

$$1. -e^{-x}$$

$$2. 10x^9 + 10^x \ln 10$$

$$3. \frac{10}{x \ln 10}$$

$$4. e^2$$

$$e \approx 2.718$$

$$5. \frac{8x}{(4x^2+1) \ln 3}$$

$$6. e^2$$

$$7. \frac{624}{5 \ln 5}$$

$$8. 0$$

$$9. \frac{1}{3} (e^{20} - e'')$$

$$10. e - \sqrt{e}$$

$$11. (x^2-3)^6 (4x-3)^8 \left( \frac{6}{x^2-3} + \frac{8}{4x-3} \right)$$

$$12. x^{3/2} e^{1-x^2} \left( \frac{3}{2x} - 2x \right)$$

$$13. (\sin x)^{3x+4} (3 \ln \sin x + (3x+4) \cot x)$$

$$14. \frac{4^{x+9}}{3^{x^2}} (\ln 4 + 2x \ln 3)$$

$$15. \sqrt{x} \left( \frac{1 - \ln x}{x^2} \right)$$

$$16. \quad \frac{1}{2}e^{2x} - \frac{1}{2}e^{-2x} + 2x + C$$

$$17. \quad \frac{1}{5} \ln(5x+1) + C$$

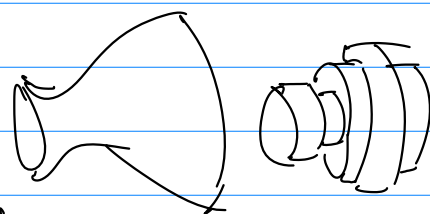
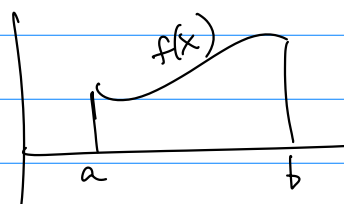
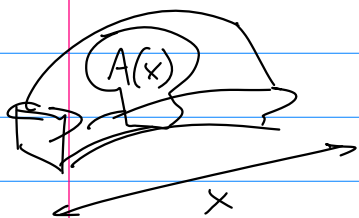
$$18. \quad \frac{18^x}{\ln 18} + C$$

$$19. \quad \int \frac{(x+1)^3}{x^2} dx = \int \frac{x^3 + 3x^2 + 3x + 1}{x^2} dx = \int x + 3 + \frac{3}{x} + \frac{1}{x^2} dx$$

$$= \frac{1}{2}x^2 + 3x + 3\ln x - \frac{1}{x} + C$$

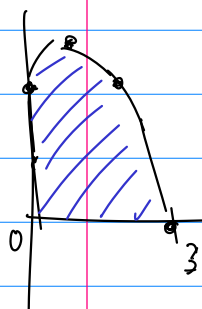
$$20. = e^{e^x} + C$$

Solid of revolution



$$V = \int_a^b \pi [f(x)]^2 dx$$

"Disk method"



$$-(x-1)^2 + 4$$

$$-(x^2 - 2x + 1) + 4$$

$$f(x) = -x^2 + 2x + 3$$



$$\int_0^3 \pi (-x^2 + 2x + 3)^2 dx$$

$$\pi \int_0^3 x^4 - 4x^3 - 2x^2 + 12x + 9 dx$$

	$-x^2$	$2x$	$3$
$-x^2$	$x^4$	$-2x^3$	$-3x^2$
$2x$	$-2x^3$	$4x^2$	$6x$
$3$	$-3x^2$	$6x$	$9$

$$\pi \left[ \frac{1}{5}x^5 - x^4 - \frac{2}{3}x^3 + 6x^2 + 9x \right]_0^3$$

$$\pi \left( \frac{1}{5}(3)^5 - 3^4 - \frac{2}{3}(3)^3 + 6(3)^2 + 9(3) \right)$$

$$\pi \left( \frac{243}{5} - 81 - 18 + 54 + 27 \right)$$

$$\pi \left( \frac{243}{5} - \frac{90}{5} \right)$$

$$= \pi \left( \frac{153}{5} \right) = \boxed{\frac{153\pi}{5}}$$

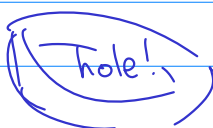
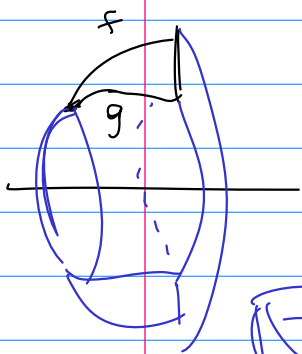
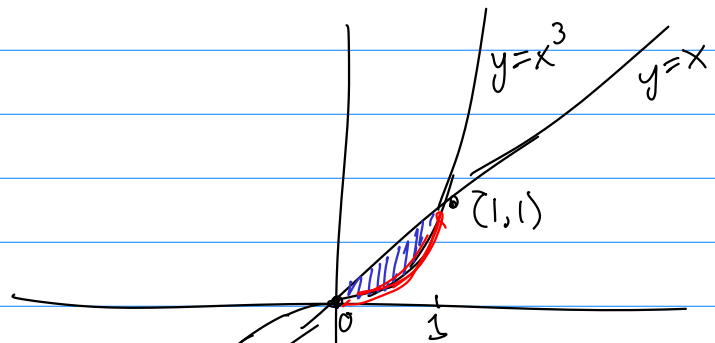
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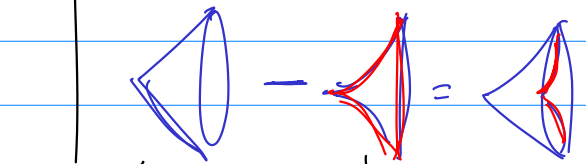
$$y = x^3$$

$$y = x$$

$$x \geq 0$$



Regions not touching the x-axis:  
Use two disk methods.



$$\int_0^1 \pi(x)^2 dx - \int_0^1 \pi(x^3)^2 dx$$

$$= \pi \int_0^1 x^2 dx - \pi \int_0^1 x^6 dx$$

$$= \pi \left. \frac{1}{3}x^3 \right|_0^1 - \pi \left. \frac{1}{7}x^7 \right|_0^1$$

$$\pi \frac{1}{3} - \pi \frac{1}{7}$$

$$= \frac{7\pi}{21} - \frac{3\pi}{21} = \boxed{\frac{4\pi}{21}}$$

$$= \int_0^1 \pi(x)^2 - \pi(x^3)^2 dx$$

$$= \int_0^1 \pi((x)^2 - (x^3)^2) dx$$

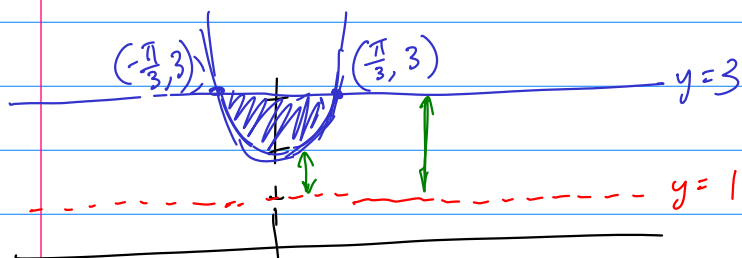
$$= \int_0^1 \pi((x - x^3)^2) dx$$

Don't!  
 $a^2 - b^2 \neq (a-b)^2$

#13

$$y = 1 + \sec x$$

$$y = 3$$

about  $y = 1$ how far is  $y = 3$  from  $y = 1$ ?

$$f(x) = 3 - 1 = 2$$

how far is  $1 + \sec x$  from  $1$ ?

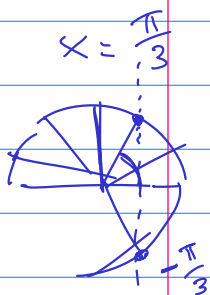
$$g(x) = 1 + \sec x - 1 = \sec x$$

Intersections?

$$1 + \sec x = 3$$

$$\sec x = 2$$

$$\cos x = \frac{1}{2}$$



$$\frac{\sin \frac{\pi}{3}}{\cos \frac{\pi}{3}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$$

$$= \sqrt{3}$$

$$\int_{-\pi/3}^{\pi/3} \pi [f(x)]^2 dx - \int_{-\pi/3}^{\pi/3} \pi [g(x)]^2 dx$$

$$\pi \int 2^2 dx - \pi \int (\sec x)^2 dx$$

$$\pi \left[ 4x \right]_{-\pi/3}^{\pi/3} - \pi \left[ \tan x \right]_{-\pi/3}^{\pi/3}$$

$$\pi \left( 4 \frac{\pi}{3} - 4 \left( -\frac{\pi}{3} \right) \right) - \pi \left( \tan \frac{\pi}{3} - \tan \left( -\frac{\pi}{3} \right) \right)$$

$$\pi \left( \frac{4\pi}{3} + \frac{4\pi}{3} \right) - \pi \left( \sqrt{3} - \left( -\frac{\sqrt{3}}{3} \right) \right)$$

$$\pi \frac{8\pi}{3} - \pi (2\sqrt{3})$$

$$\pi \left( \frac{8\pi}{3} - 2\sqrt{3} \right)$$

$\tan$  is odd  
 $\tan(-x)$   
 $= -\tan(x)$

#9

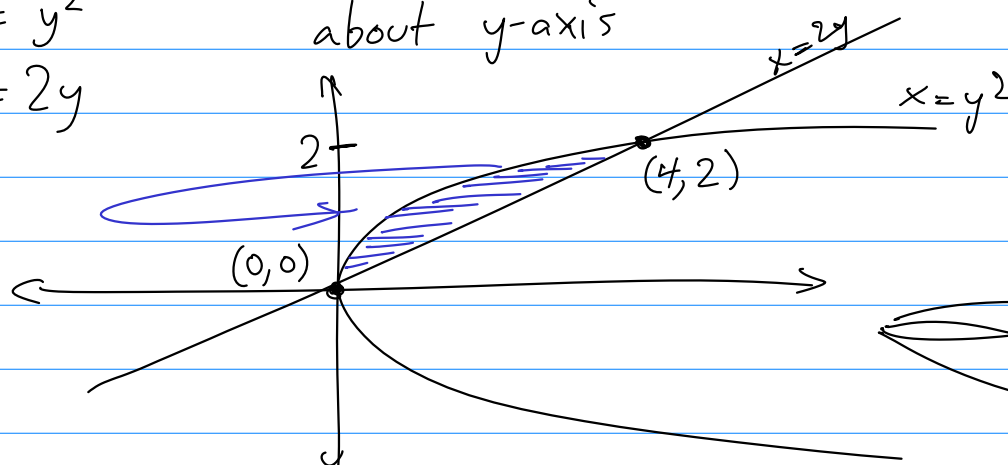
$$x = y^2$$

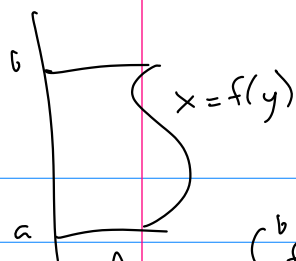
$$x = 2y$$

about  $y$ -axis

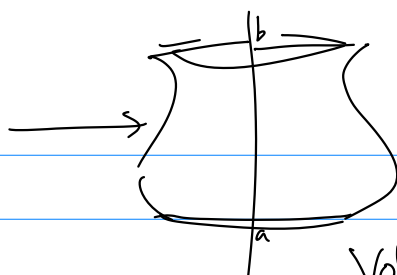
$$y^2 = 2y$$

$$y = 2$$





$$\text{Area} = \int_a^b f(y) dy$$



$$\text{Volume} = \int_a^b \pi (f(y))^2 dy$$

$$\int_0^2 \pi (2y)^2 dy - \int_0^2 \pi (y^2)^2 dy$$

$$\pi \int_0^2 4y^2 dy - \pi \int_0^2 y^4 dy$$

$$\pi \left. \frac{4}{3} y^3 \right|_0^2 - \pi \left. \frac{1}{5} y^5 \right|_0^2$$

$$\pi \frac{4}{3} (8) - \pi \frac{1}{5} (32)$$

$$\pi \left( \frac{32}{3} - \frac{32}{5} \right)$$

$$32\pi \left( \frac{1}{3} - \frac{1}{5} \right)$$

$$32\pi \left( \frac{5}{15} - \frac{3}{15} \right)$$

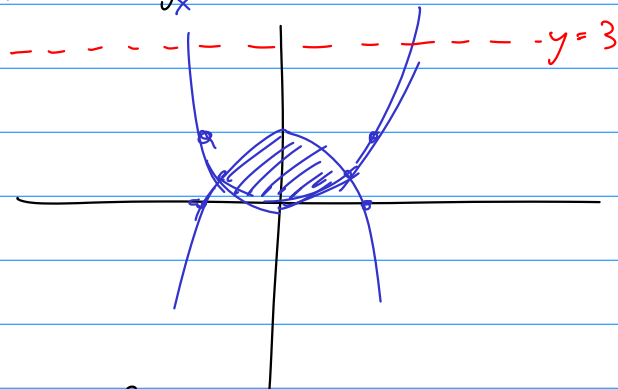
$$32\pi \frac{2}{15} = \boxed{\frac{64\pi}{15}}$$

#17

$$y = x^2$$

$$y = 1 - x^2$$

about  $x=3$



$$\begin{aligned} y &= x^2 \\ y &= 1 - x^2 \end{aligned}$$

$$y=3$$

Intersections

$$x^2 = 1 - x^2$$

$$2x^2 = 1$$

$$x^2 = \frac{1}{2}$$

$$x = \pm \sqrt{\frac{1}{2}}$$

How far is  $y = x^2$  from  $y = 3$ ?

$$f(x) = x^2 - 3$$

How far is  $y = 1 - x^2$  from  $y = 3$ ?

$$g(x) = 1 - x^2 - 3$$

$$= -2 - x^2$$

Negative radius? It will get squared away.

$$\int_{-\sqrt{\frac{1}{2}}}^{\sqrt{\frac{1}{2}}} \pi (-2 - x^2)^2 dx$$

smaller  
order backwards?

$$\int_{-\sqrt{\frac{1}{2}}}^{\sqrt{\frac{1}{2}}} \pi (x^2 - 3)^2 dx$$

larger

Just take abs value.

$$\pi \int (x^2+2)^2 dx - \pi \int (x^2-3)^2 dx$$

$$\pi \left( \int x^4 + 4x^2 + 4 dx - \int x^4 - 6x^2 + 9 dx \right)$$

$$\pi \left( \int_{-\sqrt{\frac{1}{2}}}^{\sqrt{\frac{1}{2}}} (x^4 + 4x^2 + 4) - (x^4 - 6x^2 + 9) dx \right)$$

$$\pi \left( \int_{-\sqrt{\frac{1}{2}}}^{\sqrt{\frac{1}{2}}} 10x^2 - 5 dx \right)$$

$$\pi \left( \left. \frac{10}{3}x^3 - 5x \right|_{-\sqrt{\frac{1}{2}}}^{\sqrt{\frac{1}{2}}} \right)$$

$$\pi \left( x \left( \frac{10}{3}x^2 - 5 \right) \right|_{-\sqrt{\frac{1}{2}}}^{\sqrt{\frac{1}{2}}} \right)$$

$$\pi \left[ \sqrt{\frac{1}{2}} \left( \frac{10}{3} \frac{1}{2} - 5 \right) - \sqrt{\frac{1}{2}} \left( \frac{10}{3} \frac{1}{2} - 5 \right) \right]$$

$$\pi \left[ 2 \sqrt{\frac{1}{2}} \left( \frac{5}{3} - 5 \right) \right]$$

$$\sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\pi \cancel{2} \frac{\sqrt{2}}{\cancel{2}} \left( \frac{5}{3} - \frac{15}{3} \right)$$

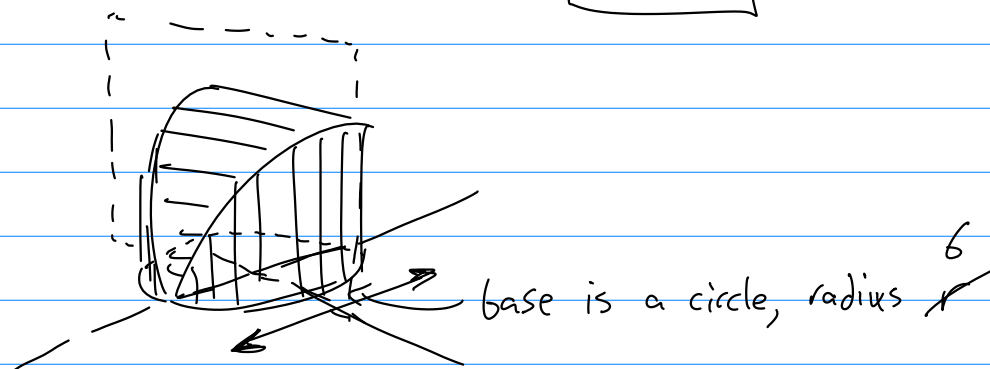
$$\pi \sqrt{2} \left( -\frac{10}{3} \right)$$

$$= \frac{-10\pi\sqrt{2}}{3}$$

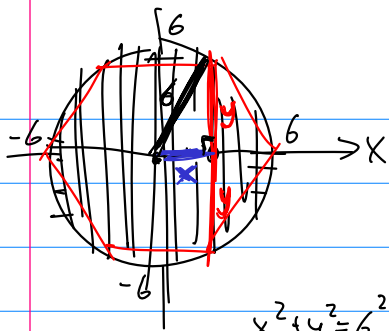
$$\text{Volume } \oplus : \boxed{\frac{10\pi\sqrt{2}}{3}} \text{ units}^3$$

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#54



Top view



$$\int_{-6}^6 4(6^2 - x^2) dx$$

$A(x) = ?$  side length for a given  $x = ?$

$$x^2 + y^2 = 6^2$$

$$y^2 = 6^2 - x^2$$

$$y = \sqrt{6^2 - x^2}$$

$$s(x) = 2(\sqrt{6^2 - x^2})$$

$$A(x) = 4(6^2 - x^2)$$