If f poly/rational,

Direct subst: $\lim_{x \to a} f(x) = f(a)$

for x in domain of f.

$f(x)$ cont at $\underline{a}$ if: $\lim_{x \to a} f(x) = f(a)$.

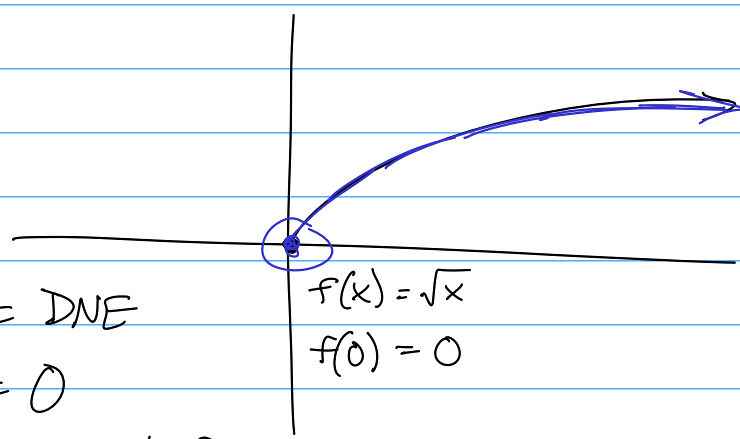
$\Rightarrow$ If f poly/rational, f is continuous on its domain.

If f and g are functions continuous at $\underline{a}$,
Then the following are also cont at $\underline{a}$:

$f+g \qquad f-g \qquad cf \qquad fg \qquad \frac{f}{g}$ (if $g(a) \neq 0$)

const

pg 88: Cont on their domains:
- Polynomials
- Rational funcs
- Root funcs
- Trig funcs



$f(x) = \sqrt{x}$

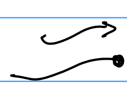
$f(0) = 0$

$\lim_{x \to 0} f(x) = DNE$

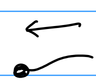
$f(0) = 0$

$\sqrt{x}$ is Not continuous at 0.

BUT...



continuous at $\underline{a}$ from the left if
$\lim_{x \to a^-} f(x) = f(a)$

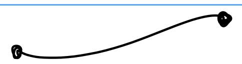


continuous at $\underline{a}$ from the right if
$\lim_{x \to a^+} f(x) = f(a)$

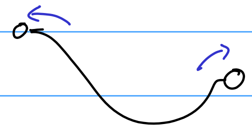
$f(x) = \sqrt{x}$ is cont. at 0 from the right.

We say f(x) is continuous on an interval if it is continuous at every number in the interval.

If f(x) is defined on only one side of an endpoint, we understand "continuous at the endpoint" to mean cont. from the right or left.



continuous.



continuous on its domain.

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pg 92 + Def of continuity + Continuity theorems
+ Properties of limits

#14 $f(x) = \underbrace{3x^4 - 5x + \sqrt[3]{x^2+4}}$ $a = 2$

$3(2)^4 - 5(2) + \sqrt[3]{2^2+4}$

$= 48 - 10 + \sqrt[3]{8} = 40$

$$\lim_{x \to 2} 3x^4 - 5x + \sqrt[3]{x^2+4}$$

$$= \lim_{x \to 2} (3x^4 - 5x) + \lim_{x \to 2} \sqrt[3]{x^2+4}$$

DSP pg 64 "Root Law"

$$48 - 10 + \sqrt[3]{\lim_{x \to 2} (x^2+4)}$$

$$48 - 10 + \sqrt[3]{8} = \underline{40}$$

$\lim_{x \to 2} f(x) = f(2)$, $\therefore$ f(x) is continuous at $a = 2$.

18

cont at $x=-2$?

$$f(x) = \begin{cases} \frac{1}{x+2} & \text{if } x \neq -2 \\ 1 & \text{if } x = -2 \end{cases}$$

$f(-2) = 1$

$$\lim_{x \to -2^+} f(x) = \infty$$

limit DNE. $\therefore$ discontinuous at $x=-2$

$$\frac{1}{-1.9+2} = \frac{1}{0.1} = 10$$

$$f(x) = \begin{cases} \frac{1-x}{x^2-1} & \text{if } x \neq 1 \\ \frac{1}{2} & \text{if } x = 1 \end{cases}$$

Cont at 1?

$f(1) = \frac{1}{2}$

$$\lim_{x \to 1} \frac{1-x}{x^2-1} = \lim_{x \to 1} \frac{\overset{-(x-1)}{\cancel{1-x}}}{(x-1)(x+1)} = \lim_{x \to 1} \frac{-1}{x+1} = -\frac{1}{2}$$

Cont at 1? No. $\lim_{x \to 1} f(x) \neq f(1)$.

---

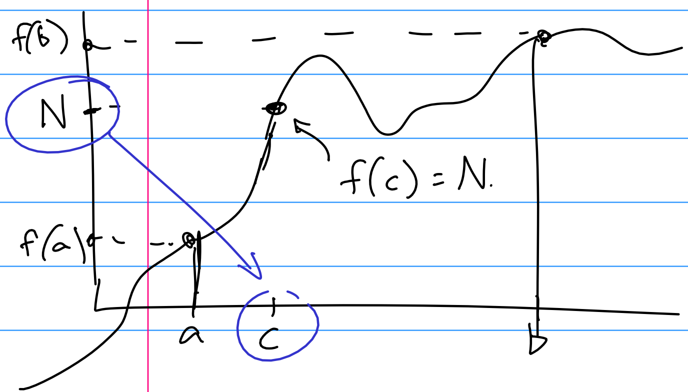
pg 89 If g cont. at a and f cont. at $g(a)$,
then $(f \circ g)(x)$ is continuous at a.

DSP now works for
poly $+$
rat $-$
trig $\times$
root $\div$
$\circ$

Intermediate Value Thm

If
- Suppose f is cont. on $[a, b]$
- Let N be any number between $f(a)$ and $f(b)$
- $f(a) \neq f(b)$

Then There exists a c in (a, b) such that $f(c) = N$.

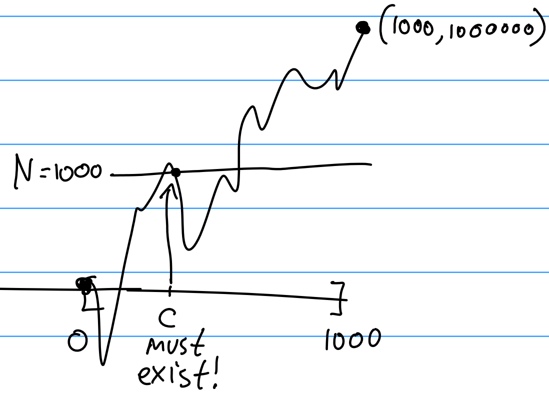


51 If $f(x) = x^2 + 10\sin x$, show there is a number c such that $f(c) = 1000$.

$f(x)$ is cont. everywhere since it is the sum of a poly and a const × a trig function, both of which are cont. on their domains.

$a = 0 \qquad f(0) = 0$

$b = 1000 \qquad f(1000) \approx 1000000$

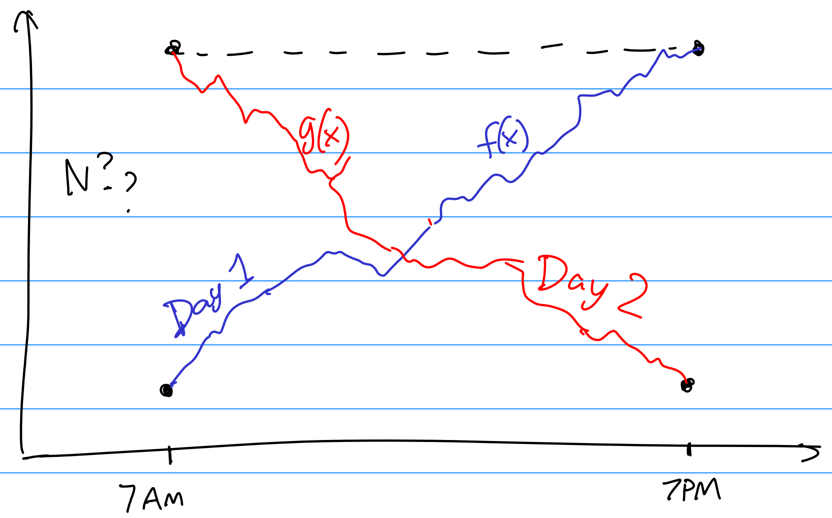


let $[a, b] = [0, 1000]$. Then $f(a) = 0$ and $f(b) \approx 1000000$.

Let $N = 1000$ $\left(f(a) < 1000 < f(b)\right)$.

Then by IVT, there exists a c such that $f(c) = 1000$. $\square$

#73

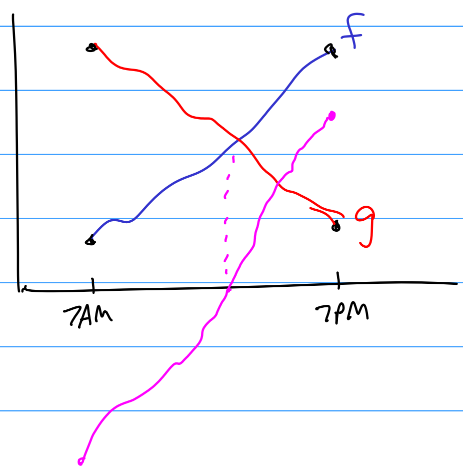


$f(x)$ and $g(x)$ are both continuous.

If $f(c) = g(c)$, then c represents the time of day when the monk retraces his path.

$\underline{f(x) - g(x)}$
continuous.

interval.
$[7AM, 7PM]$

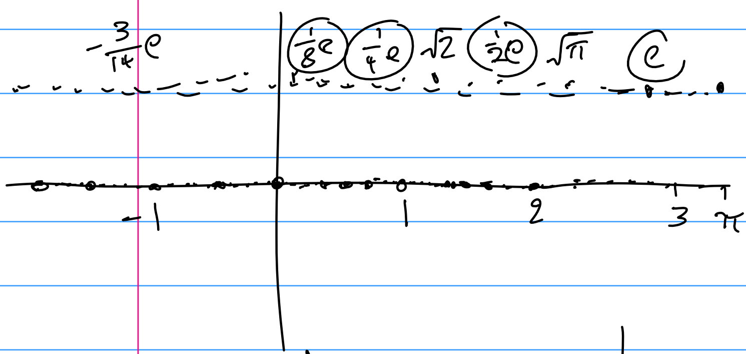


$(f-g)(7AM) < 0$
$(f-g)(7PM) > 0$
Let $N = 0$

Then by IVT, there exists a c in $[7AM, 7PM]$ such that
$f(c) - g(c) = 0 \quad \therefore \quad f(c) = g(c)$, that is,
a time and place the monk crosses on both days.

Where is f continuous?

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ 1 & \text{if } x \text{ is irrational} \end{cases}$$



$f(0) = 0$

$\lim_{x \to 0} f(x) =$ DNE?

since we can find sequences which evaluate to 0, or which evaluate to 1

Defined everywhere — Continuous Nowhere!!