Master Thesis – Prove Problem

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1 Introduction

This article is used to explain the difficulty I confront about proving the soundness of generated model. In the first phrase of algorithm, the existing model, positive event log and negative event log are used to generate a new directly-follows graph. Based on Inductiver Miner, this graph is transformed into a sound petri net without long-term dependency.

In the next phase, our algorithm focuses on detecting and adding long-term dependency in Petri net. We define, if the supported connection on the set pair of xor branches is over a threshold, pair has significant correlation.Therefore, this pair has long-term dependency.

During the implementation, it comes clear that supported connection only on the positive and negative event log is not enough, since the existing model can keep some directly-follows relations about xor branches which do not show in the positive event log or shows only in negative event log. Consequently, when we detect this long-term dependency on those xor branches, there is no evidence of long-term dependency on those xor branches. It results in an unsound model as shown in Fig , since those xor branches can't get fired to consume the tokens generated from the choices before.

2 Problem Description

To make the generated Petri net sound, we propose a method to incorporate the existing model on the long-term dependency detection. The definition **??** is rephrased into weight.

Definition 2.1 (Rephrased Correlation of xor branch)**.** *The correlation for two branches is expressed into*

$$Wlt(XORB_X, XORB_Y) = Wltext(XORB_X, XORB_Y) + Wltpos(XORB_X, XORB_Y) - Wltneg(XORB_X, XO$$

, where $W_l text(XORB_X, XORB_Y) = \frac{1}{\#XORB_Y}$*,* $\#XORB_Y$ *means the number of possible directly-follows xor branches* $XORB_Y$ *after* $XORB_X$*.*

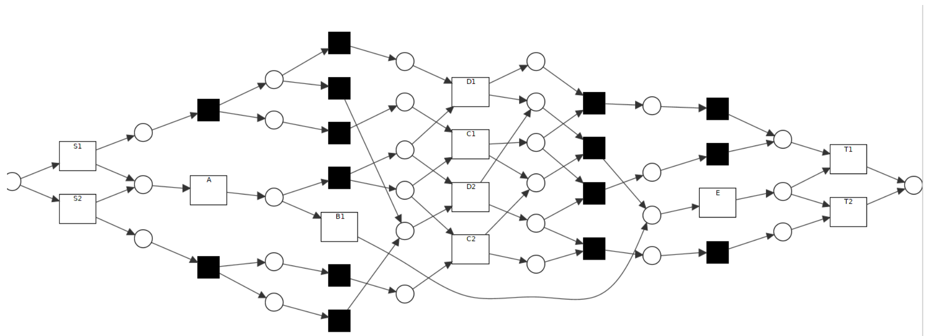

Figure 1: Unsound Repaired Model at Transition B1.

$Wltpos(XORB_X, XORB_Y) = \frac{F_{pos}(XORB_X, XORB_Y)}{F_{pos}(XORB_X, *)}$,
$Wltneg(XORB_X, XORB_Y) = \frac{F_{neg}(XORB_X, XORB_Y)}{F_{neg}(XORB_X, *)}$,

The $F_{pos}(XORB_X, XORB_Y)$ and $F_{neg}(XORB_X, XORB_Y)$ are the frequency of the coexistence of $XORB_X$ and $XORB_Y$, respectively in positive and negative event log.

With this rephrased definition, to make the model sound, we need to prove, if there is a xor branch $XORB_Y$ in the generated process tree, there must exist one long-term dependency related to it, $\exists XORB_X, Wlt(XORB_X, XORB_Y) > lt - threshold$. We formalize this problem. Else, the model can't be sound!!

Theorem 2.1. *Given a process tree, a pair of xor branch set,* (B_A, B_B) *with* $B_A = XORB_{X1}, XORB_{X2}, ...XORB_{Xm}, B_B = XORB_{Y1}, XORB_{Y2}, ...XORB_{Yn}$, *the obligatory part between* B_A *and* B_B *is marked M, it is to prove::*
for one xor branch $XORB_Y$*, if* $W(M, XORB_Yj) > threshold$*,*
then there exists one $XORB_Xi$ *with*

$$Wlt(XORB_Xi, XORB_Yj) > lt_threshold$$

.

Given a simplified scenes, it is listed in Fig 2, if there exists directly-follows relation of M and Y1, then there must exist one long-term dependency of Xi and Y1.

The definition of $W(M, XORB_Yj)$ is reviewed below.

Definition 2.2 (Assign new weights to graph G_{new})**.** *there are three weights from* G_{pos}*,* G_{neg} *and* G_{ext}*, the new weight is*

- *For one directly-follows relation,*

$$Weight(E_{G_{new}}(A, B)) = Weight(E_{G_{pos}}(A, B)) + Weight(E_{G_{ext}}(A, B)) - Weight(E_{G_{neg}}(A, B))$$

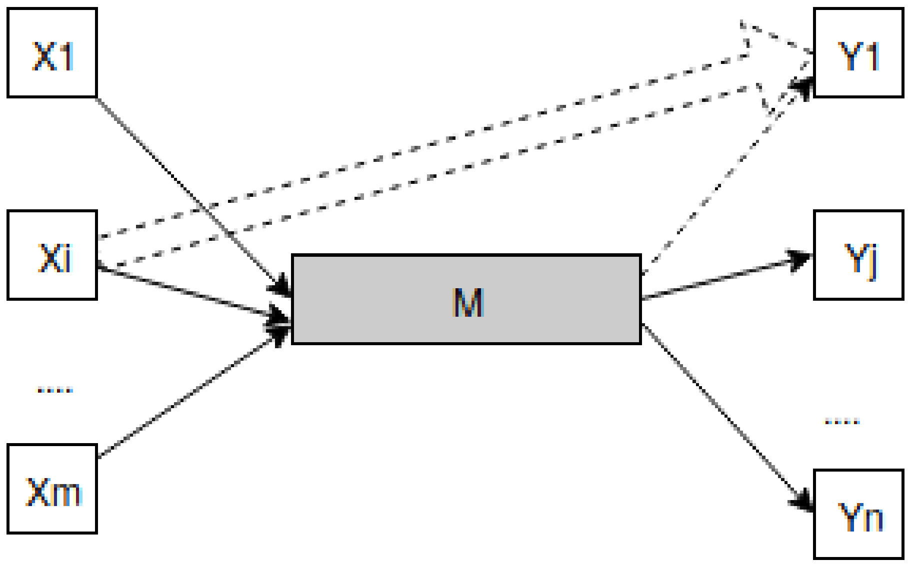

Figure 2: Unsound Repaired Model at Transition B1.

- *Given a directly-follows graph G(L), the weight of each directly-follows relation is defined as*

$$Weight(E(A,B)) = \frac{Cardinality(E(A,B))}{Cardinality(E(A,*))}$$

When prove by contradiction, we assume that the opposite proposition is true. If it shows that such an assumption leads to a contradiction, then the original proposition is valid.

Theorem 2.2. *for one xor branch $XORB_Y$, if $W(M, XORB_Y j) > threshold$, there exists no one $XORB_X i$ with*

$$Wlt(XORB_X i, XORB_Y j) < lt - threshold$$

.

Or we change to another thinking way to get the relation of threshold and lt-threshold, such that we have the theorem valid. Then we rephrase the question into

Theorem 2.3 (Another way of thinking)**.** *Given a process tree, a pair of xor branch set, (B_A, B_B) with $B_A = XORB_{X1}, XORB_{X2}, ...XORB_{Xm}, B_B = XORB_{Y1}, XORB_{Y2}, ...XORB_{Yn}$, the obligatory part between B_A and B_B is marked M. If,*
for one xor branch $XORB_Y$, if $W(M, XORB_Y j) > threshold$,
there exists one $XORB_X i$ with

$$Wlt(XORB_X i, XORB_Y j) > lt - threshold$$

What is the relation of threshold and lt-threshold??

If we expand the theorem, we need to prove

Theorem 2.4 (Relation of threshold and lt-threshold)**.** *What is the relation of threshold and lt-threshold, to make the following proposition valid. If*

$$W(M, XORB_Y j) > threshold$$
$$Weight(E_{G_{new}}(M, XORB_{Yj})) = Weight(E_{G_{pos}}(M, XORB_{Yj}))$$
$$+Weight(E_{G_{ext}}(M, XORB_{Yj})) - Weight(E_{G_{neg}}(M, XORB_{Yj})) > threshold$$
$$\frac{1}{|Y*|} + \frac{\sum_{Xi} Cardinality(M, Yj|Xi)}{\sum_{Xi} Cardinality(M, Y*|Xi)} - \frac{\sum_{Xi} Cardinality(M, Yj|Xi)\prime}{\sum_{Xi} Cardinality(M, Y*|Xi)\prime}$$
$$\text{Then, exist one Yj with}$$
$$Wlt(Xi, Yj) > lt - threshold$$
$$Wltext(Xi, Yj) + Wltpos(Xi, Yj) - Wltneg(Xi, Yj) > lt - threshold$$
$$\frac{1}{|Y*|} + \frac{Cardinality(M, Yj|Xi)}{Cardinality(M, Y*|Xi)} - \frac{Cardinality(M, Yj|Xi)\prime}{Cardinality(M, Y*|Xi)\prime} > lt - threshold$$
$$\text{Or } \textbf{there is a contradiction when all Yj}$$
$$Wlt(Xi, Yj) < lt - threshold$$
$$\sum_{Xi} Wlt(Xi, Yj) < |X*|lt - threshold$$
$$\frac{|X*|}{|Y*|} + \sum_{Xi} \frac{Cardinality(M, Yj|Xi)}{Cardinality(M, Y*|Xi)} - \sum_{Xi} \frac{Cardinality(M, Yj|Xi)\prime}{Cardinality(M, Y*|Xi)\prime} < |X*|lt - threshold$$

$Cardinality(M, Yj|Xi)$ means the frequency of coexistence of M and Yj given Xi in the trace in positive, while $Cardinality(M, Yj|Xi)\prime$ represents the frequency in negative. $Cardinality(M, Y*|Xi)$ is the sum frequency of set $Y1, ..Yj, ..Yn$, it equals to

$$Cardinality(M, Y*|Xi) = \sum_{Yi} Cardinality(M, Yj|Xi)$$