Introduction to Modern Macroeconomics

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Quantitative Dynamic Macroeconomics

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Tilburg University

Understanding Society

Course Overview - Organizational Information

- **Course name:** Quantitative Dynamic Macroeconomics
- **Course code:** 30L309
- **Language:** English
- **Workload:** 168 hours (6 ECTS)

- **Lecturer:** Dr. Konstantin Gantert
- **Assistant:** Tulio Bouzas

- **Teaching materials:** See Canvas!
- **Schedule:** See MyTimetable (Rooster)!
- **Further info:** Please read the Syllabus (on Canvas) carefully and completely!

Course Learning Goals

After *successful completion of the course*, the student will be able to:

CG 1 implement the structure of macroeconomic research papers, especially of model-based quantitative research papers.

CG 2 illustrate how the assumptions of "Dynamic Stochastic General Equilibrium" models define the model environment and its outcomes.

CG 3 apply mathematical methods to solve macroeconomic models described by systems of linear difference equations.

CG 4 compute software-based simulations, statistics, and visualizations of "Dynamic Stochastic General Equilibrium" models.

CG 5 organize the building blocks of "Dynamic Stochastic General Equilibrium" models such as Real-Business-Cycle and New-Keynesian models.

CG 6 assess economic transmission channels and policy based on computer simulations of "Dynamic Stochastic General Equilibrium" models.

Table of Contents for the Course

Problem Sets and Examination

Problem Sets:

- Weekly pen-&-paper exercises.
- Homework exercises are (partly) discussed in the tutorials.
- Any method used will be discussed in the tutorials at least once.
- Sample solutions for homework exercises.

Examination Composition

- Exam on block 1 and 2 (65%)
- Group policy paper with coding exercise in block 3 (35%)

Outline for this Lecture

What you have seen before

1. National accounting
2. AS-AD(IS-MP) model
3. Solow growth model
4. Lagrange method

What you will see today

1. Macro data & AS-AD short-comings [CG 1]
2. Macro policy in practice [CG 6]
3. Discussion of assignment 0 [CG 1, 5]
4. Static CiA macroeconomic model [CG 1, 5]

Big Picture of the Lecture:

1. Why do we need dynamic macroeconomic models to explain business cycle fluctuations?
2. How do we go from microeconomics to aggregate macroeconomic models?

Motivation:

Why do we need Dynamic Macroeconomic Models?

Dynamic Patterns: Long-Term Growth and Short-Term Fluctuation

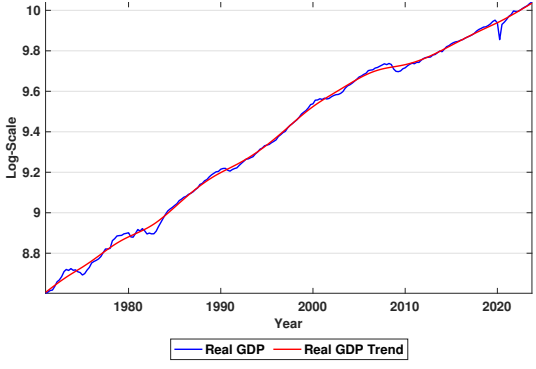

Figure 1: Real GDP in the US (1980-2023)

Some real GDP growth statistics:

- Average growth rate: 2.56%
- Autocorrelation (trend growth):
  - 1-period lag: 0.99
  - 8-period lag: 0.76
  - 16-period lag: 0.35

How to describe the growth patterns?

- We need a model that makes time explicitly in the long-run.
- Solow growth model!
- But what about the short-run and microfoundations?

Dynamic Patterns: The Business Cycle - Booms and Busts

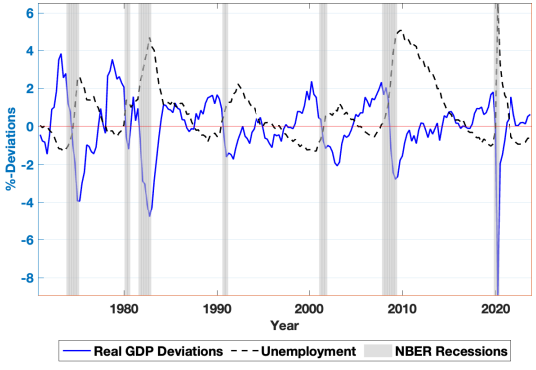

Figure 2: Quarterly Real GDP Deviations from the Long-Term Trend in the US (1980-2023)

Some business cycle statistics:

- Standard deviation:
  - 1.56% (GDP); 1.62% (UE)
- Autocorrelation (Quarters):
  - 1-lag: 0.70 (GDP); 0.67 (UE)
  - 2-lag: 0.50 (GDP); 0.45 (UE)
  - 3-lag: 0.30 (GDP); 0.30 (UE)

How to describe the fluctuations?

- AS-AD model? Does not show dynamics - only short- and long-run!
- We need a model that makes time explicit in the short-run.

Microfoundations: Inflation and Unemployment - The Phillips Curve

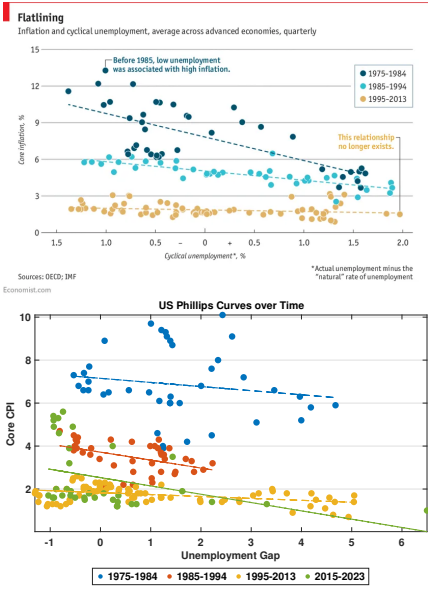

Figure 3: The flattening of the Phillips curve

Can we derive (aggregate) decision rules from the data?

- Phillips (1958) shows a statistical pattern between inflation and unemployment.
- The AS-AD model uses such statistical relationships as fundamental equations.
- What if underlying institutions, preferences, or technologies change?
- Is this how agents behave? Or should we base our decisions on micro-foundations?

Microfoundations: The Impact of Inflation Expectations

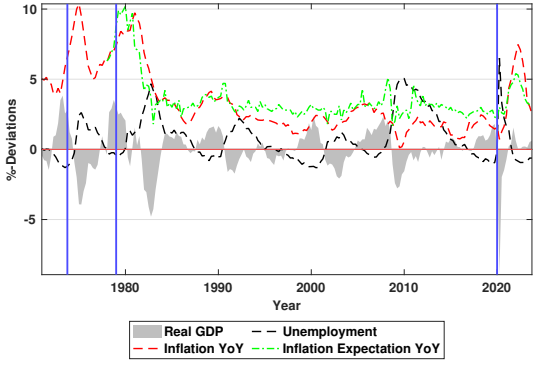

Figure 4: Fluctuations in real GDP and inflation expectations as drivers of inflation for US quarterly data (1980-2023).

A typical **Phillips curve**:

$$\pi = \kappa \times \tilde{GDP} + \xi + \pi_E$$

$\rightarrow$ What determines expecations, π_E?

Expectation formation:

- Agents observe the past and form expectations about the future.
- How do agents form expectations about the future?
  - $\rightarrow$ Rational expectations
  - $\rightarrow$ Adaptive expectations

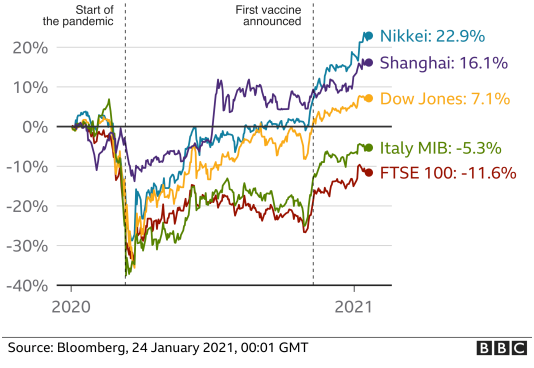

Figure 5: The impact of Covid on the stock market

Economic impact of unexpected shocks:

- (Long-term) Expectations about new viruses exist.
- However, realization of new viruses is normally unexpected.
- Significant drop in the value of stock markets as Covid unfolds.
- Also, unexpected increase in stock market values as vaccine was introduced.

Stochastic Elements: How to model the Russian Invasion of Ukraine?

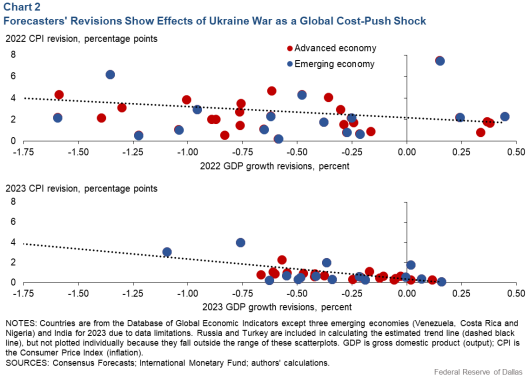

Figure 6: Forecast revision for inflation and GDP growth after the Russian invasion of Ukraine

Economic impact of unexpected shocks:

- Widely unexpected event: How did it change the economics of the world?
- How to model this event? Long-term vs short-term change?
- How do the economic dynamics of this shock look like?

$\Rightarrow$ Both the dynamic transition of the shock and expectations about future shocks need to be taken into account!

Critique of the AS-AD Model - What this Course is about

- **Ad-hoc Modeling:** No modeling of the deep/fundamental parameters. Models are not based on economic reasoning, but on statistical correlations.
  - → **Lucas (1976) Critique:** Conditional (on policy changes) forecasts need structural models. Estimated regularities break down if policy tries to exploit them.
- **Static Model:** Short-, medium-, and long-run are three different models. Transitions are fuzzy and up to idiosyncratic interpretation.
- **Perfect Foresight:** There is no uncertainty in this model. As there is no explicit time, this is hard to model. Nevertheless, uncertainty is an important driver of business cycles.

⇒ In this course, we learn how to construct Dynamic Stochastic General Equilibrium (DSGE) models, how to simulate them using a computer, and how to conduct policy analysis with those models!

Which Topics
do we cover in this Course?

How does Money affect the Macroeconomy?

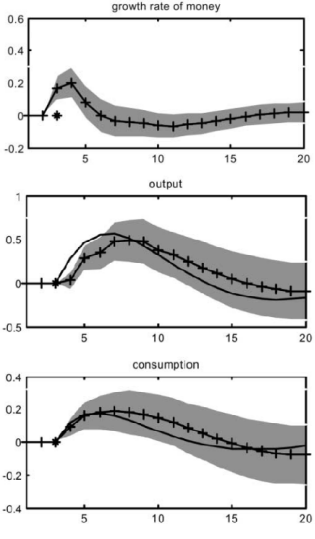

Figure 7: Impulse response functions for money, output and consumption (Christiano et al. (2005))

Money drives output in the short-run:

- What are the channels through which money affects the real economy?
- What is the long-run impact of money on the real economy?
- How high is optimal money supply?

How should Central Banks optimally set their Interest Rate?

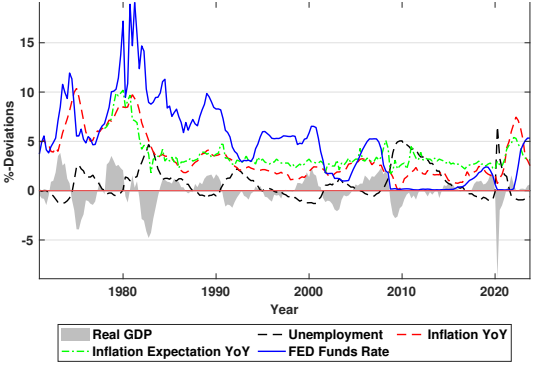

Figure 8: FED funds rate and U.S. year-on-year inflation rates, quarterly data (1980-2023).

Central banks stablize the business cycle:

- What is an appropriate policy target?
- Do people trust their central bank to achieve this target?
- What instruments stabilize the economy best? What are potential tradeoffs?

What is the Impact of different Forms of Taxation?

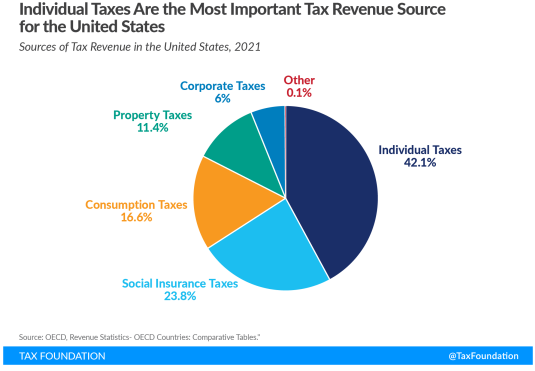

Figure 9: Tax revenue shares in the US (2021)

The macroeconomic impact of taxes:

- How should the government raise its tax revenue?
  - → Individual vs consumption taxes?
  - → Capital vs income taxes?
- What is an optimal amount of overall taxation?
- Do taxes impact business cycle fluctuations?

Fiscal Policy over the Business Cycle

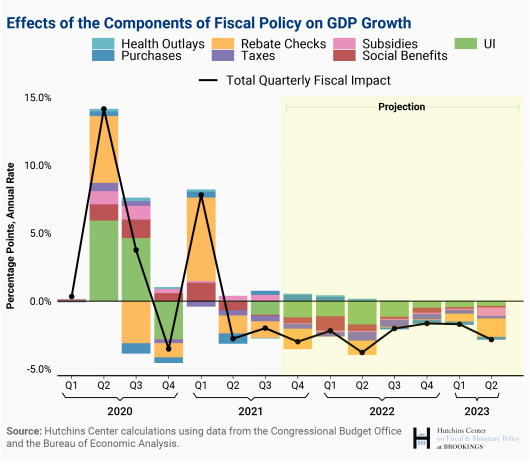

Figure 10: Effects of fiscal policy on GDP growth in the US

The economic use of governments:

- What economic purpose does government spending have?
- How large should government spending be overall?
- Do governments follow countercyclical fiscal policy? Is it optimal to do so?
- Can governments persistently influence the economy?

A Basic Approach to Intertemporal Macroeconomics

Assignment 0

The Cash-in-Advance Model

Basic Setup (Cole (2020) chapter 2)

How to Introduce Money in a Macroeconomic Model?

In a frictionless world, barter is generally preferred to money as money has no value.

However:

- People cannot track other's identities, hence (long-term) contracts are difficult.
- Trade often is sequential involving people that either want to buy or sell a good.

$\Rightarrow$ Barter is not possible, as there is no goods-for-goods exchange anymore.

$\Rightarrow$ Money acts as a medium of exhange. **Simplest friction: Cash-in-Advance (CiA)!**

The use of money is here by assumption!

- Hence, it is a poor model for understanding the role of money.
- However, the assumption is accurate enough to understand the impact of changes in money supply and other exogenous variables.

CiA Model: Basic Structure of the Economy

Our model economy consists of different households, firms, and a government:

- There are only household-firm pairs, hence no individual firms (for now).
- In each period, a household can produce one of many types of goods, $i \in I$.

Each household is composed of two agents: A seller and a buyer.

- A buyer takes available household cash to buy all the types of goods the household desires.
- A seller tries to sell some of the household good, i, saving the rest to consume themselves.

The model markets are framed by properties and institutions:

- Each goods market $i \in I$ has lots of buyers and sellers $\Rightarrow$ **markets are competitive!**

CiA Model: Goods and Asset Markets

To keep things simple, we will assume that the supply and demand of goods is symmetric:

- All prices, $P(i)$, that clear these markets will be identical.

There is an asset market where households exchange money and financial assets:

- We assume that the only type of asset is a one-period pure discount bond.
- The presence of this bond will give us an interest rate that we can determine.
- Monetary policy is conducted by changing the money supply or through the bonds market.

CiA Model: Production and Utility

Production function:

- Each household produces the good at the beginning of each period by

$$y_i = ZL, \tag{1}$$

where $Z > 0$ is productivity of the economy, and $L \geq 0$ is household hours worked.

Utility function:

- Each household has the following preferences over its consumption in a period:

$$u(C) \text{ concave, and } C = \left\{ \frac{1}{\#I} \sum_{i \in I} C(i)^{\rho} \right\}^{1/\rho}, \tag{2}$$

where $\#I$ is the number of different goods and $\rho \in (0, 1)$ defines their substitutability.

- A concave utility function implies $u'(C) > 0$ and $u''(C) < 0$.

CiA Model: Labor Disutility and Payoff

Labor disutility:

- Household preferences over labor supply in a period are given by $v(L)$.
- $v(L)$ is convex; specifically it follows that $v' > 0$ and $v'' > 0$.
- The cost of additional labor effort is positive and increasing in its level.

Household payoff function:

- The total payoff per period for a household is given by

$$\mathbb{U} = u(C) - v(L). \tag{3}$$

- Overall utility increases in consumption and decreases in labor effort!

CiA Model: Timing

Timing within each period

1. The household starts a period with M units of money.
2. It exerts labor effort L to produce its good.
3. The seller and buyer split up and go to their respective markets.
4. The seller and buyer come back together in the asset market.
5. They jointly consume the consumption good.
6. The period ends.

$\Rightarrow$ The timing of the model *will end up mattering quite a bit.*

The Cash-in-Advance Model

Static Version (Cole (2020) chapter 2)

Starting Point and Simplification

Simplification:

Static version of the household's problem, hence we will only look at the current period taking the future as given (we move to the dynamic version later on)!

Properties of the static model:

- Denote by $V(W)$ the future value of wealth, W.
- We do not yet distinguish among the types of wealth.
- We use continuation payoff functions to avoid infinite horizon models.

The household can be thought of as choosing (control variables!) how much

- to consume of each of the different goods, $\{C(i)\}_{i \in I}$,
- to work to produce their production good, $L(j)$,
- wealth, W, for next period in order to maximize its payoff.

Household Utility Maximization

Formalization of the **optimization problem** for a household whose production type is j:

$$\max_{\{C(i)\}_{i\in I}, L(j), W} \; u(C) - v(L(j)) + V(W)$$

subject to

- a **cash-in-advance condition**, which states that the household can only spend as much as it has in cash to buy goods (the notation I/j means the set I less element j.):

$$M \geq \sum_{i\in I/j} P(i)C(i)$$

- a **budget constraint**, which states that the household's net period wealth is whatever it has left out of its money holdings and the proceeds of what it sells in the goods market:

$$W \leq P(j)\left[ZL(j) - C(j)\right] + \left[M - \sum_{i\in I/j} P(i)C(i)\right]$$

Lagrangian of the Household Utility Maximization

To address this multi-dimensional maximization problem, we **form the Lagrangian:**

$$\begin{aligned}\mathcal{L} = & \max_{\{C(i)\}_{i\in I}, L(j), W} \min_{\lambda,\mu} u(C) - v(L(j)) + V(W) \\ & + \lambda \left\{ M - \sum_{i \in I/j} P(i) C(i) \right\} \\ & + \mu \left\{ P(j) \left[Z L(j) - C(j) \right] + \left[M - \sum_{i \in I/j} P(i) C(i) \right] - W \right\}.\end{aligned} \tag{4}$$

λ and μ are the **"penalty prices"** that we attach to violations of the constraints:

- The impact of the penalties comes through violations of the conditions, and can be minimized by setting them so there is no violation.
- In this maximization we are simultaneously trying to minimize the impact of these penalty prices on the overall objective and maximize the value of the objective in terms of the direct choice variables.

First-Order Conditions for Consumption

The **first-order conditions (FOCs)** will include the consumption conditions for each type of consumption, or

$$u'(C)\left\{\frac{1}{\#I}\sum_i C(i)^\rho\right\}^{(1-\rho)/\rho}\frac{1}{\#I}C(i)^{\rho-1} \;=\; (\lambda+\mu)\,P(i)$$

for each $i \neq j$.

For their own good j, the condition is

$$u'(C)\left\{\frac{1}{\#I}\sum_i C(i)^\rho\right\}^{(1-\rho)/\rho}\frac{1}{\#I}C(j)^{\rho-1} = \mu P(j).$$

- Note that this implies that the household will consume more of its own good to the extent that the cash-in-advance constraint binds and $\lambda > 0$.
- This pattern arrises because holding money has a cost due to its poor return.

First-Order Conditions for Labor and Wealth

Optimal condition for labor effort:

$$v'(L(j)) = \mu P(j) Z. \tag{5}$$

Optimal choice of wealth:

$$V'(W) = \mu. \tag{6}$$

We can use this last result to replace μ in the consumption and labor conditions.

The Cash-in-Advance Model

Symmetric Goods Market (Cole (2020) chapter 2)

Modification of CiA Constraint

Simplifying Modification

Going forward, we will not be interested in the extent to which households buy more of their own good because they don't need to use cash. Focus a model on what you want to study. Detail for details sake is bad.

Assume that you have to use cash even to buy your own consumption good. So, the constraints become

$$M \geq \sum_{i \in I} P(i)C(i)$$

and

$$W \leq P(j)ZL(j) + \left[M - \sum_{i \in I} P(i)C(i) \right].$$

Symmetric Prices across Goods

Given that the prices of the consumption good are the same $P(i) = \tilde{P}$ and $C(i) = C$:

- The derivative of the inside aggregator w.r.t. $C(i)$ is given by:

$$\left\{\frac{1}{\#I}\sum_i C(i)^\rho\right\}^{(1-\rho)/\rho} C(i)^{\rho-1} = \left\{\frac{1}{\#I}\#I\right\}^{(1-\rho)/\rho} \{C^\rho\}^{(1-\rho)/\rho} C^{\rho-1}$$
$$= \{C^\rho\}^{(1-\rho)/\rho} C^{\rho-1} = C^0 = 1$$

- Then, note that if you buy dC of each of our consumption goods, the increase in the composite is also dC, and if all of the prices are the same, and the change in $\mathcal{L}$ is:

$$\sum_i u'(C)\frac{1}{\#I}dC = \sum_i (\lambda+\mu)\,\tilde{P}dC \text{ or}$$
$$u'(C) = (\lambda+\mu)\,\#I * \tilde{P}$$

- Denote the price of a unit of the composite good (which means one of each individual good) by $P = \#I\tilde{P}$. It follows:

$$u'(C) = (\lambda+\mu)\,P \tag{7}$$

Binding CiA Constraint and Penalty Prices

If the cash-in-advance constraint *does not bind*, then $\lambda = 0$, and

$$u'(C) = V'(W)P, \tag{8}$$

If the cash-in-advance constraint *does bind*, then $\lambda > 0$,

$$C = \frac{M}{P}, \tag{9}$$

and λ will be chosen so that

$$\lambda = \frac{u'(C)}{P} - V'(W).$$

Optimal Labor Allocation in the Symmetric Equilibrium

By again using the FOC for wealth, we get a **optimal choice of labor:**

$$v'(L(j)) \;=\; V'(W)P(j)Z, \tag{10}$$

which says that the optimal labor choice is to set the marginal disutility of effort equal to the nominal marginal labor product, $P(j)Z$, times the marginal value of nominal wealth, $V'(W)$.

We want to *rewrite this in terms of the composite price*:

$$v'(L(j)) \;=\; V'(W)PZ/\#I.$$

Then if we change Z to $Z * \#I$, we get

$$v'(L) = V'(W)PZ.$$

The Cash-in-Advance Model

Asset Markets (Cole (2020) chapter 2)

Introducing the Asset Market

We now extend our simple model to incorporate an asset market at the end of the period:

- Households can exchange money for bonds and vice versa.
- Households can buy and sell government bonds, which are just another form of bond.
- We now have to distinguish between two types of wealth: Money and bonds!
  - $\rightarrow$ Next period's money and bond positions are denoted by B' and M'.
  - $\rightarrow$ Future household payoff is denoted by $V(B', M')$.
- All bonds are pure discount bonds:
  - $\rightarrow$ The payoff is \$ 1 for each unit of the bond and the cost is q per unit today.
  - $\rightarrow$ Note that $\frac{1}{q} = R$ is the gross interest rate offered by the bond.

Asset Market Lagrangian

The household faces a **budget constraint** in the bond market, which we can write as

$$PZL + [M - PC] + B \;\geq\; M' + qB'.$$

With these changes we can **rewrite its Lagrangian** as

$$\begin{aligned}\mathcal{L} \;=\; \max_{\{C(i)\}_{i\in I},L(j),M',B'} \min_{\lambda,\mu}\; & u(C) - v(L(j)) + V(M',B') \\ & + \lambda\{M - PC\} \\ & + \mu\{PZL + [M - PC] + B - M' - qB'\}.\end{aligned}$$

Static CiA Model: Asset Market First-Order Conditions

The **FOC for money**, M', is

$$-\mu + V_1(M', B') = 0.$$

Similarly, the **FOC for bonds**, B', is

$$-\mu q + V_2(M', B') = 0,$$

These two conditions differ in important ways.

- Bonds are cheaper per unit of future value to the extent that $q < 1$.
- They contribute future value in different ways to the extent that money and bonds in the future are imperfect substitutes.
- To determine how their future value differs, we need to move on to a *genuine dynamic version of the model* $\rightarrow$ **next lecture!**

Conclusion

Summarizing the Lecture

Can you summarize the three main aspects of the lecture?

Concluding Remarks

Big Picture of the Lecture:

1. Why do we need dynamic macroeconomic models to explain business cycle fluctuations?
2. How do we go from microeconomics to aggregate macroeconomic models?

We started out by criticizing older macroeconomic models. In particular:

- **Ad-hoc Modeling:** We have started to use micro-founded macroeconomics where the starting point is each single household and firm.
- **Static Models:** We will introduce dynamic models in the second lecture.
- **Perfect Foresight:** We will introduce random processes and expectations in the third lecture.

Getting Started with Matlab

Checklist: Getting Started with Matlab

1. Do you all have installed Matlab? Did you have any issues?

2. Have you tried the tutorial "Getting Started with Matlab" or the "playground AI"?

3. Let's check out the two-period model from assignment 0 in Matlab together!

References

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- Lucas Jr, R. E. (1976, January). Econometric policy evaluation: A critique. In Carnegie-Rochester conference series on public policy (Vol. 1, pp. 19-46). North-Holland.
- Phillips, A. W. (1958). The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom 1861-1957. Economica 283.
- FRED Economic Data. St Louis FED. Data retrived on. https://fred.stlouisfed.org/
  - → Real gross domestic product. Code: GDPC1
  - → Nominal gross domestic product. Code: GDP
  - → Real private investment. Code: GPDIC1
  - → Real private consumption. Code: PCECC96
  - → Real government consumption expenditures and gross investment. Code: GCEC1
  - → Unemployment rate. Code: UNRATE
  - → Natural unemployment rate. Code: NROU
  - → NBER based recessions indicator. Code: USREC
  - → University of Michigan: Inflation expectation. Code: MICH
  - → Federal funds effective rate. Code: FEDFUNDS
  - → Core CPI inflation. Code: BPCCRO1Q156NBEA