oint.

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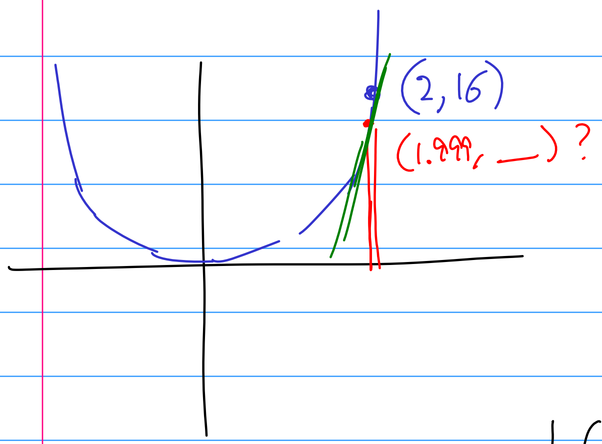
#23 $(1.999)^4$ $f(x) = x^4$



Linearization of x^4 at $\overset{x=2}{2}$.

we need $f'(2)$

$f'(x) = 4x^3$
$f'(2) = 32$

$L(x) =$ line through $(2,16)$ with slope 32

(x_0, y_0) slope m...

$L(x) = f(a) + f'(a)(x-a)$
becomes
$L(x) = y_0 + m(x - x_0)$
point-slope formula.

$L(x) = 16 + 32(x-2)$

To approximate $(1.999)^4$,

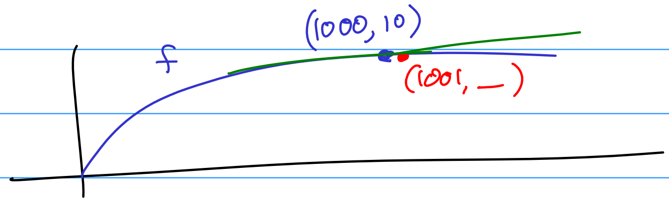
$L(1.999) = 16 + 32(1.999 - 2)$

$= 16 + 32(-.001)$

$= 16 - .032$

$$\begin{array}{r} \overset{5}{1\cancel{6}}.\overset{9}{\cancel{0}}\overset{9}{\cancel{0}}\overset{10}{\cancel{0}} \\ -\ .032 \\ \hline 15.968 \end{array}$$

#25 $\sqrt[3]{1001}$ $f(x) = \sqrt[3]{x}$



$x^{1/3}$

$f'(x) = \frac{1}{3}x^{-2/3}$

$f'(1000) = \frac{1}{3}(1000)^{-2/3}$

$a^{\text{power}/\text{root}}$

$= \frac{1}{3} \frac{1}{1000^{2/3}}$

$1000^{2/3} = \left(\sqrt[3]{1000}\right)^2$

$= 10^2 = 100$

$f'(1000) = \frac{1}{3} \frac{1}{100} = \frac{1}{300}$

Point $(1000, 10)$, slope $\frac{1}{300}$

$L(x) = y_0 + m(x - x_0)$

$L(x) = 10 + \frac{1}{300}(x - 1000)$

$\frac{1}{3} \cdot \frac{1}{100}$

$L(1001) = 10 + \frac{1}{300}(1001 - 1000)$

$(.3333) \cdot \frac{1}{100}$

$= .00333\ldots$

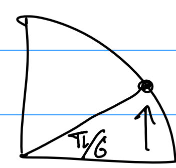
$= 10 + \frac{1}{300}$

$= \underline{10.00333}$

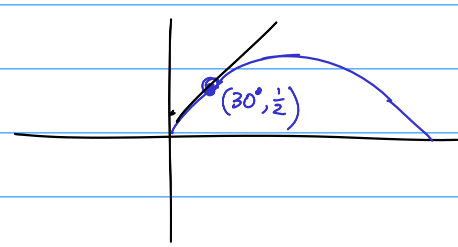
#28 Tweak

$\sin(29^\circ)$

$f(x) = \sin(x)$



close to $\underbrace{\left(30^\circ, \frac{1}{2}\right)}_{\text{point}}$



$f'(x) = \cos(x)$

$f'(30^\circ) = \cos(30^\circ) = \underbrace{\frac{\sqrt{3}}{2}}_{\text{slope}}$

$\frac{d}{dx}(\sin(x)) = \cos x$ ONLY if x is in radians.

$L(x) = y_0 + m(x - x_0)$

$\frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$

$29^\circ - 30^\circ = -1^\circ$

$L(29^\circ) = \frac{1}{2} + \frac{\sqrt{3}}{2}(-1^\circ)$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2}\left(-\frac{\pi}{180}\right)$$

$$= \frac{1}{2} - \frac{\sqrt{3}\pi}{360} \quad \ldots \quad \text{etc}$$

Req calc now... won't assign this on a test

$$\boxed{} \approx .48488$$

$\sin x \approx x \qquad \cos x \approx 1$

when x is near 0!

Differentials

Let dx be an independent variable. (Even indep of x)

Then: $dy = f'(x) \cdot dx$

$$\frac{dy}{dx} = f'(x)$$

dy is dependent on x and dx.

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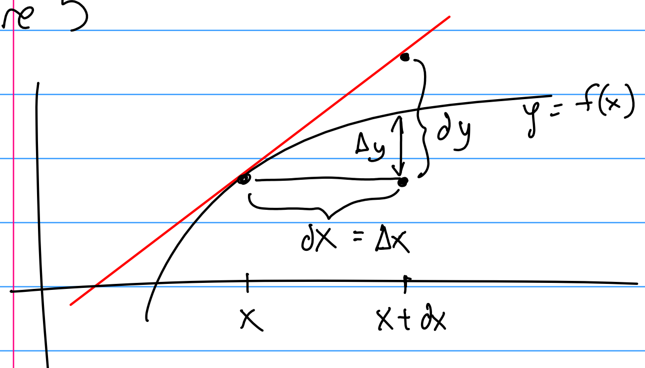
Figure 5



$$dy = f'(x)\,dx$$

This has meaning! (Ch 4)

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#15

$f(x) = y = \tan x \qquad x = \frac{\pi}{4} \qquad dx = -0.1$

$$dy = f'(x)\,dx$$

$$dy = \sec^2 x\,dx$$

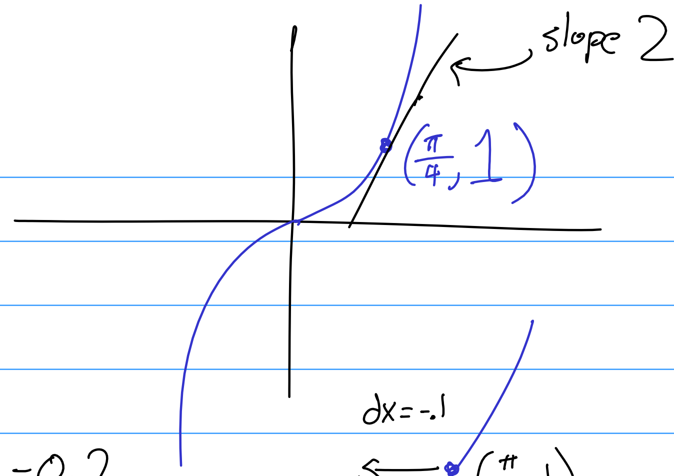
$$dy = \sec^2\left(\frac{\pi}{4}\right)(-0.1)$$

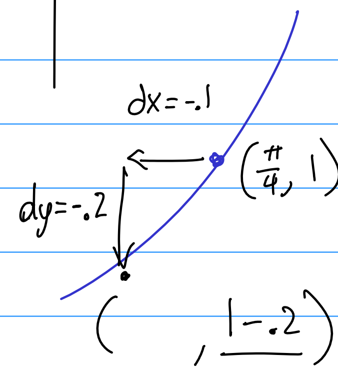
$$dy = \left(\frac{1}{\cos\left(\frac{\pi}{4}\right)}\right)^2(-0.1)$$

$$dy = \left(\frac{1}{\sqrt{2}/2}\right)^2(-0.1)$$

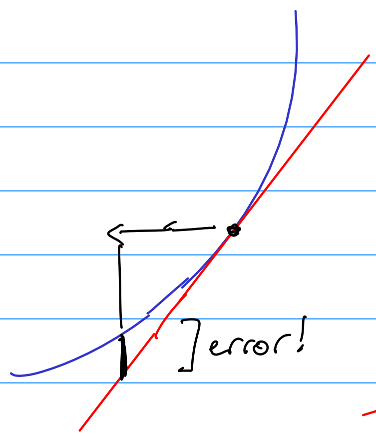
$$= \left(\frac{2}{\sqrt{2}}\right)^2(-0.1)$$

$$= \frac{4}{2}(-0.1) = 2(-0.1) = -0.2$$



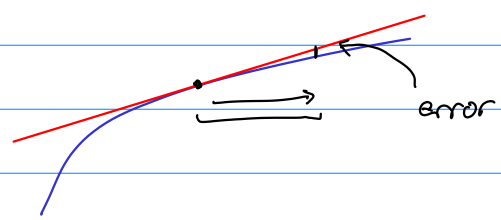


We approximated: $\tan\left(\frac{\pi}{4} - .1\right) \approx .8$



actual value = .81763

Not as good an approx as prev problems.



#18

$$f(x) = y = \frac{x+1}{x-1} \qquad x = 2 \qquad dx = 0.05$$

$$\frac{dy}{dx} = f'(x)$$

$$dy = f'(x)\,dx$$

$$f'(x) = \frac{(x-1)(1) - (x+1)(1)}{(x-1)^2}$$

$$= \frac{\cancel{x} - 1 - \cancel{x} - 1}{(x-1)^2} = \frac{-2}{(x-1)^2}$$

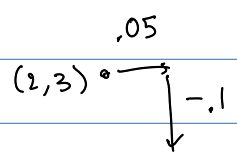
$$dy = \frac{-2}{(x-1)^2}\,dx$$

$$= \frac{-2}{(2-1)^2}(0.05)$$

$$= -2(0.05) = -0.1 = dy$$

point $f(2) = \frac{2+1}{2-1} = 3$

(2, 3)



approx $\frac{2.05+1}{2.05-1} \approx \boxed{2.9}$

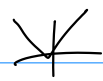
$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

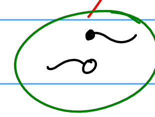
2.1 – 2.5 All derivatives have basis in the definition
Def gives us the rules.

2.2 A function is diff'able at a point c if $f'(c)$ exists.

At a point

Diff'able $\Rightarrow$ Continuous $\Rightarrow$ Defined
$\not\Leftarrow$ $\not\Leftarrow$


$y = |x|$



Nondiff'able $\not\Rightarrow$ discont $\not\Rightarrow$ Undef
$\Leftarrow$ $\Leftarrow$

2.6 $\frac{dy}{dx}$ for an implicit curve:
Derive all with respect to x, solve for $\frac{dy}{dx}$.

2.8 Get a fact relating all important quantities in the problem.
Derive all with respect to t (time)
Every variable can be a function of time!
Plug in known values and solve.

2.9 know how to find a linear equation using a point and slope.

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$$f(x) = \frac{2}{\sqrt{x^2-5}} = 2(x^2-5)^{-1/2} \quad \text{easier}$$

Prob due tonight: "~~a, b, and~~ h"
Let h be the distance between the ships. Find $\frac{dh}{dt}$.