Integrals wkst

1. 160

2. -12

3. 8

4. -60

5. $\frac{1}{2}x^6 - \frac{5}{2}x^2 + 9x + C$

---

6. 0

7. 108

8. 3

9. 29

10. $\frac{1}{22}(x^2-1)^{11} + C$

---

11. 168

12. $\frac{-1}{30(5x+6)^6}$

13. 6

14. $-x^{-4} + \frac{5}{3}x^{-3} + C$

15. 0

---

16. $(u = \tan x)$ $\quad \frac{1}{3}\tan^3 x + C$

17. $(u = 1+9x^2)$ $\quad \frac{1}{27}(1+9x^2)^{3/2} + C$

18. 0

19. $\frac{2}{7}x^{7/2} - \frac{4}{3}x^{3/2} + 2x^{1/2} + C$

20. $(u = \sec x)$ 7

---

21. $(u = 2-x)$ $\quad \frac{4}{9}(2-x)^9 - (2-x)^8$

22. $\sin x + \cot x + C$

23. $(u = x^2+1)$ $\quad 609$

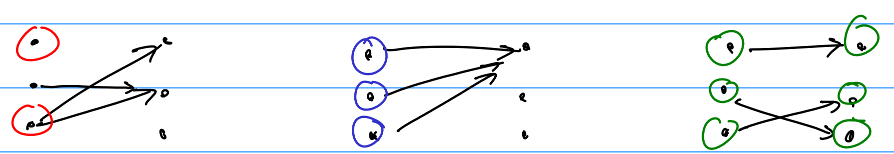
24. $u = x-3$ $\quad \frac{2}{7}(x-3)^{7/2} + \frac{12}{5}(x-3)^{5/2} + 6(x-3)^{3/2} + C$

25. $\int_8^{18} 1\, dx = 10$

6.1 Find the derivative of an inverse of a function. (No need to memorize this formula.)

$$\underset{\text{Domain}}{x} \xrightarrow{f} \underset{\text{Range}}{y}$$

Function: Relation in which each element of the domain points to exactly one element of the range.



Not a function $\qquad$ function $\qquad$ One-to-one function

A function is One-to-one if there exists an inverse.

Inverse: f has an inverse f^{-1} if:

$$f(f^{-1}(x)) = x \quad \text{for all } x \text{ in domain of } f^{-1}$$

and

$$f^{-1}(f(x)) = x \quad \text{for all } x \text{ in domain of } f.$$

$f(x)$	$f^{-1}(x)$
$x+3$	$x-3$
$3x$	$\frac{x}{3}$
x^3	$\sqrt[3]{x}$
3^x	$\log_3 x$

In general:

$f(x) = 2x^3 - 5$ $\qquad f^{-1}(x)$?

$y = 2x^3 - 5$ $\qquad$ Let $f(x) = y$

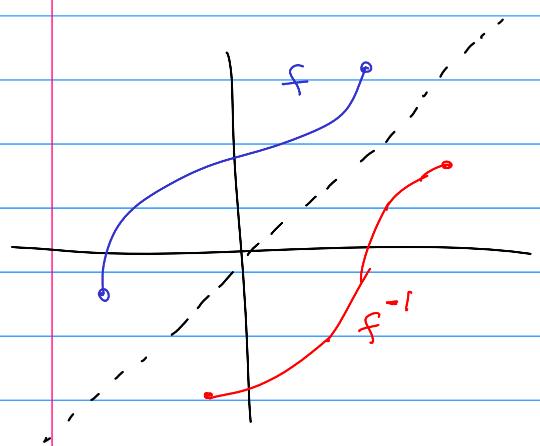
$x = 2y^3 - 5$ $\qquad$ Swap x & y

$x + 5 = 2y^3$ $\qquad$ Solve for y

$\frac{x+5}{2} = y^3$

$y = \sqrt[3]{\frac{x+5}{2}}$

$f^{-1}(x) = \sqrt[3]{\frac{x+5}{2}}$ $\qquad$ Let $y = f^{-1}(x)$



f^{-1} is the reflection of f over the line $y = x$.

How do we find the derivative of the inverse?

$$(f^{-1})'(x) = \underline{\qquad}$$

Def Inverse $\quad f(f^{-1}(x)) = x$

$$\frac{d}{dx}\left[f(f^{-1}(x))\right] = \frac{d}{dx}(x)$$

$$f'(f^{-1}(x)) \cdot [f^{-1}(x)]' = 1 \qquad \text{chain rule}$$

$$\boxed{(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}}$$

$f(x) = x^2$ $\qquad f'(x) = 2x$ $\qquad \frac{1}{2}x^{-1/2} = \frac{1}{f'(\sqrt{x})}$

$f^{-1}(x) = \sqrt{x}$ $\qquad (f^{-1})'(x) = \frac{1}{2}x^{-1/2}$

$$\frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}} \quad \checkmark$$

$f(x) = \sin x$ $\qquad f'(x) = \cos x$ $\qquad (f^{-1})'(x) = \frac{1}{f'(\sin^{-1}x)}$

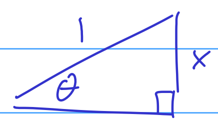
$f^{-1}(x) = \sin^{-1}x$ $\qquad (f^{-1})'(x) = \underline{\ ?\ }$

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\cos(\sin^{-1}x)}$$

$\cos(\sin^{-1}x) = ?$ $\longrightarrow$ $\cos(\theta) = \sqrt{1-x^2}$

Let $\theta = \sin^{-1}x$

$\sin\theta = \frac{x}{1} =$



Pyth: $\sqrt{1-x^2}$

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$$

39 $\quad f(x) = 3x^3 + 4x^2 + 6x + 5$ $\qquad$ Find $(f^{-1})'(5)$.

$$= \frac{1}{f'(f^{-1}(5))}$$

We need f, and f', but f^{-1} ??

We can do this just knowing $f^{-1}(5)$.

$$f^{-1}(5) = c$$

$$5 = f(c) \qquad c = 0 \text{ by observation}$$