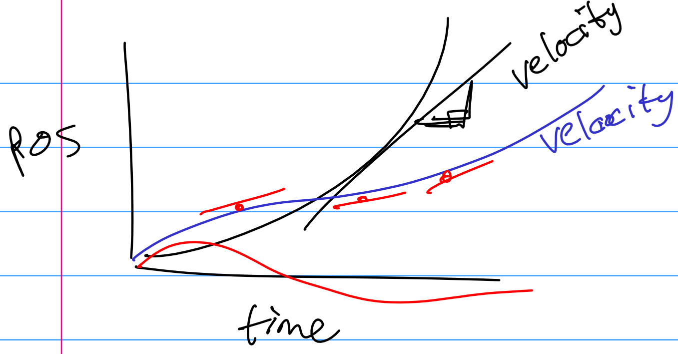


$p(x)$ = position

$v(x)$ = velocity

$a(x)$ = acceleration

$p'(x) = v(x)$ = rate of change of position

$v'(x) = a(x)$

$$p''(x) = a(x)$$

The second derivative of position is acceleration.
(Deriv of deriv)

$a'(x) = j(x)$ "jerk"

Third derivative

$$p'''(x) = j(x)$$

"Newton" notation

Liebniz notation

$y = f(x)$

d is not a variable
dx, dy are variables.

First deriv

$$y' = f'(x) = \frac{dy}{dx} = \frac{df}{dx}$$

$$= \frac{d}{dx}(y) = \frac{d}{dx}(f(x))$$

$\frac{d}{dx}(\ \ )$ is like a function.

Second deriv

$$y'' = f''(x) = \frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{d}{dx}[y]\right]$$

Third deriv

$$y''' = f'''(x) = \frac{d^3y}{dx^3}$$

Nth deriv $\quad y^{(n)} = y^{[n]} = f^{(n)}(x) = f^{[n]}(x) = \dfrac{d^{(n)}y}{dx^{(n)}} = \dfrac{d^{[n]}y}{dx^{[n]}}$

Name	$f(x)$	$f'(x)$	$F(x)$
Constant	k	0	
Identity	x	1	
Linear	$mx+b$	m	
Power	x^n real n	nx^{n-1}	proof pg 131
Root	$\sqrt[n]{x} = x^{1/n}$	see above	
Reciprocal	$\frac{1}{x} = x^{-1}$	$\frac{-1}{x^2}$	

pg 131

$x^2 \to 2x$

$x^3 \to 3x^2$

$x^4 \to 4x^3$

Suppose

$f'(x) = 3x^2+8$

$[3f(x)]' = ?$

const mult

$f(x) = x^2 \qquad f(x) = 3x^2$

$f'(x) = 2x \qquad 6x$

$$\boxed{f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}}$$

But does this always work?

Deriv of $c\,f(x)\ldots \quad [c\,f(x)]' = \lim_{h\to 0} \dfrac{c\,f(x+h) - c\,f(x)}{h}$

$[c \cdot f(x)]' = c \cdot f'(x)$!

$$= \lim_{h\to 0}\left(c \cdot \frac{f(x+h)-f(x)}{h}\right)$$

$$= c \cdot \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = c\,f'(x)$$

Name	$f(x)$	$f'(x)$
Const mult	$k \cdot g(x)$	$k \cdot g'(x)$
sum/diff	$g(x) \pm h(x)$	$g'(x) \pm h'(x)$

pg 140

#3 $f(x) = 5.2x + 2.3$

$f'(x) = 5.2$ Linear

#5 $f(t) = 2t^3 - 3t^2 - 4t$

$f'(t) = (2t^3)' - (3t^2)' - (4t)'$ sum/diff

$2(t^3)' - 3(t^2)' - 4(t)'$ const mult

$2(3t^2) - 3(2t) - 4(1)$ power rule

$= \underline{6t^2 - 6t - 4}$

#11 $F(r) = \frac{5}{r^3} = 5r^{-3}$

$F'(r) = (5r^{-3})'$

$5(r^{-3})'$ const mult

$5(-3r^{-4}) = \underline{-15r^{-4} = \frac{-15}{r^4}}$

either is good

#17 $y = \frac{x^2 + 4x + 3}{\sqrt{x}} = \frac{x^2 + 4x + 3}{x^{1/2}}$?

$y = \frac{x^2 + 4x + 3}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{(x^2 + 4x + 3)x^{1/2}}{x}$

$x^a x^b = x^{a+b}$

$$= \frac{x^{5/2} + 4x^{3/2} + 3x^{1/2}}{x} = \frac{x\left(x^{3/2} + 4x^{1/2} + 3x^{-1/2}\right)}{x}$$

$$y = \underline{x^{3/2} + 4x^{1/2} + 3x^{-1/2}}$$

$$\frac{x^2+4x+3}{\sqrt{x}} = \frac{x^2}{\sqrt{x}} + \frac{4x}{\sqrt{x}} + \frac{3}{\sqrt{x}}$$

$$x^{3/2} + 4x^{1/2} + 3x^{-1/2}$$

$$y' = \frac{3}{2}x^{1/2} + 4\left(\frac{1}{2}x^{-1/2}\right) + 3\left(-\frac{1}{2}x^{-3/2}\right)$$

$$= \underline{\frac{3}{2}x^{1/2} + 2x^{-1/2} - \frac{3}{2}x^{-3/2}}$$

✓

$$\left[(x^2)(x^3)\right]' = (x^2)'(x^3)' \ ?$$

$$(x^5)' = 2x \cdot 3x^2$$

$$5x^4 \neq 6x^3$$

Name	$f(x)$	$f'(x)$
Product	$g(x)\,h(x)$	$g'(x)h(x) + g(x)h'(x)$ or $g'h + h'g$
Quotient	$\frac{g(x)}{h(x)}$	$\frac{hg' - gh'}{h^2}$

Low d(hi) less hi d(low)
over (low)^2 must go

Name	$f(x)$	$f'(x)$
Trig	$\sin(x)$	$\cos(x)$
	$\cos(x)$	$-\sin(x)$
Cofunctions have negative derivatives.	$\tan(x)$	$\sec^2(x)$
	$\sec(x)$	$\sec(x)\tan(x)$
	$\csc(x)$	$-\csc(x)\cot(x)$
	$\cot(x)$	$-\csc^2(x)$

sin → cos → −sin → −cos → sin

2 secants and a tangent

(co)

(co)

---

pg 150

#1 $f(x) = x^2 \sin(x)$ product rule

const mult $5x^2$

$x^2 (\underbrace{\sin(x)}_{\text{trig}})' + (x^2)' \sin(x)$

$x^2\cos(x) + 2x\sin(x)$

#6 $g(t) = 4 \underset{\text{trig}}{\sec t} + \underset{\text{trig}}{\tan t}$

$4 \sec t \tan t + \sec^2 t$

#11 $f(\theta) = \dfrac{\sin\theta}{1+\cos\theta}$ Quotient rule

Low d(hi) less hi d(low)

$f'(\theta) = \dfrac{(1+\cos\theta)(\sin\theta)' - \sin\theta(1+\cos\theta)'}{(1+\cos\theta)^2}$

over low² must go

$$\frac{(1+\cos\theta)\cos\theta - \sin\theta(-\sin\theta)}{(1+\cos\theta)^2}$$ Deriving done...

$$\frac{\cos\theta + \cos^2\theta + \sin^2\theta}{(1+\cos\theta)^2}$$ ...okay simplifying...

$$\frac{\cos\theta + 1}{(1+\cos\theta)^2}$$ ...better...

$$\frac{1}{1+\cos\theta}$$:)

Reference Page 2 — Trigonometry

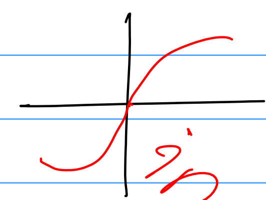
Memorize these ->

- ~~LoS~~
- ~~LoC~~
- ~~+ -~~
- ~~Double~~
- ~~Half~~

$$\sin\theta = \frac{1}{\csc\theta} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}$$



$\sin(-\theta) = -\sin\theta$

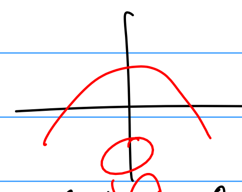
$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

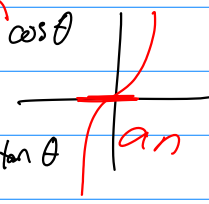
$$1 + \cot^2\theta = \csc^2\theta$$

co's together

The tangent needs a boost.



$\cos(-\theta) = \cos\theta$



$\tan(-\theta) = -\tan\theta$