

The Answer for the $Q_1(t)$:

[t u v]= [3.0472e-01 1.0972e-02 5.3370e-01]

The point on the polynomial surface is (0.05486, 2.6685, 3.1221)

The point on the query curve is (1.5236, 1.5236, 4.1172)

Distance=2.1115

The Answer for the $Q_2(t)$

[t u v]=[2.0109e-01 6.4025e-03 5.1381e-01]

The point on the polynomial surface is (0.032012, 2.5690, 3.0295)

The point on the query curve is (1.0055, 1.0055, 4.3306)

Distance=2.2399

The following is some discussion.

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M-file for $Q_1(t)$

```
%t,u,v are variables
syms u v t
syms h real
digits(8)
%h is step size
%u,v is from 0 to 1
x=5*u;
y=5*v;
z=(-0.38)+25/3*x+4/3*y-25/3*x*x-10/3*x*y;
%Q1t, t is from 0 to 1
qx=5*t;
qy=5*t;
qz=25/6*t*t-25/6*t+5;
distancesquare=(x-qx)^2+(y-qy)^2+(z-qz)^2;
%diff(distance,'t'),diff(distance,'u'),diff(distance,'v');
Ft=diff(distancesquare,'t');
Fu=diff(distancesquare,'u');
Fv=diff(distancesquare,'v');
%t0,u0,v0 are initial point;
t0=0.5;
u0=0.5;
v0=0.5;
for i=1:200
%deltat,deltau,deltav are numbers of partial derivation
deltat=eval(subs(Ft,[t,u,v],[t0,u0,v0]));
deltau=eval(subs(Fu,[t,u,v],[t0,u0,v0]));
deltav=eval(subs(Fv,[t,u,v],[t0,u0,v0]));
```

```

%t1,u1,v1 are variables
t1=t0+deltat*h;
u1=u0+deltau*h;
v1=v0+deltav*h;
newdistancesquare=subs(distancesquare,[t,u,v],[t1,u1,v1]);
%new t0,u0,v0
g=diff(newdistancesquare,'h');
h0=eval(solve(g==0,h));
[r,c]=size(h0);
if r~= 1

h0=h0.*((t0+deltat*h0>=0).*(t0+deltat*h0<=1).*(u0+deltau*h0>=0).*(u0+deltau*h0<=1).*(v
0+deltav*h0>=0).*(v0+deltav*h0<=1));
    h0(find(h0==0))=[];
end
t0=t0+deltat*h0;
u0=u0+deltau*h0;
v0=v0+deltav*h0;
end
point_tuv=[t0,u0,v0]
distance=sqrt(eval(subs(distancesquare,[t,u,v],[t0,u0,v0])))
.....

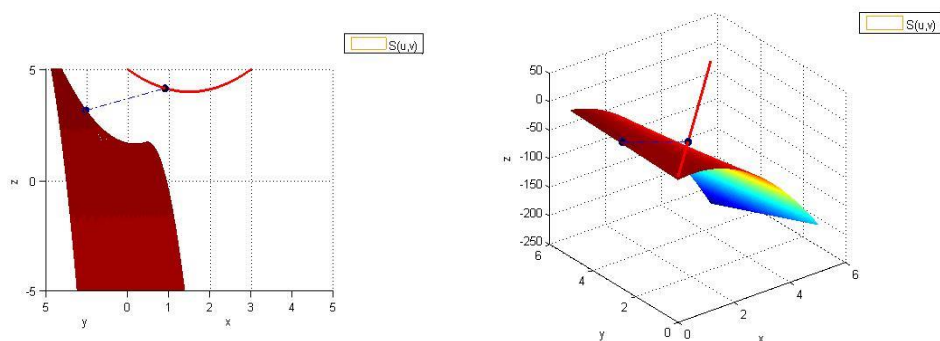
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Result

Point [t u v]= [3.0472e-01 1.0972e-02 5.3370e-01]

Distance=2.1115

Figure1



Note-The choice of initial point :

The choice of the initial point can somehow make the programme not workable.

Although in most case, the t,u,v will converge to the point [3.0472e-01 1.0972e-02

5.3370e-01], if the initial point $[t_0 \ u_0 \ v_0]$ is very close to $[1 \ 1 \ 1]$ or $[0 \ 0 \ 0]$ the programme may not converge or it may converge to some point that t, u, v are out of the range $[0 \ 1]$.

For example:

1. If the initial point is $[0 \ 0 \ 0]$ then the result converges to $[-7.9366e-03 \ 1.7293e-01 \ -4.0682e-01]$. I tried to increase the number of iterations, but it still converge to some point that the t, u, v go out of the range $[0 \ 1]$.
2. If the initial point is $[0.9 \ 0.9 \ 0.9]$ then the program cannot run because in one of the iteration there are two h meet the two requirements, which are $g'(h)=0$ and the new $[t \ u \ v]$ in the range of $[0 \ 1]$.
3. If the initial point is some point that is not too close to the $[0 \ 0 \ 0]$ and $[1 \ 1 \ 1]$, such as $[0.2 \ 0.2 \ 0.2], [0.3 \ 0.3 \ 0.3], [0.5 \ 0.5 \ 0.5], [0.7 \ 0.7 \ 0.7]$ the result all converges to $[3.0472e-01 \ 1.0972e-02 \ 5.3370e-01]$.
4. However if the initial point is the exact point $[3.0472e-01 \ 1.0972e-02 \ 5.3370e-01]$ then it still converge to itself $[3.0472e-01 \ 1.0972e-02 \ 5.3370e-01]$.

The previous discussion shows that my program is sensitive to the initial point.

I haven't figured out how big my program's converge interval is, for the initial point.

.....
M-file for $Q_2(t)$

```
%t,u,v h are variables
syms u v t
syms h real
digits(8)
%h is step size
%u,v is from 0 to 1
x=5*u;
y=5*v;
z=(-0.38)+25/3*x+4/3*y-25/3*x*x-10/3*x*y;
%Q1t, t is from 0 to 1
qx=5*t;
qy=5*t;
qz=53.976*t^4-104.64*t^3+64.256*t^2-12.589*t+5;
distancesquare=(x-qx)^2+(y-qy)^2+(z-qz)^2;
%partial derivative,expression;
Ft=diff(distancesquare,'t');
Fu=diff(distancesquare,'u');
Fv=diff(distancesquare,'v');
%t0,u0,v0 are initial point , numbers;
t0=0.5;
u0=0.5;
v0=0.5;
for i=1:100
%deltat,deltau,deltav are numbers of partial derivation
deltat=eval(subs(Ft,[t,u,v],[t0,u0,v0]));
```

```

deltau=eval(subs(Fu,[t,u,v],[t0,u0,v0]));
deltav=eval(subs(Fv,[t,u,v],[t0,u0,v0]));
%t1,u1,v1 are variables
t1=t0+deltat*h;
u1=u0+deltau*h;
v1=v0+deltav*h;
newdistancesquare=subs(distancesquare,[t,u,v],[t1,u1,v1]);
%new t0,u0,v0
g=diff(newdistancesquare,'h');
h0=eval(solve(g==0,h));
[r,c]=size(h0);
if r~= 1

h0=h0.*((t0+deltat*h0>=0).*(t0+deltat*h0<=1).*(u0+deltau*h0>=0).*(u0+deltau*h0<=1).*(v
0+deltav*h0>=0).*(v0+deltav*h0<=1));
    h0(find(h0==0))=[];
end
%h0 多解, 考虑 t0,u0, v0 大小约束
t0=t0+deltat*h0;
u0=u0+deltau*h0;
v0=v0+deltav*h0;
end
point_tuv=[t0,u0,v0]
distance=sqrt(eval(subs(distancesquare,[t,u,v],[t0,u0,v0])))
.....

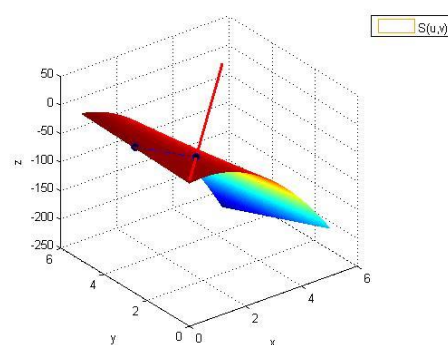
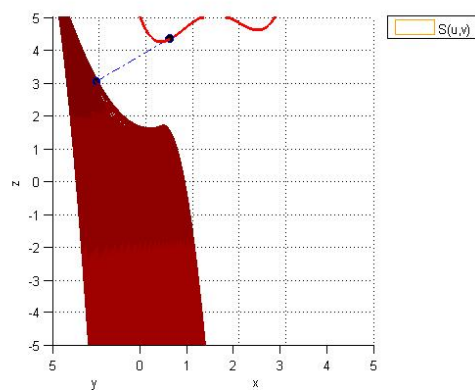
```

Result

Point [t u v] =[2.0109e-01 6.4025e-03 5.1381e-01]

Distance=2.2399

Figure 2



Note-The choice of initial point :

The same question appears. The program still is sensitive to the initial point for some extreme case. But in most case it can converge to [2.0109e-01 6.4025e-03 5.1381e-01].