The Wave Function:

$\Psi(x,t)$ obeys Schrodinger's equation

$$\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1 \qquad \rho(x,t) = |\Psi(x,t)|^2$$

$$\langle \hat{x} \rangle = \int_{-\infty}^{\infty} x|\Psi(x,t)|^2 dx; \qquad \langle \hat{p} \rangle = \int_{-\infty}^{\infty} \Psi^*(x,t)\frac{\hbar}{i}\frac{\partial}{\partial x}\Psi(x,t)dx;$$

$$\langle Q(\hat{x},\hat{p}) \rangle = \int_{-\infty}^{\infty} \Psi^*(x,t)Q(x,\frac{\hbar}{i}\frac{\partial}{\partial x})\Psi(x,t)dx \qquad J(x,t) = \frac{i\hbar}{2m}\left(\Psi\frac{\partial \Psi^*}{\partial x} - \Psi^*\frac{\partial \Psi}{\partial x}\right)$$

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right]\Psi(x,t) \qquad \hat{\mathbf{H}}\psi(x) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right]\psi(x) = E\psi(x)$$

$$\Psi(x,t) = \psi(x)\phi(t) \qquad \phi(t) = e^{-iEt/\hbar}$$

Properties of the Solution of 1D stationary Schrodinger Equation:

1. For 1D potential, all stationary solutions are non-degenerate.
2. Stationary square integrable solution exist only for E ¿ minV(x)
3. If V(x) is real, then Ψ(x) can be taken to be real.
4. Eigenvalues of a Hermitian Hamiltonian are all real.
5. The eigenfunctions of a Hermitian operator form a complete orthogonal basis set.
6. 1D Solution is real up to an over all phase.
7. For a given 1D even potential the stationary states are either even or odd.
8. The wave function and its first order space derivative is continuous all over and in particular at the boundaries of a finite potential.
9. At boundaries with Dirac delta function potential, the first space derivative of the wavefunction is discontinuous.
10. Physical solution should be finite all over space, no blow ups, in particular at infinity.
11. The number of nodes (zeros) of the eigenfunction increases by one unit as we move from the ground state (zero nodes) to higher excited states.
12. Bound states exist only for confining potential (classically between turning points of the potential).

Complete Basis Set:

Given that:

$$H\psi_n(x) - E_n\psi_n(x) \qquad \int \phi_n^*(x)\phi_m(x)dx = \delta_{nm}$$

Then, where $\{\phi_n\}$ is a complete set:

$$\psi(x) = \sum_n c_n\phi_n(x)$$

$$\int \psi^*(x)\psi(x)dx = \sum_n |c_n|^2 = 1$$

$$E = \int \psi_n^*(x)H\psi_m(x)dx = \sum_n |c_n|^2 E_n$$

$$\Psi(x,0) = \psi(x) = \sum_n c_n\phi_n(x) \implies \Psi(x,t) = \sum_n c_n e^{-iEt/\hbar}\phi_n(x)$$

$$a_n = \int \phi_n^*\Psi(x,0)dx$$

Commutator Properties:

$$[A, A] = 0$$

$$[A, B] = -[B, A]$$

$$[A + B, C] = [A, C] + [B, C]$$

$$[A, [B, C]] + [B, [C, A]] + [C, [A, B]] = 0$$

Miscellaneous:

$$\int_{-\infty}^{\infty} e^{-a(x+b)^2}\, dx = \sqrt{\frac{\pi}{a}} \qquad\qquad \delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$$