Chapter 1: All objectives assessed by Exam 1

- ~~1.1 Determine whether a relation (verbal, numeric, visual, algebraic) is a function.~~
- ~~1.1 Find the implied domain of a function.~~
- 1.1 Determine whether a function is even, odd, or neither.
- ~~1.2,3 Given a graph of a transformed parent function, describe the transformations and write the functi~~
- 1.3 Compose two or more functions.
- 1.3 Decompose a function into two or more functions.
- 1.5 Determine the (one-, two-sided) limit of a function, if it exists.
- 1.5 Describe behavior of functions near asymptotes, using limits. $\lim_{x\to 2^+}\left(\frac{1}{x-2}\right)$
- 1.6 Calculate limits using limit laws.
- ~~1.7 Rigorously prove a limit or a limit law.~~
- 1.8 Determine whether a function is continuous (at a point, in an interval, everywhere).

Chapter 2: All objectives assessed by Exam 2

- 2.2 Determine if a function is differentiable (at a point, in an interval, everywhere).
- 2.3,4,5 MEMORIZE: Basic derivatives and derivative rules. Trig
- 2.3,4,5 Find derivatives of functions.
- 2.6 Find dy/dx of an equation using implicit differentiation.
- 2.8 Solve related-rates problems.
- 2.9 Find the linearization of a function at a point.
- 2.9 Use a linearization to estimate the value of a function near a point.

Chapter 3: All objectives assessed by Exam 3

- 3.1 Find the local and absolute minimum/maximum of a function on an interval.
- 3.2 Find numbers that satisfy the conclusions of Rolle's Thm and the MVT. (No need to memorize the theo
- 3.3 Based on f'(x) and f''(x), determine properties (concavity, inc/dec) of f(x).
- 3.4 Find limits at infinity and horizontal asymptotes of graphs. $\lim_{x\to\infty, -\infty}$
- 3.5,6 Sketch curves using information obtained from algebra and calculus.
- 3.7 Solve optimization problems.

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#41

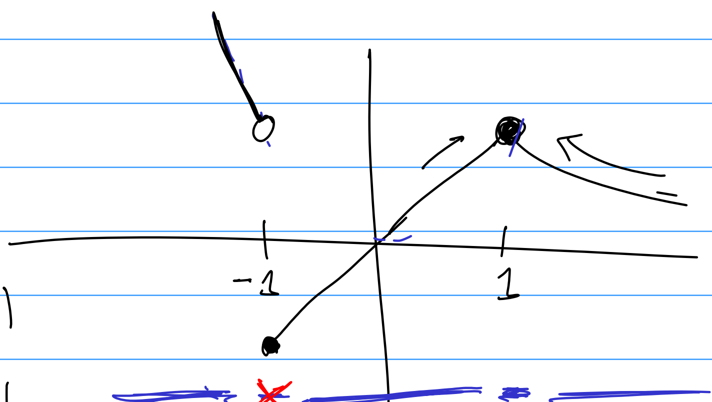
$$f(x)=\begin{cases} x^2 & \text{if } x<-1 \\ x & \text{if } -1\le x<1 \\ 1/x & \text{if } x\ge 1\end{cases}$$

$\lim_{x\to -1^-} f(x) = 1$

$\lim_{x\to -1^+} f(x) = -1$

$f(-1) = -1$

$\lim_{x\to -1} f(x) = \text{DNE}$



$\lim_{x\to 1^-} f(x) = 1$

$\lim_{x\to 1^+} f(x) = 1$

$\lim_{x\to 1} f(x) = 1$

$f(1) = 1$

poly   poly   rational expression

Where is f continuous?

limit doesn't exist

$\lim_{x\to 1} f(x) = f(1)$

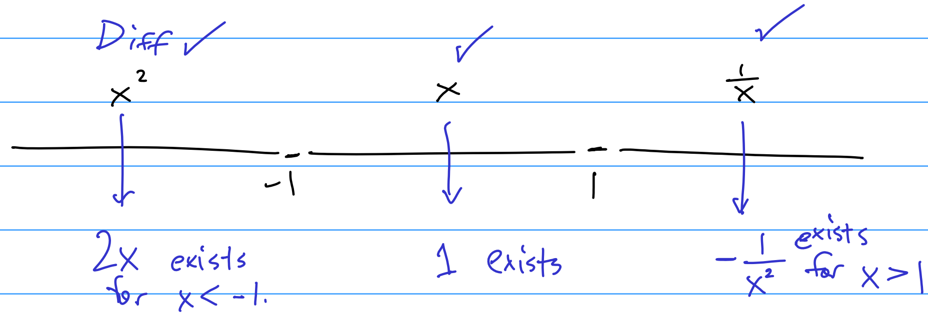
continuous

$(-\infty,-1)\cup(-1,\infty)$

$$f(x)=\begin{cases} x^2 & \text{if } x<-1 \\ x & \text{if } -1\le x<1 \\ 1/x & \text{if } x\ge 1 \end{cases}$$

Where is f diff'able?

f is diff'able at c if $f'(c)$ exists.



$$f'(c)=\lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$$

Does $\displaystyle\lim_{h\to 0^-}\frac{f(-1+h)-f(-1)}{h} = \lim_{h\to 0^+}\frac{f(-1+h)-f(-1)}{h}$?

$$\lim_{h\to 0^-}\frac{(h-1)^2-1}{h} = \lim_{h\to 0^+}\frac{-1+h--1}{h} \; ?$$

$$\lim_{h\to 0^-}\frac{h^2-2h+1-1}{h} = \lim_{h\to 0^+}\frac{h}{h} \; ?$$

$$\lim_{h\to 0^-}(h-2) = \lim_{h\to 0^+} 1 \; ?$$

DSP

$$-2 \neq 1$$

DSP

$$\lim_{x\to a} f(x) = f(a)$$

If f is: polynomial, rational, root, trig (and a is in the domain of f)

Continuous on their domains

pg 61 #32

$$\lim_{x\to 3^-}\frac{\sqrt{x}}{(x-3)^5} \quad ?$$

2.9 is close to 3 on the left

$$\frac{\sqrt{2.9}}{(2.9-3)^5} = \frac{\sqrt{2.9}}{(-.1)^5} = \frac{\sqrt{2.9}}{-.00001}$$

$$= -\infty$$

Pg 168 #59

points on $x^2y^2 + xy = 2$

where slope of tangent line is -1.

$\frac{d}{dx}(y) = \frac{dy}{dx}$

$$\frac{d}{dx}\left[x^2y^2\right] + \frac{d}{dx}\left[xy\right] = \frac{d}{dx}\left[2\right]$$

$$\frac{d}{dx}\left[(xy)^2\right] + \underbrace{1\cdot y + x\frac{dy}{dx}}_{} = 0$$

$$2xy\left[xy\right]' + y + x\frac{dy}{dx} = 0$$

$$2xy\left(y + x\frac{dy}{dx}\right) + \left(y + x\frac{dy}{dx}\right) = 0$$

solve for $\frac{dy}{dx}$

$$(2xy + 1)\left(y + x\frac{dy}{dx}\right) = 0$$

$$y + x\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} = -y$$

$$\frac{dy}{dx} = -\frac{y}{x} = -1$$

$$\frac{y}{x} = 1$$

$$\boxed{y = x}$$ True of all points we are looking for.

$x^2y^2 + xy = 2$

$x^4 + x^2 = 2$

$x^4 + x^2 - 2 = 0$

$(x^2 - 1)(x^2 + 2) = 0$

$x^2 = 1$ $\quad$ $\cancel{x^2 = -2}$

$x = \pm 1$ $\quad$ $(1,1), (-1,-1)$

$$y\cos x = x^2 + y^2$$

$$\frac{d}{dx}(y\cos x) = \frac{d}{dx}(x^2) + \frac{d}{dx}(y^2)$$

$$\frac{dy}{dx}\cos x + \underbrace{y(-\sin x)} = 2x + \underbrace{2y\frac{dy}{dx}}$$

$$\frac{dy}{dx}\cos x - 2y\frac{dy}{dx} = 2x + y\sin x$$

$$\boxed{\frac{dy}{dx} = \frac{2x + y\sin x}{\cos x - 2y}}$$

R total resistance

#39 R_1 R_2 in parallel

$R_1 \nearrow 0.3\ \Omega/s$

$R_2 \nearrow 0.2\ \Omega/s$

$R \nearrow$? when $R_1 = 80\ \Omega$, $R_2 = 100\ \Omega$

Fact:

Anything can be a function of time.

$$\frac{1}{R(t)} = \frac{1}{R_1(t)} + \frac{1}{R_2(t)}$$

Derive with respect to t (time).

$$\frac{d}{dt}(R^{-1}) = \frac{d}{dt}(R_1^{-1} + R_2^{-1})$$

$$-1R^{-2}\frac{dR}{dt} = -R_1^{-2}\frac{dR_1}{dt} + -R_2^{-2}\frac{dR_2}{dt}$$

$$\frac{1}{R} = \frac{1}{80} + \frac{1}{100}$$

$$\frac{1}{R} = \frac{5}{400} + \frac{4}{400}$$

$$\frac{1}{R} = \frac{9}{400}$$

$$R = \frac{400}{9}$$

General $$\frac{1}{R^2}\frac{dR}{dt} = \frac{1}{R_1^2}\frac{dR_1}{dt} + \frac{1}{R_2^2}\frac{dR_2}{dt}$$

Specific to problem... $$\frac{1}{R^2}\frac{dR}{dt} = \frac{1}{R_1^2}0.3 + \frac{1}{R_2^2}0.2$$

$$\frac{1}{R^2} = \left(\frac{9}{400}\right)^2$$

...at a specific time $$\frac{1}{R^2}\frac{dR}{dt} = \frac{1}{80^2}0.3 + \frac{1}{100^2}0.2$$

$$\frac{dR}{dt} = \left(\frac{0.3}{80^2} + \frac{0.2}{100^2}\right)\left(\frac{400^2}{9^2}\right)$$

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θ ~~for~~
Max amount of water?

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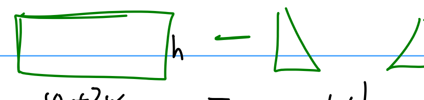
$\theta \to$ Amount of water ?

$A(\theta) =$ 

$(10 + 2w)h - wh$

$10h + 2wh - wh$

$10h + wh$

$A(\theta) = (10 + w)h$

$A(\theta) = (10 + 10\cos\theta)\,10\sin\theta$

$\tan\theta = \frac{h}{w}$

$\sin\theta = \frac{h}{10}$

$\cos\theta = \frac{w}{10}$

$h = 10\sin\theta$

$w = 10\cos\theta$