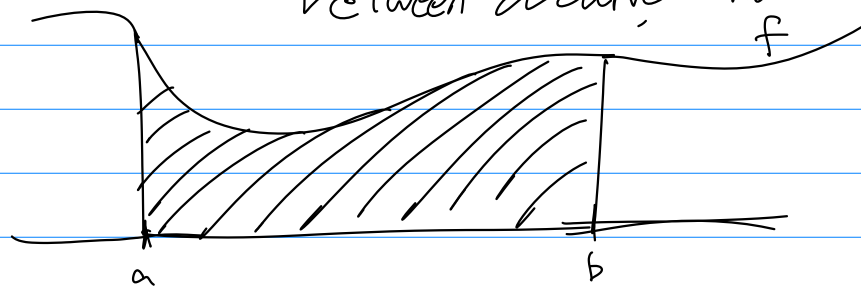
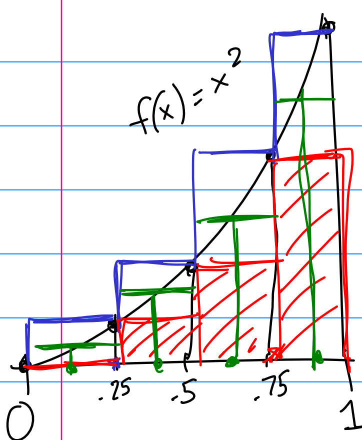
Antideriv's
Summations

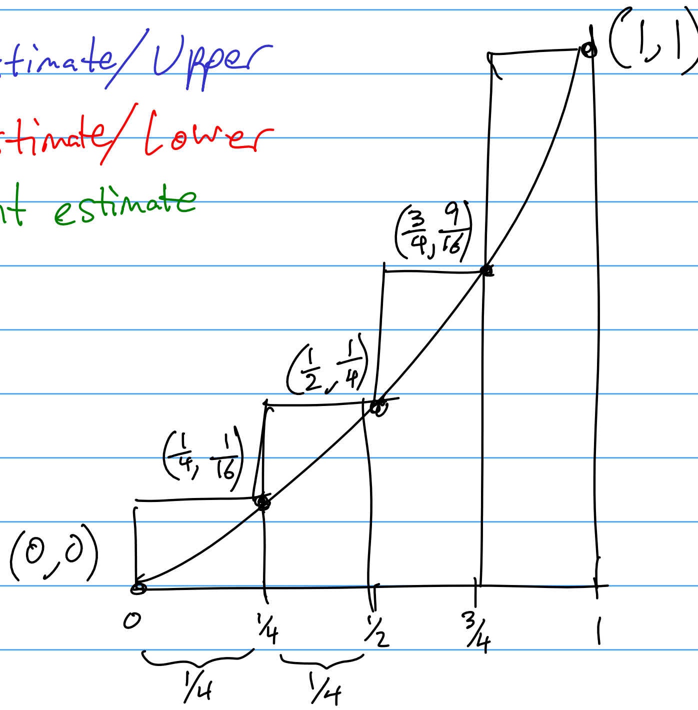
4.1 - Integral

An integral measures the area ~~under a curve.~~
between a curve and the x-axis.





Right estimate/Upper
Left estimate/Lower
Midpoint estimate



Upper =

$$\underset{w}{\frac{1}{4}}\cdot\underset{h}{\frac{1}{16}} + \underset{w}{\frac{1}{4}}\cdot\underset{h}{\frac{1}{4}} + \underset{w}{\frac{1}{4}}\cdot\underset{h}{\frac{9}{16}} + \underset{w}{\frac{1}{4}}\cdot\underset{h}{1}$$

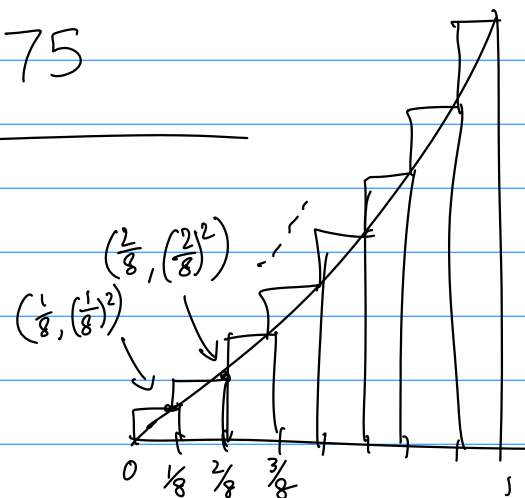
$$\frac{1}{4}\left(\frac{1}{16}+\frac{\cancel{1}^{4}}{\cancel{4}_{16}}+\frac{9}{16}+\cancel{1}^{16}_{16}\right)$$

$$=\frac{1}{4}\left(\frac{30}{16}\right) = \frac{30}{64} = .46875$$

$$\text{Upper} = \frac{1}{8}\left(\frac{1}{8}\right)^2 + \frac{1}{8}\left(\frac{2}{8}\right)^2 + \dots$$

$$= \frac{1}{8}\left(\left(\frac{1}{8}\right)^2 + \left(\frac{2}{8}\right)^2 + \dots \left(\frac{8}{8}\right)^2\right)$$

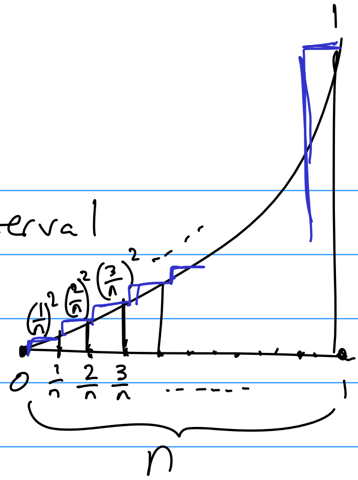
$$= \frac{1}{8}\left(\frac{1^2+2^2+3^2+4^2+5^2+6^2+7^2+8^2}{8^2}\right)$$



$$\sum_{i=1}^{8} i^2 = \frac{\cancel{8}^{4}(\cancel{9}^{3})(17)}{\cancel{6}_{\cancel{3}}} = 204$$

$$\frac{204}{8^3} = .3984375$$

n subintervals — every subinterval has width $\frac{1}{n}$



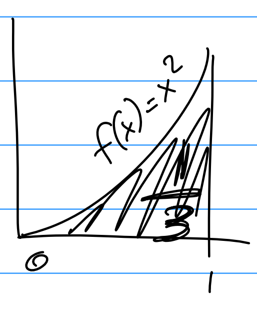
$$\frac{1}{n}\cdot\left(\frac{1}{n}\right)^2 + \frac{1}{n}\cdot\left(\frac{2}{n}\right)^2 + \frac{1}{n}\cdot\left(\frac{3}{n}\right)^2 + \dots + \frac{1}{n}\left(\frac{n}{n}\right)^2$$

$$\frac{1}{n}\left(\frac{1^2}{n^2} + \frac{2^2}{n^2} + \frac{3^2}{n^2} + \dots + \frac{n^2}{n^2}\right)$$

$$\frac{1}{n}\left(\frac{\sum_{i=1}^{n} i^2}{n^2}\right) = \frac{\left(\frac{n(n+1)(2n+1)}{6}\right)}{n^3} = \frac{n(n+1)(2n+1)}{6n^3}$$

Exact area... $\lim_{n\to\infty} \frac{n(2n^2+3n+1)}{6n^3}$

$$= \lim_{n\to\infty} \frac{2n^3+3n^2+n}{6n^3} = \frac{2}{6} = \boxed{\frac{1}{3}}\ !!$$



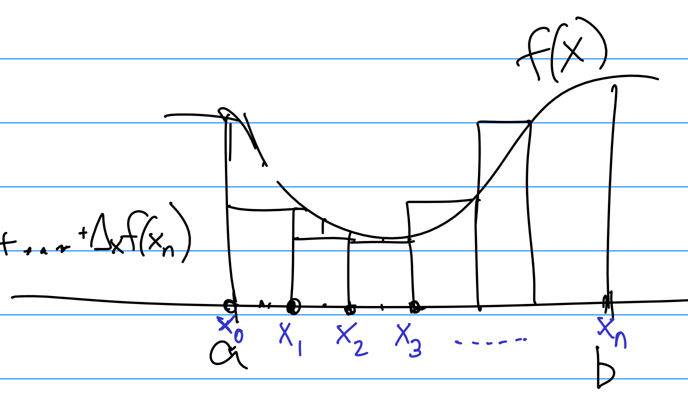
(right)
Area estimate =

$$= \Delta x\, f(x_1) + \Delta x\, f(x_2) + \Delta x\, f(x_3) + \dots + \Delta x\, f(x_n)$$

$$= \Delta x\left(f(x_1) + f(x_2) + \dots f(x_n)\right)$$

$$= \Delta x\left(\sum_{i=1}^{n} f(x_i)\right)$$

$x_i = a + i\Delta x$



n subintervals
width $= \frac{b-a}{n} = \Delta x$

or $\sum_{i=1}^{n} f(x_i)\Delta x$ ← estimate...

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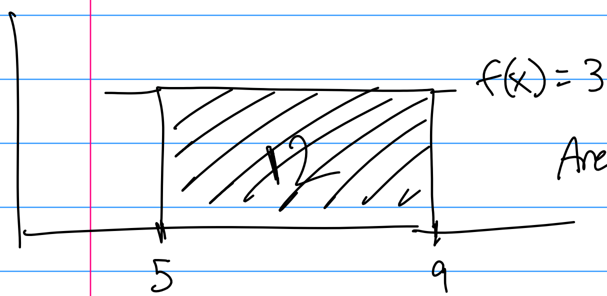
$\Delta x = \frac{b-a}{n}$

$x_i = a + i\Delta x$

Exact Integral: $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$



$\Delta x = \frac{9-5}{n} = \frac{4}{n}$

Area $= \int_5^9 3\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$

$= \lim_{n\to\infty} \sum_{i=1}^{n} 3\frac{4}{n}$

$= \lim_{n\to\infty} 12 \sum_{i=1}^{n} \frac{1}{n}$

$= \lim_{n\to\infty} 12\left(n \cdot \frac{1}{n}\right) = \lim_{n\to\infty} 12 = \boxed{12}$

Constant: $f(x) = c$

$\int_a^b c\,dx = (b-a)c$

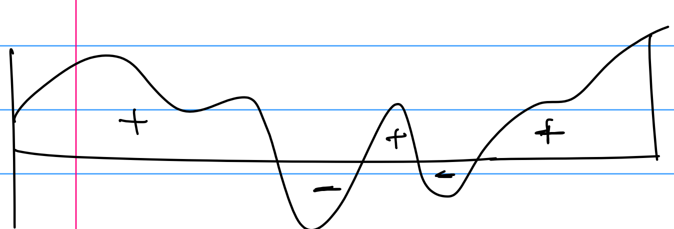
What if $f(x) = -3$?

$\int_5^9 (-3)\,dx = (9-5)(-3)$

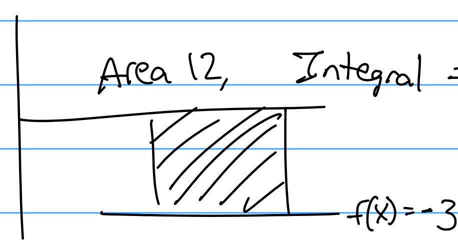
$= 4(-3) = -12$

AREA is positive...

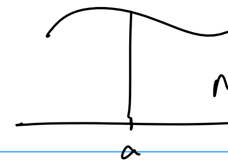
Integrals can be positive, negative, or 0.



Area 12, Integral = −12.

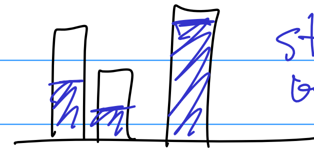


$$\int_a^a f(x)\,dx = 0$$



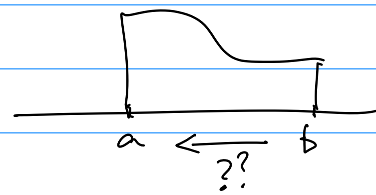
no area.

$$\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$



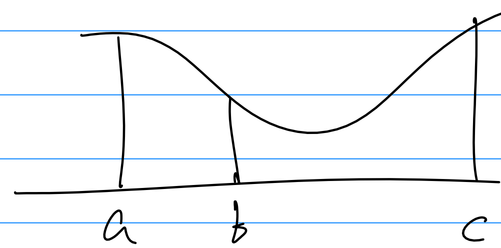
stacked bar graph

$$\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$$



$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$



Suppose $\int_{-2}^{9} f(x) = 100$ and $\int_{5}^{9} f(x) = -18$.

$$\int_{-2}^{5} f(x)\,dx = 118 \qquad \int_9^9 f(x)\,dx = 0$$

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$$\int_9^5 f(x)\,dx = 18 \qquad \int_{-2}^{9} (f(x)+3)\,dx$$

$$= \int_{-2}^{9} f(x)\,dx + \int_{-2}^{9} 3\,dx$$

$$100 + 3\cdot 11 = \boxed{133}$$

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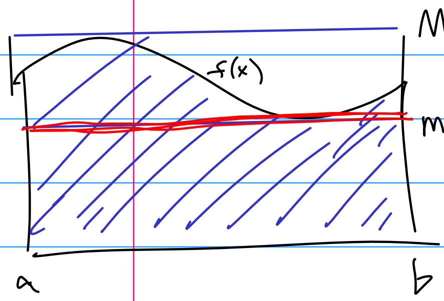

If $m \le f(x) \le M$ on $[a,b]$, then

$$m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$$

* If $f(x) \ge 0$ on $[a,b]$, then $\int_a^b f(x)\,dx \ge 0$.

* If $f(x) \ge g(x)$ on $[a,b]$, then $\int_a^b f(x)\,dx \ge \int_a^b g(x)\,dx$

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~~#33~~ #55 $\int_0^4 (x^2-4x+4)\,dx \ge 0$?

Is $x^2-4x+4 \ge 0$? on $[0,4]$

$(x-2)(x-2) \ge 0$?

$(x-2)^2 \ge 0$ is true.

#62 Estimate $\int_0^2 \underbrace{(x^3-3x+3)}_{f(x)}\,dx$

Max/min on $[0,2]$?

$f'(x) = 3x^2 - 3 = 0$

$3x^2 = 3$

$x^2 = 1$

$x = \cancel{\pm} 1$

$f(0) = 3$

$f(1) = 1 \leftarrow$ min

$f(2) = 5 \leftarrow$ max

$1 \cdot 2^{[0,2]} \rightarrow \underbrace{2 \le \int_0^2 (x^3-3x+3)\,dx \le 10}$ $\leftarrow 5 \cdot 2^{[0,2]}$