

Quantitative Dynamic Macroeconomics

– Assignment 07: Sticky Prices and Dynare –

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Exercise 1 Derive the sticky prices FOC

Suppose an intermediate firm wants to maximize its profits by choosing a price and a quantity of goods to sell. Assume that the cost minimization with respect to the input factors capital and labor is already solved. Its FOCs are given by

$$r_t = \alpha \frac{Y_t(i)}{K_t(i)} mc_t(i), \quad (1)$$

$$w_t = (1 - \alpha) \frac{Y_t(i)}{L_t(i)} mc_t(i), \quad (2)$$

where $mc_t(i)$ is firm real marginal cost and to be solved for in the following exercise. The Lagrangian for the profit maximization of the intermediate firm is given by

$$\begin{aligned} \mathcal{L} = & \max_{Y_t(i), P_t(i)} \mathbb{E}_t \sum_{t=1,2} \beta^{t-1} \frac{\mu_t}{\mu_1} [P_t(i)Y_t(i) - W_t L_t(i) - P_t r_t K_t(i)] + \mathbb{E}_t \beta^2 \frac{\mu_3}{\mu_1} \mathbb{V}_F(P_2, P_3) \\ & + \sum_{t=1,2} \Phi_t \left[Y_t(i) - (Z_t L_t(i))^{1-\alpha} K_t(i)^\alpha \left(1 - \frac{\kappa}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 \right) \right] \\ & + \sum_{t=1,2} \varphi_t \left[P_t(i) - \left(\frac{Y_t}{Y_t(i)} \right)^{1-\rho} P_t \right], \end{aligned}$$

where we assume that firms discount the future with the same rate as households, $\beta^{t-1} \frac{\mu_t}{\mu_1}$, where μ_t is the household budget constraint penalty price. Further, we assume that adjusting prices creates quadratic costs. Production follows a Cobb-Douglas production function and each firm produces one variety of a differentiated good where the demand equation is given as the second constraint.

Derive the first-order conditions for $Y_t(i)$ and $P_t(i)$ and solve for a Phillips curve representation.

Hint: Be careful, $P_t(i)$ appears not only in the current period!