
Model Repair by Incorporating Negative Instances In Process Enhancement

Master Thesis

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Abstract

Based on business execution history recorded in event logs, Process Mining provides visual insights on the business process and supports process analysis and enhancements. It bridges the gap between traditional business process management and advanced data analysis techniques such as data mining and gains more interest in recent years.

Process enhancement, as one of the main focuses in process mining, improves the existing processes according to actual business execution in the form of event logs. The records in an event log can be classified as positive and negative according to predefined Key Performance Indicators, e.g., the logistic time, and production cost in manufacturies. Most of the current enhancement techniques only consider positive instances from an event log to improve the model, while the value hidden in negative instances is simply neglected.

This thesis provides a novel strategy that considers not only the positive instances and the existing model but also incorporates negative information to enhance a business process. Those factors are balanced on directly-follows relations of activities and generate a process model. Subsequently, long-term dependencies of activities are detected and added to the model, in order to block negative instances and obtain a higher precision.

We validate the ability of our methods to incorporate negative information with synthetic data at first. The results showed that our method is able to overcome the shortcomings of current repair techniques and provide models with higher precision in given situations. Furthermore, we conducted experiments on real life data with the scientific workflow platform KNIME and verified feasibility of our proposed method in reality.

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Chapter 1

Introduction

Process mining is a relatively new discipline that bridges the gap of data mining and business process management. The objective of process mining is to support the analysis of business processes, provide valuable insights in processes and further improve the business execution based on the business execution data which are recorded in event logs. According to [1], process mining techniques are divided into three categories: *process discovery*, *conformance checking*, and *process enhancement*. *Process discovery* techniques derive visual models from event logs of information systems, aiming at a better understanding of real business processes. *Conformance checking* analyzes the deviations between a reference process model and observed behaviors driven from its execution. *Process enhancement* adapts and improves existing process models by extending the models with additional data perspectives or repairing the reference models to accurately reflect observed behaviors.

Most of the organizations have predefined process execution rules of activities which are captured in a process model. One execution of related activities in this model is called a trace. A trace which has no deviation to the model is a fitting trace. However, in practice, business processes often encounter exceptional situations where it is necessary to execute process differing from the reference model. With accumulation of deviations, the reference process model needs amending to reflect reality.

Basically, one can apply process discovery techniques again on the event log to obtain a new model. This method is referred as process rediscovery. Without consideration of the reference models, the rediscovered models are likely to differ a lot from the reference ones. As a result, in situations where the improved model is required to be as similar as possible to the original model, the rediscovery method tends to fail without considering the impact from the existing model [2].

To improve reference models and as well keep the similarity, **model repair** techniques are proposed in [2]. As one part of process enhancement [2], it analyzes the workflow deviations between an event log and a process model, and improves the model by adding subprocesses on the model. However, the repair techniques in [2, 5] only focus on the workflow of the process model and repair models without consideration of the data performance.

As known, organizations are goal-oriented and aim to have high performance according to a set of Key Performance Indicator(KPI)s, e.g., the production time for a car industry, the

logistic cost for importer companies. Process models which leading to high performance are preferred in organizations. However, little research in process mining is conducted on the basis of business performance [3]. The authors of [3] point out several contributions, e.g., [4] to consider business performance into process mining. The work in [4] divides deviations of model and the event log into positive and negative according to certain KPIs. Then it applies repair techniques in [2] only with positive deviations which are deviations leading to positive KPI outcomes. This reason behind this approach is to avoid introducing negative instances into the repaired model.

However, the current repair methods have limits. Model repair techniques fix the model by adding subprocesses. They guarantee that the repaired model replays the whole event log but overgeneralizes the model, such that more behaviors are allowed than expected. Furthermore, the model complexity increases due to the repair techniques in [2]. The work in [4] takes the performance into account but only uses deviations with positive performance outcomes to repair the model. Without consideration negative information, it is difficult for the repaired model to block the behavior which leads to negative performance.

In the following sections, motivating examples are given to describe those limits of the current repair techniques in several situations. Base on those limits, we propose research questions to overcome those limits and define our research scope. At the end, we give the outline for the whole thesis.

1.1 Motivating Examples

For better understanding, examples are extracted from the common master thesis procedure to illustrate those situations. The process is described by the Petri net M_0 in Figure 1.1. Activities are denoted as rectangle and connected by circles called places. The directed arcs in the Petri net indicate the execution order of activities. And the black dots in places of the model represents current execution states. In addition, those dots can be substituted by other symbols due to visualization tools, like numbers in places.

According to the model, a student can **select existing topics** or **create new topics**. Those two activities are exclusive. The exclusive choice relation also exist in **learn existing tools** or **explore new tools**. Activities **development** and **write thesis** are executed currently. Activity **presentation** is necessary to complete a master thesis project. For convenience, alphabets are used to represent the corresponding activities and annotated in the model later.

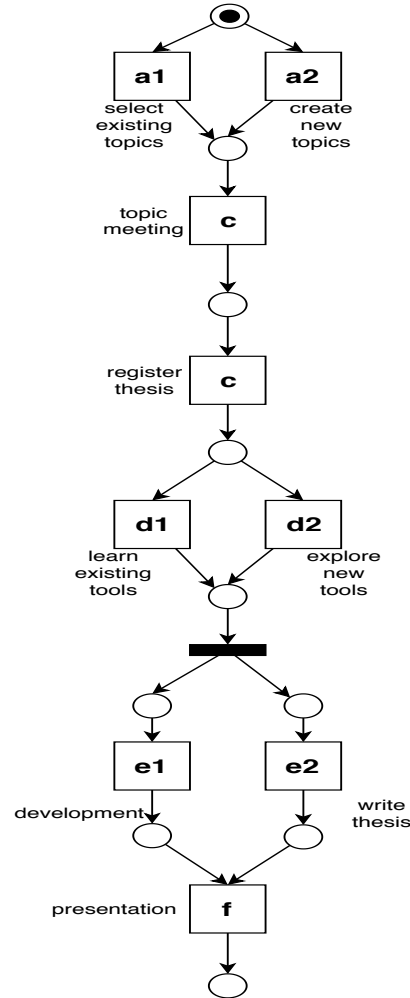


Figure 1.1: The original master study process M_0 .

1.1.1 Situation 1: Add Subprocesses as Loops

One obvious shortcoming of repair techniques in [5] is that subprocesses for deviations are created and added as loops, when the deviations start and end at the same place. If there is only one such subprocess, it is fixed as a sequence in the model with post process. Yet, the algorithm does not discover orders among different subprocesses at overlapping locations. Therefore, the subprocesses are kept in a loop form, which causes a lower precision of the repaired model. An example is given with respect to the master study.

In some universities, before registering a master thesis, the activities **write proposal**, represented by **x1** and **check course requirement**, represented by **x1**, are necessary for the whole procedure. We assume that traces with either of those activities are considered as positive.

For convenience, the real process is recorded in the event log L_1 in the following way. Traces with the same activities order and the same label are aggregated together. The corresponding sum and label are listed above the whole sequence. For example, the first item in L_1 indicates that there are 50 traces which follow the order as $\langle a1, b, \mathbf{x1}, c, d1, e1, e2, f \rangle$ and lead to positive performance outcomes.

$$L_1 := \{ \langle a1, b, \mathbf{x1}, c, d1, e1, e2, f \rangle^{50, pos}, \\ \langle a1, b, \mathbf{x2}, c, d2, e1, e2, f \rangle^{50, pos}, \}$$

By using rediscovery techniques– the Inductive Miner for Infrequent and noise threshold with 20, a new model $M_{1.2}$ based on the positive instances of L_1 is generated in Figure 1.2a. **x1** and **x2** position with exclusive choices as implied by event log L_1 . However, **a1** is kept in the new model and **a2** has been deleted, because no traces in L_1 includes **a2**.

Because the existing repair techniques [5] do not consider the performance of traces in the event log, all instances with negative labels are ignored to compute the deviations. L_1 has deviations of **x1**, **x2** at the same place without order. The corresponding subprocesses for them are added in the loop as shown in Figure 1.2b, which brings more behavior than expected into model.

The repair algorithm in [4] builds upon [5] and considers the performance of the event log. However, the repaired model is the same as the one in Figure 1.2b. The reasons are: (1) there is no deviation from negative factors. (2) positive deviations are used to add subprocesses in the same way as [5].

Compared to the model $M_{1.1}$ and $M_{1.2}$ in Figure 1.2 where the two extra activities are shown in loops, or the main original structure has been changed, we expect the subprocesses with **x1**, **x2** can be added in sequence without affecting the main structure. In this way, the repaired model has a higher precision, meanwhile, it holds the similarity to reference model.

1.1.2 Situation 2: Unable to Adapt Model with Fit Traces

This situation describes the existing problem in the current methods that fitting traces with negative performance outcomes cannot be used to repair a model. Given an actual

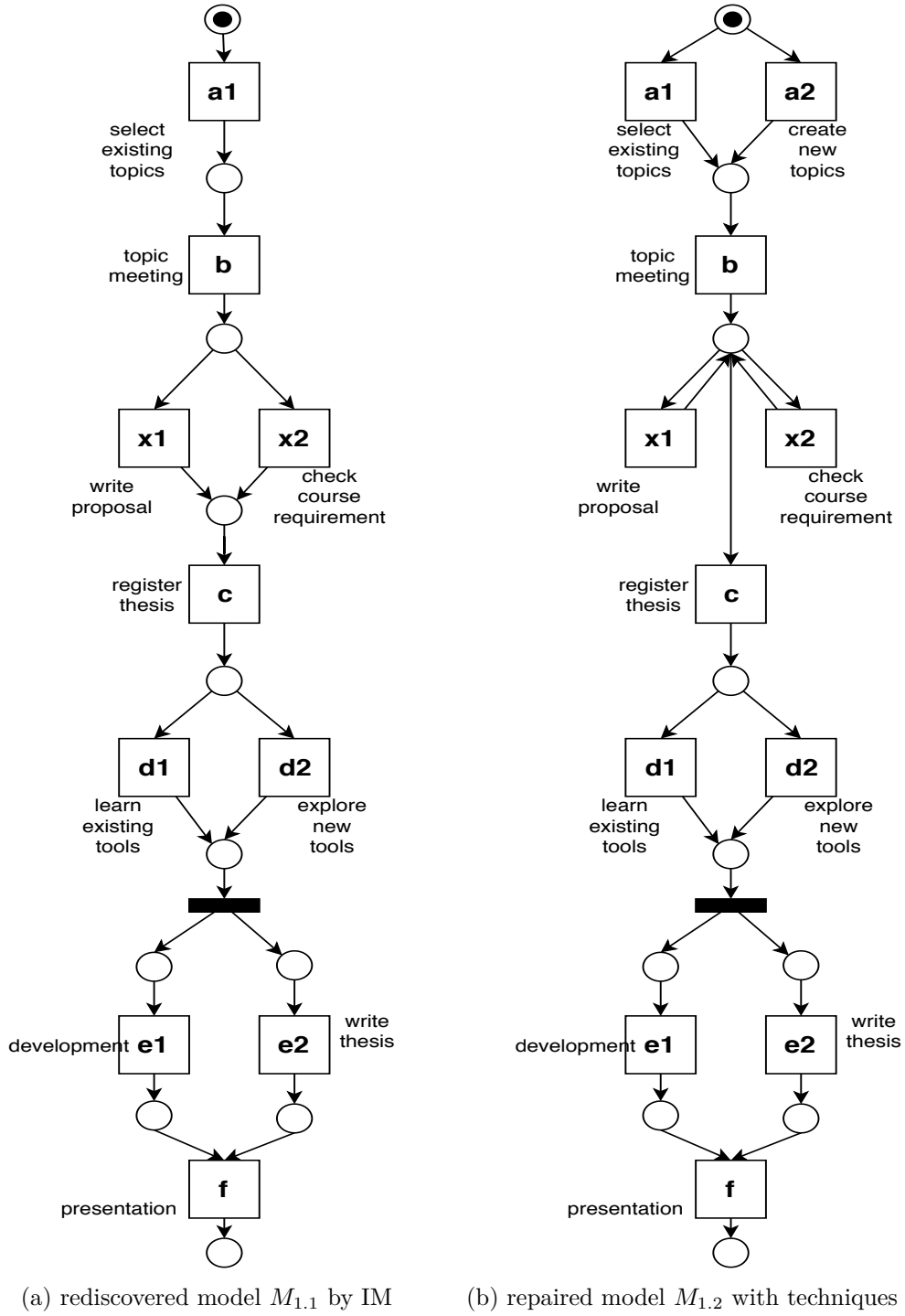


Figure 1.2: Motivating examples for situation 1 where $M_{1,1}$ is rediscovered by Inductive Miner-Infrequent with threshold 0.2, and $M_{1,2}$ is repaired with techniques in [5] by adding subprocess in the form of loops.

event log L_2 , The execution order of **development**, **write thesis** affects the performance outcomes. When **development** is executed before **write thesis**, more positive instances are given. In contrast, when **write thesis** is executed before **development**, more negative instances are caused.

$$L_2 := \{ \langle a1, b, c, d2, \mathbf{e1}, \mathbf{e2}, f \rangle^{30, pos}, \\ \langle a2, b, c, d1, \mathbf{e1}, \mathbf{e2}, f \rangle^{20, pos}; \\ \langle a2, b, c, d2, \mathbf{e2}, \mathbf{e1}, f \rangle^{10, pos}; \\ \langle a1, b, c, d2, \mathbf{e2}, \mathbf{e1}, f \rangle^{20, neg}, \\ \langle a1, b, c, d1, \mathbf{e2}, \mathbf{e1}, f \rangle^{20, neg}; \\ \langle a2, b, c, d1, \mathbf{e1}, \mathbf{e2}, f \rangle^{5, neg} \}$$

Compared to M_0 , the event log L_2 contains no deviation. When we apply the techniques in [5] and [4] to repair the model, the model remains unchanged. Also, IM mines the same model as the original one. Apparently, the fact that those two methods can not incorporate the negative information in fitting traces causes this shortcoming. A model as M_2 is expected because it enforces the positive instances and avoids the negative instance. Unfortunately, the current methods do not allow us to obtain such results.

1.1.3 Situation 3: Unable to Detect Long-term Dependency

Another problem is that current methods are unable to detect the long-term dependency in the Petri net, which causes a lower precision. The long-term dependency describes the phenomenon that the execution of an activity decides the execution of activities that do not follow directly. Due to the long distance of this dependency, current methods cannot detect it and improve the precision by adding long-term dependency on the model. One example on M_0 is used to demonstrate this problem.

Given the time consumption on the whole study as the KPI, if the total sum goes over one threshold, the trace is negative, else as positive. Since the activity **create new topics** usually demands new knowledge rather than **checking the existing tools**. If students choose to learn existing tools, it's possibly not useful and time-wasting. In the other case, if we select existing topics with existing background, it saves time when we directly learn the existing tools. According to this performance standard, we classified those event traces into positive and negative shown above. An event log L_3 is given in the following.

$$L_3 := \{ \langle \mathbf{a1}, b, c, \mathbf{d1}, e1, e2, f \rangle^{50, pos}, \\ \langle \mathbf{a2}, b, c, \mathbf{d2}, e1, e2, f \rangle^{50, pos}; \\ \langle \mathbf{a1}, b, c, \mathbf{d2}, e1, e2, f \rangle^{50, neg}, \\ \langle \mathbf{a2}, b, c, \mathbf{d1}, e1, e2, f \rangle^{50, neg} \}$$

As observed, **a1** decides **d1** while **a2** decides **d2**. They have long-term dependencies. However, there are no deviations of the model and event log L_3 according to the algorithms in [5] and [4]. Also, the Inductive Miner can not detect long-term dependency. Therefore, the original model stays the same and negative instances can not be blocked.

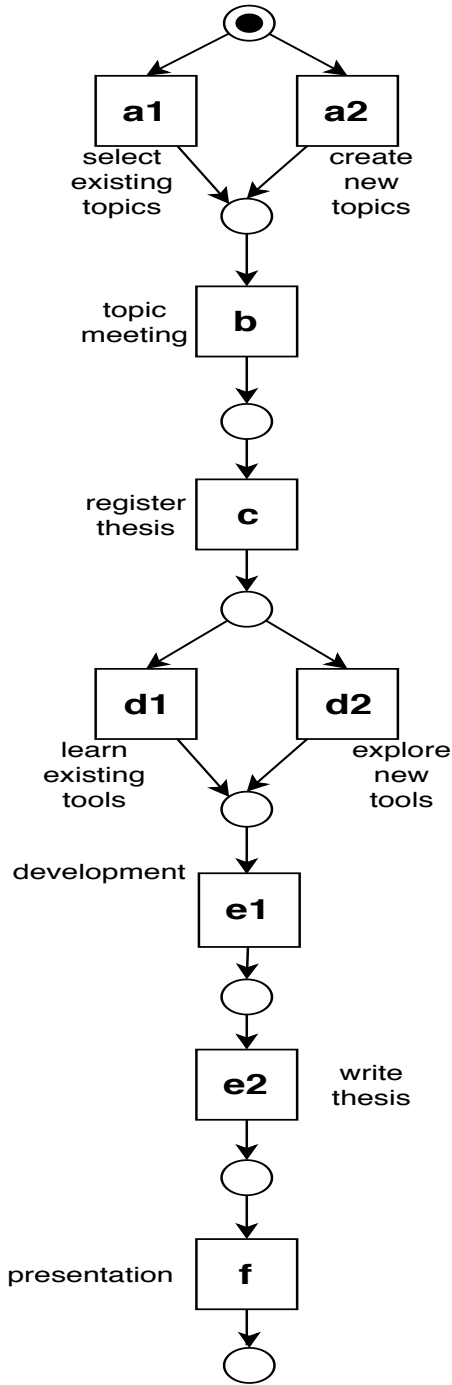
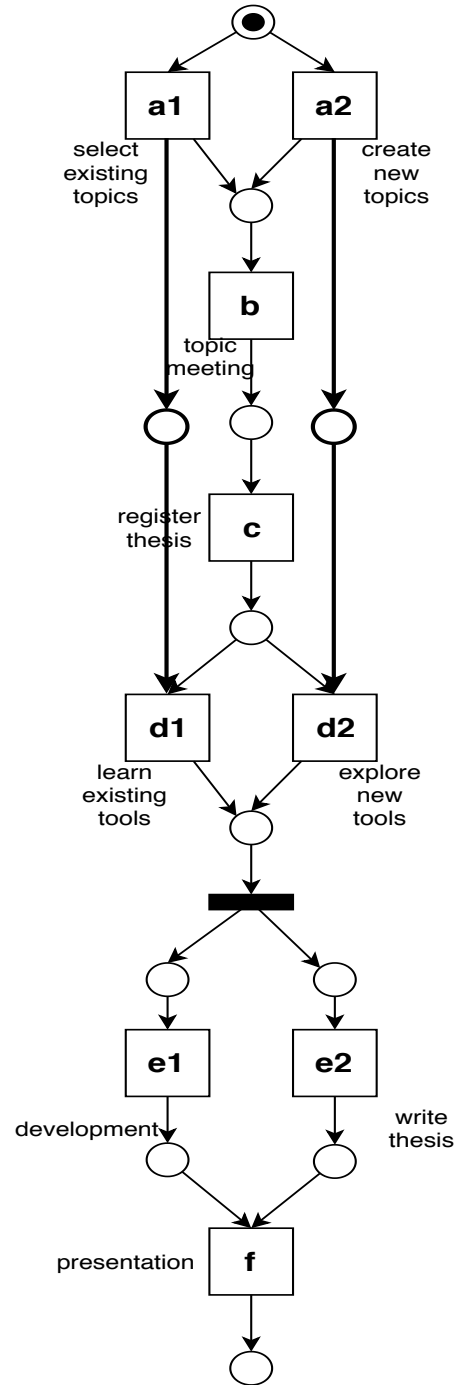
(a) model M_2 with order change(b) model M_3 with long-term dependency

Figure 1.3: Motivating examples for situation 2 and 3. Current repair techniques keep the original models unchanged and can not output M_2 and M_3 . M_2 with order changes of $e1$ and $e2$ is able to enforce positive and block negative instances. M_3 with long-term dependency leads to better precision.

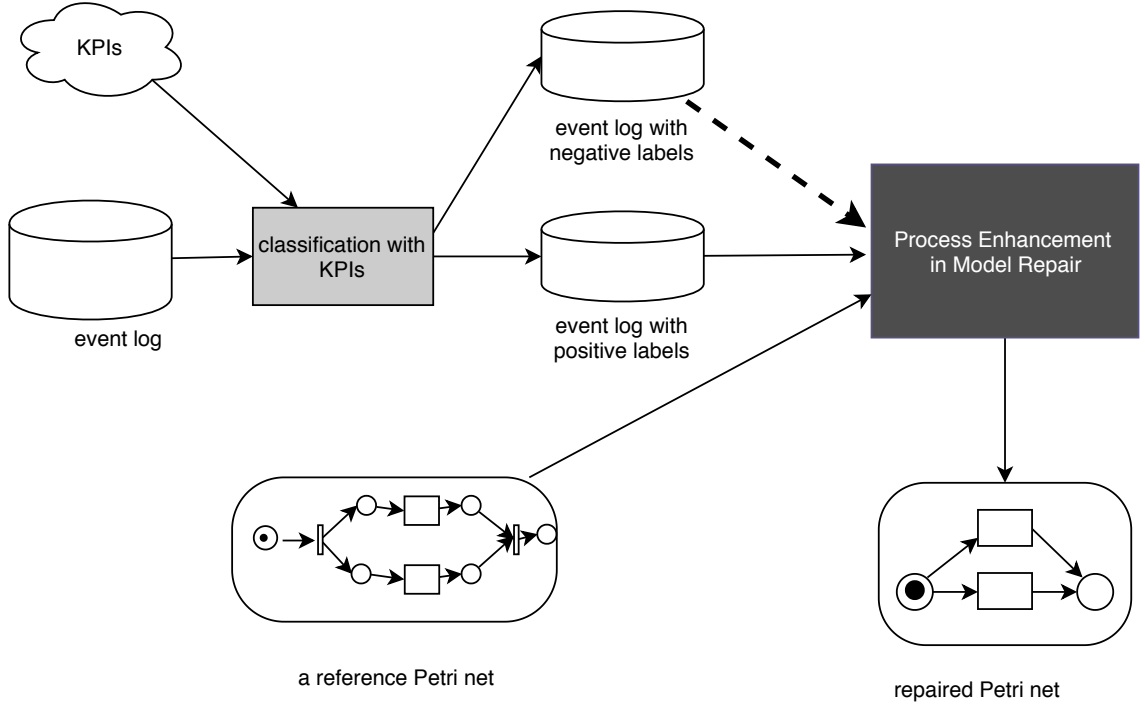


Figure 1.4: The scope shows the inputs for our problem is an event log and a process model in Petri net. According to certain KPIs, the event log is split into two event logs that are passed to the model repair techniques shown in black box. The output is also defined to be in the form of a Petri net.

Clearly, the use of negative information can bring significant benefits, e.g., enable a controlled generalization of a process model: the patterns to generalize should never include negative instances. This leads to the demand of improving current repair model techniques with incorporating negative instances. In the next section, the demand is analyzed and defined in a formal way.

1.2 Research Scope And Questions

After analyzing the current model repair methods, we limit our research scope as shown in Figure 1.4. The inputs for our research are one existing process model M , an event log L . According to predefined KPIs, each trace in event log is classified into positive or negative. After applying repair techniques in the black box, the model should be improved to enforce the positive instances while disallowing negative instances, with the condition that the generated model ought to be as similar to the original model as possible.

In this scope, we come up with several research questions listed in the following.

RQ1: How to overcome the shortcomings of current repair techniques in situations 1-3 above?

RQ2: How to balance the impact of the reference model, negative and positive instances

together to repair the model?

RQ3: How to block negative instances from the model while enforcing the positive ones?

In this thesis, we propose an innovative solution for the repair techniques in a black box in Figure 1.4. It analyzes process performance on trace level and balances the existing model, positive traces and negative traces on directly-follows relation, to incorporate all the factors on model generation.

1.3 Outline

This thesis aims to answer the questions presented in section 1.2 in the remaining chapters and provides a solution for the black box. Chapter 2 introduces the related works. Chapter 3 recalls the basic notions on process mining and list the preliminaries to solve the problem. Chapter 4 firstly presents an original framework to incorporate negative information into model repair. Based on this framework, a concrete algorithm is proposed. In Chapter 5, screenshots of the implementation tools are shown to demonstrate the usage. Chapter 6 answers the last question RQ3, by conducting a bundle of experiments. Later, results are analyzed and discussed. At last, we summarize our work in Chapter 7.

Chapter 2

Related Work

To update an existing process model in organizations, there are two strategies, rediscovery and process enhancement. Process rediscovery applies the discovery techniques on the actual event log to mine a new model. Process enhancement improves the model based on not only the actual event log but also the existing model.

Process discovery has been intensively researched in the past two decades and many algorithms have been proposed [6]. Directly-follows methods [7, 8] investigate the order of activities in the traces and extract higher relations which are used subsequently to build process models. State-based methods like [9, 10] build a transition system to describe the event log, and then extract Petri net places from the related state regions. Language-based algorithms use integer linear system to represent the place constraint where the token at one place can never go negative. By solving the system, a petri net is created. Its representative techniques are Integer Linear Programming (ILP) Miner [11]. Other methods due to [12] include search-based algorithms, like Genetic Algorithm Miner [13], and heuristic-based algorithms, like Heuristics Miner [14]. Among those discovery methods, Inductive Miner bases on the directly-follows relation and is widely applied [8] in practice.

Conformance checking analyzes the deviations between a reference process model and observed behaviors driven from its execution. To find commonalities and discrepancies, token-based replay starts from the initial place of the Petri net model and fires the corresponding activities in the event log. If the final place is reached without any missing tokens which are added to enable activities or any remaining tokens that are produced by the firing of activities, the trace is fitting without any deviations. Alignment-based conformance checking consists of moves which represent the matching situations between the transitions of a model and the events in event log. By minimizing the costs with different types of moves, it finds the optimal alignment between the trace and the model and detect the discrepancies.

When the actual event log differs a lot from the referred process model, it is suitable to use the rediscovery method to improve the business execution. However, in some cases, the process enhancement focuses to extend or improve an existing process model by using an actual event log [1]. Besides extending the model with more data perspectives, model repair is another type of enhancement. It modifies the model to reflect observed behavior while keeping the model as similar as possible to the original one [2].

In [2], model repair is firstly introduced into process enhancement. By using conformance checking, the deviations of the event log and process model are detected. The consecutive deviations in log are only collected in the form of subtraces at a specific location Q in the model. Subsequently, the subtraces are grouped into sublogs that share the same location Q for subprocess discovery. In the earlier version in [2], the sublogs are obtained in a greedy way, while in [5], sublogs are gathered by using ILP Miner to guarantee the fitness. Additional subprocesses are introduced into the existing model to ensure that all traces fit the model, but it introduces more behavior into the model and lowers the precision.

Later, compared to [2, 5], where all deviations are incorporated in model repair, [4] considers the impact of negative information. In [4], the deviations of the model and event log are firstly analyzed, in order to find out which deviations enforce the positive performance. Given a trace and a selected KPI, an observation instance is built to correlate activities in event logs which are deviating from models. Based on the observation instance, a set of rules are derived in the form of a decision tree. According to the rules, the original event log is divided into sublogs with traces matching the rules. The sublogs are then repaired with trace deviations which lead to positive KPI outputs. Following repair, the sublogs are merged as the input for model repair in [5]. According to the study case in [4], [4] provides a better result than [5] on the aspect of performance.

As described above, the state-of-the-art repair techniques are based on positive instances, meanwhile, the negative information is neglected. This causes difficulty to balance the fitness and precision of those models. Likewise, little research gives a try to incorporate negative information in multiple forms on process discovery.

In [15], the negative information is artificially generated by analyzing the available event set before and after one position and represented in the form of the complement of positive event sets. Based on the positive and negative event sets, Inductive Logic Programming is applied to detect the preconditions for each activity. Those preconditions are then converted to Petri net after applying a pruning and post-process step. In [16], the author improves the method in [15] by assigning weights on artificial events with respect to an unmatching window, in order to offer generalization on the model.

The work in [17] uses traces in the event log with negative outcomes as negative information. It extends the techniques of numerical abstract domains and Satisfiability Modulo Theories (SMT) proposed in [18] to incorporate negative information for model discovery. Each trace as positive or negative is transformed as one point in n -dimensional space, n is the number of distinct activities. By adding places between transitions, it limits the transition execution by demanding tokens on those added places. This limit can be reflected by the sequence of traces. In the linear system, those limits are represented into a set of inequalities and in a form of a convex polyhedron in n -dimensional space. Given half-space hypotheses, SMT solves the inequalities and gives the limits on the process model. Before SMT, negative information is incorporated to shift and rotate the polyhedron, which limits the generalization of the solution space. Because half-space is used, this method can not deal with negative instances overlapped on positive instances.

However, the field of model repair which considers the negative information is new. Furthermore, the idea to incorporate negative instances on trace level into model repair is innovative.

Chapter 3

Preliminaries

This chapter introduces the most important concepts and notations that are used in this thesis. Firstly, the event data and process models typically used in process mining are described. Later, details of Inductive Miner techniques are listed.

3.1 Event Log

Business processes in organizations can be reflected by the execution of related activities. The historical execution data is usually stored as event logs in information systems and can be used by process mining techniques to analyze, understand, and improve the business execution. To specify the event log, we begin with formalizing various notations [6].

Definition 3.1 (Event). \mathcal{A} is the universe of activities. An event e for an activity $a \in \mathcal{A}$ corresponds to one execution of this activity. The event set for a process is a finite set \mathcal{E} .

An event is characterized by attributes, like a timestamp, activity name, associated costs, etc. The executions of activities in a process are recorded as events. The set of events for a complete process case is a trace. Furthermore, the multiset of traces combines as an event log for the process.

Definition 3.2 (Trace and Event Log). A trace σ is a finite sequence of events, $\sigma \in \mathcal{E}^*$. An event log L is a multiset of traces, $L \in \mathcal{B}(\mathcal{E}^*)$.

A trace also has a set of attributes, like its unique identifier, the cost. In this thesis, we extend this definition to handle traces with performance output according to certain KPIs. We call a trace labeled when this trace has a positive or negative label according to certain KPIs. Correspondingly, an event log is labeled if all of its traces have labels according to certain KPIs.

3.2 Process Models

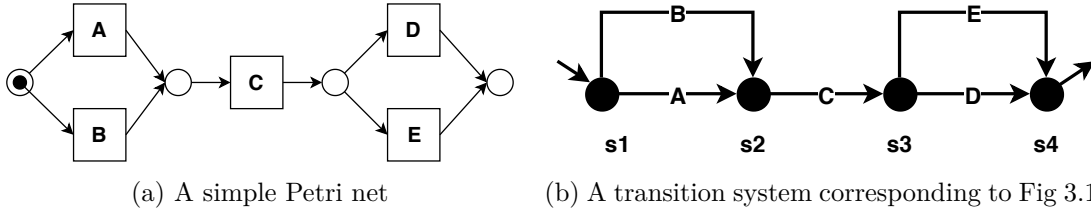
After gathering an event log from information systems, process mining can discover a process model based on the event log. With this process model, the understanding of

the business process can be improved. To describe the process, multiple process modeling languages are proposed in the past years, e.g, Petri net, BPMN models, etc.

Among those model languages, Petri nets have been studied thoroughly. Petri nets are capable to capture concurrent systems in a compact manner. Process trees are based on a tree structure to organize the event relation. They are simple to understand in comparison to other models, like BPMN models. In this thesis, Petri nets and process trees are used to represent our process.

3.2.1 Petri Net

A Petri net is a bipartite graph which consists of **transitions** represented by a square and **place** by a circle. **Tokens** in black dots are put in places to express the dynamic states of a Petri net. An Petri net example is shown in Figure 3.1a. This Petri net is a workflow net with a token at the source place and satisfy the soundness conditions.



In the following, we define Petri nets in a formal way.

Definition 3.3 (Petri net). A Petri net is a tuple $N = (P, T, F)$ where $P \cap T = \emptyset$. F is the set of arcs to connect places and transitions, $F \subseteq (P \times T) \cup (T \times P)$.

Further, we can assign activity labels to transitions in Petri nets to describe the corresponding activities in business process. Usually, transitions with activity labels are seen *observable actions*. To represent particular transitions that are not observable, we reserve label τ for them and name such transitions *silent or invisible* transitions. A Petri net with labels are called labeled Petri nets and can be formalized in Definition 3.4.

Definition 3.4 (A labeled Petri net). A labeled Petri net is a tuple $N = (P, T, F, \Sigma, \lambda)$, where Σ is a set of activity labels, λ is a function defined as $\lambda : T \rightarrow \Sigma \cup \tau$. A transition t with $\lambda(t) = \tau$ is a silent or an invisible transition.

To express the dynamic states of Petri nets, we introduce a concept called **marking** and extend the form $N = (P, T, F)$ and define a marked Petri net.

Definition 3.5 (Marked Petri nets). A marked Petri net is 4-tuple nets, $N = (P, T, F, M)$ where $M \in \mathbb{B}(\mathbb{P})$ is a place multiset called marking, which denotes the number of tokens in places.

The firing sequence of transitions from Petri net corresponds to business execution in reality. A transition can be fired if it is in an enabled state where all the input places for this transition hold at least one token. After firing the transition, the token in the input places are consumed and new tokens are generated in the output places for this transition. Before we formalize the firing rule, we need several concepts below.

Definition 3.6 (Input and output nodes). For a node x in Petri net $N = (P, T, F)$, the set of its input nodes is denoted as $\bullet x = \{y | (y, x) \in F\}$. The set of its output nodes is

denoted as $x\bullet = \{y | (x, y) \in F\}$.

Definition 3.7 (Firing rule). In a marked Petri net $N = (P, T, F, M)$, a transition $t \in T$ is enabled, written as $N[t]$, if and only if $\bullet t \subseteq M$. After firing t , the marking changes in this way, $M \Rightarrow (M \setminus \bullet t) \cup t\bullet$.

In real life, a subclass of Petri nets known as Workflow nets is often used to model business process.

Definition 3.8 (Workflow nets). A workflow net is a labeled Petri net $N = P, T, F, \lambda$ with constraints:

- P contains a source place denoted as i , where $\bullet i = \emptyset$.
- P contains one sink place denoted as o , where $o\bullet = \emptyset$.
- When connecting the sink place to source place by an arc, this net seen as a graph becomes strongly connected.

However, not every Workflow net represents a correct process. To perform the business on an enterprise level, we define a minimum correctness criterion, known as soundness [19].

Definition 3.9 (Soundness). A Petri net is sound if and only if it satisfies the following conditions.

- Safeness. Places cannot hold multiple tokens at the same time.
- Proper completion. If the sink place is marked, all other places are empty.
- Option to complete. It is always possible to reach the final marking from any reachable marking.
- No dead parts. For any transition, there exists a path from source to sink place through it.

3.2.2 Transition System

A transition system is a basic process model. As displayed in Figure 3.1b, it is composed of *states* and *transitions*. *States* are represented by black circles and include an initial and a final state. The initial state **s1** in Figure 3.1b is denoted with an input arc without any label and the final state **s4** is with an output arc without label. *Transitions*, also called actions, correspond to activities in the processes. They connect states by an arc.

Definition 3.10 (Transition System). A transition system is a triple $TS = (S, A, T)$ where S is the set of states, A is the set of activities, $T \subseteq S \times A \times S$ is the set of transitions, $(s_i, a, s_j) \in T$ is represented as $s_i \xrightarrow{a} s_j$.

The example models in Figure 3.1b are mapped to each other. There are 5 transitions and four states in the transition system, which correspond to the transitions and places in Petri net. Transition systems are able to describe the dynamic behavior and can easily be translated to Petri nets. Yet, they have difficulty to express concurrency of transitions.

3.2.3 Process Tree

Process tree is a tree and is suitable to represent a sound WF-net by construction according to [6]. Moreover, process tree is easy to understand. Therefore, we use it in this thesis.

Definition 3.11 (Process Tree). Let $A \subseteq \mathbb{A}$ be a finite set of activities with silent transition $\tau \notin \mathbb{A}$, $\oplus \subseteq \{\rightarrow, \times, \wedge, \circ\}$ be the set of process tree operators.

- $Q = a$ is a process tree with $a \in A$, and
- $Q = \oplus(Q_1, Q_2, \dots, Q_n)$ is a process tree with $\oplus \in \oplus$, and Q_i is also a process tree, $1 \leq i \leq n$.

For convenience, given a process tree $Q = \oplus(Q_1, Q_2, \dots, Q_n)$, we denote Q the parent for Q_1, Q_2, \dots, Q_n , Q_i as one child of Q . For a node in a tree without children, we call it a **leaf**; Else, we call them **blocks**.

Process tree operators represent different block relations of each subtree. Their semantics are standardized in [20, 21] and compared with Petri net in Figure 3.2 [21]. We provide an informal description in the following.

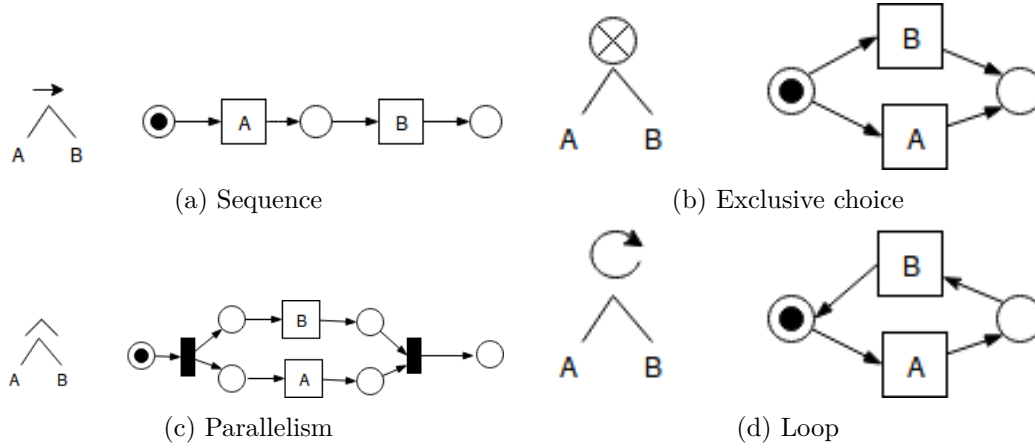


Figure 3.2: Translation of process tree operators to Petri net

Definition 3.12 (Operator Semantics). The semantics of operators $\oplus \subseteq \{\rightarrow, \times, \wedge, \circ\}$ are defined firstly in the context of its structure. The semantics with respect to a trace σ are also given.

- if $Q = (a)$, the tree only has one leaf node.
- if $Q = \rightarrow(Q_1, Q_2, \dots, Q_n)$, the subtrees have a sequential relation and are executed in order of Q_1, Q_2, \dots, Q_n .
- if $Q = \times(Q_1, Q_2, \dots, Q_n)$, the subtrees have an exclusive choice relation and only one subtree of Q_1, Q_2, \dots, Q_n can be executed.
- if $Q = \wedge(Q_1, Q_2, \dots, Q_n)$, the subtrees have a parallel relation and Q_1, Q_2, \dots, Q_n they can be executed in parallel.
- if $Q = \circ(Q_1, Q_2, \dots, Q_n)$, the subtrees have a loop relation for $\{Q_1, Q_2, \dots, Q_n\}$. $\{Q_2, \dots, Q_n\}$

have an exclusive relation. Q_1 executes at first and is triggered again once one subtree of $\{Q_2, ..Q_n\}$ executes.

According to the corresponding semantic relations, a process tree can be easily transformed into Petri net. In Figure 3.3b, it is the process model in process tree which describes the same process as in Figure 3.1a.

To describe the execution of a process tree Q , we need concepts called **start nodes** and **end nodes**. The start nodes represent the firstly executed leaf set in Q ; Similarly, the end nodes represent the last executed leaf set in Q .

Definition 3.13 (Start node and end node set). Given a process tree Q , its start node $S(Q)$ and end node set $E(Q)$ is defined as following.

- $Q = (a) \Rightarrow S(Q) = \{(a)\} = E(Q)$;
- $Q = \rightarrow (Q_1, Q_2, ..Q_n) \Rightarrow S(Q) = S(Q_1); E(Q) = E(Q_n)$. The start node set is the start nodes of its first child, and the end node set is the end nodes from its last child.
- $Q = \times (Q_1, Q_2, ..Q_n) \Rightarrow S(Q) = \cup_{i \in \{1,..n\}} S(Q_i), E(Q) = \cup_{i \in \{1,..n\}} E(Q_i)$.
For an exclusive choice block, its start and end node sets are the union of start and end nodes from each child; Any set from the union is an option start node set to represent the execution of Q .
- $Q = \wedge (Q_1, Q_2, ..Q_n) \Rightarrow S(Q) = \prod_i S(Q_i), E(Q) = \prod_i E(i)$.
For parallel relation, the start nodes are all the start nodes from each child and end nodes are all the end nodes from each child.
- $Q = \circ (Q_1, Q_2, ..Q_n) \Rightarrow S(Q) = S(Q_1), E(Q) = E(Q_1)$
The start node sets and end node sets depend on its first child.

3.3 Inductive Miner

Among multiple discovery techniques to mine a process model from an event log. On one hand, it guarantees the construction of a sound models. On the other hand, it is flexible and scalable w.r.t. an event log. In this section, we explain its algorithm in detail.

3.3.1 Construct a Directly-Follows Graph

At the start, an event log L is scanned to extract the *directly-follows relation* of events which describes the execution order of pair activities. We denote directly-follows relation in an event log by $>_L$.

For example, given an event log

$$L_{IM} = \{ \langle a, c, d \rangle^{20}, \langle b, c, e \rangle^{10}, \langle a, c, e \rangle^{20}, \langle b, c, d \rangle^{10} \},$$

a is directly followed by b and c , denoted as $a >_L b, a >_L c$; d directly follows b and c , $b >_L d, c >_L d$.

later, those directly-follows relations are combined together to build a directly-follows graph with frequency. According to [6, 8], formal definitions for the concepts above are given.

Definition 3.14 (Directly-follows Graph). The directly-follows relation $a >_L b$ is satisfied iff

$$\exists \sigma \in L, 1 \leq i < |\sigma|, \sigma(i) = a \text{ and } \sigma(i+1) = b.$$

A directly-follows graph of an event log L is $G(L) = (A, F, A_{start}, A_{end})$ where A is the set of activities in L , $F = \{(a, b) \in A \times A | a >_L b\}$ is the directly-follows relation set, A_{start}, A_{end} are the set of start and end activities respectively, $A_{start} = \{a | \exists \sigma \in L, a = \sigma(1)\}$, $A_{end} = \{a | \exists \sigma \in L, a = \sigma(|\sigma|)\}$.

What's more, the frequency information of the directly-follows relation is stored to express the relation strength and denoted as cardinality.

Definition 3.15 (Cardinality in a directly-follows graph). Given a directly-follows graph $G(L) = (A, F, A_{start}, A_{end})$ derived from an event log L , the cardinality in $G(L)$ is defined respectively on the F , A_{start} , and A_{end} .

- $\forall (a, b) \in F, c(a, b) = \sum_{\sigma \in L} |\{i \in \{1, 2, \dots, |\sigma|\} | \sigma(i) = a \text{ and } \sigma(i+1) = b\}|$ is the frequency for directly-follows relation $a >_L b$.
- $\forall a \in A_{start}, c(a) = \sum_{\sigma \in L} |\{\sigma | \sigma(1) = a\}|$ is the frequency for a start activity.
- $\forall a \in A_{end}, c(a) = \sum_{\sigma \in L} |\{\sigma | \sigma(|\sigma|) = a\}|$ is the frequency for an end activity.

According to definitions above, we obtain a directly-follows graph from event log L_{IM} as shown in the Figure 3.3a. Cardinality information is listed on the connections of activities.

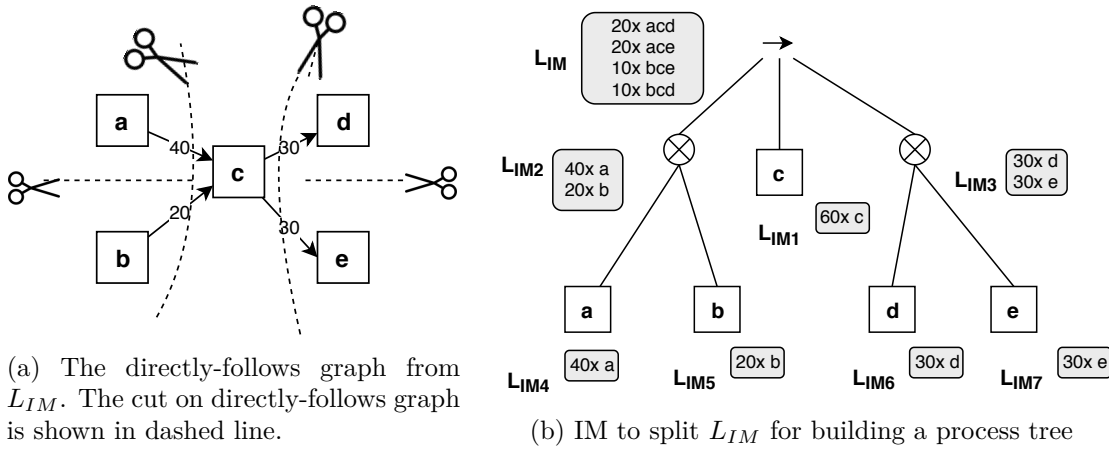


Figure 3.3: Inductive Miner to discover a process tree from event log L_{IM}

3.3.2 Split The Event Log Into Sublogs

Based on the directly-follows graph, the Inductive Miner finds the most prominent cut which is applied afterwards to split the event log into smaller sublogs. Corresponding to process tree operators $\{\rightarrow, \times, \wedge, \odot\}$, cuts consist of *exclusive-choice cut*, *sequence cut*, *parallel cut* and *redo-loop cut*. They are selected in the following order. Firstly, a maximal

exclusive-choice cut is tried to split the directly-follows graph; if it is not available, then a maximal sequence cut, a maximal parallel cut and a redo-loop cut are applied in sequence. As a result, sublogs are generated based on those cuts. Meanwhile, the operators which correspond to those cuts are used to build the process tree. The same procedure is applied again on the sublogs until single activity sets.

As an example, we apply the splitting on event log L_{IM} and $G(L_{IM})$ illustrated in Figure 3.3b. Firstly, the \rightarrow cut is applied and divides the whole event log into three sublogs $L_{IM1}, L_{IM2}, L_{IM3}$. Since L_{IM1} includes only one single activity, so we stop splitting and build a corresponding node in the process tree. Next, L_{IM2}, L_{IM3} are split by exclusive choice cuts separately until meeting single set in $L_{IM4}, L_{IM5}, L_{IM6}, L_{IM7}$.

Furthermore, a process tree is able to be converted into Petri net.

Chapter 4

Algorithm

This chapter begins with a general framework to repair a reference process model by incorporating the negative instances. A concrete algorithm within the framework is proposed in the subsequent sections.

4.1 General Framework for Repairing Process Models

Figure 4.1 shows our proposed framework to repair a process model with negative information. The inputs are a reference process model and a labeled event log. The reference process model can be in multiple types, e.g. Petri net, process tree. Traces in the labeled event log are classified as positive or negative in respect to some KPIs of business processes. The output is a repaired process model with the same type as the reference model.

The basic idea behind the framework is to unify the impact from the reference model M , positive sublog L^+ and negative sublog L^- into data models. Those data models are of the same type and denoted as D^M , D^+ and D^- respectively. Then we consider all impact from models and generate a new data model D^n . D^n is later transformed into a process model M^r as the output. Several post-processes are optional to improve the repaired model M^r .

After defining a data model to unify the impact and implementing the process modules in the general framework, we propose a concrete solution which also considers the negative information on model repair.

4.2 Algorithm

Given the inputs, an event log and a Petri net as the reference process model M as shown in Figure 1.4, our task is to repair the referenced Petri net based on the actual event log with consideration of negative information.

Based on this scope, the main modules in the framework in Figure 4.1 are designed and developed to repair the reference Petri net. We denote our repair algorithm as **dfg-repair**.

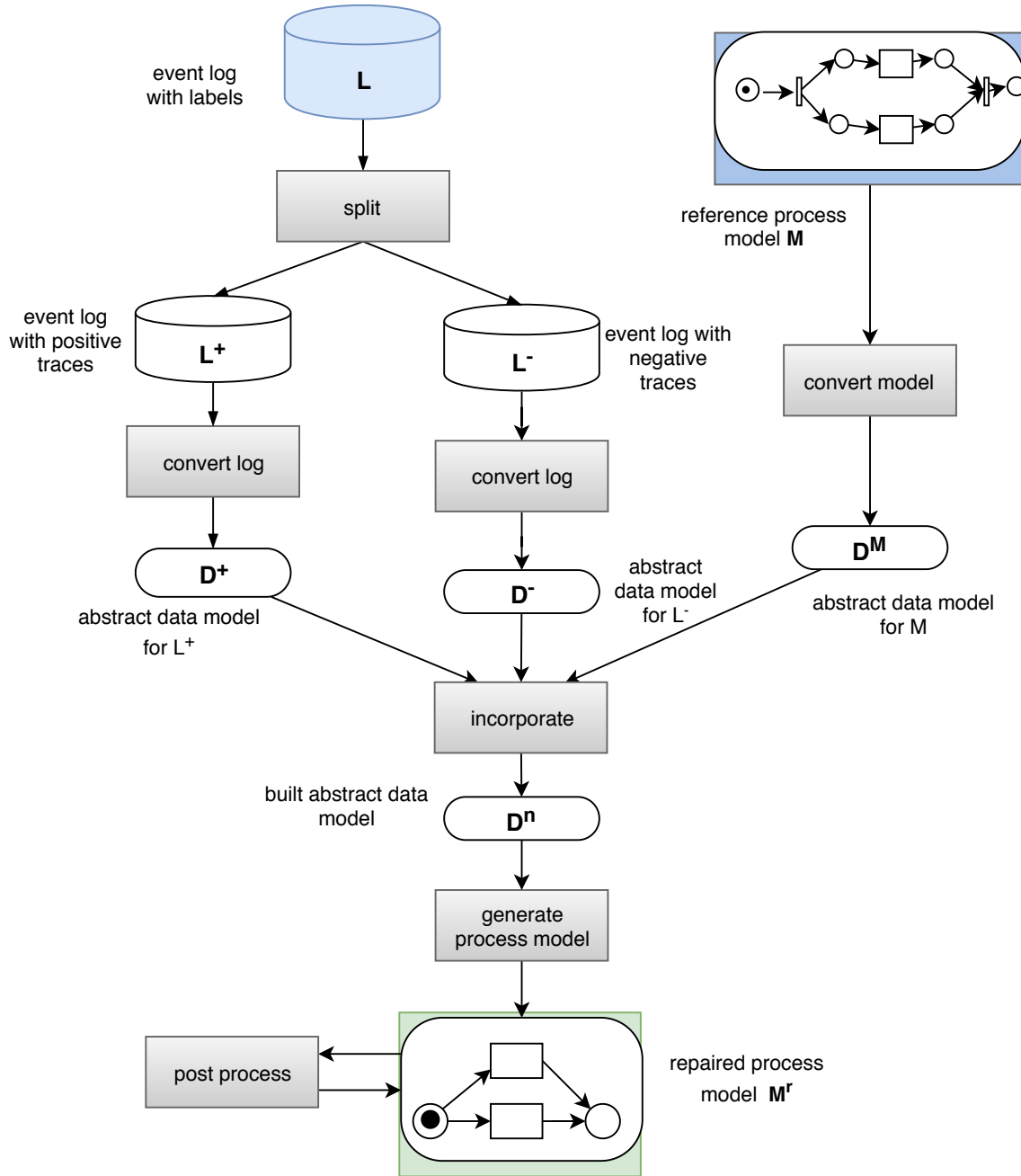


Figure 4.1: Model Repair Architecture – Rectangles represents processes and data models in eclipse shape. Input event log and the reference model are in blue, while the repaired model is in green.

First of all, we define a proper data model to represent the impact from Petri net M , and the event log L . Then, we list all the modules and describe our basic ideas to implement them.

4.2.1 Unified Data Model

We choose the directly-follows graph as the basis of our unified data model to represent the impact from the reference model and the event log. The reasons are, (1) there exist transformation algorithms to extract a directly-follows graph from an event log and to convert directly-follows graph into process tree or Petri net, which saves the development effort by reusing them; (2) the cardinality of directly-follows graphs can be used to express the impact strength.

Although we can derive three directly-follows graphs from the reference model, the positive and negative event logs respectively, their cardinalities are in different level. Therefore, to incorporate those graphs together, we introduce a concept called unified cardinality to bring all the impact onto the same level, which is the percentage to the sum of total cardinalities with a range $[0-1]$.

Definition 4.1 (Unified cardinality). Given a directly-follows graphs $G(L) = (A, F, A_{start}, A_{end})$ for a model, the unification for this graph is a function $u : F \rightarrow \mathbb{R}$ that has the following definition:

for each directly-follows relation $(a, b) \in F$,

$$u(a, b) = \frac{c(a, b)}{\sum_{(a', b') \in F} c(a', b')}$$

for start activities $a \in A_{start}$,

$$u(a) = \frac{c(a)}{\sum_{a' \in A_{start}} c(a')}$$

Similarly for end activities $a \in A_{end}$,

$$u(a) = \frac{c(a)}{\sum_{a' \in A_{end}} c(a')}$$

A directly-follows graph with unified cardinality is denoted as a unified directly-follows graph $D(L) = (A, F, A_{start}, A_{end}, u)$. After analyzing the positive, negative instances from event logs and the reference process model, $D(L_{pos})$, $D(L_{neg})$ and $D(L_{ext})$ are generated as the directly-follows graphs with unified cardinalities.

4.2.2 Modules List

After fixing the unified data models, in order to repair the reference model, the following list of modules are necessary.

- *Split event log into positive and negative sublogs* The event log L is split into an event log L^+ with only positive traces and an event log L^- with negative traces.

- *Convert event logs into unified directly-follows graphs, D^+, D^-* Two unified directly-follows graphs are generated respectively for the positive instance and negative instances from the event log.
- *Convert reference model into unified directly-follows graph D^M*
- *Incorporate unified directly-follows graphs* Three unified directly-follows graphs D^M, D^+, D^- are combined into one single directly-follows graph D^n after balancing their impact.
- *Generate process models from D^n* Process models are mined from the repaired data model D^n .
- *Post process the repaired model* Several post-processes are optionally applied on the generated process model, in order to improve the process model quality according to certain criteria.

Among those modules, for simplicity, we skip the details for the module *Split event log into positive and negative sublogs*. The details of the concrete algorithms to implement other modules are provided in the subsequent sections.

4.2.3 Convert Event Logs into Unified Directly-follows Graphs

Given an event log, to retrieve its unified directly-follows graph, we need to obtain its directly-follows graph at first. There is an existing procedure *IMLog2Dfg* from [8]. *IMLog2Dfg* traverses traces in the event log, extracts directly-follows relations of activities, and generates a directly-follows graph based on those relations.

By applying *IMLog2Dfg* separately on the event logs L^+ and L^- , we generate two directly-follows graphs $G(L_{pos})$ and $G(L_{neg})$. In the next step, the cardinalities from those graphs are unified according to Definition 4.1 and become a part of unified data model $D(L_{pos})$ and $D(L_{neg})$.

4.2.4 Convert Reference Model into Unified Directly-follows Graph

The basic idea behind this convert is to transform a reference process model into a directly-follows graph and then unify this directly-follows graph into D^M .

To generate a directly-follows graph from a Petri net, we investigate the execution order of activities with help of a transition system. In a transition system, the set of activities directly before a state have to be executed to reach this state, while the activities after this state are enabled to execute only after reaching this state. This implies the set of the activities before the state is executed before the set after the state. By analyzing the transition system, directly-follows relations between activities are extracted and used later to a directly-follows graph for the reference model.

From the positive and negative event logs, we can get the cardinality for corresponding directly-follows graph to represent the strength of this directly-follows relation. However, when the existing model is transformed into directly-follows graph $G(L_{ext})$, there is no point to assign cardinality on each edge. So we just set cardinality with 1 for each

arc. Based on this cardinality assignment, we attain the unified data model D^M for the reference Petri net.

4.2.5 Incorporate Unified Directly-follows Graphs

After the unification, the impact from the existing model, the impact from positive and negative instances are in the same level and represented in D^M , D^+ , and D^- . The strategy to incorporate those three models is that directly-follow relations are added into the repaired data model D^n if the total support from D^M and D^+ exceeds the rejection force from D^- . The other directly-follows relations are rejected. Namely, we balance all impact by subtracting the unified cardinality of D^- from the sum of unified cardinality in D^M and D^+ .

Definition 4.2 (Incorporating method). For any directly-follows relation from D^M , D^+ , and D^- , we balance all forces on it in the following way.

- For one directly-follows relation,

$$u^n(a, b) = u^M(a, b) + u^+(a, b) - u^-(a, b)$$

- For a start activity $a \in A_{start}^M \cup A_{start}^+ \cup A_{start}^-$,

$$u^n(a) = u^M(a) + u^+(a) - u^-(a)$$

- For an end activity $a \in A_{end}^M \cup A_{end}^+ \cup A_{end}^-$

$$u^n(a) = u^M(a) + u^+(a) - u^-(a)$$

In real life, there exists various needs to address the impact either from the existing model, the positive instances or the negative instances. To meet this requirement, three control parameters w^M , w^+ , and $w^- \in [0, 1]$ are assigned respectively to each unified cardinality from the existing model, and positive and negative instances. The weighted unification is modified in the way bellow.

Definition 4.3 (Weighted incorporating method). Given the control weight w^M , w^+ , and $w^- \in [0, 1]$, the weighted incorporating method to balance forces from D^M , D^+ , and D^- for D_n is defined below.

- For one directly-follows relation,

$$u_w^n(a, b) = w^M \cdot u^M(a, b) + w^+ \cdot u^+(a, b) - w^- \cdot u^-(a, b)$$

- For a start activity $a \in A_{start}^M \cup A_{start}^+ \cup A_{start}^-$,

$$u_w^n(a) = w^M \cdot u^M(a) + w^+ \cdot u^+(a) - w^- \cdot u^-(a)$$

- For an end activity $a \in A_{end}^M \cup A_{end}^+ \cup A_{end}^-$

$$u_w^n(a) = w^M \cdot u^M(a) + w^+ \cdot u^+(a) - w^- \cdot u^-(a)$$

By adjusting the weights of w^M , w^+ , and w^- , different focus can be reflected by the model. For example, by setting $w^M = 0, w^+ = 1, w^- = 1$, the existing model is ignored in the repair, while the original model is kept in situation $w^M = 1, w^+ = 0, w^- = 0$.

Next, we filter the directly-follows relation according to its weighted cardinality. If the cardinality over one certain threshold, it indicates a significant support to add this relation into the repaired data model D^n . By adding those directly-follows relation, we build a unified directly-follow graph D^n over this threshold.

4.2.6 Generate Process Models from D^n

The result of the last step above is a unified directly-follows graph D^n with weighted cardinality. To generate a process model from it, we convert D^n firstly into a general directly-follows graph G^n . Analyzing the weighted cardinality, we transform the weighted cardinality by multiplying the number of traces of the labeled event log.

Later, based on G^n , an existing procedure called *Dfg2ProcessTree* is applied to mine process models like process tree and Petri net [8]. It finds the most prominent split from the set of exclusive choice, sequence, parallelism, and loop splits on a directly-follows graph. Afterward, the corresponding operator to the split is used to build a block-structured process model called a process tree. Iteratively, the split sub graphs are passed as inputs for the same procedure until one single activity is reached and no split is available. A process tree is output as the mined process model and can be converted into another process model called Petri net.

4.2.7 Post Process on the Process Model

Due to the intrinsic characters of Inductive Miner, the dependency from activities which are not directly-following can not be discovered. To improve precision of the generated model, we propose one post process to add the long-term dependencies to the model. Additionally, another post process to simplify the model is introduced by deleting redundant silent transitions and places.

4.2.7.1 Add Long-term Dependency

Obviously, long-term dependency relates to the structure of choices in process model, such as exclusive choice, loop, and or structure. Due to the complexity of handling long-term dependency on or and loop structure, we skip them but focus only on the long-term dependency in the **exclusive choice structure**.

The generated process tree mined by Inductive Miner is used as an intermediate process model to detect long-term dependency. A process tree is (1) easy to extract the exclusive choice structure, since it is block-structured. (2) easy to transform a process tree to a Petri net as the final model.

An exclusive choice structure is represented as **xor block** in a process tree. For the sake of convenience, we name a subtree of an xor block as one **xor branch**. For two arbitrary

xor branches X, Y with long-term dependency, denoted as $X \rightsquigarrow Y$, they have to satisfy the conditions:

- they have a sequential order;
- they have significant correlation. In this thesis, we assume, the more frequent they are executed in the same traces, the more significant correlation they have.

In the following, the definition for the order of nodes in process tree is given and applied on the xor branches in this thesis.

Definition 4.4 (Order of nodes in process tree). Node X is before node Y , written in $X \prec Y$, if X is always executed before Y . In the context of process tree structure, $X \prec Y$, if their least common ancestor, i.e., a process tree node as an ancestor of X and Y and closest to X and Y , is a sequential node, and X positions before Y .

To define the correlation of xor branches, several concepts listed below are necessary.

Definition 4.5 (The execution of a process tree Q). With respect to a trace $\sigma \in L$, a process tree Q is executed in σ , denoted as $\sigma \models Q$, when

- for $Q = (a)$, if $\exists i, 1 \leq i \leq |\sigma|, \sigma(i) = a$.
- for $Q = \rightarrow (Q_1, Q_2, \dots, Q_n)$, if $\forall Q_i, \sigma \models Q_i$ is executed in order.
- for $Q = \times (Q_1, Q_2, \dots, Q_n)$, if $\exists Q_i, \sigma \models Q_i$.
- for $Q = \wedge (Q_1, Q_2, \dots, Q_n)$, if $\forall Q_i, \sigma \models Q_i$.
- for $Q = \circ (Q_1, Q_2, \dots, Q_n)$, if $\sigma \models Q_1$ and $\exists Q_i, \sigma \models Q_i \Rightarrow \sigma \models Q_1$ after Q_i .

With the definition above, we can formalize the frequency of xor branches in the event log as below.

Definition 4.6 (Xor branch frequency). The frequency for an xor branch X in event log L is the count of traces, $f : X \rightarrow N$.

$$f_L(X) = \sum_{\sigma \in L} |\{\sigma | \sigma \models X\}|$$

For multiple xor branches, the frequency of their coexistence in event log L is defined as the count of traces with all the occurrence of xor branches X_i ,

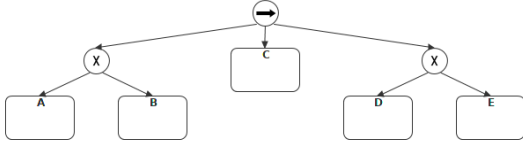
$$f_L(X_1, X_2, \dots, X_n) = \sum_{\sigma \in L} |\{\sigma | \forall X_i, \sigma \models X_i\}|$$

As an example, given the same model $M4$ and reviewed here in Figure 4.2a, and a labeled event log L ,

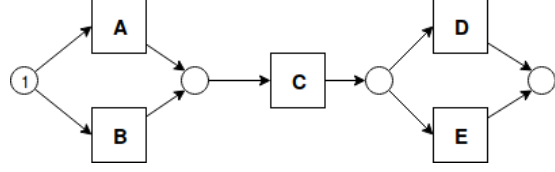
$$L := \{ \langle A, C, D \rangle^{10, pos}, \langle A, C, E \rangle^{30, pos}, \langle B, C, E \rangle^{10, pos}, \langle B, C, D \rangle^{40, neg} \}$$

the frequencies of the coexistence of different xor branches are $f_{L+}(A, D) = 10$, $f_{L+}(A, E) = 30$, $f_{L+}(B, E) = 10$, and $f_{L-}(B, D) = 40$.

The frequency of the coexistence of multiple xor branches in positive and negative event logs reflects the correlation of those xor branches. The long-term dependency in the existing model also affects the long-term dependency in the repaired model. However, since the repaired model possibly differs from the existing model, the impact of long-term



(a) The reviewed process tree PT



(b) The reviewed Petri net

dependency from the existing model becomes difficult to detect. With time limit, we only consider the impact from positive and negative instances on the long-term dependency.

Definition 4.7 (Correlation of xor branch). The correlation to express dependency of two branches $X, Y \in BB(Q)$ is expressed into

$$d(X, Y) = d^+(X, Y) - d^-(X, Y)$$

where

$$d^+(X, Y) = \frac{f_{L^+}(X, Y)}{\sum_{Y' \in BB(Q), Y' \neq X} f_{L^+}(X, Y')}, \quad d^-(X, Y) = \frac{f_{L^-}(X, Y)}{\sum_{Y' \in BB(Q), Y' \neq X} f_{L^-}(X, Y')}.$$

In this thesis, we call the correlation strong with respect to one threshold t , if

$$d(X, Y) > t.$$

Following the example above, the correlations of their xor branches are $d(A, D) = 0.2, d(A, E) = 0.6, d(B, E) = 0.2, d(B, D) = -1$. Since the xor branches of those pairs are already in order, if the threshold for long-term dependency is set as $t = 0$, we get a long-term dependency set, $LT = \{A \rightsquigarrow D, A \rightsquigarrow E, B \rightsquigarrow E\}$, for the process tree.

4.2.7.2 Cases Analysis

There are various types of long-term dependencies that are able to happen for two xor blocks. To explain those situations better, we define concepts called sources and targets of long-term dependency and then give an example of one Petri net with long-term dependency.

Definition 4.8 (Source and target set of long-term dependency). The source set of the long-term dependency in two xor blocks is the set of all xor branches, $LT_S := \{X | \exists Y, X \rightsquigarrow Y \in LT\}$, and target set is $LT_T := \{Y | \exists X, X \rightsquigarrow Y \in LT\}$.

For one xor branch $X \in S$, the target xor branch set relative to it with long-term dependency is $LT_T(X) = \{Y | X \rightsquigarrow Y \in LT\}$. Similarly, the source set related to one target xor branch is $LT_S(Y) = \{X | X \rightsquigarrow Y \in LT\}$.

At the same time, we use S and T to represent the set of xor branches for source and target xor block with long-term dependency. Given a process tree in Figure 4.2a, two xor blocks are contained in the model. They are able to have the following long-term dependencies.

1. $LT = \{A \rightsquigarrow D, A \rightsquigarrow E, B \rightsquigarrow D, B \rightsquigarrow E\}$.
 $LT_S = \{A, B\}, LT_T = \{D, E\}, |LT| = |S| \cdot |T|$, which is a full long-term dependency where all combinations of source and target xor branches have significant correlation.

2. $LT = \{A \rightsquigarrow D, A \rightsquigarrow E, B \rightsquigarrow E\}$.
 $LT_S = \{A, B\}, LT_T = \{D, E\}$ $LT_S = S$ and $LT_T = T, |LT| < |S| \cdot |T|$. it does not cover all combinations. But for one xor branch $X \in S, LT_T(X) = T$, it has all the full long-term dependency with T .
3. $LT = \{A \rightsquigarrow D, B \rightsquigarrow E\}$.
 $LT_S = \{A, B\}, LT_T = \{D, E\}$ $LT_S = S$ and $LT_T = T, |LT| < |S| \cdot |T|$. For all xor branch $X \in S, LT_T(X) \subsetneq T$, none of xor branch X has long-term dependency with T .
4. $LT = \{A \rightsquigarrow D, B \rightsquigarrow D\}$.
 $LT_S = S, LT_T \subsetneq T$. There exists at least one xor branch $Y \in T$ which has no long-term dependency on it.
5. $LT = \{A \rightsquigarrow D, A \rightsquigarrow E\}$.
 $LT_S \subsetneq S, LT_T = T$. There exists at least one xor branch in source $X \in S$ which has no long-term dependency on it.
6. $LT = \{A \rightsquigarrow E\}$.
 $LT_S \subsetneq S, LT_T \subsetneq T$. There exists at least one xor branch in source $X \in S$ and one xor target xor branch which has no long-term dependency on it.
7. $LT = \emptyset$. There is no long-term dependency on this set.

In the listed situations, situation 1 is a full long-term dependency, which allows all combinations like the original models, and no changes are necessary on the model. Situation 7 has no long-term dependency, where the correlation of them is either not so strong from the positive instances or rejected from negative instances. In this thesis, we do not take those two situations into consideration.

4.2.7.3 Way to Express Long-term Dependency

After detecting long-term dependencies with xor branches in process tree, it is not possible to express those dependencies on the process tree by using the current process tree operators. Moreover, the final output of our task demands a Petri net. Therefore, we convert the generated process tree into a Petri net and express long-term dependency on the Petri net with corresponding structures.

According to [9], adding places to transitions in Petri net can limit the model behavior. Any transition demands a token from its input places to fire. When there is no token at this place, the transitions are not enabled. By adding extra places to the Petri net, it can block negative behaviors which are not expected in the aspect of business performance.

Long-term dependency limits the available choices to fire transitions after the previous xor branch executes. So to express long-term dependency, our basic idea is to add places connected to transitions in the Petri net. In addition, because one xor branch can be as a source to multiple long-term dependencies and one xor branch can be as a target to multiple long-term dependencies, silent transitions are also needed to explicitly address a long-term dependency.

Firstly, the long-term dependency set from process tree is mapped to the corresponding Petri net. For an arbitrary long-term dependency set $LT = \{X_i \rightsquigarrow Y_j | 1 \leq i \leq m, 1 \leq$

$j \leq n\}$ with $S = \{X_1, X_2, \dots, X_m\}$ and $T = \{Y_1, Y_2, \dots, Y_n\}$ in a Petri net, we add places after the source xor branches in Petri net, $P_S = \{p_{X_i} | X_i \in LT_S\}$, and places before target xor branches, $P_T = \{p_{Y_j} | Y_j \in LT_T\}$. For each long-term dependency $X_i \rightsquigarrow Y_j$ in LT, there is silent transition t with $p_{X_i} \rightarrow t \rightarrow p_{Y_j}$. The steps to add silent transitions and places according to the long-term dependency are listed in algorithm 1.

Algorithm 1: Add long-term dependency between pure xor branch

```

1  Y is dependent on X;
2  if X is leaf then
3    | One place is added after this leaf;
4  end
5  if X is Seq then
6    | Add a place after the end node of this branch;
7    | The node points to the new place;
8  end
9  if X is And then
10   | Create a place after the end node of every children branch in this And xor
    | branch;
11   | Combine all the places by a silent transition after those places;
12   | Create a new place directly after silent transition to represent the And xor
    | branch;
13 end
14 if Y is leaf then
15   | One place is added before this leaf;
16 end
17 if Y is Seq then
18   | Add a place before the end node of this branch;
19   | The new place points to this end node;
20 end
21 if Y is And then
22   | Create a place before the end node of every children branch in this And xor
    | branch;
23   | Combine all the places by a silent transition before those places;
24   | Create a new place directly before silent transition to represent the And xor
    | branch;
25 end
26 Connect the places which represent the X and Y by creating a silent transition.

```

Reviewing the example of process tree of Figure 4.2a, it has long-term dependency set $LT = \{A \rightsquigarrow D, A \rightsquigarrow E, B \rightsquigarrow E\}$. To express long-term dependency, the process tree is firstly transformed into the Petri net as Figure 4.2b. Then two extra places are added respectively after A and B; Next, two places before D and E are created to express that the xor branches are involved with long-term dependency. At end, for each long-term dependency, a silent transition is generated to connect the extra places after the source xor branch to the place before target place. The generated model with long-term dependency is displayed in Figure 4.3.

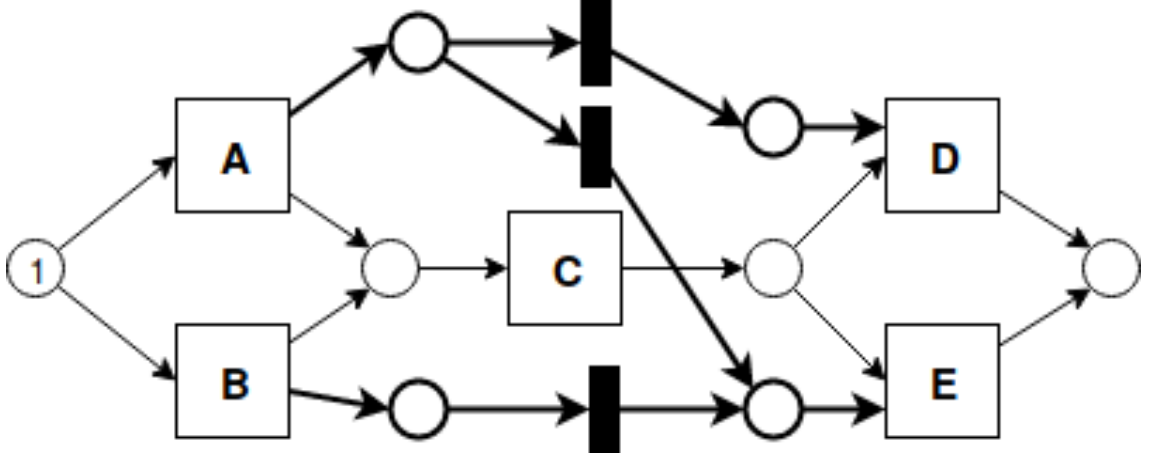


Figure 4.3: Model with long-term dependency $LT = \{A \rightsquigarrow D, A \rightsquigarrow E, B \rightsquigarrow E\}$.

4.2.7.4 Soundness Analysis

With Algorithm 1 by adding silent transitions and places to express long-term dependency, the model soundness can be violated. In the following section, we discuss the soundness in different situations.

Given a Petri net with long-term dependency $LT = \{X_i \rightsquigarrow Y_j | 1 \leq i \leq m, 1 \leq j \leq n\}$ on two xor blocks $S = \{X_1, X_2, \dots, X_m\}$ and $T = \{Y_1, Y_2, \dots, Y_n\}$. After applying the Algorithm 1, $P_S = \{p_{X_i} | X_i \in LT_S\}$, $P_T = \{p_{Y_j} | Y_j \in LT_T\}$, and silent transitions $E = \{\epsilon | p_{X_i} \rightarrow \epsilon \rightarrow p_{Y_j}\}$ are added.

The Petri net is sound if and only if (1) the soundness outside xor blocks with long-term dependency is not violated; and (2) soundness between xor blocks is kept. In the following, we check the model soundness with long-term dependency after applying Algorithm 1.

Soundness outside xor blocks.

Proof: The added silent transitions and places do not violate the execution outside of the xor blocks, because the extra tokens that are generated due to long-term dependency are constrained in the xor blocks, and it does not affect the token flows outside. As we know, the original model is sound. So the soundness outside xor blocks is not violated.

Soundness inside xor blocks.

For the xor blocks $S = \{X_1, X_2, \dots, X_m\}$ and $T = \{Y_1, Y_2, \dots, Y_n\}$ with long-term dependency $LT = \{X_i \rightsquigarrow Y_j | 1 \leq i \leq m, 1 \leq j \leq n\}$, only one xor branch can be fired in S. Without loss of generality, X_i is assumed to be enabled. After firing X_i , the marking distribution on the extra places are

$$M(p_{X_i}) = 1; \quad \forall p_{X_{i'}} \in P_S, i' \neq i, M(p_{X_{i'}}) = 0$$

If $LT_S = S, LT_T = T$, adding the long-term dependency in this situation does not violate the model soundness, we prove it in the following part.

- Safeness. Places cannot hold multiple tokens at the same time.

For all extra places p_{X_i} and p_{Y_j} ,

$$\forall p_{X_i} \in P_S, \sum M(p_{X_i}) = 1$$

Because $LT_S = S$, $X_i \in S$, so $X_i \in LT_S$, there exists one Y_j with $X_i \rightarrow Y_j$ and one $\epsilon, p_{X_i} \rightarrow \epsilon \rightarrow p_{Y_j}$. After firing X_i , the transition ϵ becomes enabled. After executing ϵ , the marking distribution turns to

$$M(p_{Y_j}) = 1; \quad \forall p_{Y_{j'}} \in P_T, j' \neq j, M(p_{Y_{j'}}) = 0$$

the marking distribution in the extra places are always

$$\sum M(p_{X_i}) \leq 1, \sum M(p_{Y_j}) \leq 1$$

- Proper completion. If the sink place is marked, all other places are empty. After firing Y_j , all the extra places hold no token. So it does not violate proper completion.
- Option to complete. It is always possible to reach the final marking just for the sink place. There is always one Y_j enabled after firing X_i to continue the subsequent execution.
- No dead part. For any transition there is a path from source to sink place. Because all $Y_j \in T$ are also in LT_T , there exists at least one $X_i \in S$ with long-term dependency with Y_j . After X_i is fired, one token is generated on the extra place p_{X_i} and can be consumed by silent transition ϵ in $p_{X_i} \rightarrow \epsilon \rightarrow p_{Y_j}$ to produce a token in p_{Y_j} , which enables xor branch Y_j and leaves no dead part.

In other situation, the model becomes unsound.

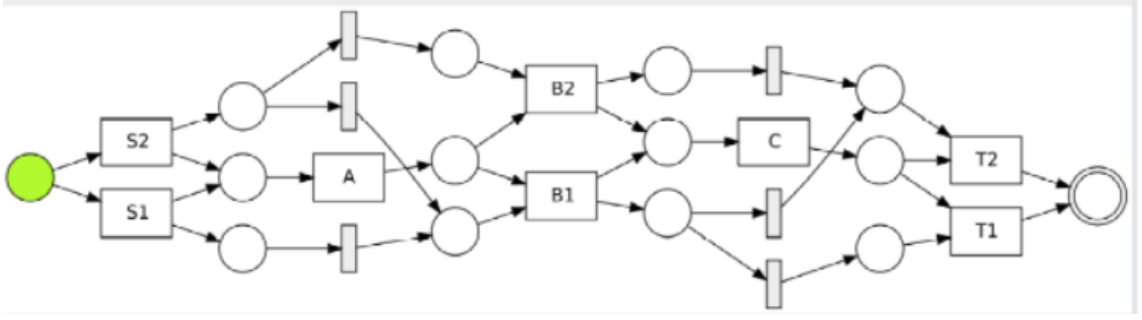
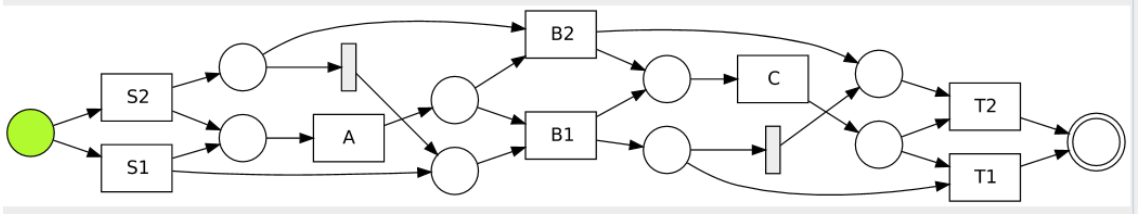
If $LT_S \neq S$, or $LT_T \neq \emptyset$, there exists one xor branch X_i with $X_i \notin LT_S$. When X_i is fired, it generates one token at place p_{X_i} , this token cannot be consumed by any Y_j . So it violates the proper completion.

If $LT_T \neq T$, there exists one $Y_j \notin LT_T$, $\nexists X_i, X_i \rightsquigarrow Y_j$, so with two input places but $Token(p_{Y_j}) = 0$, Y_j becomes a dead part, which violates soundness again. As a conclusion, to keep Petri net with long-term dependency sound, only situations with $LT_S = S, LT_T = T$ are considered.

4.2.8 Reduce Silent Transitions

Our method to represent long-term dependency can introduce redundant silent transitions and places, which complicates the model. So, we post process the Petri net with long-term dependency and delete redundant silent transitions and places in it. Silent transitions and places of a Petri net are redundant when the Petri net preserves the behavior without those transitions and places [22, 23]. There is an existing algorithm developed in the work of [23]. We reuse it as one post process in our thesis.

One example is given in the following graph. M_{lt} in Figure 4.4a has long-term dependency expressed in the silent transitions and places. The silent transition for $S2 \rightsquigarrow B1$ and silent transition for $B1 \rightsquigarrow T2$ belongs to the case (1). So they are kept in the model, while the other silent transitions are deleted. After reducing the redundant silent transitions, the model becomes M_r shown in Figure 4.4b. Those two models have the same behavior, yet the reduced model is simpler.

(a) A Petri net M_{lt} with redundant silent transitions(b) Petri net M_r with reduced silent transitions

4.2.9 Concrete Architecture

At last, we assemble all the modules together and give an overview architecture of our repair techniques. We reuse existing modules in gray rectangles in Figure 4.5, e.g. IM-Log2Dfg to convert an event log into directly-follow graph, Petrinet2TransitionSystem to transform a Petri net into a transition system. The other modules are programmed according to our specific needs and achieve the repair algorithm mentioned before. To achieve a more precise Petri net, the module to add long-term dependency becomes a necessary part. Yet, reduction on redundant places and silent transitions is optional.

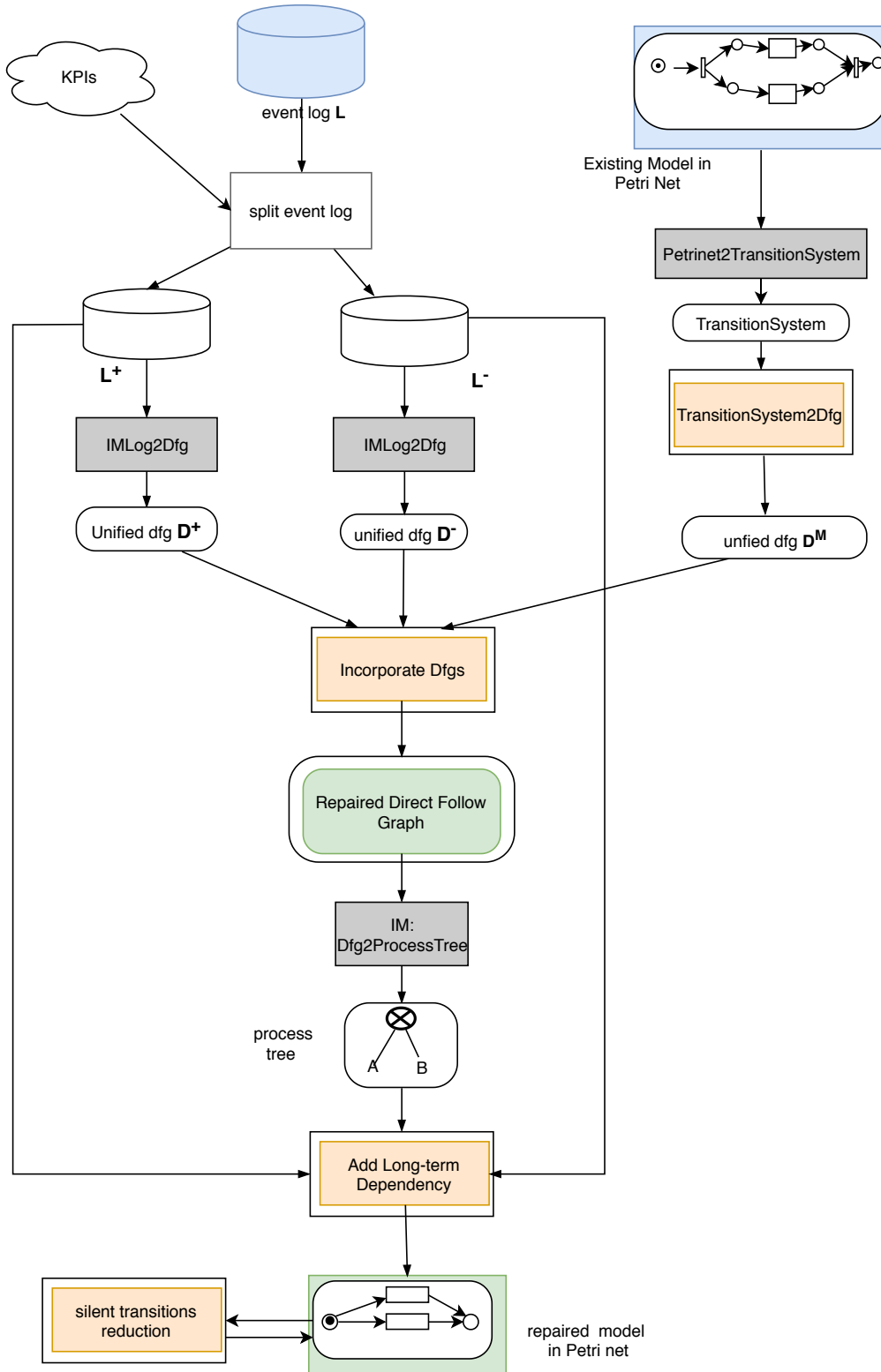


Figure 4.5: Model Repair Architecture – Rectangles represents processes and output data in eclipse shape, especially customized processes and data are in doubled eclipses. Input event log and existing model are in blue, KPIs are in cloud. The output is a petri net in green.

Chapter 5

Implementation

In this chapter, we begin with the introduction of implementation platforms for our methods and then show the use of those applications step by step.

5.1 Implementation Platforms

5.1.1 Process Mining Platform – ProM

ProM is an open-source process mining tool in Java that is extensible by adding a set of plug-ins [24]. ProM supports a wide variety of process mining techniques and is usually used for academic research. We implement the algorithm on ProM 6.8, which is the latest stable version. The corresponding plugin is *Repair Model By Kefang* and has been released online [25].

5.1.2 KNIME

KNIME Analytics Platform is an open-source software to help researchers analyze data. Multiple modules are integrated into this platform for loading, transforming and processing data. Researchers can achieve their goals by creating visual workflows composed of expected modules implemented as nodes with an intuitive, drag and drop style graphical interface, rather than focusing on any particular application area.

The reasons to integrate our techniques into KNIME are (1) KNIME is widely used in scientific research and benefits the application of our techniques; (2) KNIME supports automation of task workflow, which helps conduct more efficient experiments. However, the integration requires additional development effort.

5.2 Generate a Petri net

Firstly two dialogs are popped up to set the arguments, such as the event classifier to generate directly-follows graphs from event logs. Subsequently, a dialog is shown to set

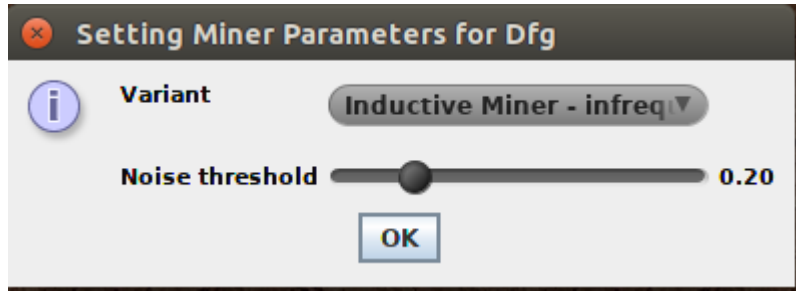


Figure 5.1: Inductive Miner Parameter Setting

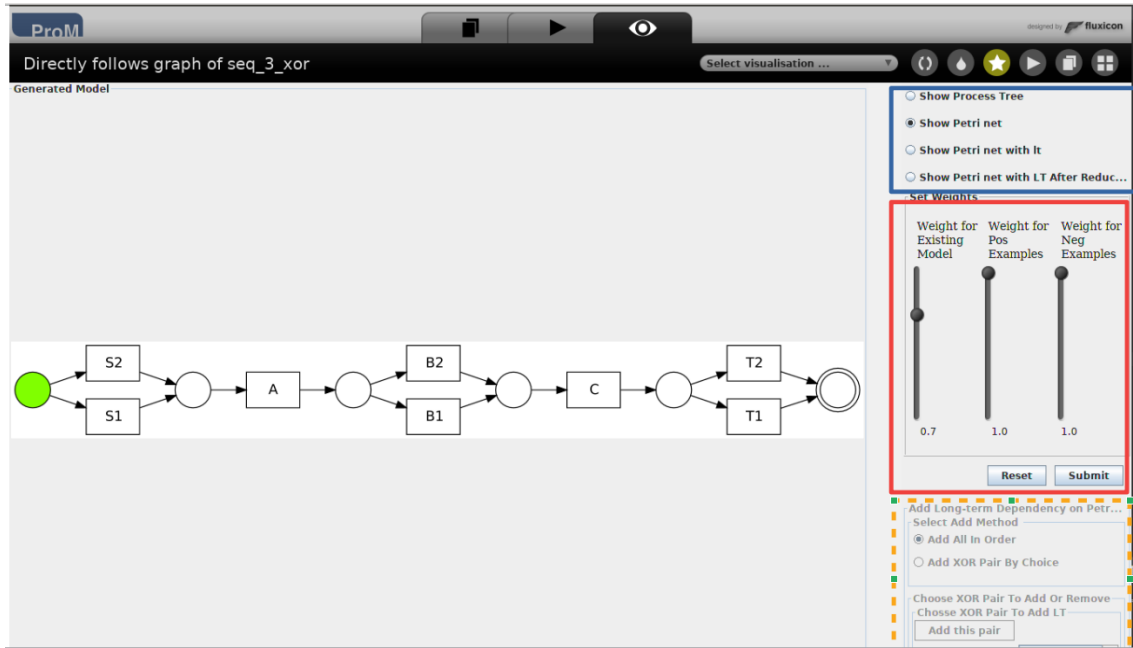


Figure 5.2: Generated Petri net without long-term dependencies

the Inductive Miner parameters. The parameters include the Inductive Miner variant and the noise threshold to filter the data. The dialog is displayed in Figure 5.1.

After setting the parameters, process models of process tree and Petri net without long-term dependencies can be generated by Inductive Miner and displayed in the result view in Figure 5.2. The left side is the model display area, where the right panel is to set the control parameters for the existing model, positive or negative instances. By adjusting the setting interactively, more flexibility is allowed in plug-in to repair model. By default, the generated model type and the weight sliders are enabled at first. The control panel for adding long-term dependencies is only triggered after choosing the option to repair model with long-term dependencies.

The model type is selected in the blue rectangle marked in Figure 5.2. It has 4 options to control the generated model type. Currently, the option "Show Petri net" is chosen, so the constructed model is a Petri net without long-term dependencies. The weights sliders are in a red rectangle. They adjust the weights for the existing model, positive and negative instances. Once those options are submitted, different process models are mined under different weights. The rectangle in orange are the invisible part to control

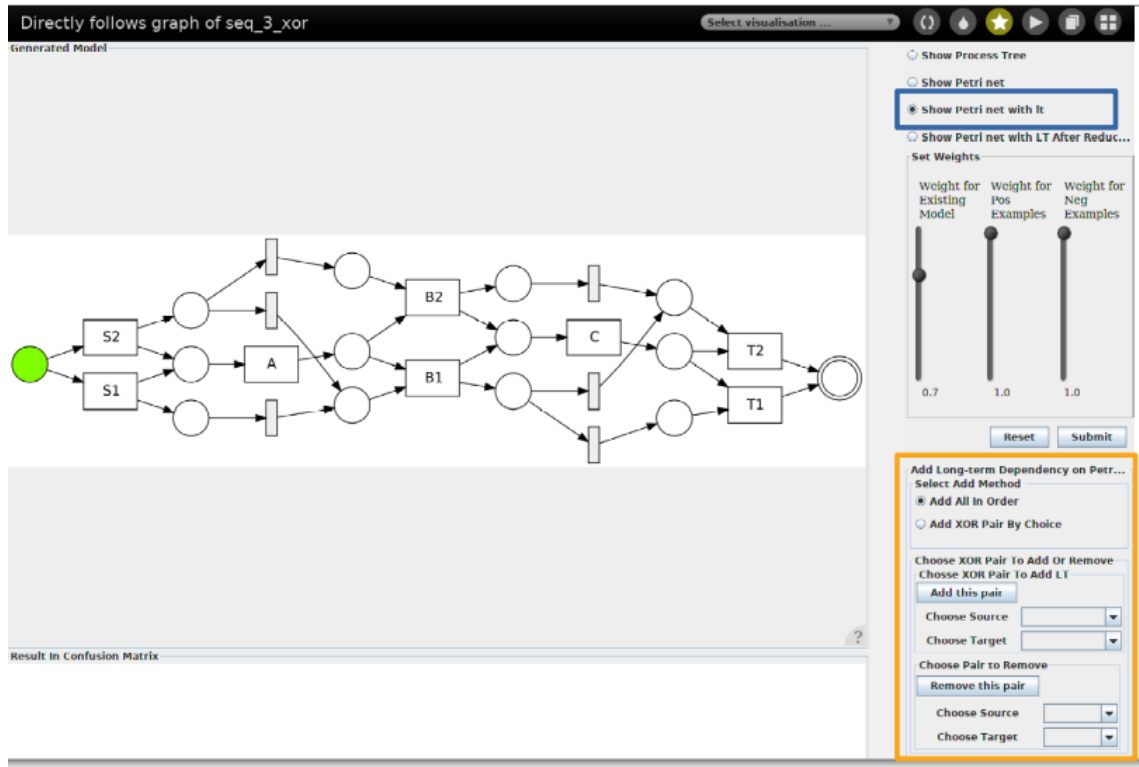


Figure 5.3: Petri Net with long-term dependencies

long-term dependencies options. It will be discussed in the next section.

5.3 Post Process to Add Long-term Dependencies

If we want to repair the Petri net with long-term dependencies, one post procedure is triggered to add long-term dependencies. This program in the background detects and puts places and silent transitions on Petri net directly mined from Inductive Miner to add long-term dependencies. As comparison, the same weight setting is kept like the Figure 5.2, but the option to show a Petri net with long-term dependencies is chosen. The resulted model is displayed in Figure 5.3.

Meanwhile, the control part of adding long-term dependencies turns visible in the orange rectangle like in Figure 5.3. It has two main options, one is to consider all long-term dependencies existing in the model, the other is to choose the part manually. It allows more flexibility for users. Below those two options are the manual selection panels, including a control part to add and remove long-term dependencies in two arbitrary xor blocks. As an example, the blocks Xor(S1,S2) and Xor(T1,T2) are chosen to add long-term dependencies. It results in the model in Figure 5.4.

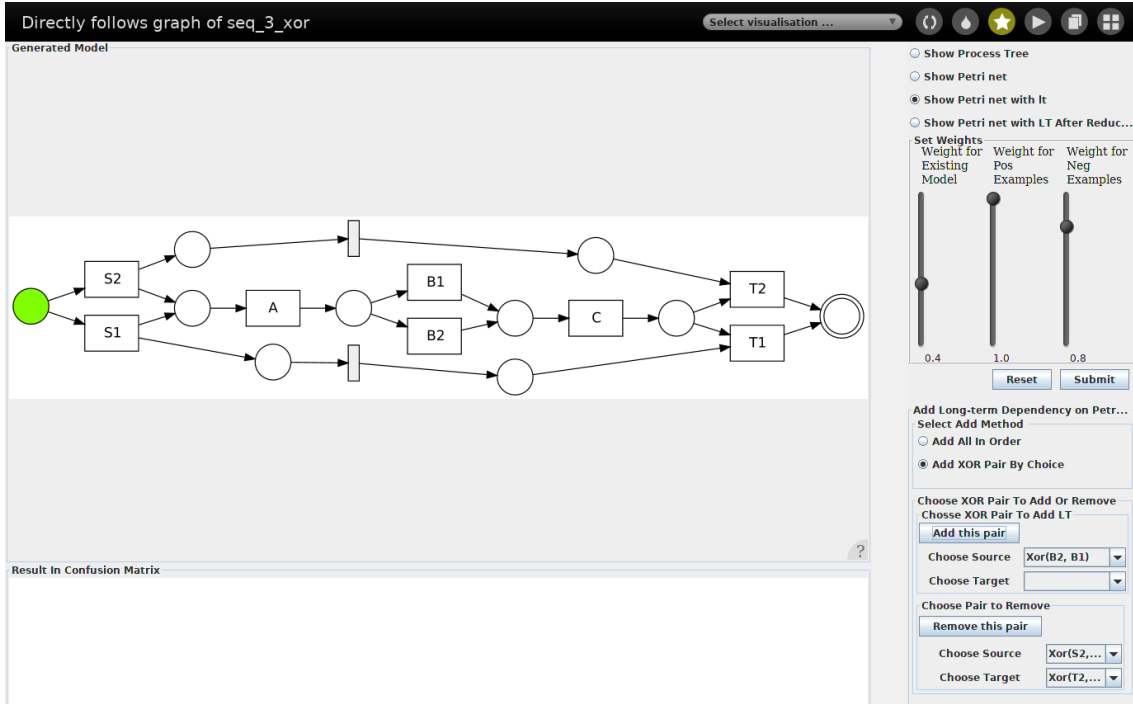


Figure 5.4: Petri net with selected long-term dependencies

5.4 Post Process to Reduce Redundant Silent Transitions and Places

By choosing the option of *Petri net with LT After Reducing* in model type, silent transitions and places are reduced to simplify the model. Under the same setting in Figure 5.2, the simpler model in Figure 5.5 is constructed, after the post processing of reducing silent transitions.

5.5 Additional Feature to Show Evaluation Result

Another feature in this plugin is to show the evaluation result based on confusion matrix. With the brief evaluation result, it helps set the parameters for the optimal Petri net.

After creating the current model in the left view, the evaluation program in background uses the event log and the current Petri net in the view as inputs. A naive fitness checking is applied on the repaired model with the event log. This procedure is based on the existing plugin in ProM – **PNetReplayer**. This plugin checks if the trace fits the model and give out the one possible deviation with minimal cost.

In our implementation, any deviation cost is set 1. Based on the deviation result and the label information on each trace, a confusion matrix is generated. Moreover, relative measurements like recall, precision are calculated and shown in the bottom of the left view in Figure 5.6. If the button of green rectangle in the right view *Show Confusion Matrix* is pressed again, the program is triggered again and generates a new confusion matrix result

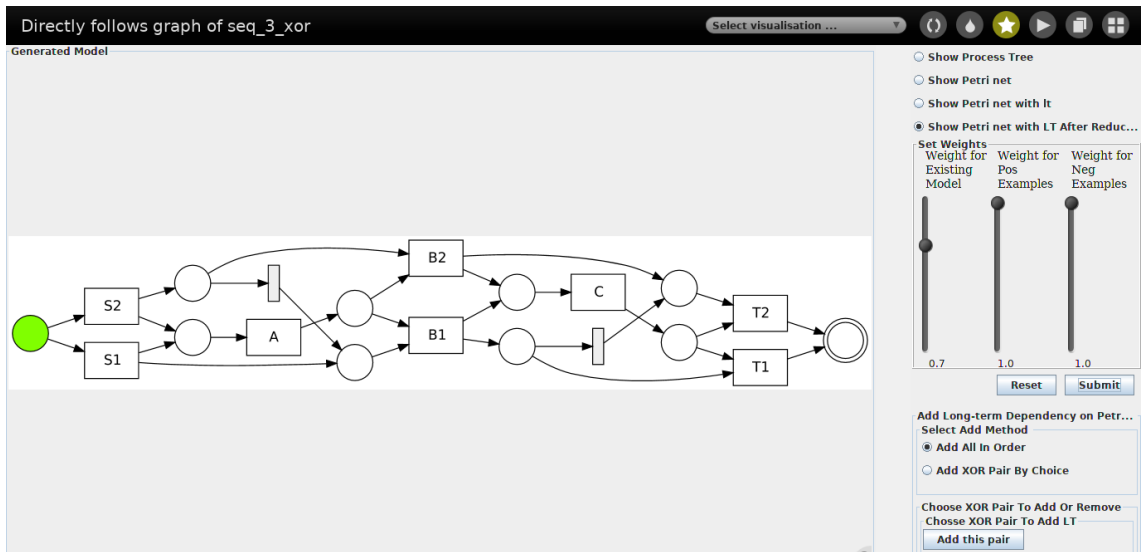


Figure 5.5: Petri net after reducing the silent transitions

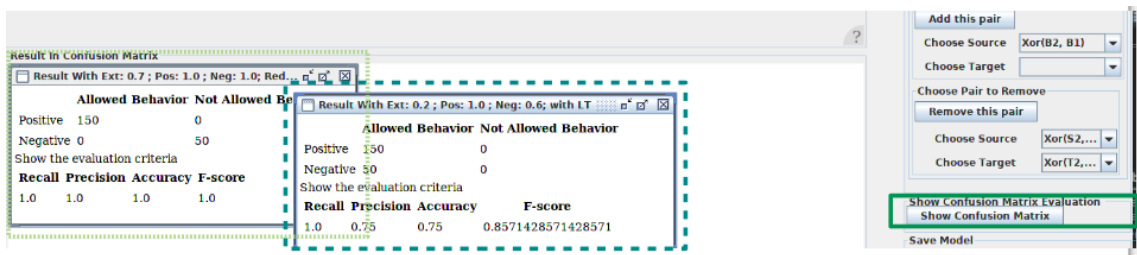


Figure 5.6: Generated Process Tree Model

in the dark green dashed rectangle which will be listed above the previous result in the dashed green area.

5.6 Integration into KNIME

Nodes in the workflow represents different modules corresponding the plugins in ProM. Each node has certain input ports on the left side to represent the required parameters and output ports on the right to pass processed results. By connecting the ports between nodes, data are passed and processed by one node after another. To integrate our algorithm into KNIME, other related modules on process mining are necessary, which can be divided into the following categories:

1. Data importer and exporter. The importers and exporters for event logs, process trees and Petri nets are implemented to load and save basic data for Process Mining.
2. Event logs manipulation. Nodes for splitting, sampling and assigning labels to event logs are implemented to benefit our experiments.
3. Classic discovery algorithms. Inductive Miner and Alpha Miner are integrated into KNIME to provide baselines for our algorithm.

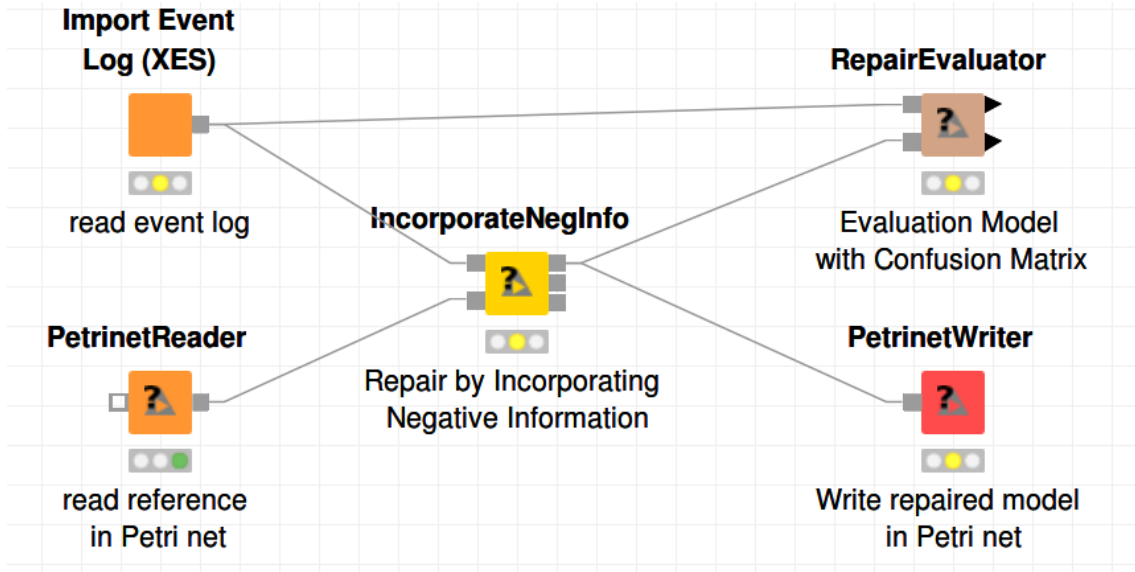


Figure 5.7: Integration of our repair techniques into KNIME

4. Model enhancement. Our proposed method is integrated in KNIME to repair model in Petri net.

To integrate our repair algorithm from ProM into KNIME, we need to create the workflow in the Figure 5.7. After reading a Petri net by *PetrinetReader* and an event log by *Import Event Log(XES)*, Node *IncorporateNegInfo* applies the algorithm in this thesis to repair a model in Petri net with incorporating negative information. The outputs have different kinds of Petri nets to match the ones generated in ProM, e.g., reduced Petri net with long-term dependencies, Petri net without long-term dependencies. In addition, we can evaluate our repaired model by using the node *RepairEvaluator*. At last, we can save the repaired Petri net by *PetrinetWriter*.

Chapter 6

Evaluation

In this chapter, we evaluate the proposed repair techniques based on the quality of repaired models. At first, we define the evaluation criteria. Next, we briefly introduce the test platforms KNIME and relevant ProM plugins tools. Then, we conduct two kinds of tests. One is based on the demo example proposed in the Section 1.1 part, and the other is based on real life data.

6.1 Evaluation Criteria

We evaluate the repair techniques based on the quality of repaired models with respect to the given event logs. In process mining, there are four quality dimensions generally used to compare the process models with event logs.

- **Fitness.** It quantifies the extent how well the model reproduces traces in the event log which is used to build the model.
- **Precision.** It quantifies the extent how the discovered model limits the completely unrelated behaviors that maybe show up in the event log.
- **Generalization.** It addresses the over-fitting problem when a model strictly matches to only seen behaviors but is unable to generalize the behaviors which are not seen in the event log.
- **Simplicity.** It captures the model complexity. According to Occam's razor principle, the model should be as simple as possible.

The four traditional quality criteria are proposed in the environment where only positive instances are available. Therefore, when it comes to the model performance, where negative instances are also possible, the measurement metrics need to be adjusted.

With labeled traces in the event log, the repaired model can be seen as a binary prediction model where the positive instances are supported while the negative ones are rejected. Consequently, the model evaluation becomes a classifier evaluation and confusion matrix is applied in our experiments.

Table 6.1: Confusion Matrix

		repaired model	
		allowed behaviors	not allowed behaviors
actual data	positive instance	TP	FN
	negative instance	FP	TN

Confusion matrix has a long history to evaluate the performance of a classification model. A confusion matrix is a table with columns to describe the prediction model and rows for actual classification of data. As seen as a binary classifier, the repaired model produces four outcomes according to the confusion matrix – true positive, true negative, false positive and false negative, which is shown in the Table 6.1.

- True Positive (TP): The execution allowed by the process model has a positive performance outcome.
- False Positive (FP): The execution allowed by the process model has a negative performance outcome.
- True Negative (TN): The negative instance is blocked by the process model.
- False Negative (FN): The positive instance is blocked by the process model.

Various measurements can be derived from confusion matrix. According to our application, the following criteria are chosen. Generally, there is a trade-off between the quality criteria. So the measurements below are only used to evaluate specific aspects of repair techniques.

- Recall. It represents the true positive rate and is calculated as the number of correct positive predictions divided by the total number of positive instances.

$$Recall = \frac{TP}{TP + FN}$$

- Precision. It describes the ability of the repaired model to produce positive instances.

$$Precision = \frac{TP}{TP + FP}$$

- Accuracy. It is the proportion of true positive and false results to the total results. In our case, it measures how well a model correctly allows the positive instances or blocks the negative instances.

$$Accuracy = \frac{TP + TN}{TP + TN + FP + FN}$$

- F-score is is the harmonic mean of precision and recall.

$$F_1 = \frac{2 * Recall * Precision}{Precision + Recall}$$

6.2 Experiment Platforms

KNIME, as a scientific workflow analytic platform, supports automating the workflow testing, which helps us repeat experiments efficiently. Yet, the integration of traditional process mining plugins into KNIME is out of our capability due to the time limit. Therefore, partial experiments with current repair techniques are still conducted in ProM.

6.2.1 KNIME

KNIME supports automating workflow for testing mainly through the following mechanisms.

- **Loop control structure.** KNIME provides a bunch of control nodes which support re-executing workflow. Two nodes *Loop Start* and *Loop End* explicitly express the beginning and end of a loop structure, where the workflow between those two nodes is the loop body and is executed recursively in a fixed number, or until certain conditions are met. In our test, we repeat our repair techniques for different parameter settings by applying loop structure into KNIME workflow.
- **Flow variables.** Flow variables are used inside a KNIME workflow to parameterize node settings dynamically. By introducing flow variables into loop body, tests with different settings are able to conduct automatically.

Furthermore, there are nodes provided by KNIME to optimize the value of some parameters with respect to a cost function. As long as the cost function is provided, KNIME is able to automatically optimize the corresponding parameters.

6.2.2 Experiments with ProM Plugins

Due to the frequent exceptions on the plugin for repair techniques in [4], we exclude tests with this repair method and conduct experiments with the following types.

- **Type 1 Inductive Miner** bases only on the positive event log to discover a model. The default setting with infrequent variant and noise threshold 20% is chosen. Later, the mined model is checked on the labeled event logs with positive and negative instances. This method is abbreviated as IM.
- **Type 2 Repair Model** from [5] is applied on the positive event log to discover a model. The default setting of this plugin is chosen. Later, the mined model is checked on the labeled event with positive and negative instances. This method is abbreviated as Fahland, named after the name of the main author.
- **Type 3 Dfg-Repair from our thesis** is applied on the labeled event log with positive and negative instances. Default setting for the control parameters is 1.0 while the parameters to generate Petri nets from the directly-follows graph are set as the same as experiment Type 1. Later, the repaired model is evaluated on the labeled data.

6.3 Experiment Results

Our experiment contains two main parts. One is to verify if our method overcomes the limits of current repair algorithms. This experiment is based on the synthetic data and models in Section 1.1. The other experiment is based on real life data, in order to test the feasibility of our repair techniques.

6.3.1 Test on Demo Example

In this part, experiments are performed on the motivating examples which are listed in Section 1.1. Thereby, we are able to answer whether our repair method overcomes shortcomings of current techniques which are mentioned in the Section 1.1.

6.3.1.1 Answer to Situation 1

Situation 1 shows the drawbacks of current repair methods [5] that unexpected behaviors are introduced into the model by adding subprocesses in the form of loops. Moreover, rediscover strategy with IM does not take the original model into account and generates a new model that deviates from the original model.

Given the process model M_0 and the event log L_1 , additional activities **x1** and **x2** in L_1 good performance and need to be added into the model M_0 . Applying proposed repair techniques Dfg-repair, we obtain the repaired model listed in Figure 6.1a. The parameters for our method are set in the following : weight for the existing model is 0.45, weight for positive examples is 1.0, the Inductive Miner for Infrequent is chosen and has a noise threshold of 20%, which is the same setting as the rediscovery method by Inductive Miner in Section 1.1.

As seen in Figure 6.1a, the subprocesses for **x1** and **x2** are added in a sequence with others. In this way, $M_{1,3}$ is able to reflect the deviations in positive instances while staying similar to the reference model M_0 . Compared to techniques in [5], it increases the precision without loops.

6.3.1.2 Answer to Situation 2

Situation 2 describes the inability of current repair methods, i.e., fitting traces with negative performance outcomes cannot be used to repair a model. The execution order of **e1**, **e2** affects the performance outcomes and **e1** is expected to be placed before **e2**. Without negative information, the repaired models have the same structure as the reference ones, because the execution of **e2** before **e1** also brings the positive outcomes.

If we apply our repair methods on the model M_0 and event log L_2 , with 1.0 for all control weights, and the same Inductive Miner-Infrequent with noise 20, the repaired model $M_{2,3}$ is obtained. In $M_{2,3}$, **e1** is executed before **e2**. It shows that our method is able to incorporate the negative information and balance the forces from the existing model, positive and negative instances.

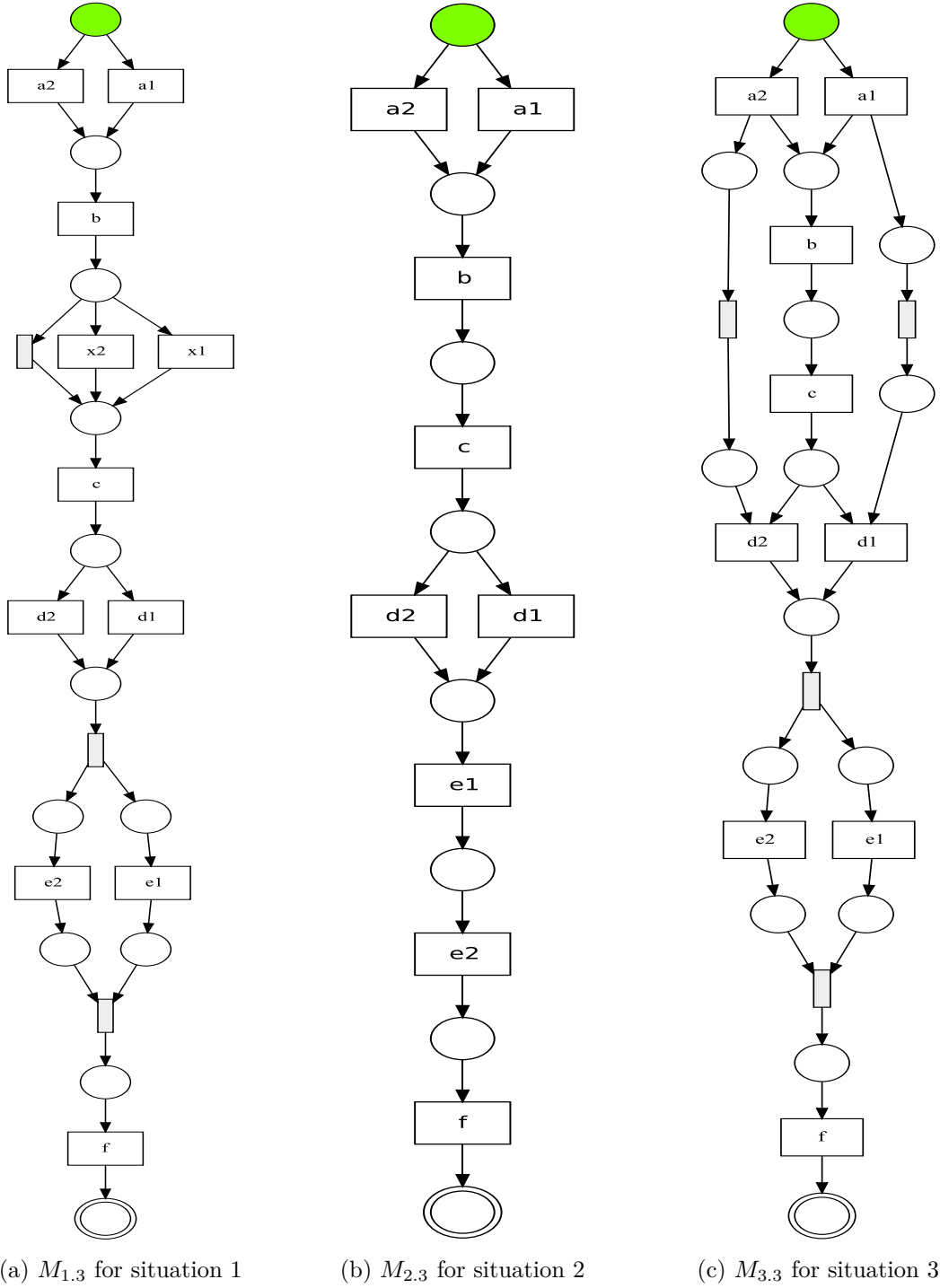


Figure 6.1: repaired models with our techniques for situations 1, 2 and 3 in Section 1.1 part. The green place is the initial marking of the Petri net and the doubled place is the final marking.

6.3.1.3 Answer to Situation 3

Situation 3 concerns long-term dependencies in Petri nets, which is not handled in current repair and rediscovery techniques. As observed in the event log, there exists long-term dependencies set, $LT = \{a1 \rightsquigarrow d1, a2 \rightsquigarrow d2\}$. By adding long-term dependencies as expected in Figure 1.3b, precision and accuracy increase, as the model limits activity selection and blocks the negative behaviors due to free execution of xor branches. Yet none of the current repair and rediscovery techniques are able to detect and add long-term dependencies in the Petri net.

In our repair techniques, long-term dependencies is taken into account. With the Petri net M_0 and event log L_3 , our methods produces the repaired model $M_{3,3}$ with long-term dependencies. Two silent transitions that are used to explicitly represent long-term dependencies can be deleted with post process to reduce the redundant silent transitions and places. After reduction, our repaired model is simplified as the model M_3 .

6.3.1.4 Comparison with Confusion Matrix

In this section, we list the evaluation results of the repaired models based on confusion matrix. In Table 6.2, for Situation 1 with only positive instances, the repair techniques give the same confusion matrix result. However, $M_{1,2}$ with loops implicates a lower precision. In Situation 2, with current techniques or rediscovery methods in IM, the model stays the same as the reference model M_0 . Since no negative instance is rejected, the recall is 1 but precision is below 0.6. In comparison, Dfg-repair uses the negative instances and adjusts the model correspondingly. Therefore, the repaired model $M_{2,3}$ has higher precision, accuracy and F1 score. In Situation 3 with long-term dependencies, our method succeeds to detect and add the long-term dependencies in the model. In this way, no false positive or false negative instances are in the confusion matrix, and the repaired model holds the highest values for all listed measurements.

Table 6.2: Test Result on BPI15-M1 data

Situation	method	Generated model	confusion matrix metrics							
			TP	FP	TN	FN	recall	precision	accuracy	F1
S1	IM	$M_{1,1}$	50	50	0	0	1	0.5	0.5	0.667
S1	Fahland	$M_{1,2}$	50	50	0	0	1	0.5	0.5	0.667
S1	Dfg-repair	$M_{1,3}$	50	50	0	0	1	0.5	0.5	0.667
S2	IM/Fahland	M_0	60	45	0	0	1	0.571	0.571	0.727
S2	Dfg-repair	$M_{2,3}$	50	5	40	10	0.833	0.909	0.857	0.870
S3	IM/Fahland	M_0	100	100	0	0	1.0	0.5	0.5	0.667
S3	Dfg-repair	$M_{3,3}$	100	0	100	0	1	1	1	1

In conclusion, our proposed method is able to conquer shortcomings of current techniques mentioned in the Section 1.1. It avoids the loops for additional activities as the work in [5]. By incorporating the negative information, it is able to output the repaired model which blocks the negative behavior. Additionally, it detects and adds long-term dependencies into model. In this way, the repaired model has better recall, and accuracy.

6.3.2 Test on Real Life Data

We choose publicly available event logs from the BPI challenge 2015 and build a data set from them to test the feasibility of proposed repair techniques.

6.3.2.1 Data Description

The data set for BPI Challenge 2015 contain 5 event logs which are provided by five Dutch municipalities respectively. Those event logs describe the building permit application around four years. We choose them as the basis to build our test data due to the following reasons.

- The event logs hold attributes as potential KPIs to classify traces. Attribute **SUMleges** which records the cost of the application is a candidate to label traces as positive or negative if its value is over the threshold. Furthermore, we take the throughput time of the application as another potential KPI. In a word, this data set provides us information to reasonably label traces.
- The five event logs describe an identical process, but include deviations caused by the different procedures, regulations in those municipalities. Also, the underlying processes have changes over four years. This data set gives us a basic process but also allows deviations of the actual event logs and predefined process. This builds the experiments environment for repair techniques.

We conduct our experiments on those event logs. However, due to the time limits, we only managed to get the results on experiments with the event log **BPIC15_1.xes.xml**. This event log includes 1199 cases and 52217 events in 398 classes. We preprocess the event log and get a proper subset of data as our user case.

Table 6.3: Test event log from real life data BPI15-1

Data ID	Data Description	Traces Num	Events Num	Event Classes
D1	heuristic filter with 40	495	9565	20
D2	apply heuristic filter on D1 with 60	378	4566	12
D3.1	classify on SUMleges; values below 0.7 as positive	349	6744	20
D3.2	classify on SUMleges; values above 0.7 as negative	146	2811	20
D3.3	union of D3.1 and D3.2	495	9596	20
D4.1	classify on throughput time; values below 0.7 as positive	349	6744	20
D4.2	classify on throughput time; values above 0.7 as negative	146	2811	20
D4.3	union of D4.1 and D4.2	495	9596	20

We filter the raw event log by *Filter Log By Simple Heuristic* in ProM with the following setting: 40% threshold for the start, end activities and the other events. We get

the event log *D1*. After this, we calculate the throughput time for each trace and add it as a trace attribute **throughput time**. Then we classify traces according to **SUMleges** and **throughput time** separately. When our performance goal is to reduce the cost of application, if **SUMleges** of one trace is over 70% of the whole traces, this trace is treated as negative, else as positive. The similar strategy is applied on the attribute **throughput time**. A trace with **throughput time** higher than 70% of all traces is considered as a negative instance. Following this preprocess, we gain event logs in Table 6.3 available for our tests.

Table 6.4: Generated reference models for test

Model ID	Used Data	Setting	Event Class	Confusion Matrix Evaluation								
				Data	TP	FP	TN	FN	recall	precision	accuracy	F1
M1	D1	IM-infrequent: Noise Setting: 20	20	D3.3	112	40	106	237	0.321	0.737	0.440	0.447
				D4.3	131	21	128	215	0.379	0.862	0.523	0.526
M2	D1	IM-infrequent: Noise Setting: 50	20	D3.3	106	39	107	243	0.304	0.731	0.430	0.429
				D4.3	125	20	129	221	0.361	0.862	0.513	0.509
M3	D2	IM-infrequent: Noise Setting: 20	12	D3.3	0	0	146	349	0	NaN	0.295	0
				D4.3	0	0	149	346	0	NaN	0.301	0
M4	D2	IM-infrequent: Noise Setting: 50	12	D3.3	0	0	146	349	0	NaN	0.295	0
				D4.3	0	0	149	346	0	NaN	0.301	0

Based on the filtered data, we derive corresponding Petri nets as reference process models. The Table 6.4 lists the models with different settings. **IM-infrequent** is one variant of Inductive Miner working on event logs with infrequent traces. **Noise** is set as the threshold to filter out infrequent traces. After mining reference models, we compare them with corresponding event logs to get the basis lines for later evaluation.

As seen in Table 6.4, the reference models do not apply well to the corresponding event logs. M3 and M4 reject all positive instances while M1 and M2 have lower recall and F1 score. The models are in demand to reflect the reality better, and also to enforce the positive instances and avoid negative instances.

6.3.2.2 Test Result

Three types of repair techniques which are **IM**, **Fahland** and **Dfg-repair**, are applied on preprocessed event log set in Table 6.3 and models from Table 6.4. The repaired model is evaluated on the alignment-based conformance checking with cost function 1 for all deviations. The experiment results are listed in the Table 6.5. For better understanding, we give the details of one experiment set which is conducted with the reference model M3 and event log D3.1 and D3.3.

Figure 6.2a displays the reference model M3, which has 0 TP and 0 FP compared to D3.3. It implies that M3 leads to no positive performance outcomes. Firstly, the rediscovery techniques – Inductive Miner for infrequent traces with noise threshold 20% is used on the positive event log D3.1 to rediscover a model. The generated model is shown in Figure 6.2b, which has changed a lot compared to the original model M3 as shown in the Figure

6.2a. After getting the generated model, we compare it with labeled event log D3.3 and compute the confusion matrix criteria as a baseline for our comparison.

After applying the Fahland’s repair techniques from [5], the reference model M3 is repaired as Figure 6.3a. With duplicated transitions, loops and silent transitions, the repaired model is more complicated than the reference model M3. When evaluating the repaired model with confusion matrix, we get 349 TP, 145 FP, 1 TN, and 0 FN, which leads to high recall. However, the principle of Fahland repair techniques is to add subprocesses for deviations in the model. Without consideration of eliminating negative behaviors, the repaired model is likely to damage the model precision and accuracy.

Dfg-repair techniques are firstly applied with the default setting, 1 for all control parameters. It results in a model with 0 TP, 0 FP, 146 TN, and 349 FN, which contrasts the result by Fahland repair techniques. This might be caused by that the forces from the existing model and negative instances exceed the positive force, and block behaviors with positive outcomes.

As a comparison, we change the control parameter setting to 0.5 for the existing model, 1 for the positive instances, 0.5 for the negative instances. After repeating the experiment, the confusion matrix changes to 131 TP, 63 FP, 83 TN, and 217 FN, while recall increases from 0 to 0.378, accuracy from 0.294 to 0.428, and F1 from 0 to 0.485. Apparently, the quality is improved due the the new setting. The possible reason is that three forces are balanced more properly for the repaired model.

Table 6.5: Test Results on BPI15-M1 data

event log	reference model	method	confusion matrix metrics							
			TP	FP	TN	FN	recall	precision	accuracy	F1
D3.1	-	IM	137	48	118	289	0.32	0.74	0.43	0.45
D3.1	M1	Fahland	343	136	10	6	0.983	0.716	0.713	0.829
D3.3	M1	Dfg-repair:1-1-1	124	52	94	225	0.355	0.705	0.44	0.472
D3.3	M1	Dfg-repair:0.5-1-0.5	155	66	80	194	0.444	0.701	0.474	0.544
D3.1	M2	Fahland	317	133	13	32	0.908	0.704	0.667	0.793
D3.3	M2	Dfg-repair:1-1-1	124	52	94	225	0.355	0.705	0.44	0.472
D3.3	M2	Dfg-repair:0.5-1-0.5	155	66	80	194	0.444	0.701	0.475	0.544
D3.1	M3	Fahland	35	7	139	314	0.100	0.833	0.352	0.179
D3.3	M3	Dfg-repair:1-1-1	0	0	146	349	0	NaN	0.295	0
D3.3	M3	Dfg-repair:0.5-1-0.5	132	63	83	217	0.378	0.677	0.434	0.485
D3.1	M4	Fahland	0	0	146	349	0	NaN	0.294	0.0
D3.3	M4	Dfg-repair:1-1-1	0	0	146	349	0	NaN	0.294	0
D3.3	M4	Dfg-repair:0.5-1-0.5	125	59	87	224	0.358	0.679	0.428	0.469
D4.1	-	IM	131	21	128	215	0.379	0.862	0.523	0.526
D4.1	M1	Fahland	325	133	16	21	0.939	0.710	0.689	0.808
D4.3	M1	Dfg-repair:1-1-1	139	36	113	207	0.402	0.794	0.509	0.534
D4.3	M1	Dfg-repair:0.5-1-0.5	172	48	101	174	0.497	0.782	0.552	0.608
D4.1	M2	Fahland	325	130	19	21	0.939	0.714	0.695	0.811
D4.3	M2	Dfg-repair:1-1-1	139	36	113	207	0.402	0.794	0.509	0.534
D4.3	M2	Dfg-repair:0.5-1-0.5	172	48	101	174	0.497	0.782	0.552	0.608
D4.1	M3	Fahland	10	20	129	336	0.029	0.333	0.281	0.053
D4.3	M3	Dfg-repair:1-1-1	0	0	346	149	0	NaN	0.303	0
D4.3	M3	Dfg-repair:0.5-1-0.5	182	49	164	100	0.526	0.788	0.70	0.631
D4.1	M4	Fahland	5	14	135	341	0.014	0.263	0.283	0.027
D4.3	M4	Dfg-repair:1-1-1	0	0	346	149	0	NaN	0.303	0
D4.3	M4	Dfg-repair:0.5-1-0.5	172	48	101	174	0.497	0.782	0.552	0.608

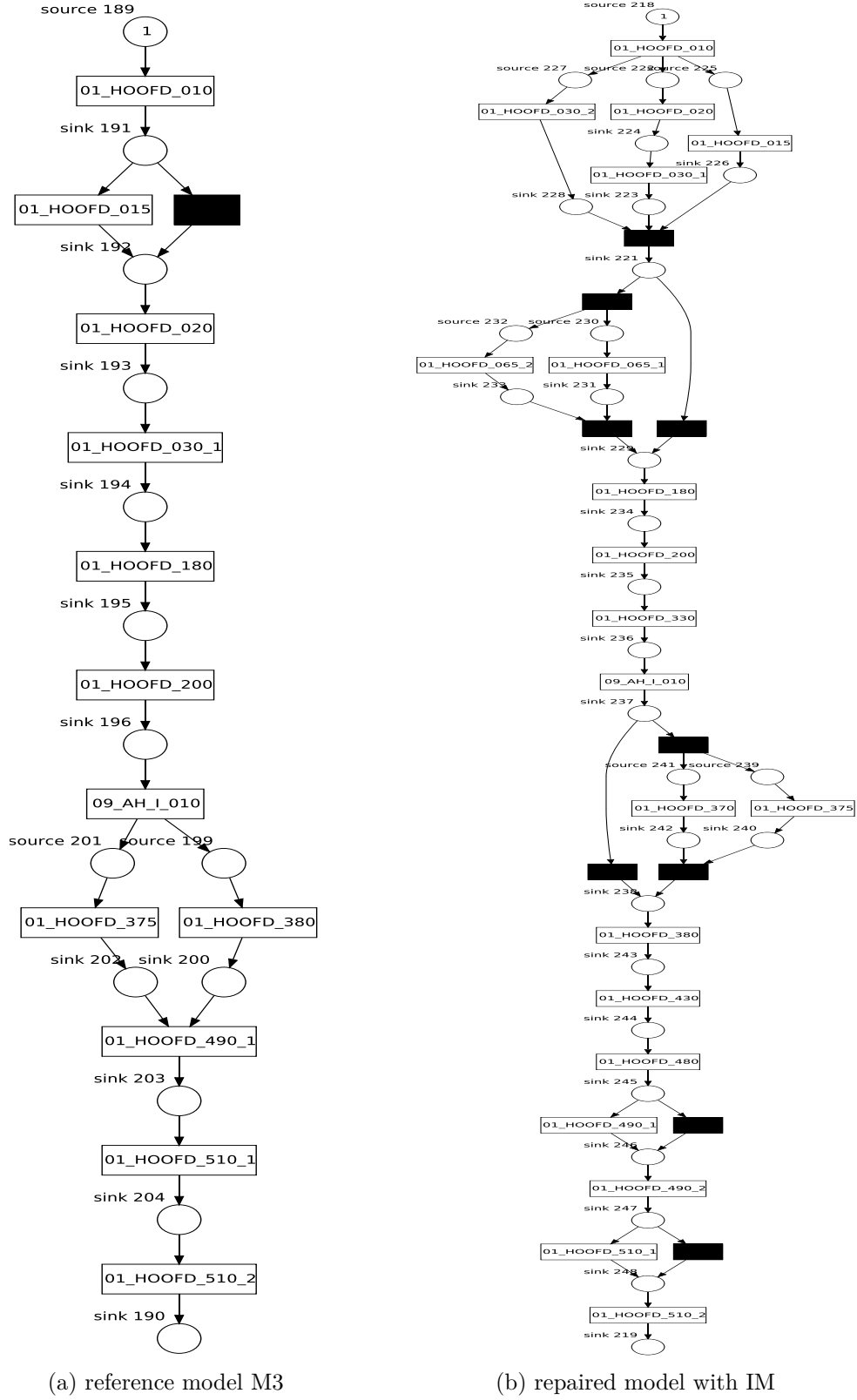


Figure 6.2: The left model is the reference model M3. The rediscovery algorithm IM generates a new model based on the positive instances which is shown on the right side.

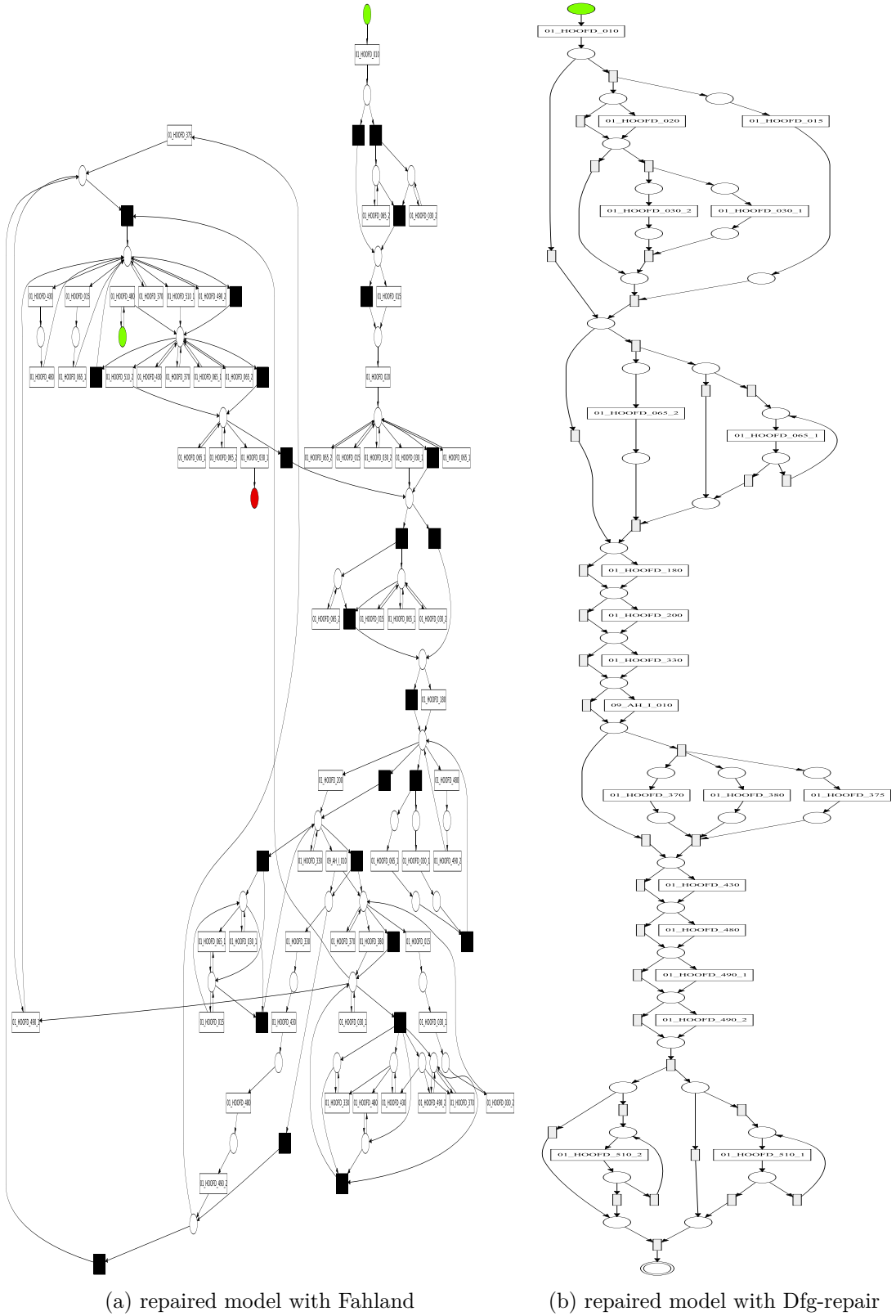


Figure 6.3: Repaired model for M_3 based on event log D3.1 with positive instances for Fahland’s repair techniques while event log D3.3 with both positive and negative instances for Dfg-repair. The weight setting for Dfg-repair is 0.5 for the existing model, 1 for positive and 0.5 for negative instances. Default setting is chosen for Fahland’s method.

With overview of results in Table 6.5, Inductive Miner rediscovers a new model from the given positive instances, while the reference models are simply ignored. As a result, the generated models probably change a lot compared to the original models. With respect to confusion matrix, the rediscovered models have similar measurement values for M1 and M2 but higher measurements than the reference models M3 and M4.

Fahland’s repair techniques from [5] tend to have high recall but low values for true and false negative on the M1 and M2. The possible reasons are that (1) M1 and M2 benefit already the positive outcomes; (2) after repairing those models by adding subprocesses, more behaviors are introduced into the model, which also allows the negative instances. In contrast, on M3 and M4 which block all the positive instances, it has low recall, but high values for TN and FN. The structures of original models dominates the performance outcomes. Since M3 and M4 blocks all positive instances, if the repaired models retains the main structure, it blocks the positive instances as well.

Dfg-repair techniques uses control parameters to balance the forces from the existing model, positive and negative instances. Therefore, with different setting, Dfg-repair techniques repair the model in different ways. To address this phenomenon, besides the default setting with value 1 for all parameters, another setting with 0.5 for the existing model, 1 for the positive instances, 0.5 for the negative instances is used to conduct experiments. Compared to the default setting, the setting with values 0.5, 1 and 0.5 results in models with higher recall, accuracy, and F1 score. The reasons behind might be that the force from negative instances affects model a lot, with the weight 1.0. It possibly blocks the behaviors which contribute to positive performance. With lower value on it, the forces from the existing model, positive and negative are balanced better and the quality of the repaired model is improved. As an example, the experiments on D3.3 and M3, or D.3. and M4, which show the quality changes due to different settings.

Except for the weight for negative instances, the weights for the existing model and positive instances also affect the quality of repaired model. To investigate the effect of those weights on the repaired model, we conducts our experiments with the following settings.

Each of three control parameters for the existing model, positive and negative instances changes value from 0.0 to 1.0 with step 0.1. With this setting, directly-follows relation is generated. Afterward, the default setting of Inductive Miner Infrequent with noise threshold 20 is used to mine Petri nets from the generated directly-follows graph. In total, 1000 experiments are conducted with proposed Dfg-repair method. After filtering out the data with missing value NaN, we average the confusion matrix results fixed with the analyzed weight, and draw plots to show the tendency of evaluation results on the weight for the existing model, positive and negative event logs, respectively.

For the experiments on the Figure 6.4, as the parameter for the existing model goes up, recall, accuracy and F1 go down firstly and then become stable. Because the reference model M3 tends to block positive behaviors from model, with the weight for the existing model going up, the force to keep the model as M3 increases. Thereby, the repaired model fits less positive instances and the recall goes lower.

Figure 6.5 displays the tendency with the weight for positive event log. When the weight rises, recall, precision and accuracy increase. The reason might be the forces from the existing model and negative event logs tend to block the positive instances. With higher values, the force for the positive instances adjusts the model with better balance. There-

Measurements change with existing weight

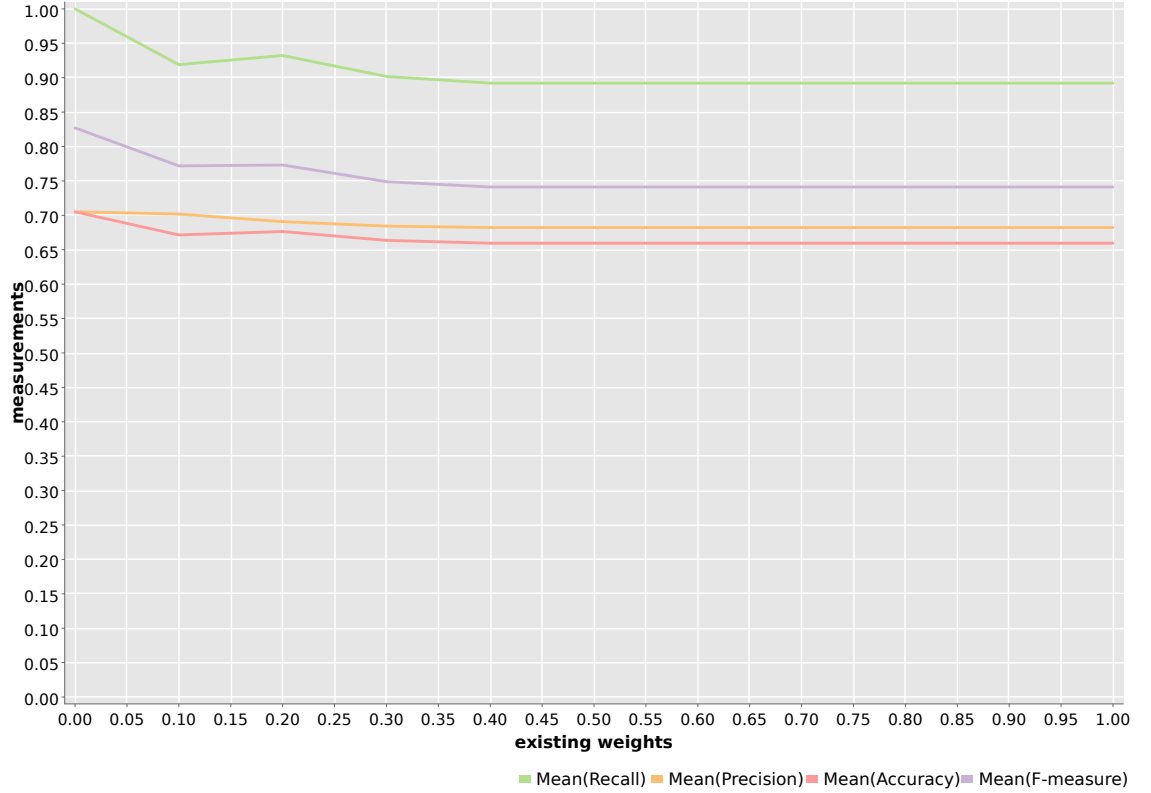


Figure 6.4: Result with weight for existing model on event log D3.3 and model M3.

fore, as the weight for positive event log increases, the repaired model allows more positive behaviors and leads to the increase of all criteria related to confusion matrix.

Figure 6.6 shows the tendency with the weight for negative event log. With lower value, the weight leads to higher recall, precision and accuracy. The negative force is possibly over the force from positive event log. When the weight increases, behaviors which contribute to positive performance are likely to be deleted from the models. In this way, the measurements go down with the weight going up.

By applying our proposed method in the real life data, it is feasible to repair model in reality. Compared to IM with the same rediscovery setting, Dfg-repair techniques are able to output models with higher recall, accuracy and F1 score. At the same time, it keeps the models as similar to the reference models, while IM simply ignores the reference models. Compared to Fahland repair techniques, which bring more behaviors into the model and cause high values with TP and FP, Dfg-repair techniques take negative information into account and are capable to produce models with higher values in TN. Moreover, the repaired models from Dfg-repair are simpler than the ones from Fahland repair techniques. With observation, Dfg-repair also runs faster.

Yet, we need to notice that the optimal control parameter setting differs in various situations. Trials are in demand to find the optimal setting for Dfg-repair techniques.

Measurements change with positive weight

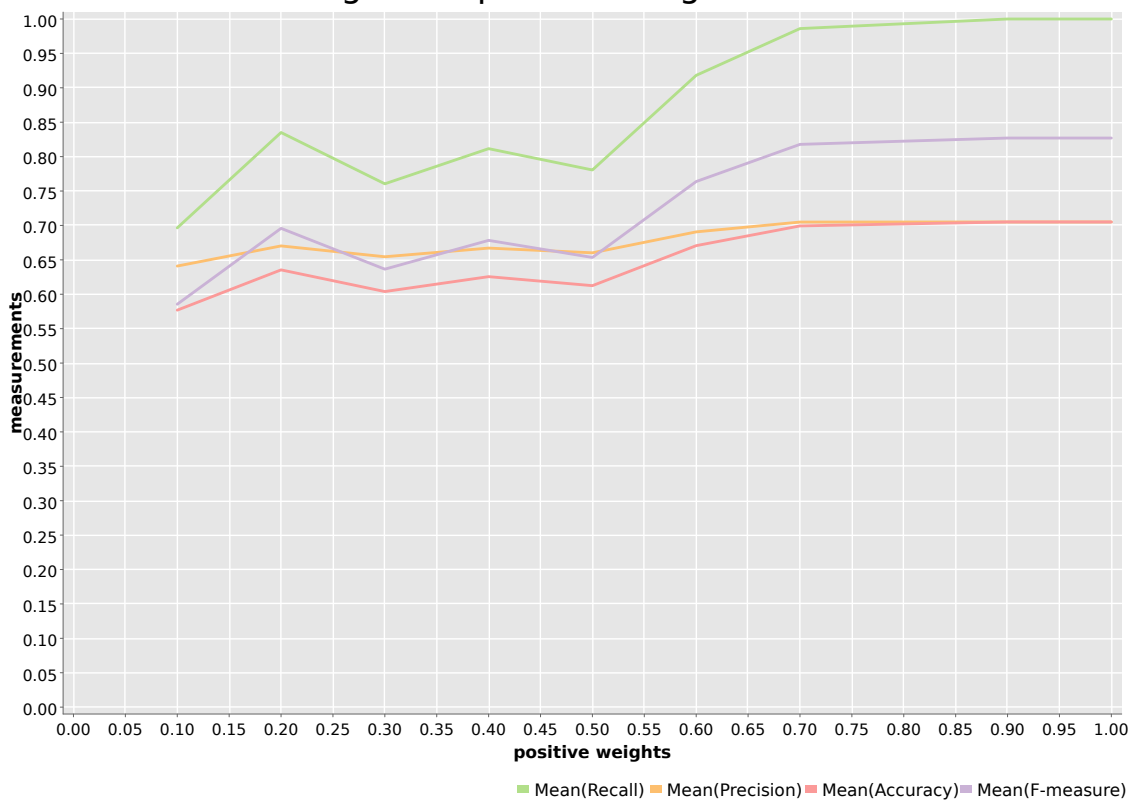


Figure 6.5: Result with weight for positive instance on event log D3.3 and model M3.

Measurements change with negative weight

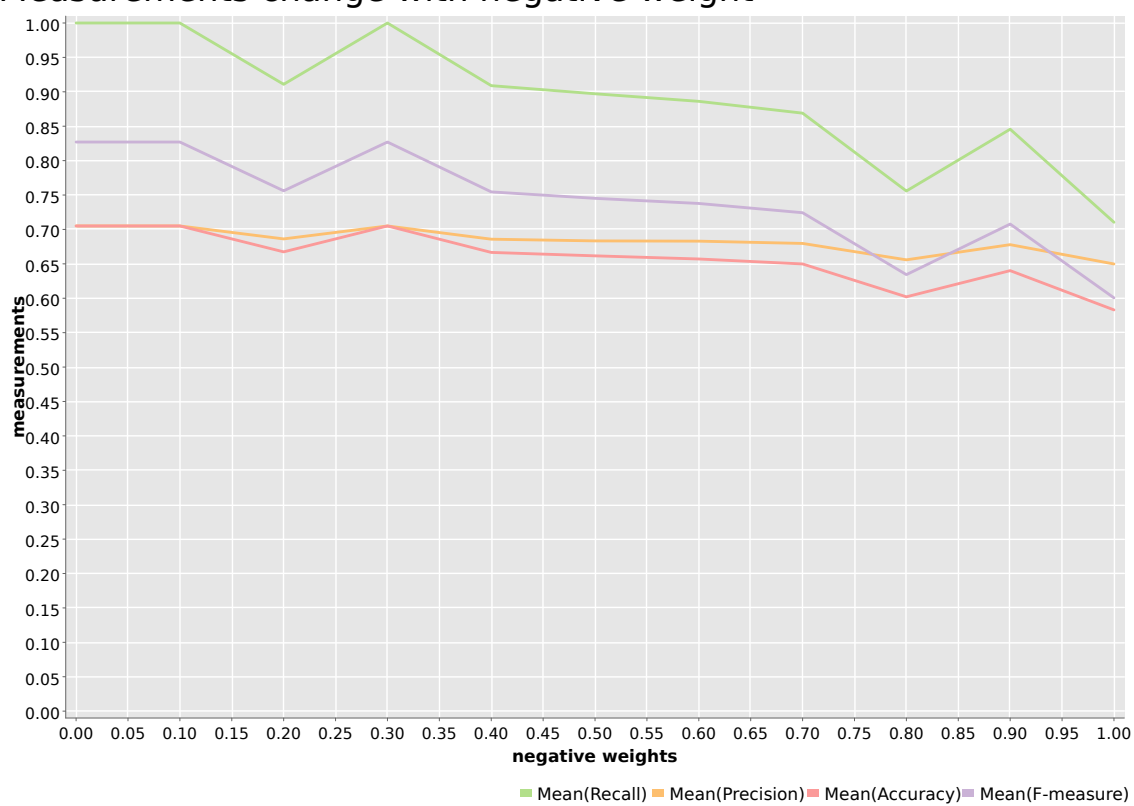


Figure 6.6: Result with weight for negative instance on event log D3.3 and model M3.

Chapter 7

Conclusion

In this thesis, we explore ways to use negative information in model repair and propose our innovative method. Firstly, we analyzed the current techniques of model repairs based on performance and detected their shortcomings. Then we proposed a general framework to incorporate the forces from the existing model, positive and negative event logs. Three abstraction data models in the same type are built to represent those forces. Later, forces are balanced based on those data models and expressed in a new data model. From this new data model, process models are discovered and converted into repaired models with the required type. Optional post processes include long-term dependency detection and silent transition reduction, which further improve the repaired models.

Moreover, we demonstrate the usage of our method by conducting experiments with synthetic data and real life data. In the situations shown with synthetic data, our method is able to overcome the shortcomings of the current repair techniques and provide repaired models with higher accuracy and precision. In experiments with the real life data, it shows that the proposed techniques are capable to produce models which better quality measurements while keeping the similarity between the repaired and reference models. Additionally, with respect to other methods, it runs faster and generates simpler models.

As future work, we consider improving the rules of balancing different forces, which choose the directly-follows relation on the simple subtraction and sum of those forces. Advanced data mining techniques such as association rules discovery, can be used on those forces to derive rules for building a process model. The same improvement can be applied on the long-term dependency discovery. Moreover, in this thesis, the long-term dependency discovery is restricted on the activities with exclusive choice relations. Later, we can extend the long-term dependencies on other possible relations. Also, we can discard the process tree as our intermediate result and detect long-term dependencies directly on the Petri net.

In conclusion, our proposed repair method is able to incorporate negative instances with the existing model and positive instances, which provides an innovative way to repair models in Process Enhancement.

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