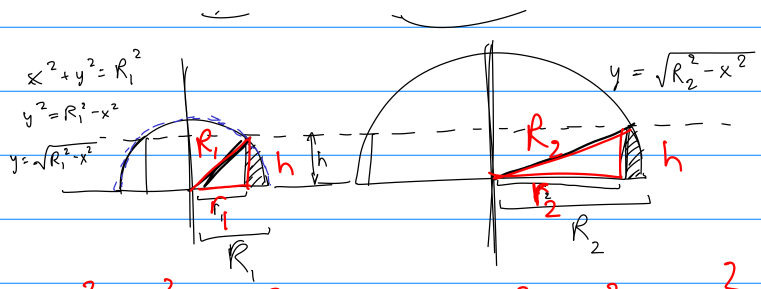
$$\frac{2}{3}\pi\left(\underbrace{R_1^2-r_1^2}_{=h^2}\right)^{3/2}$$

$$\int_{r_2}^{R_2} 2\pi x\sqrt{R_2^2-x^2}\,dx$$

$$\frac{2}{3}\pi\left(\underbrace{R_2^2-r_2^2}_{=h^2}\right)^{3/2}$$

$$\frac{2}{3}\pi\left(h^2\right)^{3/2}$$

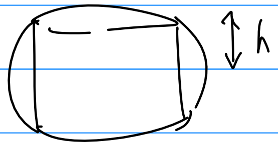
$$=\frac{2}{3}\pi h^3$$



$$r_1^2+h^2=R_1^2$$
$$h^2=R_1^2-r_1^2$$

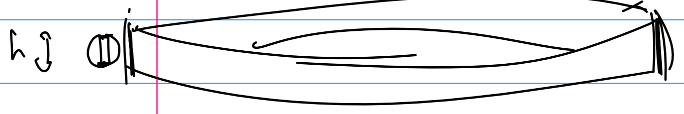
$$r_2^2+h^2=R_2^2$$
$$h^2=R_2^2-r_2^2$$

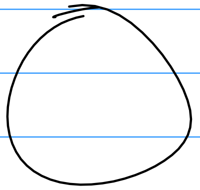
$\frac{2}{3}\pi h^3$



$V=\frac{4}{3}\pi h^3$

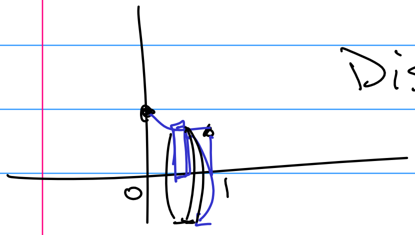
Same height → same volume!





$\frac{4}{3}\pi r^3 = V$

pg 438 #85 $y = \frac{1}{\sqrt{x+1}}$ 0 to 1 about x-axis.



Disc method

$$\int_0^1 \pi \left(\frac{1}{\sqrt{x+1}}\right)^2 dx$$

$$= \pi \int_0^1 \frac{1}{x+1} \, dx$$

$u = x+1$
$du = dx$

$$= \pi \int_1^2 \frac{1}{u} \, du$$

$$\pi \ln|u| \Big|_1^2$$

$$\pi (\ln 2 - \ln 1)$$

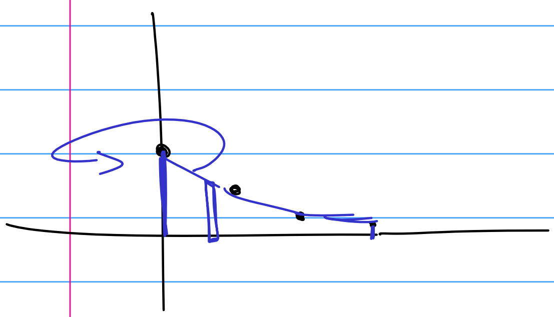
$e^{\square} = 1$

$$\pi (\ln 2 - 0)$$

$$= \boxed{\pi \ln 2}$$

#86

$y = \frac{1}{x^2+1}$ 0 to 3 about y-axis.



$$\int_0^3 2\pi x \frac{1}{x^2+1} \, dx$$

$$= \pi \int_0^3 \frac{2x}{x^2+1} \, dx$$

$u = x^2 + 1$
$du = 2x \ dx$

$$= \pi \int_{0=x}^{3=x} \frac{1}{u} \, du$$

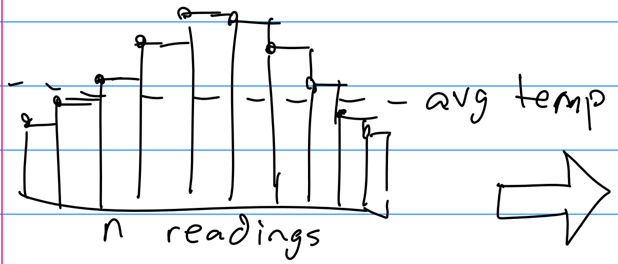
$$\pi \ \ln|u| \Big|_{0=x}^{3=x}$$

$$\pi \ln|x^2+1| \Big|_0^3$$

$$= \pi(\ln 10 - \underset{0}{\cancel{\ln 1}})$$

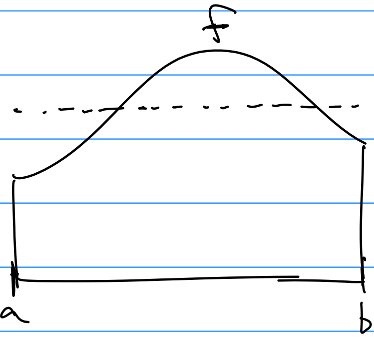
$$= \pi \ln 10$$

5.5 Find the average value of a function over an interval.



$$\frac{1}{n} \sum t_n$$

Pg 389
proof



$$f_{ave} = \frac{1}{b-a} \int_a^b f(x) \, dx$$

Pg 391
\#2

Avg value of

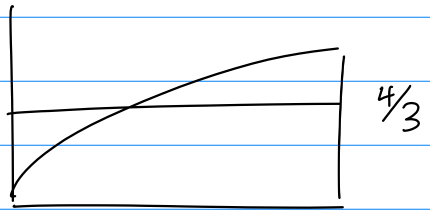
$f(x) = \sqrt{x}$ on $[0,4]$

$$= \frac{\text{Integral}}{\text{Length of interval}}$$

$$\frac{1}{4} \int_0^4 \sqrt{x} \ dx$$

$$\frac{1}{4} \left[\frac{2}{3} x^{3/2} \right]_0^4$$

$$\frac{1}{4} \ \frac{2}{3} (4)^{3/2} = \frac{1}{6} \cdot 8 = \boxed{\frac{4}{3}}$$

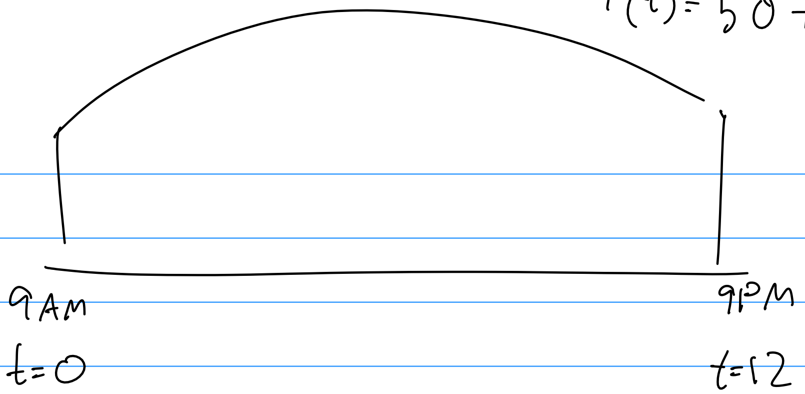


pg 391

#17

$T(t) = 50 + 14\sin\left(\frac{\pi t}{12}\right)$

t = hrs after 9AM



$$\frac{1}{12}\int_0^{12} 50 + 14\sin\left(\frac{\pi t}{12}\right) dt$$

$u = \frac{\pi t}{12}$

$du = \frac{\pi}{12} dt$

$du \frac{12}{\pi} = dt$

$$\frac{1}{12}\int_{0=t}^{12=t}\left(50 + 14\sin(u)\right) dt$$

$$\frac{1}{12}\int_{0=t}^{12=t}\left(50 + 14\sin(u)\right)\frac{12}{\pi} du$$

$$\frac{1}{\pi}\Big[50u \quad - 14\cos u\Big]_{0=t}^{12=t}$$

$u = \pi$

$u = 0$

$$\frac{1}{\pi}\Big[50u - 14\cos u\Big]_0^{\pi}$$

Do not discard!

$$\frac{1}{\pi}\Big(\left(50\pi - 14\cos\pi\right) - \left(0 - 14\cos 0\right)\Big)$$

$$\frac{1}{\pi}\Big(\left(50\pi + 14\right) - \left(-14\right)\Big)$$

$$= \frac{1}{\pi}\left(50\pi + 28\right)$$

$$= \boxed{50 + \frac{28}{\pi}}$$