Table 9.1: *Some properties of the Airy functions.*

Differential Equation: $$\frac{d^2y}{dz^2} = zy.$$

Solutions: Linear combinations of Airy functions, Ai(z) and Bi(z).

Integral Representation:

$$\mathrm{Ai}(z) = \frac{1}{\pi}\int_0^\infty \cos\left(\frac{s^3}{3} + sz\right) ds,$$

$$\mathrm{Bi}(z) = \frac{1}{\pi}\int_0^\infty \left[e^{-\frac{s^3}{3} + sz} + \sin\left(\frac{s^3}{3} + sz\right)\right] ds.$$

Asymptotic Forms:

$$\left.\begin{aligned} \mathrm{Ai}(z) &\sim \frac{1}{2\sqrt{\pi}\,z^{1/4}}\, e^{-\frac{2}{3}z^{3/2}} \\ \mathrm{Bi}(z) &\sim \frac{1}{\sqrt{\pi}\,z^{1/4}}\, e^{\frac{2}{3}z^{3/2}} \end{aligned}\right\} z \gg 0; \qquad \left.\begin{aligned} \mathrm{Ai}(z) &\sim \frac{1}{\sqrt{\pi}(-z)^{1/4}} \sin\left[\frac{2}{3}(-z)^{3/2} + \frac{\pi}{4}\right] \\ \mathrm{Bi}(z) &\sim \frac{1}{\sqrt{\pi}(-z)^{1/4}} \cos\left[\frac{2}{3}(-z)^{3/2} + \frac{\pi}{4}\right] \end{aligned}\right\} z \ll 0.$$