Quantitative Dynamic Macroeconomics

– Assignment 02: A dynamic CiA model –

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Suppose there is an economy with many households that live infinitely many periods. Each household receives utility out of consumption according to $u(C_t) = \frac{C_t^{1-\sigma}-1}{1-\sigma}$ and receives disutility out of labor supply according to $v(L_t) = \frac{\varphi}{1+\nu} L_t^{1+\nu}$. There is a period discount factor $\beta < 1$. Households have to hold cash-in-advance to be able to consume goods, $M_t = \kappa P_t C_t$, where $\kappa \leq 1$ is money velocity, which indicates for values below one that each unit of cash can be used more than once. This eases the cash-in-advance constraint somewhat. The household looses a share of his money between any two periods defined by δ_M, hence money at the beginning of next period is $\frac{M_{t+1}}{1-\delta_M}$. Alternatively, the household can buy bonds that pay one unit next period at the price of $q_t \leq 1$. Any increase in the money supply is rebated to the household by net transfers, T_t, at the end of the period. The intertemporal budget constraint sums up the different elements of income and expenditure mentioned above.

Exercise 1 Setting up and solving a dynamic model

Set up the intertemporal Lagrangian of the household optimization problem described above and derive its first-order conditions. Apply an appropriate change-in-variables to render the FOCs stationary. *Hint: Check under which conditions marginal utility is constant across time.*

Exercise 2 Change-in-variables and economic intuition

Summarize the FOCs as derived in exercise 1 as far as possible and give a short economic interpretation of the reduced-form equation(s).

Exercise 3 General equilibrium and balanced growth path

Suppose that goods markets always clear in general equilibrium, $C_t = Z_t L_t$, and that the cash-in-advance constraint always binds, $M_t = \kappa P_t C_t$. Assume further that the money stock grows according to $\bar{M}_{t+1} = (1+\tau)(1-\delta_M)\bar{M}_t$, money markets always clear in general equilibrium,

$M_t = \bar{M}_t$, and productivity grows according to $Z_{t+1} = (1 + g_{t+1})Z_t$. For labor to be constant across time and the economy being on its *balanced growth path*, we assume $\sigma = 1$ *(take this as given, you do not have to show it!)*. Derive the dynamic general equilibrium equations in labor and the nominal interest rate and solve for their balanced growth path (steady-state) analitically.

Exercise 4 Negative Interest Rates and Replacing Lost Money

Discuss the impact of the following two statements in light of the model:

1. Money authorities cannot set interest rates below zero as households would exchange all their bonds for cash.
2. If the money authority replaces the average money lost across households lump-sum back to the households, the economy would converge to the same equilibrium as if no money would be lost.