

Final Paper

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STAT 632: Linear and Logistic Regression

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Which characteristics of Electric Vehicles have a significant impact on increasing their price? Nowadays, we see Electric Vehicles on the car market more often and it is getting our attention. However, we once all wonder why the electric cars are so costly and what is the reasonable price for us to spend on one. Our curiosity led us to ask which characteristics of electric vehicles have a significant impact on their price.

The dataset we used was the “Cheapest Electric Cars” from Kaggle user KOUSTUBHK. This user scraped data from <https://ev-database.org/> in August 2021. The dataset contains 180 rows and 11 columns. Some of the columns were stored as strings, but clearly contained numeric substrings that were useful predictors. In some rows, the string “-” was used to denote a null value. These values were converted to NA during the data cleaning process.

Since we wanted to measure price, the variable **PriceinUK** was chosen to be the response variable. There data also contained **PriceinGermany**, which was another form of the desired response. Since we only needed to use one response variable, **PriceinGermany** would be considered an alternate response while **PriceinUK** would be the main response. This left the columns **Name**, **Acceleration**, **TopSpeed**, **Range**, **Efficiency**, **FastChargeSpeed**, **Drive**, and **Numberofseats** as most of the predictors.

There is still one remaining predictor. Originally, the data contained the column **Subtitle**. This column contained information about the type of vehicle as well as its battery capacity in kilowatt hours (kWh). Since all vehicles were of the same type, only the battery capacity component was useful. This numeric variable was stripped and renamed to **battery_capacity**. After this cleaning was done, the following figure was produced.

It is important to note that out of the 180 rows in the dataset, only 124 of them were complete. This is a problem because many of the missing values were for the response variables. As such, many of the missing columns had to be dropped from the dataset.

In order to select the model, we began with a full — and untransformed — additive model. The purpose of this were to recognize which variables introduced a lot of multicollinearity. From this, the scatterplot matrix, and the correlation matrix, we were able to recognize two pairs of variables that were highly correlated: **battery_capacity** and **Range**, **Acceleration** and **TopSpeed**.

We ended up dropping the variables **battery_capacity** and **Acceleration**. This was because these predictors were less accurate in later parts of the process than **Range** and **TopSpeed** respectively.

After this, there were still possible transformations needed for the predictors. Initial predictions were found by looking at marginal plots between the predictors and the response. Other transformations of the predictors would be tested and the more accurate predictions would be kept.

There were also possible transformations needed for the response variable. This was done by using a Box-Cox Power Transformation. In this case, we got $\lambda \approx -0.5$, which corresponds to an inverse square root transformation.

After this, there were still some insignificant predictors remaining. In order to choose the significant ones, stepwise selection — with BIC as the metric — was utilized. This resulted in the final model:

$$\begin{aligned} \frac{1}{\sqrt{\text{PriceinUK}}} = & \beta_0 \\ & + \beta_1 \ln \text{Range} \\ & + \beta_2 \text{TopSpeed} \\ & + \beta_3 \text{TopSpeed}^2 \\ & + \beta_4 \text{Efficiency} \\ & + \beta_5 \text{Efficiency}^2 \\ & + \epsilon \end{aligned}$$

A summary of the table can be seen below:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	0.01915	0.00154	12.43	3.386e-23
log(Range)	-0.0005637	0.0001617	-3.487	0.0006865

	Estimate	Std. Error	t value	Pr(> t)
poly(TopSpeed, 2, raw = T)1	-3.424e-05	4.669e-06	-7.334	3.07e-11
poly(TopSpeed, 2, raw = T)2	4.427e-08	9.647e-09	4.589	1.12e-05
poly(Efficiency, 2, raw = T)1	-5.274e-05	1.392e-05	-3.789	0.0002394
poly(Efficiency, 2, raw = T)2	9.774e-08	3.382e-08	2.89	0.00459

Table 2: Final Model Summary

Observations	Residual Std. Error	R^2	Adjusted R^2
124	0.0003877	0.8588	0.8528

From Table 2, we can see that although the magnitude of the coefficients are small, they are significant. This makes sense because the model is linear on an inverse square root scale.

From Figure 2, it appears that the constant variance requirement is mostly satisfied. However, when a studentized Breusch-Pagan test was used, the resulting p -value was $1.086 \cdot 10^{-4}$. This is highly significant and indicates heteroscedasticity in the model's residuals.

The next assumption that was checked was normality of residuals. From Figure 2, it appears that normality of the residuals is satisfied. In order to check, a Shapiro-Wilk normality test was used. This yielded test statistic $W = 0.97628$ and p -value 0.02778. The p -value is significant for $\alpha = 0.05$, but not for $\alpha = 0.01$. Since the test statistic is also larger than 0.95, the normality condition seems satisfied.

The model provided the following interpretations. With all other predictors in the model held fixed, the interpretation of the estimated **Range** is that a unit increase in **log(Range)** is associated with an decrease in $1/\sqrt{\text{PriceinUK}}$ by 0.0005637. With **Range** and **Efficiency** fixed, an increase in **TopSpeed** is associated with quadratic growth in $1/\sqrt{\text{PriceinUK}}$. With **Range** and **TopSpeed** fixed, an increase in **Efficiency** is associated with quadratic growth in $1/\sqrt{\text{PriceinUK}}$. It is important to note that the quadratic growth in the previous two interpretations is due to the positive sign of the coefficients for the square terms in the model.

We were also curious about the effects of outliers on our model. For our purposes, outliers were defined as points having an absolute leverage higher than twice the mean, and having an absolute standardized residual higher than 2. These outliers were identified in Table 3.

Table 3: Outliers

Name	PriceinUK
Tesla Roadster	189000
Mercedes EQV 300 Long	70665
Tesla Cybertruck Dual Motor	48000
Tesla Cybertruck Tri Motor	68000

Now that the outliers are identified, the model will be refit without them to see the effect.

From the diagnostic plots in Figure 3, the results appear similar. It appears that the constant variance requirement is mostly satisfied. However, when a studentized Breusch-Pagan test was used, the resulting p -value was $1.373 \cdot 10^{-4}$. This is slightly higher than in the original model, but is still highly significant and indicates heteroscedasticity in the model's residuals.

From looking at the plot in Figure 3, it appears that normality of the residuals is satisfied. In order to check, a Shapiro-Wilk normality test was used. This yielded test statistic $W = 0.97688$ and p -value 0.0328. This is slightly higher than in the original model and appears normal for similar reasons.

The plots diagnostic plots and hypothesis tests for the model with and without the outliers appear very similar. As such, it appears that removing outliers and refitting the model does not yield a significant effect.

We started the full model with 8 predictors and after applying transformations on the model, only 3 predictors(**Range**, **TopSpeed**, **Efficiency**) are significant from the dataset. This model predicts the prices

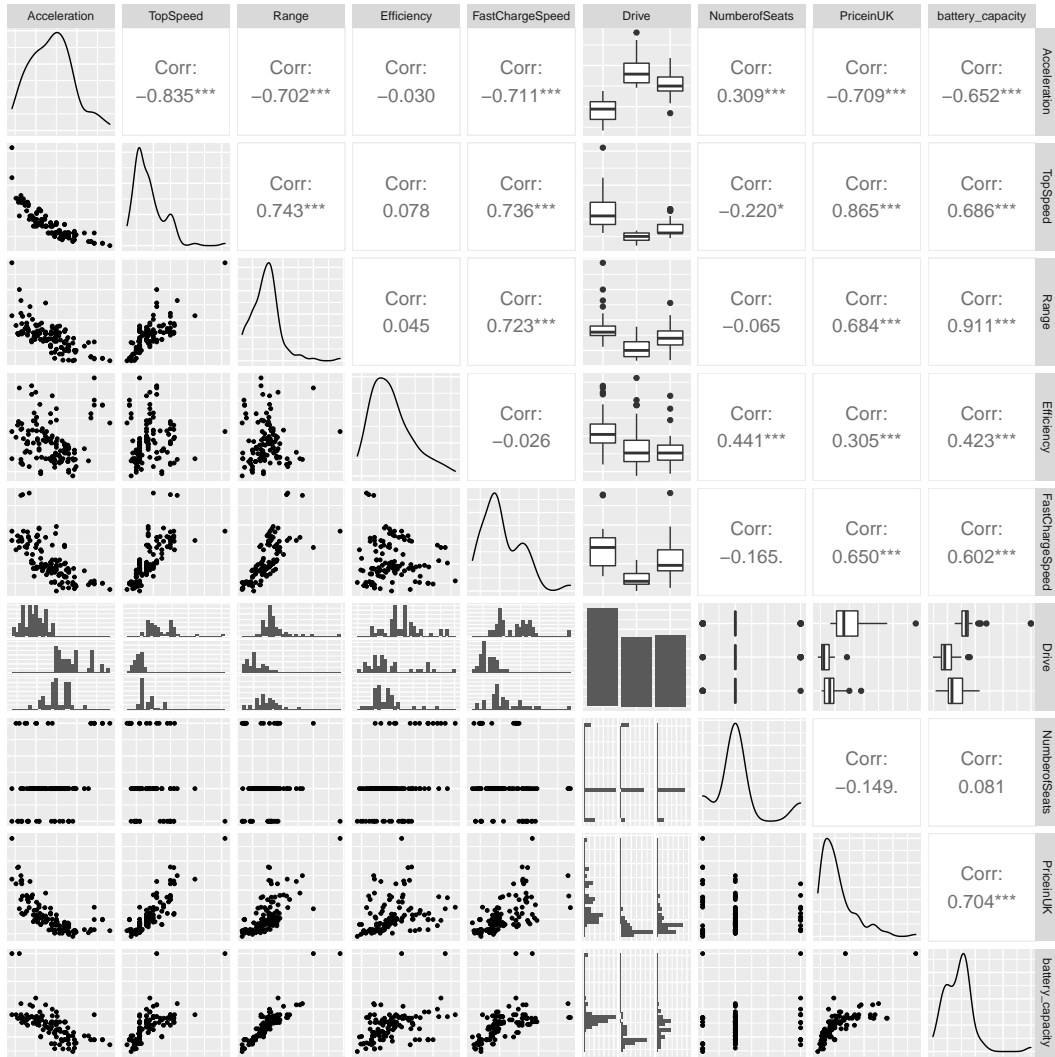


Figure 1: Correlation/Scatterplot/Density Matrix

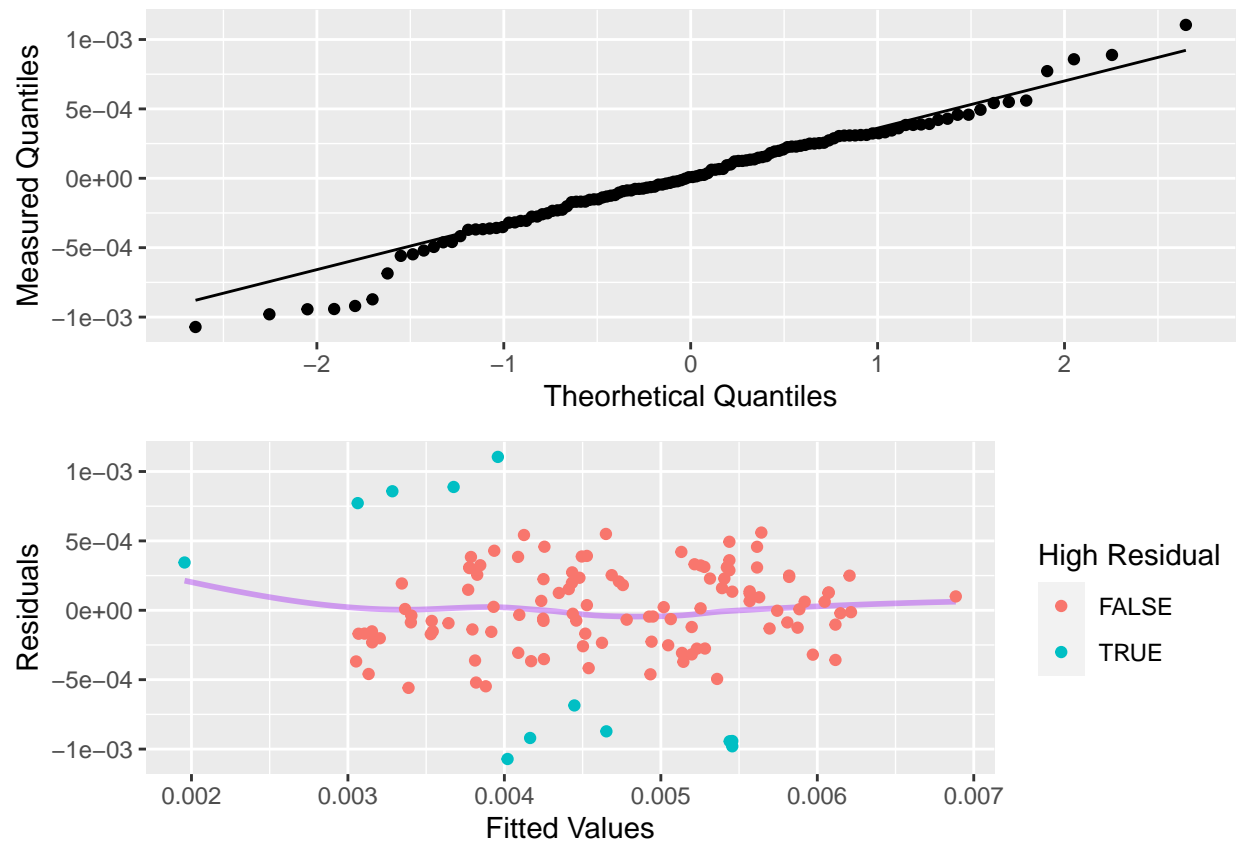


Figure 2: Plots for Regression Assumptions

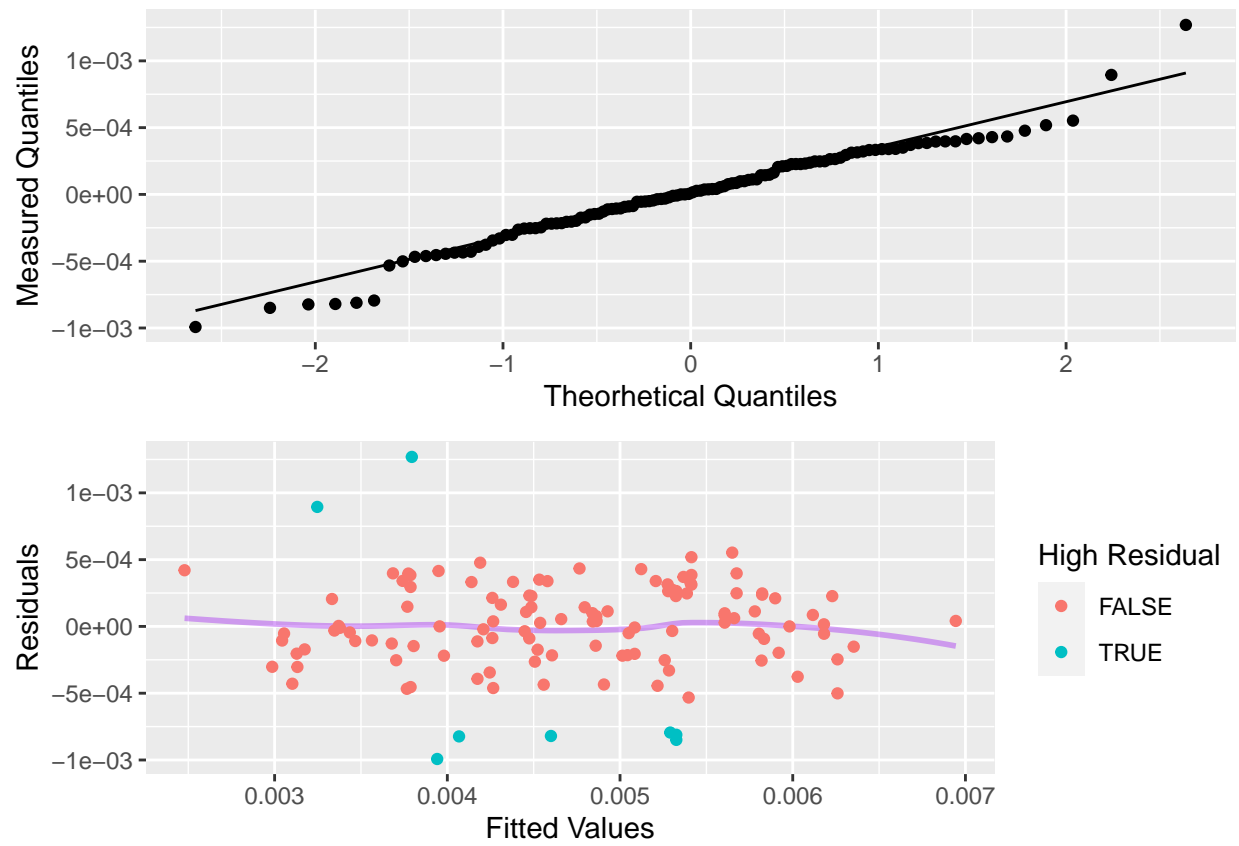


Figure 3: Refitted Diagnostic Plots without Outliers

of electric vehicles with slight margin of errors with better normality, constant-variance and r-squared over the full and reduced models. Our model also reduced the multicollinearity significantly. Since the model doesn't account for only high priced cars, it may not be a good fit for cars which are high in price. This is because, the margin of error increases with an increase in price of the car due to other multiple factors. For future study: an increase in number of observations and predictors may lead us to achieve better models for prediction.

Code Appendix

```
# sets up default settings for code chunks
knitr::opts_chunk$set(
  echo = F,
  message = F,
  warning = F
)

# loads necessary libraries
library(tidyverse)
library(GGally)

# load cleaned dataset and rename subtitle column
full_data <- read_csv("Dataset/cleaned_data.csv") %>%
  mutate(battery_capacity = Subtitle, .keep = "unused")

# drop incomplete observations
data <- drop_na(full_data)

# Plots figure 1
select(data, -Name, -PriceinGermany) %>%
  ggpairs(

    # change font size of correlation stuff
    upper = list(continuous = wrap("cor", size = 5)),

    # changes point size
    lower = list(continuous = wrap("points", size = 1)),

    # removes cluttered axis ticks
    axisLabels = "none"
  )

# stepwise selection w/ BIC
final_model <- lm(
  1 / sqrt(PriceinUK) ~
    log(Range) +
    poly(TopSpeed, 2, raw = T) +
    poly(Efficiency, 2, raw = T) +
    FastChargeSpeed +
    NumberofSeats +
    Drive,
  data
) %>%
  step(trace = 0, k = nrow(data) %>% log)

# prints model summary in table format
summary(final_model) %>%
  pandrer::pander(caption = "Final Model Summary")

# indices of high residual points
high_res <- rstandard(final_model) %>%
```



```

abs() > 2

normplot <- ggplot(mapping = aes(sample = final_model$residuals)) +
  geom_qq() +
  geom_qq_line() +
  labs(x = "Theoretical Quantiles", y = "Measured Quantiles")

cvarplot <- ggplot(mapping = aes(final_model$fitted.values, final_model$residuals)) +
  geom_line(stat = "smooth", method = "loess", alpha = 0.4, color = "purple", size = 1) +
  geom_point(aes(color = high_res)) +
  labs(x = "Fitted Values", y = "Residuals", color = "High Residual")

gridExtra::grid.arrange(normplot, cvarplot)

# shapiro.test(final_model$residuals)
# lmtest::bptest(final_model)

# get leverage from model
fm_lev <- hatvalues(final_model)
high_lev <- abs(fm_lev) > 2 * mean(fm_lev)

# print desired observations
filter(
  data,
  high_res,
  high_lev
) %>%
  select(Name, PriceinUK) %>%
  knitr::kable(tabel.envir = "figure", caption = "Outliers")

# subset of data without outliers
no_outlier_data <- filter(data, !(high_res & high_lev))

# refit model
no_outlier_model <- lm(
  1 / sqrt(PriceinUK) ~
    log(Range) +
    poly(TopSpeed, 2, raw = T) +
    poly(Efficiency, 2, raw = T),
  no_outlier_data
)

# new high residual points
high_res2 <- rstandard(no_outlier_model) %>%
  abs() > 2

normplot2 <- ggplot(mapping = aes(sample = no_outlier_model$residuals)) +
  geom_qq() +
  geom_qq_line() +
  labs(x = "Theoretical Quantiles", y = "Measured Quantiles")

cvarplot2 <- ggplot(
  mapping = aes(no_outlier_model$fitted.values, no_outlier_model$residuals)

```

```
) +  
  geom_line(stat = "smooth", method = "loess", alpha = 0.4, color = "purple", size = 1) +  
  geom_point(aes(color = high_res2)) +  
  labs(x = "Fitted Values", y = "Residuals", color = "High Residual")  
  
gridExtra::grid.arrange(normplot2, cvarplot2)  
  
# shapiro.test(no_outlier_model$residuals)  
# lmtest::bptest(no_outlier_model)
```