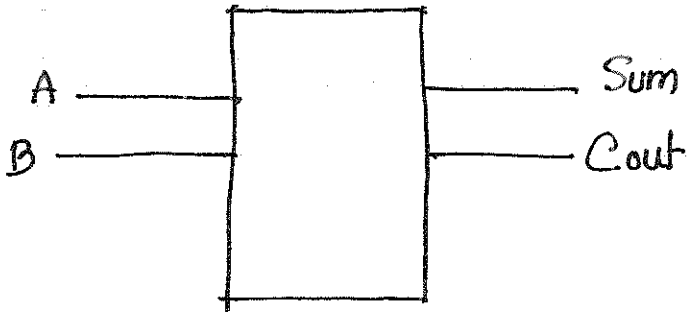
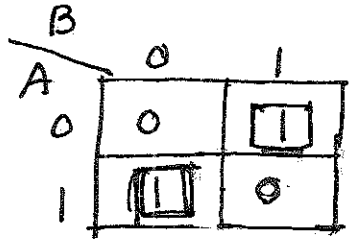
Arithmetic circuits:

Design a half adder

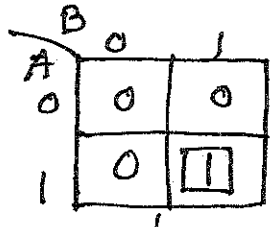


A	B	Sum	Cout
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

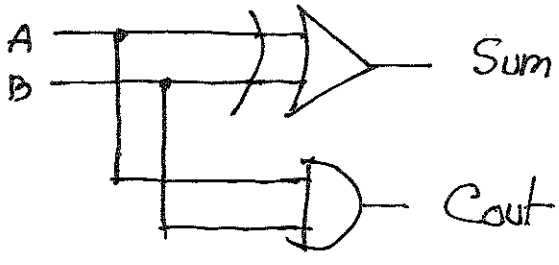


$Sum = AB' + A'B$

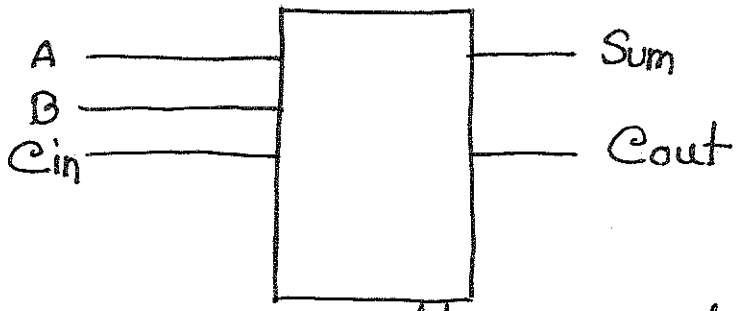
$= A \oplus B$



$Cout = AB$

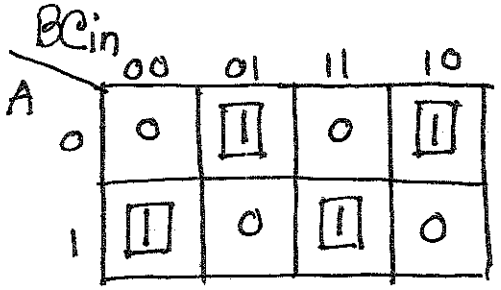


Design a full adder:

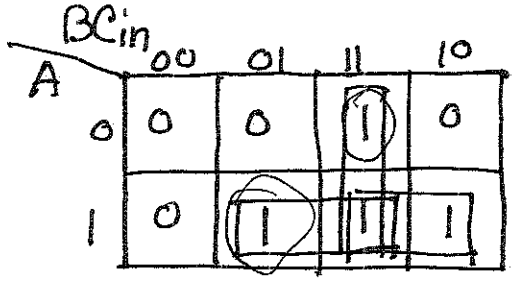


Full adder block diagram

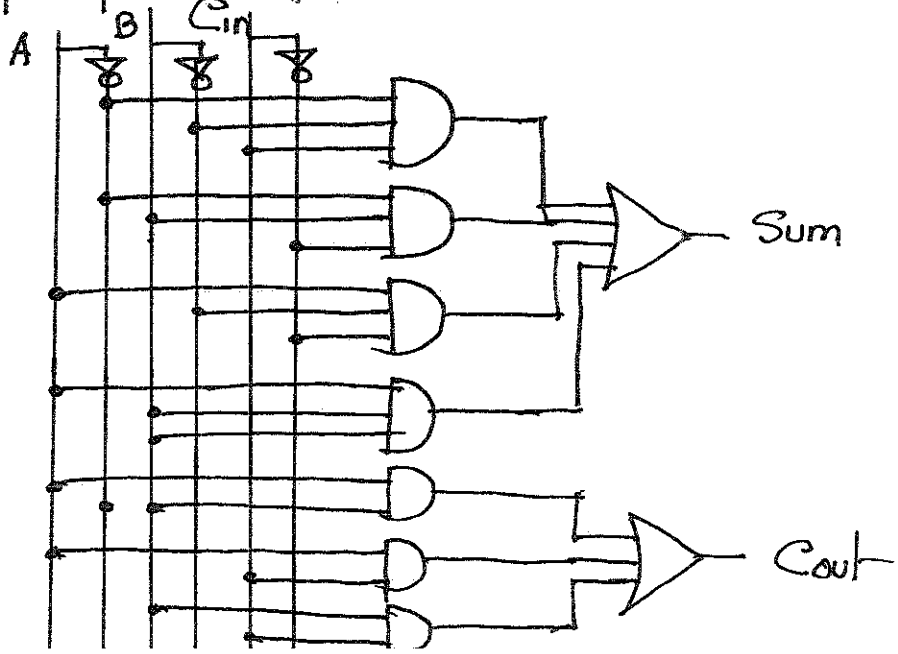
A	B	Cin	Sum	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



$Sum = A'B'Cin + A'BCin' + AB'Cin' + ABC$



$Cout = AB + ACin + BCin$



Design a full-adder with two half-adders.

A \ BCin	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$$Sum = A'B'Cin + A'BCin' + AB'Cin' + ABCin$$

$$= Cin(A'B' + AB) + Cin'(A'B + AB')$$

$$= Cin\overline{(A'B' + AB)} + Cin'(A \oplus B)$$

$$= Cin\left((A+B).(A'+B')\right)' + Cin'(A \oplus B)$$

$$= Cin\left(AA' + AB' + A'B + BB'\right)' + Cin'(A \oplus B)$$

$$= Cin(AB' + A'B)' + Cin'(A \oplus B)$$

$$= Cin(A \oplus B)' + Cin'(A \oplus B)$$

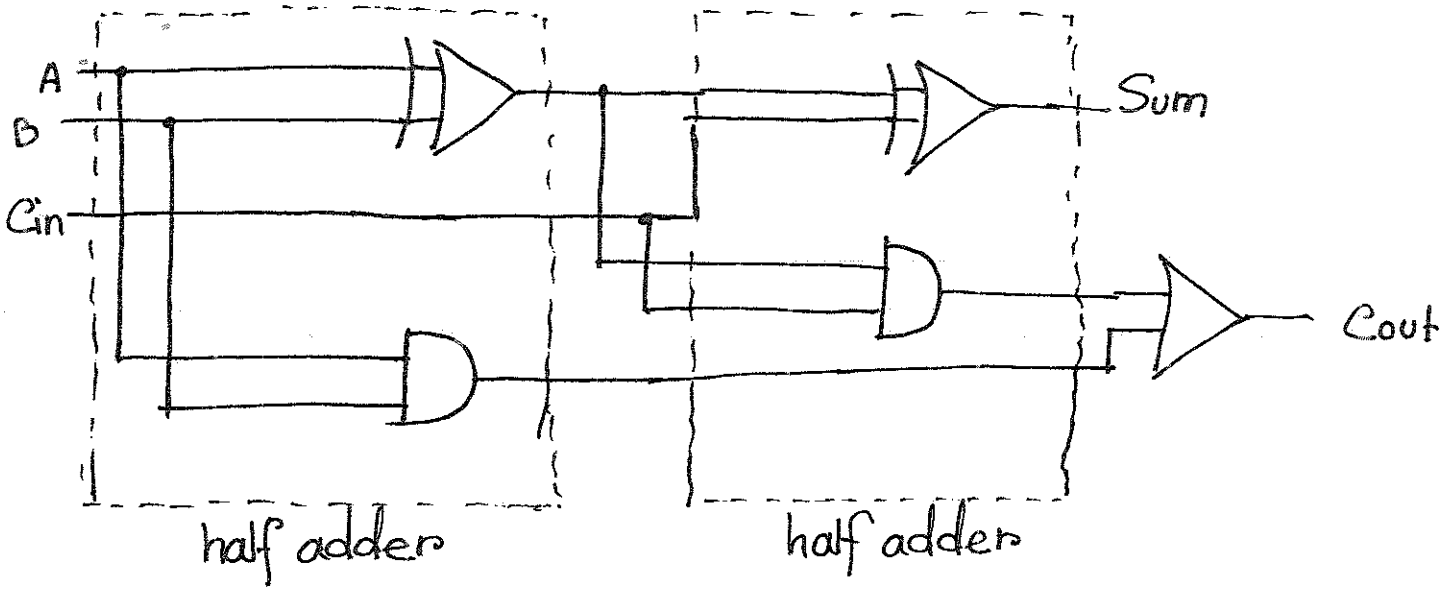
$$= A \oplus B \oplus Cin$$

A \ BCin	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$$Cout = AB'Cin + A'BCin + AB$$

$$= Cin(AB' + A'B) + AB$$

$$= Cin(A \oplus B) + AB$$



Full-adder logic diagram