CS217: Artificial Intelligence and Machine Learning (associated lab: CS240)

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Week6 of 17feb25, Predicate Calculus

Main points covered: week5 of 3feb25

Important pointes associated with FFNN BP

Local Minima

Momentum Factor

Symmetry Breaking

Hilbert's formalization of propositional calculus

1. Elements are *propositions* : Capital letters

2. Operator is only one : $\rightarrow$ (called implies)

3. Special symbol F (called 'false')

4. Two other symbols : *'('* and *')'*

5. Well formed formula is constructed according to the grammar

$$WFF \rightarrow P|F|WFF \rightarrow WFF$$

6. Inference rule : only one

Given $A \rightarrow B$ and

A

write B

known as MODUS PONENS

7. Axioms : Starting structures

A1: $(A \to (B \to A))$

A2: $((A \to (B \to C)) \to ((A \to B) \to (A \to C)))$

A3 $(((A \to F) \to F) \to A)$

This formal system defines the propositional calculus

A very useful theorem (Actually a meta theorem, called deduction theorem)

<u>Statement</u>

If

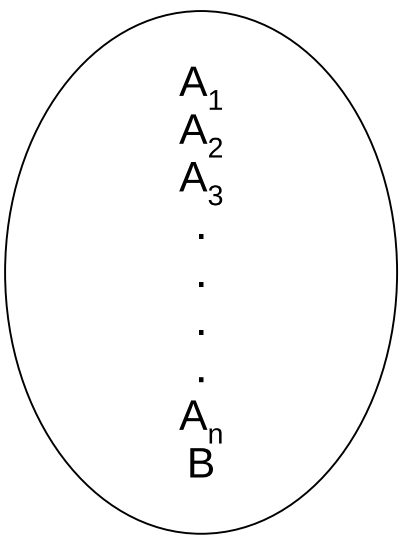
$A_1, A_2, A_3 \ldots\ldots\ldots\ldots A_n \vdash B$

then

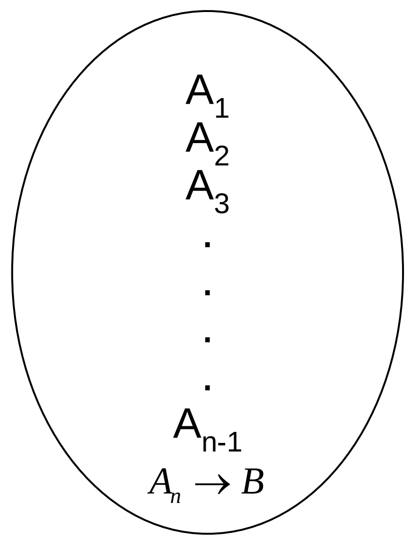
$A_1, A_2, A_3, \ldots\ldots\ldots\ldots\ldots A_{n-1} \vdash \quad A_n \to B$

$\vdash$ is read as 'derives'

Given

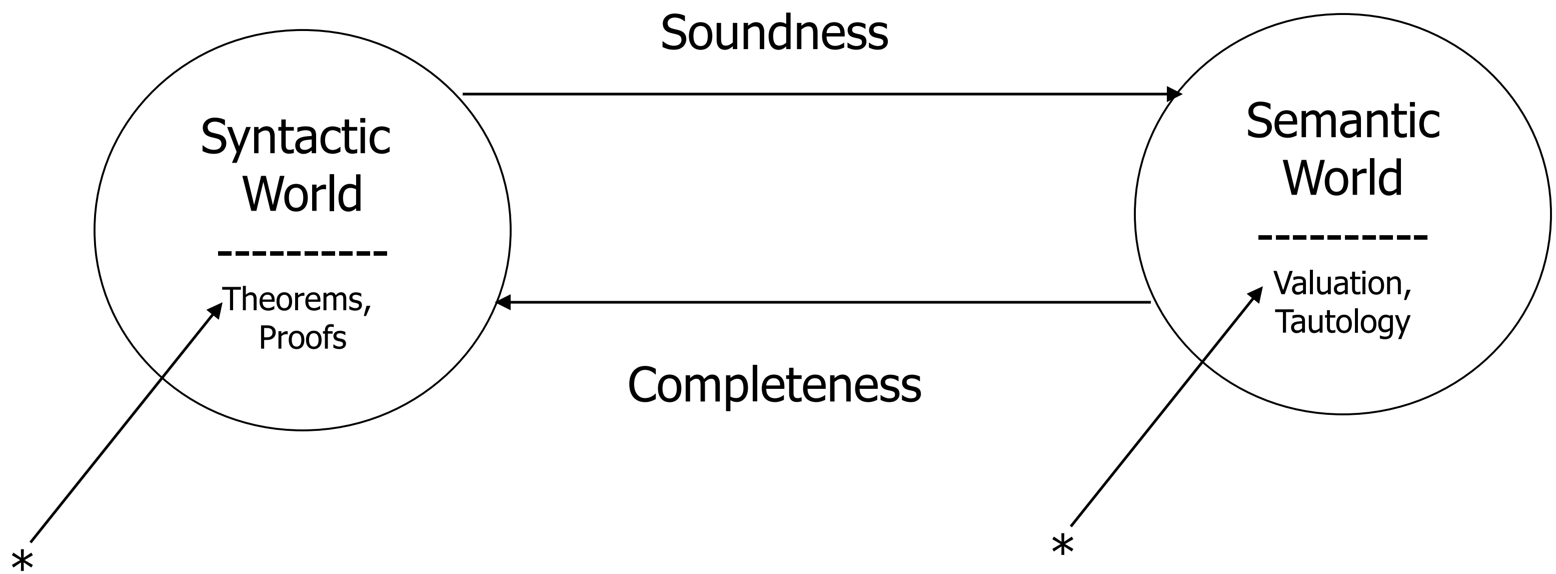


Picture 1



Picture 2

Soundness, Completeness & Consistency



Consistency

The System should not be able to

prove both P and ~P, *i.e.*, should not be

able to derive

$\boxed{F}$

End of main points

Predicate calculus

Introduce through the “Himalayan Club Example”

Himalayan Club example

- Introduction through an example *(Zohar Manna, 1974):*
  - Problem: A, B and C belong to the Himalayan club. Every member in the club is either a mountain climber or a skier or both. A likes whatever B dislikes and dislikes whatever B likes. A likes rain and snow. No mountain climber likes rain. Every skier likes snow. *Is there a member who is a mountain climber and not a skier?*
- Given knowledge has:
  - Facts
  - Rules

Example contd.

- Let *mc* denote mountain climber and *sk* denotes skier. Knowledge representation in the given problem is as follows:
  1. $member(A)$
  2. $member(B)$
  3. $member(C)$
  4. $\forall x[member(x) \rightarrow (mc(x) \vee sk(x))]$
  5. $\forall x[mc(x) \rightarrow \sim like(x,rain)]$
  6. $\forall x[sk(x) \rightarrow like(x, snow)]$
  7. $\forall x[like(B, x) \rightarrow \sim like(A, x)]$
  8. $\forall x[\sim like(B, x) \rightarrow like(A, x)]$
  9. $like(A, rain)$
  10. $like(A, snow)$
  11. Question: $\exists x[member(x) \wedge mc(x) \wedge \sim sk(x)]$
- We have to infer the 11^{th} expression from the given 10.
- Done through Resolution Refutation.

Club example: Inferencing

1. $member(A)$
2. $member(B)$
3. $member(C)$
4. $\forall x[member(x) \rightarrow (mc(x) \vee sk(x))]$
   - Can be written as
     $[member(x) \rightarrow (mc(x) \vee sk(x))]$
   - $\sim member(x) \vee mc(x) \vee sk(x)$
5. $\forall x[sk(x) \rightarrow lk(x, snow)]$
   - $\sim sk(x) \vee lk(x, snow)$
6. $\forall x[mc(x) \rightarrow \sim lk(x, rain)]$
   - $\sim mc(x) \vee \sim lk(x, rain)$
7. $\forall x[like(A, x) \rightarrow \sim lk(B, x)]$
   - $\sim like(A, x) \vee \sim lk(B, x)$

8. $\forall x[\sim lk(A,x) \rightarrow lk(B,x)]$

– $lk(A,x) \vee lk(B,x)$

9. $lk(A, rain)$

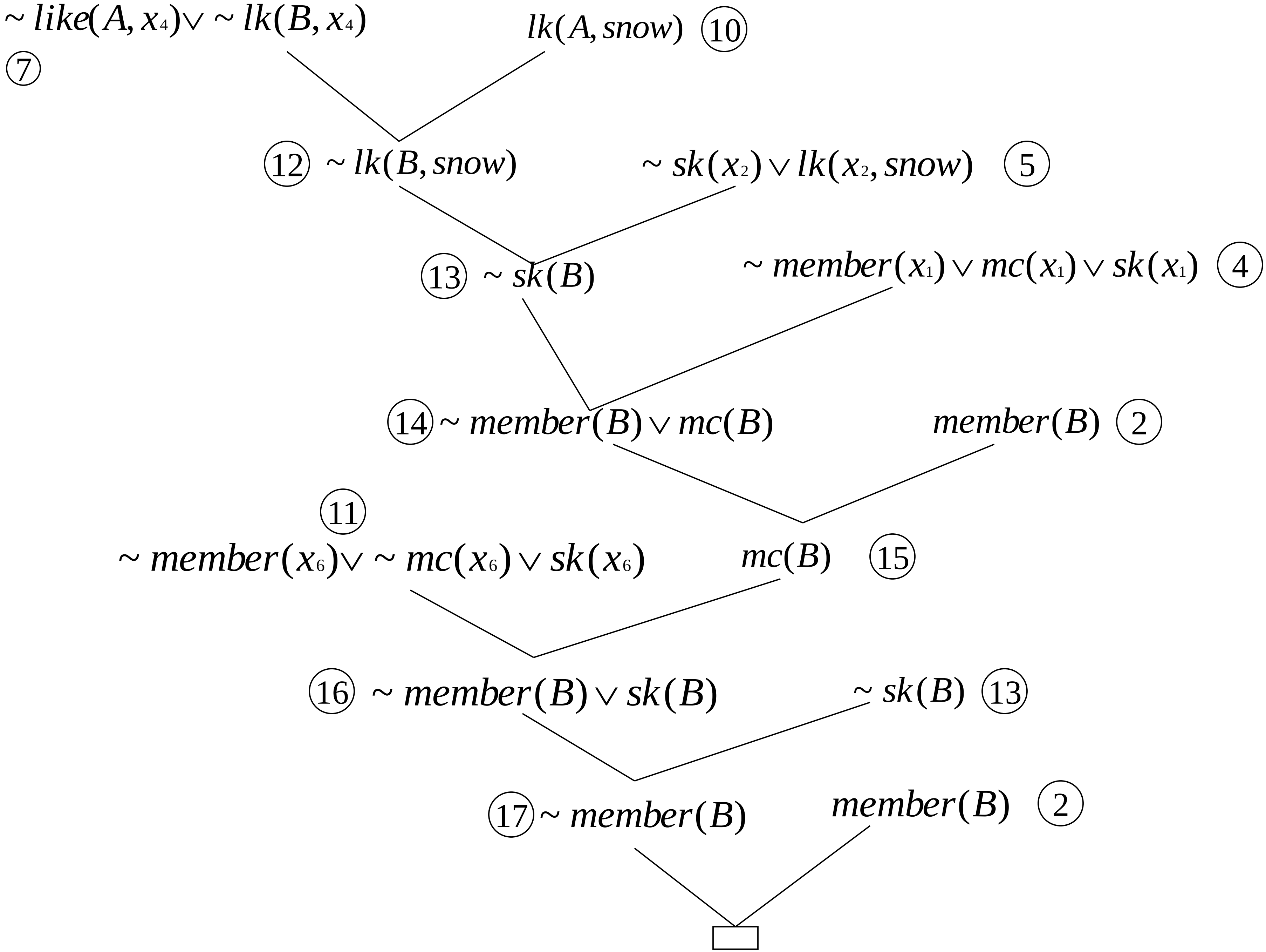
10. $lk(A, snow)$

11. $\exists x[member(x) \wedge mc(x) \wedge \sim sk(x)]$

– Negate– $\forall x[\sim member(x) \vee \sim mc(x) \vee sk(x)]$

- Now standardize the variables apart which results in the following

1. $member(A)$
2. $member(B)$
3. $member(C)$
4. $\sim member(x_1) \vee mc(x_1) \vee sk(x_1)$
5. $\sim sk(x_2) \vee lk(x_2, snow)$
6. $\sim mc(x_3) \vee \sim lk(x_3, rain)$
7. $\sim like(A, x_4) \vee \sim lk(B, x_4)$
8. $lk(A, x_5) \vee lk(B, x_5)$
9. $lk(A, rain)$
10. $lk(A, snow)$
11. $\sim member(x_6) \vee \sim mc(x_6) \vee sk(x_6)$



~ like(A, x4) ∨ ~ lk(B, x4) 7
lk(A, snow) 10
12 ~ lk(B, snow)
~ sk(x2) ∨ lk(x2, snow) 5
13 ~ sk(B)
~ member(x1) ∨ mc(x1) ∨ sk(x1) 4
14 ~ member(B) ∨ mc(B)
member(B) 2
11
~ member(x6) ∨ ~ mc(x6) ∨ sk(x6)
mc(B) 15
16 ~ member(B) ∨ sk(B)
~ sk(B) 13
17 ~ member(B)
member(B) 2

Well known examples in Predicate Calculus

- Man is mortal : rule

  $\forall x[man(x) \rightarrow mortal(x)]$

- shakespeare is a man

  man(shakespeare)

- To infer shakespeare is mortal

  mortal(shakespeare)

Predicate Calculus: origin

- Predicate calculus originated in language

Sentence

Subject Predicate

Grass is green

Subject Predicate

green(grass)

$$P : D \rightarrow \{T, F\}$$

D : Domain

Predicate Calculus: only for declarative sentences

- Is grass green? (Interrogative)
- Oh, grass is green! (Exclamatory)

Declarative Sentence

Subject Predicate

- Grass which is supple is green

$$\forall x(\mathrm{grass}(x)) \wedge \mathrm{supple}(x) \rightarrow \mathrm{green}(x))$$

Predicate Calculus: more expressive power than propositional calculus

- 2 is even and is divisible by 2: P1
- 4 is even and is divisible by 2: P2
- 6 is even and is divisible by 2: P3

Generalizing,

$$\forall x((Integer(x) \wedge even(x) \Rightarrow divides(2, x))$$

Predicate Calculus: finer than propositional calculus

1. Finer Granularity (Grass is green, ball is green, leaf is green (green(x)))
2. Succinct description for infinite number of statements which would need $\propto$ number of properties

3 place predicate
Example: x gives y to z   give(x,y,z)

4 place predicate
Example: x gives y to z through w   give(x,y,z,w)

Double causative in Hindi giving rise to higher place predicates

- जॉन ने खाना खाया
  John ne khana khaya
  John <CM> food ate
  John ate food
  eat(John, food)

- जॉन ने जैक को खाना खिलाया
  John ne Jack ko khana khilaya
  John <CM> Jack <CM> food fed
  John fed Jack
  eat(John, Jack, food)

- जॉन ने जैक को जिल के द्‌वारा खाना खिलाया
  John ne Jack ko Jill ke dvara khana khilaya
  John <CM> Jack <CM> Jill <CM> food made-to-eat
  John fed Jack through Jill
  eat(John, Jack, Jill, food)

PC primitive: N-ary Predicate

$$P(a_1, \ldots a_n)$$

$$P : D^n \rightarrow \{T, F\}$$

- Arguments of predicates can be variables and constants
- Ground instances : Predicate all whose arguments are constants

N-ary Functions

$$f : D^n \rightarrow D$$

president(India) :   Pranab Mukherjee

- Constants & Variables : Zero-order objects
- Predicates & Functions : First-order objects

Prime minister of India is older than the president of India

older(prime_minister(India), president(India))

Operators

$$\wedge \vee \sim \oplus \forall \rightarrow \exists$$

- Universal Quantifier $\exists$
- Existential Quantifier

All men are mortal

$$\forall x[man(x) \rightarrow mortal(x)]$$

Some men are rich

$$\exists x[man(x) \wedge rich(x)]$$

Tautologies

$$\sim \forall x(p(x)) \rightarrow \exists x(\sim p(x))$$

$$\sim \exists x(p(x)) \rightarrow \forall x(\sim p(x))$$

- 2nd tautology in English:
  - *Not a single man in this village is educated implies all men in this village are uneducated*
- Tautologies are important instruments of logic, but uninteresting statements!

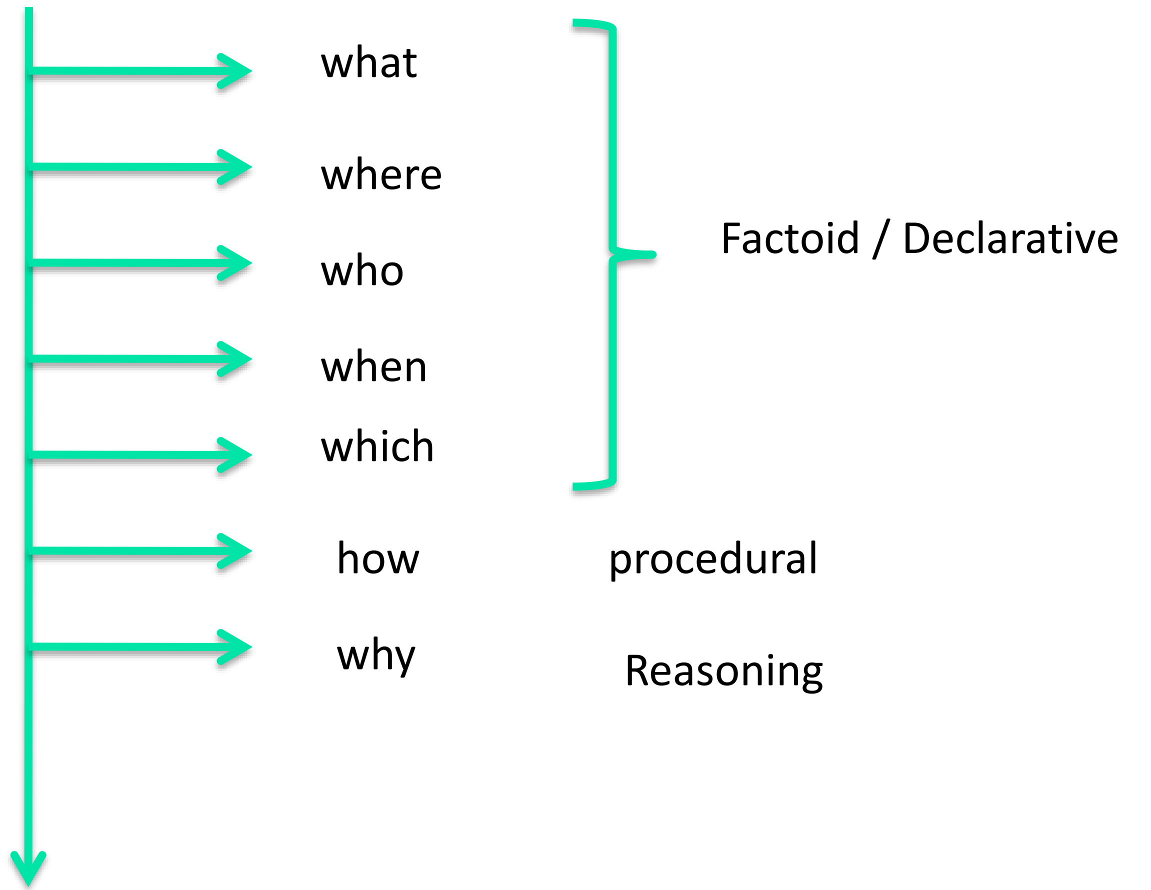
Inferencing: Forward Chaining

- $man(x) \rightarrow mortal(x)$
  - *Dropping the quantifier, implicitly Universal quantification assumed*
  - *man(shakespeare)*
- Goal mortal(shakespeare)
  - Found in one step
  - x = shakespeare, unification

Backward Chaining

- $man(x) \rightarrow mortal(x)$
- Goal mortal(shakespeare)
  - x = shakespeare
  - Travel back over and hit the fact asserted
  - man(shakespeare)

Wh-Questions and Knowledge



Fixing Predicates

- Natural Sentences

<Subject> <verb> <object>

Verb(subject,object)

predicate(subject)

Examples

- Ram is a boy
  - Boy(Ram)?
  - Is_a(Ram,boy)?

- Ram Playes Football
  - Plays(Ram,football)?
  - Plays_football(Ram)?

Knowledge Representation of Complex Sentence

- *"In every city there is a thief who is beaten by every policeman in the city"*

Knowledge Representation of Complex Sentence

- *"In every city there is a thief who is beaten by every policeman in the city"*

$$\forall x[\mathrm{city}(x) \rightarrow \{\exists y((\mathrm{thief}(y) \wedge \mathrm{lives\_in}\ (y,x)) \wedge \forall z(\mathrm{policeman}(z,x) \rightarrow \mathrm{beaten\_by}\,(z,y)))\}]$$

Insight into resolution

Resolution - Refutation

- $man(x) \rightarrow mortal(x)$
  - *Convert to clausal form*
  - $\sim man(shakespeare) \lor mortal(x)$
- Clauses in the knowledge base
  - $\sim man(shakespeare) \lor mortal(x)$
  - $man(shakespeare)$
  - $mortal(shakespeare)$

Resolution – Refutation contd

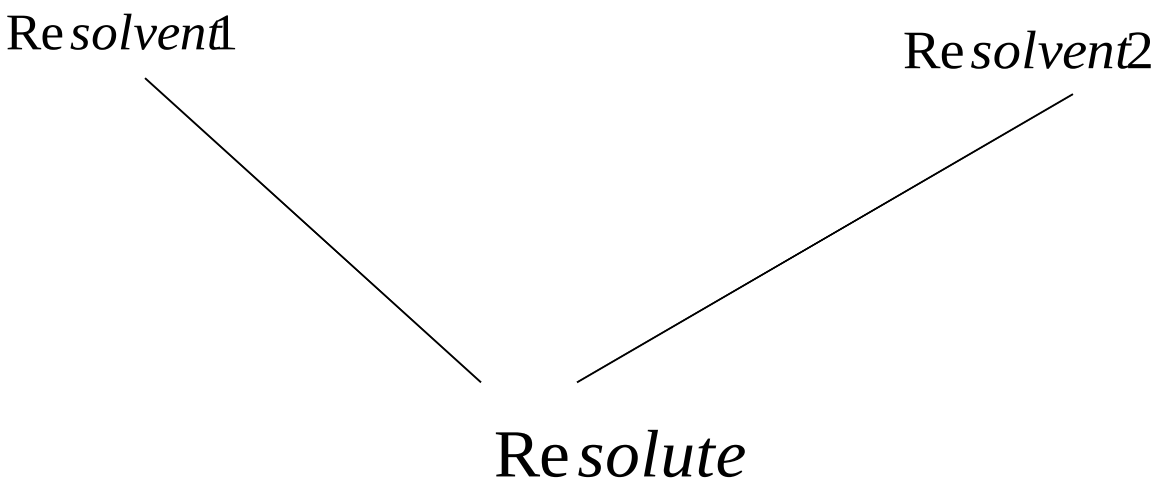
- *Negate the goal*
  - *~man(shakespeare)*
- Get a pair of resolvents

$\sim mortal(shakespeare)$ $\qquad$ $\sim man(shakespeare) \vee mortal(shakespeare)$

$\sim man(shakespeare)$ $\qquad$ $\sim man(shakespeare)$

$\square$

Resolution Tree



Search in resolution

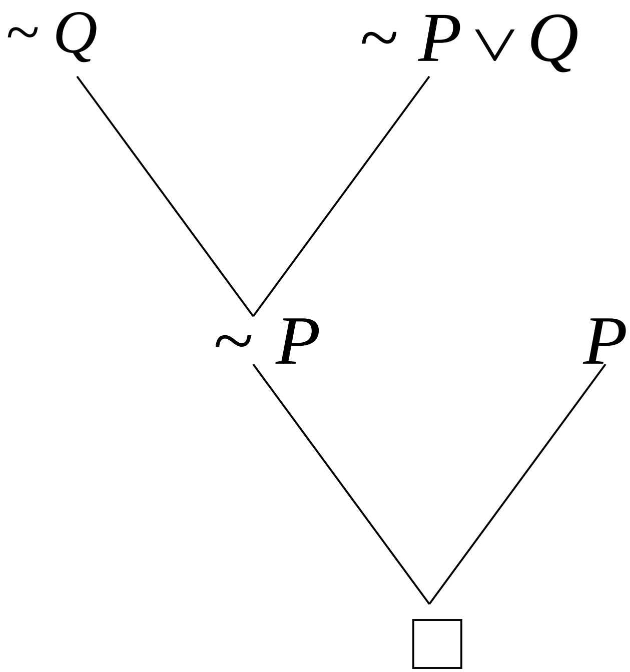
- Heuristics for Resolution Search
  - Goal Supported Strategy
    - Always start with the negated goal
  - Set of support strategy
    - Always one of the resolvents is the most recently produced resolute

Inferencing in Predicate Calculus

- Forward chaining
  - Given P, $P \rightarrow Q$, to infer Q
  - P, match *L.H.S* of
  - Assert Q from *R.H.S*
- Backward chaining
  - Q, Match *R.H.S* of $P \rightarrow Q$
  - assert P
  - Check if P exists
- Resolution – Refutation
  - Negate goal
  - Convert all pieces of knowledge into clausal form (disjunction of literals)
  - See if contradiction indicated by null clause $\square$ can be derived

1. P
2. $P \rightarrow Q$ converted to $\sim P \vee Q$
3. $\sim Q$

Draw the resolution tree (actually an inverted tree). Every node is a clausal form and branches are intermediate inference steps.



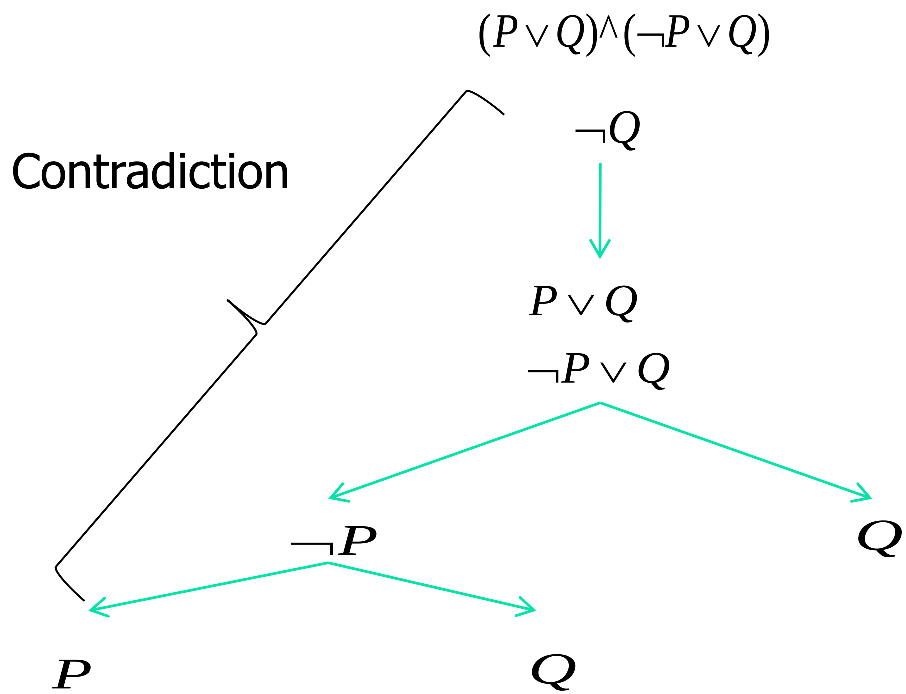
Theoretical basis of Resolution

- Resolution is proof by contradiction
- ***resolvent1 .AND. resolvent2 => resolute*** is a tautology

$$\begin{tikzcd} P \vee Q \arrow[dr] & & \neg P \vee Q \arrow[dl] \\ & Q & \end{tikzcd}$$

Tautologiness of Resolution

- Using Semantic Tree



Theoretical basis of Resolution (cont …)

- Monotone Inference
  - Size of Knowledge Base goes on increasing as we proceed with resolution process since intermediate resolvents added to the knowledge base
- Non-monotone Inference
  - Size of Knowledge Base does not increase
  - Human beings use non-monotone inference

Interpretation in Logic

- Logical expressions or formulae are "FORMS" (placeholders) for whom <u>contents</u> are created through interpretation.
- Example:

$$\exists F\left[\{F(a)=b\}\wedge\forall x\{P(x)\rightarrow\left(F(x)=g\left(x,F\left(h(x)\right)\right)\right)\}\right]$$

- This is a Second Order Predicate Calculus formula.
- Quantification on 'F' which is a function.

Examples

- Interpretation:1

  $D=N$ (natural numbers)

  $a = 0$ and $b = 1$

  $x \in N$

  $P(x)$ stands for $x > 0$

  $g(m,n)$ stands for $(m \times n)$

  $h(x)$ stands for $(x - 1)$

- Above interpretation defines **Factorial**

Examples (contd.)

- Interpretation:2

  D={strings)

  $a = b = \lambda$

  $P(x)$ stands for “x is a non empty string”

  $g(m, n)$ stands for “append head of m to n”

  $h(x)$ stands for $tail(x)$

- Above interpretation defines “reversing a string”