The authors present a novel approach in an attempt to improve latitudinal temperature gradients from spatially scarce proxy estimates.  
  
While their efforts can clearly help substantially towards better assessing and interpreting the available proxies, the authors do not provide a convincing case to support their claims regarding the early Eocene. The technicalities and implementation of the method is solid, but the results allow only for a limited assessment of its applicability. Many of the limitations considering these proxy estimates lie in the methods behind the proxies themselves, something that is not adequately addressed in my opinion. I would therefore like to see some more tests including the potential effects of e.g. seasonality, lacking upper/lower bounds, or differently-sourced temperatures (ML ocean, SST, SAT etc.).

Regardless, I believe that this study can substantially benefit the field and therefore suggest publication after some adjustments/additions.

We thank the referee for the feedback and for endorsing the publication of our manuscript after some adjustments.

We agree that the limitations regarding the proxy data primarily lie in the methods behind the proxy data. There is indeed potential evidence that seasonal effects may play a role in the higher-latitude geochemical proxy data: We have now added a figure to the Supplementary Material showing that the standard deviation of the temperature estimates provided by proxy data from the same location increases with absolute palaeolatitude.

However, the structure of our model inherently accounts for such effects by not using the proxy temperatures directly, but instead estimating a local temperature that preserves the uncertainty associated with the variation in proxy estimates. Seasonal effects would thus only be problematic if they systematically bias the temperature proxy record in one direction. The data compilation we use does not provide information that would allow us to account for this.

We recognise that time averaging also necessarily effects aggregate temperature effects, and believe that the structure of our model leads to a fair representation of the accompanying uncertainty.

As an additional test, we have now expanded our analysis to include also the uncertainties associated with each individual proxy measurement, and present these new results in the Supplementary Materials.

**General remarks**:  
  
*Introduction*

* + spatial patterns and distribution are discussed in detail, but how is temporal variation covered?  
      
    When considering proxies within a certain interval, they may represent entirely different subsets of this interval and therefore not be compatible.

This may indeed part of the reason why the standard deviation of the EECO gradient is much higher than that of the modern gradient. A more temporally finely resolved analysis would be an interesting project, but is beyond the scope of this study.

* + (L73) The introduction could use some background and references on Bayesian modelling and why one could expect this method to be a suitable tool to the problem raised.

We expanded the justification of the Bayesian method in the introduction. We provide further details in the methods and discussion sections.

*Methods*

* + I assume the authors consider near-surface, surface or mixed layer ocean temperatures? I am missing some clarification and motivation here.

We are using the sea surface data compilation by Hollis et al. 2019. We tried to minimise depth differences in the origin of the SST temperature proxy data by excluding any data labelled as thermocline or sub-thermocline, and otherwise rely on the screening of Hollis et al. 2019 for excluding data not representative of sea surface temperatures. We have not included LAT data. Realistically, the proxy compilation from Hollis et al. 2019 used herein may be more accurately characterised as including data from the mixed layer, but we use the term SST as is convential for palaeoclimate studies, including the Hollis et al. 2019 compilation. We have clarified this in the Methods section (l...)

* + Are the temperature limits mentioned considering yearly, seasonally, monthly or extreme values?  
      
    Especially towards higher latitudes, seasonal temperatures may be much more restrictive than yearly averages.

We used mean annual temperatures to define our ecological proxies, which we deem most appropriate as we are not modelling seasonal temperatures; instead the geochemical proxy compilation likely consists of temperature signals averaged across seasons, and/or from different seasons. We acknowledge that using seasonal temperatures would be preferable for a seasonally resolved analysis, which we cannot provide herein.

We have however added an additional test in which we use the temperatures of the coldest and warmest weeks/months to define the ecological proxies. (SM...)

*Model validation*

* + As the authors mention in the introduction, the spatial distribution of temperature estimates can greatly limit the skill of derived latitudinal gradients.
  + In the model validation, is there a way to not only consider the shear amount of samples, but also their spatial clustering?  
      
    Looking at many random spatial distributions of temperature estimates, one may get a too optimistic view on how well they could capture the considered gradient.

*Results*

* + Are hemispheric asymmetries considered? These may differ substantially between the current and Eocene climate.
  + Somewhat philosophical question: considering the idealised profiles in Figure 1, one would hardly see any difference between the extreme icehouse, icehouse and present climate using the adopted method to determine the latitudinal temperature gradient. Were other measures explored in this sense?
  + Looking at the main result in figure 4, I find it hard to see the added value of the method presented in this work.  
      
    The temperature estimates based on coral reefs in the tropics seem highly doubtful (this is briefly touched upon in the discussion), while information at higher latitudes is still extremely scarce.  
      
    Likely, the potential influence of seasonal biases in some high latitude proxies are potentially problematic for the method, this is again only briefly mentioned in the discussion.  
      
    In that sense, I am not convinced about the authors' claim that this method succeeds in providing an unbiased estimate of the latitudinal temperature gradient of the Early Eocene climate.

**Specific Comments**:

* + L43: and strongly limited by the possibilities to determine temperature estimates from different proxies.  
      
    This is associated with many assumptions, the most important probably being that any relations found in present experiments still hold in the distant past.
  + L55: this is indeed the case in a general sense, but there are clear exceptions e.g. near fronts (e.g. the ACC or a WBC). How can we tell whether a certain proxy is representative of the surrounding region?
  + L121: What is the motivation to use a normal distribution when it is known that the actual distribution is skewed?

Given the possibility of warmer-than-modern coral reef habitats in the early Eocene, a skewed distribution based on the modern coral reef environmental parameters would exacerbate the potential underestimation of temperatures at Eocene coral reef locations. We prefer a broad, normal distribution which acknowledges the large uncertainty inherent in this proxy.

Furthermore, the statistically derived temperature range falls well short of the potential maximum of 35.6C mentioned earlier, how is this consistent?

* + L146: What motivates the maximum value of 29.5C for mangroves? This may push tropical temperature ranges down considerably, and thus needs to be justified.

We have added a sensitivity test using a broader temperature range of 16 – 35.6 C (combining this suggestion and that of reviewer 2) for both the corals and the Avicennia-Rhizophoraceae mangrove assemblage. The result is very similar to the original analysis (Fig. S ???).

* + L210: It is unclear to me how the emulated climatic states represent a realistic simulation. If these are highly idealised, they may not be suitable to purposefully test the bayesian model.
  + L290: The us of the word 'modelled' is a bit ambiguous here, as the study still considers estimates from proxy-based data rather than numerical climate models.  
      
    The autors may instead consider using e.g. 'estimates from our Bayesian model', or 'proxy-based model estimates'.
  + L308: Considering hemispheres separately is new at this point and should therefore not be considered solely in the results section.
  + L359: A big caveat here is that the inclusion of ecological constraints is highly dependent on the underlying assumption, something I feel is not adequately addressed here.  
      
    There is yet, however, sufficient discussion on several related aspects further down.

**Figures**:

* + Figure 1: This figure needs some grid lines and coordinate labels.
  + Figure 3: As shown clearly in this figure, equatorial temperatures are on average cooler than tropical temperatures.  
      
    This may be too detailed for the scope of the study, but this would suggest using average tropical temperatures rather than equatorial ones would be better suitable to estimate the latitudinal gradient.

In our study, we simplified the shape of the gradient to assume highest temperatures at the equator. We consider this simplification justified, as the difference in averages between equatorial and the highest tropical sea surface temperatures in the modern is only ~ 0.5 C, which is much lower than the uncertainties in our reconstructed gradient.

In general, using average tropical temperatures to calculate the magnitude of the latitudinal gradient has its own problems, as the width of the tropical climate belt has varied through time, and using tropical averages may thus be influenced not only by the magnitude, but also by the shape of the gradient within the tropics.

* + Figure 2/4: It is described in the methods that the latitudinal gradient of the prior lies at middle latitudes, this is however very different from the emulated greenhouse climates.  
      
    In figure 4, we again see the maximum gradient shown at middle latitudes. This seems inconsistent with the emulator cases and also poses the question what determines this position?  
      
    A profile much like the one shown in the bottom panels of Figure 2 would likely result in highly different polar temperatures and thus gradients.

**Tables**

* + Table 1: why not use the recently published DeepMIP model results for this estimate?  
      
    In this table, it should be explained better what the gradient means and at least have units (I assume degree C?). Is this a regression, a difference between points/regions?

**Small remarks**:

* + L121: values are missing units (likewise in other distribution values further down).
  + L291: usage of degC after the brackets is a bit awkward.

**RC2**: ['Comment on egusphere-2023-1188'](https://egusphere.copernicus.org/#RC2), Anonymous Referee #2, 30 Jul 2023  [reply](https://editor.copernicus.org/index.php?_mdl=msover_md&_jrl=778&_lcm=oc116lcm117t&_acm=open&_ms=111966&p=248418&salt=9089431141350565268)

The authors provide a novel quantitative method for reconstructing Eocene temperature gradients from sparse proxy data. The paper is well written, and the implementation is mostly solid, but I am not yet convinced of their claims for the early Eocene. In particular, I’m concerned that the authors may have overestimated their model’s skill by neglecting to include proxy biases and noise in their model’s validation. I would also appreciate a bit more context and/or justification for several methodological choices – such as the choice of a logistic model, and the sensitivity to the ecological parameters. That said, I think this paper has excellent potential to improve the paleoclimate field, and I recommend its acceptance following various revisions.

We expanded the justification of the method in the introduction, and have added additional details on the choice of the logistic functions in the Methods.

Comments:

Introduction – I would like some more context on the use of Bayesian models for paleoclimate reconstruction here. How have BHM’s been used before, and why are they a good choice for this reconstruction? Same for the choice of the logistic model. I thought there were some nice points in lines 320-330 that could be useful here.

We have expanded the introduction. Bayesian hierarchical models are increasingly commonly used in (palaeo-) climatic studies, and a de-novo explanation of the Bayesian approach and philosophy is not deemed necessary here. References on Bayesian modelling are included in the Methods section.

Equations 2, 5, 6 - Similarly, I’d like more context for the choice of this model and its design. Is this logistic model’s design a common setup for paleoclimate? If so, some citations would be nice. If this is a completely novel approach, then I’d appreciate more discussion as to why the authors made these choices. By contrast, I appreciated the discussions around equations 7-10 and thought these were well justified.

Logistic functions are ideally suited for reconstructing latitudinal temperature gradients, as they are flexible, yet require few shape parameters. Despite this, logistic functions are not yet commonly used to model latitudinal temperature gradients for palaeoclimate reconstructions. We have added a justification to the methods (l...)

119 – Have the authors done any sensitivity testing of the model’s ecological constraints?

One immediate example: The minimum and maximum temperatures used to define the coral distributions (21 – 29.5 C) seem to be drawn from the mean values listed in Table 3 of Kleypas et al., 1999. However, the range of extreme values listed in that table (16 – 34.4 C) is considerably broader and could also be a reasonable choice. Does using the broader range noticeably change the results of the analysis? If so, this should be noted.

We do not agree that the extreme range (based on coldest and hottest weekly temperature) is a a good alternative to describe the modern coral habitat in terms of annual mean temperature. However, we have added a sensitivity test using the temperature range of 16 – 35.6 C for both the corals and the Avicennia-Rhizophoraceae mangrove assemblage to follow the suggestion of referee 1. The result is very similar to the original analysis (Fig. S ???).

211 - I would appreciate a slightly more detailed description of the gradients and how they were constructed.

216 – I’m concerned that this validation is neglecting the effects of bias and noise in the proxy data. The idealized gradients and limited spatial sampling are a great start, but the current setup seems to assume that proxy data is a perfect record of past temperature. In reality, this is not the case, and I would like to see the validation take this into account. Incorporating the effects of proxy seasonality and auto-regressive noise would be my two foremost concerns.

280 – I’m curious why the authors have limited the prior to the modern empirical gradient. Is there a reason for not using priors derived from Eocene climate model simulations?

We wanted to avoid letting a prior based on Eocene climate model simulations push the results in the direction of already existing, Eocene climate estimates. A prior broadly based on the modern is conservative in that it ensures that a latitudinal temperature gradient very different from the modern is inferred based mainly on the data, not on the prior – and this is what our results show.

301 – “the early Eocene data does not fit as well to the logistic latitudinal gradient model” - This begs the question of whether the logistic model is a reasonable choice here. Again, I'd suggest adding more context for the selection of this model.

This might be beyond the scope of the paper – but are there other models that might fit this data better?

Models with more parameters or non-parametric model such as GAMs or splines will be able to fit the data better. However, the high residual standard deviation (compared to the modern) is not a problem of the specific function we chose, but of the scattered data. These data are likely, as the referee has remarked earlier, influenced by seasonal and temporal signals, and possibly by miscalibrated proxies. We want to emphasise that our goal was not to achieve a perfect fit to the data, but to model the average latitudinal temperature gradient given the data and the constraints imposed by the priors and the logistic function.

Figure 3 – Please add R2, and sample size (N) to either the figure or the caption. Also, I suspect many of the grey dots are obscuring data points behind them. If this is the case, consider using a heatmap-style shading for the grey dots.

Minor notes:

83 - This sentence runs on a bit. Consider splitting.

93 - I think it would be best to reference Figure 1 in the text of this section.

121, 141, 149 – If I’m understanding this correctly, the standard deviations were selected specifically with the 97.5/95% distributions in mind. I'd suggest rewording slightly to clarify this point.

127 - Extra comma after “empirical”

138 - Remove “being”

139 - Is “ascribed” the right verb here? Perhaps “used” or “assumed” instead?

252 - Missing capitalization

254 - “where then” -> “were then”

256 - This paragraph changes tenses several times

260 - I believe this references Figure 4 before figures 2 and 3