pdf_draft

Kaleb Cervantes

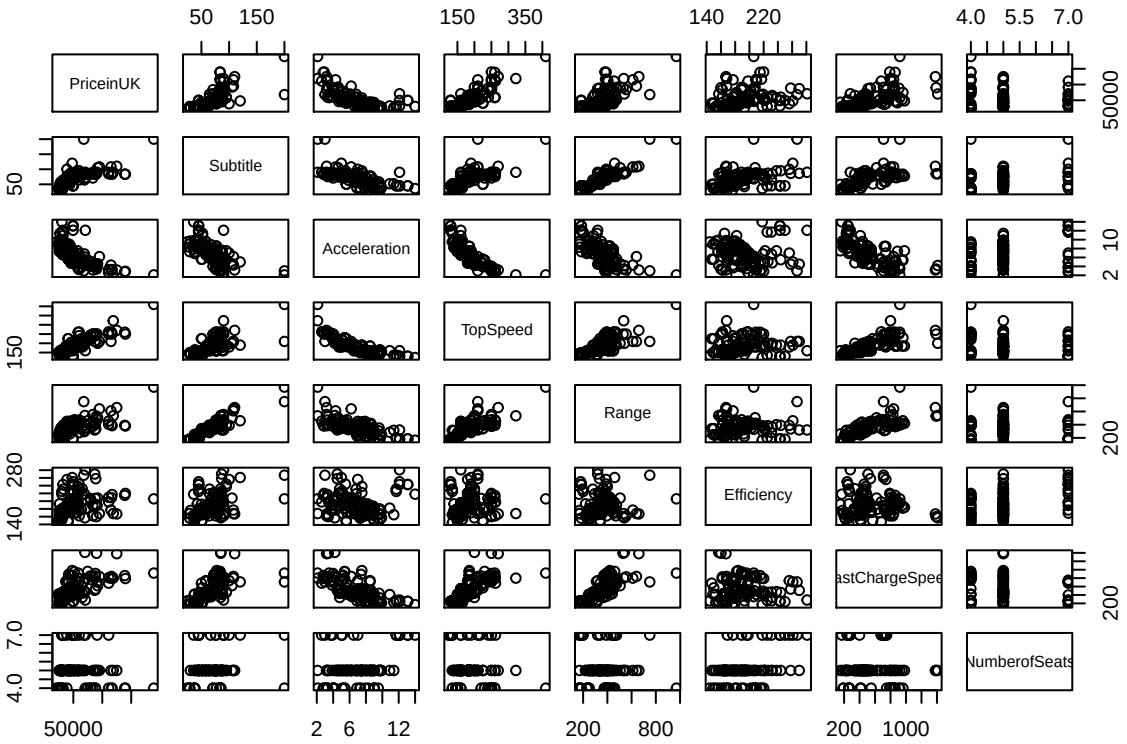
4/20/2022

Model Selection

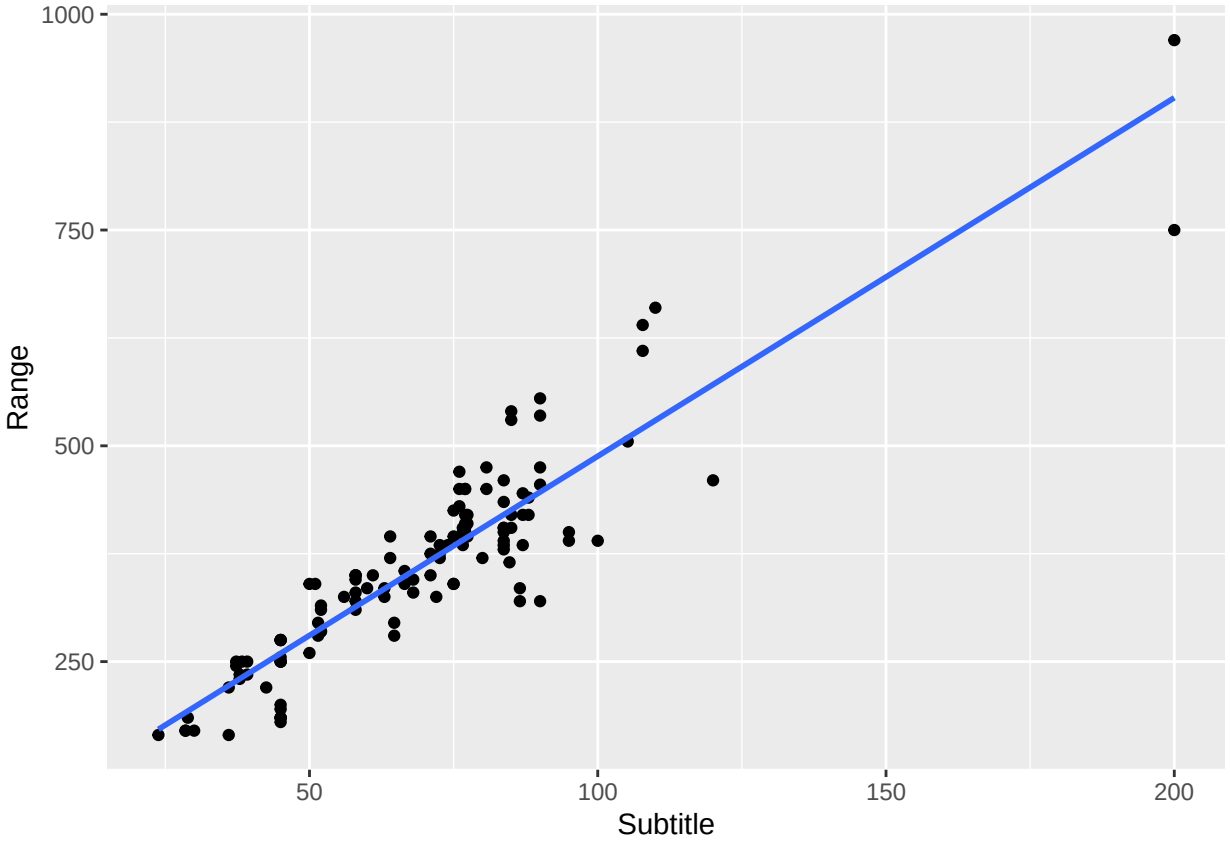
Variable Selection

The first thing was to check the correlations between some of the variables and visualize the data.

```
##                    Subtitle Acceleration    TopSpeed       Range  Efficiency
## Subtitle         1.00000000  -0.65205890  0.68551542  0.91104205  0.42266885
## Acceleration    -0.65205890   1.00000000 -0.83536291 -0.70220767 -0.03009386
## TopSpeed         0.68551542  -0.83536291  1.00000000  0.74349641  0.07810845
## Range            0.91104205  -0.70220767  0.74349641  1.00000000  0.04496225
## Efficiency       0.42266885  -0.03009386  0.07810845  0.04496225  1.00000000
## FastChargeSpeed  0.60230194  -0.71134894  0.73572843  0.72271112 -0.02642250
## NumberofSeats    0.08103089   0.30861901 -0.21958021 -0.06531154  0.44090323
## PriceinUK        0.70390802  -0.70863967  0.86502374  0.68439993  0.30531728
##                  FastChargeSpeed NumberofSeats  PriceinUK
## Subtitle               0.6023019    0.08103089  0.7039080
## Acceleration          -0.7113489    0.30861901 -0.7086397
## TopSpeed               0.7357284   -0.21958021  0.8650237
## Range                  0.7227111   -0.06531154  0.6843999
## Efficiency            -0.0264225    0.44090323  0.3053173
## FastChargeSpeed        1.0000000   -0.16516351  0.6495690
## NumberofSeats         -0.1651635    1.00000000 -0.1491998
## PriceinUK              0.6495690   -0.14919979  1.0000000
```



The most highly correlated variables are `Subtitle` and `Range`. These have a correlation of 0.91104205 and seem to have similar plots in the scatter plot matrix. A zoomed in scatter plot is included below.



When zoomed in, the two variables seem to have a linear relationship with each other. This indicates that

one of these may be dropped. `Subtitle` and `Range` have respective correlations 0.7039080 and 0.6843999 with the response. Since `Subtitle` has a stronger correlation with the response, that will be the predictor that is kept.

Transformations

In the first row of the scatter plot matrix, `TopSpeed` and `Acceleration` appear to form a parabola when next to each other. As such, a quadratic transformation will be applied to those two variables.

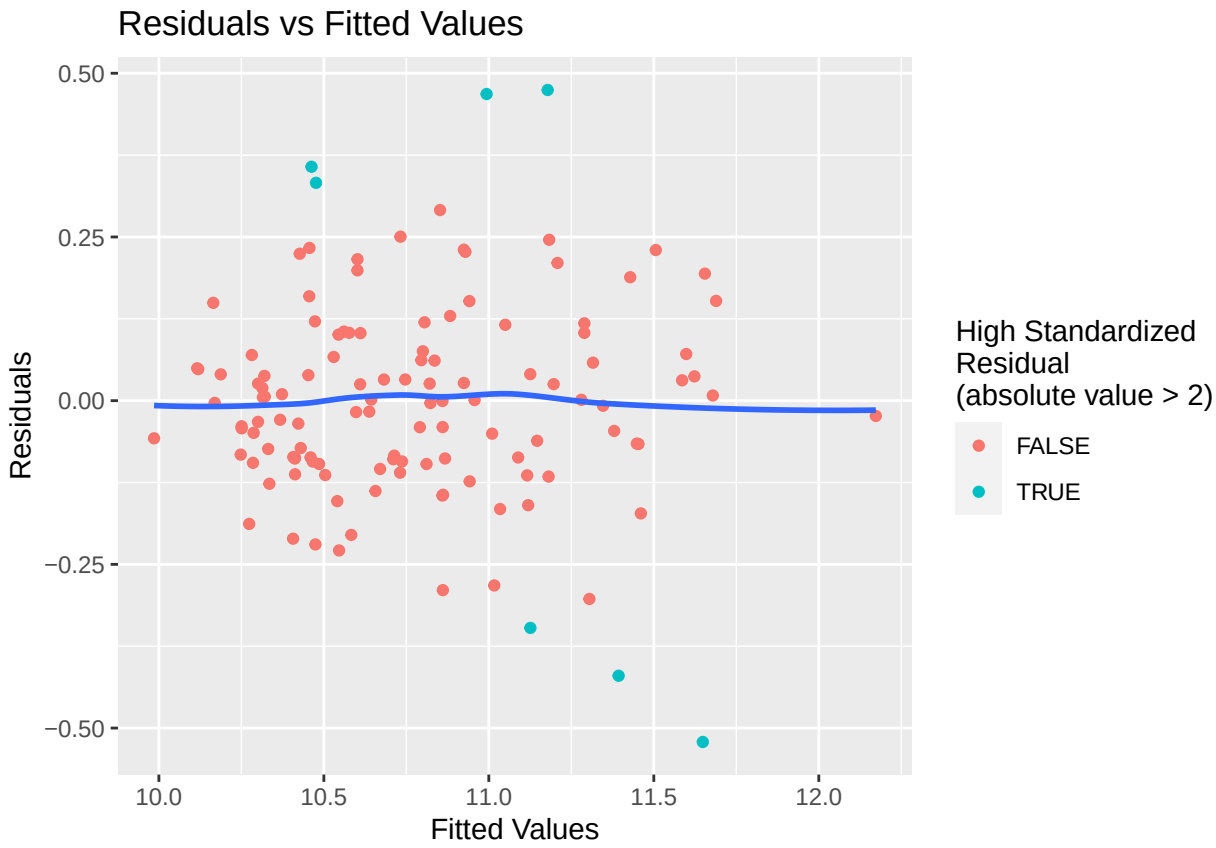
`NumberofSeats` also appears to behave as a factor in the scatter plot matrix. As such, it will be transformed into one.

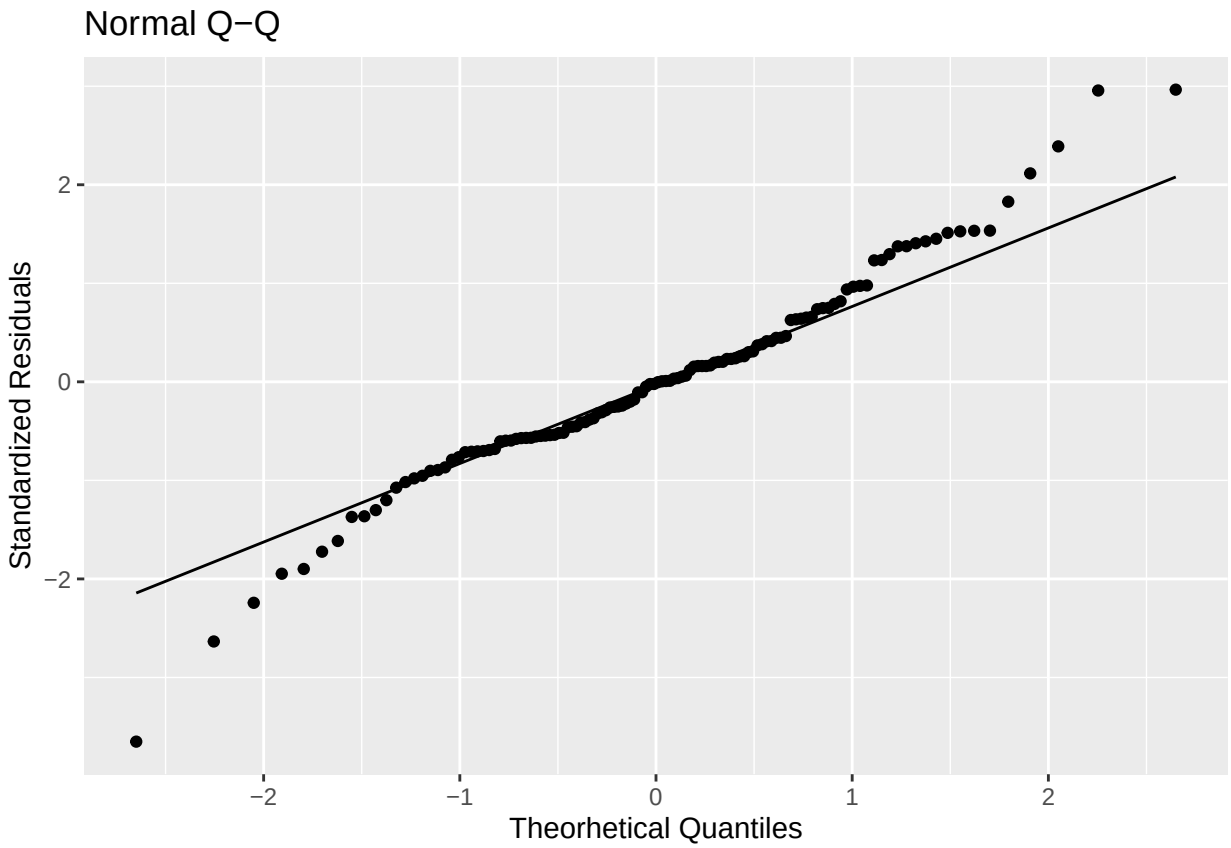
In order to handle non-constant variance, logarithmic transformations will be applied to the other predictors and response variables.

Full Model

```
##
## Call:
## lm(formula = ., data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.52123 -0.09026 -0.00196  0.08179  0.47444
##
## Coefficients:
##                                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)                      5.237e+00  9.592e-01   5.460 2.90e-07 ***
## log(Subtitle)                    4.012e-01  9.933e-02   4.039 9.86e-05 ***
## poly(Acceleration, 2, raw = T)1 -2.076e-01  5.572e-02  -3.726 0.000307 ***
## poly(Acceleration, 2, raw = T)2  1.387e-02  3.076e-03   4.510 1.61e-05 ***
## poly(TopSpeed, 2, raw = T)1      1.321e-02  3.176e-03   4.159 6.30e-05 ***
## poly(TopSpeed, 2, raw = T)2     -1.910e-05  5.803e-06  -3.291 0.001334 **
## log(Efficiency)                  5.679e-01  1.738e-01   3.267 0.001443 **
## log(FastChargeSpeed)            -6.657e-03  6.781e-02  -0.098 0.921973
## DriveFront Wheel Drive          -2.338e-02  6.980e-02  -0.335 0.738269
## DriveRear Wheel Drive           -3.726e-02  5.891e-02  -0.632 0.528352
## factor(NumberofSeats)5          -1.268e-01  4.791e-02  -2.646 0.009308 **
## factor(NumberofSeats)7          -2.770e-01  7.294e-02  -3.798 0.000238 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1665 on 112 degrees of freedom
## Multiple R-squared:  0.8815, Adjusted R-squared:  0.8698
## F-statistic: 75.71 on 11 and 112 DF,  p-value: < 2.2e-16
```

```
## `geom_smooth()` using method = 'loess' and formula 'y ~ x'
```


Residuals vs Fitted Values
0.50
0.25
0.00
−0.25
−0.50
Residuals
10.0
10.5
11.0
11.5
12.0
Fitted Values
High Standardized Residual (absolute value > 2)
FALSE
TRUE

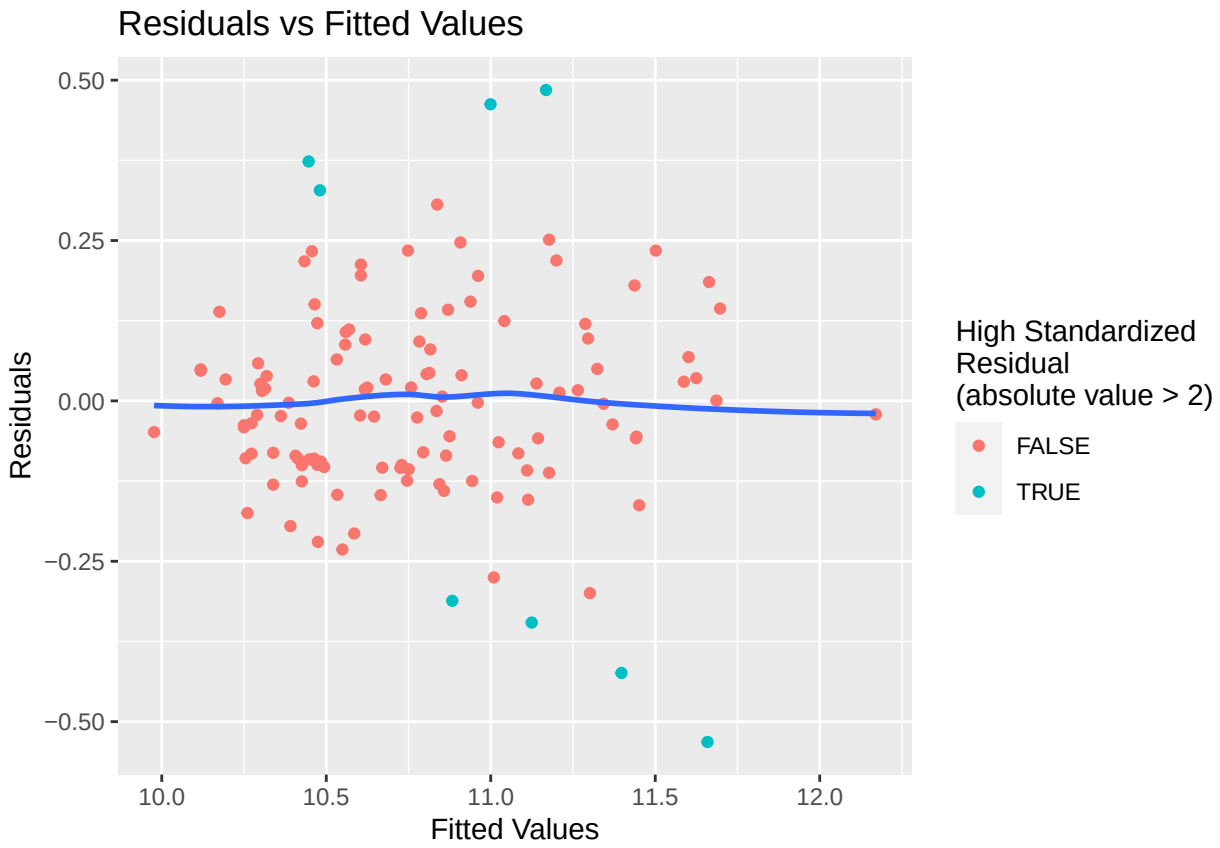

Normal Q−Q
2
0
−2
Standardized Residuals
−2
−1
0
1
2
Theorhetical Quantiles

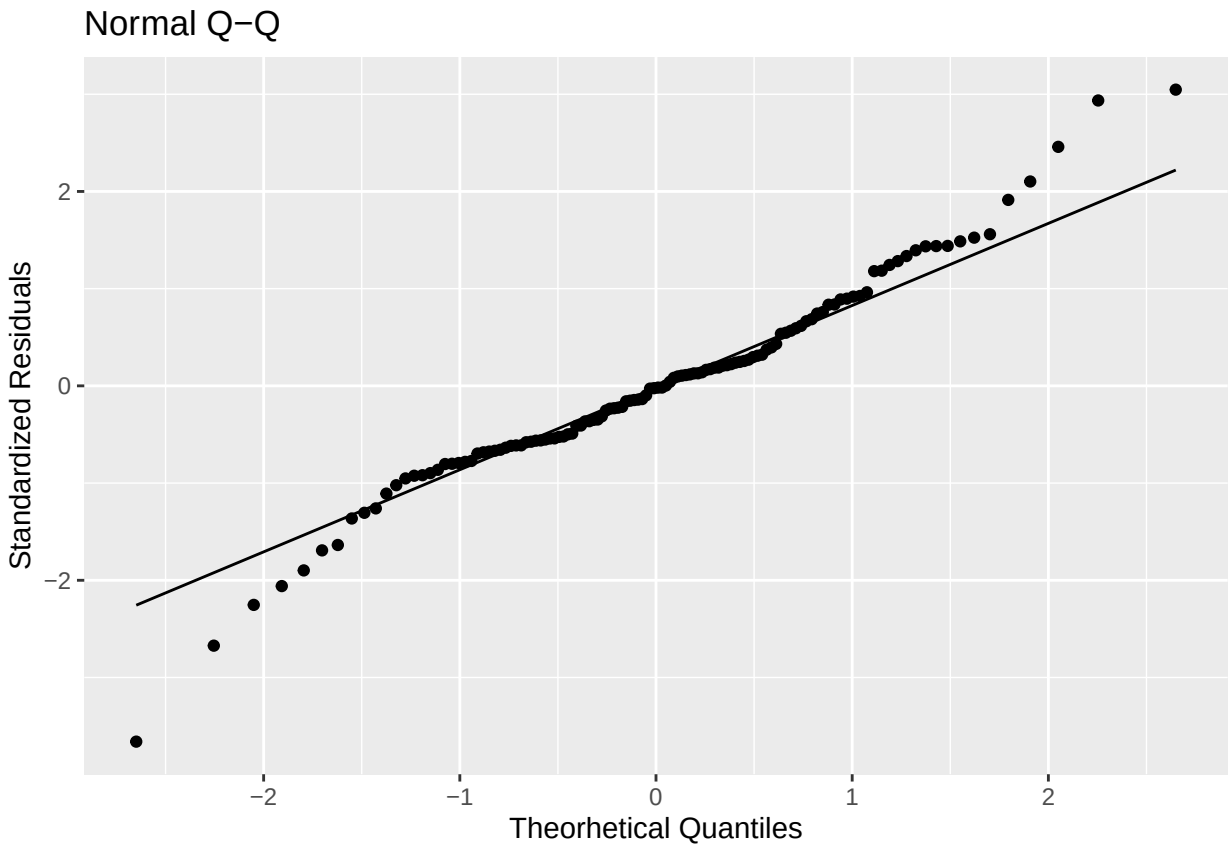
Reduced Model

This full model seems to have met the assumptions for linear regression. However, it still contains predictors that are not significant. In order to reduce the model, a step-wise process will be used with BIC as the metric.

```
##
## Call:
## lm(formula = log(PriceinUK) ~ log(Subtitle) + poly(Acceleration,
##     2, raw = T) + poly(TopSpeed, 2, raw = T) + log(Efficiency) +
##     factor(NumberofSeats), data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -0.53164 -0.09592 -0.00326  0.08894  0.48461
##
## Coefficients:
##                                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)                       5.139e+00  8.158e-01   6.299 5.67e-09 ***
## log(Subtitle)                     3.836e-01  8.049e-02   4.766 5.55e-06 ***
## poly(Acceleration, 2, raw = T)1 -2.248e-01  4.879e-02  -4.608 1.06e-05 ***
## poly(Acceleration, 2, raw = T)2  1.456e-02  2.845e-03   5.117 1.26e-06 ***
## poly(TopSpeed, 2, raw = T)1      1.280e-02  2.711e-03   4.722 6.66e-06 ***
## poly(TopSpeed, 2, raw = T)2     -1.852e-05  5.013e-06  -3.695 0.000338 ***
## log(Efficiency)                  6.144e-01  1.454e-01   4.225 4.81e-05 ***
## factor(NumberofSeats)5          -1.197e-01  4.577e-02  -2.615 0.010118 *
## factor(NumberofSeats)7          -2.726e-01  7.175e-02  -3.799 0.000234 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.1646 on 115 degrees of freedom
## Multiple R-squared:  0.881,  Adjusted R-squared:  0.8727
## F-statistic: 106.4 on 8 and 115 DF,  p-value: < 2.2e-16
```

```
## `geom_smooth()` using method = 'loess' and formula 'y ~ x'
```


Residuals vs Fitted Values
Residuals
0.50
0.25
0.00
−0.25
−0.50
10.0
10.5
11.0
11.5
12.0
Fitted Values
High Standardized Residual (absolute value > 2)
FALSE
TRUE


Normal Q−Q
Standardized Residuals
2
0
−2
−2
−1
0
1
2
Theorhetical Quantiles

This process removed `log(FastChargeSpeed)` and `Drive` from the model. The assumptions still appear to be met. In order to ensure no significant predictors were removed, a partial F-Test will be conducted between the two models.

```
## Analysis of Variance Table
##
## Model 1: log(PriceinUK) ~ log(Subtitle) + poly(Acceleration, 2, raw = T) +
##     poly(TopSpeed, 2, raw = T) + log(Efficiency) + factor(NumberofSeats)
## Model 2: log(PriceinUK) ~ log(Subtitle) + poly(Acceleration, 2, raw = T) +
##     poly(TopSpeed, 2, raw = T) + log(Efficiency) + log(FastChargeSpeed) +
##     Drive + factor(NumberofSeats)
##   Res.Df    RSS Df Sum of Sq     F Pr(>F)
## 1    115 3.1166
## 2    112 3.1036  3  0.012972 0.156 0.9256
```

Since the p-value is high, we lack statistically significant evidence that any significant predictors were dropped.

This leaves us with the formula:

$$\begin{aligned}\ln \texttt{PriceinUK} = \beta_0 &\\ &+ \beta_1 \ln \texttt{Subtitle} \\ &+ \beta_2 \texttt{Acceleration} + \beta_3 \texttt{Acceleration}^2 \\ &+ \beta_4 \texttt{TopSpeed} + \beta_5 \texttt{TopSpeed}^2 \\ &+ \beta_6 \ln \texttt{Efficiency} \\ &+ \beta_7 (\texttt{NumberofSeats} == 5) + \beta_8 (\texttt{NumberofSeats} == 7) \\ &+ \epsilon\end{aligned}$$

Using this, we can determine some of the high leverage points:

	Name	PriceinUK
3	Nissan e-NV200 Evalia	30255
14	Tesla Roadster	189000
19	Tesla Model Y Long Range Dual Motor	54000
20	Tesla Model Y Performance	60000
35	Mercedes EQV 300 Long	70665
42	Tesla Cybertruck Single Motor	39000
44	Tesla Cybertruck Tri Motor	68000
84	Tesla Model S Plaid	118980
115	Mercedes EQB 350 4MATIC	50000

and the high residual points

	Name	PriceinUK
42	Tesla Cybertruck Single Motor	39000
43	Tesla Cybertruck Dual Motor	48000
44	Tesla Cybertruck Tri Motor	68000
77	Opel Zafira-e Life L 50 kWh	49465
96	Kia EV6 GT	58295
105	Mercedes EQS 450+	95000
106	Mercedes EQS 580 4MATIC	115000

	Name	PriceinUK
115	Mercedes EQB 350 4MATIC	50000