Introduction to hypothesis testing: one sample t-tests

R-Training Workshop

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At Pwani University

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Posits

- **Learnt how to summarise continuous data**
- **Move on to**
  - **testing prior beliefs about population means using sample means**
  - **testing for differences in means for paired data**

Hypothesis testing (1)

- Have a belief that;

  population mean = particular value

- Design a study to collect data to test this
- Estimate a mean value from a sample
- Use this to test if sample data consistent with population mean = some value
- 4 keys stages to testing hypotheses

Stage 1: define hypothesis

- **We make two key statements to define**
  - **null hypothesis**
  - **alternative hypothesis**

- **Null hypothesis:**

  population mean = particular value

$$H_0 : \mu = \mu_0$$

- **Alternative hypothesis:**

  population mean ≠ particular value

$$H_1 : \mu \neq \mu_0$$

Example

- **Sample mean birth weight in 141 babies was 3.01 with s.e. 0.04**
- **Suppose prior to the study it was thought**
  - **population mean was 3.25 kg**
  - $\mu_0 = 3.25$
- **What are the null & alternative hypotheses?**
- $H_0: \mu = 3.25$ vs $H_1: \mu \neq 3.25$

Stage 2: significance

- **State level of significance**
  - **usually 5%**
- **Two-sided test**
  - **difference of the same size in either direction**
- **Assume null hypothesis is true**
- **Look for evidence in the sample data**
  - **to accept or reject the null hypothesis**

Stage 3: test statistic

- **1 sample t-test**
- **Test statistic**

= <u>(difference between sample mean & popul. mean)</u>
standard error of the sample mean

$$t = \frac{\bar{x} - \mu_0}{se(\bar{x})}$$

- **Degrees of freedom = no. of observations minus 1**

Example

- **sample mean = 3.01**
- **standard error (sample mean) = 0.04**
- **population mean = 3.25**

- $$t = \frac{(3.01 - 3.25)}{0.04} = -6.00$$
- **degrees of freedom = n – 1 = 141 - 1 = 140**

Stage 4: p-value

- **To get p-value**
  - **compare t to statistical tables or use R output**
- **Interpretation of p-value**
  - **p-value = probability of observing our data**
  - **assuming that the null hypothesis is true**
- **A low p-value implies**
  - **low prob. of observing data if null hypothesis is true**
  - **evidence to reject the null hypothesis**
  - **In R, *P_value<- pt(-6, df=(141-1))::::::::::[p<0.001]***

Interpretation

p-value

= probability of observing our data *assuming* that the null hypothesis is true

= probability of observing our data given that the population mean birth weight is 3.25 kg

< 0.001

- Conclude
  - unlikely that we would get our sample mean if the population mean birth weight was really 3.25 kg
  - have found strong evidence to suggest that the population mean $\neq$ 3.25 kg

Beware

A low p-value does not prove the alternative hypothesis:

it provides evidence to suggest it could be true

A high p-value does not prove the null hypothesis

Paired comparisons

- When it is not feasible to assume that two groups of data are independent
- Used to compare means of the same population/subjects under different conditions
- Takes the correlation into account
- The differences between paired observations are assumed to be normally distributed
- More powerful since it reduces inter-subject variability

Examples

- Pre- and –post scores for scholars receiving tutorials

- Sunburns scores for two sunblock lotions; one applied to the individual right arm and the other on the left arm

One- sample t-test for paired data (1)

- **Weight measured in same babies**
  - **at birth**
  - **prior to leaving clinic**
- **Q: mean weight at birth = mean weight on leaving clinic?**
- **Birth**
  - **mean birthweight in 122 babies = 3.03 kg**
- **Leaving clinic**
  - **mean birthweight in 122 babies = 3.07 kg**

Paired Data (2)

- **What do we think looking at those 2 mean values?**
- **Are the 2 sets of weights comparable?**
- **What is our null hypothesis?**

  H_0: mean at birth = mean on leaving

- **Equivalent to:**

  H_0: mean difference in weights = 0

Paired data (3)

- include 122 babies with 2 weight measurements
- mean difference = difference between means

  = - 0.04
- calculate s.e. of the differences in the weights
- test statistic = mean difference / s.e.(mean difference)
- t = - 0.04 / 0.01 = - 4.00
- d.f. = 122 – 1 = 121

An example in R

- A stimulus applied to 12 men to determine its effect on systolic blood pressure (BP) prior to and after application
- BP prior:20,20,21,22,23,22,27,25,27,31,30,28
- BP after:19,22,24,24,25,26,26,28,28,29,30,25

- Hypothesis?
- Is there significant effect on BP before and after application of the stimulus?

…in R?

- BP_prior_Vector<-c(,,,,,,,,,,)
- BP_after_Vector<-c(,,,,,,,,,,,)
- Ttest_paired<-t.test(BP_prior_vector,BP_after_Vector,paired=TRUE)
- Paired t-test,
- data: BP_prior_Vector, BP_after_Vector

Comparing the means of 2 groups

Statistical inference is a tool to enable comparisons to be made.

A comparison of means makes assumptions about the groups being compared:

The two groups are measured in the same way.

The data are approximately normally distributed within each group

For the comparison of 2 means, the parameter we are estimating is the difference between the means.

To do this calculate the standard error of the difference.

Summary

- **Use one sample t-test**
  - **Test if population mean = particular value**
  - **Based on prior assumption about population mean**

  - **Test if sample means measured on 2 occasions are equal**
  - **More common than previous example**
  - **Based on set of complete measurements**