Vert asymp's: occur when denominators equal 0.

Horiz asymp's: occur when $\lim_{x\to\infty} f(x)$ or $\lim_{x\to-\infty} f(x)$ exist.

Look at domain,

and optionally x-intercepts, and where f(x) is +/−.

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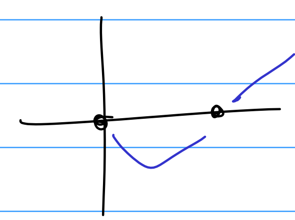
#21 $f(x) = (x-3)\sqrt{x}$

☆ Domain: $[0, \infty)$

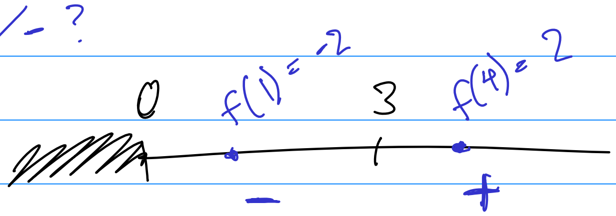
☆ x-intercepts?: $(x-3)\sqrt{x} = 0$?

$x - 3 = 0 \qquad \sqrt{x} = 0$

$x = 3 \qquad x = 0$.



☆ +/− ?



☆ Crit pts

$$f(x) = x\sqrt{x} - 3\sqrt{x}$$

$$= x^{3/2} - 3x^{1/2}$$

$$f'(x) = \frac{3}{2}x^{1/2} - \frac{3}{2}x^{-1/2} = 0 \text{ or DNE}$$

$$\frac{3}{2}\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) = 0 \text{ or DNE}$$

DNE when $x = 0$

$$\sqrt{x} - \frac{1}{\sqrt{x}} = 0$$

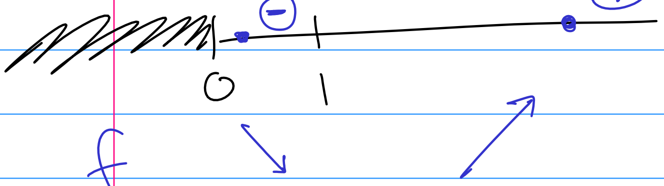
$$\sqrt{x} = \frac{1}{\sqrt{x}}$$

$$x = 1$$

Crit pts: $x = 0, 1$

$f'\left(\frac{1}{4}\right) = \frac{3}{2}\left(\frac{1}{2} - \frac{1}{1/2}\right)$ $\ominus$

$f'(4) = \frac{3}{2}\left(2 - \frac{1}{2}\right)$ $\oplus$



☆ Inc/dec

increasing on $(1, \infty)$

decreasing on $(0, 1)$

☆ Inflection pts

$$f'(x) = \frac{3}{2}x^{1/2} - \frac{3}{2}x^{-1/2}$$

$$f''(x) = \frac{3}{4}x^{-1/2} + \frac{3}{4}x^{-3/2} = 0 \text{ or DNE}$$

$$= \frac{3}{4}\left(\frac{1}{\sqrt{x}} + \frac{1}{x^{3/2}}\right) = 0 \text{ or DNE}$$

$x = 0$

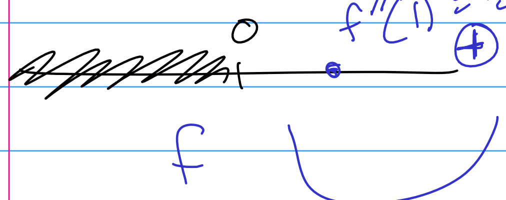
$$\frac{x^{3/2}}{x^{1/2}} = x^{2/2} = x^1$$

$$x^{3/2}\left(\frac{1}{\sqrt{x}} + \frac{1}{x^{3/2}}\right) = (0)x^{3/2}$$

$$x + 1 = 0$$

$\cancel{x = -1}$ not in domain

$f''(1) = \frac{3}{4}(1 + 1)$ $\oplus$



☆ $x = 0$ cannot be an inflection point because the concavity does not change there.

$f(x)$ is concave up on $[0, \infty)$.

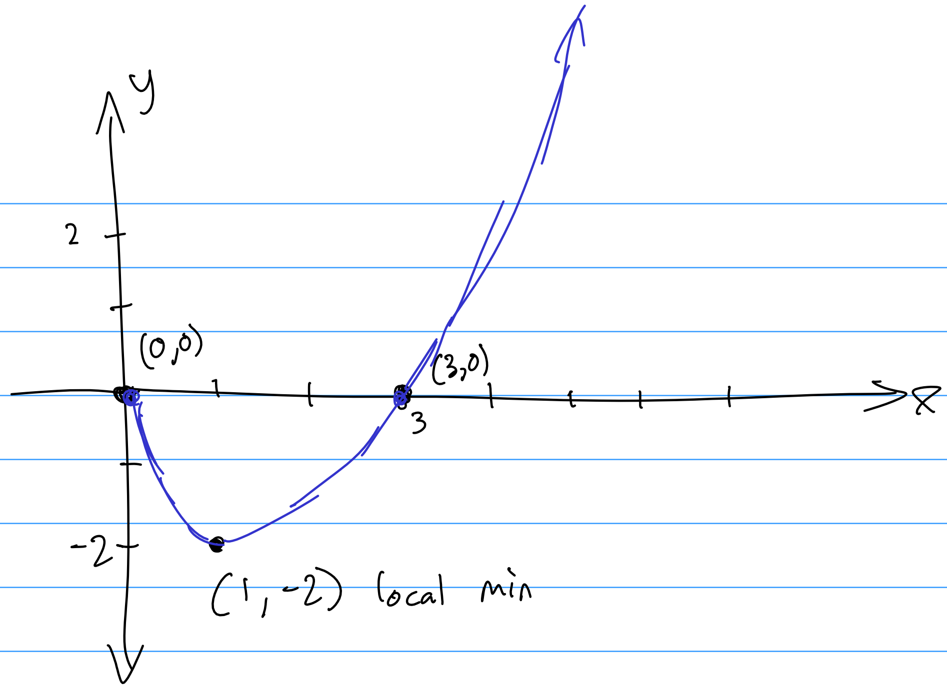
☆ min/max :

$x = 0$ ← maximum, sort of. (Not local or absolute)

$x = 1$ ←

↘ ↗ and ∪

Local minimum: $(1, -2)$



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#12 $f(x) = 1 + \frac{1}{x} + \frac{1}{x^2}$

☆ Domain: $(-\infty, 0) \cup (0, \infty)$

~~☆ x intercepts:~~

☆ Vert asymp: $x = 0$

☆ Horiz asymp: $\lim_{\substack{x \to \infty \\ x \to -\infty}} \left(1 + \frac{1}{x} + \frac{1}{x^2}\right) = \underline{1}$ $y = 1$

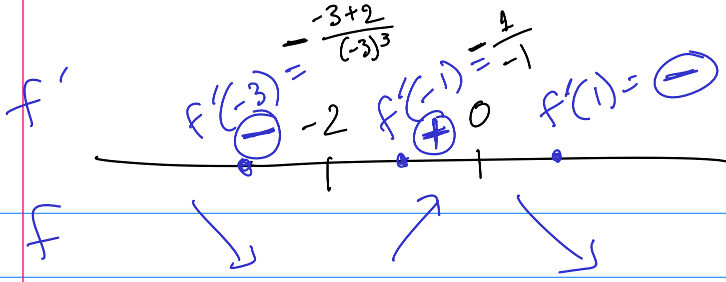
☆ Crit pts: $f(x) = 1 + x^{-1} + x^{-2}$

$$f'(x) = -x^{-2} - 2x^{-3}$$

$$= -\frac{1}{x^2} - \frac{2}{x^3}$$

$$= -\left(\frac{x}{x^3} + \frac{2}{x^3}\right) = \underline{-\frac{x+2}{x^3}} = 0 \text{ or DNE}$$

$x = -2$ $\quad$ $x = 0$



local min: $\left(-2, \frac{3}{4}\right)$ No max at $(0, f(0))$ undef.

$$1 + \frac{1}{-2} + \frac{1}{(-2)^2}$$

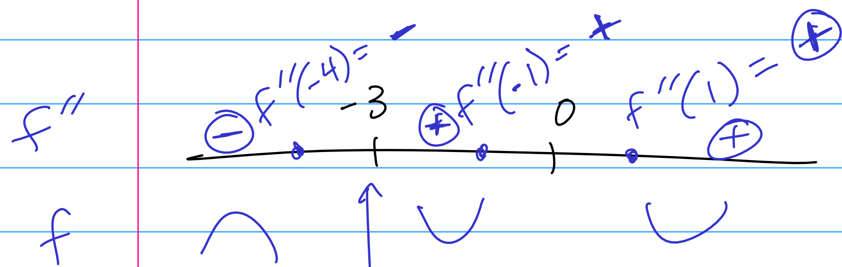
$$= 1 - \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$f'(x) = -x^{-2} - 2x^{-3}$$

$$f''(x) = 2x^{-3} + 6x^{-4} = \frac{2}{x^3} + \frac{6}{x^4} = \frac{2x}{x^4} + \frac{6}{x^4} = \frac{2x+6}{x^4} = 0 \text{ or DNE}$$

$f''(x)$ DNE when $x = 0$ $\qquad 2x + 6 = 0 \qquad 2x = -6$

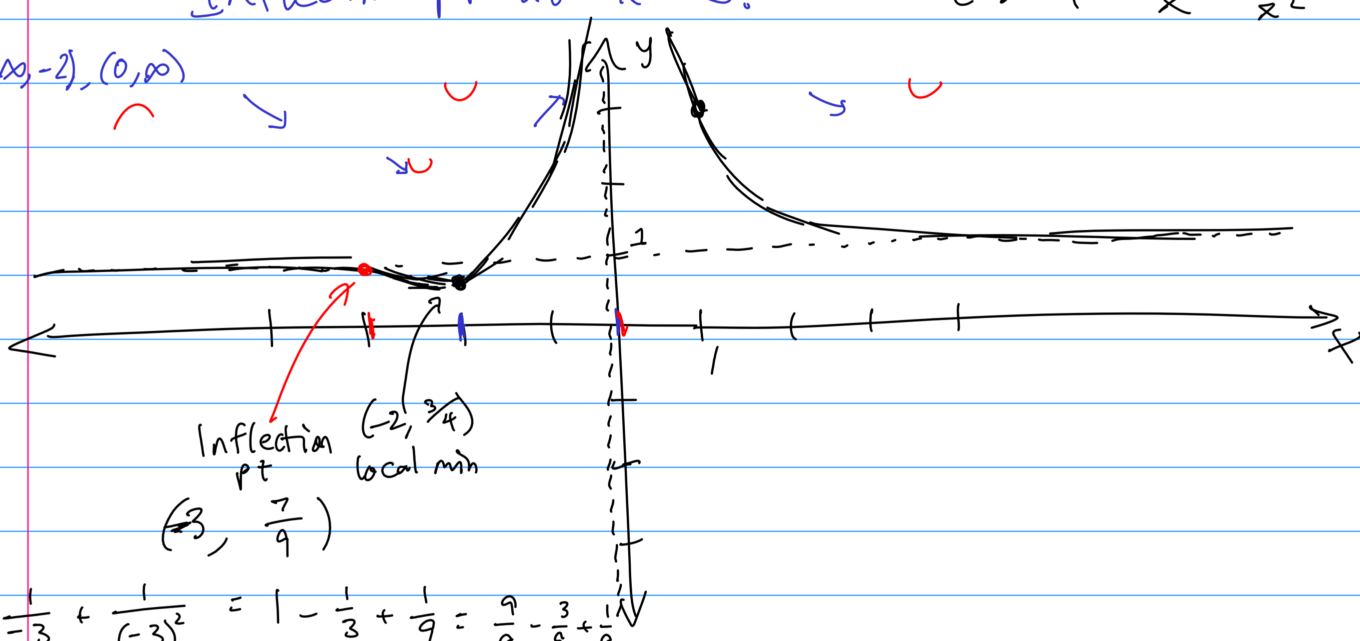
$f''(x) = 0$ when $x = -3$ $\qquad x = -3$



Inflection pt at $x = -3$.

$$f(x) = 1 + \frac{1}{x} + \frac{1}{x^2}$$

Dec $(-\infty, -2), (0, \infty)$



$\left(-3, \frac{7}{9}\right)$

$$1 + \frac{1}{-3} + \frac{1}{(-3)^2} = 1 - \frac{1}{3} + \frac{1}{9} = \frac{9}{9} - \frac{3}{9} + \frac{1}{9}$$

3.7 Solve optimization problems.

cut corners

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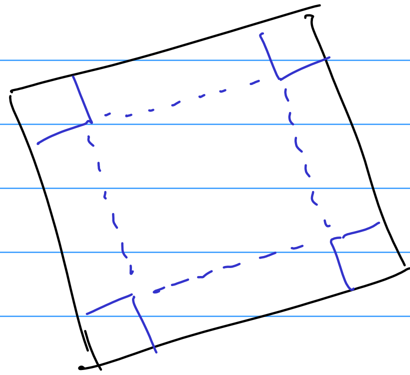
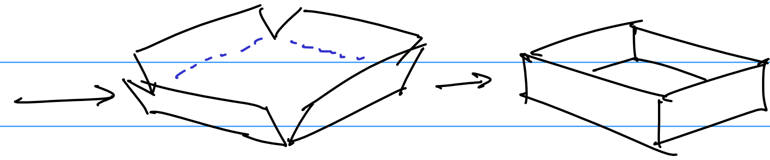
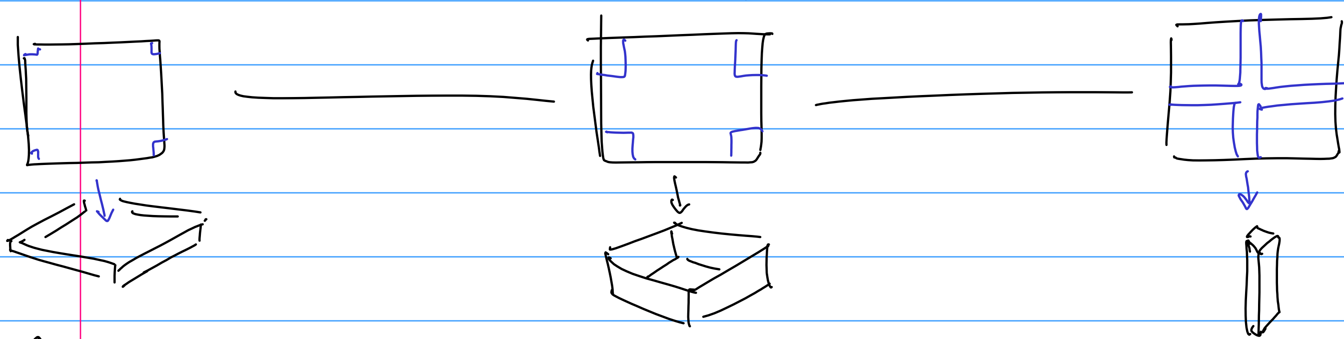
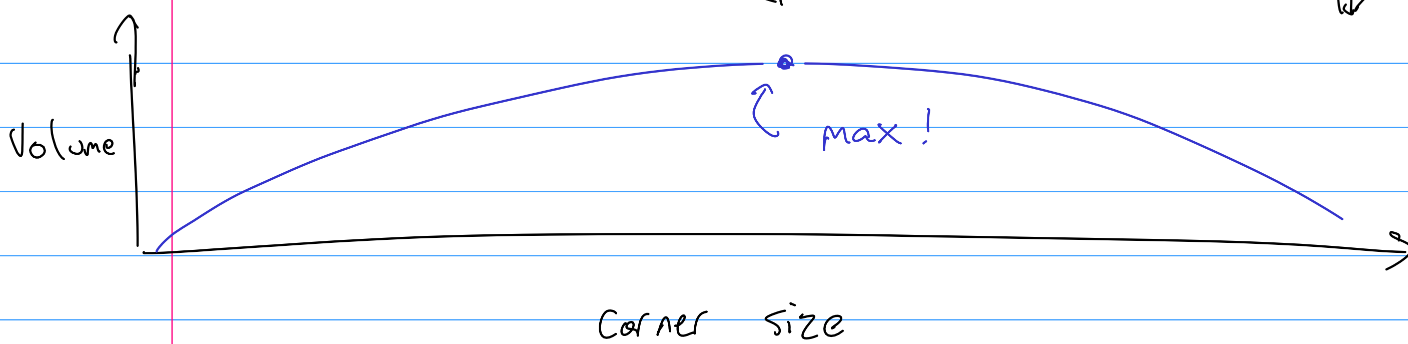
#12



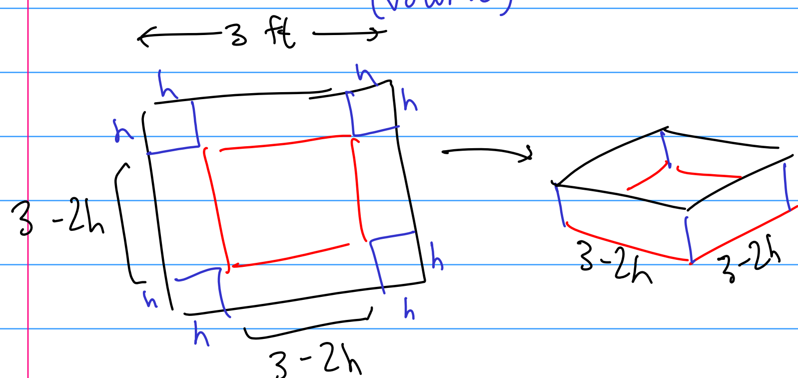


Most volume?





1. Write the thing-to-optimize as a function of one variable.
(volume) (corner size)



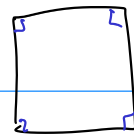
$$V = wlh$$

$$V = (3-2h)(3-2h)\,h$$

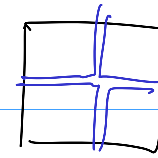
$$V = (9 - 12h + 4h^2)\,h$$

$$V = 4h^3 - 12h^2 + 9h$$

2. Pick a reasonable interval for the variable (h)



$0 < h < 1.5$



3. Do the Closed Interval Method on the function (Step 1) on the interval. (Step 2)

$V(h) = 4h^3 - 12h^2 + 9h$

$V'(h) = 12h^2 - 24h + 9 = 0$

$$h = \frac{24 \pm \sqrt{576 - 432}}{24} = \frac{24 \pm \sqrt{144}}{24} = \frac{24 \pm 12}{24} = \frac{1}{2} \text{ or } \frac{3}{2}$$

24×24: 96, 48, 576

36×12: 72, 36, 432

$576 - 432 = 144$

Crit pts: $x = \frac{1}{2}, \frac{3}{2}$

End pts: $x = 0, \frac{3}{2}$

$V = (3-2h)^2 h$

$V(0) = 0$

$V(\frac{1}{2}) = (3 - 2\cdot\frac{1}{2})^2 \frac{1}{2} = 4 \cdot \frac{1}{2} = \boxed{2 \text{ ft}^3}$ Max volume.

$V(\frac{3}{2}) = 0$

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#2 Find two numbers whose difference is 100 and whose product is a minimum.

Product $= xy$

$P = x(x-100)$

$P(x) = x^2 - 100x$

Interval: $(-\infty, \infty)$. "Numbers". No endpoints here.

we look for critical points only.

$$P'(x) = 2x - 100 = 0$$

$$2x = 100$$

$$x = 50$$

	x	$x-100$	Product
Numbers:	50	-50.	-2500