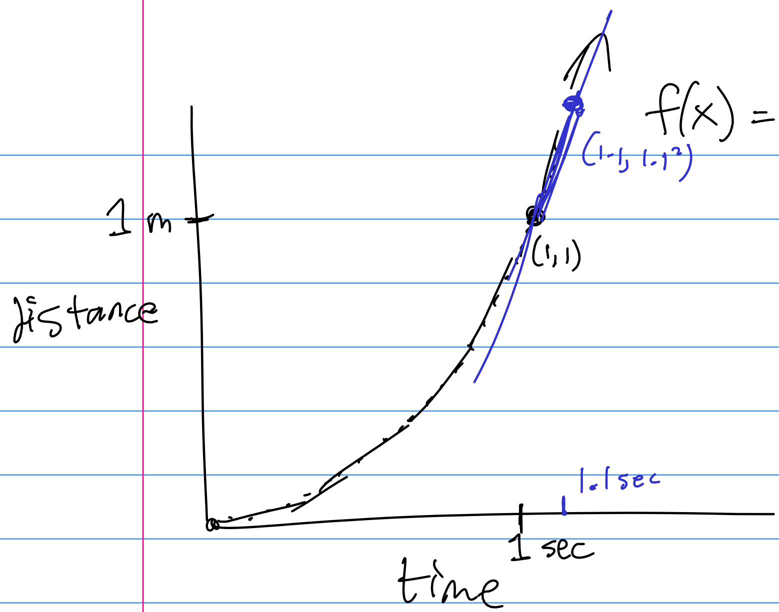
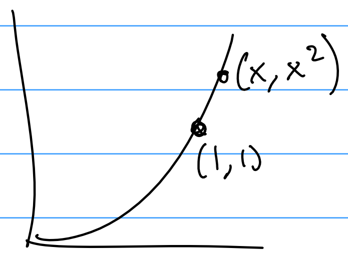


Speed estimate at $t = 1$:

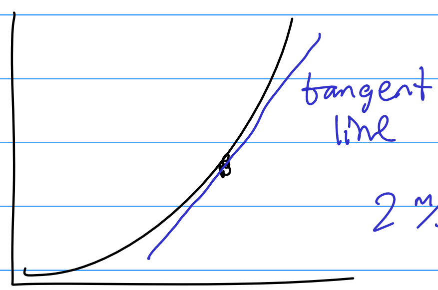
$$\text{Slope} = \frac{1.1^2 - 1}{1.1 - 1} = \frac{.21}{.1} = 2.1 \ \text{m/s}$$

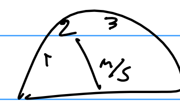
$$\frac{1.01^2 - 1}{1.01 - 1} = 2.01 \ \text{m/s}$$



$$\text{slope} = \frac{x^2 - 1}{x - 1} \qquad \lim_{x \to 1^+} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1^+} \frac{(x+1)(x-1)}{x-1}$$

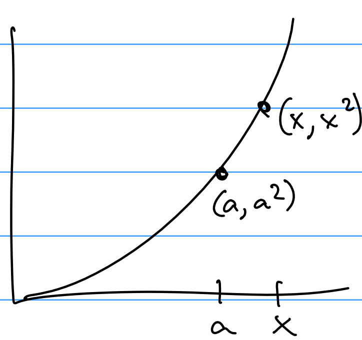
$$= \lim_{x \to 1^+} (x+1) = 2 \ \text{m/s}$$





2 m/s is the slope of the tangent line to $f(x) = x^2$ when $x = 1$.

2 m/s is the instantaneous velocity of the car at time $x = 1$.

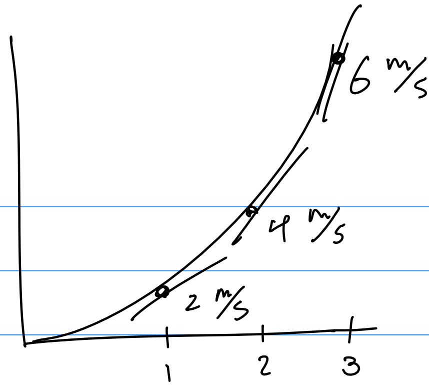


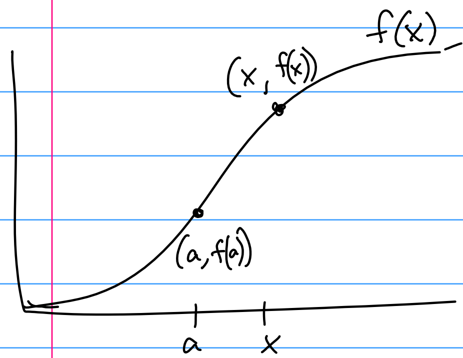
I want the inst. vel. at time $\underline{a}$

$$\text{secant slope} = \frac{x^2 - a^2}{x - a}$$

$$\text{tangent slope} = \lim_{x \to a} \frac{x^2 - a^2}{x - a} = \lim_{x \to a} \frac{(x+a)(x-a)}{x-a}$$

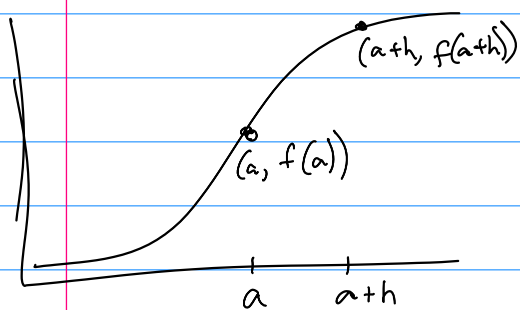
$$= \lim_{x \to a} (x + a) = \underline{\underline{2a}} \ !$$





slope of tangent line $= \lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

$(a + h - a = h)$



$$\boxed{\text{slope of tangent line} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}}$$

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#5 $\quad \overset{f(x)}{\cancel{y}} = 4x - 3x^2 \qquad \overset{a}{\downarrow}\ (2, -4)$

$f(a+h) = 4(a+h) - 3(a+h)^2$
$f(a) = 4a - 3a^2$

$$\text{slope} = \lim_{h \to 0} \frac{\left[4(a+h) - 3(a+h)^2\right] - \left[4a - 3a^2\right]}{h}$$

$$= \lim_{h \to 0} \frac{4(2+h) - 3(2+h)^2 - 4(2) + 3(4)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{8} + 4h - 3(h^2 + 4h + 4) - \cancel{8} + 12}{h}$$

$$= \lim_{h \to 0} \frac{4h - 3h^2 - 12h - \cancel{12} + \cancel{12}}{h} = \lim_{h \to 0} \frac{-3h^2 - 8h}{h}$$

$$= \lim_{h \to 0} \frac{h(-3h-8)}{h} = \lim_{h \to 0} (-3h-8) = \underline{-8}$$

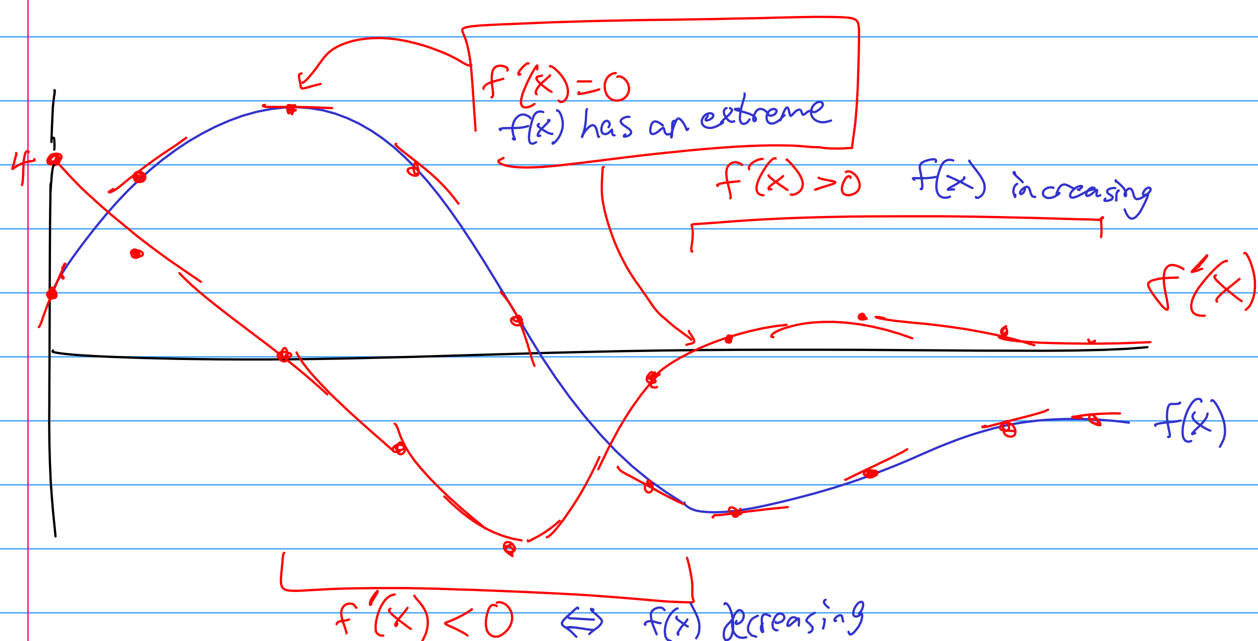
Line: slope -8
through $(2,-4)$

$$y = -8x + b$$
$$-4 = -8(2) + b$$
$$-4 = -16 + b$$
$$b = 12$$

$$\boxed{y = -8x + 12}$$

The derivative of a function $f(x)$ is ANOTHER function, called $f'(x)$.
"f prime"

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



earlier:

when
$f(x) = x^2$,
$f'(x) = 2x$.

$$f(x) = \sqrt{x} \qquad f'(x) = ?$$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

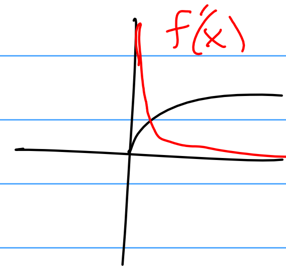
$$\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}} = \lim_{h\to 0}\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0}\frac{h}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{\sqrt{x}+\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$$

2.2 Determine if a function is differentiable (at a point, in an interval, everywhere).

$f(x)=\sqrt{x}$ $\qquad f'(x)=\frac{1}{2\sqrt{x}}$

x	$f(x)$	$f'(x)$
4 s	2 m	$\frac{1}{4}$ m/s
1 s	1 m	$\frac{1}{2}$ m/s
¼ s	$\frac{1}{2}$ m	1 m/s
0 s	0 m	$\frac{1}{0}$ m/s ?!



f(x) is defined at 0, but
f'(x) is undefined at 0...

when x=a

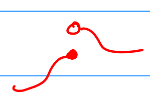
If f'(x) as a limit DNE, then we say
f(x) is nondifferentiable at a.

OR

f(x) is differentiable at a if and only if f'(a) exists.

v. derive = differentiate
n. derivative = differential
adj. derivable?? = differentiable

f(x) = √x is diff'able
when x > 0, but
nondiff'able when x = 0.



Defined at a $\not\Rightarrow$ Continuous at a $\not\Rightarrow$ Differentiable at a
$\Leftarrow$ $\qquad$ $\Leftarrow$

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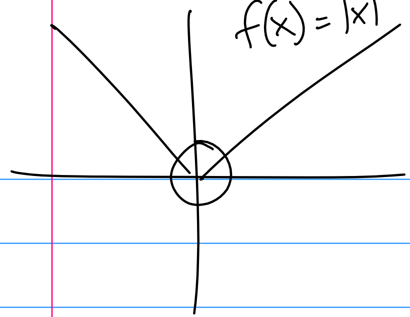


$f'(0) = ?$

$$f'(x) = \lim_{h \to 0} \frac{|x+h| - |x|}{h}$$

$$f'(0) = \lim_{h \to 0} \frac{|h| - 0}{h} = \lim_{h \to 0} \frac{|h|}{h}$$

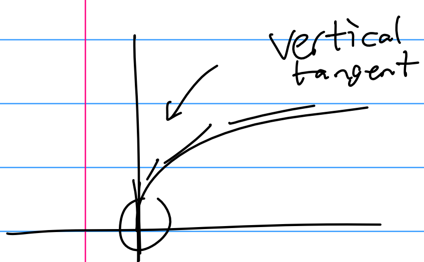
$$\lim_{h \to 0^-} \frac{|h|}{h} \qquad\qquad \lim_{h \to 0^+} \frac{|h|}{h}$$

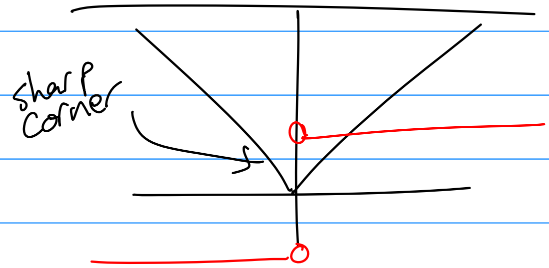
$$= \lim_{h \to 0^-} \frac{-h}{h} = -1 \quad \neq \quad = \lim_{h \to 0^+} \frac{h}{h} = 1$$

$$\lim_{h \to 0} \frac{|h|}{h} = DNE$$

$f(x) = |x|$ is not differentiable at $x = 0$.

NON DIFF'ABLES





and others...