



The properties of helium: Density, specific heats, viscosity, and thermal conductivity at pressures from 1 to 100 bar and from room temperature to about 1800 K

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Danish Atomic Energy Commission
Research Establishment Risø

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Specific Heats, Viscosity, and Thermal
Conductivity at Pressures from 1 to
100 bar and from Room Temperature
to about 1800 K**

by Helge Petersen

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Abstract

An estimation of the properties of helium is carried out on the basis of a literature survey. The ranges of pressure and temperature chosen are applicable to helium-cooled atomic reactor design. A brief outline of the theory for the properties is incorporated, and comparisons of the recommended data with the data calculated from intermolecular potential functions are presented.

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1. INTRODUCTION

Calculations of the heat transfer and the temperature in the core and the heat exchangers of a helium-cooled reactor require knowledge of some of the thermophysical properties of the helium gas, and the same applies to the treatment of observed data in heat transfer experiments.

The properties treated in this report are the density, the specific heats, the thermal conductivity, and the viscosity.

The gas properties are only known with a certain accuracy, and it is therefore necessary to survey the existing experimental and theoretical data in the literature to establish a set of recommendable data. The data resulting from this survey are presented as equations in order to facilitate the use of computers, and it is shown that very simple equations suffice to express the data with ample accuracy. Two equations are presented for each property, one in terms of the absolute temperature, T , in Kelvin and the other in terms of the ratio T/T_0 , T_0 being the absolute temperature at zero degrees Celsius, equal to 273.16 K. The ranges of pressure and temperature considered for the data are 1 to 100 bar for pressure and 273 to 1800 K for temperature.

The Prandtl number is a combination of the properties which is often used in thermodynamic calculations, and for that reason equations are also presented for this quantity.

2. SUMMARY OF RECOMMENDED DATA FOR HELIUM AT 1 TO 100 BAR AND 273 TO 1800 K

kg	unit of mass, kilogram
m	unit of length, metre
s	unit of time, second
N	unit of force, newton, $\text{kg} \cdot \text{m}/\text{s}^2$
J	unit of energy, joule, $\text{N} \cdot \text{m}$
W	unit of power, watt, J/s
P	pressure, bar, $10^5 \text{ N}/\text{m}^2 = 0.9869 \text{ phys. atm}$
T	absolute temperature, K
T_0	abs. temperature at $0^\circ\text{C} = 273.16 \text{ K}$

Gas Law of Helium

$$\frac{P \cdot 10^5}{2077.3 \cdot \rho \cdot T} = Z, \quad (3-2), (3-3)$$

where ρ is the density, and Z is the compressibility factor:

$$Z = 1 + 0.4446 \frac{P}{T^{1.2}} = 1 + 0.530 \cdot 10^{-3} \frac{P}{(T/T_0)^{1.2}}. \quad (3-18)$$

The standard deviation, σ , is about 0.03% at a pressure of 1 bar and 0.3% at 100 bar, i.e. $\sigma = 0.03 \cdot \sqrt{P}\%$. (3-20)

Mass Density of Helium

$$\begin{aligned} \rho &= 48.14 \frac{P}{T} \left[1 + 0.4446 \frac{P}{T^{1.2}} \right]^{-1} \quad (3-19) \\ &= 0.17623 \frac{P}{(T/T_0)} \left[1 + 0.53 \cdot 10^{-3} \frac{P}{(T/T_0)^{1.2}} \right]^{-1} \text{ kg/m}^3. \end{aligned}$$

The standard deviation is as for Z .

Specific Heats

$$c_p = 5195. \quad \text{J/kg} \cdot \text{K}, \quad (4-4)$$

$$c_v = 3117. \quad \text{J/kg} \cdot \text{K}, \quad (4-5)$$

$$\gamma = c_p/c_v = 1.6667. \quad (4-6)$$

The standard deviation, σ , is at 273 K about 0.05% at a pressure of 1 bar and 0.5% at 100 bar, decreasing to 0.05% at high temperature at all pressures, i.e. $\sigma = 0.05 P^{(0.6 - 0.1(T/T_0))}\%$. (4-7)

Coefficient of Dynamic Viscosity

$$\mu = 3.674 \cdot 10^{-7} T^{0.7} = 1.885 \cdot 10^{-5} (T/T_0)^{0.7} \quad \text{kg/m} \cdot \text{s}. \quad (6-1)$$

The standard deviation, σ , is about 0.4% at 273 K and 2.7% at 1800 K, i.e. $\sigma = 0.0015 T\%$. (6-5)

Coefficient of Thermal Conductivity

$$k = 2.682 \cdot 10^{-3} (1 + 1.123 \cdot 10^{-3} P) T^{(0.71 (1 - 2 \cdot 10^{-4} P))} \quad (7-1)$$

$$= 0.144 (1 + 2.7 \cdot 10^{-4} P) (T/T_0)^{(0.71 (1 - 2 \cdot 10^{-4} P))} \text{ W/m} \cdot \text{K.}$$

The standard deviation, σ , is about 1% at 273 K and 6% at 1800 K,
i. e. $\sigma = 0.0035 \text{ T\%}$. (7-4)

Prandtl Number

$$Pr = c_p \frac{\mu}{k}$$

$$Pr = \frac{0.7117}{1 + 1.123 \cdot 10^{-3} P} T^{-(0.01 - 1.42 \cdot 10^{-4} P)} \quad (8-1)$$

$$= \frac{0.6728}{1 + 2.7 \cdot 10^{-4} P} (T/T_0)^{-(0.01 - 1.42 \cdot 10^{-4} P)}$$

The standard deviation is $\sigma = 0.004 \text{ T\%}$. (8-2)

3. GAS LAW

The ideal gas law or the ideal equation of state is the p-v-T relation for an ideal gas:

$$p \cdot v = R \cdot T \quad (3-1)$$

p, pressure, N/m^2

v, specific molal volume, $\text{m}^3/\text{kg-mole}$

R, universal gas constant, $8314.5 \text{ J/kg-mole} \cdot \text{K}$

T, absolute temperature, K.

For a noble gas such as helium the real p-v-T relation deviates little from the real gas law at the pressure and the temperature in a reactor core. The deviation increases numerically with pressure and decreases with temperature. If the ideal gas law is applied to helium at a pressure of 60 bar and a temperature of 550 K, the deviation from the real gas law is 1.5%.

In calculations where such deviations are allowable, the ideal gas law can therefore be applied. The molecular weight of helium is 4.003, and

this gives the following gas law for helium when considered an ideal gas:

$$\frac{P \cdot 10^5}{2077.3 \cdot \rho \cdot T} = 1 \quad (3-2)$$

P, pressure, bar (1 bar = 10^5 N/m² = 0.9863 phys. atm)
 ρ, mass density, kg/m³.

In some calculations it might, however, be desirable to operate with an expression which gives a greater accuracy.

This is normally done by introduction of the compressibility factor, Z, in the gas law:

$$\frac{P \cdot V}{R \cdot T} = Z. \quad (3-3)$$

The compressibility factor, Z, can be determined by consideration of the virial equation of state.

An equation of state for a gas is a relationship between the characteristic force, the characteristic configuration and the temperature. For an ideal gas the pressure is the only characteristic force, the volume being the characteristic configuration, but other forces must be taken into account for a real gas if the pressure is not very low. This can be done by adding force-describing terms to the ideal equation of state.

Virial equations are power expansions in terms of volume or pressure respectively

$$p \cdot v = A \left(1 + \frac{B}{v} + \frac{C}{v^2} \dots \right) \quad (3-4)$$

$$p \cdot v = A + B \cdot p + \frac{C-B^2}{A} p^2 + \dots \quad (3-5)$$

The coefficients A, B, C and so on are named virial coefficients, i. e. they are force coefficients. The virial coefficients are obtained experimentally by fitting of the power series to measured isotherms of the gas. This gives a different set of coefficients for each isotherm, and thus the virial coefficients are temperature-dependent.

Equation (3-5) converges less rapidly than eq. (3-4) and thus requires more terms for equal precision. This is of minor importance for density determination, for which only the first correction term, the second virial coefficient, B, is significant except at extremely high pressure. The virial equation of state sufficient for density calculation of light noble gases

is then:

$$p \cdot v = A + B \cdot p. \quad (3-6)$$

A system of units often used in this equation is the Amagat units. In this system the pressure p_a is in atm, and the volume v_n is expressed in the normal molal volume as a unit, writing

$$p_a \cdot v_n = A + B \cdot p_a = A_0 \frac{T}{T_0} + B \cdot p_a, \quad (3-7)$$

A_0 is the ratio of $p_a \cdot v_n$ at zero pressure to $p_a \cdot v_n$ at the pressure of 1 atm, or

$$A_0 = 1 - B_0. \quad (3-8)$$

B_0 is the coefficient B at 0°C , measured to be between $0.5 \cdot 10^{-3}$ and $0.54 \cdot 10^{-3}$. Equation (1-7) becomes

$$p_a \cdot v_n = (1 - B_0) \frac{T}{T_0} + B \cdot p_a, \quad (3-9)$$

and eq. (1-9) can be rearranged to give the following form:

$$\frac{p_a \cdot v_n \cdot T_0}{(1 - B_0) T} = 1 + \frac{T_0 \cdot B}{T(1 - B_0)} p_a. \quad (3-10)$$

On the left-hand side the correcting term, B_0 , should not be omitted, but on the right-hand side it is negligible, so

$$\frac{p_a \cdot v_n \cdot T_0}{(1 - B_0) T} = 1 + \frac{T_0}{T} B \cdot p_a. \quad (3-11)$$

Eq. (1-9) then becomes

$$Z = 1 + \frac{T_0}{T} B \cdot p_a. \quad (3-12)$$

It is obvious that if B can be expressed with sufficient accuracy by

$$B = B_1 \left(\frac{T_0}{T} \right)^a, \quad (3-13)$$

B_1 being a constant, then a very simple equation for Z is obtained:

$$Z = 1 + B_1 \cdot p_a \left(\frac{T}{T_0} \right)^{-1-a} \quad (3-14)$$

In the following such an equation is established, and by means of this a simple equation for the density is derived.

Reported measurements of isotherms of helium are most frequently presented with the second virial coefficient, B , as a function of temperature.

The latest measurements, Stroud, Miller and Brandt, 1960, however, give Z directly, but at about room temperature only. It is notable that $(Z - 1)$ is found to be strictly proportional to the pressure over the entire pressure range investigated, 10 to 275 atm. For comparison with other measurements the results are converted into values of B by means of eq. (1-12) and plotted in fig. 1 together with older measurements as follows.

The shaded area in fig. 1 represents measurements carried out until 1940, discussed by Keesom, 1942, who adopts the dotted line as representative. Measurements reported by Schneider and Duffie, 1949, and Yntema and Schneider, 1950, are also included.

Although the reproducibility for each investigation is within 1%, the results deviate by 4% at room temperature. Bearing in mind, however, that this corresponds to deviations of only 0.3% in Z at 100 atm, it seems most likely that such deviations can occur between two different investigations or from one method of measurement to another.

The two curves b and c in fig. 1 are the results of calculations based on intermolecular potentials, as will be dealt with later.

The curve adopted for this work is the full line in the figure. This function is

$$B = 0.537 \cdot 10^{-3} \left(\frac{T}{T_0} \right)^{-0.2} \quad (3-15)$$

Eq. (3-14) is then

$$Z = 1 + 0.537 \cdot 10^{-3} p_a \left(\frac{T}{T_0} \right)^{-1.2} \quad (3-16)$$

With the pressure, P , in bar the adopted function is

$$Z = 1 + 0.53 \cdot 10^{-3} \frac{P}{(T/T_0)^{1.2}} \quad (3-17)$$

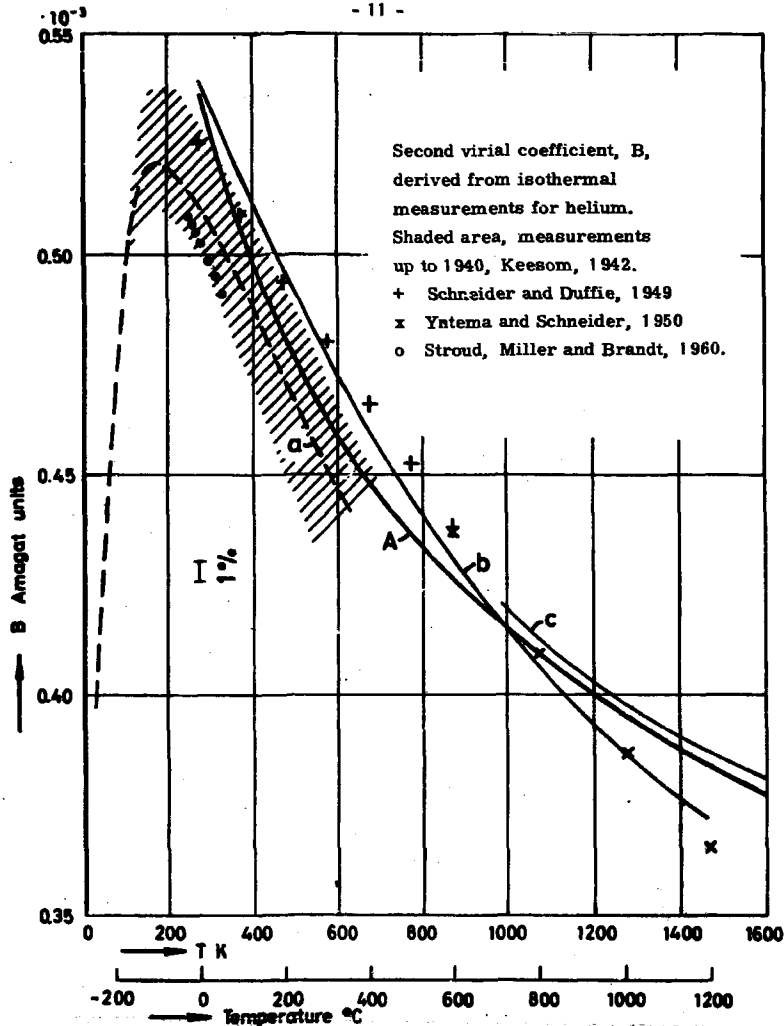


Fig. 1

a Adopted by Keesom, 1942

b Mason and Rice, 1954. "Exp-6" potential

c Amundson and Mason, 1958. Inverse power pot.

A Adopted for this work, $B = 0.537 \cdot 10^{-3} (T/T_0)^{-3}$

Then the equation of state (eq. (3-2)) for helium becomes

$$\frac{P \cdot 10^5}{2077.3 \cdot \rho \cdot T} = 1 + 0.53 \cdot 10^{-3} \frac{P}{(T/T_0)^{1.2}} \quad (3-18)$$

Mass Density

Rearrangement of eq. (3-18) gives the density of helium.

$$\rho = 48.14 \frac{P}{T} \left[1 + 0.4446 \frac{P}{T^{1.2}} \right]^{-1} \quad (3-19)$$

$$= 0.17623 \frac{P}{(T/T_0)} \left[1 + 0.53 \cdot 10^{-3} \frac{P}{(T/T_0)^{1.2}} \right]^{-1} \text{ kg/m}^3.$$

P , pressure, bar,

T , abs. temperature, K,

$T_0 = 273.16 \text{ K}.$

If the first-order term alone is used, and the conditions are limited to temperatures above 500 K and pressures up to 60 bar, the accuracy is better than 1.5%. For the full equation the standard deviation, σ , is about 0.03% at a pressure of 1 bar and 0.3% at 100 bar, i. e. $\sigma = 0.03 \sqrt{P\%}$.
(3-20)

Values of ρ calculated from eq. (3-19) agree very closely with the values calculated and tabulated by Wilson, 1960. Checks of this agreement were made at 1, 20 and 100 atm, at 40 °F and 1600 °F. The maximum deviation was 0.2%. Checks of agreement were also made with values tabulated by Holley, Worlton and Ziegler, 1959, and these showed deviations up to 0.5% at 100 bar. An analysis of the report disclosed, however, that the authors were aware of the deviation from measured values at that pressure, and that the deviation may have been allowed with the aim of establishing a formula which covered pressures up to 1000 atm.

4. SPECIFIC HEATS

At the zero condition, $p \rightarrow 0$, the molal specific heats, C_p at constant pressure and C_v at constant volume, for an ideal monatomic gas with the molecular weight M can be shown to be

$$C_p = 2.5 \text{ R/M} \quad \text{J/kg-mole} \cdot \text{K}, \quad (4-1)$$

$$C_v = 1.5 \text{ R/M} \quad \text{J/kg-mole} \cdot \text{K}. \quad (4-2)$$

The ratio C_p/C_v is correspondingly

$$\gamma = C_p/C_v = 5/3. \quad (4-3)$$

For a real gas the specific heats can be determined from measurements of the isenthalps and C_p from direct measurements, while γ can be derived indirectly from measurements of the speed of sound. In the case of helium such measurements do not appear to have been performed with an accuracy better than the actual deviation from the above-mentioned basic figures, the deviations being very small.

C_p , C_v and γ can also be derived from the virial equation of state. In such expressions the quantities depend on $\partial B/\partial T$ and $\partial^2 B/\partial T^2$, and these derivatives will differ substantially from one function to another for the second virial coefficient, B , shown in fig. 1. Calculations indicate that the deviations from the basic figures at 100 bar are about 0.5% at room temperature and about 0.05% at 1000 K. Such deviations are insignificant for engineering purposes for which reason C_p , C_v and γ can be regarded as constants and the deviations regarded as the standard deviations.

In practical units the specific heats of helium are:

$$c_p = 5195. \quad \text{J/kg} \cdot \text{K}, \quad (4-4)$$

$$c_v = 3117. \quad \text{J/kg} \cdot \text{K}, \quad (4-5)$$

and

$$\gamma = c_p/c_v = 1.6667. \quad (4-6)$$

The standard deviation is $\sigma = 0.05 P^{(0.6 - 0.1(T/T_0))\%}$.

5. CORRELATION FORMULAE FOR VISCOSITY AND CONDUCTIVITY

For the purpose of the following part of the report, which is to establish the most realistic values for the transport properties: viscosity and thermal conductivity, it would be quite satisfactory only to examine the experimental data if sufficient and reliable data were available. A great many measurements on the subject are reported, most of them at temperatures below 1000 K, and only a few at higher temperatures. The results,

however, deviate much more from one investigation to another than the claimed accuracy of each investigation accounts for, for which reason it is necessary to perform a judgement in order to select the most probable values. For guidance in performing such judgement and especially for judgement of the values at higher temperatures it is relevant to examine the results evaluated by means of the statistical mechanical theory of gases. It must, however, be pointed out that even advanced theories cannot at present predict values of μ and k unless some values are already known from experiments. The theoretical formulae derived in such a way can be used to interpolate between the known values, but cannot improve the accuracy determined by the experimental values, and extrapolation to higher temperatures is bound to be very uncertain since whole families of functions can be brought to fit the medium temperature data, giving greatly differing extrapolated values at high temperature.

In the following a brief and not at all complete evaluation of the dependencies of the viscosity, conductivity and Prandtl number on the pressure, temperature and molecular quantities is outlined.

The conductivity and the viscosity of the gas describe the overall transport rates of heat and momentum transferred within the gas by molecular transport.

The coefficient of viscosity, μ , is defined by the equation for the momentum current density, the amount of convective momentum parallel to the y-axis transferred per unit time across a unit area perpendicular to the direction in which the convective velocity changes, the x-axis,

$$j = -\mu \frac{\partial u}{\partial x} . \quad (5-1)$$

The coefficient of conductivity, k , is defined by the law of Fourier as

$$q = -k \frac{\partial T}{\partial x} . \quad (5-2)$$

q is the energy current density due to temperature gradient, i. e. the energy which crosses per unit time through a unit area perpendicular to the direction in which the energy flows, the x-axis.

The simplest model that is applied for theoretical considerations of the transport properties of gases is the so-called billiard-ball model, the molecules being regarded as rigid spheres with the diameter σ , moving freely among each other.

From the definitional equations (5-1) and (5-2) it follows that

$$j = -\frac{1}{3} n m \bar{v} \lambda \frac{\partial u}{\partial x} \quad \text{giving} \quad \mu = \frac{1}{3} n m \bar{v} \lambda,$$

and

$$q = -\frac{1}{3} n C_{\text{mol}} \bar{v} \lambda \frac{\partial T}{\partial x} \quad \text{giving} \quad k = \frac{1}{3} n C_{\text{mol}} \bar{v} \lambda,$$

where n is the number of molecules per unit volume, m the molecular mass, C_{mol} the specific heat per molecule, \bar{v} the speed which is proportional to $\sqrt{kT/m}$, and λ the mean free path proportional to $1/(n\pi\sigma^2)$. k is Boltzmann's constant.

The properties can therefore be expressed as

$$\mu = \alpha \frac{\sqrt{\pi m k T}}{\pi \sigma^2}, \quad (5-3)$$

and

$$k = \beta \frac{\sqrt{\pi m k T}}{\pi \sigma^2} \frac{C_{\text{mol}}}{m} \quad (5-4)$$

The rigorous theory for rigid-sphere molecules predicts that the values of α and β are

$$\alpha = \frac{5}{16} \quad \text{and} \quad \beta = \frac{25}{32}.$$

As a result of this theory insertion of α and β gives the following equations, expressed in practical units (σ is in Å):

$$\mu = 2.6693 \cdot 10^{-6} \frac{\sqrt{MT}}{\sigma^2} \quad \text{kg/m} \cdot \text{s}, \quad (5-5)$$

$$k = 2.5 \mu \cdot c_v \quad \text{W/m} \cdot \text{K}. \quad (5-6)$$

From this it follows that the Prandtl number is

$$Pr = \frac{\mu \cdot c_p}{k} = \frac{2c_p}{5c_v} = \frac{2}{3}. \quad (5-7)$$

These expressions show that both viscosity and conductivity increase with temperature as \sqrt{T} and are independent of the pressure. Experiments show that the conductivity is slightly dependent on pressure, which can be verified by a more elaborate theory, and that the temperature dependence is actually considerably greater. For explanation of this a more advanced molecular model must be examined, taking the intermolecular force into consideration.

The usual approach to the problem assumes an analytical form for the intermolecular potential, and after the necessary calculations the result is compared with available data for selection of the parameters of the potential in order to verify the assumed model. Such a potential is sketched in fig. 2. $\varphi(r)$ is the potential energy of two molecules at a separating distance, r , ϵ is the depth of the potential energy minimum, r_m is the value of r for the minimum, and σ is the value of r at which $\varphi(r)$ is zero.

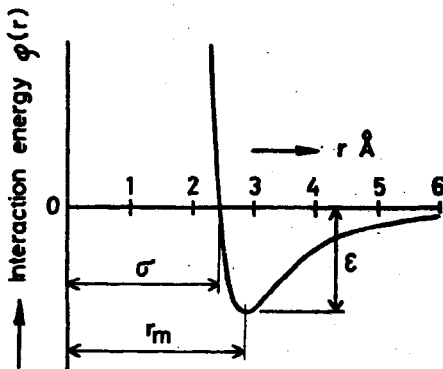


Fig. 2

Potential energies of a pair of helium atoms as a function of the separating distance, r , between the centres of the two atoms.

The general equation for the viscosity deduced from an intermolecular potential function is

$$\mu_1 = 2.6693 \cdot 10^{-6} \frac{\sqrt{M \cdot T}}{\sigma^2 (2.2)^{1/2} + (T^X)} \quad (5-8)$$

$$= 5.34 \cdot 10^{-6} \frac{\sqrt{T}}{\sigma^2 (2.2)^{1/2} + (T^X)}$$

where M is inserted with its value for helium, 4.003. Further

T^* is the reduced temperature, $T/k\epsilon$,

k , Boltzmann's constant = $1.3805 \cdot 10^{-16}$ erg/K,

ϵ , the potential parameter, erg, and

σ is the potential parameter, Å.

As it will be seen μ_1 is derived from the viscosity predicted by the rigorous rigid-sphere theory by division by the collision integral $\Omega^{(2,2)+}(T^*)$.

The required formulae for calculation of the collision integrals are discussed by Hirschfelder, Curtiss and Bird, 1954, "Molecular Theory of Gases and Liquids".

In the 3rd approximation, which is normally used, the coefficient of viscosity is given by

$$\mu_3 = \mu_1 f_\mu^{(3)}. \quad (5-9)$$

Correspondingly the coefficient of thermal conductivity is given by

$$k_3 = 2.5 \cdot c_v \cdot \mu_3 \frac{f_k^{(3)}}{f_\mu^{(3)}}. \quad (5-10)$$

The ratio $f_k^{(3)}/f_\mu^{(3)}$ is close to unity.

The equations are not corrected for quantum effects. For helium such a correction would amount at the most to 0.3% at room temperature and be vanishing at higher temperatures, and it can therefore be neglected.

Tables for the collision integral and the factors f_μ and f_k are readily available for many potential functions.

For the present work a very simple function is chosen for correlation of the experimental data of the viscosity:

$$\mu = a \cdot T^b. \quad (5-11)$$

The conductivity data are correlated with a similar equation.

Such an equation is deduced from the inverse power potential function

$$\varphi(r) = \epsilon (a/r)^b, \quad (5-12)$$

for which the collision integral is:

$$\Omega^{(2,2)+}(T^*) = W(b) T^{*2/3}. \quad (5-13)$$

The viscosity is then:

$$\begin{aligned}\mu_1 &= 5.34 \cdot 10^{-6} \frac{T^{1/2} T^{2/\delta}}{\sigma^2 \cdot W(\delta)} \\ &= 5.34 \cdot 10^{-6} \frac{(k/\epsilon)^{2/\delta}}{\sigma^2} \frac{T^{(1/2 + 2/\delta)}}{W(\delta)}.\end{aligned}\quad (5-14)$$

The quantity $W(\delta)$ only depends on δ . Its value has been calculated by several authors, latest by Le Fevre, 1958. It is unity at $\delta = \infty$, and 1.0557 at $\delta = 2$.

As will be seen, the equation has the form of eq. (5-11), $\mu = a \cdot T^b$.

For most gases this is not a very good correlation formula for the transport properties at about room temperature, while it is very good at high temperature. For helium, however, characterized by its extremely small attractive intermolecular forces, the equation is superior to most others as will be demonstrated in the next section.

As an illustration of the development in the knowledge of the transport properties of helium through the years, fig. 3 is presented.

Curve 1 represents measurements of the viscosity of helium performed up to 1940, surveyed by Keesom, 1942. Curves 2 and 3 are predicted extrapolations calculated by Amdur and Mason, 1958, and Lick and Emmons, 1965, respectively. Curve 4 is the result of the present literature survey which will be discussed later. As will be seen from curves 1 and 4 there is a trend to accept higher values of viscosity in recent years. Curve 5 represents measurements of the thermal conductivity of helium till the beginning of the fifties reported by Hilsenrath and Touloukian, 1954. Amdur and Mason (curve 2) keep the Prandtl number constant at 0.666, and Lich and Emmons (curve 3) take the Prandtl number equal to 0.6718 for the temperature range shown in the figure. Curve 6 was calculated by Mann, 1960, on the basis of measurements carried out by Mann and Blais, 1959. Curve 7 is the result of the present work. As in the case of viscosity there has been an increase in the accepted values of thermal conductivity through the years, but the two increases have not been simultaneous, and this has caused much trouble for those trying to find an intermolecular potential that might suit viscosity as well as conductivity data. In the forties the ratio of viscosity to conductivity was increasing with temperature, and this indicated that the Prandtl number would increase with temperature. In the fifties higher values of conductivity data were reported, indicating decreasing Prandtl numbers. However, more recent measurements of the viscosity

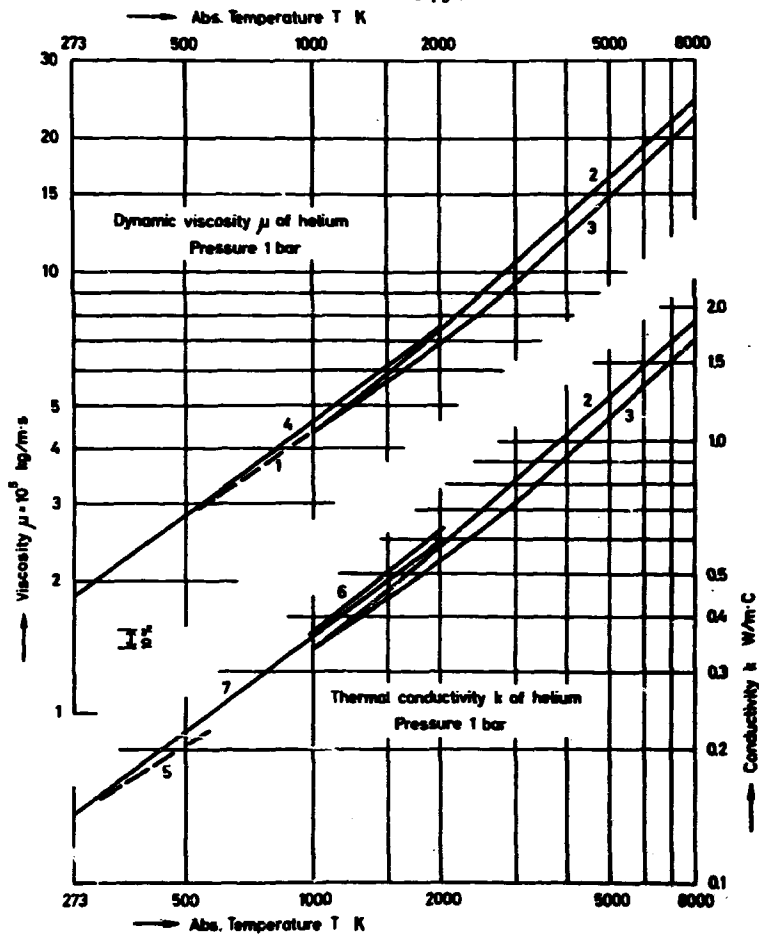


Fig. 3

Viscosity and conductivity of helium at 1 bar

- 1 - Keesom, 1942
- 2 - Amdur and Mason, 1958
- 3 - Lick and Emmons, 1965
- 4 - this work, $\mu = 1.865 \cdot 10^{-5} (T/T_0)^{0.70}$
- 5 - Hilsenrath and Toweloukian, 1954
- 6 - Maass, 1960
- 7 - this work, $k = 0.144 (T/T_0)^{0.71}$

over an extended temperature range show good agreement with the theory when treated together with newer conductivity data as pointed out in an article "Discrepancies Between Viscosity Data for Simple Gases" by H. J. M. Hanley and G. E. Childs, 1968.

6. VISCOSITY DATA

The following equation, which is of the form of eq. (5-11), is adopted for the present work as being the best expression for the viscosity data of helium:

$$\mu = 3.674 \cdot 10^{-7} T^{0.7} = 1.865 \cdot 10^{-5} (T/T_0)^{0.7} \text{ kg/m} \cdot \text{s} \quad (6-1)$$

Fig. 4 is a deviation plot based on this equation. The great number of measurements conducted in the course of time are not shown in the deviation plot, since it is felt that the reviews cited incorporate all known measurements in such a way that detailed consideration of the older individual measurements seems superfluous. Especially the analysis undertaken by professor J. Kestin, of Brown University, U.S.A., establishes the viscosity at room temperature and some hundred degrees above with a great accuracy. Therefore only the following observations and interpolation formulae are included in fig. 3, one of them, the oldest, mostly for historical reasons. The numbers refer to the numbers in the graph.

1 - Keesom, 1942, correlates the measurements performed till that time at temperatures up to 1100 K with the equation:

$$\mu = 1.894 \cdot 10^{-5} (T/T_0)^{0.647} \text{ kg/m} \cdot \text{s}.$$

2 - Mason and Rice, 1954, correlate the measurements till then with the "Exp-6" potential and find the parameters:

$$\alpha = 12.4, \quad r_m = 3.135 \text{ \AA} \quad \text{and} \quad \epsilon/k = 9.16 \text{ K},$$

and compare them with the Lennard-Jones (6-12) potential parameters:

$r_m = 2.869 \text{ \AA}$ and $\epsilon/k = 10.22$, determined by de Boer and Michels, 1939, and revised by Lundbeck, 1951, compiled by Hirschfelder, Curtiss and Bird, 1954. Both potentials were shown to give good agreement to second virial coefficients and to the data of the viscosity of helium available at that time.

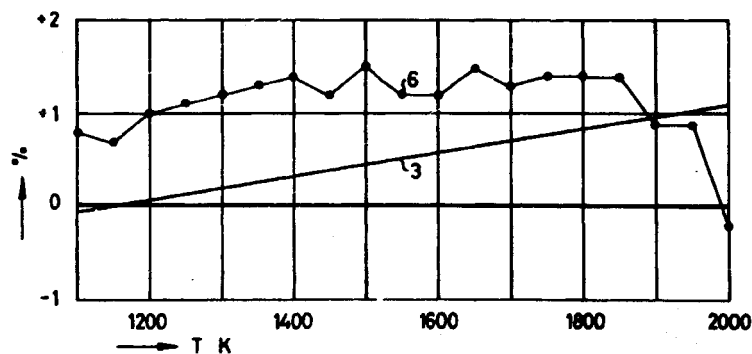
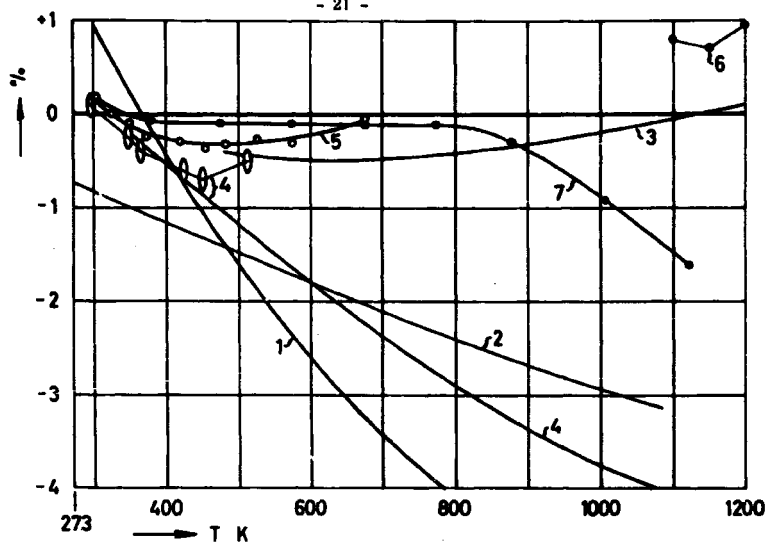


Fig. 4

Dynamic viscosity μ of helium at 1 bar
 Deviations in % from $\mu = 1.865 \cdot 10^{-5} (T/T_0)^{0.7}$

- 1 - Keesom, 1942
- 2 - Mason and Rice, 1954
- 3 - Mann, 1960
- 4 - Kestin and Leidenfröst, 1954
- 5 - DiPippo and Kestin, 1968
- 6 - Guevara, McInteer and Wageman, 1969
- 7 - K. elkar and Kestin, 1970

The two mentioned intermolecular potential functions have the following expressions. The Lennard-Jones (6-12) potential is the (6-12) form of the (6-n) family of potentials.

"Exp-6" potential:

$$\varphi(r) = \frac{\epsilon}{1 - (6/\alpha)} \left[\frac{6}{\alpha} \exp \left(\alpha \left(1 - \frac{r}{r_m} \right) \right) - \left(\frac{r_m}{r} \right)^6 \right], \quad r > r_{\max}$$

$$\varphi(r) = \infty, \quad r < r_{\max}, \quad (6-2)$$

where α is a dimensionless parameter.

The (6-n) family:

$$\varphi(r) = \epsilon \frac{n}{n-6} \left(\frac{r}{r_m} \right)^{6/(n-6)} \left[\left(\frac{r}{r_m} \right)^n - \left(\frac{r_m}{r} \right)^6 \right]. \quad (6-3)$$

For the notation see fig. 2.

3 - Mann, 1960, deduces the viscosity from measurements of thermal conductivity carried out by Mann and Blais, 1959, at high temperatures, 1200 - 2000 K, and uses the exponential potential:

$$\varphi(r) = \epsilon \cdot e^{\alpha(1-r/r_c)}. \quad (6-4)$$

This potential may give the best fit to experimental viscosity data. An alternative form, employed by Amdur and Mason, 1958, and by Monchick, 1959, is

$$\varphi(r) = \varphi_0 \cdot e^{-r/\rho}.$$

These potentials unfortunately do not give a simple equation for the viscosity suitable for engineering purposes as does the inverse power potential chosen for this work.

4 - Kestin and Leidenfrost, 1959, perform measurements by means of the oscillating disk method. The results are marked with ovals. They correlate the data together with existing measurements at higher temperatures with the "Exp-6" potential obtaining the parameters: $\alpha = 12.4$, $r_m = 3.225 \text{ \AA}$ and $\epsilon/k = 6.482 \text{ K}$. For the Lennard-Jones parameter they find: $r_m = 2.462$ and $\epsilon/k = 69.08$.

5 - DiPippo and Kestin, 1969, refine the oscillating disk method to still greater accuracy, a precision of 0.05%, and obtain the results marked with circles. They correlate with the "Exp-6" potential, with the parameters:

$\alpha = 12.4$, $r_m = 2.7254$ and $\epsilon/k = 36.12$. For the Lennard-Jones potential they determine the parameters $r_m = 2.4220$ and $\epsilon/k = 86.20$.

6 - Guevara, McInteer and Wageman, 1969. These viscosity measurements were performed by the capillary tube method in the temperature range 1100 to 2150 K relative to the viscosity at the reference temperature, 283 K. The measurements are of great reproducibility, 0.1%, and accuracy, 0.4%.

7 - Kalelkar and Kestin, 1970. In these measurements the temperature range of the oscillating disk method is extended to 1100 K. The results are represented by a (6-9.5) potential model with the parameters $\sigma = 2.215 \text{ \AA}$ and $\epsilon/k = 73.21 \text{ K}$.

The deviation plot, fig. 4, indicates that the standard deviation of the viscosity data is about 0.4% at 273 K and 2.7% at 1800 K, i. e.

$$\sigma = 0.0015 \text{ T\%}.$$

(6-5)

The potential parameters applied for fig. 4 are listed in the following table. Only the parameters of potentials for which ϵ and σ have the physical meaning ascribed to them in fig. 2 are tabulated, and this here applies to the "Exp-6" and the (6-n) potentials. The exponential and the inverse power potentials do not describe attractive intermolecular forces, and for this reason ϵ etc. for these potentials have mathematical significance only. In the table r_m is replaced by σ , the ratio σ/r_m is 0.8792 for the "Exp-6", $\alpha = 12.4$, potential and 0.8909 for the (6-12) potential.

Potential	Ref. above	Source	$\epsilon/k \text{ K}$	$\sigma \text{ \AA}$
"Exp-6", $\alpha = 12.4$	2	Mason and Rice, 1954	9.16	2.757
- " -	4	Kestin and Leidenfrost, 1959	6.48	2.836
- " -	5	DiPippo and Kestin, 1969	36.12	2.398
(6-12)	2	Mason and Rice, 1954	10.22	2.556
- " -	4	Kestin and Leidenfrost, 1959	69.08	2.211
- " -	5	DiPippo and Kestin, 1969	86.20	2.158
(6-9.5)	7	Kalelkar and Kestin, 1970	73.21	2.215

The table indicates that the newer measurements of the viscosity of helium can be represented by the potentials mentioned only if relatively high values of ϵ/k are used. The table shows values of 46 to 98 K, etc.

pared with the older values of about 10 K, which are, however, based on a much wider temperature range than the newer ones. Buckingham, Davies and Davies, 1957, further discuss the findings of Mason and Rice and others. From their detailed analyses it follows that for viscosity measurements at 2 to 260 K the value of ϵ/k is 9.7 to 10.3, derived from a modified "Exp-6" potential with $a = 13.5$, properly corrected for quantum effects. The potential also fits the older measurements of viscosity at 300 to 1000 K that are now supposed to be incorrect, but since the part of the potential in the vicinity of the energy minimum has very little bearing on the calculation of the viscosity at these temperatures, it does not imply that $\epsilon/k = 10$ is incorrect, merely that the "Exp-6" as well as the (6-n) potential are unrealistic for calculations when high as well as low temperatures are to be covered by one potential.

The two potentials used by Mason and Rice are sketched in fig. 5A. It will be seen that quantitatively they differ rather much in spite of their equal ability to express the measured values. It is interesting to compare these artificial or empirically determined potentials with interaction energy functions calculated on the basis of atomic physics alone. Such functions for helium are shown in fig. 5B; the curves correspond to different assumptions for the calculation. The agreement is good as far as the order of magnitude of the energy minimum is concerned. Fig. 5 is sketched from graphs in the monograph "Theory of Intermolecular Forces" by H. Margenau and N.R. Kestner, 1969.

The problem as to which part of a potential is significant for calculation of the viscosity at a given temperature has been discussed by Amdur and Ross, 1958, and by Kalelkar and Kestin, 1970. Amdur and Ross present a very simple solution to the problem based on the inverse power potential, mentioned earlier. Equating the quantity W in the collision integral with unity, and introducing the equivalent hard-sphere diameter, σ_0 , which at a given temperature gives the same value of the viscosity as does the actual inverse power potential, we get

$$\frac{k}{T} = \varphi(\sigma_0) \text{ erg,}$$

where $k = 1.381 \cdot 10^{-16} \text{ erg/K.}$

These considerations are valid at temperatures that are high compared with the "temperature" ϵ/k . Since this is only 10 K, i. e. $\epsilon = 0.0014 \cdot 10^{-12} \text{ erg,}$ for helium, it follows that the expression is accurate enough for helium already from room temperature and up. For the range of temperatures

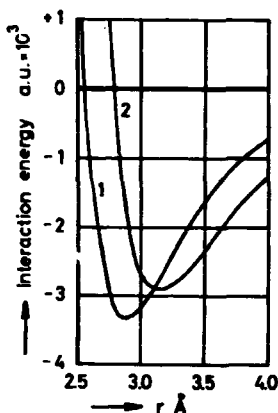


Fig. 5A. The Lennard-Jones Potential, 1, and the "Exp-6" potential, 2, for helium, Mason and Rice, 1954.

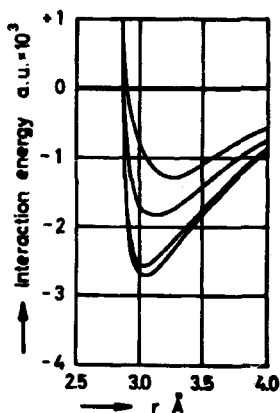


Fig. 5B. Theoretically evaluated interaction energy of helium, Margenau and Kestner, 1969.

considered in this work, namely 273 - 1800 K, the range of energies is $0.04 \cdot 10^{-12}$ to $0.25 \cdot 10^{-12}$ erg, assuming that only one point of the potential determines the viscosity. This assumption is, of course, not strictly correct since the viscosity represents a weighted average of the potential, but evidently only a limited part of the potential on each side of the point in question is important for the determination of the viscosity at the corresponding temperature.

Kalelkar and Kestin estimate the range of separation distances, r , significant for the calculation of viscosities in a given temperature range by computing the average separating distance at a given temperature. The result is very much the same as estimated by the procedure of Amdur and Ross. The temperature range 300 to 1100 K gives a range of average separation distances corresponding to energies from $0.03 \cdot 10^{-12}$ to $0.1 \cdot 10^{-12}$ erg, which should be compared with $\epsilon = 0.0014 \cdot 10^{-12}$ erg for $\epsilon/k = 10$ K.

Kalelkar and Kestin conclude that the physical significance of ϵ is distorted when ϵ/k is determined by fitting a potential of the types considered here to viscosity data at higher temperatures.

The difference between the "Exp-6" and the (6-n) potential and their ability to give a precise fit to the measurements is illustrated in figs. 6A and 6B. Fig. 6A is a plot of the deviations of the viscosity values calculated by means of a "Exp-6", $\alpha = 12$, potential from the adopted viscosity values. The parameter ϵ/k is varied from 10 to 50 K. Similarly fig. 6B is a deviation plot for the (6-9) potential with values of ϵ/k from 20 to 80. As will be seen the "Exp-6" potential is superior to the (6-9) potential, but on the other hand, the "Exp-6" potential can be fitted to within 0.5%, even if ϵ/k is arbitrarily selected between 20 and 50. This indicates that the "Exp-6" potential is rather insensitive to a variation of the parameters.

Fig. 7 is presented as an illustration of the features mentioned and also for comparison of the potentials calculated from gaseous transport properties with potentials derived from measurements of the scattering of high-velocity helium atoms performed by Amdur and Harkness, 1954, curves 1 and 2, and by Amdur, Jordan and Colgate, 1961, curve 3. Curve 4 is calculated from the "Exp-6" pot., eq. (6-2), with the parameters $\alpha = 12$, $\epsilon/k = 20$ K and $r_m = 2.925$ Å. Curve 5 is calculated from the (6-9) pot., eq. (6-3), with the parameters $\epsilon/k = 20$ K and $\sigma = 2.530$ Å. Curve 6 is the exponential potential, eq. (6-4), with the parameters $r_c = 2$, $\alpha = 10$ and $\epsilon/k = 560$. Curve 7 is the inverse power potential, eq. (5-12), $\phi(r) = 77 \cdot 10^{-12}/r^{10}$, from which the equation for the viscosity adopted for this work, eq. (6-1), can be derived by insertion of $W_{(b)} = 1.031$.

Of the potentials shown the exponential potential will give the best fit to the results of Amdur et al. when extrapolated to higher energies. However, it should be noted that none of the individual potentials seems to fit at all temperatures and for calculation of the different properties which can be computed from the intermolecular potential. As an example fig. 1 shows the second virial coefficient, and curve b by Mason and Rice fits well over the entire temperature range in spite of lack of ability of the potential to fit the new viscosity measurements. Curve c, an inverse power potential by Amdur and Mason, also fits well, but as seen from fig. 3 the potential gives too small viscosity values at 1000 K.

It may be concluded that the "Exp-6" and the (6-n) potentials, although giving very good interpolation formulae for the properties and also being useful, when mixtures of helium and other gases are treated, do not represent the intermolecular forces for helium above room temperature as

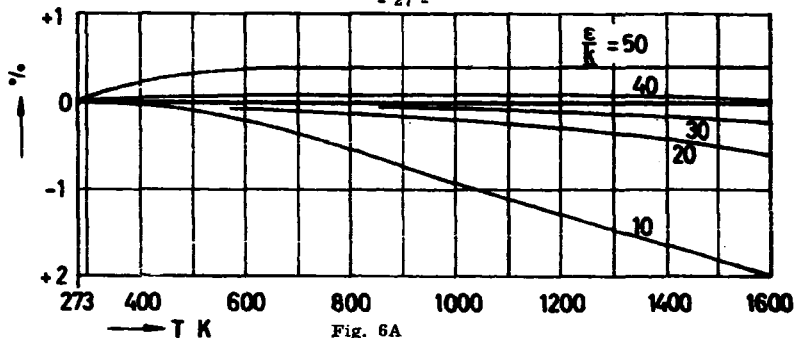


Fig. 6A

"Exp-6" potential, $\alpha = 12$

Parameters	ϵ/k K	10	20	30	40	50
	r_m Å	3.134	2.925	2.809	2.728	2.663

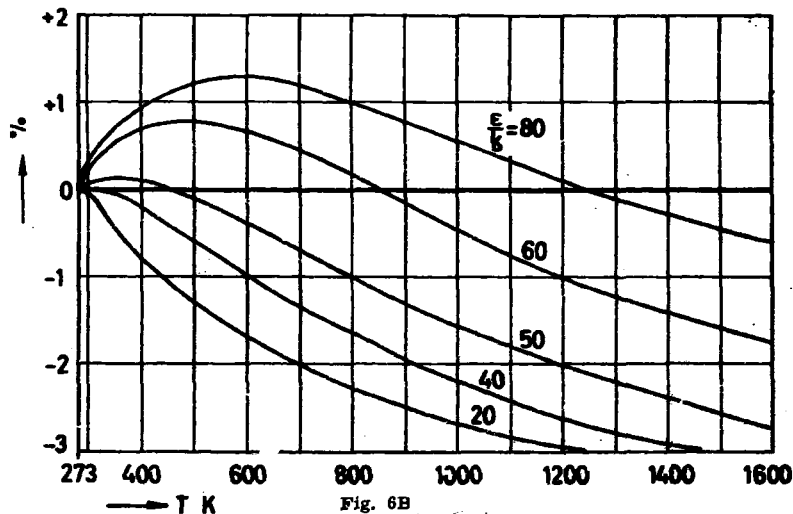


Fig. 6B

(6-9) potential

Parameters	ϵ/k K	20	40	50	60	80
	σ Å	2.530	2.372	2.320	2.268	2.200

Deviations in % from $\mu = 1.865 \cdot 10^{-5} (T/T_0)^{0.7}$
 Deviation plots of calculated viscosity coefficients

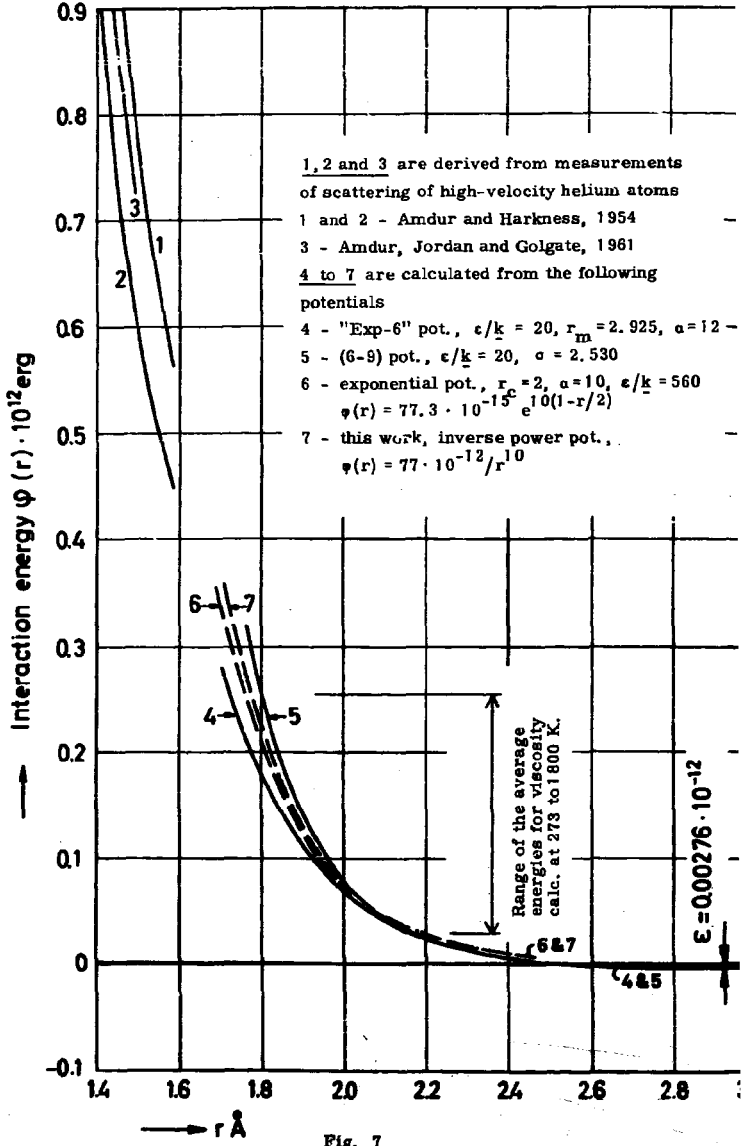


Fig. 7

Intermolecular potential functions for helium.

well as other potentials do. It seems most likely that the exponential potential is the most versatile function particularly for temperatures higher than those considered in this work. The inverse power potential chosen for this work gives a sufficiently accurate interpolation formula of ample simplicity for practical purposes.

Finally it should be noted that the viscosity is independent of the pressure within the accuracy of the measurements hitherto performed.

7. THERMAL CONDUCTIVITY DATA

The literature presents much information on the coefficient of thermal conductivity of helium based on measurements of the conductivity in the steady-state configuration in a cell with either a thin wire suspended along the centre line or concentric cylinders. The data at or below 1 bar are presented in the form of a deviation plot, fig. 8, based on the following equation adopted for this work:

$$\begin{aligned} k &= 2.682 \cdot 10^{-3} (1 + 1.123 \cdot 10^{-3} P) T^{(0.71(1 - 2 \cdot 10^{-4} P))} \\ &= 0.144 (1 + 2.7 \cdot 10^{-4} P (T/T_0)^{(0.71(1 - 2 \cdot 10^{-4} P))}) \quad \text{W/m K.} \end{aligned} \quad (7-1)$$

P is the pressure in bar.

At 1 bar the pressure term is negligible, therefore the following equation is used in fig. 8:

$$k = 0.144 (T/T_0)^{0.71}. \quad (7-2)$$

The deviations are also given for a survey of the data presented by the N. B. S., USA, 1966, curve 6, and for the expression for the conductivity based on the viscosity data, eq. (5-10):

$$k = 2.5 \, c_v \cdot \mu_3 (l_k^{(3)} / l_\mu^{(3)}) .$$

The ratio of the correcting factors is about 1.004. Insertion of this and c_v into the equation based on viscosity data gives:

$$k = 2.51 \, c_v \cdot \mu = 7824 \, \mu, \quad (7-3)$$

represented by curve 9 in the figure.

The deviation plot shows that the adopted values of the conductivity are chosen as being between the measured values and the values derived from the viscosity data, curve 9. It is felt that this is the most appropriate choice, since some uncertainty still exists as to the accuracy of both approaches to the problem. If this is taken into account, the standard deviation of the conductivity data calculated from the adopted equation can be estimated to be about 1% at 273 K and 6% at 1800 K, i. e.,

$$\sigma = 0.0035 \text{ T \%} \quad (7-4)$$

While the conductivity measurements at low pressure are complicated by the inaccuracy of the correction for the accommodation effects, measurements at high pressure are difficult to perform in steady-state experiments because of the risk of natural convection in the gas. However, the agreement of different measurements at 100 bar is reasonably good as is evident from the deviation plot, fig. 9. The pressure effect is small and seems to vanish at elevated temperatures. Fig. 9 is based on eq. (7-1) for $P = 100$, i. e.

$$k = 0.14789(T/T_0)^{0.6958} \quad (7-5)$$

The adopted formula eq. (7-1) takes the pressure effect into account.

8. PRANDTL NUMBER

The Prandtl number is

$Pr = c_p \frac{\mu}{k}$, or, by insertion of eqs. (4-4), (6-1) and (7-1):

$$\begin{aligned} Pr &= \frac{0.7111}{1 + 1.123 \cdot 10^{-3} P} T^{-(0.01 - 1.42 \cdot 10^{-4} P)} \\ &= \frac{0.6728}{1 + 2.7 \cdot 10^{-4} P} (T/T_0)^{-(0.01 - 1.42 \cdot 10^{-4} P)} \quad (8-1) \end{aligned}$$

The standard deviation is $\sigma = 0.004 \text{ T \%}$. (8-2)

Finally it should be mentioned that the conductivity and the Prandtl number can be measured indirectly by shock-tube and temperature-recovery methods. Such measurements, however, appear to be less accurate than those considered in this work for which reason a detailed analysis of the results by those methods is not incorporated herein.

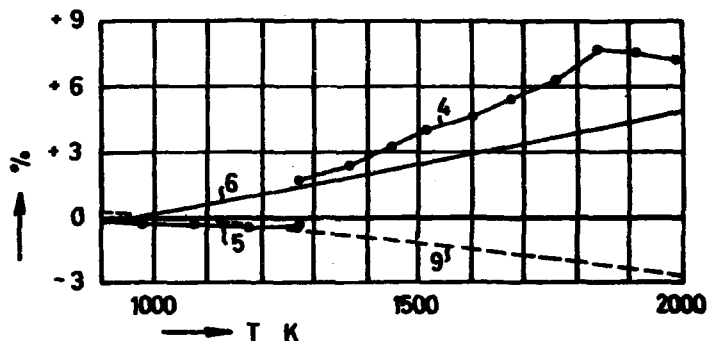
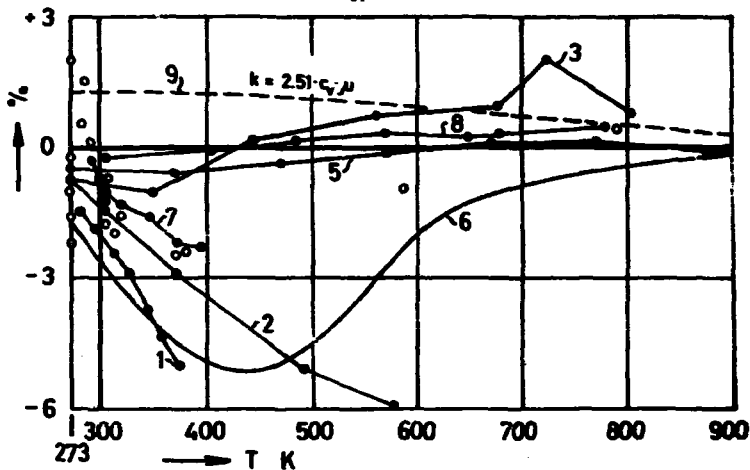


Fig. 8

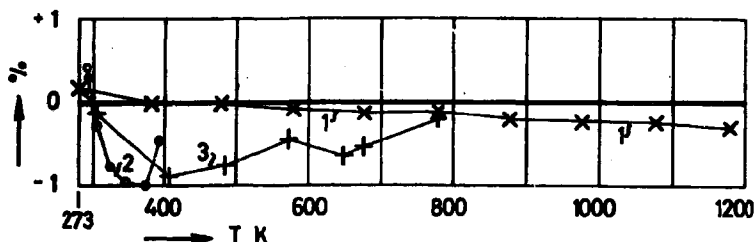
Thermal conductivity k of helium at 1 bar

Deviations in % from $k = 0.144(T/T_0)^{0.71}$

- 1 - Johnston and Grilly, 1946
- 2 - Kannuliuk and Carman, 1952
- 3 - Zaitseva, 1959
- 4 - Mann and Blais, 1959
- 5 - Vargaftik and Zimina, 1965
- 6 - Powell, Ho and Liley, 1966

- 7 - Ho and Leidenfrost, 1969
- 8 - Le Neindre, ... and Vodar, 1969
- 9 - derived from viscosity,
i.e. $k = 2.51 \cdot c_v \cdot \mu$.

Open circles are observations by nine other experimentalists.



Thermal conductivity k of helium at 100 bar

Fig. 9

Deviation in % from $k = 0.14789 (T/T_0)^{0.6958}$

1 - Vargaftik and Zimina, 1965

2 - Ho and Leidenfrost, 1969

3 - Le Neindre, and Vodar, 1969

ACKNOWLEDGEMENTS AND NOTES

The author is indebted to Mr. K. Sullivan, UKAEA, Risley, and Mr. Jan Blomstrand, OECD-H. T. R. -project DRAGON, for interest and inspiration, and to Mr. N.E. Kaiser and Mrs. Susanne Jensen for the computational handling of the data.

After the report was written a recent work on the conductivity of helium and other gases by Dr. J.W. Haarman of the Netherlands was brought to the author's attention. This work is particularly interesting as it gives high-precision measurements by the transient hot-wire method with an extremely thin wire. The results are dealt with in the last-minute note on p. 37.

A similar apparatus is at present being finally tested at the laboratories of the Research Establishment Ris5. In this set-up the data logger is a digital computer, and in this the method differs from that used by Dr. Haarman. Moreover, the apparatus is designed for larger ranges of pressure and temperature.

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Last-Minute Note

The work mentioned in section 9, Acknowledgement and Notes, by Dr. J. W. Haarman "EEN NAUWKEURIGE METHODE VOOR HET BEPALEN VAN DE WARMTEGELEIDINGSCOËFFICIËNT VAN GASSEN (DE NIET-STATIONAIRE DRAADMETHODE)", Delftsche Uitgevers Maatschappij, NV, Delft, 1969, presents measurements of the conductivity of helium at about atmospheric pressure and at temperatures from 55 to 195 °C. The measured values give the points marked 10 in the deviation plot, fig. 8a. The deviation are small and do not necessitate a change in the adopted formula, eq. (7-1), for the thermal conductivity of helium.

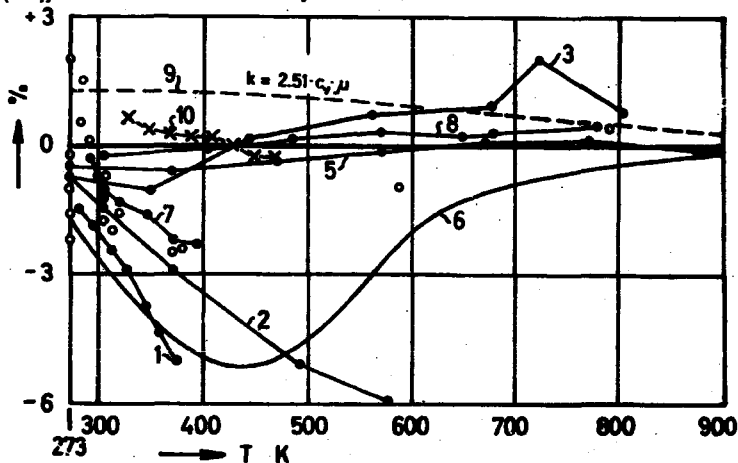


Fig. 8a. Thermal conductivity k of helium at 1 bar
Deviations in % from $k = 0.144 (T/T_0)^{0.71}$
10 - Haarman, 1969

11. TABLES OF RECOMMENDED DATA

The tables on the following sheets give values of the properties at selected pressures and temperatures.

Tables with smaller intervals of pressure and temperature, suitable for linear interpolation, are presented in Dragon Project Report No. 704 "Tables of Thermophysical Properties of Helium", issued by O. E. C. A. High Temperature Reactor Project, DRAGON, A. E. E. Winfrith, Dorchester, Dorset, England.

Temperature °C K		Pressure bar abs					
		1	20	40	60	80	100
0.00	273.16	1.00053	1.01060	1.02120	1.03190	1.04240	1.05300
26.84	300.00	1.00047	1.00947	1.01894	1.02842	1.03762	1.04736
50.00	323.16	1.00043	1.00866	1.01733	1.02599	1.03465	1.04332
76.84	350.00	1.00039	1.00787	1.01575	1.02362	1.03149	1.03936
100.00	373.16	1.00036	1.00729	1.01458	1.02187	1.02916	1.03645
126.84	400.00	1.00034	1.00671	1.01341	1.02012	1.02683	1.03353
150.00	423.16	1.00031	1.00627	1.01254	1.01881	1.02508	1.03134
176.84	450.00	1.00029	1.00582	1.01165	1.01747	1.02329	1.02911
200.00	473.16	1.00027	1.00548	1.01097	1.01645	1.02193	1.02741
226.84	500.00	1.00026	1.00513	1.01026	1.01539	1.02053	1.02566
250.00	523.16	1.00024	1.00486	1.00972	1.01458	1.01944	1.02430
276.84	550.00	1.00023	1.00458	1.00915	1.01373	1.01831	1.02368
300.00	573.16	1.00022	1.00436	1.00871	1.01307	1.01742	1.02278
326.84	600.00	1.00021	1.00412	1.00825	1.01237	1.01649	1.02162
350.00	623.16	1.00020	1.00394	1.00788	1.01182	1.01576	1.01970
376.84	650.00	1.00019	1.00375	1.00749	1.01124	1.01498	1.01873
400.00	673.16	1.00018	1.00359	1.00718	1.01077	1.01437	1.01796
426.84	700.00	1.00017	1.00343	1.00685	1.01028	1.01371	1.01713
450.00	723.16	1.00016	1.00330	1.00659	1.00989	1.01318	1.01648
476.84	750.00	1.00016	1.00315	1.00631	1.00946	1.01262	1.01577
500.00	773.16	1.00015	1.00304	1.00608	1.00912	1.01217	1.01521
526.84	800.00	1.00015	1.00292	1.00584	1.00876	1.01168	1.01460
550.00	823.16	1.00014	1.00282	1.00564	1.00846	1.01128	1.01411
576.84	850.00	1.00014	1.00271	1.00543	1.00814	1.01086	1.01357
600.00	873.16	1.00013	1.00263	1.00526	1.00789	1.01051	1.01314
626.84	900.00	1.00013	1.00253	1.00507	1.00760	1.01014	1.01267
650.00	923.16	1.00012	1.00246	1.00492	1.00738	1.00983	1.01229
676.84	950.00	1.00012	1.00238	1.00475	1.00713	1.00950	1.01188
700.00	973.16	1.00012	1.00231	1.00462	1.00692	1.00923	1.01154
726.84	1000.00	1.00011	1.00223	1.00447	1.00670	1.00893	1.01117
750.00	1023.16	1.00011	1.00217	1.00435	1.00652	1.00869	1.01087
776.84	1050.00	1.00011	1.00211	1.00421	1.00632	1.00843	1.01053
800.00	1073.16	1.00010	1.00205	1.00410	1.00616	1.00821	1.01026
826.84	1100.00	1.00010	1.00199	1.00398	1.00598	1.00797	1.00996
850.00	1123.16	1.00010	1.00194	1.00389	1.00585	1.00777	1.00971
876.84	1150.00	1.00009	1.00189	1.00378	1.00567	1.00755	1.00944
900.00	1173.16	1.00009	1.00184	1.00369	1.00553	1.00738	1.00922
926.84	1200.00	1.00009	1.00179	1.00359	1.00538	1.00718	1.00897
950.00	1223.16	1.00009	1.00175	1.00351	1.00526	1.00702	1.00877
976.84	1250.00	1.00009	1.00171	1.00342	1.00513	1.00684	1.00854
1000.00	1273.16	1.00008	1.00167	1.00334	1.00501	1.00669	1.00836
1026.84	1300.00	1.00008	1.00163	1.00326	1.00489	1.00652	1.00815
1050.00	1323.16	1.00008	1.00160	1.00319	1.00479	1.00638	1.00798
1076.84	1350.00	1.00008	1.00156	1.00312	1.00467	1.00623	1.00779
1100.00	1373.16	1.00008	1.00153	1.00305	1.00458	1.00611	1.00763
1126.84	1400.00	1.00007	1.00149	1.00298	1.00447	1.00597	1.00746
1150.00	1423.16	1.00007	1.00146	1.00292	1.00439	1.00585	1.00731
1176.84	1450.00	1.00007	1.00143	1.00286	1.00429	1.00572	1.00715
1200.00	1473.16	1.00007	1.00140	1.00281	1.00421	1.00561	1.00702
1226.84	1500.00	1.00007	1.00137	1.00275	1.00412	1.00549	1.00687
1250.00	1523.16	1.00007	1.00135	1.00270	1.00404	1.00539	1.00674
1276.84	1550.00	1.00007	1.00132	1.00264	1.00396	1.00528	1.00660
1300.00	1573.16	1.00006	1.00130	1.00259	1.00389	1.00519	1.00648
1326.84	1600.00	1.00006	1.00127	1.00254	1.00381	1.00508	1.00635
1350.00	1623.16	1.00006	1.00125	1.00250	1.00373	1.00500	1.00625
1376.84	1650.00	1.00006	1.00122	1.00245	1.00367	1.00490	1.00612
1400.00	1673.16	1.00006	1.00120	1.00241	1.00361	1.00482	1.00602
1426.84	1700.00	1.00006	1.00118	1.00236	1.00354	1.00473	1.00591
1450.00	1723.16	1.00006	1.00116	1.00233	1.00349	1.00465	1.00581
1476.84	1750.00	1.00006	1.00114	1.00228	1.00342	1.00456	1.00571
1500.00	1773.16	1.00006	1.00112	1.00225	1.00337	1.00449	1.00562
1526.84	1800.00	1.00006	1.00110	1.00221	1.00331	1.00441	1.00552

Temperature °C K		Pressure bar abs					
		1	20	40	60	80	100
0.00	273.16	0.17614	3.4877	6.2030	10.2481	13.5252	16.7364
26.84	300.00	0.16039	3.1792	6.2993	9.3620	12.3687	15.3210
50.00	323.16	0.14880	2.9537	5.8572	8.7116	11.5182	14.2781
76.84	350.00	0.13749	2.7294	5.4164	8.0622	10.6675	13.2334
100.00	373.16	0.12896	2.5615	5.0861	7.5747	10.0281	12.4469
126.84	400.00	0.12031	2.3910	4.7503	7.0786	9.3764	11.6445
150.00	423.16	0.11273	2.2611	4.4942	6.6998	8.8764	11.0306
176.84	450.00	0.10695	2.1272	4.2299	6.3085	8.3634	10.3951
200.00	473.16	0.10171	2.0237	4.0255	6.0057	7.9646	9.9027
226.84	500.00	0.09626	1.9158	3.8121	5.6892	7.5475	9.3872
250.00	523.16	0.09200	1.8315	3.6453	5.4417	7.2210	8.9335
276.84	550.00	0.08751	1.7426	3.4693	5.1805	6.8763	8.5569
300.00	573.16	0.08397	1.6725	3.3306	4.9744	6.6042	8.2200
326.84	600.00	0.08022	1.5981	3.1831	4.7552	6.3145	7.8613
350.00	623.16	0.07724	1.5390	3.0659	4.5809	6.0842	7.5759
376.84	650.00	0.07405	1.4757	2.9404	4.3943	5.8375	7.2700
400.00	673.16	0.07150	1.4252	2.8401	4.2451	5.6401	7.0252
426.84	700.00	0.06876	1.3707	2.7321	4.0843	5.4273	6.7613
450.00	723.16	0.06656	1.3270	2.6453	3.9550	5.2562	6.5490
476.84	750.00	0.06418	1.2797	2.5514	3.8151	5.0709	6.3190
500.00	773.16	0.06225	1.2415	2.4755	3.7021	4.9212	6.1331
526.84	800.00	0.06017	1.2000	2.3930	3.5792	4.7584	5.9309
550.00	823.16	0.05847	1.1663	2.3262	3.4795	4.6264	5.7669
576.84	850.00	0.05663	1.1296	2.2532	3.3707	4.4822	5.5877
600.00	873.16	0.05513	1.0998	2.1938	3.2821	4.3648	5.4418
626.84	900.00	0.05348	1.0671	2.1288	3.1851	4.2362	5.2820
650.00	923.16	0.05214	1.0404	2.0757	3.1059	4.1311	5.1514
676.84	950.00	0.05067	1.0111	2.0174	3.0189	4.0157	5.0079
700.00	973.16	0.04946	0.9871	1.9696	2.9477	3.9212	4.8903
726.84	1000.00	0.04810	0.9607	1.9170	2.8692	3.8171	4.7608
750.00	1023.16	0.04705	0.9390	1.8739	2.8047	3.7316	4.6545
776.84	1050.00	0.04584	0.9150	1.8262	2.7336	3.6372	4.5370
800.00	1073.16	0.04485	0.8953	1.7870	2.6750	3.5594	4.4403
826.84	1100.00	0.04376	0.8735	1.7436	2.6102	3.4734	4.3332
850.00	1123.16	0.04286	0.8556	1.7078	2.5568	3.4025	4.2449
876.84	1150.00	0.04186	0.8356	1.6681	2.4975	3.3238	4.1469
900.00	1173.16	0.04103	0.8192	1.6353	2.4485	3.2587	4.0660
926.84	1200.00	0.04011	0.8009	1.5989	2.3941	3.1865	3.9760
950.00	1223.16	0.03935	0.7858	1.5688	2.3491	3.1266	3.9015
976.84	1250.00	0.03851	0.7689	1.5352	2.2989	3.0600	3.8186
1000.00	1273.16	0.03781	0.7550	1.5074	2.2574	3.0048	3.7498
1026.84	1300.00	0.03703	0.7394	1.4764	2.2110	2.9435	3.6731
1050.00	1323.16	0.03638	0.7265	1.4507	2.1726	2.8921	3.6095
1076.84	1350.00	0.03566	0.7121	1.4219	2.1296	2.8351	3.5384
1100.00	1373.16	0.03506	0.7001	1.3980	2.0939	2.7876	3.4792
1126.84	1400.00	0.03438	0.6867	1.3713	2.0540	2.7345	3.4131
1150.00	1423.16	0.03382	0.6755	1.3491	2.0207	2.6904	3.3581
1176.84	1450.00	0.03320	0.6631	1.3242	1.9835	2.6409	3.2964
1200.00	1473.16	0.03268	0.6526	1.3035	1.9525	2.5997	3.2450
1226.84	1500.00	0.03209	0.6410	1.2802	1.9177	2.5554	3.1875
1250.00	1523.16	0.03160	0.6313	1.2608	1.8887	2.5149	3.1334
1276.84	1550.00	0.03106	0.6203	1.2391	1.8561	2.4716	3.0834
1300.00	1573.16	0.03060	0.6112	1.2209	1.8289	2.4354	3.0404
1326.84	1600.00	0.03009	0.6010	1.2004	1.7964	2.3948	2.9998
1350.00	1623.16	0.02966	0.5924	1.1834	1.7728	2.3609	2.9474
1376.84	1650.00	0.02917	0.5828	1.1642	1.7441	2.3227	2.8998
1400.00	1673.16	0.02877	0.5747	1.1481	1.7201	2.2907	2.8600
1426.84	1700.00	0.02832	0.5657	1.1300	1.6951	2.2548	2.8151
1450.00	1723.16	0.02794	0.5581	1.1149	1.6704	2.2246	2.7776
1476.84	1750.00	0.02751	0.5495	1.0978	1.6449	2.1907	2.7353
1500.00	1773.16	0.02715	0.5424	1.0835	1.6235	2.1622	2.6998
1526.84	1800.00	0.02674	0.5343	1.0674	1.5994	2.1372	2.6598

Multiply tabulated values by 10^{-5} to obtain viscosity in kg/m \cdot s

Temperature °C	Temperature K	Pressure bar abs 1 - 100
0.00	273.16	1.8648
26.84	300.00	1.9912
50.00	323.16	2.0976
76.84	350.00	2.2181
100.00	373.16	2.3199
126.84	400.00	2.4355
150.00	423.16	2.5333
176.84	450.00	2.6448
200.00	473.16	2.7393
226.84	500.00	2.8472
250.00	523.16	2.9389
276.84	550.00	3.0436
300.00	573.16	3.1328
326.84	600.00	3.2348
350.00	623.16	3.3217
376.84	650.00	3.4212
400.00	673.16	3.5061
426.84	700.00	3.6094
450.00	723.16	3.6864
476.84	750.00	3.7817
500.00	773.16	3.8630
526.84	800.00	3.9564
550.00	823.16	4.0363
576.84	850.00	4.1279
600.00	873.16	4.2063
626.84	900.00	4.2964
650.00	923.16	4.3735
676.84	950.00	4.4622
700.00	973.16	4.5380
726.84	1000.00	4.6253
750.00	1023.16	4.7000
776.84	1050.00	4.7860
800.00	1073.16	4.8586
826.84	1100.00	4.9444
850.00	1123.16	5.0170
876.84	1150.00	5.1007
900.00	1173.16	5.1724
926.84	1200.00	5.2549
950.00	1223.16	5.3257
976.84	1250.00	5.4072
1000.00	1273.16	5.4772
1026.84	1300.00	5.5578
1050.00	1323.16	5.6269
1076.84	1350.00	5.7065
1100.00	1373.16	5.7749
1126.84	1400.00	5.8537
1150.00	1423.16	5.9213
1176.84	1450.00	5.9992
1200.00	1473.16	6.0662
1226.84	1500.00	6.1433
1250.00	1523.16	6.2096
1276.84	1550.00	6.2860
1300.00	1573.16	6.3516
1326.84	1600.00	6.4272
1350.00	1623.16	6.4922
1376.84	1650.00	6.5672
1400.00	1673.16	6.6316
1426.84	1700.00	6.7078
1450.00	1723.16	6.7857
1476.84	1750.00	6.8633
1500.00	1773.16	6.9466
1526.84	1800.00	6.9796

Temperature °C K		Pressure bar abs					
		1	20	40	60	80	100
0.00	273.16	0.1440	0.1449	0.1457	0.1465	0.1472	0.1479
26.84	300.00	0.1539	0.1548	0.1557	0.1565	0.1572	0.1579
50.00	323.16	0.1622	0.1632	0.1640	0.1648	0.1656	0.1662
76.84	350.00	0.1717	0.1727	0.1735	0.1743	0.1751	0.1757
100.00	373.16	0.1797	0.1807	0.1815	0.1824	0.1831	0.1837
126.84	400.00	0.1888	0.1897	0.1906	0.1915	0.1922	0.1928
150.00	423.16	0.1965	0.1975	0.1984	0.1992	0.1999	0.2005
176.84	450.00	0.2053	0.2062	0.2071	0.2079	0.2087	0.2093
200.00	473.16	0.2127	0.2137	0.2146	0.2154	0.2161	0.2167
226.84	500.00	0.2212	0.2222	0.2231	0.2239	0.2246	0.2252
250.00	523.16	0.2284	0.2294	0.2303	0.2311	0.2318	0.2324
276.84	550.00	0.2367	0.2377	0.2386	0.2394	0.2401	0.2407
300.00	573.16	0.2437	0.2447	0.2456	0.2464	0.2471	0.2477
326.84	600.00	0.2518	0.2528	0.2537	0.2544	0.2551	0.2557
350.00	623.16	0.2587	0.2596	0.2605	0.2613	0.2620	0.2625
376.84	650.00	0.2665	0.2675	0.2684	0.2691	0.2698	0.2703
400.00	673.16	0.2732	0.2742	0.2751	0.2758	0.2765	0.2770
426.84	700.00	0.2809	0.2819	0.2827	0.2835	0.2841	0.2846
450.00	723.16	0.2875	0.2884	0.2893	0.2901	0.2907	0.2912
476.84	750.00	0.2950	0.2960	0.2968	0.2976	0.2982	0.2986
500.00	773.16	0.3014	0.3024	0.3033	0.3040	0.3046	0.3050
526.84	800.00	0.3088	0.3098	0.3106	0.3113	0.3119	0.3124
550.00	823.16	0.3152	0.3161	0.3169	0.3176	0.3182	0.3186
576.84	850.00	0.3224	0.3234	0.3242	0.3249	0.3254	0.3258
600.00	873.16	0.3286	0.3296	0.3304	0.3311	0.3316	0.3320
626.84	900.00	0.3358	0.3367	0.3375	0.3382	0.3387	0.3390
650.00	923.16	0.3419	0.3428	0.3436	0.3442	0.3447	0.3451
676.84	950.00	0.3489	0.3498	0.3506	0.3512	0.3517	0.3520
700.00	973.16	0.3549	0.3558	0.3566	0.3572	0.3577	0.3580
726.84	1000.00	0.3618	0.3627	0.3635	0.3641	0.3645	0.3648
750.00	1023.16	0.3678	0.3687	0.3694	0.3700	0.3704	0.3707
776.84	1050.00	0.3746	0.3755	0.3762	0.3768	0.3772	0.3774
800.00	1073.16	0.3804	0.3813	0.3820	0.3826	0.3830	0.3832
826.84	1100.00	0.3872	0.3880	0.3887	0.3893	0.3896	0.3898
850.00	1123.16	0.3929	0.3938	0.3945	0.3950	0.3954	0.3955
876.84	1150.00	0.3996	0.4004	0.4011	0.4016	0.4019	0.4021
900.00	1173.16	0.4053	0.4061	0.4068	0.4073	0.4076	0.4077
926.84	1200.00	0.4118	0.4126	0.4133	0.4138	0.4141	0.4142
950.00	1223.16	0.4175	0.4183	0.4189	0.4194	0.4196	0.4197
976.84	1250.00	0.4239	0.4247	0.4254	0.4258	0.4260	0.4261
1000.00	1273.16	0.4295	0.4303	0.4309	0.4313	0.4315	0.4316
1026.84	1300.00	0.4359	0.4367	0.4373	0.4377	0.4379	0.4379
1050.00	1323.16	0.4414	0.4422	0.4427	0.4431	0.4433	0.4433
1076.84	1350.00	0.4478	0.4485	0.4491	0.4494	0.4496	0.4495
1100.00	1373.16	0.4532	0.4539	0.4545	0.4548	0.4549	0.4549
1126.84	1400.00	0.4595	0.4602	0.4607	0.4610	0.4611	0.4611
1150.00	1423.16	0.4648	0.4655	0.4661	0.4664	0.4665	0.4664
1176.84	1450.00	0.4711	0.4717	0.4722	0.4725	0.4726	0.4725
1200.00	1473.16	0.4764	0.4771	0.4775	0.4778	0.4778	0.4777
1226.84	1500.00	0.4825	0.4832	0.4836	0.4839	0.4839	0.4837
1250.00	1523.16	0.4878	0.4884	0.4889	0.4891	0.4891	0.4889
1276.84	1550.00	0.4939	0.4945	0.4949	0.4951	0.4951	0.4949
1300.00	1573.16	0.4997	0.4997	0.5001	0.5003	0.5003	0.5000
1326.84	1600.00	0.5051	0.5057	0.5061	0.5063	0.5062	0.5059
1350.00	1623.16	0.5103	0.5109	0.5113	0.5114	0.5113	0.5110
1376.84	1650.00	0.5163	0.5169	0.5172	0.5173	0.5172	0.5169
1400.00	1673.16	0.5214	0.5220	0.5223	0.5224	0.5223	0.5219
1426.84	1700.00	0.5273	0.5279	0.5282	0.5283	0.5282	0.5277
1450.00	1723.16	0.5333	0.5339	0.5342	0.5343	0.5342	0.5337
1476.84	1750.00	0.5393	0.5398	0.5401	0.5401	0.5400	0.5395
1500.00	1773.16	0.5453	0.5459	0.5461	0.5461	0.5460	0.5454
1526.84	1800.00	0.5504	0.5509	0.5511	0.5511	0.5510	0.5504

Temperature		Pressure bar abs					
°C	K	1	20	40	60	80	100
0.00	273.16	0.6727	0.6687	0.6648	0.6613	0.6580	0.6551
26.84	300.00	0.6720	0.6682	0.6645	0.6612	0.6581	0.6554
50.00	323.16	0.6715	0.6679	0.6643	0.6611	0.6582	0.6556
76.84	350.00	0.6710	0.6675	0.6641	0.6610	0.6583	0.6558
100.00	373.16	0.6706	0.6672	0.6639	0.6610	0.6585	0.6560
126.84	400.00	0.6701	0.6668	0.6637	0.6609	0.6584	0.6562
150.00	423.16	0.6698	0.6666	0.6635	0.6608	0.6584	0.6563
176.84	450.00	0.6694	0.6663	0.6634	0.6608	0.6585	0.6565
200.00	473.16	0.6690	0.6660	0.6632	0.6607	0.6585	0.6566
226.84	500.00	0.6687	0.6658	0.6631	0.6607	0.6586	0.6568
250.00	523.16	0.6684	0.6656	0.6629	0.6606	0.6586	0.6569
276.84	550.00	0.6680	0.6653	0.6628	0.6606	0.6587	0.6570
300.00	573.16	0.6678	0.6651	0.6627	0.6605	0.6587	0.6571
326.84	600.00	0.6675	0.6649	0.6625	0.6605	0.6587	0.6573
350.00	623.16	0.6672	0.6647	0.6624	0.6605	0.6588	0.6574
376.84	650.00	0.6669	0.6645	0.6623	0.6604	0.6588	0.6575
400.00	673.16	0.6667	0.6644	0.6622	0.6604	0.6588	0.6576
426.84	700.00	0.6664	0.6642	0.6621	0.6603	0.6589	0.6577
450.00	723.16	0.6662	0.6640	0.6620	0.6603	0.6589	0.6578
476.84	750.00	0.6660	0.6638	0.6619	0.6603	0.6589	0.6579
500.00	773.16	0.6658	0.6637	0.6618	0.6602	0.6590	0.6580
526.84	800.00	0.6656	0.6635	0.6617	0.6602	0.6590	0.6581
550.00	823.16	0.6654	0.6634	0.6616	0.6602	0.6590	0.6581
576.84	850.00	0.6652	0.6632	0.6615	0.6601	0.6590	0.6582
600.00	873.16	0.6650	0.6631	0.6615	0.6601	0.6591	0.6583
626.84	900.00	0.6648	0.6630	0.6614	0.6601	0.6591	0.6584
650.00	923.16	0.6646	0.6629	0.6613	0.6601	0.6591	0.6585
676.84	950.00	0.6644	0.6627	0.6612	0.6600	0.6591	0.6585
700.00	973.16	0.6643	0.6626	0.6612	0.6600	0.6592	0.6586
726.84	1000.00	0.6641	0.6625	0.6611	0.6600	0.6592	0.6587
750.00	1023.16	0.6640	0.6624	0.6610	0.6600	0.6592	0.6587
776.84	1050.00	0.6638	0.6622	0.6609	0.6599	0.6592	0.6588
800.00	1073.16	0.6636	0.6621	0.6609	0.6599	0.6593	0.6589
826.84	1100.00	0.6635	0.6620	0.6608	0.6599	0.6593	0.6589
850.00	1123.16	0.6633	0.6619	0.6607	0.6599	0.6593	0.6590
876.84	1150.00	0.6632	0.6618	0.6607	0.6599	0.6593	0.6591
900.00	1173.16	0.6631	0.6617	0.6606	0.6598	0.6593	0.6591
926.84	1200.00	0.6629	0.6616	0.6606	0.6598	0.6594	0.6592
950.00	1223.16	0.6628	0.6615	0.6605	0.6598	0.6594	0.6592
976.84	1250.00	0.6626	0.6614	0.6604	0.6598	0.6594	0.6593
1000.00	1273.16	0.6625	0.6613	0.6604	0.6598	0.6594	0.6593
1026.84	1300.00	0.6624	0.6612	0.6603	0.6597	0.6594	0.6594
1050.00	1323.16	0.6623	0.6611	0.6603	0.6597	0.6594	0.6595
1076.84	1350.00	0.6621	0.6611	0.6602	0.6597	0.6595	0.6595
1100.00	1373.16	0.6620	0.6610	0.6602	0.6597	0.6595	0.6596
1126.84	1400.00	0.6619	0.6609	0.6601	0.6597	0.6595	0.6596
1150.00	1423.16	0.6618	0.6608	0.6601	0.6596	0.6595	0.6597
1176.84	1450.00	0.6617	0.6607	0.6600	0.6596	0.6595	0.6597
1200.00	1473.16	0.6616	0.6605	0.6600	0.6596	0.6595	0.6598
1226.84	1500.00	0.6615	0.6606	0.6599	0.6596	0.6596	0.6598
1250.00	1523.16	0.6614	0.6605	0.6599	0.6596	0.6596	0.6598
1276.84	1550.00	0.6612	0.6604	0.6598	0.6596	0.6596	0.6599
1300.00	1573.16	0.6611	0.6603	0.6598	0.6595	0.6596	0.6599
1326.84	1600.00	0.6610	0.6603	0.6597	0.6595	0.6596	0.6600
1350.00	1623.16	0.6609	0.6602	0.6597	0.6595	0.6596	0.6600
1376.84	1650.00	0.6608	0.6601	0.6597	0.6595	0.6596	0.6601
1400.00	1673.16	0.6607	0.6600	0.6596	0.6595	0.6597	0.6601
1426.84	1700.00	0.6606	0.6600	0.6596	0.6595	0.6597	0.6602
1450.00	1723.16	0.6606	0.6599	0.6595	0.6595	0.6597	0.6602
1476.84	1750.00	0.6604	0.6598	0.6595	0.6594	0.6597	0.6602
1500.00	1773.16	0.6604	0.6598	0.6594	0.6594	0.6597	0.6603
1526.84	1800.00	0.6603	0.6597	0.6594	0.6594	0.6597	0.6603