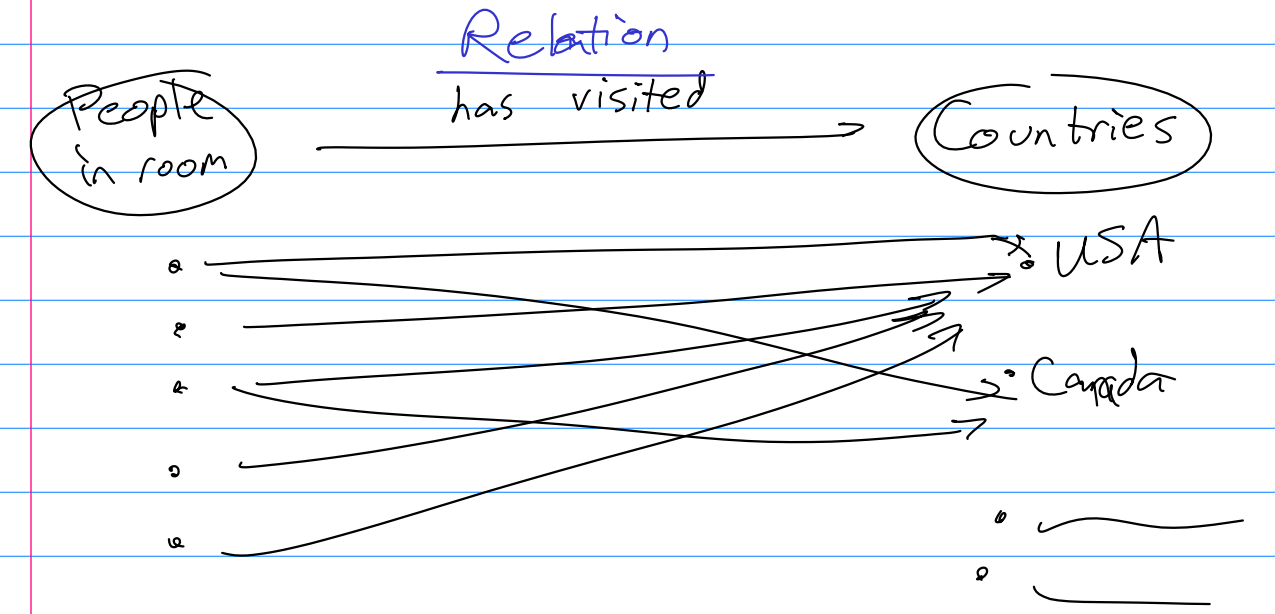
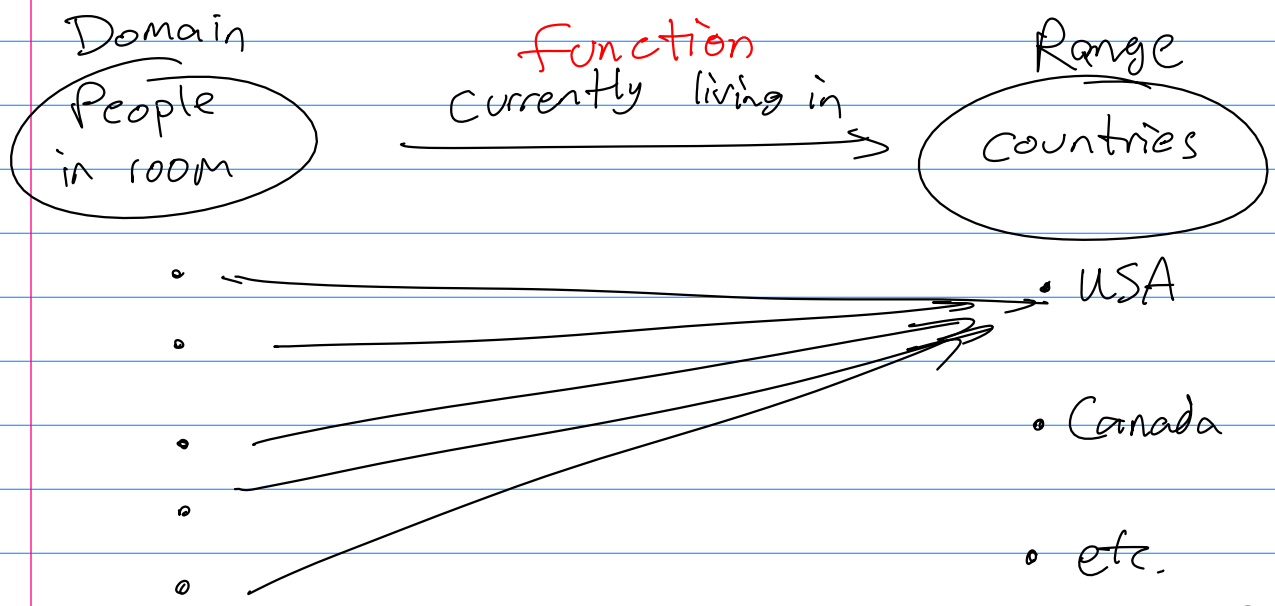
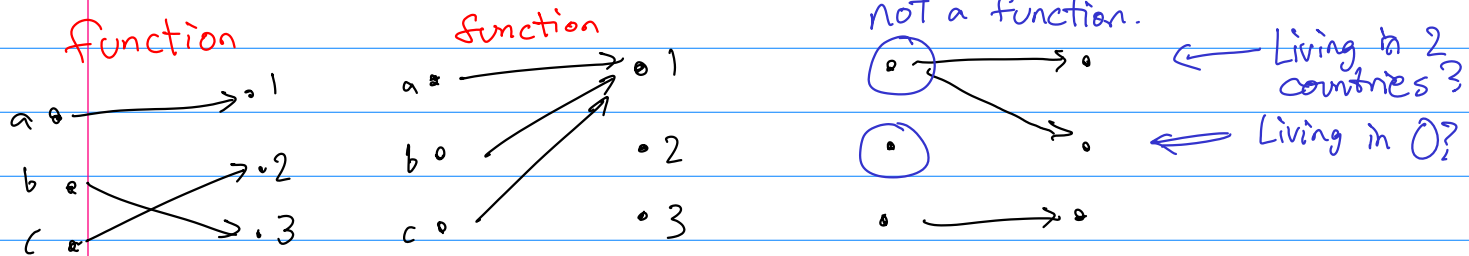
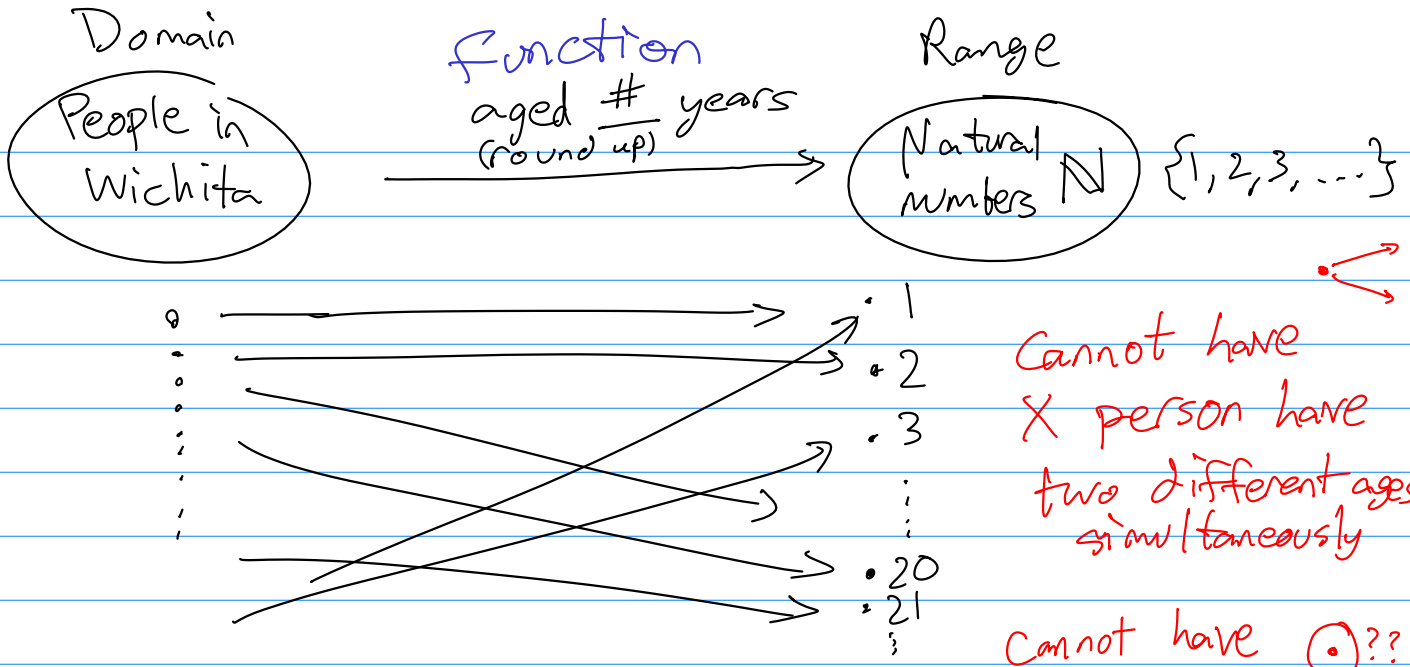
1.1 Determine whether a relation (verbal, numeric, visual, algebraic) is a function.

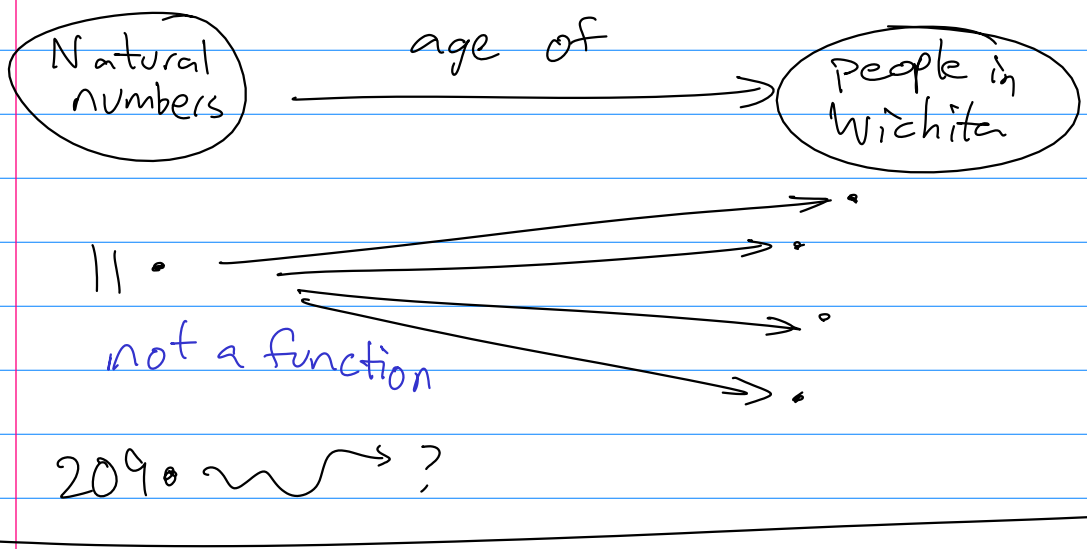


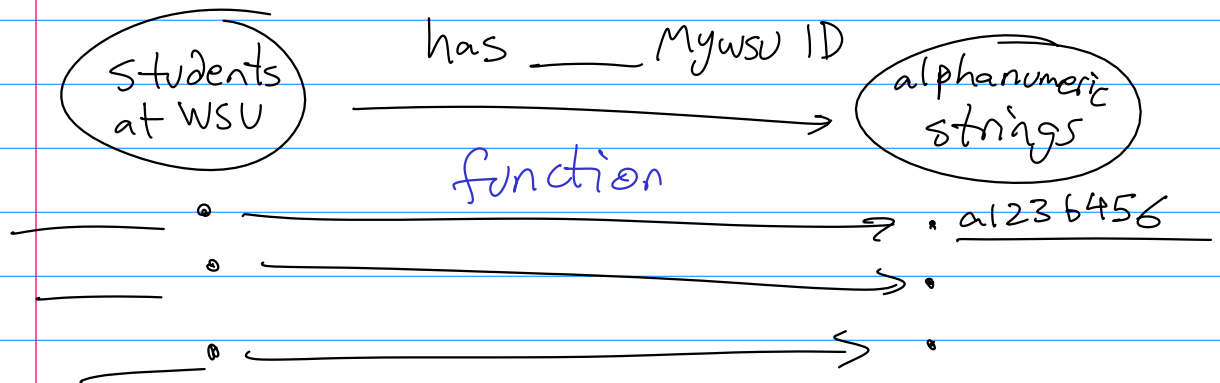


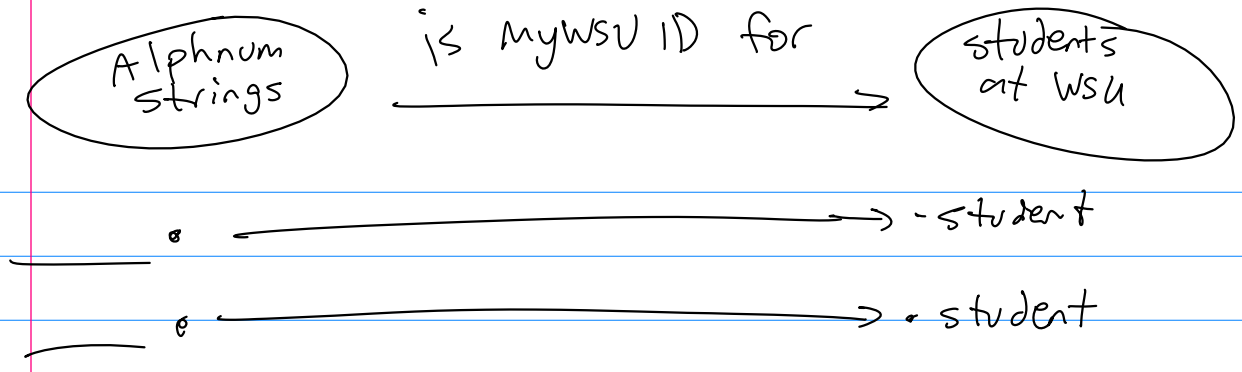
A function is a relation from a set D (the domain) to a set R (the range) such that:
every element of D is mapped to exactly one element of R.



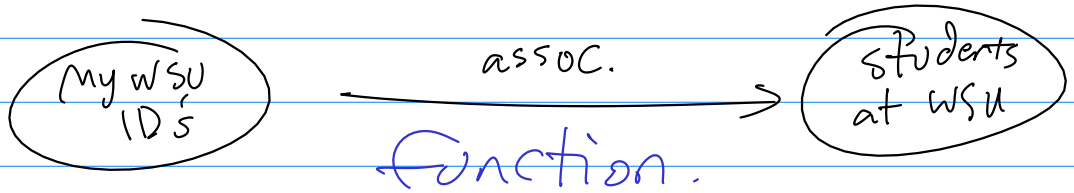

Domain
People in Wichita
function
aged # years (round up)
Range
Natural numbers ℕ
{1, 2, 3, ...}
1
2
3
20
21
Cannot have X person have two different ages simultaneously
Cannot have ⊙?? X person having no age.


Natural numbers
age of
People in Wichita
11
not a function
209
?


students at WSU
has ___ MyWSU ID
alphanumeric strings
function
a1236456



not quite a function

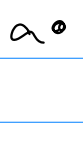


function.

Verbal "The function that maps each real number to its square."

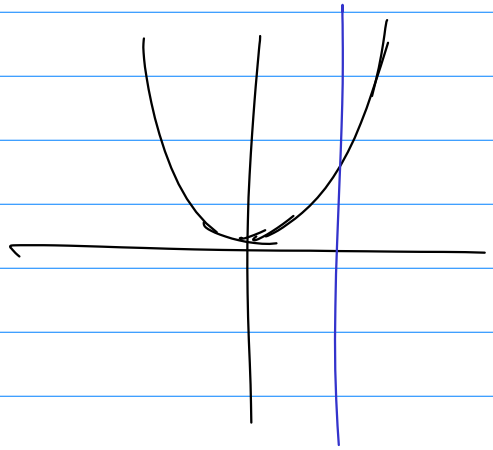
- lots of writing
- possible ambiguity

Numeric $(0,0)$, $(2,4)$, $(-3,9)$, etc.



- cannot list an infinite set

Graphic



vertical line test.

+ visual!
- inexact

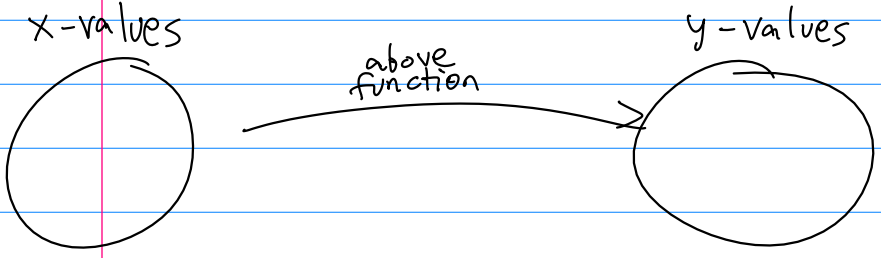
Algebraic $y = x^2$

or

$f(x) = x^2$

+ most compact
+ very easy to verify that this is a function

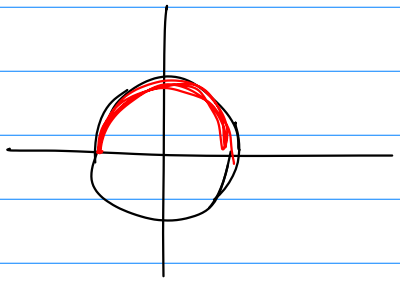
$y = \{ x^2, 4, x, 7, x^3, 9, -2, \text{etc} \}$



But $x^2 + y^2 = 1$ ....

$y^2 = 1 - x^2$

$y = \sqrt{1-x^2}$



Set of points, but y is not a function of x.

function, but not the same as the original.

1.1 Find the implied domain of a function.

$f(x) = x^2$

Implied domain:
Start with all real numbers,
Then delete any numbers x for which f(x) is undefined.

Domain: All reals. ($\mathbb{R}$)

$$f(x) = \frac{7x}{x^2 - 9}$$

$$(x+3)(x-3)$$

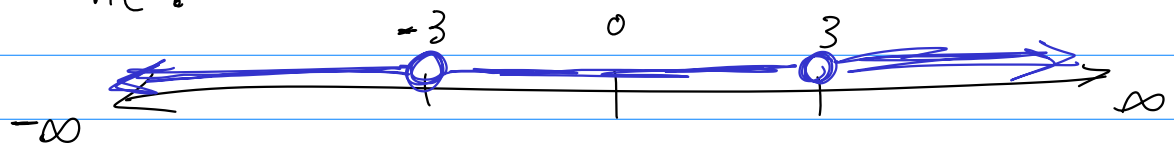
Bad values:
± 3

How to express the good values (Domain)?

Set notation: $\{x \in \mathbb{R} \mid x \neq 3, -3\}$

All x in real numbers such that condition(s).

Number line:



( ) ○ open dot "excludes" -3 not in the set

[ ] ● closed dot "includes" -2.999 is

Interval notation:

$$(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$$

Union symbol, joins two sets.

$f(x) = \sqrt{5 - 2x}$

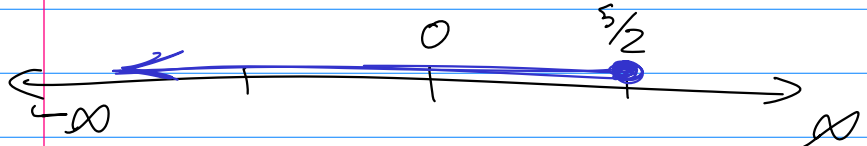
Bad values:

$5 - 2x < 0$

$5 < 2x$

$\frac{5}{2} < x$

Good values: $x \leq \frac{5}{2}$

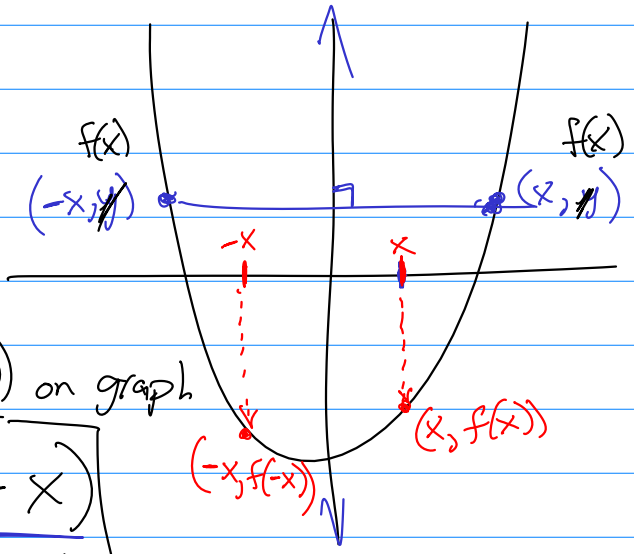


$(-\infty, \frac{5}{2}]$

∞ and $-\infty$ are not real numbers so they are always excluded ( ).

1.1 Determine whether a function is even, odd, or neither.

An even function
- has a graph that is symmetric across the y-axis.



$(-x, f(x))$ and $(x, f(x))$ on graph

Def: $f(x) = f(-x)$ for all x (in the domain of f)

Is $f(x) = |x| - 2$ even?

$f(3) = |3| - 2 = 1$

$f(-3) = |-3| - 2 = 1$

a few examples are not enough!

a, an arbitrary real number.

$f(a) = f(-a)$?

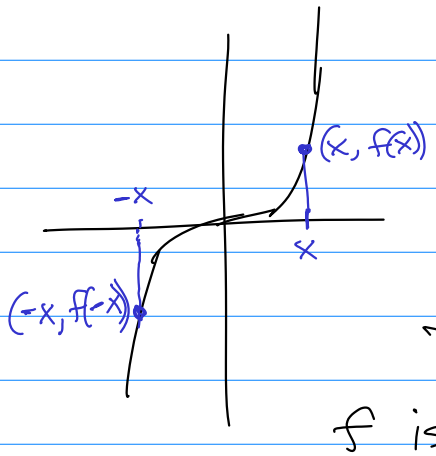
$|a| - 2 \qquad |-a| - 2 = |a| - 2$

By a property of absolute value

Therefore f is even.

Odd functions

have a graph that is symmetric 180° about the origin.



$f(x)$ and $f(-x)$ are opposites, or:

Def:
f is odd if $f(-x) = -f(x)$ for all x in the domain of f

Is $f(x) = x^3 - x$ odd?

$f(-x) = (-x)^3 - (-x)$

$= -x^3 + x$

$x^3 - x$ and $-x^3 + x$ are opposites.

Therefore, $f(x)$ is odd.

$f(-x) = -f(x)$

$-x^3 + x = -(x^3 - x)$ ✓