

Quantitative Dynamic Macroeconomics

– Assignment 00: Intertemporal Approach –

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The contents for this assignment should have been covered in previous macroeconomics courses (1st or 2nd year) in your studies. If not, you can read up on it e.g. in the following books:

- Jones, Charles I. (2020). *Macroeconomics*. Fifth edition. W.W. Norton & Company. Chapters 16-18.
- Burda, Michael & Charles Wyplosz (2022). *Macroeconomics - A European Text*. Eighth edition. Oxford University Press. Chapters 6-8.

The household lives for two periods. Life-time utility is given by the intertemporal utility function

$$U(c_1, c_2) = \left[\frac{c_1^{1-\sigma} - 1}{1-\sigma} - \psi \frac{h_1^{1+\nu}}{1+\nu} \right] + \beta \left[\frac{c_2^{1-\sigma} - 1}{1-\sigma} - \psi \frac{h_2^{1+\nu}}{1+\nu} \right],$$

where $c_1, c_2 > 0$ is consumption in period 1 and 2, $h_1, h_2 > 0$ is total hours worked in period 1 and 2, $0 < \beta < 1$ is the period discount factor, and $\sigma > 0$ describes the intertemporal elasticity of substitution of the household. $\psi > 0$ determines labor supply disutility, and $\nu > 0$ determines its elasticity. The household savings s_1 can be invested for a fixed interest rate $r \geq 0$. The income of the household in both periods is governed by a production function, $F(k_t, h_t)$, with capital and total hours worked as its input. In the production function, $z > 0$ is a fixed technology parameter, and $0 < \alpha < 1$ determines the capital elasticity of production. The capital stock, $\bar{k}_1 > 0$, in period 1 is exogenously given. The capital stock, k_2 , in period 2 is created by household investments, i_1 , in period 1. The budget constraints are given as follows

$$\begin{aligned} c_1 + i_1 + s_1 &= F(\bar{k}_1, h_1) \\ c_2 &= (1 + r) s_1 + F(k_2, h_2) \\ F(k_t, h_t) &= z k_t^\alpha h_t^{1-\alpha} \\ i_1 &= k_2 \end{aligned}$$

Exercise 1

Derive the consolidated budget constraint using all the information available.

Exercise 2

Set up the Lagrangian of the optimization problem. Define the endogenous and exogenous variables of the model.

Exercise 3

Derive the first-order conditions (FOCs) of the model. Solve for the consumption, labor, and capital FOCs and give an economic interpretation.

Exercise 4

Solve as far as possible to derive the utility maximizing consumption levels c_1^*, c_2^* , the utility maximizing total hours worked h_1^*, h_2^* , and the utility maximizing capital stock k_2^* . Is it possible to derive an analytical solution?