

# CS 480 - Homework #1 Solutions

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## 1) Show the assertions

**Assertion 1.** *If  $h_1(n), h_2(n)$  are admissible then  $h(n) = \max(h_1(n), h_2(n))$  is admissible.*

*Proof.* Let  $h_1, h_2$  describe admissible heuristics. By definition,  $h_1(n) \leq C(n)$  and  $h_2(n) \leq C(n)$  where  $C(n)$  is the actual cost to get from node  $n$  to the goal node. Without loss of generality, assume  $h_1 \leq h_2(n)$  for some node  $n^*$ .

if

$$h(n) = \max(h_1(n), h_2(n))$$

then

$$h(n^*) = h_2(n^*)$$

since

$$h_2(n^*) \leq C(n^*)$$

$$h(n^*) \leq C(n^*)$$

and  $h(n)$  is therefore admissible. □

**Assertion 2.** *If  $h_1(n), h_2(n)$  are consistent then  $h(n) = \max(h_1(n), h_2(n))$  is consistent.*

*Proof.* Let  $h_1, h_2$  describe consistent heuristics. By definition,

$$h_1(n_j) \leq C(n_{j+1}, n_j) + h_1(n_{j+1})$$

$$h_2(n_j) \leq C(n_{j+1}, n_j) + h_2(n_{j+1})$$

where  $C(n_{j+1}, n_j)$  is the cost to get from node  $n_j$  to  $n_{j+1}$ . We will also define

$$h(n) = \max(h_1(n_j), h_2(n_j))$$

There will be two cases: one where one heuristic is greater than both nodes, and one where the greater heuristic switches between nodes.

Case 1: Without loss of generality, assume

$$\begin{aligned}h_1(n_j) &\leq h_2(n_j) \\ h_1(n_{j+1}) &\leq h_2(n_{j+1})\end{aligned}$$

then

$$h(n_j) = h_2(n_j) \text{ and } h(n_{j+1}) = h_2(n_{j+1})$$

Since  $h_2$  is known to be consistent,  $h$  is also consistent.

Case 2: Without loss of generality, assume

$$\begin{aligned}h_1(n_j) &\leq h_2(n_j) \\ h_2(n_{j+1}) &\leq h_1(n_{j+1})\end{aligned}$$

then

$$h(n_j) = h_2(n_j) \text{ and } h(n_{j+1}) = h_1(n_{j+1})$$

additionally,

$$C(n_{j+1}, n_j) + h_2(n_{j+1}) \leq C(n_{j+1}, n_j) + h_1(n_{j+1})$$

since  $h_2$  is consistent,

$$h_2(n_j) \leq C(n_{j+1}, n_j) + h_2(n_{j+1}) \leq C(n_{j+1}, n_j) + h_1(n_{j+1})$$

Therefore, by substitution,  $h$  is consistent.

□

## 2) Exhibit the search tree

• Starting State:

2	8	3
1	6	5
7		5

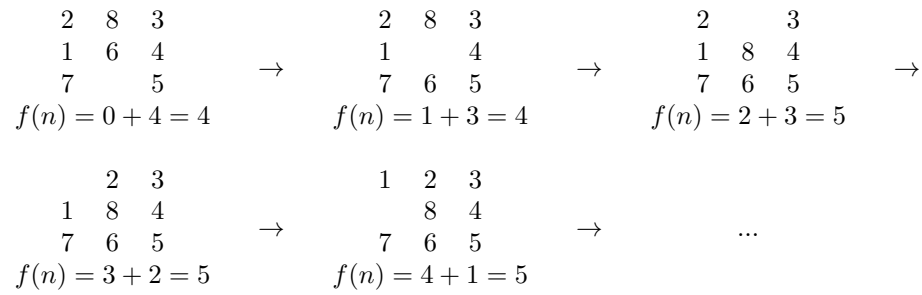
• Goal State:

1	2	3
8		4
7	6	5

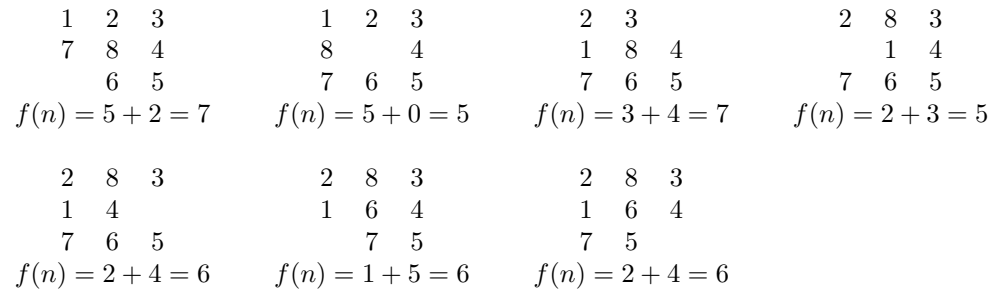
• Note: The state sets and search paths were generated programmatically. The code is available for review on [github](#)<sup>[?]</sup>

### 1. A\* search algorithm incorporating the Hamming distance heuristic

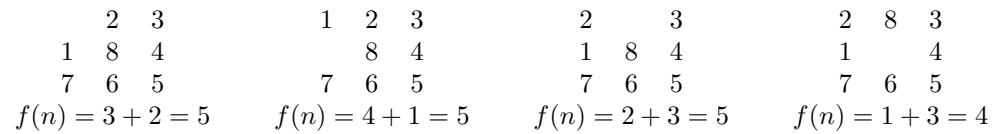
(a) A\* search path (max depth of 3):



(b) Current open set:



(c) Current closed set:



2. A\* search algorithm incorporating the Manhattan distance algorithm

(a) A\* search path (max depth of 3):

$$\begin{array}{ccccc}
 \begin{array}{ccc} 2 & 8 & 3 \\ 1 & 6 & 4 \\ 7 & & 5 \end{array} & \rightarrow & \begin{array}{ccc} 2 & 8 & 3 \\ 1 & & 4 \\ 7 & 6 & 5 \end{array} & \rightarrow & \begin{array}{ccc} 2 & & 3 \\ 1 & 8 & 4 \\ 7 & 6 & 5 \end{array} & \rightarrow \\
 f(n) = 0 + 5 = 5 & & f(n) = 1 + 4 = 5 & & f(n) = 2 + 3 = 5 & \\
 \\
 \begin{array}{ccc} 2 & 3 & \\ 1 & 8 & 4 \\ 7 & 6 & 5 \end{array} & \rightarrow & \begin{array}{ccc} 1 & 2 & 3 \\ & 8 & 4 \\ 7 & 6 & 5 \end{array} & \rightarrow & \dots & \\
 f(n) = 3 + 2 = 5 & & f(n) = 4 + 1 = 5 & & & 
 \end{array}$$

(b) Current open set:

$$\begin{array}{cccc}
 \begin{array}{ccc} 1 & 2 & 3 \\ 7 & 8 & 4 \\ & 6 & 5 \end{array} & \begin{array}{ccc} 1 & 2 & 3 \\ 8 & & 4 \\ 7 & 6 & 5 \end{array} & \begin{array}{ccc} 2 & 3 & \\ 1 & 8 & 4 \\ 7 & 6 & 5 \end{array} & \begin{array}{ccc} 2 & 8 & 3 \\ & 1 & 4 \\ 7 & 6 & 5 \end{array} \\
 f(n) = 5 + 2 = 7 & f(n) = 5 + 0 = 5 & f(n) = 3 + 4 = 7 & f(n) = 2 + 5 = 7 \\
 \\
 \begin{array}{ccc} 2 & 8 & 3 \\ 1 & 4 & \\ 7 & 6 & 5 \end{array} & \begin{array}{ccc} 2 & 8 & 3 \\ 1 & 6 & 4 \\ & 7 & 5 \end{array} & \begin{array}{ccc} 2 & 8 & 3 \\ 1 & 6 & 4 \\ 7 & 5 & \end{array} & \\
 f(n) = 2 + 5 = 7 & f(n) = 1 + 6 = 7 & f(n) = 1 + 6 = 7 & 
 \end{array}$$

(c) Current closed set:

$$\begin{array}{cccc}
 \begin{array}{ccc} 2 & 3 & \\ 1 & 8 & 4 \\ 7 & 6 & 5 \end{array} & \begin{array}{ccc} 1 & 2 & 3 \\ & 8 & 4 \\ 7 & 6 & 5 \end{array} & \begin{array}{ccc} 2 & & 3 \\ 1 & 8 & 4 \\ 7 & 6 & 5 \end{array} & \begin{array}{ccc} 2 & 8 & 3 \\ 1 & & 4 \\ 7 & 6 & 5 \end{array} \\
 f(n) = 3 + 2 = 5 & f(n) = 4 + 1 = 5 & f(n) = 2 + 3 = 5 & f(n) = 1 + 4 = 5
 \end{array}$$

### 3) Justify a true/false answer

a) False. For a weighted, directed graph, increasing all of the weights of the edges by some amount  $A$  would cause the cost of the shortest path from node  $s$  to node  $g$  to increase by an amount proportional to the number of steps in that path. If the shortest path is not also the path with the fewest steps and  $A$  is of sufficient magnitude, the path with the fewest steps from  $s$  to  $g$  would become the shortest path.

b) True. When best first search is implemented using an admissible heuristic function, it is the A\* algorithm, but unless the heuristic function is also consistent, the possibility remains that a node may be discovered more than once and its cost improved.

#### 4) Pancake sorting problem

a) The heuristic function  $h(x) = 1/2 * \text{the number of breakpoints}$  is admissible. Each step between states costs 1, and each step away from the goal node can increase the number of breakpoints by 2, or  $h(x)$  by 1. Because the heuristic and the cost to get to the goal increase at the same rate as we move further from the goal in our state space,  $h(x)$  can never overestimate cost, and is therefore admissible.

Furthermore,  $h(x)$  is consistent. Because the cost of going from one state to another is always 1, and the maximum increase in  $h(x)$  between nodes is 1,  $h(x_j) \leq h(x_{j+1}) + 1$  where  $x_{j+1}$  is one step closer to the goal than  $x_j$ .

b) The search tree using A\* with  $1\ 3\ 6\ 4\ 2\ 5$  as the initial state. States expanded from the initial state indicate the input required to get to them, as well as their  $h, g$ , and  $f$  A\* values. The arrows and letters indicate which states were expanded in each iteration. In the case of a tie,  $h(x)$  is used as a tie breaker. The goal node, which is found on the fourth iteration, is underlined. All states that are not struck-out are in the open set when the algorithm finishes and those that are struck-out are in the closed set.

1st iteration:

~~$1, 2 : < 136425 > h(x) : 2, g(x) : 0, f(x) : 2 < - 1$~~

2nd iteration:

$1, 2 : < 316425 > h(x) : 2, g(x) : 1, f(x) : 3$   
 $1, 3 : < 631425 > h(x) : 2, g(x) : 1, f(x) : 3$   
 $1, 4 : < 463125 > h(x) : 2, g(x) : 1, f(x) : 3$   
 $1, 5 : < 246315 > h(x) : 2, g(x) : 1, f(x) : 3$   
 $1, 6 : < 524631 > h(x) : 2, g(x) : 1, f(x) : 3$   
 $2, 3 : < 163425 > h(x) : 2, g(x) : 1, f(x) : 3$   
 $2, 4 : < 146325 > h(x) : 2, g(x) : 1, f(x) : 3$   
 $2, 5 : < 124635 > h(x) : 2, g(x) : 1, f(x) : 3$   
 $2, 6 : < 152463 > h(x) : 2, g(x) : 1, f(x) : 3$   
 $3, 4 : < 134625 > h(x) : 2, g(x) : 1, f(x) : 3$   
 ~~$3, 5 : < 132465 > h(x) : 1, g(x) : 1, f(x) : 2 < - 2$~~   
 $3, 6 : < 135246 > h(x) : 2, g(x) : 1, f(x) : 3$   
 $4, 5 : < 136245 > h(x) : 2, g(x) : 1, f(x) : 3$   
 $4, 6 : < 136524 > h(x) : 2, g(x) : 1, f(x) : 3$   
 $5, 6 : < 136452 > h(x) : 2, g(x) : 1, f(x) : 3$

3rd iteration:

$1, 2 : < 312465 > h(x) : 1, g(x) : 2, f(x) : 3$   
 $1, 3 : < 231465 > h(x) : 1, g(x) : 2, f(x) : 3$   
 $1, 4 : < 423165 > h(x) : 1, g(x) : 2, f(x) : 3$   
 $1, 5 : < 642315 > h(x) : 2, g(x) : 2, f(x) : 4$   
 $1, 6 : < 564231 > h(x) : 1, g(x) : 2, f(x) : 3$   
 ~~$2, 3 : < 123465 > h(x) : 0, g(x) : 2, f(x) : 2 < - 3$~~

$2, 4 :< 142365 > h(x) : 1, g(x) : 2, f(x) : 3$   
 $2, 5 :< 164235 > h(x) : 2, g(x) : 2, f(x) : 4$   
 $2, 6 :< 156423 > h(x) : 1, g(x) : 2, f(x) : 3$   
 $3, 4 :< 134265 > h(x) : 1, g(x) : 2, f(x) : 3$   
 $3, 6 :< 135642 > h(x) : 2, g(x) : 2, f(x) : 4$   
 $4, 5 :< 132645 > h(x) : 1, g(x) : 2, f(x) : 3$   
 $4, 6 :< 132564 > h(x) : 1, g(x) : 2, f(x) : 3$   
 $5, 6 :< 132456 > h(x) : 1, g(x) : 2, f(x) : 3$

4th iteration:

$1, 2 :< 213465 > h(x) : 1, g(x) : 3, f(x) : 4$   
 $1, 3 :< 321465 > h(x) : 1, g(x) : 3, f(x) : 4$   
 $1, 4 :< 432165 > h(x) : 0, g(x) : 3, f(x) : 3$   
 $1, 5 :< 643215 > h(x) : 1, g(x) : 3, f(x) : 4$   
 $1, 6 :< 564321 > h(x) : 0, g(x) : 3, f(x) : 3$   
 $2, 4 :< 143265 > h(x) : 1, g(x) : 3, f(x) : 4$   
 $2, 5 :< 164325 > h(x) : 1, g(x) : 3, f(x) : 4$   
 $2, 6 :< 156432 > h(x) : 1, g(x) : 3, f(x) : 4$   
 $3, 4 :< 124365 > h(x) : 1, g(x) : 3, f(x) : 4$   
 $3, 5 :< 126435 > h(x) : 1, g(x) : 3, f(x) : 4$   
 $3, 6 :< 125643 > h(x) : 1, g(x) : 3, f(x) : 4$   
 $4, 5 :< 123645 > h(x) : 1, g(x) : 3, f(x) : 4$   
 $4, 6 :< 123564 > h(x) : 1, g(x) : 3, f(x) : 4$   
 $5, 6 :< 123456 > h(x) : 0, g(x) : 3, f(x) : 3 < - 4$  - Goal Node

The code<sup>[?]</sup> used to produce these results did not run the  $A^*$  algorithm, but rather defined the game logic and printed out state and heuristic information.

## References

- [1] Aman Gill, CS480HW1 Git repository.  
<https://github.com/WorldDotHack/CS480HW1>
- [2] Kyle Janssen, PancakeNumbers Git repository.  
<https://github.com/WizardCode/PancakeNumbers>