3.9 Find antiderivatives of functions. (General and specific)
3.9 MEMORIZE: Basic antiderivatives and antiderivative rules.

Pg 282 Find general antideriv.

#2 $f(x) = x^2 - 3(x) + 2$

$F(x) = \frac{1}{3}x^3 - 3 \cdot \frac{1}{2}x^2 + 2x$

$\boxed{= \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x + C}$

check: ✓ Does $F'(x) = f(x)$?

$\frac{1}{3}3x^2 - \frac{3}{2}2x + 2 + 0$

$x^2 - 3x + 2$

#5 $f(x) = x(12x+8)$ ← No "product rule" for antideriv's!

$f(x) = 12x^2 + 8x$

$F(x) = 4x^3 + 4x^2 + C$

#12 $f(x) = \sqrt[3]{x^2} + x\sqrt{x}$ $\quad x^1 \cdot x^{1/2}$

$x^{2/3} + x^{3/2}$

$\boxed{F(x) = \frac{3}{5}x^{5/3} + \frac{2}{5}x^{5/2} + C}$

#13 $f(x) = \frac{10}{x^9} = 10x^{-9}$

$$F(x) = \frac{10}{-8}x^{-8} = \underbrace{-\frac{5}{4}x^{-8} + C = -\frac{5}{4x^8} + C}_{\checkmark}$$

#16

$f(t) = 3\cos t - 4\sin t$

$F(t) = 3\sin t - 4(-\cos t)$

$= 3\sin t + 4\cos t + C$

pg 283 Find $f(x)$

#29 $f'(x) = 1 + 3\sqrt{x}$ $f(4) = 25$

$f'(x) = 1 + 3x^{1/2}$

$f(x) = x + 3\frac{2}{3}x^{3/2} + C$

$f(x) = x + 2x^{3/2} + C$

With more facts, we may be able to find a specific antiderivative.

$f(4) = 4 + 2(4)^{3/2} + C = 25$ solve for c.

$4 + 2\cdot 8 + c = 25$

$20 + c = 25$

$\boxed{c = 5}$

$$\boxed{f(x) = x + 2x^{3/2} + 5}$$

\# 27 $f'''(t) = 12 + \sin t$

$f''(t) = 12t - \cos t + c$

$f'(t) = 6t^2 - \sin t + ct + d$

$f(t) = 2t^3 + \cos t + \frac{1}{2}ct^2 + dt + q$

most general.

\# 35

$f''(x) = -2 + 12x - 12x^2$ $f(0)=4,\ f'(0)=12$

$f'(x) = -2x + 6x^2 - 4x^3 + c$

$f'(0) = \cancel{-2(0)} + \cancel{6(0)^2} - \cancel{4(0)^3} + c = 12$

$c = 12$

$f'(x) = -2x + 6x^2 - 4x^3 + 12$

$f(x) = -x^2 + 2x^3 - x^4 + 12x + d$

$f(0) = 0 \quad 0 \quad 0 \quad 0 + d = 4$

$f(x) = -x^4 + 2x^3 - x^2 + 12x + 4$

Find $f(x)$

\# 39

$f''(x) = 4 + 6x + 24x^2$ $f(0)=3$ $f(1)=10$

$f'(x) = 4x + 3x^2 + 8x^3 + c$???

$f(x) = 2x^2 + x^3 + 2x^4 + cx + d$

$f(0) = 0 + 0 + 0 + 0 + d = 3$

$f(x) = 2x^4 + x^3 + 2x^2 + cx + 3$

$f(1) = 2 + 1 + 2 + c + 3 = 10$

$c = 2$

$f(x) = 2x^4 + x^3 + 2x^2 + 2x + 3$

What if ....

system!
$$\begin{cases} 8c + d = 5 \\ 2c + d = 13 \end{cases} \quad ?$$

subst, elim

AppxE: Express sums of sequences in sigma notation.
AppxE: Calculate sums of sequences.

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Ending value for iterator.

"sigma"

$$\sum_{i=3}^{9} \underbrace{i^2 + 1} = \underline{?}$$

iterator, always increases by 1.

sequence definition

$5 + 7 + 9 + 11 + 13 + 15$

$= \sum_{i=1}^{6} \boxed{}$ expression such that when $i=1$, exp $= 5$

$i=2$ exp $= 7$

$i=3$ exp $= 9$

Can start anywhere. 0 or 1 is almost always good.

$$\sum_{i=1}^{6} 2i+3$$

$i=6$ $\quad exp=15$

i	~~$i+2$~~	$2i+3$
1	~~3~~	5
2	~~4~~	7
3	~~5~~	9

$i=$ $\quad$ 0 $\;$ 1 $\;$ 2 $\;$ 3 $\;$ 4 $\;$ 5 $\;$ 6 $\;$ 7

$\quad\;\;$ ~~1~~ $\;$ ~~2~~ $\;$ ~~3~~ $\;$ ~~4~~ $\;$ ~~5~~ $\;$ ~~6~~ $\;$ ~~7~~ $\;$ ~~8~~

$$\boxed{2^i} \quad \underbrace{1+2+4+8+16+32+64+128}_{\sum_{i=0}^{7} 2^i}$$

$i=$ $\quad$ 1 $\;$ 2 $\;$ 3 $\;$ 4

$2^{i-1} =$ 1 $\;$ 2 $\;$ 4 $\;$ 8

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$i=$ $\quad$ 1 $\quad$ 2 $\quad$ 3 $\quad$ 4 $\quad$ . . . . $\quad$?

#14 $\quad \frac{i+2}{i+6} \quad \frac{3}{7} + \frac{4}{8} + \frac{5}{9} + \frac{6}{10} + \ldots + \frac{23}{27}$

$$\sum_{i=1}^{21} \frac{i+2}{i+6}$$

$i=$ $\quad$ 1 $\quad$ 2 $\quad$ 3 $\quad$ 4 $\quad$ . . . $\quad$ n

#15 $\quad 2i = 2+4+6+8+\ldots+2n$

$$\sum_{i=1}^{n} 2i$$

#3 $\quad$ expand

$$\sum_{i=4}^{6} 3^i = \underline{3^4+3^5+3^6}$$

Sum of numbers from 1 to n:

$$\sum_{i=1}^{n} i = \frac{n\cdot(n+1)}{2}$$

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$$\sum_{i=1}^{n} 1 = \overbrace{1+1+1+\dots+1}^{n} = n$$

$$\sum_{i=1}^{n} c^{(\text{const})} = cn$$

$$\boxed{\sum_{i=1}^{n} i = \frac{n(n+1)}{2}}$$ memorize

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

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$\sum$ distributes across $+/-$

$$\sum_{i=p}^{q} a_i + \sum_{i=p}^{q} b_i = \sum_{i=p}^{q}(a_i + b_i)$$

And const mults can float.

$$\sum c a_i = c \sum a_i$$

$$\sum_{i=1}^{1000} i(i+1) = 1\cdot 2 + 2\cdot 3 + 3\cdot 4 + 4\cdot 5 + \dots 1000\cdot 1001$$

$$= ??$$

$$= \sum_{i=1}^{1000} i^2 + i$$

$$= \sum_{i=1}^{1000} i^2 + \sum_{i=1}^{1000} i$$

$$\frac{\cancel{1000}^{500}(1001)(\cancel{2001}^{667})}{\cancel{6}\ \cancel{3}} + \frac{\cancel{1000}^{500}(1001)}{\cancel{2}}$$

$$\begin{array}{r} 667 \\ 3\overline{)2001} \\ -18 \\ \hline 20 \\ -18 \\ \hline 21 \end{array}$$

$$1001\cdot 500\cdot 667 + 1001\cdot 500\cdot 1$$

$$1001\cdot 500\cdot 668$$

$$\begin{array}{r} 668 \\ \times 1001 \\ \hline 668668 00 \\ \times \quad 5 \\ \hline 40 \\ 30 \\ 30 \\ 3340 \end{array}$$

$$\boxed{334334000}$$