Lecture 04: Monetary Policy, Price-Setting, and Information Frictions

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Understanding Society

Outline for this Lecture

What have we seen so far

1. Stochastic CiA Model [CG 1, 2, 3, 5]
2. Markov Process Computer Model [CG 3, 4]
3. AR(1) Process Computer Model [CG 3, 4]
4. Calculating Model Statistics [CG 2, 3, 4]

What we will see today

1. Monetary policy rules [CG 6]
2. Monopolistic competition [CG 1, 3, 5]
3. Information frictions [CG 1, 5]
4. Impulse response functions [CG 3, 4]

Big Picture of the Lecture:

1. How to set monetary policy goals, targets, and instruments?
2. How do households set their goods prices under market power?
3. What role does uncertainty within period money growth play for aggregate demand?

Money Supply Rules and Interest Rate Rules

Cole (2020) chapter 9

US Central Bank Policy through Time

We can order US central bank policy by the Federal Reserve in roughly five episodes since the second world war:

- **Mid 1960s to 1979:** Inflation targeting regime with aim to create high inflation for driving down unemployment (and finance Vietnam war).
- **Oct 1979 to Oct 1982:** Price level targeting in a regime of monetary tightening as inflation (expectations) were high alongside high unemployment.
- **Oct 1982 to 2008:** Interest rate targeting with explicit price stability and employment targets (Great Moderation).
- **2009 to 2015:** Zero interest rate policy (and quantitative easing) following the Great Recession (financial crisis that started end of 2006).

How is monetary policy in the US (the EMU) conducted since then?

Augmented Model

We augment our stochastic CiA model to include a policy margin through **preference shocks**!

Modified flows of utility:

$$u(C_t) - (1 + \Upsilon_t)\, v(L_t) \tag{1}$$

- Υ_t is a shock to the **disutility of labor** making households more or less willing to work.

The **only FOC that changes** is the labor FOC given by:

$$(1 + \Upsilon_t)\, v'(L(s^t))L(s^t) \;=\; \frac{\beta}{1 + \tau_t(s^t)} \tag{2}$$

- AR(1)-process: $\Upsilon_t = \rho_\Upsilon \Upsilon_{t-1} + \sigma_\Upsilon \upsilon_t$
- The implied level of labor supply is efficient.

However, we assume that the central bank wants to stabilize the economy at its original level.

$$v'(\bar{L})\bar{L} \;=\; \frac{\beta}{1 + \bar{\tau}} \tag{3}$$

Money Supply Rules

Central bank **sets their money supply** given expectations about $\tilde{\Upsilon}_t$ to achieve its target $\bar{L}$:

$$\left(1+\tilde{\Upsilon}_t\right) v'\left(\bar{L}\right) \bar{L} = \frac{\beta}{1+\tau\left(\tilde{\Upsilon}_t\right)} \tag{4}$$

where $\tilde{\Upsilon}_t = \mathbb{E}_t \Upsilon_t$ (shocks this period are unexpected). Monetary policy ...

- smoothes out changes in Υ_t to the extend that they are expected.
- is backward-looking as the shock process and its implications for the economy depend on the persistence parameter of the AR(1) process.

What if there is a persistent shift in Υ_t that the central bank does not see?

Inflation Targeting

A central bank targets some level of the inflation rate which is given in the model by

$$1 + \pi_t = \frac{P_{t+1}}{P_t} = \frac{M_{t+1} C_t}{M_t C_{t+1}} = \frac{(1+\tau_t)\, L(\Upsilon_t, \tau_t)}{(1+g_{t+1})\, L(\Upsilon_{t+1}, \tau_{t+1})} \tag{5}$$

In order for the central bank to be able to implement this policy rule, it needs to ...

- forecast the future growth rate, g_{t+1}.
- forecast the future labor supply disutility, Υ_{t+1}.
- commit to its own future money growth rate, τ_{t+1}.

Can a central bank accurately and persistently forecast the future?

Do households and firms belief the central bank is comitted to its future interest rates?

Interest Rate Rules

A central bank sets the penalty price of the budget constraint by setting the interest rate:

$$\begin{aligned} \mu(s^t) &= \frac{\sum_{s^{t+1}\in S^{t+1}(s^t)} \mu(s^{t+1})}{q_t} \\ \Leftrightarrow \qquad \mu_t &= \mathbb{E}_t \frac{\mu_{t+2}}{q_t \cdot q_{t+1}} = \mathbb{E}_t \frac{\mu_{t+T+1}}{\prod_{i=0}^{T} q_{t+i}} \end{aligned} \tag{6}$$

Implications:

- Future interest rates, q_{t+1}, impact μ_t the same as current interest rates.
- However, future interest rates, q_{t+1}, will also affect future μ_{t+1}.
- **Forward guidance:** The fact that future interest rates affect today's economy encourages central banks even more to forecast the future.

Does the central bank understand the impact of interest rates on the economy?

Monetary Policy in the CiA Model

What are the goals, targets, and instruments used by a central bank?

- **Money supply:** Do we observe unbiased shocks in real-time?
- **Inflation targeting:** Do we observe unbiased shocks in real-time and can forecast the future accurately?
- **Interest rate:** Do we understand the link between interest rates and targets variables (inflation or employment stability) in our ecomnomy correctly?

Taylor Rule

John B. Taylor described and estimated a rule for US monetary policy in a series of papers. The rule named after him describes consistently how the FED behaves:

$$i_t = \pi_t + r_t^* + 0.5\,(\pi_t - \pi_t^*) + 0.5\,(y_t - \bar{y}_t) \quad (7)$$

The Demand Function of Differentiated Goods

Cole (2020) chapter 10.1

Money and the Business Cycle

- Many people believe that increases in money ...
  - $\rightarrow$ ... increases nominal demand,
  - $\rightarrow$ ... and this in turn increases output and employment.
- This is a bit surprising because ...
  - $\rightarrow$ ... if money doubles prices double and nothing really has changed.
  - $\rightarrow$ ... money in the long-run seems neutral for this reason.
- Many argue that some element of "surprise" is important here.

The Element of Surprise in Money Increases

Do surprise increases in money supply have an impact on the economy?

- **Historical evidence:**
  - → The long-depression in the UK after WWI.
  - → The rebound when inflation returned during the Great Depression.
- **Evidence from vector-auto-regressions (VARs):**
  - → This is a standard sort of VAR equation

$$y_t = A + By_{t-1} + C(m_t - E_{t-1}m_t) + Dx_t + \epsilon_t$$

  - → We need assumptions to determine money growth expectations and to estimate their effect.
  - → Different approaches, and at least some suggest reasonably sizable and moderately persistent effects.
- **Further method:** Directly date "surprise" changes in central bank policy and see what happened.

⇒ This evidence led to the development of New Keynesian models with price-setting frictions. **We will adjust our model accordingly!**

Step 1: Putting Price-Setting in our Model

Starting point: Changing the model setup!

- Go back to the original set-up (lecture 1) with multiple goods indexed by $i \in I$.
- Each good is produced by an individual producer, which chooses her price $P(i)$.
- Each producer has to meet any demand once the price is set.
- The **composite good** is determined by an aggregator of the individual goods:

$$C = \left\{ \frac{1}{\# I} \sum_{i \in I} C(i)^{\rho} \right\}^{\frac{1}{\rho}}$$

$\rightarrow$ The household's preferences are again given by $u(C)$.

$\rightarrow$ Individual production is $C(i)$, labor is $C(i)/Z$, and revenue is $P(i)C(i)$.

Step 2: Deriving the Demand Function (1/5)

Derive the optimal composition of individual goods given some fixed money holdings.

- Lagrangian of a household's optimal expenditure problem: spending M to maximize C:

$$\mathcal{L} \;=\; \max_{C(i)} \min_{\lambda} \left\{ \frac{1}{\#I} \sum_{i \in I} C(i)^{\rho} \right\}^{\frac{1}{\rho}} + \lambda \left\{ M - \sum_{i} P(i) C(i) \right\}.$$

- The **first-order condition** is given by

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial C_t(i)} : \quad & \left\{ \frac{1}{\#I} \sum_{i \in I} C(i)^{\rho} \right\}^{\frac{1}{\rho}-1} \frac{1}{\#I} C(i)^{\rho-1} \;=\; \lambda P(i) \\ \Longrightarrow \qquad & C^{1-\rho} \frac{1}{\#I} C(i)^{\rho-1} = \lambda P(i). \end{aligned}$$

- Further note:

$$\frac{\partial \mathcal{L}}{\partial M} \;=\; \lambda,$$

so λ is the value of money in terms of a unit of composite consumption.

Step 2: Deriving the Demand Function (2/5)

A **price index** is a way of saying what something costs:

- A price index $\bar{P}$ for composite consumption says that C units cost $\bar{P} \times C$ units of money.
- While λ says 1 unit of money will get me λ units of C, so $\bar{P} = \lambda^{-1}$.

$\Rightarrow$ Inserting the price index $\bar{P}^{-1}$ for λ in our FOC, we get that

$$\frac{1}{\#I} C(i)^{\rho-1} = \frac{P(i)}{\bar{P}} C^{\rho-1}. \tag{8}$$

Step 2: Deriving the Demand Function (3/5)

- Rearranging (8), we get a nice form for the **demand function**:

$$\begin{aligned} C(i) &= \left[\#I \frac{P(i)}{\bar{P}}\right]^{\frac{1}{\rho-1}} C \\ &= D\left(\frac{P(i)}{\bar{P}}\right) \end{aligned} \tag{9}$$

- From (9) we can derive the **slope of the demand function**:

$$\frac{dD\left(\frac{P(i)}{\bar{P}}\right)}{dP(i)} = D\left(\frac{P(i)}{\bar{P}}\right) \times \frac{1}{\rho-1} P(i)^{-1}$$

- The **elasticity** is then given by:

$$\frac{1}{\rho-1}$$

Step 2: Deriving the Demand Function (4/5)

But what exactly is our price index $\bar{P}$?

- For everything to add up:

$$\begin{aligned} M \;=\; \bar{P}C \;&=\; \sum_{i \in I} P(i)C(i) \;=\; \sum_{i \in I} P(i)\left[\#I\lambda P(i)\right]^{\frac{1}{\rho-1}} C \\ \implies \bar{P} &= \sum_{i \in I} P(i)\left[\#I\frac{P(i)}{\bar{P}}\right]^{\frac{1}{\rho-1}} \\ \implies \bar{P} &= \left\{(\#I)^{\frac{1}{\rho-1}} \sum_{i} P(i)^{\frac{\rho}{\rho-1}}\right\}^{\frac{\rho-1}{\rho}} \end{aligned} \tag{10}$$

Step 2: Deriving the Demand Function (5/5)

There's a units problem here since

$$C = \left\{ \frac{1}{\#I} \sum_{i \in I} C(i)^{\rho} \right\}^{\frac{1}{\rho}}$$

implies that one unit of each good i leads to one unit of the composite good.

- So in total it takes $\#I$ units of the different goods to get the composite.
- Hence, when all prices are symmetric, $P(i) = P$, it follows $\bar{P} = \#I \times P$.
- This is something we saw before and just need to adjust Z

$$PZ = \bar{P}Z/\#I.$$

- We ignore this small distinction going forward because it won't matter for anything.

Optimal Price Setting

Cole (2020) chapter 10.1

Step 3: The Household's 2-Period Problem (1/4)

$$\max_{\{C_t, P_t(i), M_{t+1}, B_{t+1}\}_{t=1,2}} \quad u(C_1) - v\left(D\left(\frac{P_1(i)}{\bar{P}_1}\right)/Z_1\right) + \beta\left[u(C_2) - v\left(D\left(\frac{P_2(i)}{\bar{P}_2}\right)/Z_2\right)\right] + \beta^2 V(M_3, B_3)$$

subject to

$$\begin{aligned} M_t &\geq \bar{P}_t C_t \\ P_t(i) D\left(\frac{P_t(i)}{\bar{P}_t}\right) + \left[M_t - \bar{P}_t C_t\right] + B_t + T_t &\geq M_{t+1} + q_t B_{t+1} \end{aligned}$$

$\rightarrow$ **Assumption:** We impose the condition that the household must work enough to satisfy demand for its product given the price it sets and the overall price index $\bar{P}$.

Step 3: The Household's 2-Period Problem (2/4)

Three of our FOCs are **completely unchanged!**

- The **FOC for consumption** is given by:

$$\beta^{t-1}u'(C_t) - \bar{P}_t[\lambda_t + \mu_t] \stackrel{!}{=} 0$$

$$\implies \frac{\beta^{t-1}}{Z_t L_t} = \bar{P}_t[\lambda_t + \mu_t] \quad \text{in equilibrium with log preferences}$$

$$\implies \beta^{t-1} = \bar{M}_t[\lambda_t + \mu_t] \quad \text{because the CiA says } \bar{P}_t C_t = \bar{M}_t = \bar{P}_t Z_t L_t$$

$\rightarrow$ $\bar{M}_t$ is whatever amount of money they have in the goods market.

- The **FOC for money** is given by:

$$-\mu_t + [\mu_{t+1} + \lambda_{t+1}] \stackrel{!}{=} 0 \qquad \implies \mu_t = \frac{\beta^t}{\bar{M}_{t+1}}$$

- The **FOC for bonds** is given by:

$$-q_t\mu_t + \mu_{t+1} = 0$$

- The **FOC for pricing** is then given by:

$$
\begin{aligned}
0 \overset{!}{=}\ & -\beta^{t-1} v'\left(\frac{D\left(\frac{P_t(i)}{\bar{P}_t}\right)}{Z_t}\right) \frac{1}{Z_t} \frac{d}{dP_t(i)} D\left(\frac{P_t(i)}{\bar{P}_t}\right) \\
& + \mu_t \left[D\left(\frac{P_t(i)}{\bar{P}_t}\right) + P_t(i) \frac{d}{dP_t(i)} D\left(\frac{P_t(i)}{\bar{P}_t}\right) \right]
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow \quad 0 \overset{!}{=}\ & -\beta^{t-1} v'\left(\frac{D\left(\frac{P_t(i)}{\bar{P}_t}\right)}{Z_t}\right) \frac{1}{Z_t} D\left(\frac{P_t(i)}{\bar{P}_t}\right) \frac{1}{\rho - 1} P_t(i)^{-1} \\
& + \mu_t \left[D\left(\frac{P_t(i)}{\bar{P}_t}\right) + D\left(\frac{P_t(i)}{\bar{P}_t}\right) \frac{1}{\rho - 1} \right]
\end{aligned}
$$

$$
\Leftrightarrow \quad 0 \overset{!}{=} -\beta^{t-1} v'\left(\frac{D\left(\frac{P_t(i)}{\bar{P}_t}\right)}{Z_t}\right) \frac{1}{Z_t} P_t(i)^{-1} + \mu_t \rho.
$$

Step 3: The Household's 2-Period Problem (4/4)

- By rearranging our **FOC for pricing**, we finally get:

$$\beta^{t-1} v' \left(\frac{D\left(\frac{P_t(i)}{\bar{P}_t}\right)}{Z_t} \right) \frac{1}{\rho} = Z_t P_t(i) \mu_t \tag{11}$$

$\rightarrow$ This condition replaces our FOC for labor in the original model.

$\rightarrow$ But, our FOCs for (composite) consumption, money and bonds are essentially unchanged.

Can you give an economic interpretation of the FOC for pricing and explain why households can charge a price markup?

Step 4: Reconstructing our Fundamental Equation (1/3)

Assumption: All households are symmetric, hence they have the same pricing rule:

$$P_t(i) = P_t(i') = \bar{P}_t$$

It follows:

- Production and consumption across goods will be symmetric:

$$\frac{D\left(\frac{P_t(i)}{\bar{P}_t} = 1\right)}{Z_t} = \frac{C_t}{Z_t} = L_t$$

- The CiA constraint will be assumed to bind:

$$\bar{P}_t = \frac{\bar{M}_t}{Z_t L_t}$$

Solution approach:

- We can boil things down to a system of two equations, like in the stochastic model.
- We can substitute out for our multipliers.

Step 4: Reconstructing our Fundamental Equation (2/3)

- The **symmetric labor FOC** becomes:

$$\beta^{t-1} v' \left(\frac{D\left(\frac{P_t(i)}{\bar{P}_t}\right)}{Z_t} \right) \frac{1}{\rho} = Z_t P_t(i) \mu_t$$

$$\Leftrightarrow \quad \beta^{t-1} v' \left(L_t \right) \frac{1}{\rho} = Z_t \bar{P}_t \frac{\beta^t}{\bar{M}_{t+1}}$$

Using the CiA constraint, $\bar{P}_t = \frac{\bar{M}_t}{Z_t L_t}$, we get our **modified labor supply condition:**

$$\frac{1}{\rho} v'(L(s_t)) L(s_t) = \frac{\beta}{(1 + \tau_t(s^t))} \tag{12}$$

- The second equation is our **interest rate condition:**

$$q(s^t) = \frac{\beta}{(1 + \tau_t(s^t))} \frac{\sum_{s^{t+1} \in S^{t+1}(s^t)} \left[v'(L(s_{t+1})) L(s_{t+1}) \right] \Pr(s_{t+1} | s_t)}{v'(L(s_t)) L(s_t)}$$

$\rightarrow$ Because the $\frac{1}{\rho}$ terms cancel out, this condition is also unchanged!

Step 4: Reconstructing our Fundamental Equation (3/3)

Let's take stock: **What is the impact of price setting on the model equilibrium?**

- The labor equation implies:

$$L(s_t) \;=\; \left[\frac{\rho\beta}{(1+\tau_t(s^t))}\right]^{\frac{1}{1+\gamma}}$$

  → ρ increases labor supply. Hence, a *low substitutability of goods* lowers labor supply.

  → Money growth *lowers* labor and output.

- The price level is given by $\bar{P}_t = \frac{\bar{M}_t}{Z_t L_t}$. So the change in prices is given by:

$$\frac{\bar{P}_{t+1}}{\bar{P}_t} \;=\; (1+\tau_t)\frac{Z_t L_t}{Z_{t+1}L_{t+1}}$$

  → ρ does not have an impact on price changes.

  → The loss from rising prices coming from $1+\tau_t$ is unchanged.

Adding an Information Friction

Cole (2020) chapter 10.2

Changing the Timing of Money Injection

Why does a money injection not drive up aggregate demand?

- A *key problem* is that money growth is coming after the goods market clears and hence generates no increase in current demand.
- *Another key problem* is that the time t injection now scales up P_{t+1} and hence lowers the return you earn from holding money between today and tomorrow.
- **Change in assumption:** Now assume that the transfer occurs at the beginning of the period and before the shopper gets to the good market.

Household Optimization with Information Friction

The **household optimization problem** is given by:

$$\max_{\{C_t, P_t(i), M_{t+1}, B_{t+1}\}_{t=1,2}} \quad u(C_1) - v\left(\frac{D\left(\frac{P_1(i)}{\bar{P}_1}\right)}{Z_1}\right) + \beta\left[u(C_2) - v\left(\frac{D\left(\frac{P_2(i)}{\bar{P}_2}\right)}{Z_2}\right)\right] + \beta^2 V(M_3, B_3)$$

subject to

$$M_t + T_t \geq \bar{P}_t C_t \quad \text{and}$$

$$P_t(i) D\left(\frac{P_t(i)}{\bar{P}_t}\right) + \left[M_t + T_t - \bar{P}_t C_t\right] + B_t \geq M_{t+1} + q_t B_{t+1}$$

$\Rightarrow$ This leaves *all* our FOCs **unchanged** from the first take!

Implications for Money Transfers at the Beginning of the Period (1/2)

- Assume that prices are equal across goods again and the **CiA constraint** binds. It follows:

$$\bar{P}_t = \frac{\bar{M}_t(1+\tau_t)}{Z_t L_t}$$

- Taking $\frac{D\left(\frac{P_t(i)}{\bar{P}_t}\right)}{Z_t} = L_t$ into account for the **labor FOC**, we get:

$$\beta^{t-1} v'(L_t)\frac{1}{\rho} = Z_t P_t(i)\mu_t$$

Implications for Money Transfers at the Beginning of the Period (2/2)

- If we use the *FOCs for money and consumption*, the **labor FOC** becomes:

$$\beta^{t-1} v'(L_t)\frac{1}{\rho} = Z_t P_t(i)\left[\mu_{t+1} + \lambda_{t+1}\right]$$
$$= \frac{\bar{P}_t}{\bar{P}_{t+1}} Z_t \beta^t u'(C_{t+1}) = \frac{\bar{P}_t}{\bar{P}_{t+1}} Z_t \beta^t \frac{1}{Z_{t+1} L_{t+1}}$$

$\rightarrow$ This is the same equation as before (given that everyone sets the same price $\bar{P}$)

- When we substitute in for prices using the new condition we get:

$$v'(L_t)\frac{1}{\rho} = \frac{1}{1+\tau_{t+1}} \frac{Z_{t+1} L_{t+1}}{L_t} \beta \frac{1}{Z_{t+1} L_{t+1}}$$

- Even if we bring back uncertainty, we are going to get:

$$L_t^{1+\gamma} = \rho\beta\mathbb{E}\left\{\frac{1}{1+\tau_{t+1}}\right\}$$

Changing the Timing of Money Injection Once More

Why does a money injection still not drive up aggregate demand?

- The time t injection now scales up P_t and P_{t+1} by an equal amount.
- Hence, it has no effect on the return from holding money between today and tomorrow.
- Also no demand increase today.

What assumption is necessary to produce aggregate demand effects?

- **Now assume:** The transfer occurs after they set prices, but **right before** the shopper goes to the goods market.
- **Assume:** Households do not know the transfer at the time they set prices. They forecast anticipated transfers and factor that into setting prices.

⇒ The level of demand now depends explicitly on the realized money supply shock, and to emphasize this, we write:

$$D\left(\frac{P_t(i)}{\bar{P}_t}, \tau_t\right) \tag{13}$$

Forecasting Money Supply Growth (1/2)

- Let forecasted money supply injection be $\bar{\tau}_t$. It follows that forecasted level of consumption demand is given by:

$$D\left(1, \bar{\tau}_t\right) = \bar{C}_t = \frac{M_t(1+\bar{\tau}_t)}{\bar{P}_t}$$

  → The last equality follows from the CiA constraint holding as an equality at the forecasted level of the money supply.

- The **actual** level of consumption is given by:

$$C_t = \frac{M_t(1+\tau_t)}{\bar{P}_t} = \frac{M_t(1+\tau_t)}{M_t(1+\bar{\tau}_t)}\bar{C}_t = \frac{(1+\tau_t)}{(1+\bar{\tau}_t)}\bar{C}_t \tag{14}$$

  → Consumption today will be higher than forecasted if $\tau_t > \bar{\tau}_t$ and lower if the reverse is true.

Forecasting Money Supply Growth (2/2)

- Since **consumption is produced with labor**, this implies that ...
  - $\rightarrow$ ... forecasted labor is given by $Z_t \bar{L}_t = \bar{C}_t$,
  - $\rightarrow$ ... while actual labor is given by $Z_t L_t = C_t$.

- From (14), it follows that the **actual level of labor input** is given by:

$$L_t = \frac{(1+\tau_t)}{(1+\bar{\tau}_t)} \bar{L}_t \tag{15}$$

  - $\rightarrow$ Labor will be higher in response to an inflation surprise.

- Hence, (15) allows us to think in terms of a labor effort target $\bar{L}$, and the impact of the deviation in money growth relative to its forecast on actual labor.

Discussion: The Impact of Surprise Inflation

Take five minutes and discuss with your neighbors the following points:

1. Why is there a positive impact of surprise inflation on aggregate demand?
2. How long does this effect last? And how can we extend it?
3. How long does the negative impact of surprise inflation on labor supply last?

Putting the Pieces Together

Cole (2020) chapter 10.3

Household Optimization Problem with Forecasted Money Growth

The **household objective function** is given by:

$$\max_{P_1(i)} \mathbb{E}_1 \left\{ \max_{C_1, M_1, B_1} u(C_1) - v\left(\frac{D\left(\frac{P_1(i)}{\bar{P}_1}, \tau_1\right)}{Z_1}\right) \right.$$
$$\left. + \beta \max_{P_2(i)} \mathbb{E}_2 \left\{ \max_{C_2, M_2, B_2} u(C_2) - v\left(\frac{D\left(\frac{P_2(i)}{\bar{P}_2}, \tau_2\right)}{Z_2}\right) + \ldots \right\} \right\}$$

Timing of the decisions:

1. The household chooses its price $P_t(i)$ knowing τ_{t-1} and being able to infer $\bar{P}_t$ hence $\mathbb{E}_1$.
2. It finds out τ_t and chooses $\{C_t, M_t, B_t\}$ knowing also q_t.
3. In period $t+1$ it will start over choosing $P_{t+1}(i)$ knowing no more than it did at the end of period t hence $\mathbb{E}_2$.

Optimal Price Setting with Forecasted Money Growth (1/2)

Focus on the parts that involve P_1 in the Lagrangian and ignore the others to get:

$$\mathcal{L} = \max_{P_1(i)} \mathbb{E}_1 \left\{ -v\left(\frac{D\left(\frac{P_1(i)}{\bar{P}_1}, \tau_1\right)}{Z_1}\right) + \mu_1(\tau_1) P_1(i) D\left(\frac{P_1(i)}{\bar{P}_1}, \tau_1\right) \right\}$$

The **first-order condition** for the price is:

$$\sum_{\tau_1} \left\{ -v'\left(\frac{D\left(\frac{P_1(i)}{\bar{P}_1}, \tau_1\right)}{Z_1}\right) \frac{1}{Z_1} \frac{\partial}{\partial P_1(i)} D\left(\frac{P_1(i)}{\bar{P}_1}, \tau_1\right) \right.$$
$$\left. + \mu_1(\tau_1) D\left(\frac{P_1(i)}{\bar{P}_1}, \tau_1\right) + \mu_1(\tau_1) P_1 \frac{\partial}{\partial P_1(i)} D\left(\frac{P_1(i)}{\bar{P}_1}, \tau_1\right) \right\} Pr\{\tau_1\} \overset{!}{=} 0$$

$\rightarrow$ We know the partial and the terms are the same as we saw before on slide 13.

Optimal Price Setting with Forecasted Money Growth (2/2)

We substitute to get:

$$\sum_{\tau_1}\Bigg\{-v'\left(\frac{D\left(\frac{P_1(i)}{\bar{P}_1},\tau_1\right)}{Z_1}\right)\frac{1}{Z_1}D\left(\frac{P_1(i)}{\bar{P}_1}\right)\frac{1}{\rho-1}P_1(i)^{-1}$$
$$+\mu_1(\tau_1)D\left(\frac{P_1(i)}{\bar{P}_1},\tau_1\right)+\mu_1(\tau_1)P_1(i)D\left(\frac{P_1(i)}{\bar{P}_1}\right)\frac{1}{\rho-1}P_1(i)^{-1}\Bigg\}Pr\{\tau_1\} \;=\; 0$$

Make use of the fact (as before) that (i) $P_t(i)=\bar{P}_t$ since all prices are the same in equilibrium, and (ii) that $D\left(\frac{P_t(i)}{\bar{P}_t},\tau_t\right)=Z_tL_t$, to get:

$$\sum_{\tau_t}\left\{-\beta^{t-1}v'\left(L_t\right)L_t\frac{1}{\rho-1}\bar{P}_t^{-1}+\mu_t(\tau_t)Z_tL_t+\mu_t(\tau_t)Z_tL_t\frac{1}{\rho-1}\right\}Pr\{\tau_t\} \;=\; 0$$
$$\Leftrightarrow \quad \sum_{\tau_t}\left\{-\beta^{t-1}v'\left(L_t\right)+\mu_t(\tau_t)\bar{P}_tZ_t\rho\right\}L_t(\tau_t)Pr\{\tau_t\} \;=\; 0$$

Certainty Equivalence

- Assume we have a random variable x and we want to know:

$$\mathbb{E}\Big\{F(x)\Big\} = \sum_x F(x)Pr\{x\}$$

- We can approximate things as follows:

$$\begin{aligned}\mathbb{E}\Big\{F(x)\Big\} &\approx \sum_x \left\{\begin{array}{c} F(\mathbb{E}\{x\}) + F'(\mathbb{E}\{x\})(x - \mathbb{E}\{x\}) \\ +\frac{1}{2}F''(\mathbb{E}\{x\})(x - \mathbb{E}\{x\})^2 \end{array}\right\} Pr\{x\} \\ &= F(\mathbb{E}\{x\}) + F''(\mathbb{E}\{x\}) \times \frac{\sigma_x^2}{2}\end{aligned}$$

- Linear approximation methods lead to certainty equivalence. We are going to fudge in our first-order conditions by saying that:

$$\mathbb{E}\Big\{F(x)\Big\} = F(\mathbb{E}\{x\})$$

Applying Certainty Equivalence to our Model

- Assume that we use certainty equivalence for expected labor, $\bar{L}_t$. It follows:

$$-\beta^{t-1}v'(\bar{L}_t) + \mathbb{E}\{\mu_t\}\bar{P}_t Z_t \rho \;=\; 0$$

- We know what $\bar{P}_t$ is and also:

$$-\mu_t \;=\; \mathbb{E}\{\frac{\beta^t}{\bar{M}_{t+1}}\} \quad\Longrightarrow\quad \mathbb{E}\{\mu_t\} \;=\; \mathbb{E}\frac{\beta^t}{\bar{M}_{t+1}}$$

- Plugging in yields:

$$-\beta^{t-1}v'(\bar{L}_t) + \left\{\frac{\beta^t}{M_{t-1}(1+\bar{\tau}_t)(1+\bar{\tau}_{t+1})}\right\}\frac{M_{t-1}(1+\bar{\tau}_t)}{Z_t\bar{L}_t}Z_t\rho \;=\; 0$$

$$\Leftrightarrow \quad -v'(\bar{L}_t)\bar{L}_t + \beta\frac{1}{1+\bar{\tau}_{t+1}}\rho \;=\; 0 \quad\Longrightarrow\quad \bar{L}_t \;=\; \left[\frac{\beta\rho}{1+\bar{\tau}_{t+1}}\right]^{1/(1+\gamma)}$$

Actual vs. Forecasted Labor Supply

- Forecasted labor supply depends on the model and expected money growth:

$$\bar{L}_t = \left[\frac{\beta\rho}{1+\bar{\tau}_{t+1}}\right]^{1/(1+\gamma)}$$

- Actual labor supply depends on forecasted labor supply and money growth surprises:

$$L_t = \frac{(1+\tau_t)}{(1+\bar{\tau}_t)}\bar{L}_t$$

- Actual money supply growth is given by:

$$\tau_t = \rho_\tau \tau_{t-1} + B_\tau + \sigma_\tau \varepsilon_{\tau,t}$$

- Expected (in time t) money supply growth today and tomorrow is given by:

$$\bar{\tau}_t = \rho_\tau \tau_{t-1} + B_\tau$$

$$\bar{\tau}_{t+1} = \rho_\tau(\rho_\tau \tau_{t-1} + B_\tau) + B_\tau$$

$\Rightarrow$ So expected money growth in the form of $\rho_\tau^2 \tau_{t-1}$ lowers labor, but surprise money shocks in the form of $\sigma_\tau \varepsilon_{\tau,t}$ raise labor and output.

Simulating and Analyzing the Economy

Cole (2020) chapter 10.3

Building Up All Variables from Start to Finish

1. Determine our forecasts for money supply growth, τ:

$$\begin{aligned}\bar{\tau}_t &= \rho_\tau \tau_{t-1} + B_\tau \\ \bar{\tau}_{t+1} &= \rho_\tau(\rho_\tau \tau_{t-1} + B_\tau) + B_\tau\end{aligned}$$

2. Determine our target labor level:

$$\bar{L}_t = \left[\frac{\beta\rho}{1+\bar{\tau}_{t+1}}\right]^{1/(1+\gamma)}$$

3. Determine the price index:

$$\bar{P}_t = \frac{M_t(1+\bar{\tau}_t)}{Z_t \bar{L}_t}$$

4. Determine realized labor:

$$L_t = \frac{(1+\tau_t)}{(1+\bar{\tau}_t)}\bar{L}_t$$

Some Simulations of our Model Economy

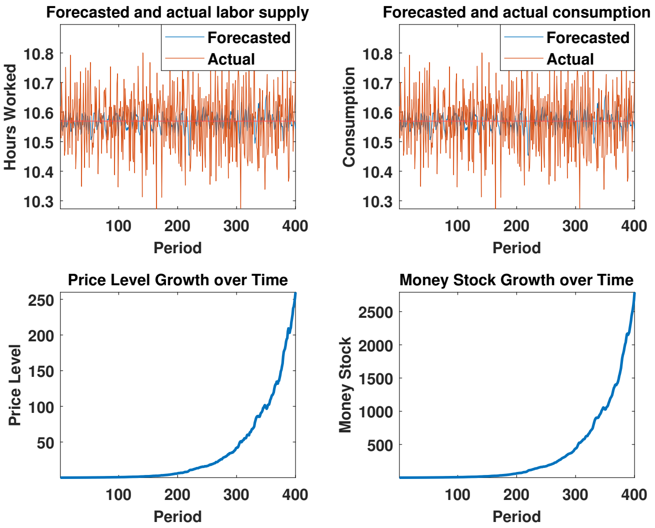

Figure 1: Model simulations: 400 periods, $\rho = 11$, $\beta = 0.98$, $Z = 1$, $B_\tau = 0.01$, $\rho_\tau = 0.5$, $\sigma_\tau = 0.01$.

Impulse Response Functions

Impulse response functions (IRFs) are a nice way to **understand the dynamic implications of a model**. They are constructed as follows:

1. The prior value of the random variable is set equal to its mean:

$$\tau_0 = \frac{B_\tau}{1-\rho_\tau}$$

2. In period 1, $\varepsilon_{\tau,1} = 1$, so:

$$\tau_1 = \frac{B_\tau}{1-\rho_\tau} + C$$

3. After period 1 the shock is set to zero, so:

$$\tau_t = \frac{B_\tau}{1-\rho_\tau} + \rho_\tau^{t-1} C \quad \forall t \geq 1$$

4. We then push these shocks through our model to trace what happens in period $t \geq 1$.
5. For each period we calculate the variables using the approach from slide 37.

Impulse Response Functions of a Surprise Money Growth Shock

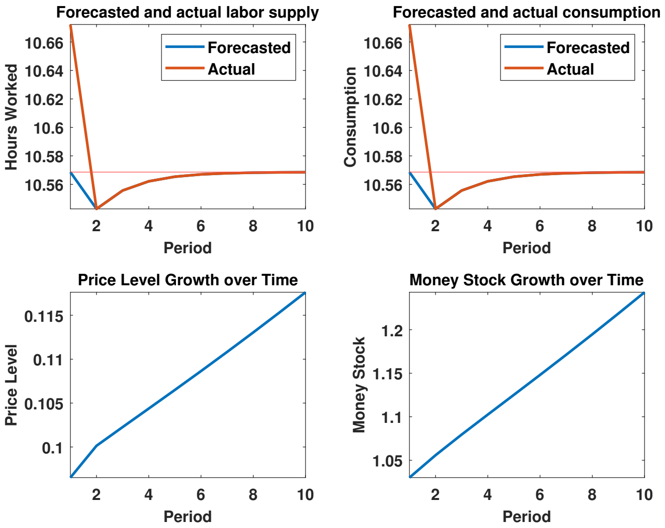

Figure 2: Impulse response functions: 40 periods (10 periods shown), $\rho = 11$, $\beta = 0.98$, $Z = 1$, $B_\tau = 0.01$, $\rho_\tau = 0.5$, $\sigma_\tau = 0.01$.

Conclusion

Summarizing the Lecture

Can you summarize the three main aspects of the lecture?

Concluding Remarks

Big Picture of the Lecture:

1. How to set monetary policy goals, targets, and instruments?
2. How do households set their goods prices under market power?
3. What role does uncertain within period money growth play for aggregate demand?

- **Monetary policy goals, targets, and instruments:**
  - → Implement observables targets with instruments of which we understand their economic channels well.
- **Monopolistic competition and market power:**
  - → Each producer can set its own prices and has some market power to charge markups.
  - → Yet, no aggregate demand effects of surprise money growth!
- **Information frictions and surprise inflation:**
  - → Surprise money growth can lead to an increase in aggregate demand.
  - → Surprise money injections must come after prices are set and before households buy goods.
- **Impulse response functions:**
  - → An important tool to disect and analyze the impact of a single shock on the model economy.

References

- Harold L. Cole (2020). Monetary and Fiscal Policy through a DSGE Lens. Oxford University Press.
- Taylor, J. B. (1993, December). Discretion versus policy rules in practice. In Carnegie-Rochester conference series on public policy (Vol. 39, pp. 195-214). North-Holland.