

Lecture 07: A New-Keynesian Business Cycle Model

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Quantitative Dynamic Macroeconomics

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Understanding Society

Outline for this Lecture

What we have seen so far

1. Stochastic CiA model with capital [CG 1, 5]
2. State-space and recursive methods [CG 1, 5]
3. Approximation & undet. coeff. [CG 3, 4]
4. Calibration & quantitative analysis [CG 4, 5]

What we will see today

1. Price Adjustment Costs [CG 1, 4, 5]
2. Natural Rate Hypothesis [CG 5, 6]
3. Simulations and Analysis [CG 1, 6]

Big Picture of the Lecture:

1. How to model persistent aggregate demand effects in our model economy?
2. What impact does monetary policy have on our model economy?

Motivation for Sticky Prices

Money Non-Neutrality in the Short-Run

Motivation: Short-Comings of the CiA Model/RBC

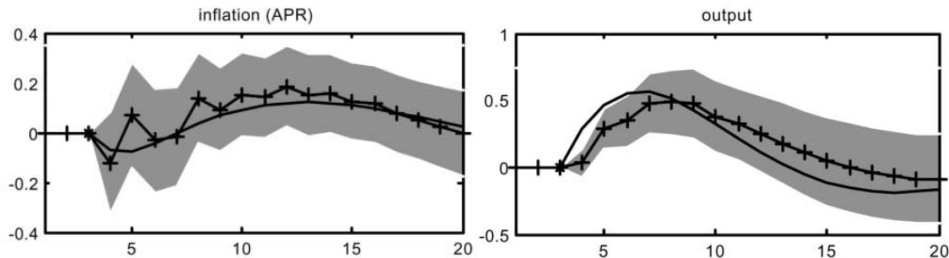
Our CiA/RBC model has dealt with many issues of earlier macroeconomic models, such as:

- ▶ Micro-foundations and dynamic modeling
- ▶ Random processes and rational expectations
- ▶ General equilibrium

Why do we need another model then? **Short-comings of the CiA/RBC model:**

- ▶ **Predominant Productivity Shocks:** Traditional Keynesians see productivity change as a source of long-run growth, not of short-run cycles, which are attributed to demand fluctuations.
- ▶ **Efficiency of Markets:** There is no role for stabilization policy in a frictionless model. Keynes saw recessions as episodes of inefficient use of resources.
- ▶ **Money:** The CiA/RBC model has a limited role for money and nominal variables. Only money growth has a one-period effect on real variables. This implies a limited role for monetary policy.

Motivation: Evidence on Money Non-Neutrality



SOURCE: Christiano et al. (2005). NOTE: VAR-estimated IRFs to an expansionary monetary policy shock.

- **Money Non-Neutrality:** Real variables as output (also consumption, investment) increase following an expansionary monetary policy shock.
- **Sticky Prices:** Inflation shows a hump-shaped behavior with high autocorrelation.

Ten Facts on Prices and their Implications for Macro Models (1/2)

1. **Prices change at least once a year:**

→ US CPI changes every 4 month, US PPI every 6-8 month, EU CPI is even stickier.

2. **Sales and product turnover are often important for micro price flexibility:**

→ Micro price flexibility does not imply macro price flexibility.

3. **Reference prices are stickier and more persistent than regular prices:**

→ Reference pricing behavior may suggest some form of sticky plan and/or sticky information.

4. **There is substantial heterogeneity in the frequency of price change across goods:**

→ Service prices are stickier than those of goods. Among goods, "raw" goods (energy, fresh produce) are more flexible than "processed" goods.

5. **More cyclical goods change prices more frequently:**

→ Prices change more frequently in categories with more procyclical real consumption growth.

Ten Facts on Prices and their Implications for Macro Models (2/2)

6. Price changes are big on average, but many small changes occur:

→ Many price changes are larger than needed to keep up. However, many very small price changes happen too. There is a "missing middle".

7. Relative price changes are transitory:

→ Relative price movements tend to fade over time - they are less persistent than random walk.

8. Price changes are typically not synchronized over the business cycle:

→ Periods of greater macro volatility may exhibit more synchronization.

9. Neither frequency nor size is increasing in the age of a price:

→ Hazard rate of price changes is falling over the first few months and largely flat afterward.

10. Price changes are linked to wage changes:

→ Firms with a higher share of labor costs in total make less frequent price adjustments, potentially resulting from the fact that wages adjust less frequently than other input prices.

See Klenow and Malin (2010) for a full analysis of prices.

What are common ways of modeling sticky prices in the literature?

- ▶ **Information problems:** Asymmetric information concerning the money supply, or household demand. The information might be unavailable, or it is available but too costly (e.g. Mankiw and Reis, 2002).
- ▶ **Non-Walrasian features:** Markets are determined by prices and a second component, e.g. search costs for trade opportunities, simultaneously (e.g. Kiyotaki and Wright, 1989).
- ▶ **Adjustment costs:** It is costly to change prices. There is a trade-off between additional demand and adjustment costs (i.e. Rotemberg, 1982).
- ▶ **Pricing rules:** Firms follow rules to change prices:
 - State-dependent pricing: Adjust when prices are relatively far away from some pre-specified target price (i.e. Caplin and Spulber, 1987).
 - Time-dependent pricing: Adjust after a pre-specified amount of periods. (i.e. Fisher, 1977; Taylor, 1980; Calvo, 1983).

Frictional Price Setting

Applying Rotemberg (1982) to our Model

Model Overview - Five Types of Agents in the Model Economy

1. Representative household:

- Buy consumption and investment goods.
- Supply labor and capital to firms.
- Holds bonds, money, and capital as financial assets.
- Receives government revenues and firm profits lump-sum at the end of the period.

2. Representative final good firm:

- Aggregate many different intermediate goods to one final good.

3. Intermediate good firms of many different types $i \in I$:

- Employ labor and capital to produce intermediate goods.
- Sets intermediate good prices, but price adjustment is costly (Rotemberg (1982)).

4. The government (fiscal policy):

- Taxes labor and capital income of households. Rebates it lump-sum.

5. The central bank (monetary policy):

- Sets the nominal interest rate.

Differentiating Between Individual and Aggregate Variables

Remember from the lecture on taxes

Firms are separate entities and input factors are traded on competitive markets. Households and firms choose individual levels, while market rates are determined by aggregate levels of labor and capital.

Underlying assumptions:

- ▶ w and r are the prices such that labor and capital markets clear.
- ▶ Many households and firms: Nobody is large enough to have an impact on market prices.
- ▶ Every firm uses labor and capital from many households.
- ▶ Every household supplies labor and capital to many firms.

Household Utility Maximization

The *representative household* maximizes his utility by choosing

$$\max_{C_t, L_t, B_{t+1}, K_{t+1}} \mathbb{E}_t \sum_{t=1,2} \beta^{t-1} \left[\log(C_t) - \frac{\varphi}{1+\gamma} L_t^{1+\gamma} \right] + \mathbb{E}_t \beta^2 \mathbb{V}_H(B_3, K_3)$$

subject to

$$\begin{aligned} \frac{M_t}{\nu} &= P_t C_t, \\ q_t B_{t+1} + M_{t+1} &= P_t [(1 - \tau_L) w_t L_t + (1 - \tau_K) r_t K_t] + M_t + B_t + T_t + \Pi_t \\ &\quad - P_t C_t - P_t [K_{t+1} - (1 - \delta) K_t], \\ K_{t+1} &= (1 - \delta) K_t + X_t. \end{aligned}$$

Atomistic households

Household income depends on the market rates for labor, $(1 - \tau_L)w_t$, and capital, $(1 - \tau_K)r_t$. Like in the lecture on taxes, the labor and capital decisions of households do not affect their market rates.

Household First-Order Conditions

Applying a change-in-variables, $\mu_t = \frac{\beta^{t-1}}{P_t C_t} \tilde{\mu}_t$, to render the model stationary, we are left with the *usual household first-order conditions*:

$$w_t = \varphi L_t^\gamma \frac{C_t}{(1 - \tau_L) \tilde{\mu}_t} \quad (1)$$

$$\tilde{\mu}_t = \beta \mathbb{E}_t \frac{P_t}{P_{t+1}} \frac{C_t}{C_{t+1}} \frac{1}{\nu} [1 - (1 - \nu) \tilde{\mu}_{t+1}] \quad (2)$$

$$q_t = \beta \mathbb{E}_t \frac{P_t}{P_{t+1}} \frac{C_t}{C_{t+1}} \frac{\tilde{\mu}_{t+1}}{\tilde{\mu}_t} \quad (3)$$

$$1 = \beta \mathbb{E}_t [(1 - \tau_K) r_{t+1} + (1 - \delta)] \frac{C_t}{C_{t+1}} \frac{\tilde{\mu}_{t+1}}{\tilde{\mu}_t} \quad (4)$$

Final Good Firms as Goods Bundler

Final good firms maximize profits by bundling *differentiated goods* to one good according to

$$\max_{Y_t, Y_t(i)} \left[P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \right]$$

subject to household preferences

$$Y_t = \left[\int_0^1 Y_t(i)^\rho di \right]^{\frac{1}{\rho}}$$

which they take into account as they sell the final good to households later on.

Optimal demand for good i (first-order condition):

$$P_t(i) = \left[\frac{Y_t}{Y_t(i)} \right]^{1-\rho} P_t \quad (5)$$

- If $P_t(i) > P_t$, demand is **not** zero. If $P_t(i) < P_t$, demand is **not** equal to C_t .
- **Monopolistic Competition** (Dixit & Stiglitz, 1977): Goods are imperfect substitutes (set by ρ)!

Intermediate Good Firms: Production and Price Setting

A intermediate good firm of type i maximizes its profits by choosing

$$\max_{Y_t(i), L_t(i), K_t(i), P_t(i)} \mathbb{E}_t \sum_{t=1,2} \beta^{t-1} \frac{\mu_t}{\mu_1} \{P_t(i) Y_t(i) - W_t L_t(i) - P_t r_t K_t(i)\} + \mathbb{E}_t \beta^2 \frac{\mu_3}{\mu_1} \mathbb{V}_F(P_2, P_3)$$

subject to

$$Y_t(i) = [Z_t L_t(i)]^{1-\alpha} K_t(i)^\alpha \left[1 - \frac{\kappa}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 \right],$$

$$P_t(i) = \left[\frac{Y_t}{Y_t(i)} \right]^{1-\rho} P_t,$$

and where $\Gamma(P_t(i), P_{t-1}(i)) = \frac{\kappa}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2$.

Properties of the firm:

- It anticipates the demand function of the final good firm.
- It maximizes its profits by **setting prices** under quadratic price adjustment costs, Γ_t .
- Firms are owned by households, hence they discount the future the same way, $\beta^{t-1} \frac{\mu_t}{\mu_1}$.

Intermediate Good Firms: First-Order Conditions

Input factor demand functions:

$$w_t = (1 - \alpha) \frac{Y_t(i)}{L_t(i)} mc_t(i) \quad (6)$$

$$r_t = \alpha \frac{Y_t(i)}{K_t(i)} mc_t(i) \quad (7)$$

Price setting condition:

$$\begin{aligned} \kappa \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right) \frac{P_t(i)}{P_{t-1}(i)} &= \frac{1 - \frac{\rho}{mc_t(i)}}{1 - \rho} \\ &+ \mathbb{E}_t q_t \kappa \left(\frac{P_{t+1}(i)}{P_t(i)} - 1 \right) \left(\frac{P_{t+1}(i)}{P_t(i)} \right)^2 \frac{Y_{t+1}(i)}{Y_t(i)} \frac{mc_{t+1}(i)}{mc_t(i)} \end{aligned} \quad (8)$$

- **Steady-state:** Price markup, $\mathcal{M} = \frac{1}{mc} = \frac{1}{\rho}$, that depends on elasticity of substitution, ρ .
- **Dynamics:** Markup fluctuates over the business cycle due to sticky prices.

Representative agents:

- ▶ All households have symmetric preferences and characteristics.
- ▶ All intermediate firms have the same technologies.

Equilibrium - all markets clear:

- ▶ Good markets clear, $C_t + X_t = Y_t = \int_0^1 Y_t(i) di$.
- ▶ Labor markets clear, $L_t^D = w_t = L_t^S$.
- ▶ Capital rental markets clear, $K_t^D = r_t = K_t^S$.
- ▶ Money markets clear, $M_t = \bar{M}_t$.
- ▶ The nominal bond price, q_t , adjusts such that bonds are in zero supply, $B_t = 0$.

Assumptions about tax and profit distribution:

- ▶ Government revenues are rebated to the households lump-sum, $\tau_L w_t L_t + \tau_K r_t K_t = T_t$.
- ▶ Firm profits, Π_t , are redistributed to all households equally.

Summary of the Model in Symmetric Equilibrium

$$\text{Labor market: } \varphi L_t^\gamma \frac{C_t}{(1 - \tau_L) \tilde{\mu}_t} = (1 - \alpha) \frac{Y_t}{L_t} m c_t \quad (9)$$

$$\text{Capital market: } \tilde{\mu}_t = \beta \mathbb{E}_t \left[(1 - \tau_K) \alpha \frac{Y_{t+1}}{K_{t+1}} m c_{t+1} + (1 - \delta) \right] \frac{C_t}{C_{t+1}} \tilde{\mu}_{t+1} \quad (10)$$

$$\text{Bond market: } q_t = \beta \mathbb{E}_t (1 + \pi_{t+1}) \frac{C_t}{C_{t+1}} \frac{\tilde{\mu}_{t+1}}{\tilde{\mu}_t} \quad (11)$$

$$\text{Money market: } 1 + \tau_t = \mathbb{E}_t (1 + \pi_{t+1}) \frac{C_{t+1}}{C_t} \quad (12)$$

$$\text{Phillips curve: } \Gamma'(\pi_t) = \frac{1 - \frac{\rho}{m c_t}}{1 - \rho} + \mathbb{E}_t q_t (1 + \pi_{t+1}) \frac{Y_{t+1}}{Y_t} \frac{m c_{t+1}}{m c_t} \Gamma'(\pi_{t+1}) \quad (13)$$

$$\text{Resource constraint: } C_t = Y_t - K_{t+1} + (1 - \delta) K_t \quad (14)$$

$$\text{Production function: } Y_t = [Z_t L_t]^{1-\alpha} K_t^\alpha \quad (15)$$

→ We use $\frac{P_{t+1}}{P_t} = 1 + \pi_{t+1}$ for the inflation rate.

→ We have 8 endogenous variables, but only 7 equilibrium equations: **Indeterminacy!**

→ We need to define a monetary policy rule!

Monetary Policy and the Natural Interest Rate

Setting the Nominal Interest Rate

Central Bank Policy according to Taylor (1993, 1999):

$$\frac{1 + r_{B,t}}{1 + \bar{r}_B} = \left(\frac{1 + \pi_t}{1 + \bar{\pi}} \right)^{\theta_\pi} \times \left(\frac{1 + Y_{Gap,t}}{1 + Y_{Gap}} \right)^{\theta_{Gap}} \times \exp(M_t) \quad (16)$$

→ Estimated equation that represents interest rate setting behavior of modern central banks!

- **Policy Coefficients:** $\theta_\pi, \theta_{Gap} \geq 0$. Determine the reaction function of monetary policy.
 - **Taylor principle:** To ensure determinacy of the economy, we must set $\theta_\pi > 1$ (credible threat). Otherwise, there are sunspot equilibria!
- **Target Shock:** $M_t \in \mathbb{R}$. Summarizes deviations from (expected) monetary policy.
- **Interest Rate Target:** $\bar{r}_B \in \mathbb{R}$. Target steady-state nominal interest rate.

What determines \bar{r}_B and $Y_{Gap,t}$?

Determining the Optimal Nominal Interest Rate Target

The **steady-state nominal interest rate** is given by the bond Euler equation

$$1 + \bar{r}_B = \frac{1 + \bar{\pi}}{\beta} (1 + g)$$

Equivalently, the **natural interest rate** (social planner) is given by

$$1 + r_B^* = \frac{1}{\beta} (1 + g)$$

Therefore, the **long-run stable intertemporal output allocation** is given by a Fisher equation

$$1 + \bar{r}_B = (1 + r_B^*) (1 + \bar{\pi}),$$

where

- r_B^* determines the fundamental intertemporal features of the economy (social planner),
- $\bar{\pi}$ is a target inflation rate (set by the central bank).

What determines 1) the natural interest rate and 2) the target inflation rate?

Drivers of the Natural Interest Rate and Target Inflation

Possible drivers of the **natural interest rate** in reality:

► **NK Model:** Household discount rate across time, $\frac{1}{\beta} - 1$.

► **OLG Models:** Demographics and the global savings glut.

► **TFP Growth:** Slowdown in productivity growth, g .

⇒ It becomes increasingly clear that r_B^* **has decreased** over the last 30 years.

(Laubach & Williams (2003); Holston, Laubach & Williams (2017)).

⇒ This pushes \bar{r}_B closer to the **zero lower bound**!

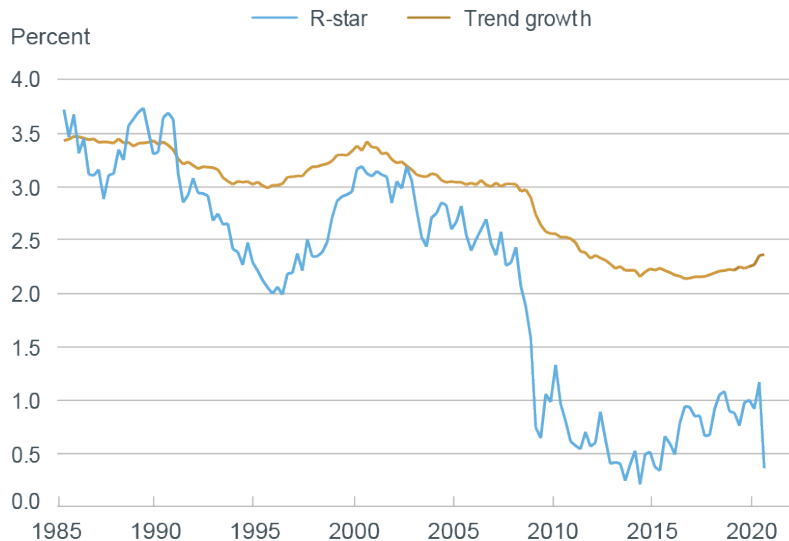
Possible drivers of the **target inflation rate** in reality:

► Allows asymmetric downward rigid prices and wages to adjust.

► Creates head-space to prevent deflationary spirals.

► (De-)anchoring of inflation expectations in periods of high inflation.

The U.S. Natural Interest Rate (Laubach & Williams, 2003)



Source: Laubach and Williams (2003).

Note: We plot estimates of the natural rate of interest (r -star) along with those for the trend growth rate of the U.S. economy, a source of change driving r -star.

The Output Gap: Definition

The output gap is given by:

$$Y_{Gap,t} = \frac{\tilde{Y}_t}{\tilde{Y}_{N,t}} - 1 \quad (17)$$

- ▶ The output gap is defined as actual output relative to natural (potential) output.
- ▶ The natural rate of the economy prevails **absent any frictions or imperfections** in the market economy. HERE: $\rho = 1$, and $\kappa = 0$. (This economy is equivalent to the social planner solution.)
- ▶ Deviations from potential output, $\tilde{Y}_{N,t}$, follow from sticky prices over the business cycle.
 - Positive deviations indicate the economy being above its long-run potential.
 - Negative deviations indicate the economy being below its long-run potential.

A Calibrated NK Model and the Data

Policy Functions, State Variables, and Exogenous Shocks

We solve the dynamic model by its **policy functions** (symmetric for natural rate model):

$$\begin{aligned}C_t &= \Lambda_C(K_t, Z_t, M_t) & \tilde{\mu}_t &= \Lambda_{\tilde{\mu}}(K_t, Z_t, M_t) \\L_t &= \Lambda_L(K_t, Z_t, M_t) & mc_t &= \Lambda_{mc}(K_t, Z_t, M_t) \\K_{t+1} &= \Lambda_K(K_t, Z_t, M_t) & \pi_t &= \Lambda_{\pi}(K_t, Z_t, M_t) \\Y_t &= \Lambda_Y(K_t, Z_t, M_t) & r_{B,t} &= \Lambda_r(K_t, Z_t, M_t)\end{aligned}$$

Forward-looking variables:

- ▶ $\mathbb{E}_t C_{t+1}$: Intertemporal consumption allocation by the Euler equation!
- ▶ $\mathbb{E}_t \pi_{t+1}$: Intertemporal pricing by the Phillips curve!
- ▶ $\mathbb{E}_t \tilde{\mu}_{t+1}$: Intertemporal asset allocation by the capital Euler equation!

Exogenous Shock Processes:

$$\begin{aligned}Z_t &= \rho_Z Z_{t-1} + \varepsilon_{Z,t}, \quad \varepsilon \sim \mathcal{N}(0, \sigma_{Z,t}^2) \\M_t &= \rho_M M_{t-1} + \varepsilon_{M,t}, \quad \varepsilon \sim \mathcal{N}(0, \sigma_{M,t}^2)\end{aligned}$$

⇒ We ask Dynare to do all those calculations for us!

Calibrating our Model Economy

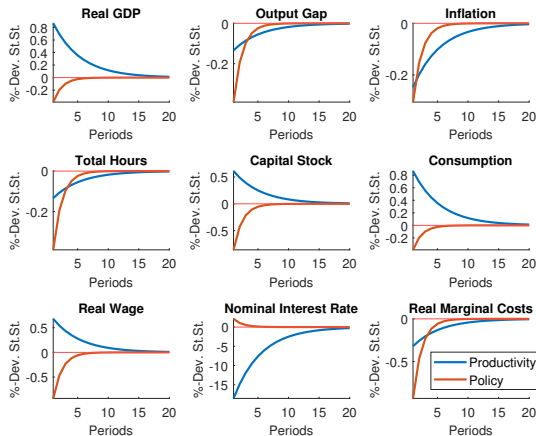
Variable	Value	Variable	Value
β	0.98	σ_Z	0.01
γ	$\frac{1}{0.72}$	σ_M	0.005
δ	0.03	ρ_Z	0.8
α	$\frac{1}{3}$	ρ_M	0.5
ρ	0.9	κ	60
τ_L	0.2	τ_K	0.1

Table 1: Calibration overview

- ▶ We assume a zero inflation steady-state, $\pi^* = 0$.
- ▶ There is no exogenous growth of productivity or money supply in this model!
- ▶ $\beta = 0.98$ implies 8.4% nominal interest per year.
- ▶ $\rho = 0.9$ implies average price markups of 11%.
- ▶ $\alpha = \frac{1}{3}$ implies roughly one-third capital income share (markups distort it a bit).

Impulse Response Functions for a Model with Marginal Capital

We assume here: $\alpha \approx 0$ and $\delta = 1$!



An increase in productivity, Z_t , leads to:

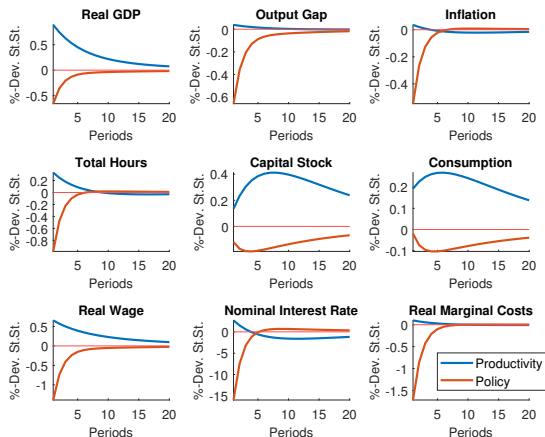
- $mc_t \downarrow \Rightarrow \pi_t \downarrow \Rightarrow Y_t, C_t, K_t \uparrow$
- $\pi_t \downarrow \Rightarrow r_{B,t} \downarrow$
- $\pi_t \downarrow$ (but sticky!) $\Rightarrow Y_{Gap,t} \downarrow$
- $w_t \uparrow$, but sticky $\pi_t \downarrow \Rightarrow L_t \downarrow$

A rise in interest rates, $r_{B,t}$, leads to:

- $C_t \downarrow \Rightarrow Y_t, K_t, L_t \downarrow$
- $Y_t \downarrow \Rightarrow mc_t, \pi_t \downarrow$
- $\pi_t \downarrow$ (but sticky!) $\Rightarrow Y_{Gap,t}, w_t \downarrow$

Impulse Response Functions for a Model with Capital

We assume here: $\alpha = \frac{1}{3}$ and $\delta = 0.03$! Hence, K_t becomes a state variable!



An increase in productivity, Z_t , leads to:

- ▶ $Y_t, K_t, C_t \uparrow$ as before!
- ▶ $Z_t \uparrow$ saved for later through $K_t \uparrow \uparrow$
- ▶ It follows
 $Y_t \uparrow \uparrow \Rightarrow m_{C_t}, \pi_t, L_t, Y_{Gap,t} \uparrow$
- ▶ $\pi \uparrow \Rightarrow r_{B,t}$
- ▶ Higher persistence b/c $\delta < 1$

A rise in interest rates, $r_{B,t}$, leads to:

- ▶ Mostly symmetric to case w/o capital.
- ▶ Higher persistence through $\delta < 1$.
- ▶ $Y_t, \pi_t \downarrow \downarrow \Rightarrow r_{B,t} \downarrow$!

Conclusion

Can you summarize the three main aspects of the lecture?

Big Picture of the Lecture:

1. How to model persistent aggregate demand effects in our model economy?
2. What impact does monetary policy have on our model economy?

The **New Keynesian Model** allows for persistent aggregate demand effects:

- ▶ **Monopolistic competition:** Firms are price setters and charge a price markup.
- ▶ **Sticky prices:** There is a trade-off between optimal prices and adjustment costs!

A **cyclical output gap** indicates a sub-optimal short-run equilibrium:

- ▶ Sticky prices prevent instantaneous price adjustment leading to distorted input allocation.

Monetary policy can control the nominal interest rate:

- ▶ It has real (persistent) effects as prices are sticky.
- ▶ Inflation targeting can stabilize the economy.

New Neoclassical Synthesis: Prices are sticky in the short-run, but flexible in the long-run!

References

- ▶ Dixit, A. K., & Stiglitz, J. E. (1977). Monopolistic competition and optimum product diversity. *The American economic review*, 67(3), 297-308.
- ▶ Cole, Harold L. (2020). *Monetary and Fiscal Policy through a DSGE Lens*. Oxford University Press.
- ▶ Greenwood, J., Hercowitz, Z., & Huffman, G. W. (1988). Investment, capacity utilization, and the real business cycle. *The American Economic Review*, 402-417.
- ▶ Holston, K., Laubach, T., & Williams, J. C. (2017). Measuring the natural rate of interest: International trends and determinants. *Journal of International Economics*, 108, 59-75.
- ▶ Christiano, L. J., Eichenbaum, M., & Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of political Economy*, 113(1), 1-45.
- ▶ Galí, J. (2015). *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*. Princeton University Press.
- ▶ Klenow, P. J., & Malin, B. A. (2010). Microeconomic evidence on price-setting. In *Handbook of monetary economics* (Vol. 3, pp. 231-284). Elsevier.
- ▶ Laubach, T., & Williams, J. C. (2003). Measuring the natural rate of interest. *Review of Economics and Statistics*, 85(4), 1063-1070.
- ▶ Rotemberg, J. J. (1982). Monopolistic price adjustment and aggregate output. *The Review of Economic Studies*, 49(4), 517-531.
- ▶ Taylor, J. B. (1993). Discretion versus policy rules in practice. In *Carnegie-Rochester conference series on public policy* (Vol. 39, pp. 195-214). North-Holland.
- ▶ Taylor, J. B. (1999). A historical analysis of monetary policy rules. In *Monetary policy rules* (pp. 319-348). University of Chicago Press.