Quantitative Dynamic Macroeconomics

– Assignment 06: Solving the Money and Capital Model –

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The model in this assignment is build on the model from assignment 05. The only change is that there are productivity shocks to a productivity process that follows an AR(1)-process given by $Z_t = \rho_Z Z_{t-1} + \epsilon_{Z,t}$, where $0 \leq \rho_Z \leq 1$, and $\epsilon_{Z,t} \sim \mathcal{N}\left(0, \sigma_Z^2\right)$. There is a cash-in-advance constraint and money supply given by

$$\begin{aligned}
(1-\delta_M)M_t &= \bar{P}_t C_t, \\
\bar{M}_{t+1} &= (1+\tau)(1-\delta_M)\bar{M}_t,
\end{aligned}$$

where money loss, δ_M, and money growth, τ, are both constant. Production is given by a Cobb-Douglas function

$$Y_t = \left(Z_t \bar{L}_t\right)^{1-\alpha} \bar{K}_t^{\alpha},$$

where $Z_t > 0$ is the productivity level, $\bar{L}_t$ is aggregate labor traded on the input market, $\bar{K}_t$ is the aggregate capital stock traded on the input market, and $0 < \alpha < 1$ is a parameter. The capital stock of each household changes over time accroding to the capital law of motion, $K_{t+1} = (1 - \delta_K)K_t + X_t$, where $0 \leq \delta_K \leq 1$ is the capital depreciation rate, K_t is the individual capital stock of a household, and X_t are individual investments into the individual capital stock. The reduced-form of the system of model equations is given by (see assignment 05 for detailed derivations)

$$\begin{aligned}
\varphi L_t^{\nu} &= (1-\alpha) Z_t^{1-\alpha} \left(\frac{K_t}{L_t}\right)^{\alpha} \frac{1}{C_t} \frac{\beta}{1+\tau}, && (1) \\
\mathbb{E}_t \frac{C_{t+1}}{C_t} &= \beta \mathbb{E}_t \left[\alpha Z_{t+1}^{1-\alpha} \left(\frac{L_{t+1}}{K_{t+1}}\right)^{1-\alpha} + (1-\delta_K)\right], && (2) \\
\left(Z_t L_t\right)^{1-\alpha} K_t^{\alpha} &= C_t + X_t \; = \; C_t + K_{t+1} - (1-\delta_K)K_t. && (3)
\end{aligned}$$

Exercise 1 Linearizing the System of Model Equations

The system of model equations is a system of non-linear difference equations. As a first step to solve this model and to be able to simulate it across time, we need to linearize it. Use a first-order Taylor approximation for all model equations necessary to solve the model. *Hint: Don't forget to calculate the steady-state first in order to solve for any steady-state variables in the linearized equations.*

Exercise 2 ADDITIONAL: Method of Undetermined Coefficients

Use the method of undetermined coefficients to solve for the policy functions of the model. Use the following steps to derive a solution to the system of linear difference equations:

a) Define appropriate guesses for the policy functions of all endogenous variables of the model. *Hint: Remember, a policy function determines a control variable based on all endogenous and exogenous state variables of the model in period t.*

b) Plug the guesses into the system of linear difference equations and sort the terms appropriately. *Hint: Select all parameters and coefficients for each state variable. There are two blocks of state variables: endogenous states and exogenous states. Sort the coefficients of the state variables accordingly.*

c) Solve for the deterministic part of the model. *Hint: Make sure that you select the stable root for the undetermined coefficient of capital.*

d) Solve for the stochastic part of the model, using the already solved-for coefficients from c).

e) Solve for the policy function of output using the production function of the model and the policy functions you have derived in c) and d).

Exercise 3 Simulating the Model using Dynare

Use the Dynare file provided for the model discussed. Try to understand the different building blocks of the code. Simulate the economy and plot impulse response functions. Try out different calibration values and check out the Dynare manual sections 4.1-4.5, 4.8, 4.10, 4.11, 4.13: https://www.dynare.org/manual/