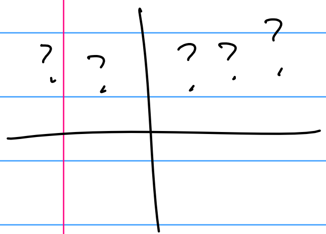
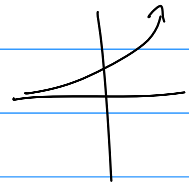
als.
6.2,3,4 Find derivatives and integrals of exponential and logarithmic functions.

$f(x) = b^x$ Exponential function

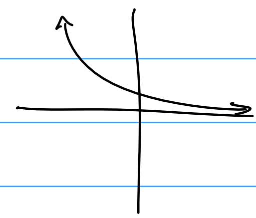
$b < 0$



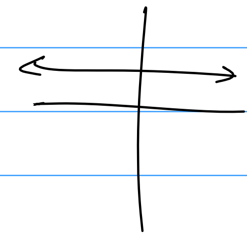
$\underline{b > 1}$ Growth



$\underline{0 < b < 1}$ Decay



$b = 1$



Integer power

$2^3 = 8$

$2.5^3 = 15.625$

$(\sqrt{2})^3 \approx 2.828$

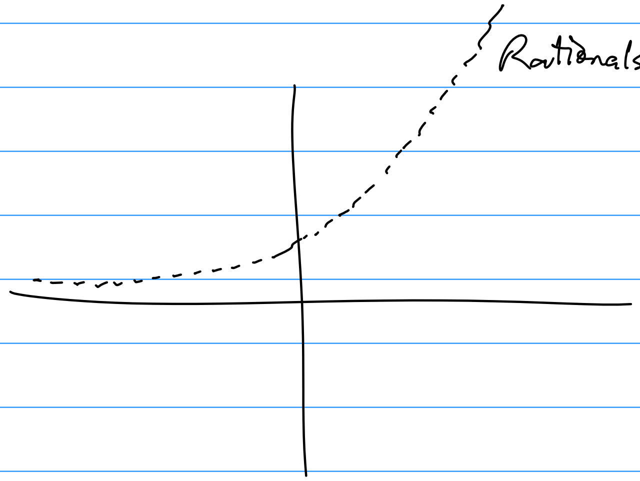
Rational power

$16^{3/4} = 8$

$7^{3/4} \approx 4.304$

Irrational power

$2^{\sqrt{2}} \approx 2.665$



$2^{\sqrt{2}} = ?$

$2^1 < 2^{\sqrt{2}} < 2^2$

$2^{1.4} < 2^{\sqrt{2}} < 2^{1.5}$

$2^{1.41} < 2^{\sqrt{2}} < 2^{1.42}$

$\vdots \qquad \vdots$

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$$b^x = \lim_{r \to x} b^r, \quad r \text{ rational}$$

Exp's grow very very fast!

$$(1.1)^x \underset{\text{eventually}}{>} x^{1000000} + 9x^{479} + 3$$

2^x 1 2 4 8 16 32 64 128

exp! 1 2 4 8 16 32 64

x^2 1 4 9 16 25 36

linear 3 5 7 9 11

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \qquad f(x) = b^x \ldots$$

$$f'(x) = \lim_{h\to 0} \frac{b^{x+h} - b^x}{h} = \lim_{h\to 0} \frac{b^x b^h - b^x}{h} = \lim_{h\to 0} \frac{b^x(b^h-1)}{h}$$

$$[b^x]' = b^x \lim_{h\to 0} \frac{b^h-1}{h}$$

$[b^x]'$ is proportional to b^x !

$$\lim_{h\to 0} \frac{b^h-1}{h} = ?$$

Depends on b.

If $b=2$, $\lim \approx 0.69$ ⟵ $\lim = 1$?

If $b=3$, $\lim \approx 1.10$

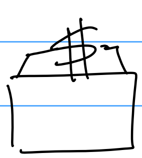
If $b=4$, $\lim \approx 1.38$

What is e?

e is the number such that $\lim_{h\to 0} \frac{e^h-1}{h} = 1$.

---

e also arises in compound interest.



100% return yearly!

$\$1000 \xrightarrow{\text{1 year}} \$2000$

Compounded twice a year? $\quad \$1000 \xrightarrow[50\%]{6\text{ mo}} \$1500 \xrightarrow[50\%]{6\text{ mo}} \$2250$

Monthly? $\quad \$1000 \xrightarrow[\left(\frac{100}{12}\right)\%]{1\text{ mo}} \quad 1000\left(1+\frac{1}{12}\right)\left(1+\frac{1}{12}\right)\left(1+\frac{1}{12}\right)\ldots \longrightarrow$

$$1000\left(1+\frac{1}{12}\right)^{12} \approx \$2613$$

Daily? $\quad \$1000\left(1+\frac{1}{365}\right)^{365} \approx \$2714$

10000 times/year $\quad \underbrace{1000\left(1+\frac{1}{10000}\right)^{10000}}_{} \approx \$2718$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \approx 2.71828 = e$$

Does this e satisfy

$$\lim_{h\to 0}\frac{e^h-1}{h} = 1?$$

Let $n = \frac{1}{s}$. $\quad n\to\infty$, $s \to 0$

$$\lim_{\frac{1}{s}\to\infty}(1+s)^{1/s}$$

$$= \lim_{\cancel{s}\,h\to 0}\left(1+\cancel{s}\,h\right)^{1/\cancel{s}\,h}$$

$$\lim_{h\to 0}\frac{\left((1+h)^{1/h}\right)^h - 1}{h}$$

$$= \lim_{h\to 0}\frac{1+h-1}{h} = \frac{h}{h} = 1!$$

$$\boxed{e \approx 2.718281828}$$

$$\frac{d}{dx}\left[e^x\right] = e^x$$

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$\underline{\underline{e \text{ is a constant!!}}} \approx 2.718$

#31 $f(x) = e^5 \qquad f'(x) = 0$

#32 $k(r) = \underbrace{e^r}_{\text{Exp}} + \underbrace{r^e}_{\text{Power}} \qquad k'(r) = \underbrace{e^r}_{\text{Exp rule}} + \underbrace{er^{e-1}}_{\text{power rule}}$

#33 $f(x) = \underbrace{(3x^2 - 5x)\,e^x}_{\text{product rule}}$

$$f'(x) = \underbrace{(6x-5)}_{\text{deriv}}e^x + (3x^2-5x)\underbrace{e^x}_{\text{deriv}} \quad ✓$$

$$= e^x(6x - 5 + 3x^2 - 5x)$$

$$= \underline{e^x(3x^2 + x - 5)}$$

#37 $y = e^{\tan\theta} = f(g(\theta))$

chain rule.

outer: $e^\theta = f(\theta) \qquad f'(\theta) = e^\theta$

inner: $\tan\theta = g(\theta) \qquad g'(\theta) = \sec^2\theta$

$$y' = f'(g(\theta)) \cdot g'(\theta) = f'(\tan\theta)\sec^2\theta$$

$$= e^{\tan\theta}\sec^2\theta$$

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$\int e^x\,dx = e^x + C$

#83 $\int_0^1 x^e + e^x\,dx$

$$\frac{1}{e+1}x^{e+1} + e^x \Big|_0^1$$

$$\left(\frac{1}{e+1}(1)^{e+1} + e^1\right) - \left(\frac{1}{e+1}(0)^{e+1} + e^0\right)$$

$$\left(\frac{1}{e+1} + e\right) - (0 + 1)$$

$$\underline{\frac{1}{e+1} + e - 1}$$

	$f(x)$	$f'(x)$	$F(x)$
Exp	e^x	e^x	$e^x + C$
	$e^{g(x)}$	$e^{g(x)} g'(x)$	______
	b^x	$\boxed{}$	$\boxed{}$

Inverse of exp function: logarithmic function.

☆ $b^x = y \iff x = \log_b(y)$

$2^3 = 8 \longleftrightarrow 3 = \log_2(8)$

☆ $f(x) = b^x \Rightarrow f^{-1}(x) = \log_b x.$ $\Rightarrow$ $f(f^{-1}(x)) = b^{\log_b(x)} = x$

$f^{-1}(f(x)) = \log_b(b^x) = x$

☆ $\log_b x$ is the power to which b must be raised in order to equal x.

6.3 Evaluate logarithms.

$\log_b b = 1$

$\log_3 81 = 4$ $\qquad$ $\log_7 7 = 1$

$81 = 3^4$ $\qquad$ $7 = 7^1$

$\log_2 32 = 5$

$32 = 2^5$

$\log_4 1 = 0$

$1 = 4^0$

$\log_{16} 4 = \frac{1}{2}$

$4 = 16^{1/2}$

$\log_8 32 = \frac{5}{3}$

$32 = 8^{5/3}$

$\log_3 \frac{1}{3} = -1$

$\log_5 \frac{1}{125} = -3$

$\log_2 0 = \text{undef}$

$0 = 2^{\square}$

$\log_2(-1) = \text{undef}$

$\boxed{\log_b 0 \ \text{undef}}$

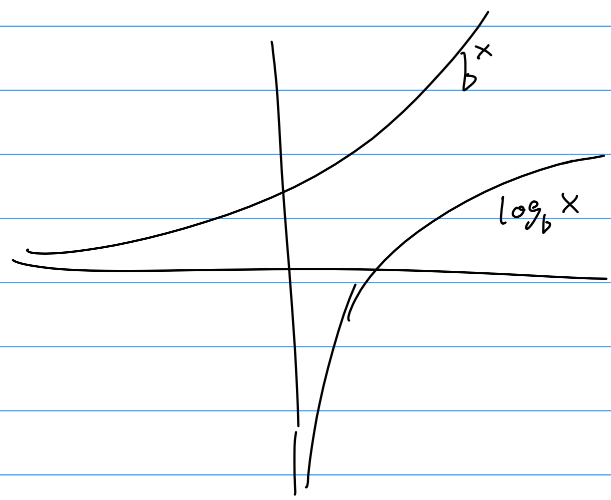
$\boxed{\log_b(-a) \quad \text{undef}}$

$\cancel{\log_{-3} 7}$

$b > 0.$

Logarithmic function

$f(x) = \log_b(x)$ $\quad 0 < b$ and $b \neq 1$.



Grows very very slowly.

"$\log(\log(x)) < 5$"

6.3 Expand and collect logarithmic expressions. (Have the logarithm laws memorized!)

Log rules (not derivative rules!)

$\log_b(xy) = \log_b x + \log_b y$ $\qquad b^x b^y = b^{x+y}$

$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$ $\qquad b^x / b^y = b^{x-y}$

$\log_b(x^n) = n \cdot \log_b(x)$ $\qquad (b^x)^n = b^{xn}$

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#10 $\log_{10}\left(\sqrt{\dfrac{x-1}{x+1}}\right) = \dfrac{1}{2}\log_{10}\left(\dfrac{x-1}{x+1}\right) =$

$$\underline{\frac{1}{2}\left(\log_{10}(x-1) - \log_{10}(x+1)\right)}$$

There is no rule for $\log_b(x+y)$.

Condense

#13 $2\ln x + 3\ln y - \ln z$

$\ln(x^2) + \ln(y^3) - \ln(z)$

$\ln(x^2y^3) - \ln(z)$

$$\underline{\ln\left(\frac{x^2y^3}{z}\right)}$$

$\ln(x) = \log_e(x)$
"natural log"

$\log x = \log_{10}(x)$
no subscript
"common logarithm"

$\log 1000 = 3$ $\qquad$ $\ln(e^5) = 5$

Change-of-base law:

$$\boxed{\log_b x = \frac{\log x}{\log b} = \frac{\ln x}{\ln b} = \frac{\log_c x}{\log_c b}}$$

$\log_2 10$ on calc? $= \dfrac{\log 10}{\log 2} = \dfrac{1}{\log 2} \approx \dfrac{1}{.301} = 3.321$