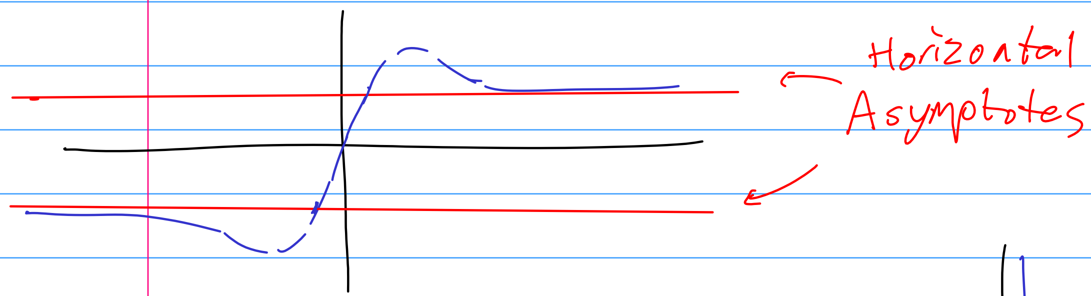
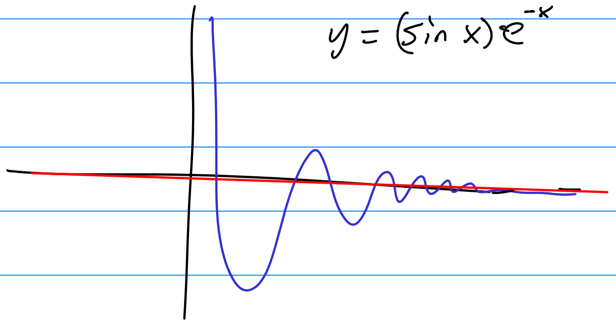
3.4 Find limits at infinity and horizontal asymptotes of graphs.





$y = f(x)$ has a horizontal asymptote of $y = L$

if:

$$\lim_{x\to\infty} f(x) = L \quad \text{and/or} \quad \lim_{x\to-\infty} f(x) = L$$

$\lim_{x\to\infty} f(x) = L$ if we can make $f(x)$ as close as we want to L by making x large enough.

$\lim_{x\to-\infty} f(x) = L$ " " " " " " " " " by making x large negative enough.

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pg 234 If $r > 0$ is rational, then:

$$\lim_{x\to\infty} \frac{1}{x^r} = 0$$

$$\lim_{x\to-\infty} \frac{1}{x^r} = 0 \quad (\text{as long as } x^r \text{ is defined for all } x)$$

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#9 $$\lim_{x\to\infty} \frac{(3x-2)}{(2x+1)} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \qquad = \frac{3}{2}\ ?$$

$$= \lim_{x\to\infty} \frac{3 - \frac{2}{x}}{2 + \frac{1}{x}}$$

The limit laws apply, even when $\to \infty$.

$$= \frac{\lim_{x\to\infty} 3 - \lim_{x\to\infty} \frac{2}{x}}{\lim_{x\to\infty} 2 + \lim_{x\to\infty} \frac{1}{x}}$$

By our new limit law

$$= \frac{3 - 2\cdot 0}{2 + 0}$$

$$= \boxed{\frac{3}{2}}$$

#13 $$\lim_{t\to\infty} \frac{\sqrt{t} + t^2}{2t - t^2} \cdot \frac{\frac{1}{t^2}}{\frac{1}{t^2}} = \lim_{t\to\infty} \frac{\frac{1}{t^{3/2}} + 1}{\frac{2}{t} - 1}$$

$$\frac{\sqrt{t}}{t^2} = \frac{t^{1/2}}{t^2} = t^{-3/2}$$

$$= \frac{\lim_{t\to\infty} \frac{1}{t^{3/2}} + \lim_{t\to\infty} 1}{2\lim_{t\to\infty} \frac{1}{t} - \lim_{t\to\infty} 1} = \frac{0 + 1}{2(0) - 1} = -1$$

#11 $$\lim_{x\to-\infty} \frac{x-2}{x^2+1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} \qquad = 0?\ \ 1?$$

$$= \lim_{x\to-\infty} \frac{\frac{1}{x} - \frac{2}{x^2}}{1 + \frac{1}{x^2}}$$

$$= \frac{\lim \frac{1}{x} - \lim \frac{2}{x^2}}{\lim 1 + \lim \frac{1}{x^2}} = \frac{0 - 0}{1 + 0} = \frac{0}{1} = 0$$

#17

$$\lim_{x\to\infty} \frac{\sqrt{1+4x^6}}{2-x^3} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to\infty} \frac{\sqrt{(1+4x^6)}\sqrt{\left(\frac{1}{x^6}\right)}}{\frac{2}{x^3} - 1}$$

$$= \lim_{x\to\infty} \frac{\sqrt{\frac{1}{x^6} + 4}}{\frac{2}{x^3} - 1} = \frac{\sqrt{\lim_{x\to\infty} \frac{1}{x^6} + \lim_{x\to\infty} 4}}{\lim_{x\to\infty} \frac{2}{x^3} - \lim_{x\to\infty} 1} = \frac{\sqrt{4}}{-1} = -2$$

#18

$$\lim_{x\to-\infty} \frac{\sqrt{1+4x^6}}{2-x^3} = \ldots \text{ same steps? } = -2?$$

Graph indicates 2...

A problem which asks $\lim_{x\to-\infty} f(x)$

$$= \lim_{-x\to-\infty} f(-x)$$

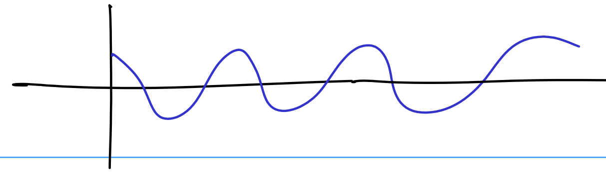
But when $-x \to -\infty$, $x \to \infty$!

$$= \lim_{x\to\infty} f(-x)$$

$$\lim_{x\to-\infty} \frac{\sqrt{1+4x^6}}{2-x^3} = \lim_{-x\to-\infty} \frac{\sqrt{1+4(-x)^6}}{2-(-x)^3} = \lim_{x\to\infty} \frac{\sqrt{1+4x^6}}{2+x^3}$$

$$\text{(same steps)} = \frac{\sqrt{\lim_{x\to\infty} \frac{1}{x^6} + 4}}{\lim_{x\to\infty} \frac{2}{x^3} + 1} = \frac{\sqrt{4}}{1} = \boxed{2}$$

$$\lim_{x\to\infty} \cos x = \text{DNE}$$



$$\lim_{x\to\infty} x^3+3 = \infty$$

$$\lim_{x\to-\infty} \left(-2x^5+4x^2\right) = \infty$$

$$\lim_{x\to-\infty} x^3-2x = -\infty$$

$-2(-1000)^5$

$= +$

$$\lim_{x\to-\infty} x^4 = \infty$$

Limits of polys: Look at leading term, leading coefficient, and $x\to\infty$, $x\to-\infty$.

Limits of rationals: $\lim_{x\to\infty} \frac{P(x)}{Q(x)}$

$\deg(P)=\deg(Q)$: Ratio of leading Coefficients.

$$\lim_{x\to\infty} \frac{3x^6-5}{4x-7x^6} = \frac{3}{-7}$$

$\deg(P) < \deg(Q)$: 0.

$$\lim_{x\to\infty} \frac{x^{5/2}+1}{x^3+1} = 0$$

$\frac{5}{2} < 3$

$\deg(P) > \deg(Q)$: ratio of $\frac{\lim P}{\lim Q}$. $= \pm\infty$

$$\lim_{x\to\infty} \frac{x^7-3x}{2x^5+1} = \infty$$

$$\lim_{x\to-\infty} \frac{-x^4+3x^3-1}{5x^3+2} = \frac{-\infty}{-\infty} = \infty$$

Curve Analysis

- Inc/Dec intervals
- Critical points
- Conc up/down intervals
- Inflection points
- Local min/max
- Horiz/vert asymps

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#14

$$y = \frac{1}{x^2-4}$$

$$y' = \frac{\cancel{(x^2-4)(1)'} - 1(x^2-4)'}{(x^2-4)^2}$$

$$y' = \frac{-2x}{(x^2-4)^2} = \underline{0} \text{ or } \underline{\text{DNE}}$$

$-2x = 0$

$x = 0$

$(x^2-4)^2 = 0$

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

crit pts: $0, \pm 2$

$y'(-3) = \frac{6}{25}$, $y'(-1) = \frac{2}{9}$, $y'(1) = \frac{-2}{9}$, $y'(3) = \frac{-6}{25}$

y': $+$ | -2 | $+$ | 0 | $-$ | 2 | $-$

y: $\nearrow$ $\nearrow$ $\searrow$ $\searrow$

$x - 3$

$x(-3)$

$$y'' = \frac{(x^2-4)^2(-2) - (-2x)\,2(x^2-4)\,2x}{(x^2-4)^4}$$

$$y'' = \frac{-2(x^2-4)^2 + 8x^2(x^2-4)}{(x^2-4)^4}$$

$$y'' = \frac{-2(x^2-4) + 8x^2}{(x^2-4)^3} \cdot \cancel{\frac{(x^2-4)}{(x^2-4)}}$$

$$y'' = \frac{-2x^2 + 8 + 8x^2}{(x^2-4)^3} = \frac{8 + 6x^2}{(x^2-4)^3} = \underline{0} \text{ or } \underline{\text{DNE}}$$

Possible inflection pts:

$8 + 6x^2 = 0$

$6x^2 = -8$

$\cancel{x^2 = -\frac{8}{6}}$

no sol's

$(x^2-4)^3 = 0$

$x^2 - 4 = 0$

$x^2 = 4$

$x = \pm 2$

$y''(-3) = +$, $y''(0) = \frac{8}{-64}$, $y''(3) = +$

y'': $+$ | -2 | $-$ | 2 | $+$

y: $\cup$ $\cap$ $\cup$

$$y = \frac{1}{x^2-4}$$

-2, 2 not in domain

$\cancel{-2}, 0, \cancel{2}$: local mins/maxs?

Around 0, we have $\nearrow \searrow$
and $\cap$

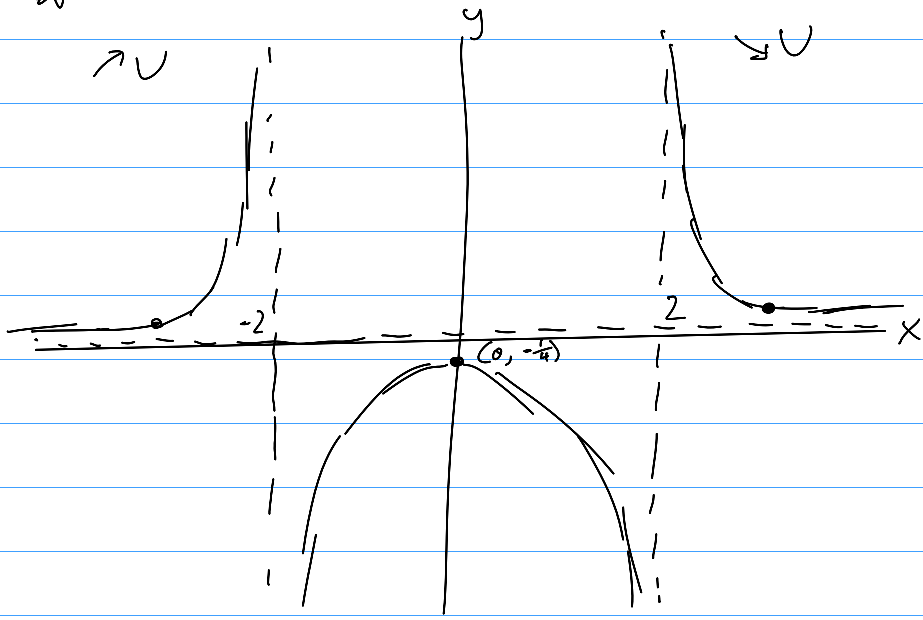
so $(0, -\frac{1}{4})$ is a local max.

Vert asymps: $x = 2, -2$

Horiz asymps:

$$\lim_{x \to \infty} \frac{1}{x^2-4} = 0$$

$y = 0$



$f(3) = \frac{1}{5}$

$f(-3) = \frac{1}{5}$