Trip Production & Attraction

-Multi Parameter Linear Regression-

$$y_i = x_{0i} + \beta_1 x_{1i} + \beta_2 x_{2i} + \cdots + \beta_n x_{ni}$$

i= Zone Index

n= Variable Index

β_n = Coefficient of Variable *n*

y_i= Trip Production & Attraction of Zone *i*

x_{ni} = Variable *n* (Population, Labor Force etc.) of Zone *i*

Trip Distribution

-Gravity Model-

$$T_{ij} = A_i * B_j * O_i * D_j * f(c_{ij})$$

$$A_i = \frac{1}{\sum_j B_j * D_j * f(c_{ij})}$$

$$B_j = \frac{1}{\sum_i A_i * O_i * f(c_{ij})}$$

$$f_{Combined}\left(c_{ij}\right) = c_{ij}^{\alpha} * e^{-\beta * c_{ij}}$$

i= Origin Zone Index

j= Destination Zone Index

T_{ij}= Total Trips Between Origin *i* and Destination *j*

O_i= Total Trip Production by Zone *i*

D_j= Total Trip Attraction to Zone *j*

A_i= Balancing Factor Assuring $\sum_j T_{ij} = O_i$

B_j= Balancing Factor Assuring $\sum_i T_{ij} = D_j$

$f(c_{ij})$= Deterrence Function Between Zones *i* and *j*

e= Mathematical Constant (2.71828)

α, β= Coefficient

Modal Split (Mode Choice)

-Multinomial Logit Model-

$$T_{ij}^{m} = \frac{e^{-U_{ij}^{m}}}{\sum_m e^{-U_{ij}^{m}}} * \sum_m T_{ij}^{m}$$

$$U_{ij}^{m} = a_1 * x_{1ij}^{m} + a_2 * x_{2ij}^{m} + \cdots + a_n * x_{nij}^{m}$$

i= Origin Zone Index

j= Destination Zone Index

m= Trip Mode

n= Variable Index

e= Mathematical Constant (2.71828)

a_n= Coefficient of Variable *n*

T_{ij}^{m}= Total Trips of Mode *m* Between Zones *i* and *j*

U_{ij}^{m}= Total Utility of Mode *m* Between Zones *i* and *j*

x_{nij}^{m}= Utility Variable *n* of Mode *m* Between Zones *i* and *j* (Trip Cost, Fare, In-Vehicle Travel Time etc.)

Assignment Matrice

-Linear Algebra-

$$T_{ij}^{m} = \frac{\sum_p T_{ij}^{m,p} * PHF_{ij} * PCU^{m}}{Occ^{m}}$$

i= Origin Zone Index

j= Destination Zone Index

p= Trip Purpose

m= Trip Mode

T_{ij}^{m}= Total Vehicle Trip (as Per Car Unit) of Mode *m,* Between Zones *i* and *j*

$T_{ij}^{m,p}$= Trips of Mode *m* and Purpose *p,* Between Zones *i* and *j*

PHF_{ij}= Peak Hour Factor Between Zones *i* and *j*

PCU^{m}= Passenger Car Equavalent (Per Car Unit) of Mode *m*

Occ^{m}= Occupancy of Mode *m*

Volume-Delay Relationship of Links

-BPR Function-

$$t_c = t_0 * \left(1 + \alpha * \left(\frac{V}{C}\right)^{\beta}\right)$$

t_c= Current (Updated) Travel Time

t_0= Free Flow Travel Time

V= Volume (veh/hr)

C= Capacity (veh/hr)

α, β= Coefficient

User Equilibrium Traffic Assignment

-Nonlinear Mathematical Optimization-

- Source for Notation: Mathew T. V., Rao K.V.K., 2007, Traffic Assignment, Introduction to Transportation Engineering, NPTEL May 7, 2007-

$$\text{Minimize } Z = \sum_a \int_0^{x_a} t_a(x_a)dx,$$

$$\text{Subject to } \sum_k f_k^{rs} = q_{rs} : \forall r, s$$

$$x_a = \sum_r \sum_s \sum_k \delta_{a,k}^{rs} f_k^{rs} : \forall a$$

$$f_k^{rs} \geq 0 : \forall k, r, s$$

$$x_a \geq 0 : a \in A$$

$$\delta_{a,k}^{r,s} = \begin{cases} 1 \text{ if link a belongs to path k,} \\ 0 \qquad\qquad\qquad\qquad \text{otherwise} \end{cases}$$

k= Path

x_a= Equilibrium Flows in Link a

t_a= Travel Time on Link a

f_k^{rs}= Flow on Path k Connecting O-D Pair r-s

q_{rs}= Trip Rate Between r and s

$\delta_{a,k}^{rs}$= Definitional Constraint

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