Introduction to Digital Design: Exam I

Time: 50 minutes **Total points: 50**

Do NOT seek help from others. Ask the exam proctor if you have any questions. Show all steps with your solutions for full credits. Use additional papers if needed. Make sure you write your name on each page and return all of them.

NAME: KEY

1. (2+2+2 points) Convert the decimal number **125**
   a. Binary
   b. Octal
   c. Hexadecimal

   Show your work

$125/2 = 62$ r 1 LSB

$62/2 = 31$ r 0

$31/2 = 15$ r 1

$15/2 = 7$ r 1

$7/2 = 3$ r 1

$3/2 = 1$ r 1

$1/2 = 0$ r 1 MSB

STOP

Binary 1111101_2

Octal 175_8

Hexadecimal $7D_{16}$

2. (4+4+2 points) What is the maximum and minimum (Signed 2's complement) binary number that can be expressed with 7 bits? Convert the maximum binary number into hexadecimal number.

Maximum no. with 7 bits $= (2^{(n-1)} - 1) = (2^{(7-1)} - 1) = 2^6 - 1$
$= 64 - 1 = 63$

Minimum no. with 7 bits $= -2^{n-1} = -2^{7-1} = -2^6 = -64$

$2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$
0 1 1 1 1 1 1 → +63
1 0 0 0 0 0 0 → -64

Maximum Binary (base 2): $0111\ 1111_2$

Minimum Binary (base 2): 1000000

Hexadecimal equivalent of maximum binary $3F_{16}$

3. (2+2 points) Show how the decimal numbers below are stored in 2's complement format? (8 bits)

$2^7\ 2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

a. +29 0 0 0 1 1 1 0 1 → +29

b. -49 0 0 1 1 0 0 0 1 → +49
1 1 0 0 1 1 1 0 → Invert the bits
+ 1 → Add 1
1 1 0 0 1 1 1 1
-128+64+8+4+2+1 = -49

4. ((1+1+1+1+1) * 2) Points) Add the two pairs of 6-bit **signed (2's complement)** numbers. Show the both operands and the result of each addition in decimal as well as binary. Indicate if there is overflow.

a.

```
 111
 010111 ——> +23
 001110 ——> +14
 ------     ----
 100101     +37
```

Looks like -27; shoul be +37; overflow
sum of two positive numbers look negative.

b.

```
 1
 110011 ——> -13
 110100 ——> -12
 ------     ----
 100111     -25
```

5. (10 points) For each of the following pairs of integers, subtract the second from the first. Show the operands and the answers in decimal. Indicate overflow where appropriate

a.

```
 111010 ——> -6
 000111 ——> +7
 ------
```

$-6-(7)$
$=-6+(-7)$

```
 000111 ——> +7
 111000
      1
 ------
 111001 ——> -7
```

```
 1 1
 111010 ——> -6
 111001 ——> -7
 ------     ----
 110011     -13
```

b. $100100 \longrightarrow -28$
$011000 \longrightarrow +24$

$-28-(+24)$
$= -28+(-24)$

$011000 \rightarrow +24$

100111

$+\ 1$

$101000\ (-24)$

$100100 \rightarrow -28$
$101000 \rightarrow -24$
$+$
$001100 \rightarrow -52$

Looks like +12; should be -52; overflow. Sum of two negative numbers look posit

6. (4+6 Points) Derive the logical expression from the truth table. Specify the number of gates and inputs (e.g. 2-input AND) for the functions. Draw the circuit diagram.

a	b	c	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

$f = a'b'c + a'bc + ab'c' + abc' + abc$

5 → 3 input AND gates
1 → 5 input OR gate
3 inverter