Supplementary material for  
Bayesian multi-proxy reconstruction of the early Eocene latitudinal temperature gradient

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## Hierarchical, Bayesian model for latitudinal temperature gradients

**Model structure (Fig 2).** We model the mean temperature () at location as a function of absolute latitude () with a logistic regression (growth curve or Richard's curve) of the form:

where and denote the lower and upper asymptote, respectively, specifies the latitude of maximal growth, the most quickly with latitude, denotes the growth rate (Fig. 2), denotes the residual standard deviation, and denotes the number of locations.

We infer from individual temperature observations , derived from geochemical data, at location as

where is the number of observations at each location, and is the estimated standard deviation of the temperatures at location .

Similarly, is inferred for locations with ecological proxies from the associated normal temperature distributions with a given mean and standard deviation, and , as

This structure implies that is not fixed at the mean proxy temperature at location , , but is drawn towards the overall logistic regression curve, i.e. towards . The pull towards tends to be high when is low, when the observations are scattered, i.e.  is high, and/or when the overall standard deviation is low. In practice, this has the desirable consequence that locations with few observations and large temperature differences between observations have less influence on the overall regression than well-sampled locations with consistent temperature observations.

**Priors (Fig2x).** In the Bayesian framework, priors need to be placed on the unknown parameters of a model. We placed weakly informative, conjugate inverse-gamma priors on and :

We set , allowing these priors to be quickly overwhelmed by the data as and increase, as we have little *a priori* knowledge of these parameters.

In contrast, we put informative priors on the regression coefficients , , and , based on physical principles, and vaguely based on the modern climate system:

**A.** Predicted seawater surface temperatures are not allowed to be , the freezing point of sea water. The highest prior density of is placed around , and it slowly tapers off towards higher temperatures. This shape is achieved by placing a skew-normal prior on the lower asymptote, specified as

where , , and are the location, scale and shape parameters.

**K.** Input of solar energy decreases from the tropics to the poles. Hence, the latitudinal temperature gradient is broadly negative, i.e. temperature decreases with absolute latitude. This is achieved by setting . The prior on the upper asymptote is a truncated normal distribution with the mean set to of the modern SST gradient, with a broad standard deviation:

The distribution is truncated to the left at , but not truncated to the right ().

**M.** The steepness of the gradient is presumed to be highest in mid-latitudes; this is expressed with a normal prior on with the mean set to of the modern SST gradient:

**B.** The steepness or growth rate of the gradient is constrained to be and to not be exceedingly high, as oceanic and atmospheric heat transfer is bound to prevent very abrupt SST changes across latitudes. A gamma-distributed prior of the form

was placed on . The shape and rate parameters and were chosen such that the highest prior density is at of the modern SST gradient, .

## Convergence checks

To ensure the convergence and mixing of the chains, we inspected traceplots of all the unknown parameters of the model. [Add figures]

# References

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