(a) From the above PDF we can determine the value of c by integrating the PDF and setting it equal to 1, yielding

$$\int_0^2 cx\,dx = 2c = 1. \tag{2}$$

Therefore $c = 1/2$.

(b) $\mathrm{P}[0 \leq X \leq 1] = \int_0^1 \frac{x}{2}\,dx = 1/4$.

(c) $\mathrm{P}[-1/2 \leq X \leq 1/2] = \int_0^{1/2} \frac{x}{2}\,dx = 1/16$.

(d) The CDF of X is found by integrating the PDF from 0 to x.

$$F_X(x) = \int_0^x f_X(x')\,dx' = \begin{cases} 0 & x < 0, \\ x^2/4 & 0 \leq x \leq 2, \\ 1 & x > 2. \end{cases} \tag{3}$$

Problem 4.3.2 Solution

From the CDF, we can find the PDF by direct differentiation. The CDF and corresponding PDF are

$$F_X(x) = \begin{cases} 0 & x < -1, \\ (x+1)/2 & -1 \leq x \leq 1, \\ 1 & x > 1, \end{cases} \tag{1}$$

$$f_X(x) = \begin{cases} 1/2 & -1 \leq x \leq 1, \\ 0 & \text{otherwise}. \end{cases} \tag{2}$$

Problem 4.3.3 Solution

We find the PDF by taking the derivative of $F_U(u)$ on each piece that $F_U(u)$ is