Quantitative Dynamic Macroeconomics

– Assignment 04: Surprise money losses and aggregate demand –

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fall semester 2023, this version: September 29, 2025

The model in this assignment builds on the model derived in assignment 3. There are two changes: 1) We allow each household to set the price of its own good. 2) We adjust the timing of the realization of the money loss shock, $\delta(s^t)$, which happens after prices are set, but before the household goes to the goods market to buy consumption goods. *Hint: Be careful to take the timing changes into account!*

For the sake of clarity, we present the model setup in detail once more. Assume that each household can set the price of its good. As in assignment 3, it chooses consumption, $C(s^t)$, money holdings, $M(s^t)$, bond holdings, $B(s^t)$, and the price of its good, $P(i, s^t)$, to maximize utility given by

$$\mathbb{U}_t \;=\; \mathbb{E}\sum_{t=1,2}\beta^{t-1}\left[\log(C(s^t)) - \frac{\varphi}{1+\nu}\left(\frac{D\left(\frac{P(i,s^t)}{\bar{P}(s^t)}\right)}{Z(s^t)}\right)^{1+\nu}\right] + \beta^2\mathbb{E}_t\mathbb{V}\left(M(s^2), B(s^2), s_3\right)$$

subject to the cash-in-advance constraint (adjusted for the change in timing)

$$\left(1-\delta(s^t)\right)M(s^{t-1}(s^t)) \;=\; \bar{P}(s^t)C(s^t),$$

and the budget constraint

$$\begin{aligned} &P(i,s^t)D\left(\frac{P(i,s^t)}{\bar{P}(s^t)}\right) + \left(1-\delta(s^t)\right)M(s^{t-1}(s^t)) + B(s^{t-1}(s^t)) + T(s^t) \\ &= \; \bar{P}(s^t)C(s^t) + M(s^t) + q(s^t)B(s^t), \end{aligned}$$

where $P(i, s^t)$ is the price set by household i in state s^t, $\bar{P}(s^t)$ is the aggregate price level, $0 < \beta < 1$, and $\varphi, \nu > 0$. Assume that the productivity level is fixed, $Z(s^t) = 1$, that the money growth rate is fixed at $\bar{\tau}$, and that the money loss rate, $\delta(s^t)$, follows an AR(1) process given by

$$\delta(s^t) \;= \rho_\delta\delta(s^{t-1}(s^t)) + B_\delta + \sigma_\delta \times \epsilon,$$

where $\epsilon \sim \mathcal{N}(0, 1)$. The money stock law of motion can then be described by

$$\bar{M}(s^t) = (1+\tau)(1-\delta(s^t))M(s^{t-1}(s^t)).$$

Assume that the optimal pruchase allocation for the household regarding different types of goods is already solved. The resulting demand function is given by

$$D\left(\frac{P(i,s^t)}{\bar{P}(s^t)}\right) = \left[\#I\frac{P(i,s^t)}{\bar{P}(s^t)}\right]^{\frac{1}{\rho-1}} C(s^t),$$

where $\#I$ denotes the amount of different types of goods, and $\rho > 0$. Further, in case you are stuck, you can assume that in equilibrium all households are symmetric, hence $P(i,s^t) = P(s^t) = \bar{P}(s^t)$, effectively setting $\#I = 1$.

Exercise 1 Deriving the pricing equation

Suppose that an appropriate change-in-variables for the penalty prices of the optimization of the Lagrangian is given by

$$\begin{aligned}\lambda(s^t) &= \tilde{\lambda}(s^t)\frac{\beta^{t-1}Pr(s^t)}{(1-\delta(s^t))M(s^{t-1}(s^t))},\\ \mu(s^t) &= \tilde{\mu}(s^t)\frac{\beta^{t-1}Pr(s^t)}{(1-\delta(s^t))M(s^{t-1}(s^t))},\end{aligned}$$

where $Pr(s^t)$ is the probability of observing history state s^t. Set up the Lagrangian and solve for the model first-order conditions. Use the change-in-variables as proposed to summarize the system of equations as far as possible. *Hint: Be careful about the expectations operators as we have changed the timing for the realization of the shock to $\delta(s^t)$, which is only realized within each period now.*

Exercise 2 ADDITIONAL: Solving the model with certainty equivalence

Solve the model from exercise 1 using the given Matlab file. All parameters and settings are already written down in the Matlab file. Fill the gaps in order to get a fully functioning Matlab file. This contains the following tasks:

1. Set up shock vectors for 1) a simulation with continuous draws from the shock distribution and for 2) impulse response functions for each shock assuming non-correlated realizations of the random variables across shocks.
2. Write a function that calculates a solution to both the expected/forecasted model as well as the actual model outcome. Assume that certainty equivalence holds. Use a WHILE-loop to calculate the model outcome across time.
3. Make sure that you update all stock variables in between two periods of the model.
4. Use structure-arrays (e.g. "output.labor") to summarize all output from the model function and return it back to the main Matlab file.
5. Have a look at the figures created by Matlab and give an economic interpretation of the results.