ELEC9123: Design Task B (Wireless Communications)

Performance Analysis Verification via Monte Carlo Simulations

Term T2, 2025

1 Background

This **computer simulation**-based design project is aimed at understanding the various performance metrics in wireless communication channels experiencing fading and interference due to noise while incorporating the random locations of the users. Further, to verify these performance metrics, Monte Carlo simulations need to be performed for different system settings. You will use MATLAB or the mentioned computer simulations. Below, after briefly discussing the mentioned simulation settings, we discuss the adopted wireless channel models.

1.1 Monte Carlo Simulations

Monte Carlo simulation is a technique used to investigate how a model responds to randomly generated input and involves the following three key steps:

- Firstly, generate *N* samples (generally very high in the order of millions) at random.
- For each of these *N* inputs, run simulations based on a computerised model of the system being analysed.
- Assemble and evaluate the simulated outputs and compare them against the analytical or theoretical results.
 Some examples of assembling operations include the probability distribution, mean, maximum, or minimum of the output samples.

1.2 Wireless Channel Models

We have considered the following six wireless channel models involving:

- Additive White Gaussian Noise (AWGN) only,
- Laplace distributed Impulsive Noise only,
- · Rayleigh fading and AWGN,
- Rician fading and AWGN,
- Randomly deployed users over Rician Fading and AWGN channels.
- Randomly deployed users over Rayleigh Fading and AWGN channels.

1.3 Objectives

This task will help with simulating the random variable following different distributions, along with the implementation of various wireless communication models, including noise, fading, and propagation losses. Also, the transformation of random variables will be investigated along with the key concepts like bit error rate (BER) and outage probability. The simulation-based verification of these two metrics under varying communication models involves a very different methodology to be studied.

In the next section, we discuss the analytical results for the considered performance metrics for the binary phase-shift keying (BPSK) system.

2 Performance Analysis for BPSK System

We have considered the following two performance metrics:

- BER,
- · Outage probability,

Next, before presenting the analytics expressions for these two metrics under different channel models, we discuss some distribution expressions that these metrics will use.

2.1 Probability Distribution Models

As discussed here, we have six random distributions to consider one by one.

2.1.1 Complex Gaussian Noise

AWGN is modelled as a zero-mean circularly symmetric complex Gaussian with a finite variance that represents the underlying noise power as denoted by N_0 . Here, circularly symmetric implies that both real and imaginary terms have zero mean and equally share the variance between them.

2.1.2 Laplace Distribution for Impulsive Noise

Additive impulse noise can be modelled using a Laplacian probability density function (PDF) as defined below

$$f_n(x) = \frac{1}{\sqrt{2N_0}} e^{-\sqrt{\frac{2}{N_0}}|x|}, \quad -\infty \le x \le \infty$$
(1)

where N_0 is the variance of the noise, and its cumulative distribution function (CDF) is defined as follows

$$F_n(x) = \Pr(n \le x) = \begin{cases} \frac{1}{2} e^{\sqrt{\frac{2}{N_0}} x}, & x \le 0, \\ 1 - \frac{1}{2} e^{-\sqrt{\frac{2}{N_0}} x}, & x > 0. \end{cases}$$
 (2)

2.1.3 Rayleigh Fading

We can denote the wireless channel power gain a, including both the square of the magnitude of the fading coefficient, which is exponentially distributed owing to Rayleigh fading, and the path loss in terms of distance d=1m and path loss exponent α . So, the probability density function (PDF) $f_{a_r}(x)$ of a_r for unit distance separation between transmitter and receiver, i.e., d=1m, is given by

$$f_{a_r}(x) = \frac{1}{\gamma_b} e^{-\frac{1}{\gamma_b}x}, \quad x \ge 0.$$
 (3)

where signal-to-noise-ratio (SNR) $a_r \triangleq |h_s|^2 \frac{\mathbb{E}\{|x|^2\}}{N_0}$, with $\mathbb{E}\{a_r\} = \gamma_b$ denoting the mean of a_r . Here, with $|h_s|$ being Rayleigh distributed and $\mathbb{E}\{|x|^2\}$ denoting power of the transmitted signal x, a_r and $|h_s|^2$ both follow exponential distribution. Also, we assume that $\mathbb{E}\{|h_s|^2\} = 1$ and $\frac{\mathbb{E}\{|x|^2\}}{N_0} = \gamma_b$. Furthermore, h_s is a zero-mean circularly symmetric complex Gaussian distributed random variable with unit variance. More details on this fading and path loss can be found in [R1].

2.1.4 Rician Fading

For the scenarios with a strong line of sight (LoS) component as compared to the other scattered components, the amplitude of the channel gain coefficient is characterized by a Rician distribution and is described by a parameter called Rice factor K, which is the ratio between the power in the direct LoS path and the power in the other, scattered paths. We can define this underlying channel gain coefficient h whose magnitude or amplitude |h| follows Rician distribution as follows

$$h = \sqrt{\frac{K}{K+1}} \, h_d + \sqrt{\frac{1}{K+1}} \, h_s \tag{4}$$

where h_d represents a unit magnitude deterministic complex scalar containing the LoS and the specular components of the channel, and h_s denotes the scattered components of the channel, which is a zero-mean unit variance circular symmetric complex Gaussian random variable [R5, Section II]. Here again, we can denote the wireless channel power gain $|h|^2$ including both the square of the magnitude of the fading coefficient as, which is noncentral- χ^2 -squared distributed with two degrees of freedom owing to Rician fading and the path loss in terms of distance d=1m and path loss exponent α . So, the probability density function (PDF) $f_{a_R}(x)$ of the SNR a_R for unit distance separation between transmitter and receiver, i.e., d=1m, is given by

$$f_{a_R}(x) = \frac{(K+1)}{\gamma_b} e^{-\frac{1}{2} \left(\frac{2(K+1)x}{\gamma_b} + 2K\right)} I_0\left(\sqrt{\frac{4K(K+1)x}{\gamma_b}}\right), \quad x \ge 0.$$
 (5)

where $a_R \triangleq |h|^2 \frac{\mathbb{E}\{|x|^2\}}{N_0}$, with $\mathbb{E}\{a_R\} = \gamma_b$ denoting the mean of a_R . Here, with |h| being Rician distributed and $\mathbb{E}\{|x|^2\}$ denoting power of the transmitted signal x, a_R and $|h|^2$ both follow noncentral- χ^2 -squared distribution with two degrees of freedom. Also, we assume that $\mathbb{E}\{|h|^2\} = 1$ and $\frac{\mathbb{E}\{|x|^2\}}{N_0} = \gamma_b$. Lastly, it may be recalled that Rician fading for K = 0 reduces to Rayleigh fading.

2.1.5 Uniform Deployment of Users

Considering that the transmitter is placed at the center of a circular field of radius R meters (m) and the users (or receivers) are uniformly deployed inside it, the PDF $f_d(x)$ of the distance d between this center-place transmitter and a randomly deployed user is given by

$$f_d(x) = \frac{2x}{R^2}, \quad 0 \le x \le R. \tag{6}$$

More details on this distribution can be found in [R2].

2.1.6 Combined Effect of Fading and Random Deployment

Noting that $\frac{a_r}{d^{\alpha}}$ represents the overall channel power gain, we first use the following PDF of $d^{-\alpha}$ to eventually obtain the desired PDF $f_{\frac{a_r}{d^{\alpha}}}(x)$ for randomly deployed users over Rayleigh fading as mentioned after it.

$$f_{d^{-\alpha}}(x) = \frac{2x^{-\frac{1}{\alpha}}}{R^2} \left| \frac{\partial x^{-\frac{1}{\alpha}}}{\partial x} \right| = \frac{2x^{-\frac{\alpha+2}{\alpha}}}{\alpha R^2}, \quad R^{-\alpha} \le x \le \infty.$$
 (7)

$$f_{\frac{a_r}{d^{\alpha}}}(x) = \int_{R^{-\alpha}}^{\infty} \frac{1}{y} f_{d^{-\alpha}}(y) f_{a_r}\left(\frac{x}{y}\right) dy$$

$$= \frac{2 \left(\gamma_b\right)^{\frac{2}{\alpha}}}{\alpha R^2 r^{\frac{\alpha+2}{\alpha}}} \left(\Gamma\left(\frac{\alpha+2}{\alpha}\right) - \Gamma\left(\frac{\alpha+2}{\alpha}, \frac{R^{\alpha}x}{\gamma_b}\right)\right), \quad x \ge 0.$$
(8)

where $\Gamma(s,x) = \int_x^\infty t^{s-1} \mathrm{e}^{-t} \mathrm{d}t$ is upper incomplete gamma function with $\Gamma(s) = \Gamma(s,0)$ being ordinary gamma function.

Likewise, the PDF of the overall channel power gain for randomly deployed users over the Rayleigh fading scenario is as follows:

$$f_{\frac{a_R}{d^{\alpha}}}(x) = \int_{R^{-\alpha}}^{\infty} \frac{1}{y} f_{d^{-\alpha}}(y) f_{a_R}\left(\frac{x}{y}\right) dy.$$

$$\tag{9}$$

2.2 Bit Error Rate (BER)

BER for BPSK transmission is studied for all three wireless channel models, and the underlying expressions are reproduced below. For more details on the derivation, refer to [R3].

2.2.1 AWGN only

$$P_{b_A} = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\gamma_b}\right). \tag{10}$$

2.2.2 Laplacian Impulse Noise only

$$P_{b_L} = \frac{1}{2} e^{-\sqrt{2\gamma_b}}. (11)$$

2.2.3 Rayleigh fading and AWGN

$$P_{b_{AF_r}} = \int_0^\infty \frac{1}{2} \operatorname{erfc}\left(\sqrt{x}\right) f_{a_r}(x) dx$$

$$= \frac{1}{2} \left(1 - \sqrt{\frac{\gamma_b}{1 + \gamma_b}}\right). \tag{12}$$

2.2.4 Rician fading and AWGN

$$P_{b_{AF_R}} = \int_0^\infty \frac{1}{2} \operatorname{erfc}\left(\sqrt{x}\right) f_{a_R}(x) dx. \tag{13}$$

2.2.5 Randomly deployed users over Rayleigh fading and AWGN channels

$$P_{b_{AF_rR}} = \int_0^\infty \frac{1}{2} \operatorname{erfc}\left(\sqrt{x}\right) f_{\frac{a_r}{d^\alpha}}(x) dx. \tag{14}$$

2.2.6 Randomly deployed users over Rician fading and AWGN channels

$$P_{b_{AF_{R}R}} = \int_{0}^{\infty} \frac{1}{2} \operatorname{erfc}\left(\sqrt{x}\right) f_{\frac{a_{R}}{d^{R}}}(x) dx. \tag{15}$$

2.3 Outage Probability

It represents that the probability of the underlying rate is less than an acceptable threshold of \overline{C} bps/Hz. For more details, refer to [R4].

2.3.1 Rayleigh fading

$$P_{out}^{F_r} = \Pr\left\{\log_2\left(1 + \gamma_b |h_s|^2\right) \le \overline{C}\right\} = \Pr\left\{|h_s|^2 \le \frac{2^{\overline{C}} - 1}{\gamma_b}\right\} = 1 - e^{-\frac{2^{\overline{C}} - 1}{\gamma_b}}.$$
 (16)

Here, we assumed d = 1 m for simplicity.

2.3.2 Rician fading

$$P_{out}^{F_R} = \Pr\left\{\log_2\left(1 + \gamma_b|h|^2\right) \le \overline{C}\right\} = \Pr\left\{|h|^2 \le \frac{2^{\overline{C}} - 1}{\gamma_b}\right\}$$

$$= 1 - Q_1\left(\sqrt{2K}, \sqrt{2(K+1)\frac{2^{\overline{C}} - 1}{\gamma_b}}\right). \tag{17}$$

Here, $Q_1(\cdot, \cdot)$ is Marcum Q-function and we assumed d = 1 m for simplicity.

2.3.3 Randomly deployed users over Rayleigh fading channels

$$P_{out}^{F_rR} = \Pr\left\{\log_2\left(1 + \gamma_b \frac{|h_s|^2}{d^\alpha}\right) \le \overline{C}\right\} = \Pr\left\{\frac{a_r}{d^\alpha} \le 2^{\overline{C}} - 1\right\}$$
$$= \int_0^{2^{\overline{C}} - 1} f_{\frac{a_r}{d^\alpha}}(x) \, dx. \tag{18}$$

2.3.4 Randomly deployed users over Rician fading channels

$$P_{out}^{F_R R} = \Pr\left\{\log_2\left(1 + \gamma_b \frac{|h|^2}{d^{\alpha}}\right) \le \overline{C}\right\} = \Pr\left\{\frac{a_R}{d^{\alpha}} \le 2^{\overline{C}} - 1\right\}$$
$$= \int_0^{2^{\overline{C}} - 1} f_{\frac{a_R}{d^{\alpha}}}(x) \, dx. \tag{19}$$

3 Design Task Requirements

Considering the default system parameters as $\overline{C}=1.2$ bps/Hz, γ_b varying between 0 dB and 15dB in the steps of 1 dB, R=3 m, $\alpha=2.2$, $h_d=1$, K=5, $N_0=1$, and $\mathbb{E}\left\{|h_s|^2\right\}=1$, perform the following five tasks.

- 1. Generate the following six simulation environments
 - Laplacian Noise channels
 - AWGN channels
 - Rayleigh fading and AWGN channels
 - Rician fading and AWGN channels
 - Randomly deployed users over Rayleigh fading and AWGN channels
 - Randomly deployed users over Rician fading and AWGN channels
- 2. Verify the generated Laplacian random variables by comparing the underlying simulated PDF and CDF with the corresponding analytical ones

- 3. Obtain the simulated results for the following performance metrics:
 - BER in all six simulated environments
 - Outage probability over the four simulated environments as grouped under the following two categories
 - Rayleigh fading channels with both unit distance (known, d = 1) and randomly deployed users in a circular region of radius R,
 - Rician fading channels with both unit distance (known, d=1) and randomly deployed users in a circular region of radius R.
- 4. Compare the Simulated Results with Analytical Solutions. Make sure that the estimates from the simulation function are close to the exact analytical values. Further, the percentage deviation of the simulated results from the ones obtained using the closed-form expressions is obtained. Also, check if this percentage error or gap can be reduced by increasing the number of random samples generated.
- 5. Generate the following three plots:
 - BER (P_b) vs SNR (γ_b) for environments affected only by noise, containing four curves: analytical and simulated results for BER in wireless channel settings impacted by AWGN and Laplacian noise
 - BER (P_b) vs SNR (γ_b) containing eight curves: analytical and simulated results for BER in four wireless channel settings
 - Rayleigh fading and AWGN,
 - Rician fading and AWGN,
 - Randomly deployed users over Rician Fading and AWGN channels.
 - Randomly deployed users over Rayleigh Fading and AWGN channels
 - Outage probability (P_{out}) vs SNR (γ_b) containing eight curves: analytical and simulated results in four wireless channel settings
 - Rayleigh fading
 - Rician fading
 - Randomly deployed users over Rician Fading channels.
 - Randomly deployed users over Rayleigh Fading channels

4 TLT Assessment Structure

To pass the course, you need to solve this task according to the **Tiered Learning taxonomy (TLT)** guidelines and present it to an instructor/demo at one of the lab exercises that offer an examination. If you choose to do designs above Level 2, you should follow them in a sequential order. For instance, you should finish Level 2 before trying Level 3 and so on. The highest score you can get is based on the level you choose. Explicitly following the TLT, this task is designed to provide an increasing complexity from Pass (Level 2) to High Distinction (Level 5) levels, as shown below:

- Level 2 (Pass): If you correctly implement the first two simulation environments (Laplacian Noise and AWGN channels) in terms of the underlying BER simulation and analyses, along with verifying the generated Laplacian random variables by comparing the simulated PDF and CDF with the corresponding analytical ones, the maximum mark you can achieve is 64.
- Level 3 (Credit): If you complete the correct implementation of Level 2 along with that of Rayleigh fading and AWGN channels and conduct the verification of outage probability and BER analyses, the maximum mark you can achieve is 74.
- Level 4 (Distinction): If you complete the correct implementation of Level 3 and randomly deploy users over Rayleigh fading and AWGN channels, along with the verification of the underlying outage probability and BER analyses, the maximum mark you can achieve is 84.
- Level 5 (High Distinction): If you complete the correct implementation of Level 4 and randomly deploy users over the Rician fading and AWGN channels, along with the verification of the underlying outage probability and BER analyses, the maximum mark you can achieve is 100.

5 Useful Facts

- Random number for any distribution can be generated using the rand function in MATLAB for generating the uniform random variable between 0 and 1. The trick is based on the fact that the cumulative distribution function (CDF) also lies in the range between 0 and 1. The formula for generating the random variable x having the CDF F is given by $x = F^{-1}$ (rand).
- For example with CDF $F(x) = 1 e^{-\lambda x}$ of exponentially distributed random variable having mean $\frac{1}{\lambda}$, the formula for generating the exponentially distributed random variable x is given by $x = -\frac{1}{\lambda} \log (\text{rand})$.
- The number of random samples to be generated for simulation should be sufficiently large to ensure that at least about 10⁶ are considered for obtaining the simulated results, which are in a close match to those obtained analytically. The improvement in accuracy can be observed by increasing the number of random samples to 10⁷.

6 Reference Materials

Though most of the required details that should be sufficient for completing this project are mentioned in this document itself, for further details, interested students are referred to read the following:

[R1] A. Goldsmith, Chapters 2 and 3 of Wireless Communications. Cambridge University Press. 2005.

[R2] Eric W. Weisstein, *Disk Point Picking*. From MathWorld-A Wolfram Web Resource. Weblink: https://mathworld.wolfram.com/DiskPointPicking.html

[R3] V. Meghdad, Lecture Notes on BER calculation. Weblink: https://www.unilim.fr/pages_perso/vahid/notes/ber_awgn.pdf

[R4] I-H. Wang Slides 39 and 40 of Lecture 4 in Capacity of Wireless Channels. Weblink: https://unsw-my.sharepoint.com/:b:/g/personal/z3528746_ad_unsw_edu_au/ESsYyxLfUAJGn-whZKc7V7YBeyO8_6YLswqDXxEVHSQFfg?e=iGO0xk

[R5] S. Kashyap, E. Björnson and E. G. Larsson, "On the Feasibility of Wireless Energy Transfer Using Massive Antenna Arrays," *IEEE Transactions on Wireless Communications*, vol. 15, no. 5, pp. 3466-3480, May 2016

7 Attendance Requirements

- Student attendance in each laboratory session (Wednesday and Friday) will be recorded.
- In order to receive a grade for this design task, it is mandatory to attend all required lectures/labs, including
 the lab assessments.
- To ensure consistency and fairness among different simulation and hardware-focused students, all the labbased questions will be answered or addressed only during the two three-hour weekly lab sessions.

8 Design Journal Submission

- It is mandatory to maintain a journal or report describing your design process, implementation code, and results, which must be documented via an MS Word or .pdf file.
- The design journal must include a working and well-commented MATLAB file that meets the task requirements. Submit this file via the **Moodle Design Task X submission link**, where X is 1, 2, or 3 depending on whether you chose this task as your first, second, or third design task.
- Please ensure that the MATLAB simulation file is formatted according to the requirements listed in Section 3.
- Your journal should already have all the subparts required to showcase the necessary functionality, along with detailed comments explaining all the steps. You may be penalised if significant additions or changes are required to show functionality or demonstration.
- The simulation file must be saved as a '.m' file (MATLAB file). The name of this file must be: zID_LastName_DTB_2025.m. Here if any student has written their code in multiple MALTAB files, then they can compress all the .m together in a single zip folder while exactly mentioning which file to run and what does each .m file is doing by adding a README text file explaining the role of each .m file. The name of this zip folder, in this case, must be: zID_LastName_DTB_2025.zip

- Your simulation file should already have all the subparts required to showcase the necessary functionality, along with detailed comments explaining all the steps. You may be penalised if significant additions or changes are required to show functionality as per the mentioned requirements.
- The brief portable document format (.pdf) report summarising the outcomes of this Design task and your steps in obtaining them must be saved as a '.pdf' file (readable text file). The name of this file must be: zID_LastName_DTB_2025.pdf

9 Design Task Schedule and Assessment Timeline

- Each design task must be completed over a span of six lab sessions. Term 2 is structured to accommodate three design tasks to be selected by each student out of the 7 available options, with the following schedule:
 - Task 1: Week 1 Wednesday lab session to Week 3 Friday lab session,
 - Task 2: Week 4 Friday lab session to Week 7 Wednesday lab session,
 - Task 3: Week 8 Wednesday lab session to Week 10 Friday lab session.
- Each task has a specific assessment session and submission deadline:
 - If this task is completed as Task 1:
 - * Assessment: Wednesday lab session of Week 4,
 - * Deadline: 11:59 PM, Tuesday, June 24, 2025.
 - If this task is completed as Task 2:
 - * Assessment: Friday lab session of Week 7,
 - * Deadline: 11:59 PM, Thursday, July 17, 2025.
 - If this task is completed as Task 3:
 - * Assessment: Wednesday lab session of Week 11,
 - * Deadline: 11:59 PM, Tuesday, August 12, 2025.

10 Assessment Criteria and Marking Rubric

• The assessment breakdown for each design task, showing different components involved, is shown in Table 1.

Table 1: The assessment breakdown for Design Task B

Deliverable	Percentage of Assignment Grade
Demonstration of Correct Working Design	20%
Explanation of the Implementation and Results	40%
Ability to Answer Questions	20%
Design Journal or Notebook	20%

- The Correct Working Design component, worth 20%, requires that **all objectives are accurately met** according to the selected level, as described in Section 4.
- The Explanation of Implementation and Results is evaluated based on the following four aspects:
 - Clarity and explanation of plots how well the plots are presented and interpreted.
 - Code efficiency and commenting the quality, readability, and documentation of the code.
 - Working code and its explanation whether the code runs correctly and is clearly explained.
 - Explanation of Results how well the outcomes are analysed and related to the design objectives.
- The "Ability to Answer Questions" component is based on the quality of responses to the **two to four questions** based on the design task.
- The Design Journal component is evaluated based on four equally weighted areas. First, students must provide a brief problem description that clearly outlines the design task. Second, they should present their understanding and logic, explaining their approach and reasoning in a structured and coherent manner. Third, the journal should include a demonstration of results, showcasing the outcomes of their design work with appropriate interpretation. Finally, students must include project management details, such as weekly progress updates, a declaration of any AI tools used, and proper referencing of all sources.

- Overall, during the assessment, students will need to demonstrate the functionality of the MATLAB code, explain
 their design journal and design to the lab demonstrator, and answer any questions they may have about their
 design.
- All design tasks in this course must be completed **individually**. Copying from others is not permitted and may result in a failing grade, reinforcing the importance of academic integrity.
- Please note that design tasks must be submitted by the set deadline, and extensions will not be granted, except in cases of approved special consideration, which may allow students to redo the task during Weeks 11 and 12.
- Passing Requirement and Hurdle: To pass this course, you must:
 - Achieve an overall course mark of 50% or higher, where this overall mark is calculated as follows:
 - * 30% from each of the three design tasks (totaling 90%),
 - * 10% from the lab exam, which will be conducted during the Friday lab session of Week 11, and
 - Pass at least two out of your three design tasks. This means you can fail only one design task.
- **Redo Opportunity**: If you fail a design task, you will have a chance to redo it during Weeks 11 and 12.
 - Lab sessions for redoing tasks will run from 9:00 AM to 3:00 PM on:
 - * 6-hour lab session on Wednesday of Week 11
 - * 6-hour lab sessions on Wednesday and Friday of Week 12
 - If your resubmitted task meets the requirements for a satisfactory grade, you will receive a mark of 50% for that task.
 - The grading and assessment will be conducted on the Friday lab session of Week 12 after the student submits their task during the same session.
 - Please be aware that you are allowed to **redo ONLY one design task**.
- Supplementary Lab Sessions: The above-mentioned three 6-hour lab sessions in Weeks 11 and 12 can also be used as make-up sessions for students who missed earlier labs due to approved special consideration. In such cases, students will be assessed normally and will receive their actual earned mark, rather than the fixed 50% awarded for task redos. Please be aware that students are strictly permitted to complete only one design task during the supplementary lab sessions in Weeks 11 and 12. No exceptions will be made beyond this single opportunity.
- If needed, students may be given access to open lab sessions to work outside regular lab hours. However, lecturer and demonstrator support will only be available during the two scheduled weekly lab sessions.
- **Formal feedback** will be provided well within two weeks of the relevant submission date through Moodle, followed by a brief discussion during the lab session.
- Note: Use of AI tools (such as ChatGPT or Copilot) for your learning in this course is allowed, but you are responsible for everything you produce designs, reports, graphs, etc. (keeping in mind that the output from AI tools can be incorrect). Any use of AI tools must be declared.