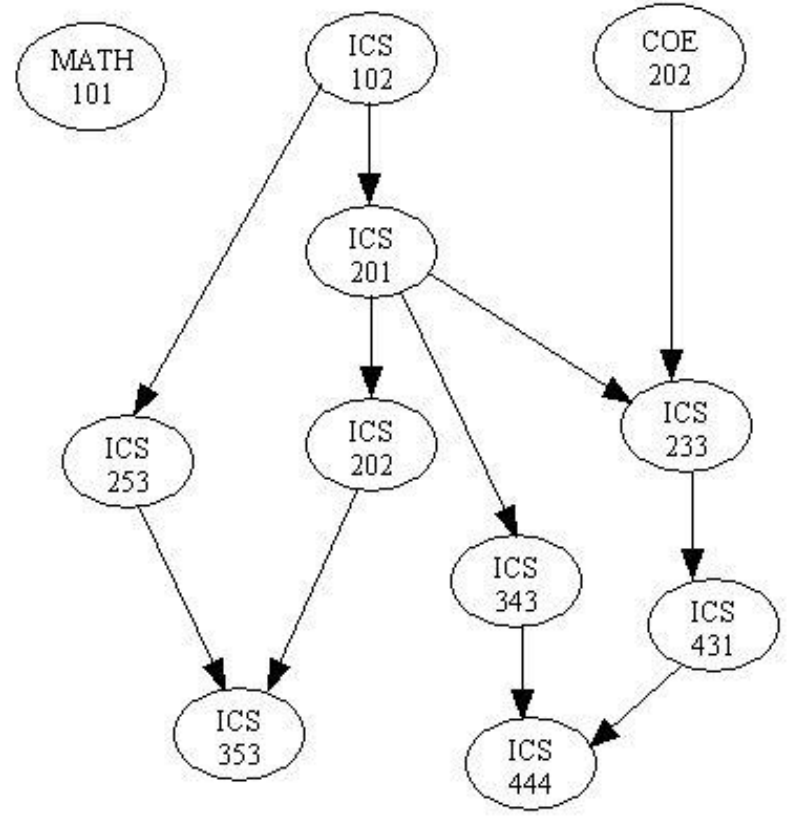
Topological sort

Motivation

Given a set of tasks with dependencies,

is there an order in which we can complete the tasks?

Cycles in dependencies can cause issues...



Definition of topological sorting

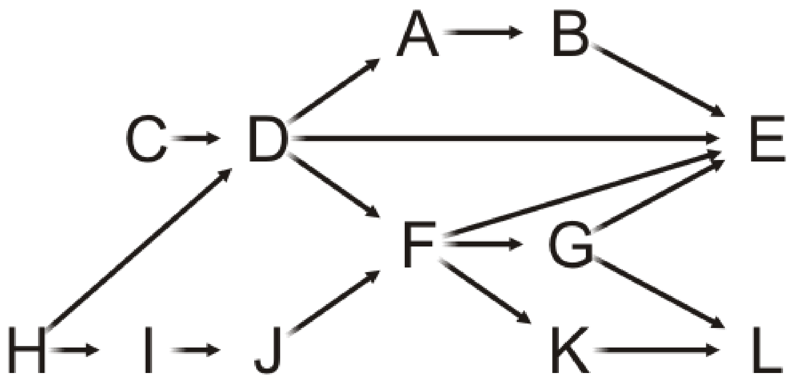
A topological sorting of the vertices in a DAG is an ordering

$$v_1, v_2, v_3, \ldots, v_{|V|}$$

such that v_j appears before v_k if there is a path from v_j to v_k

Given this DAG, a topological sort is

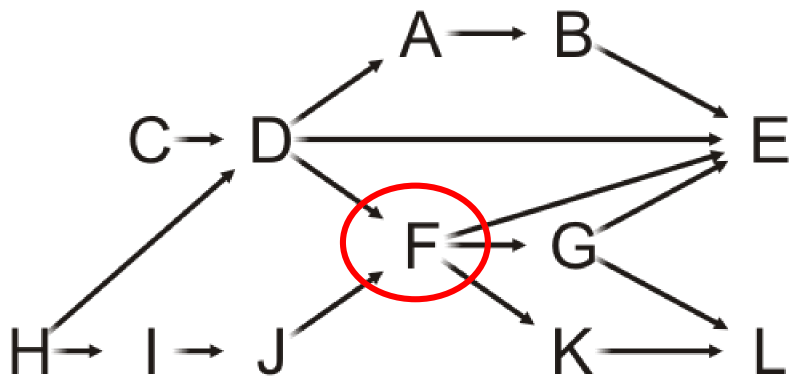
H, C, I, D, J, A, F, B, G, K, E, L



Example

For example, there are paths from H, C, I, D and J to F, so all these must come before F in a topological sort

H, **C**, **I**, **D**, **J**, A, **F**, B, G, K, E, L



Clearly, this sorting need not be unique

Applications

Consider someone is getting ready for a dinner out

He must wear the following:

- jacket, shirt, socks, tie, shoes etc.

There are certain constraints:

- the tie really should go on after the shirt,
- socks are put on before shoes

Applications

C++ header and source files have `#include` statements

- A change to an included file requires a recompilation of the current file
- On a large project, it is desirable to recompile only those source files that depended on those files which changed
- For large software projects, full compilations may take hours

Different scheduling programs in operating system

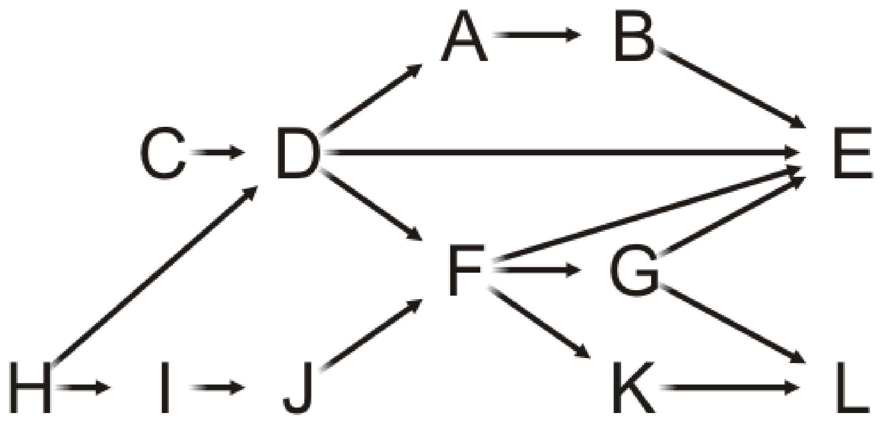
Topological Sort

Idea:

- Given a DAG V, make a copy W and iterate:
  - Find a vertex v in W with in-degree zero
  - Let v be the next vertex in the topological sort
  - Continue iterating with the vertex-induced sub-graph $W \setminus \{v\}$

On this graph, iterate the following $|V| = 12$ times

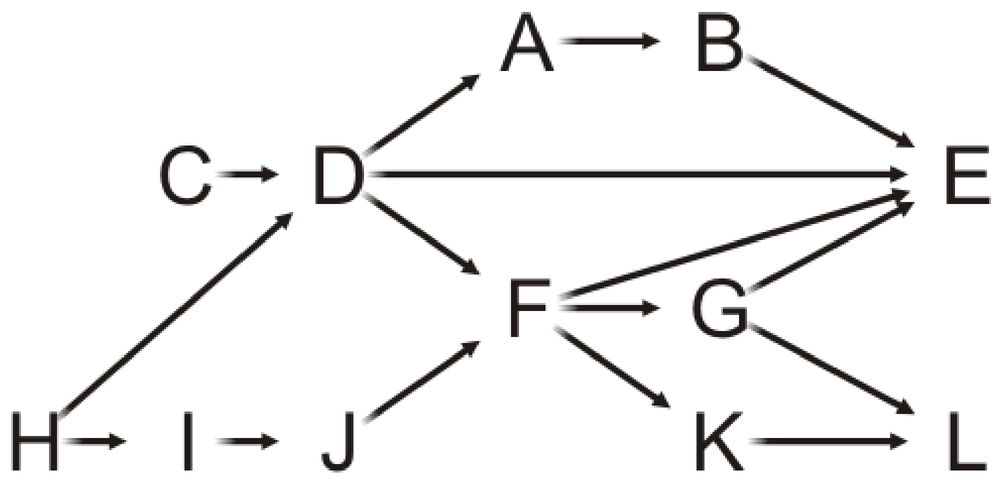
Choose a vertex v that has in-degree zero
Let v be the next vertex in our topological sort
Remove v and all edges connected to it



Example

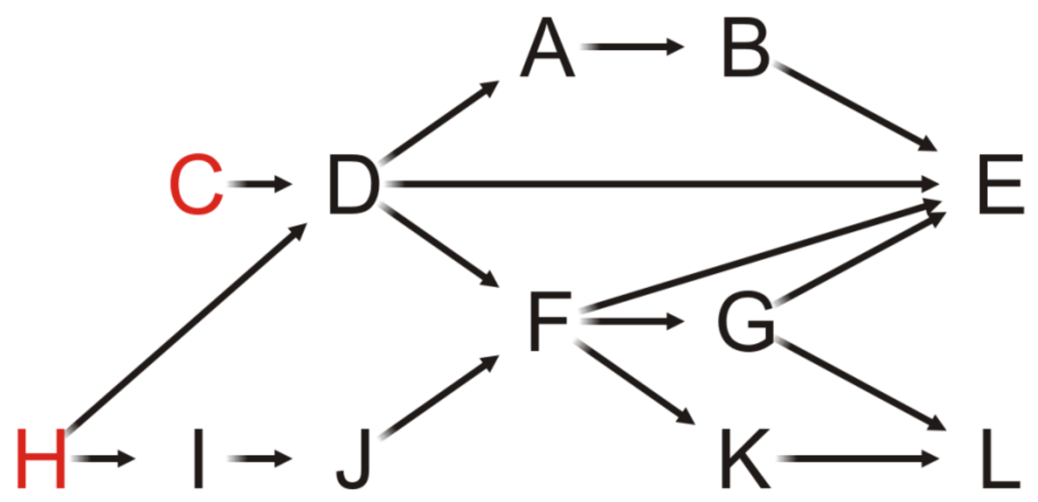
Let’s step through this algorithm with this example

- Which task can we start with?



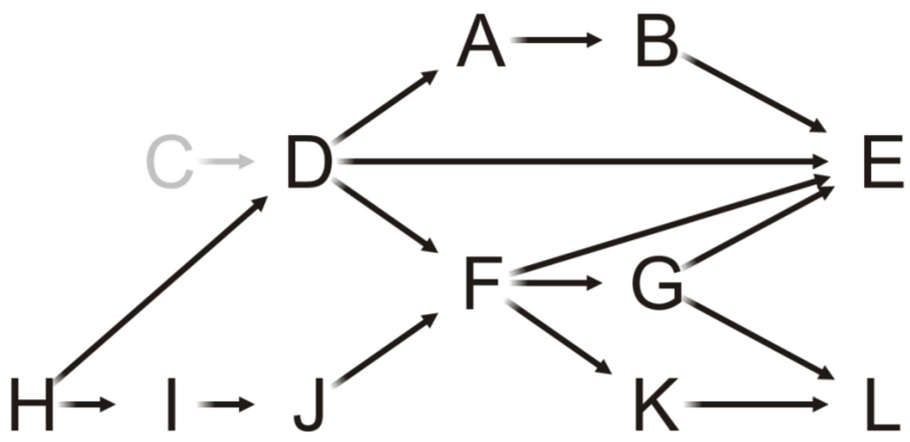
Example

Of Tasks C or H, choose Task C



Example

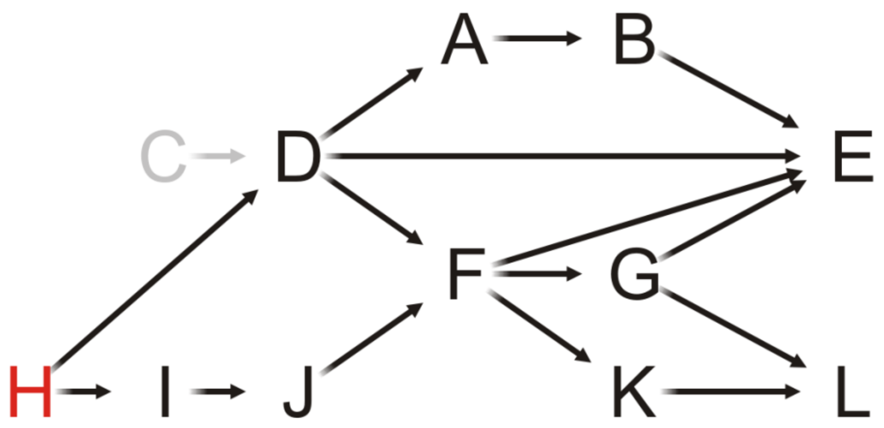
Having completed Task C, which vertices have in-degree zero?



C

Example

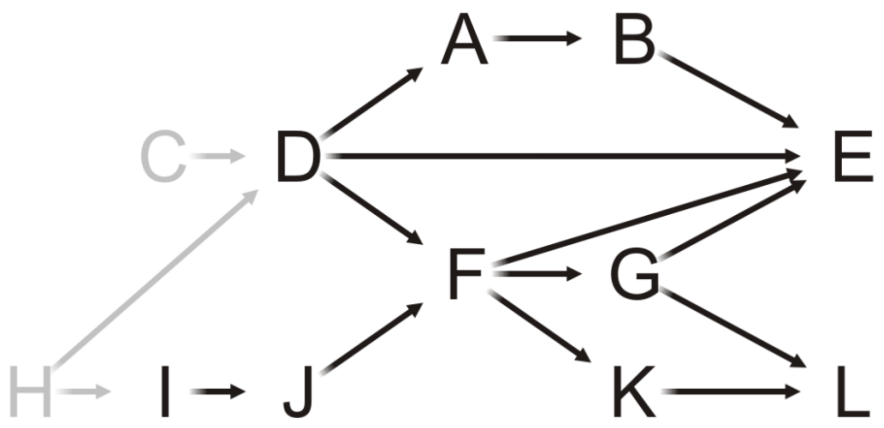
Only Task H can be completed, so we choose it



C

Example

Having removed H, what is next?

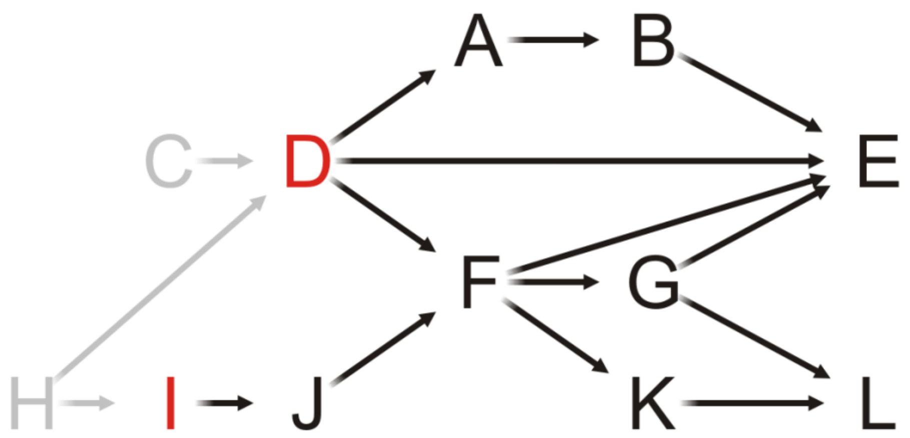


C, H

Example

Both Tasks D and I have in-degree zero

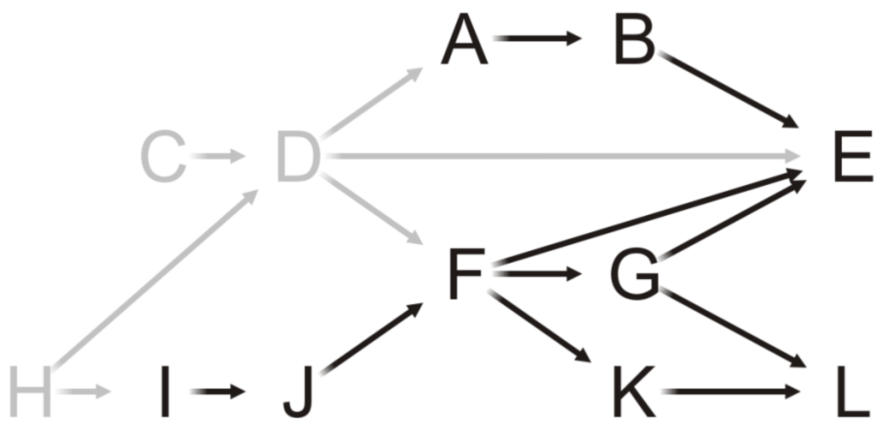
- Let us choose Task D



C, H

Example

We remove Task D, and now?

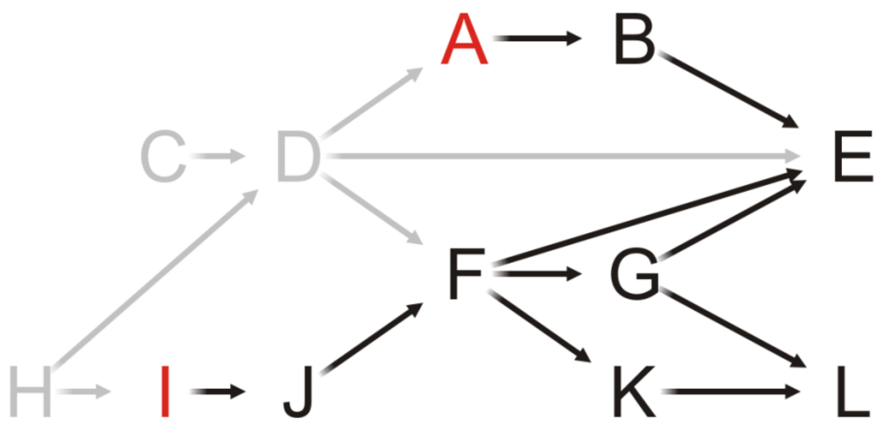


C, H, D

Example

Both Tasks A and I have in-degree zero

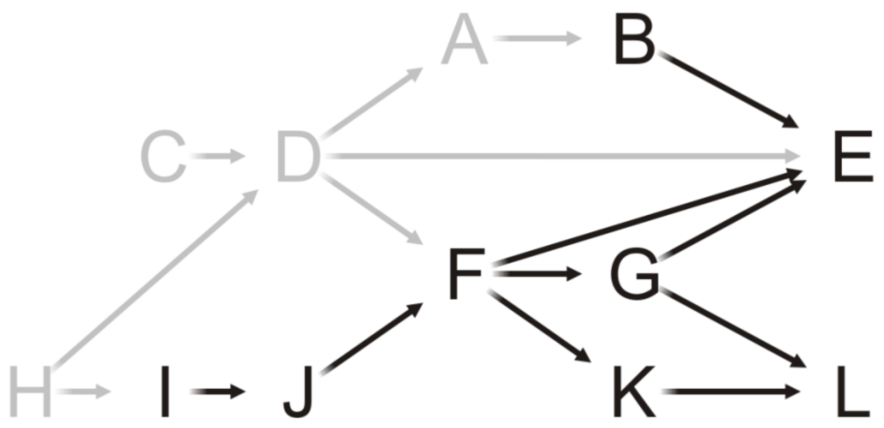
- Let's choose Task A



C, H, D

Example

Having removed A, what now?

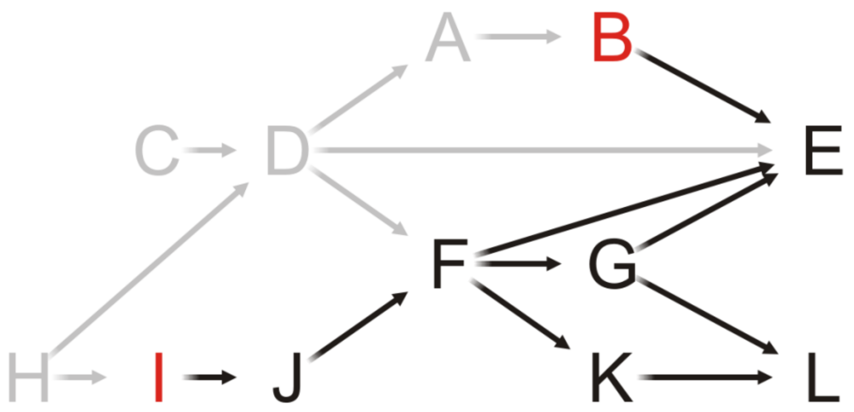


C, H, D, A

Example

Both Tasks B and I have in-degree zero

- Choose Task B

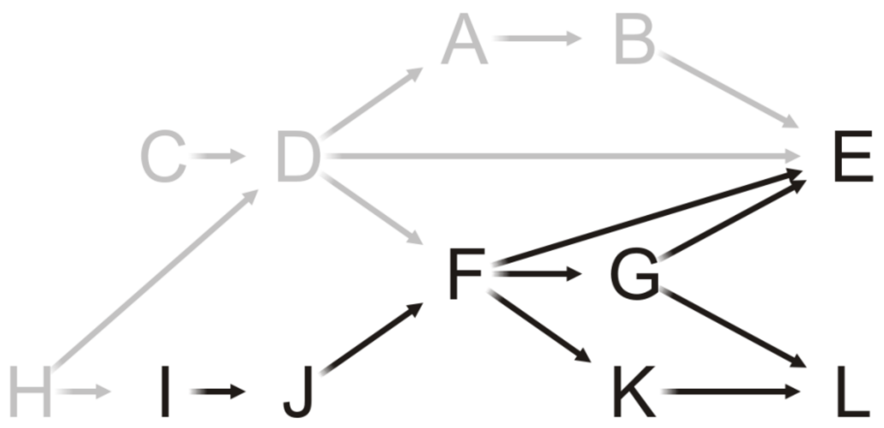


C, H, D, A

Example

Removing Task B, we note that Task E still has an in-degree of two

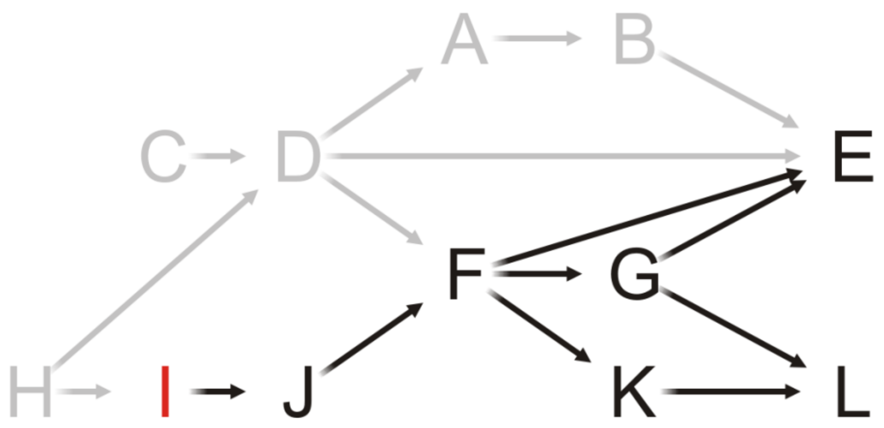
- Next?



C, H, D, A, B

Example

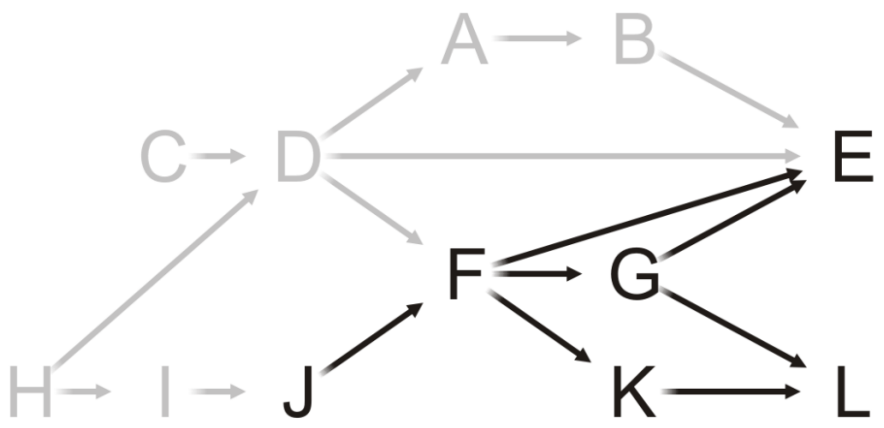
As only Task I has in-degree zero, we choose it



C, H, D, A, B

Example

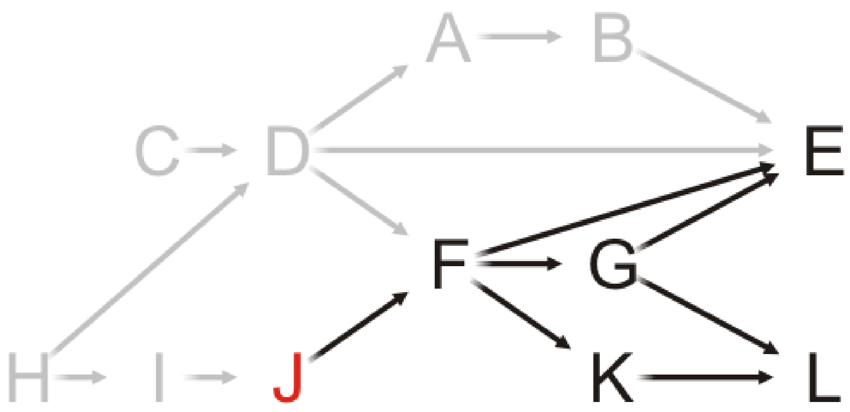
Having completed Task I, what now?



C, H, D, A, B, I

Example

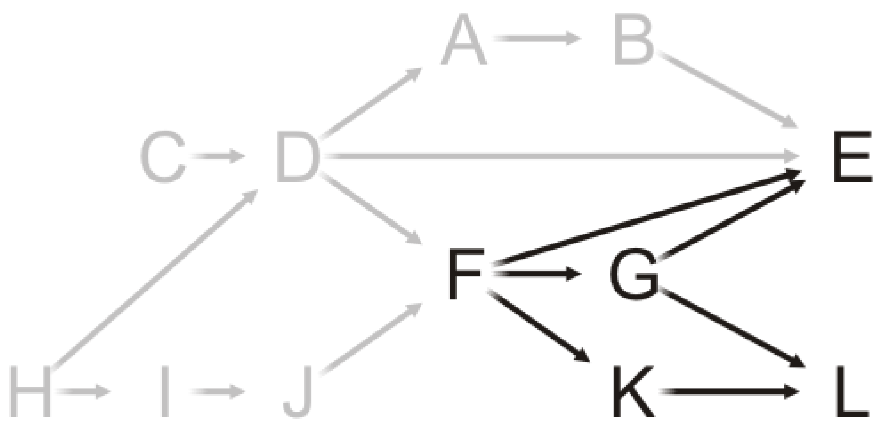
Only Task J has in-degree zero: choose it



C, H, D, A, B, I

Example

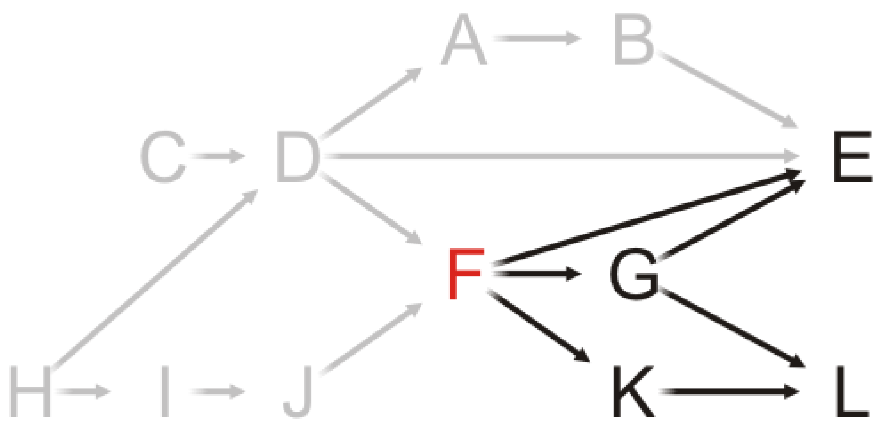
Having completed Task J, what now?



C, H, D, A, B, I, J

Example

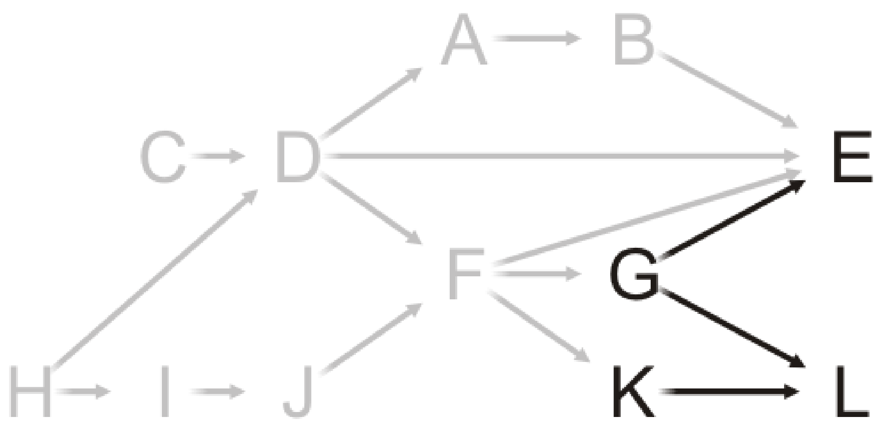
Only Task F can be completed, so choose it



C, H, D, A, B, I, J

Example

What choices do we have now?

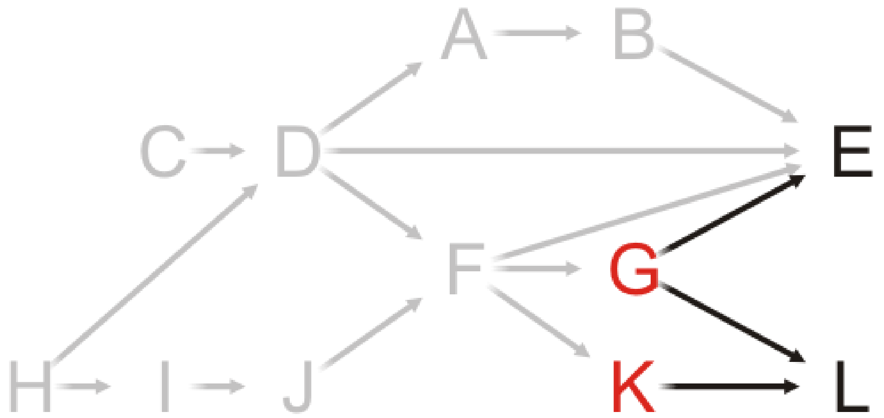


C, H, D, A, B, I, J, F

Example

We can perform Tasks G or K

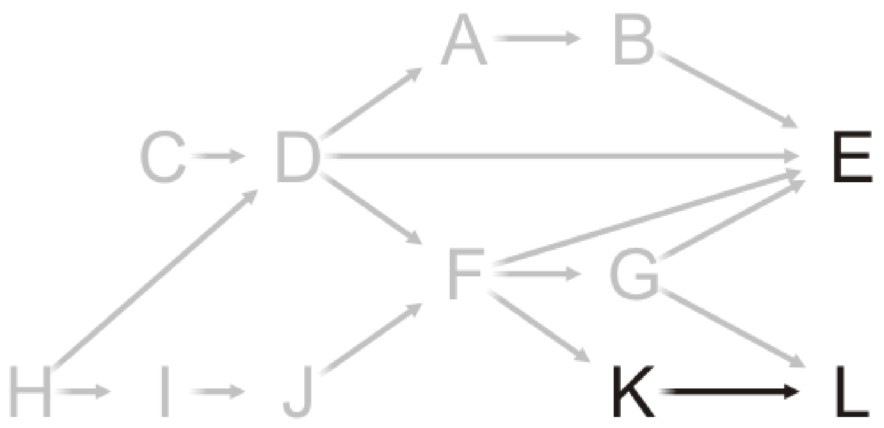
- Choose Task G



C, H, D, A, B, I, J, F

Example

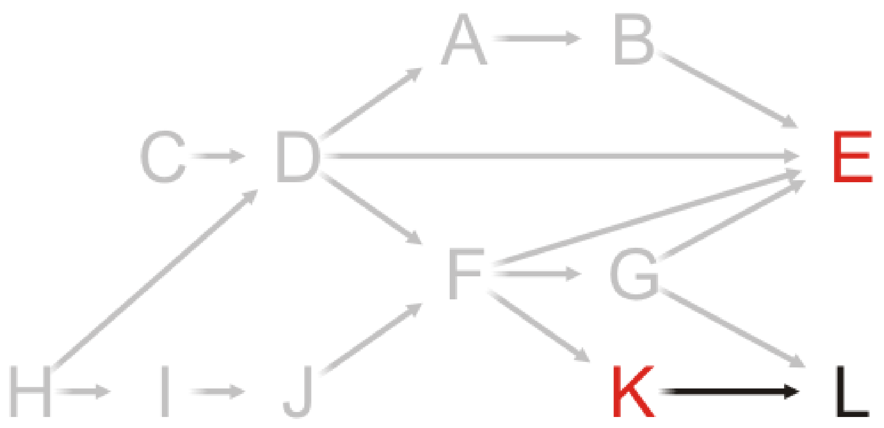
Having removed Task G from the graph, what next?



C, H, D, A, B, I, J, F, G

Example

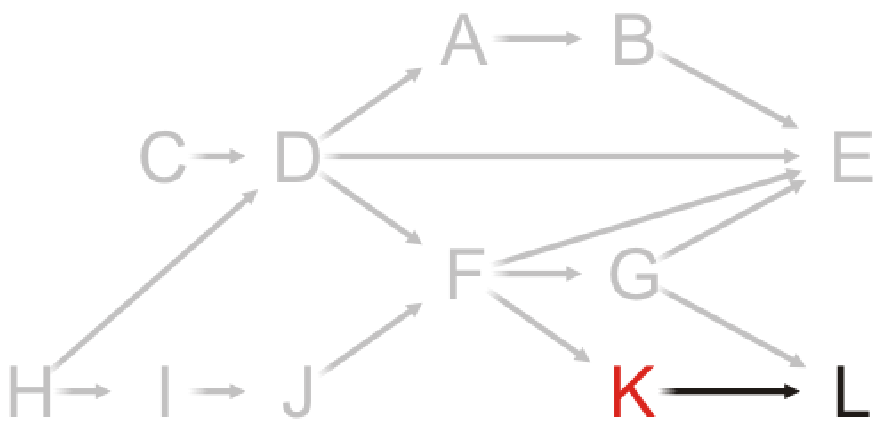
Choosing between Tasks E and K, choose Task E



C, H, D, A, B, I, J, F, G

Example

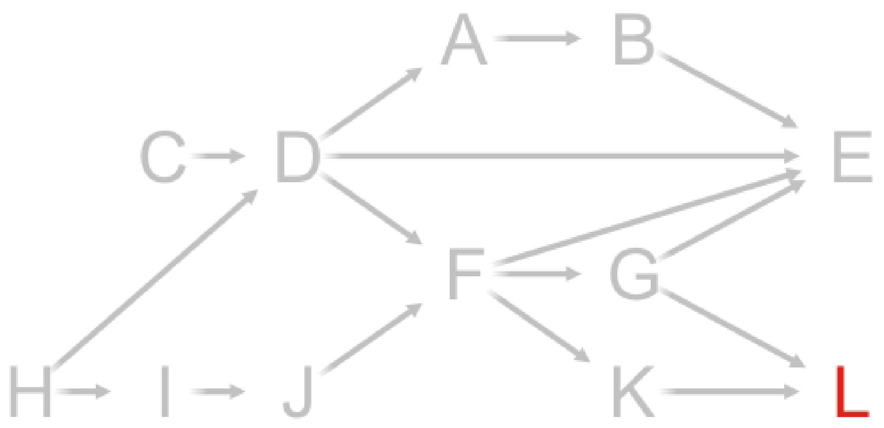
At this point, Task K is the only one that can be run



C, H, D, A, B, I, J, F, G, E

Example

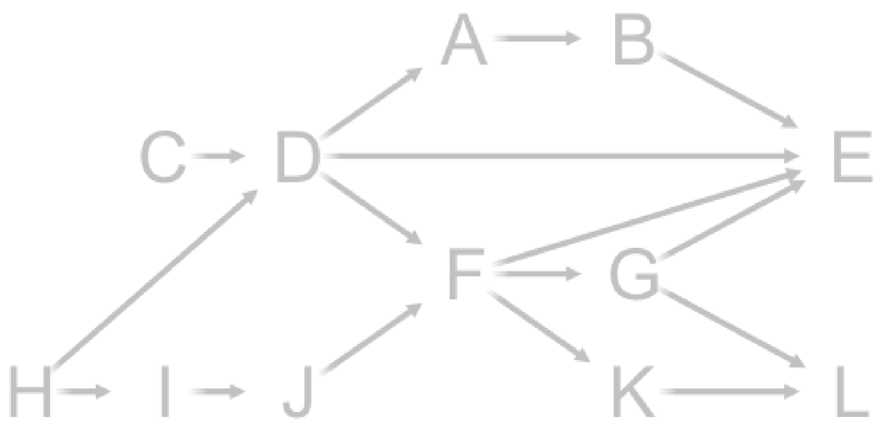
And now that both Tasks G and K are complete, we can complete Task L



C, H, D, A, B, I, J, F, G, E, K

Example

There are no more vertices left

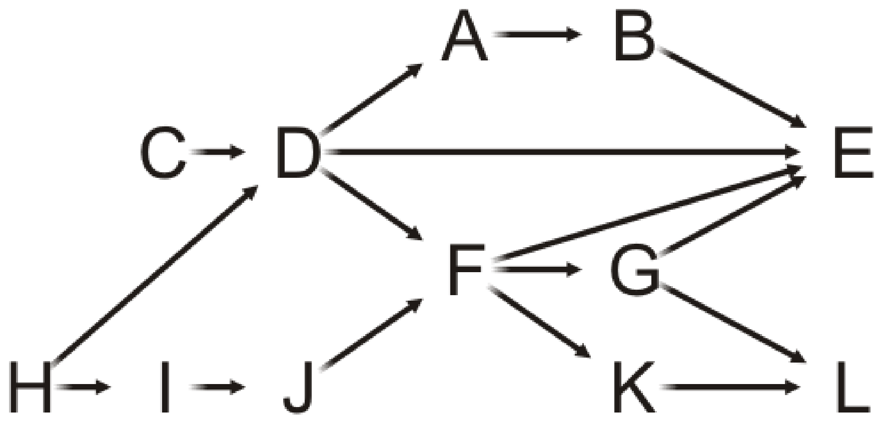


C, H, D, A, B, I, J, F, G, E, K, L

Example

Thus, one possible topological sort would be:

C, H, D, A, B, I, J, F, G, E, K, L

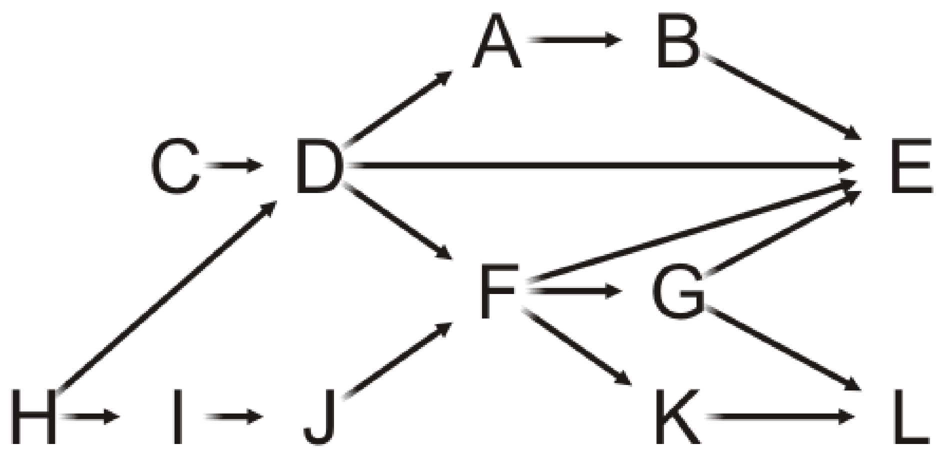


Example

Note that topological sorts need not be unique:

C, H, D, A, B, I, J, F, G, E, K, L

H, I, J, C, D, F, G, K, L, A, B, E

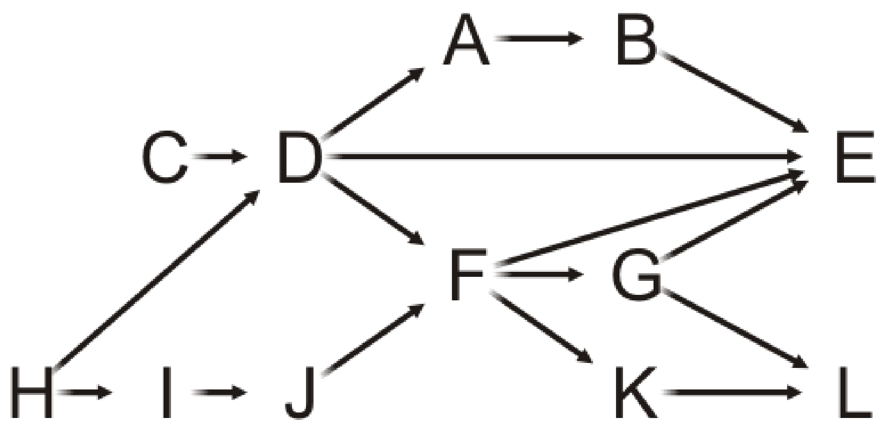


Analysis

What are the tools necessary for a topological sort?

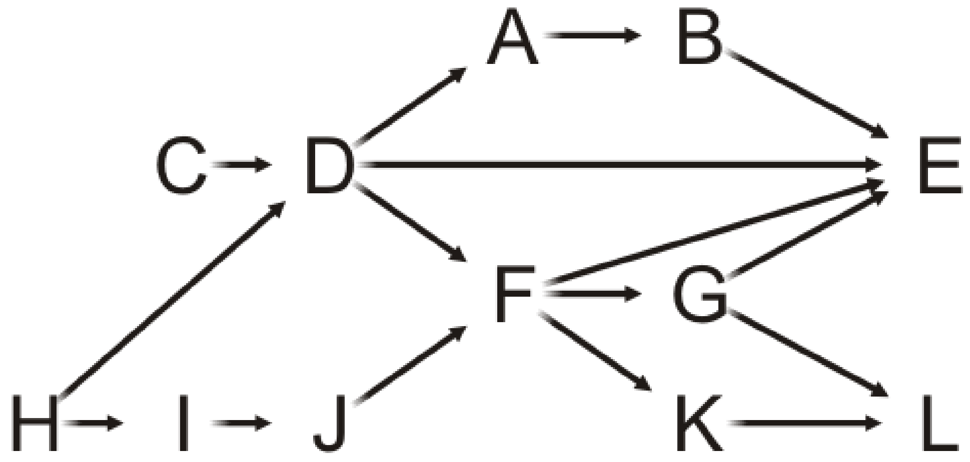
- We must know and be able to update the in-degrees of each of the vertices
- We could do this with a table of the in-degrees of each of the vertices
- This requires $\Theta(|V|)$ memory

A	1
B	1
C	0
D	2
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2



Analysis

We must iterate at least $|V|$ times, so the run-time must be $\Omega(|V|)$

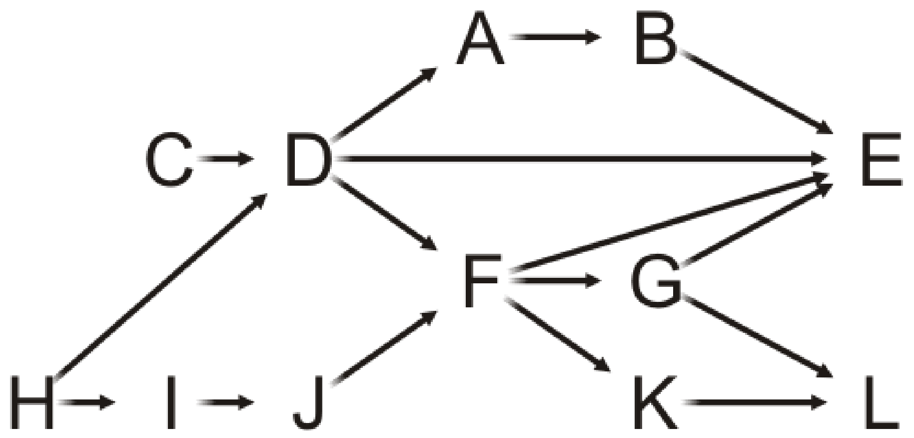


A	1
B	1
C	0
D	2
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2

Analysis

We need to find vertices with in-degree zero

- We could loop through the array with each iteration
- The run time would be $O(|V|^2)$

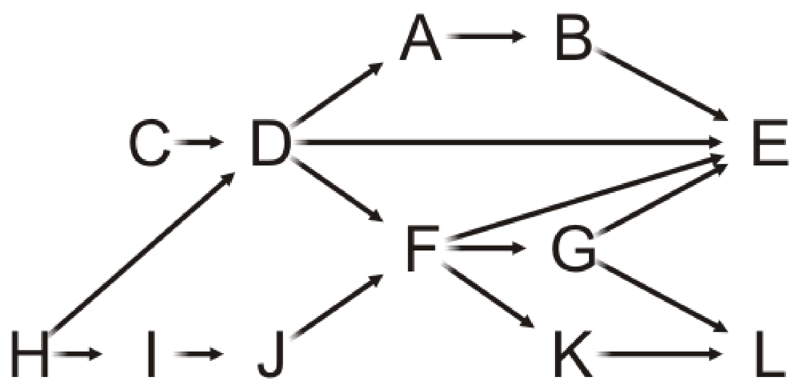


A	1
B	1
C	0
D	2
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2

Analysis

What did we do with tree traversals?

- Use a queue (or other container) to temporarily store those vertices with in-degree zero
- Each time the in-degree of a vertex is decremented to zero, push it onto the queue



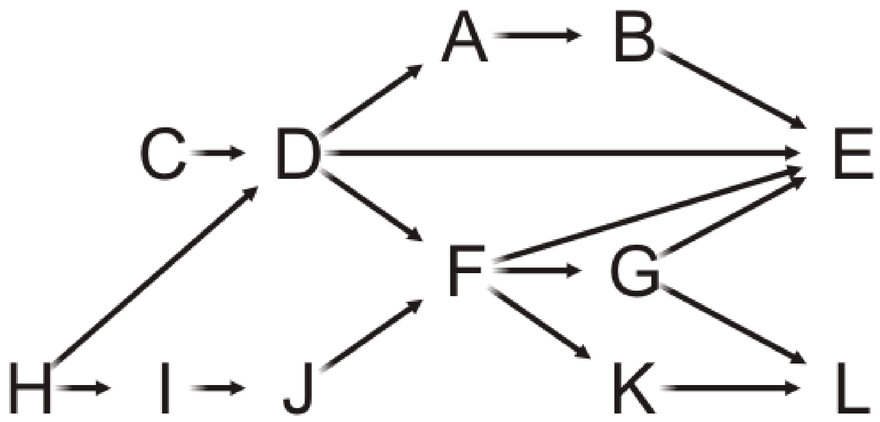
A	1
B	1
C	0
D	2
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2

Analysis

What are the run times associated with the queue?

- Initially, we must scan through each of the vertices: $\Theta(|V|)$
- For each vertex, we will have to push onto and pop off the queue once, also $\Theta(|V|)$

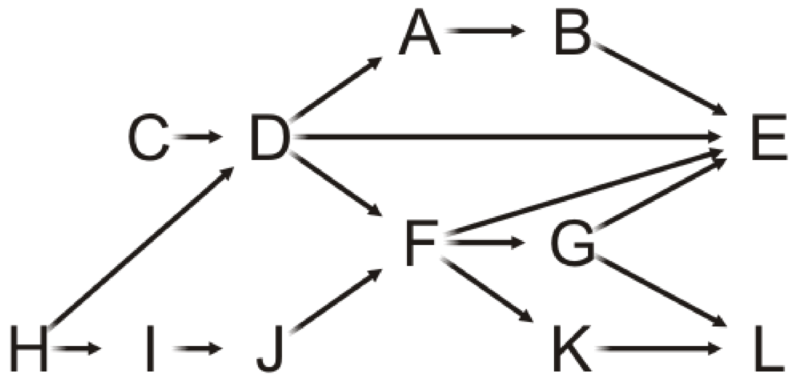
A	1
B	1
C	0
D	2
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2



Analysis

Finally, each value in the in-degree table is associated with an edge

- Here, $|E| = 16$
- Each of the in-degrees must be decremented to zero
- The run time of these operations is $\Omega(|E|)$
- If we are using an adjacency matrix: $\Theta(|V|^2)$
- If we are using an adjacency list: $\Theta(|E|)$



A	1
B	1
C	0
D	2
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2 +

16

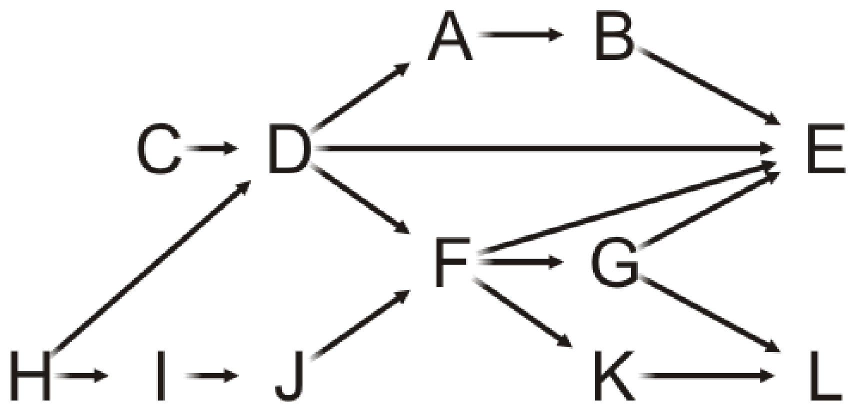
Analysis

Therefore, the run time of a topological sort is:

$\Theta(|V| + |E|)$ if we use an adjacency list

$\Theta(|V|^2)$ if we use an adjacency matrix

and the memory requirements is $\Theta(|V|)$

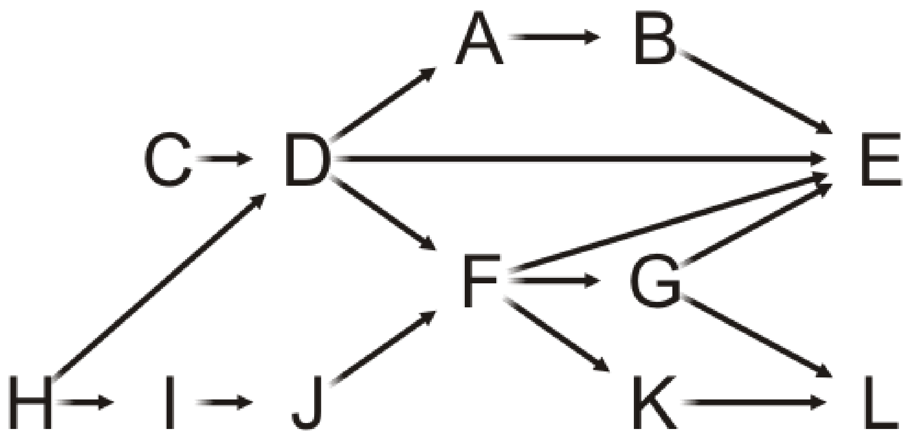


A	1
B	1
C	0
D	2
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2

Analysis

What happens if at some step, all remaining vertices have an in-degree greater than zero?

Consequence: we now have an $\Theta(|V| + |E|)$ algorithm for determining if a graph has a cycle



Implementation

Thus, to implement a topological sort:

- Allocate memory for and initialize an array of in-degrees
- Create a queue and initialize it with all vertices that have in-degree zero

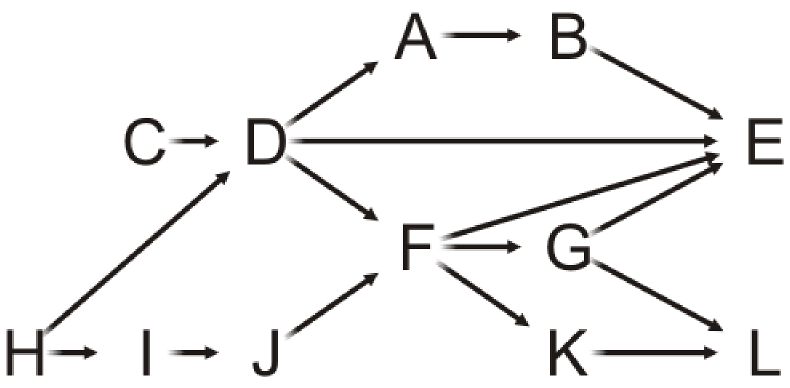
While the queue is not empty:

- Pop a vertex from the queue
- Decrement the in-degree of each neighbor
- Those neighbors whose in-degree was decremented to zero are pushed onto the queue

Example

With the previous example, we initialize:

- The array of in-degrees
- The queue



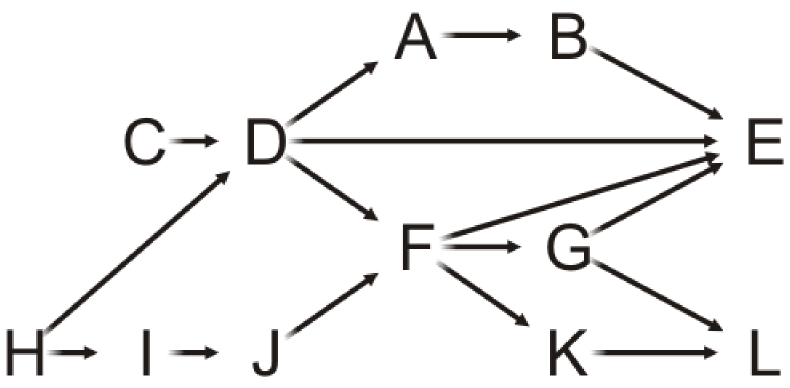
A	1
B	1
C	0
D	2
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2

Queue:

The queue is empty

Example

Stepping through the table, push all source vertices into the queue

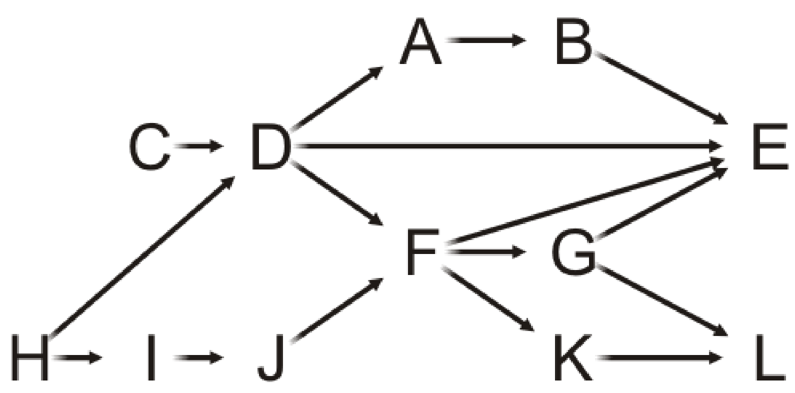


A	1
B	1
C	0
D	2
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2

The queue is empty

Example

Stepping through the table, push all source vertices into the queue



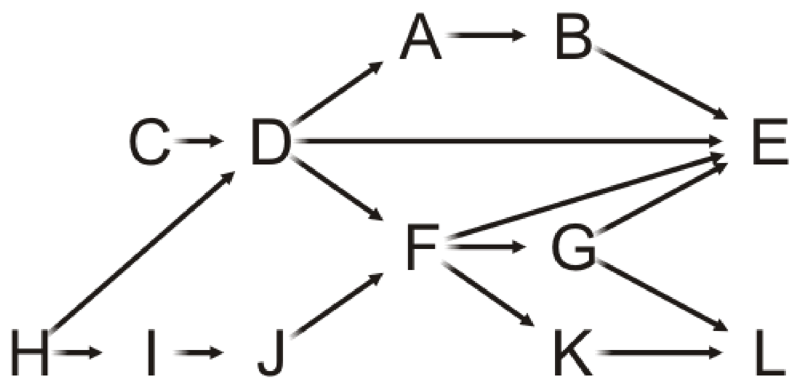
A	1
B	1
C	**0**
D	2
E	4
F	2
G	1
H	**0**
I	1
J	1
K	1
L	2

Queue: | **C** | **H** | | | | | | | | | | |

The queue is empty

Example

Pop the front of the queue



A	1
B	1
C	0
D	2
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2

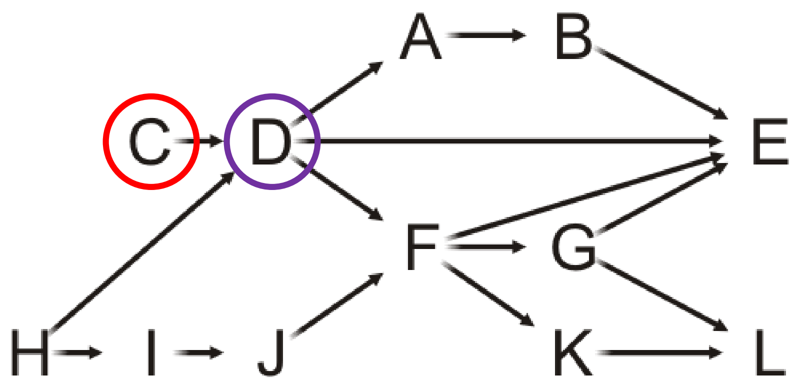
Queue:

C	H										

Example

Pop the front of the queue

- C has one neighbor: D



A	1
B	1
C	**0**
D	**2**
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2

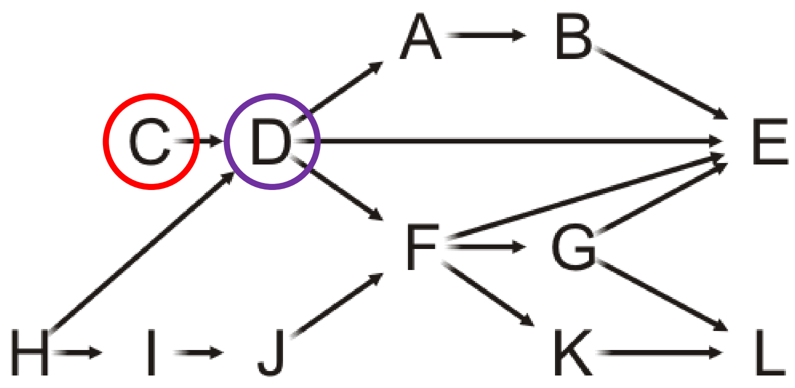
Queue:

C	H										

Example

Pop the front of the queue

- C has one neighbor: D
- Decrement its in-degree

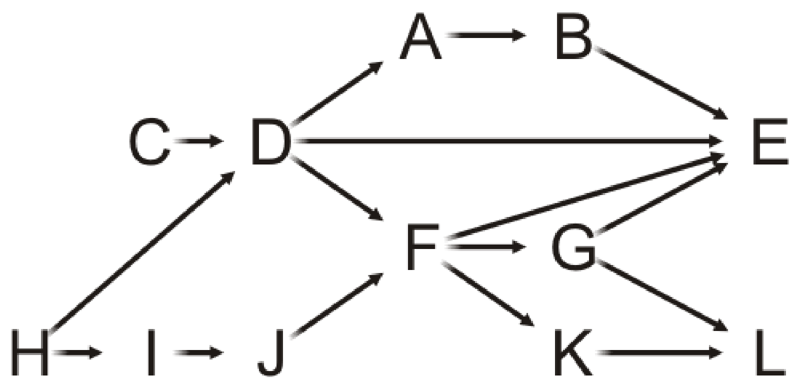


A	1
B	1
C	**0**
D	**1**
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2

Queue: | C | H | | | | | | | | | | |

Example

Pop the front of the queue



A	1
B	1
C	0
D	1
E	4
F	2
G	1
H	0
I	1
J	1
K	1
L	2

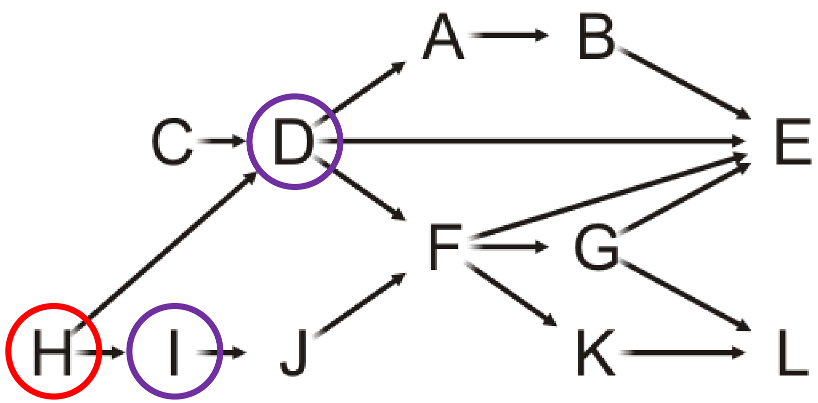
Queue:

C	H										

Example

Pop the front of the queue

- H has two neighbors: D and I



A	1
B	1
C	0
D	**1**
E	4
F	2
G	1
H	**0**
I	**1**
J	1
K	1
L	2

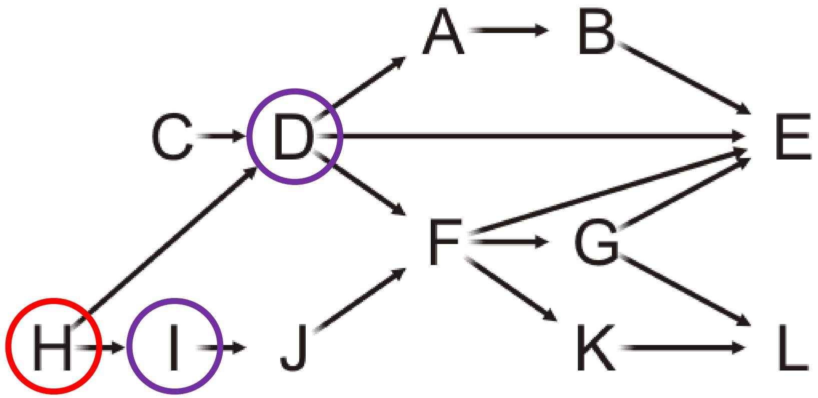
Queue:

C	H										

Example

Pop the front of the queue

- H has two neighbors: D and I
- Decrement their in-degrees



Queue:

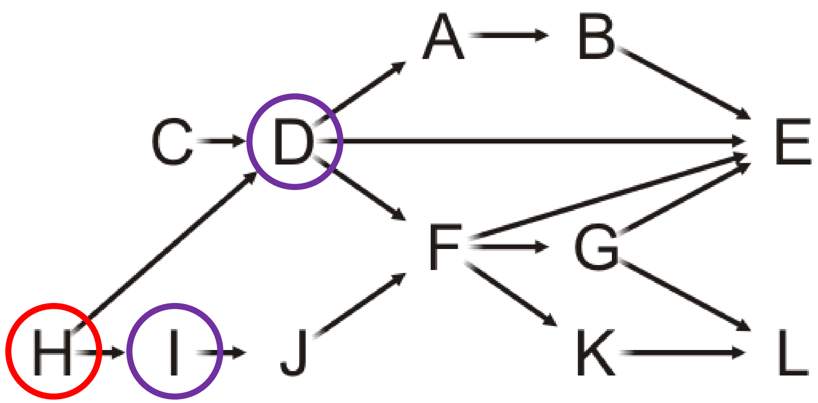
C	H										

A	1
B	1
C	0
D	**0**
E	4
F	2
G	1
H	**0**
I	**0**
J	1
K	1
L	2

Example

Pop the front of the queue

- H has two neighbors: D and I
- Decrement their in-degrees
  - Both are decremented to zero, so push them onto the queue



Queue:

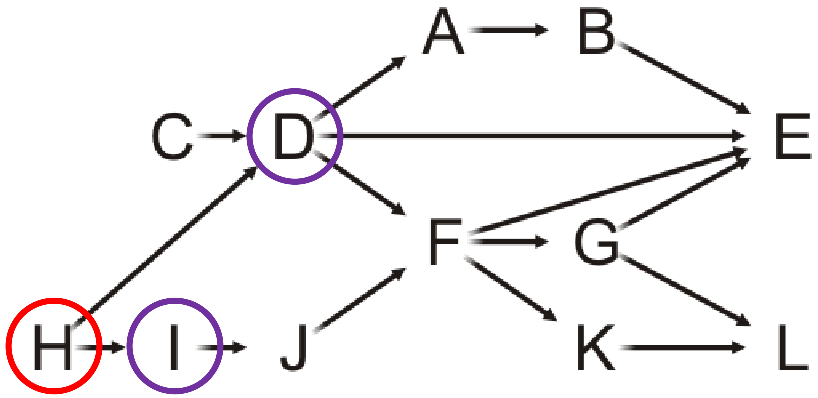
C	H										

A	1
B	1
C	0
D	**0**
E	4
F	2
G	1
H	**0**
I	**0**
J	1
K	1
L	2

Example

Pop the front of the queue

- H has two neighbors: D and I
- Decrement their in-degrees
  - Both are decremented to zero, so push them onto the queue



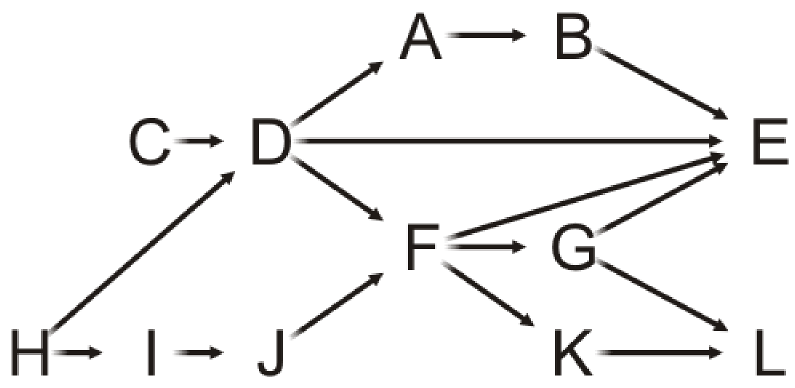
Queue:

C	H	D	I								

A	1
B	1
C	0
D	**0**
E	4
F	2
G	1
H	**0**
I	**0**
J	1
K	1
L	2

Example

Pop the front of the queue



A	1
B	1
C	0
D	0
E	4
F	2
G	1
H	0
I	0
J	1
K	1
L	2

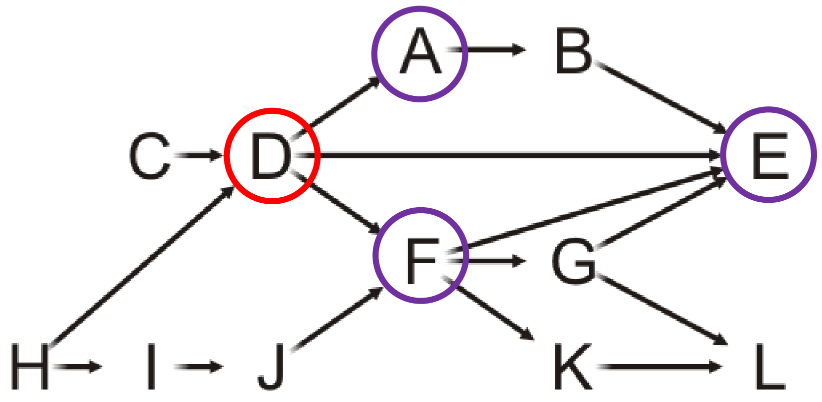
Queue:

C	H	D	I								

Example

Pop the front of the queue

- D has three neighbors: A, E and F

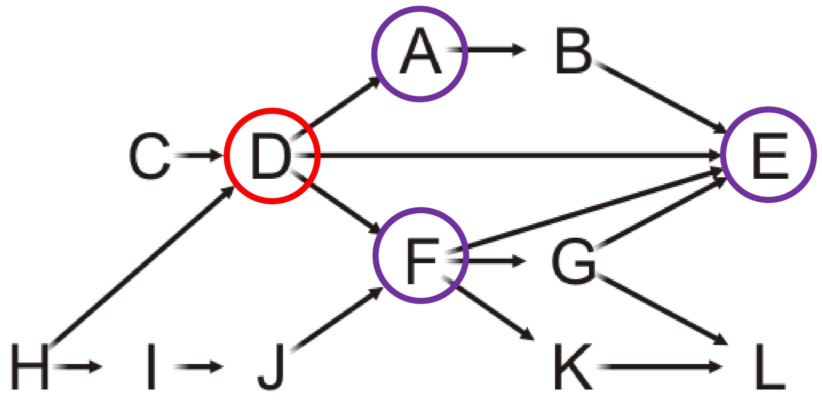


A	1
B	1
C	0
D	0
E	4
F	2
G	1
H	0
I	0
J	1
K	1
L	2

Example

Pop the front of the queue

- D has three neighbors: A, E and F
- Decrement their in-degrees



Queue:

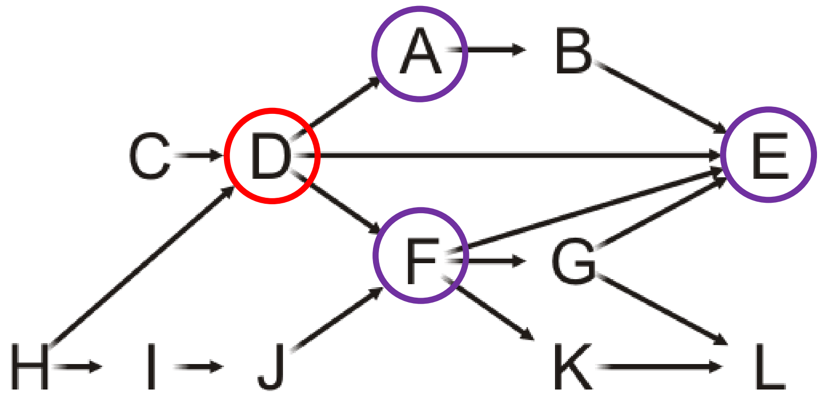
C	H	D	I								

A	**0**
B	1
C	0
D	**0**
E	**3**
F	**1**
G	1
H	0
I	0
J	1
K	1
L	2

Example

Pop the front of the queue

- D has three neighbors: A, E and F
- Decrement their in-degrees
    - A is decremented to zero, so push it onto the queue



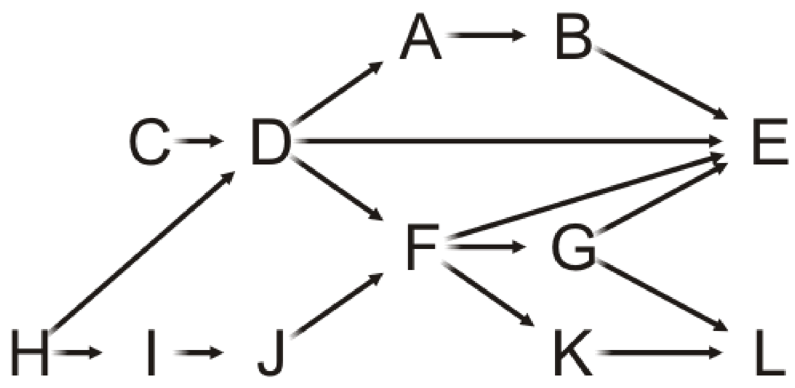
A	0
B	1
C	0
D	0
E	3
F	1
G	1
H	0
I	0
J	1
K	1
L	2

Queue:

C	H	D	I	A							

Example

Pop the front of the queue



A	0
B	1
C	0
D	0
E	3
F	1
G	1
H	0
I	0
J	1
K	1
L	2

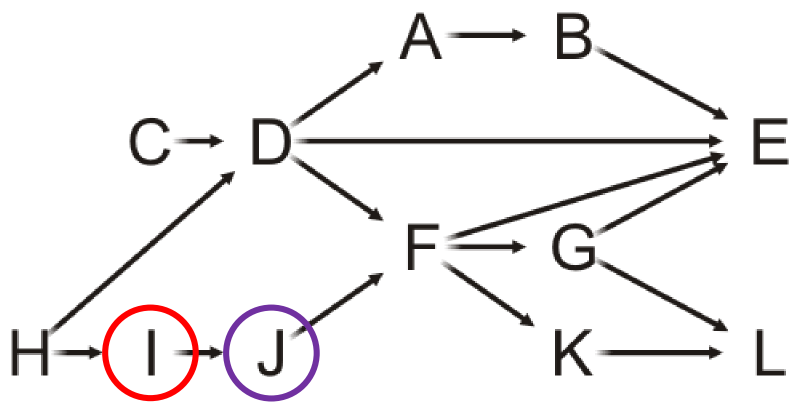
Queue:

C	H	D	I	A							

Example

Pop the front of the queue

- I has one neighbor: J



A	0
B	1
C	0
D	0
E	3
F	1
G	1
H	0
I	**0**
J	**1**
K	1
L	2

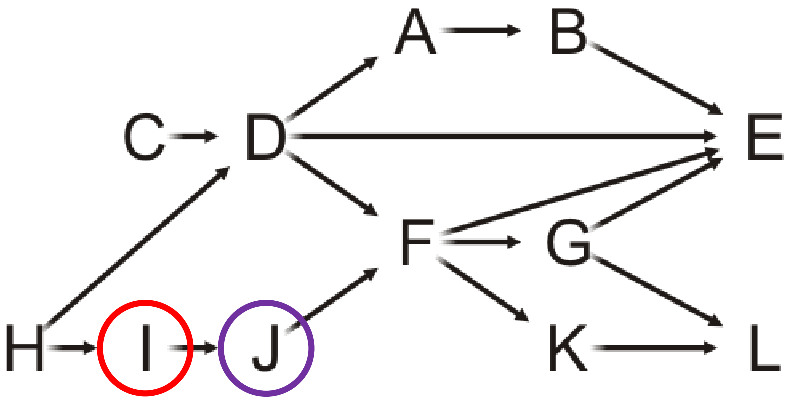
Queue:

C	H	D	I	A							

Example

Pop the front of the queue

- I has one neighbor: J
- Decrement its in-degree



Queue:

C	H	D	I	A							

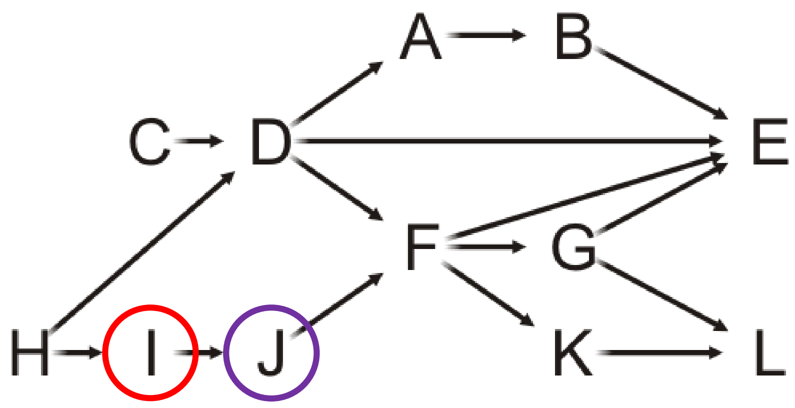
A	0
B	1
C	0
D	0
E	3
F	1
G	1
H	0
I	**0**
J	**0**
K	1
L	2

Example

Pop the front of the queue

- I has one neighbor: J
- Decrement its in-degree
  - J is decremented to zero, so push it onto the queue

A	0
B	1
C	0
D	0
E	3
F	1
G	1
H	0
I	**0**
J	**0**
K	1
L	2

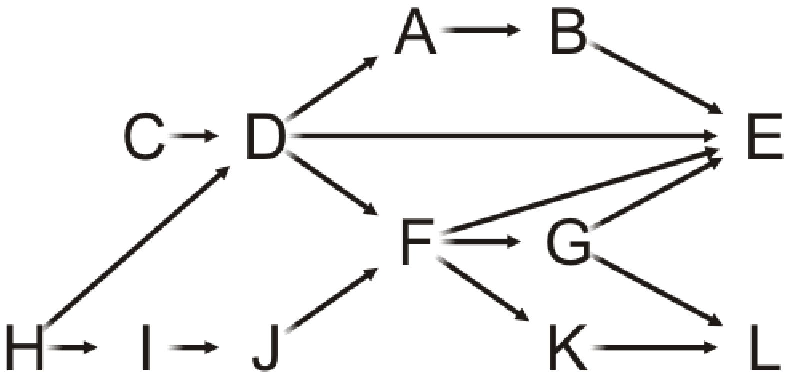


Queue:

C	H	D	I	A	J						

Example

Pop the front of the queue



A	0
B	1
C	0
D	0
E	3
F	1
G	1
H	0
I	0
J	0
K	1
L	2

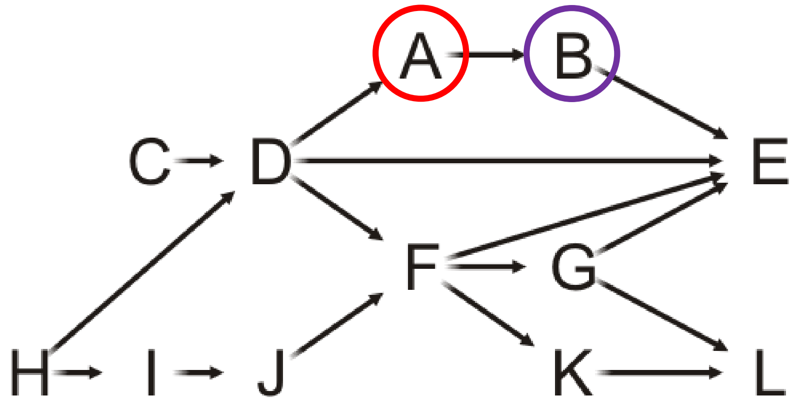
Queue:

C	H	D	I	A	J						

Example

Pop the front of the queue

- A has one neighbor: B



A	0
B	1
C	0
D	0
E	3
F	1
G	1
H	0
I	0
J	0
K	1
L	2

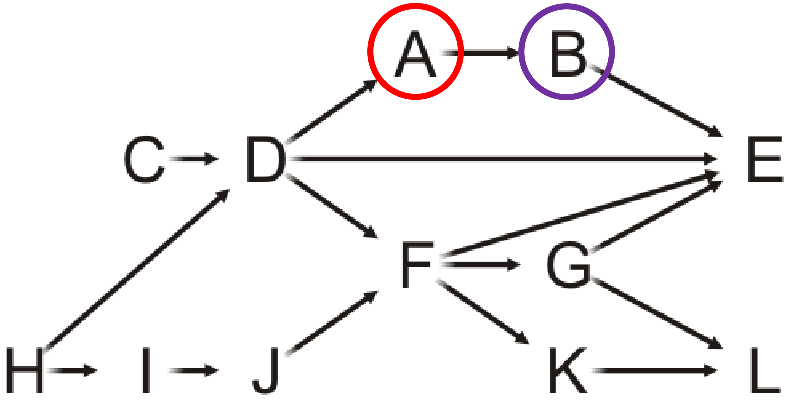
Queue:

C	H	D	I	A	J						

Example

Pop the front of the queue

- A has one neighbor: B
- Decrement its in-degree



Queue:

C	H	D	I	A	J						

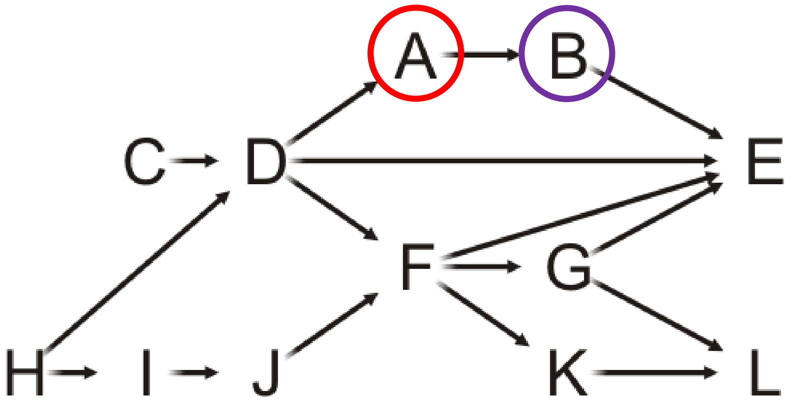
A	0
B	0
C	0
D	0
E	3
F	1
G	1
H	0
I	0
J	0
K	1
L	2

Example

Pop the front of the queue

- A has one neighbor: B
- Decrement its in-degree
  - B is decremented to zero, so push it onto the queue

A	0
B	0
C	0
D	0
E	3
F	1
G	1
H	0
I	0
J	0
K	1
L	2

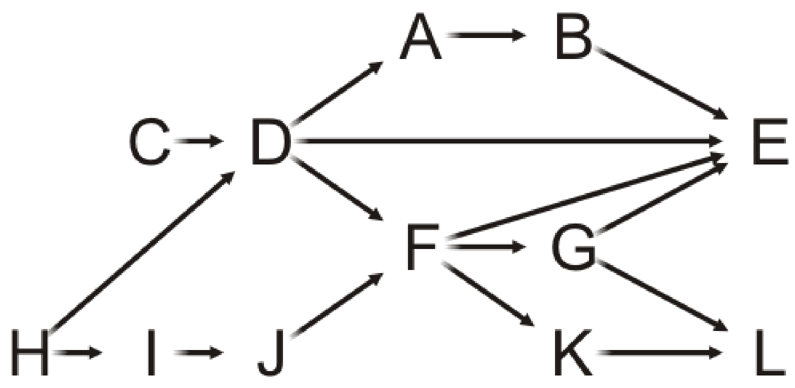


Queue:

C	H	D	I	A	J	B					

Example

Pop the front of the queue



A	0
B	0
C	0
D	0
E	3
F	1
G	1
H	0
I	0
J	0
K	1
L	2

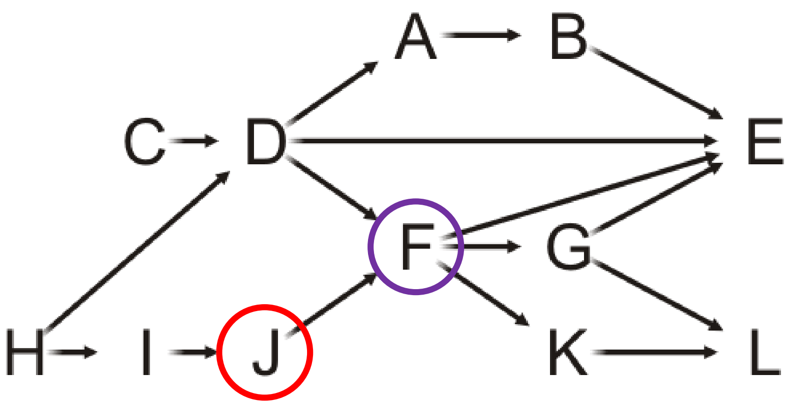
Queue:

C	H	D	I	A	J	B					

Example

Pop the front of the queue

- J has one neighbor: F



A	0
B	0
C	0
D	0
E	3
F	**1**
G	1
H	0
I	0
J	**0**
K	1
L	2

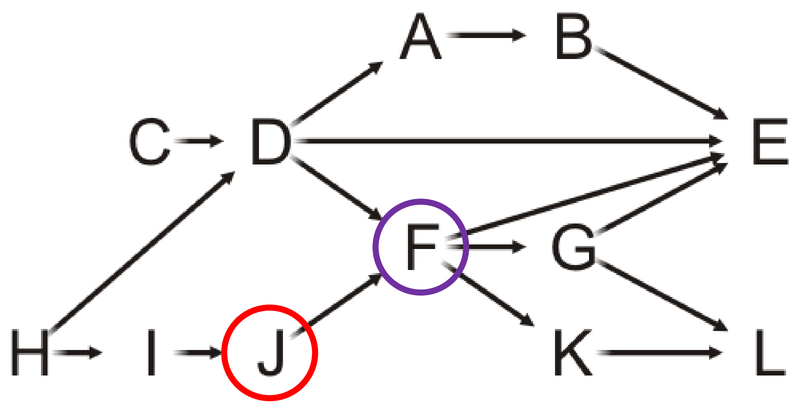
Queue:

C	H	D	I	A	J	B					

Example

Pop the front of the queue

- J has one neighbor: F
- Decrement its in-degree



Queue:

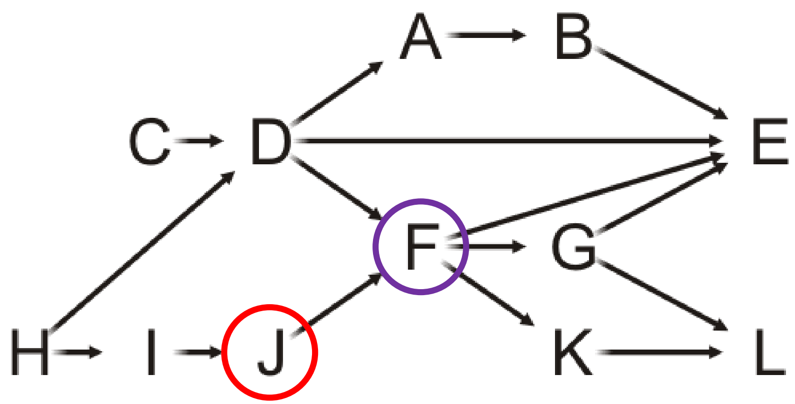
C	H	D	I	A	J	B					

A	0
B	0
C	0
D	0
E	3
F	**0**
G	1
H	0
I	0
J	**0**
K	1
L	2

Example

Pop the front of the queue

- J has one neighbor: F
- Decrement its in-degree
  - F is decremented to zero, so push it onto the queue



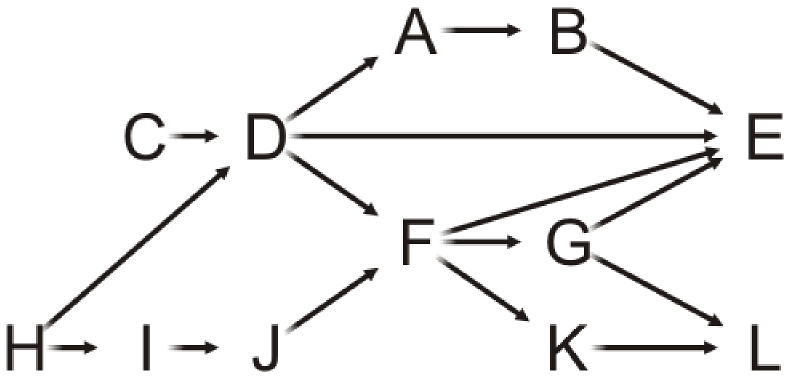
Queue:

C	H	D	I	A	J	B	F				

A	0
B	0
C	0
D	0
E	3
F	**0**
G	1
H	0
I	0
J	**0**
K	1
L	2

Example

Pop the front of the queue



A	0
B	0
C	0
D	0
E	3
F	0
G	1
H	0
I	0
J	0
K	1
L	2

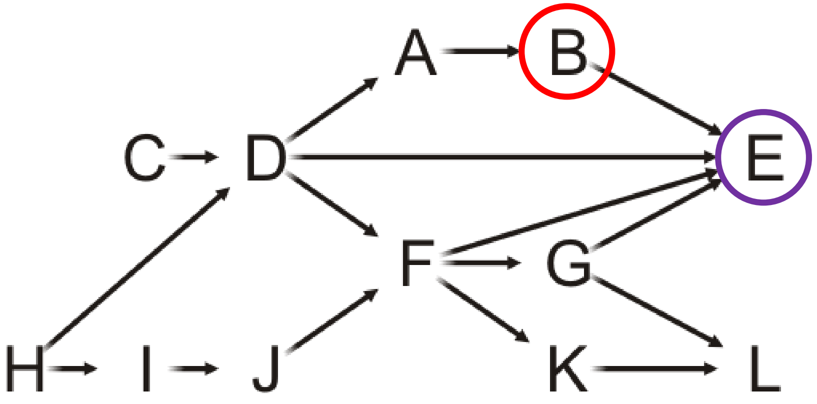
Queue:

C	H	D	I	A	J	B	F				

Example

Pop the front of the queue

- B has one neighbor: E



A	0
B	**0**
C	0
D	0
E	**3**
F	0
G	1
H	0
I	0
J	0
K	1
L	2

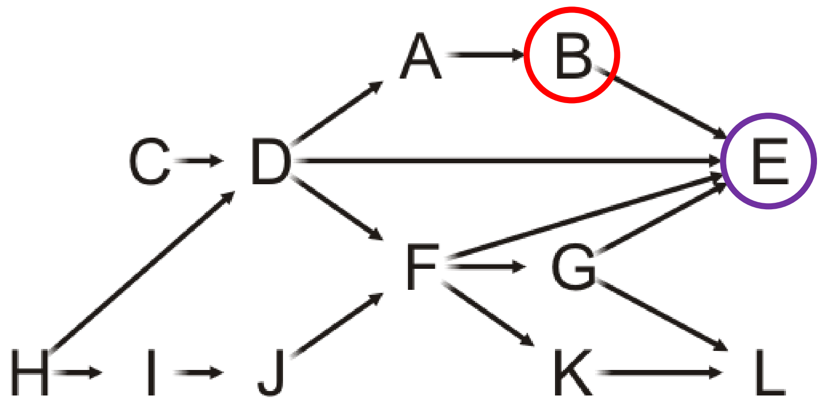
Queue:

C	H	D	I	A	J	B	F				

Example

Pop the front of the queue

- B has one neighbor: E
- Decrement its in-degree



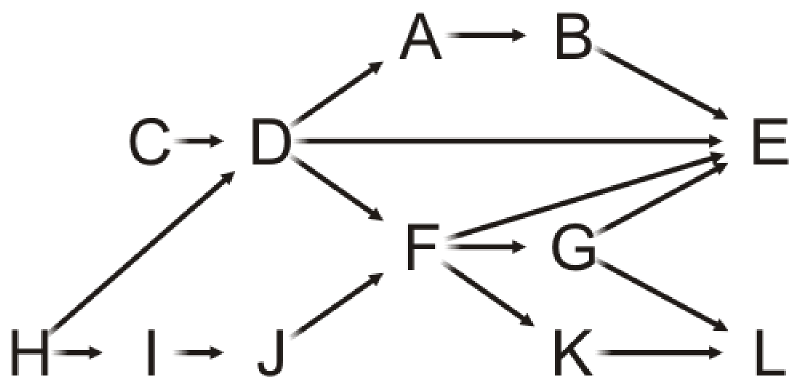
A	0
B	**0**
C	0
D	0
E	**2**
F	0
G	1
H	0
I	0
J	0
K	1
L	2

Queue:

C	H	D	I	A	J	B	F				

Example

Pop the front of the queue

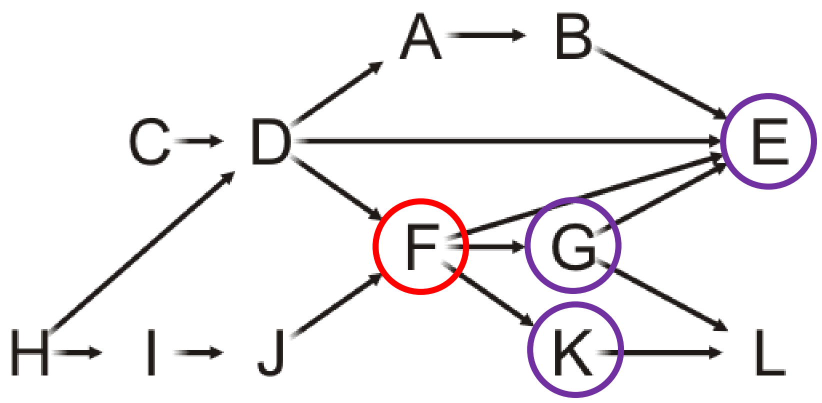


A	0
B	0
C	0
D	0
E	2
F	0
G	1
H	0
I	0
J	0
K	1
L	2

Example

Pop the front of the queue

- F has three neighbors: E, G and K



A	0
B	0
C	0
D	0
E	**2**
F	**0**
G	**1**
H	0
I	0
J	0
K	**1**
L	2

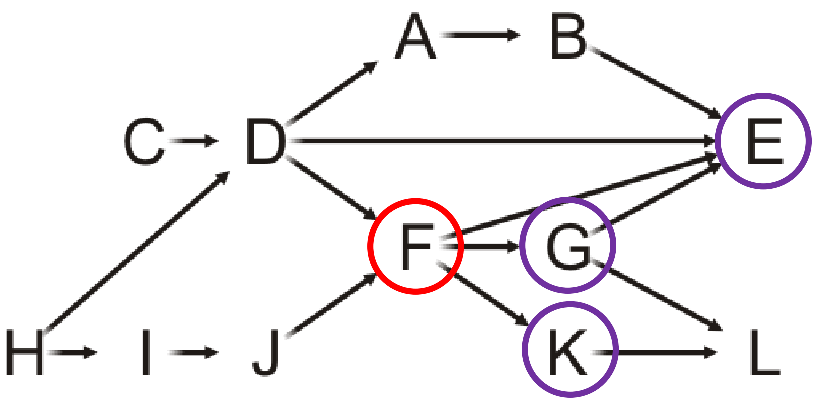
Queue:

C	H	D	I	A	J	B	F				

Example

Pop the front of the queue

- F has three neighbors: E, G and K
- Decrement their in-degrees



Queue:

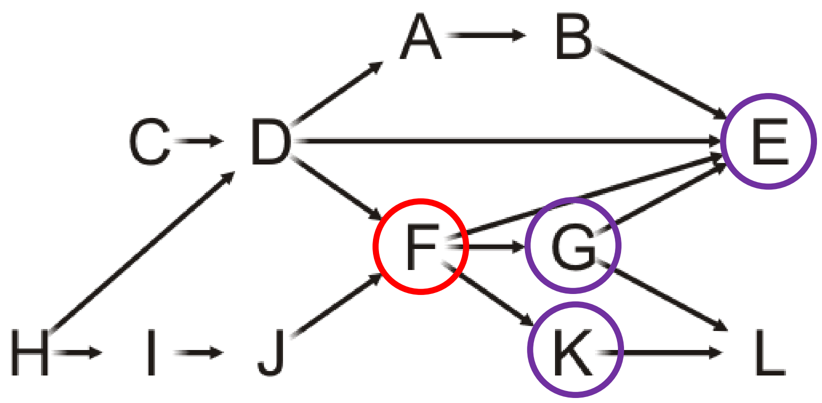
C	H	D	I	A	J	B	F				

A	0
B	0
C	0
D	0
E	**1**
F	**0**
G	**0**
H	0
I	0
J	0
K	**0**
L	2

Example

Pop the front of the queue

- F has three neighbors: E, G and K
- Decrement their in-degrees
  - G and K are decremented to zero,
    so push them onto the queue



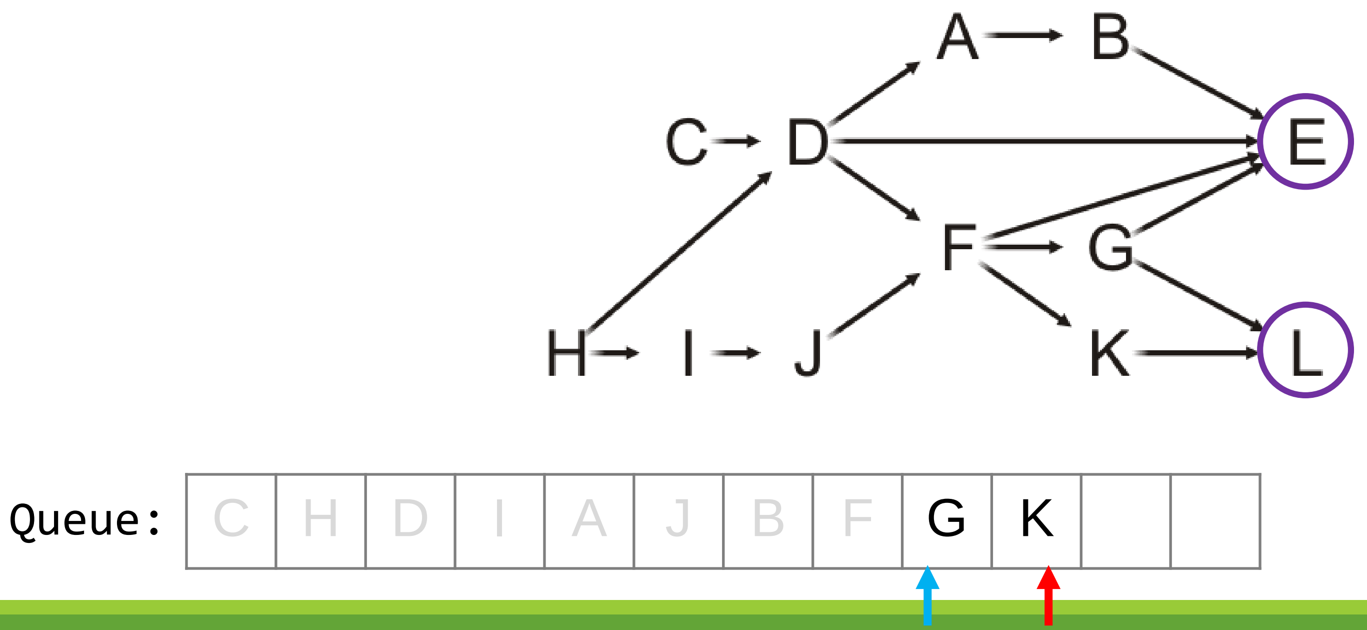
A	0
B	0
C	0
D	0
E	1
F	0
G	0
H	0
I	0
J	0
K	0
L	2

Queue:

C	H	D	I	A	J	B	F	G	K		

Example

Pop the front of the queue

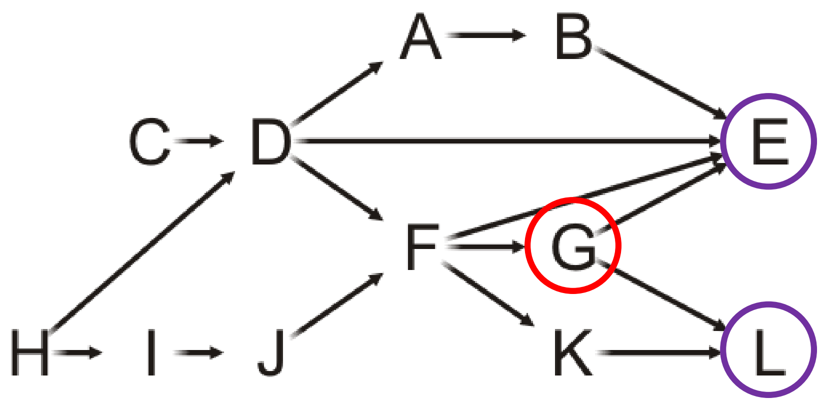


A	0
B	0
C	0
D	0
E	1
F	0
G	0
H	0
I	0
J	0
K	0
L	2

Example

Pop the front of the queue

- G has two neighbors: E and L



Queue:

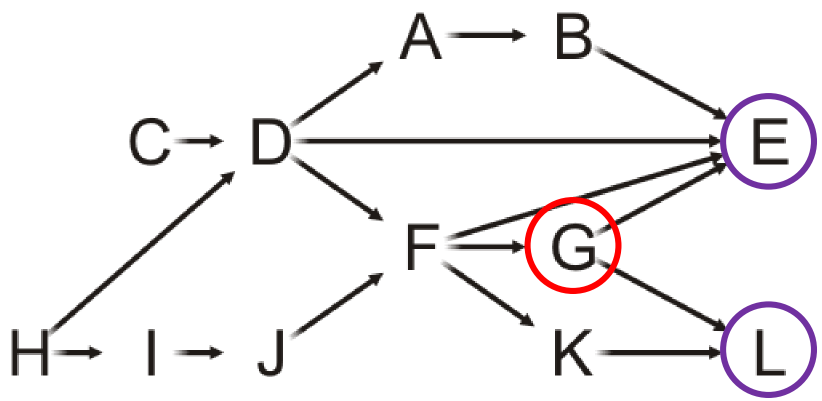
C	H	D	I	A	J	B	F	G	K		

A	0
B	0
C	0
D	0
E	**1**
F	0
G	**0**
H	0
I	0
J	0
K	0
L	**2**

Example

Pop the front of the queue

- G has two neighbors: E and L
- Decrement their in-degrees



Queue:

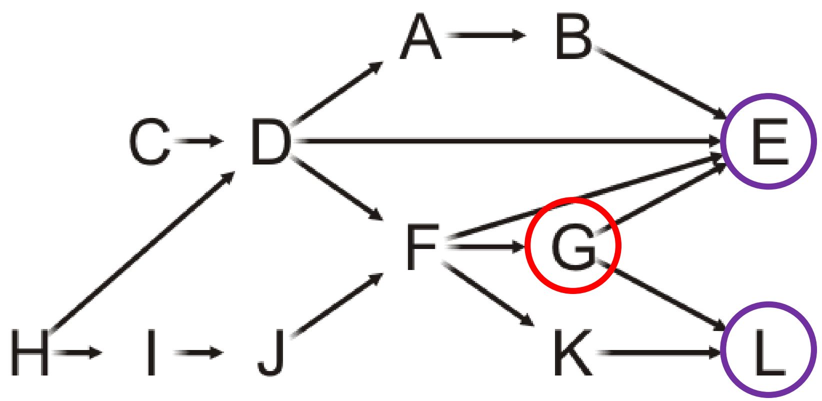
C	H	D	I	A	J	B	F	G	K		

A	0
B	0
C	0
D	0
E	**0**
F	0
G	**0**
H	0
I	0
J	0
K	0
L	**1**

Example

Pop the front of the queue

- G has two neighbors: E and L
- Decrement their in-degrees
  - E is decremented to zero, so push it onto the queue



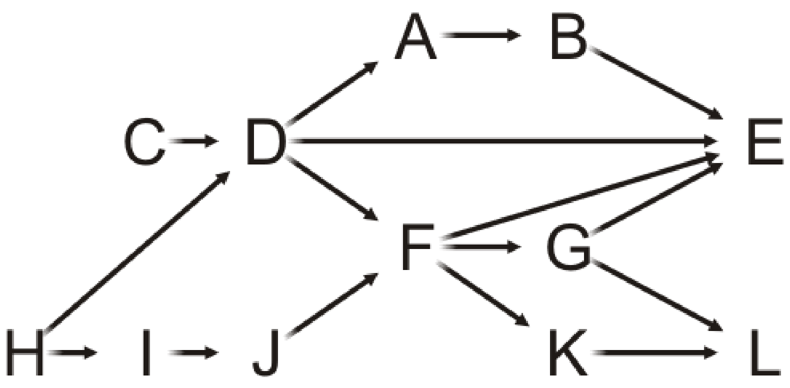
Queue:

C	H	D	I	A	J	B	F	G	K	E	

A	0
B	0
C	0
D	0
E	**0**
F	0
G	**0**
H	0
I	0
J	0
K	0
L	**1**

Example

Pop the front of the queue



A	0
B	0
C	0
D	0
E	0
F	0
G	0
H	0
I	0
J	0
K	0
L	1

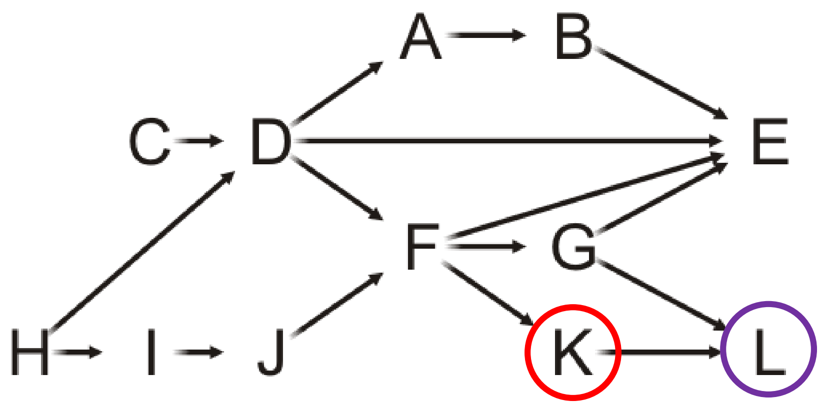
Queue:

C	H	D	I	A	J	B	F	G	K	E	

Example

Pop the front of the queue

- K has one neighbors: L



Queue:

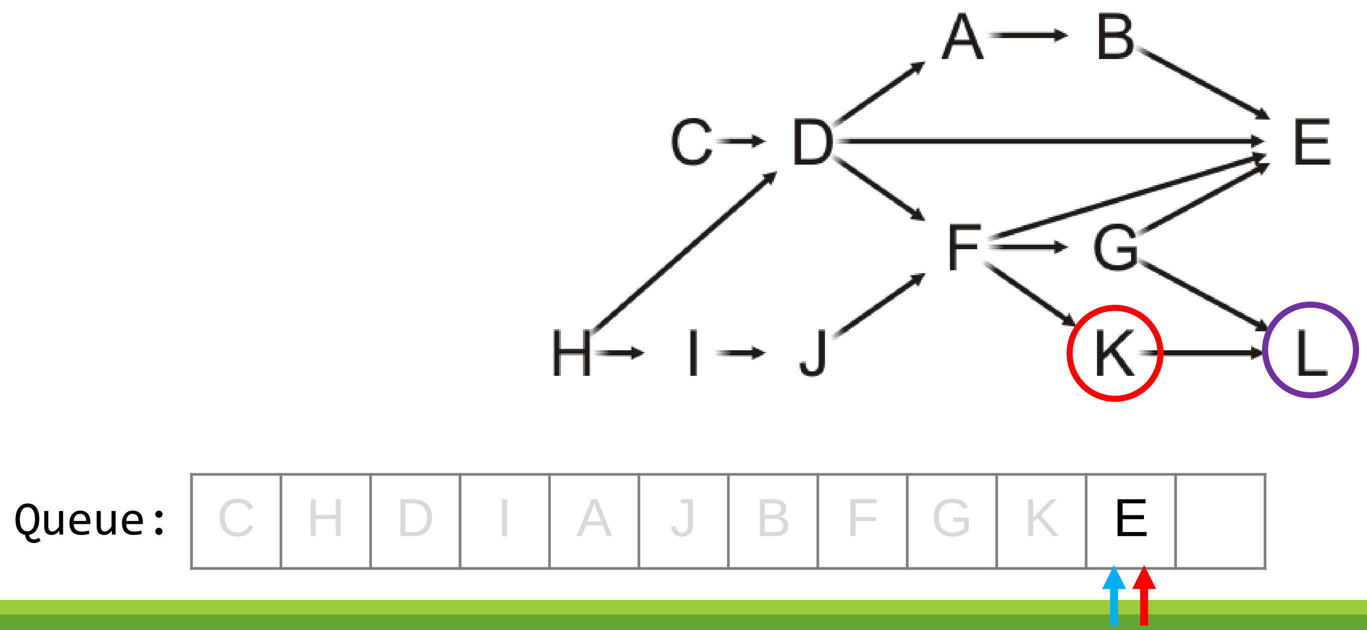
C	H	D	I	A	J	B	F	G	K	E	

A	0
B	0
C	0
D	0
E	0
F	0
G	0
H	0
I	0
J	0
K	**0**
L	**1**

Example

Pop the front of the queue

- K has one neighbors: L
- Decrement its in-degree

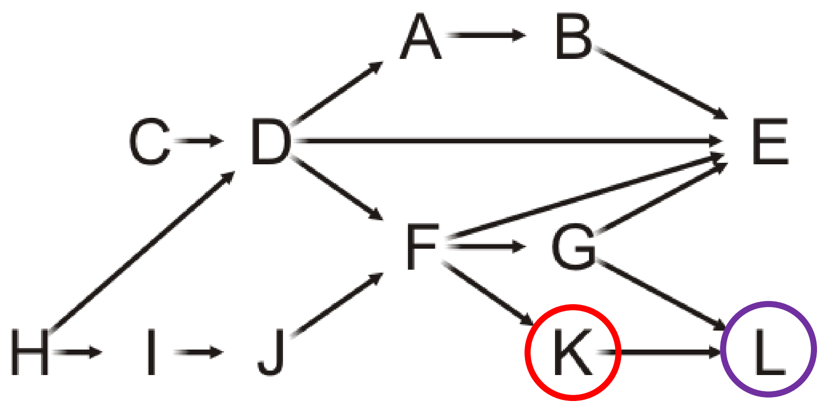


A	0
B	0
C	0
D	0
E	0
F	0
G	0
H	0
I	0
J	0
K	**0**
L	**0**

Example

Pop the front of the queue

- K has one neighbors: L
- Decrement its in-degree
  - L is decremented to zero, so push it onto the queue



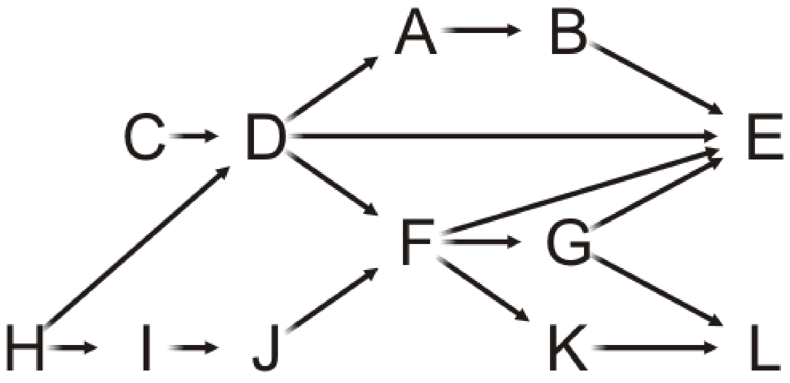
Queue:

C	H	D	I	A	J	B	F	G	K	E	L

A	0
B	0
C	0
D	0
E	0
F	0
G	0
H	0
I	0
J	0
K	**0**
L	**0**

Example

Pop the front of the queue



A	0
B	0
C	0
D	0
E	0
F	0
G	0
H	0
I	0
J	0
K	0
L	0

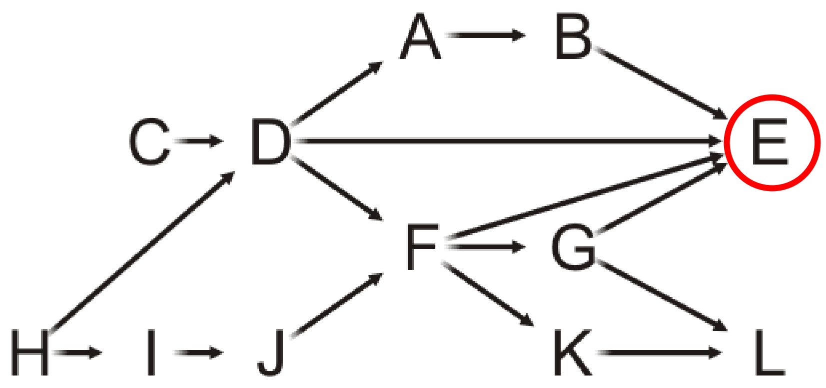
Queue:

C	H	D	I	A	J	B	F	G	K	E	L

Example

Pop the front of the queue

- E has no neighbors—it is a *sink*



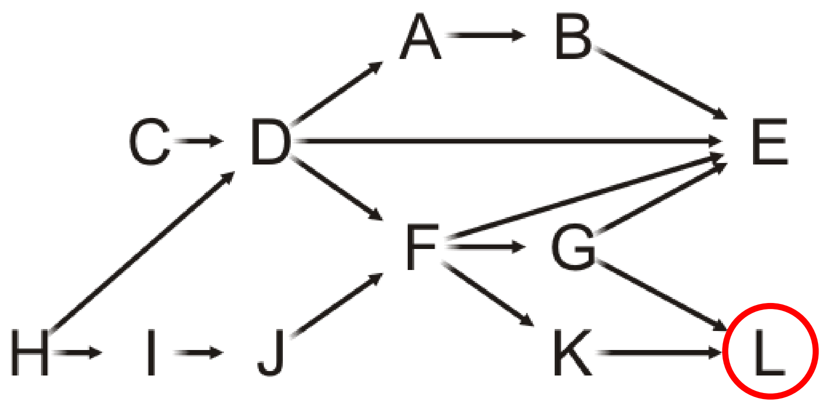
A	0
B	0
C	0
D	0
E	**0**
F	0
G	0
H	0
I	0
J	0
K	0
L	0

`Queue:`

C	H	D	I	A	J	B	F	G	K	E	L

Example

Pop the front of the queue



A	0
B	0
C	0
D	0
E	0
F	0
G	0
H	0
I	0
J	0
K	0
L	0

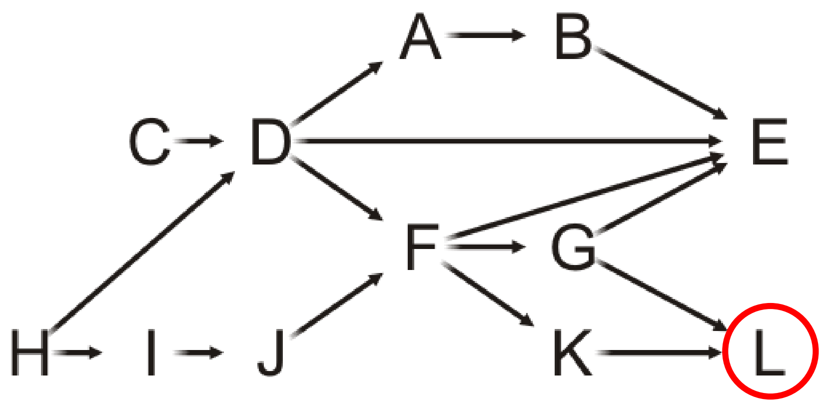
Queue:

C	H	D	I	A	J	B	F	G	K	E	L

Example

Pop the front of the queue

- L has no neighbors—it is also a *sink*



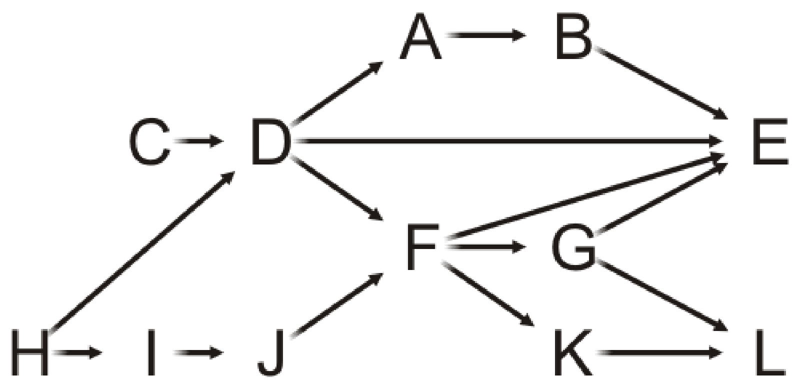
A	0
B	0
C	0
D	0
E	0
F	0
G	0
H	0
I	0
J	0
K	0
L	**0**

Queue:

C	H	D	I	A	J	B	F	G	K	E	L

Example

The queue is empty, so we are done



Queue:

C	H	D	I	A	J	B	F	G	K	E	L

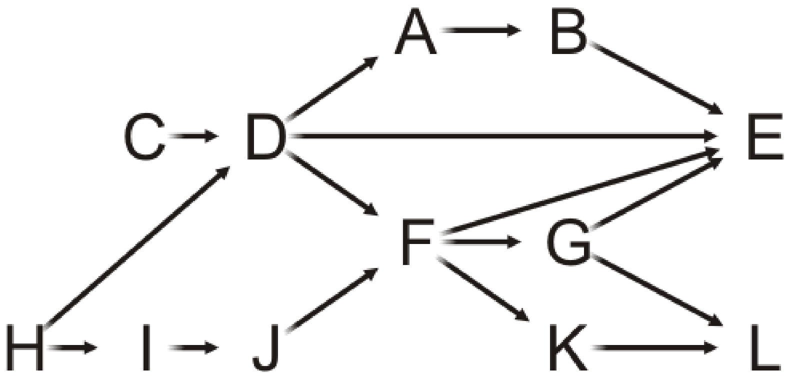
A	0
B	0
C	0
D	0
E	0
F	0
G	0
H	0
I	0
J	0
K	0
L	0

Example

We deallocate the memory for the temporary in-degree array

A	0
B	0
C	0
D	0
E	0
F	0
G	0
H	0
I	0
J	0
K	0
L	0

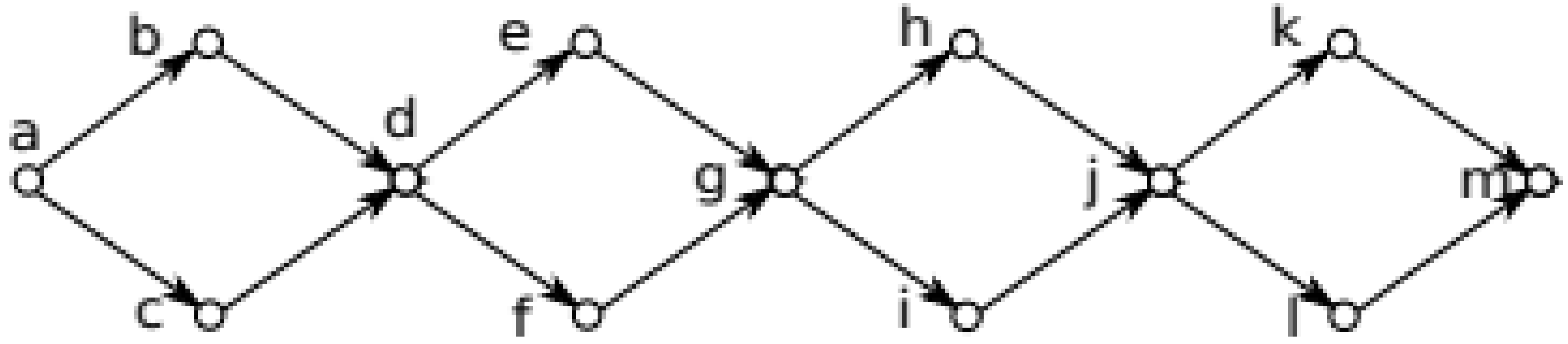
The array stores the topological sorting



C	H	D	I	A	J	B	F	G	K	E	L

Question

Give a topological sort of the following graph 2*2*2*2=16

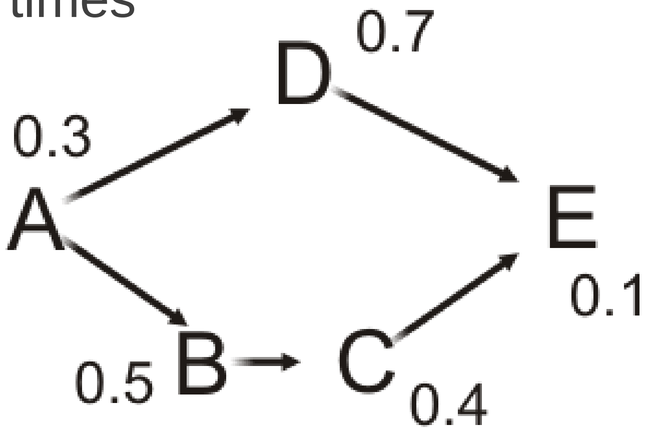


What is the number of different topological sorts of this graph? (Bonus Mark Question) (Negative mark for wrong answer)

Critical path

Suppose each task has a performance time associated with it

- If the tasks are performed serially, the time required to complete the last task equals to the sum of the individual task times

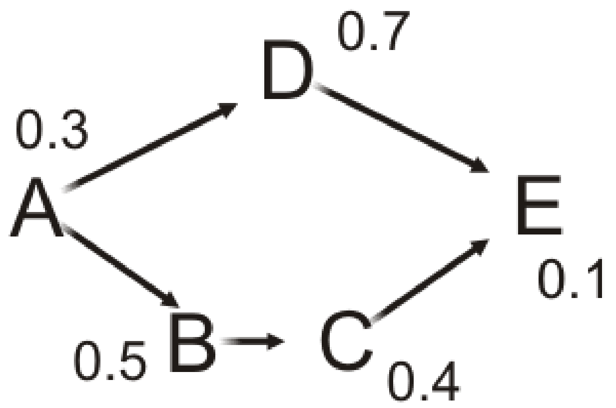


- These tasks require $0.3 + 0.7 + 0.5 + 0.4 + 0.1 = 2.0\text{ s}$ to execute serially

Critical path

Suppose two tasks are ready to execute

- We could perform these tasks in parallel

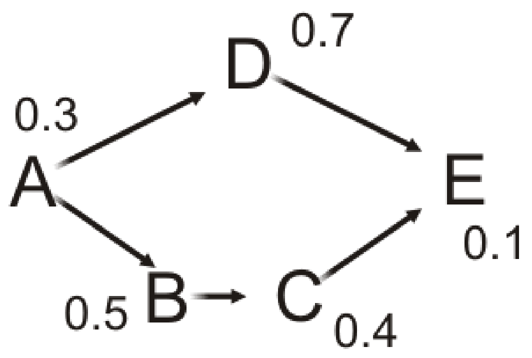


- Computer tasks can be executed in parallel (multi-processing)
- Different tasks can be completed by different teams in a company

Critical path

Suppose Task A completes

- We can now execute Tasks B and D in parallel



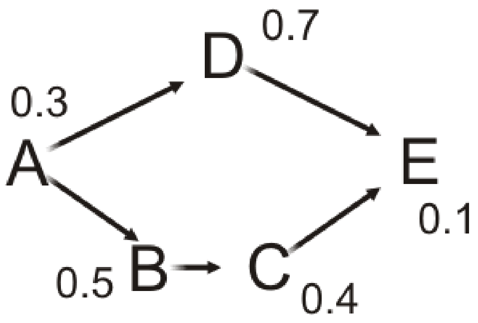
- However, Task E cannot execute until Task C completes, and Task C cannot execute until Task B completes
  - The least time in which these five tasks can be completed is
    $$0.3 + 0.5 + 0.4 + 0.1 = 1.3 \text{ s}$$
  - This is called the *critical time of all tasks*
  - The path (A, B, C, E) is said to be the *critical path*

Finding the critical path

Tasks that have no prerequisites have a critical time equal to the time it takes to complete that task

For tasks that depend on others, the critical time will be:
- The maximum critical time that it takes to complete a prerequisite
- Plus the time it takes to complete this task

In this example, the critical times are:
- Task A completes in $0.3\ \mathrm{s}$
- Task B must wait for A and completes after $0.8\ \mathrm{s}$
- Task D must wait for A and completes after $1.0\ \mathrm{s}$
- Task C must wait for B and completes after $1.2\ \mathrm{s}$
- Task E must wait for both C and D, and completes after $\max(1.0,\ 1.2) + 0.1 = 1.3\ \mathrm{s}$



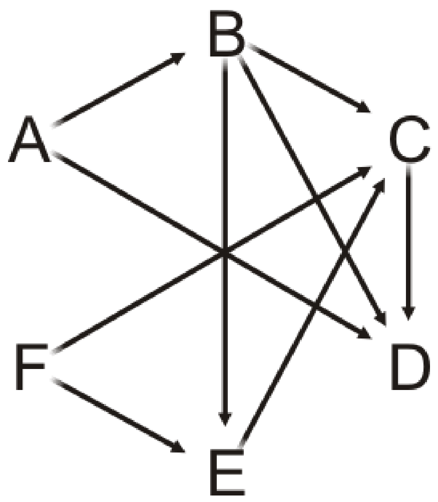
Finding the critical path

Thus, we require more information:

- We must know the execution time of each task
- We will have to record the critical time for each task
  - Initialize these to zero
- We will need to know the previous task with the longest critical time to determine the critical path
  - Set these to null

Finding the critical path

Suppose we have the following times for the tasks



Queue

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	0.0	Ø
B	1	6.1	0.0	Ø
C	3	4.7	0.0	Ø
D	3	8.1	0.0	Ø
E	2	9.5	0.0	Ø
F	0	17.1	0.0	Ø

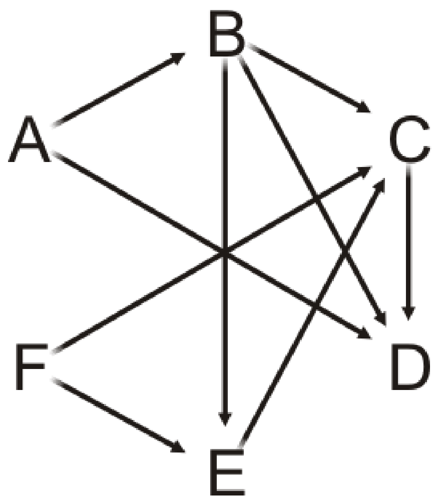
Finding the critical path

Each time we pop a vertex v, in addition to what we already do:

- For v, add the task time onto the critical time for that vertex:
  - That is the critical time for v
- For each adjacent vertex w:
  - If the critical time for v is greater than the currently stored critical time for w
    - Update the critical time with the critical time for v
    - Set the previous pointer to the vertex v

Finding the critical path

So we initialize the queue with those vertices with in-degree zero



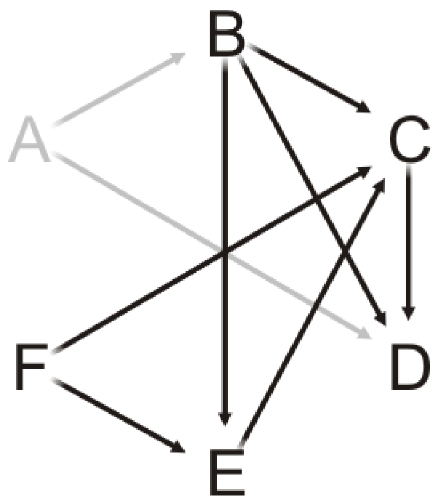
Queue

A	F		

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	0.0	Ø
B	1	6.1	0.0	Ø
C	3	4.7	0.0	Ø
D	3	8.1	0.0	Ø
E	2	9.5	0.0	Ø
F	0	17.1	0.0	Ø

Finding the critical path

Pop Task A and update its critical time 0.0 + 5.2 = 5.2



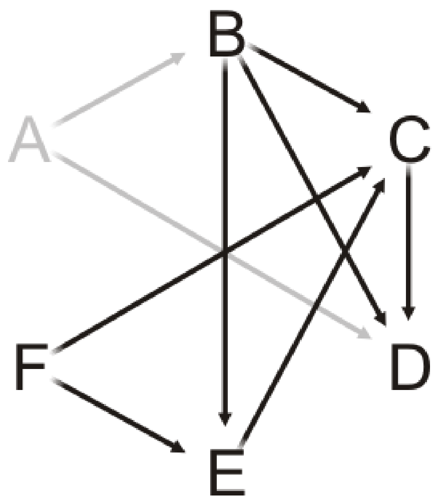
Queue

F			

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	0.0	Ø
B	1	6.1	0.0	Ø
C	3	4.7	0.0	Ø
D	3	8.1	0.0	Ø
E	2	9.5	0.0	Ø
F	0	17.1	0.0	Ø

Finding the critical path

Pop Task A and update its critical time 0.0 + 5.2 = 5.2



Queue

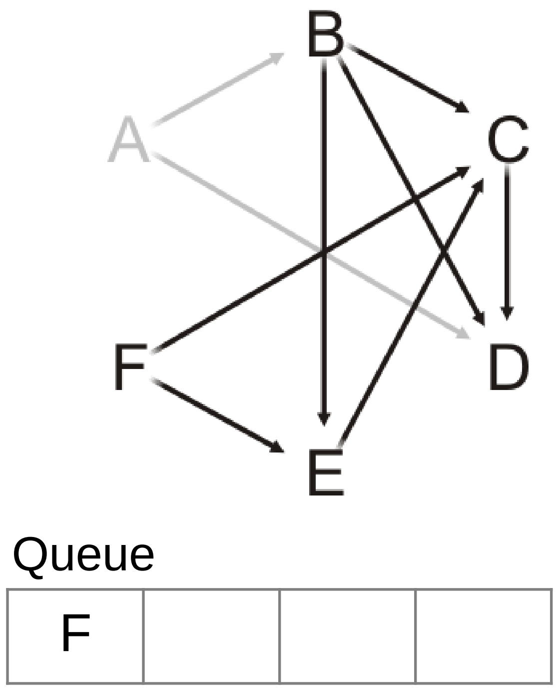
F			

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	**5.2**	Ø
B	1	6.1	0.0	Ø
C	3	4.7	0.0	Ø
D	3	8.1	0.0	Ø
E	2	9.5	0.0	Ø
F	0	17.1	0.0	Ø

Finding the critical path

For each neighbor of Task A:

- Decrement the in-degree, push if necessary, and check if we must update the critical time

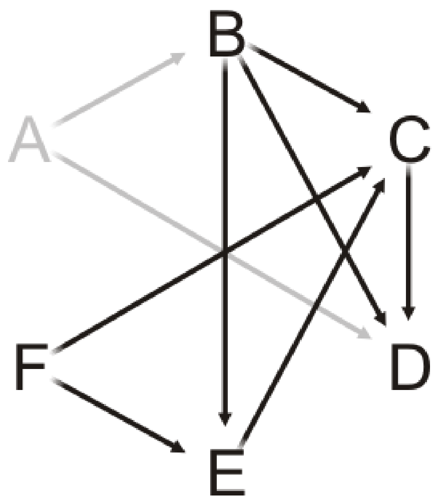


Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	1	6.1	0.0	Ø
C	3	4.7	0.0	Ø
D	3	8.1	0.0	Ø
E	2	9.5	0.0	Ø
F	0	17.1	0.0	Ø

Finding the critical path

For each neighbor of Task A:

- Decrement the in-degree, push if necessary, and check if we must update the critical time



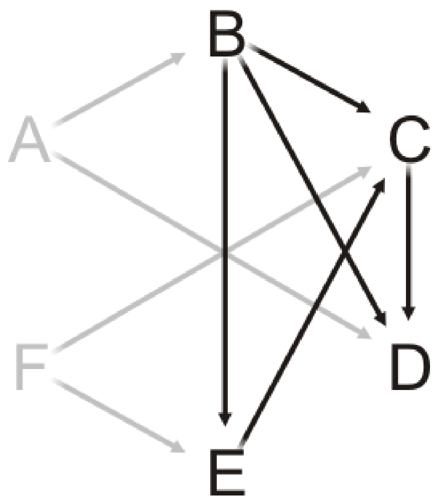
Queue

F	**B**		

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	**0**	6.1	**5.2**	**A**
C	3	4.7	0.0	Ø
D	**2**	8.1	**5.2**	**A**
E	2	9.5	0.0	Ø
F	0	17.1	0.0	Ø

Finding the critical path

Pop Task F and update its critical time 0.0 + 17.1 = 17.1



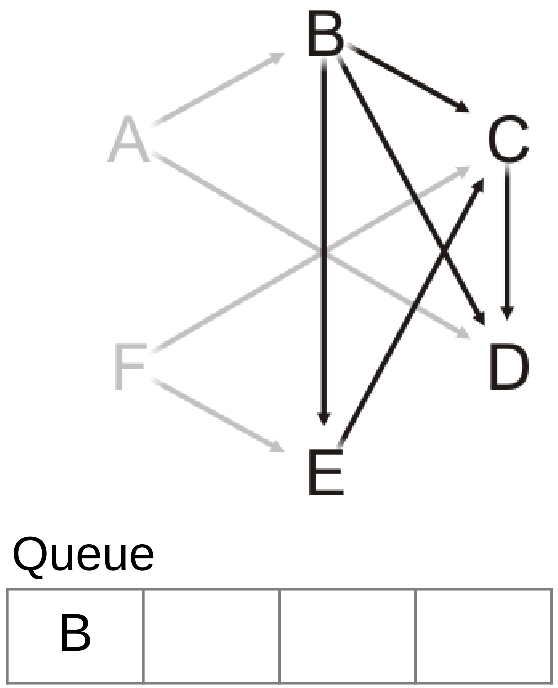
Queue

B			

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	5.2	A
C	3	4.7	0.0	Ø
D	2	8.1	5.2	A
E	2	9.5	0.0	Ø
F	0	17.1	0.0	Ø

Finding the critical path

Pop Task F and update its critical time 0.0 + 17.1 = 17.1

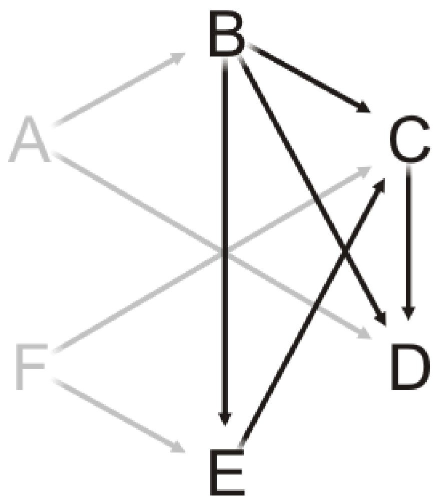


Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	5.2	A
C	3	4.7	0.0	Ø
D	2	8.1	5.2	A
E	2	9.5	0.0	Ø
F	0	17.1	**17.1**	Ø

Finding the critical path

For each neighbor of Task F:

- Decrement the in-degree, push if necessary, and check if we must update the critical time



Queue

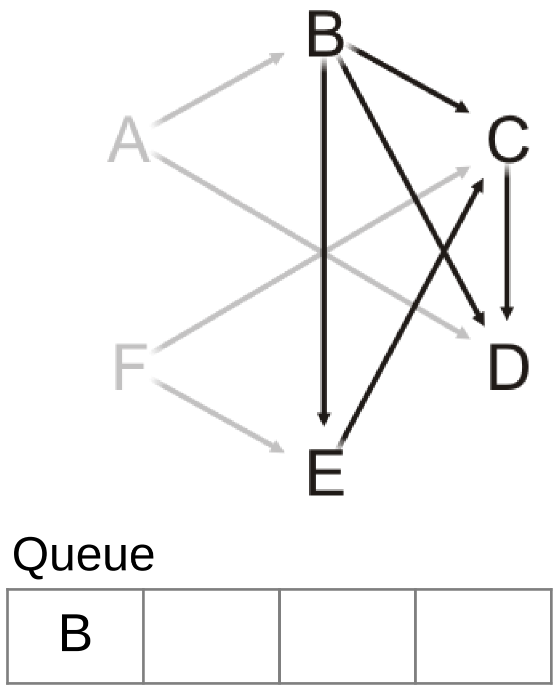
B			

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	5.2	A
C	3	4.7	0.0	Ø
D	2	8.1	5.2	A
E	2	9.5	0.0	Ø
F	0	17.1	17.1	Ø

Finding the critical path

For each neighbor of Task F:

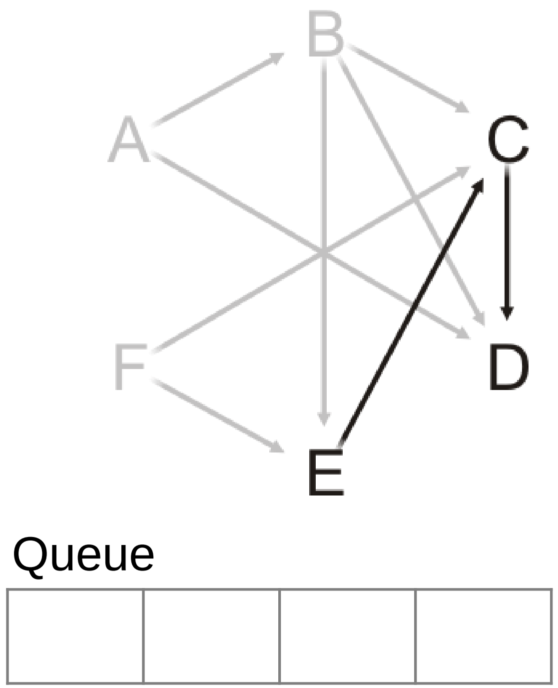
- Decrement the in-degree, push if necessary, and check if we must update the critical time



Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	5.2	A
C	**2**	4.7	**17.1**	**F**
D	2	8.1	5.2	A
E	**1**	9.5	**17.1**	**F**
F	0	17.1	17.1	Ø

Finding the critical path

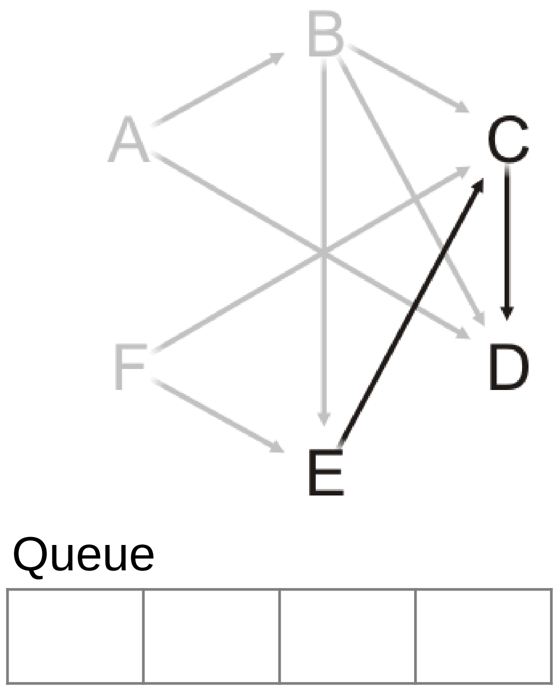
Pop Task B and update its critical time 5.2 + 6.1 = 11.3



Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	5.2	A
C	2	4.7	17.1	F
D	2	8.1	5.2	A
E	1	9.5	17.1	F
F	0	17.1	17.1	Ø

Finding the critical path

Pop Task B and update its critical time 5.2 + 6.1 = 11.3

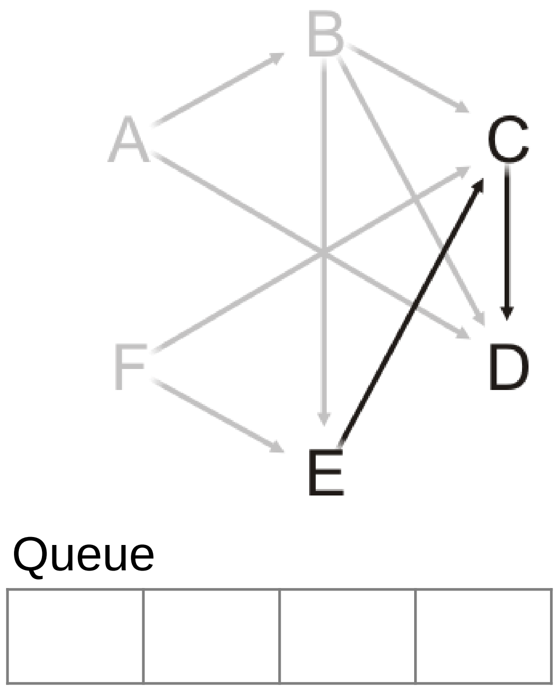


Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	**11.3**	A
C	2	4.7	17.1	F
D	2	8.1	5.2	A
E	1	9.5	17.1	F
F	0	17.1	17.1	Ø

Finding the critical path

For each neighbor of Task B:

- Decrement the in-degree, push if necessary, and check if we must update the critical time

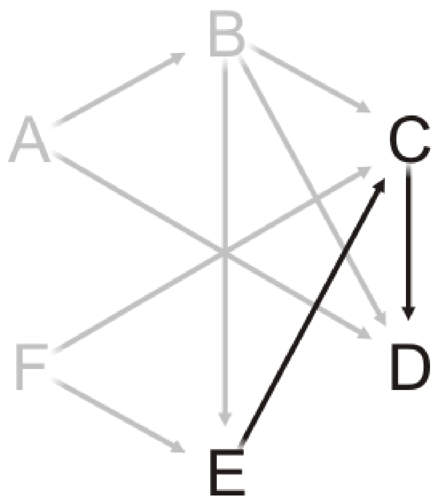


Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	2	4.7	17.1	F
D	2	8.1	5.2	A
E	1	9.5	17.1	F
F	0	17.1	17.1	Ø

Finding the critical path

For each neighbor of Task B:

- Decrement the in-degree, push if necessary, and check if we must update the critical time
- Both C and E are waiting on F



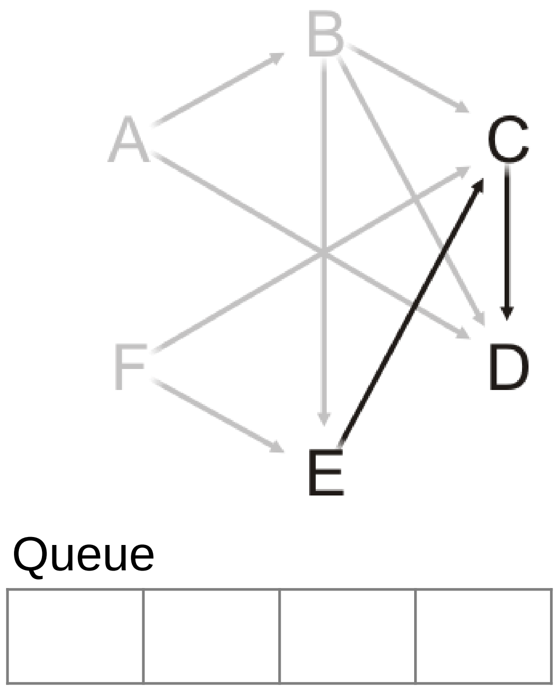
Queue

E			

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	1	4.7	17.1	F
D	1	8.1	**11.3**	**B**
E	0	9.5	17.1	F
F	0	17.1	17.1	Ø

Finding the critical path

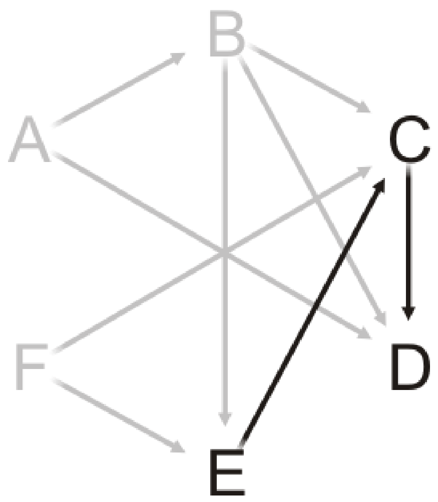
Pop Task E and update its critical time 17.1 + 9.5 = 26.6



Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	1	4.7	17.1	F
D	1	8.1	11.3	B
E	0	9.5	17.1	F
F	0	17.1	17.1	Ø

Finding the critical path

Pop Task E and update its critical time 17.1 + 9.5 = 26.6



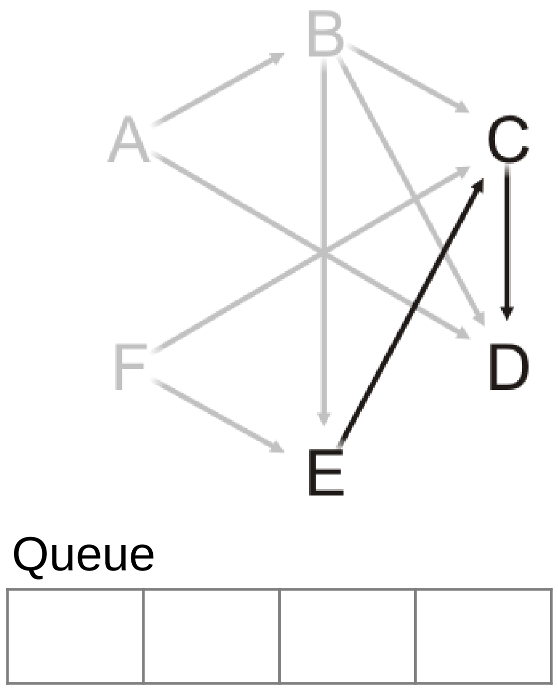
Queue

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	1	4.7	17.1	F
D	1	8.1	11.3	B
E	0	9.5	**26.6**	F
F	0	17.1	17.1	Ø

Finding the critical path

For each neighbor of Task E:

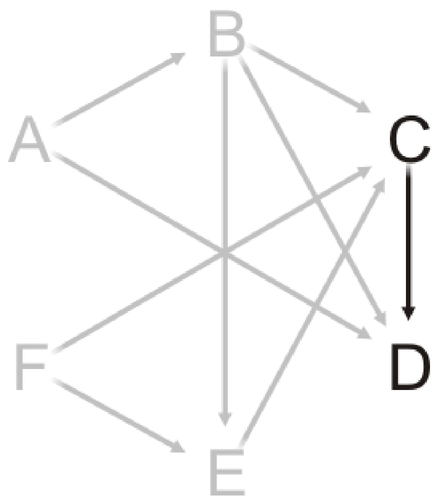
- Decrement the in-degree, push if necessary, and check if we must update the critical time



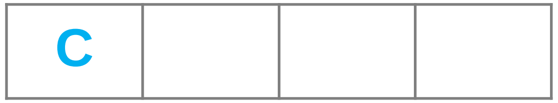
Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	1	4.7	17.1	F
D	1	8.1	11.3	B
E	0	9.5	26.6	F
F	0	17.1	17.1	Ø

Finding the critical path

For each neighbor of Task E:
- Decrement the in-degree, push if necessary, and check if we must update the critical time



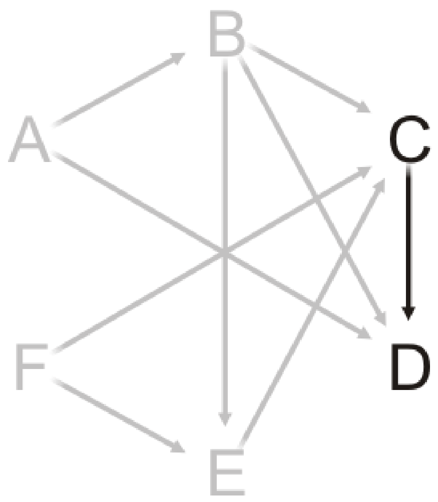
Queue



Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	**0**	4.7	**26.6**	**E**
D	1	8.1	11.3	B
E	0	9.5	26.6	F
F	0	17.1	17.1	Ø

Finding the critical path

Pop Task C and update its critical time 26.6 + 4.7 = 31.3

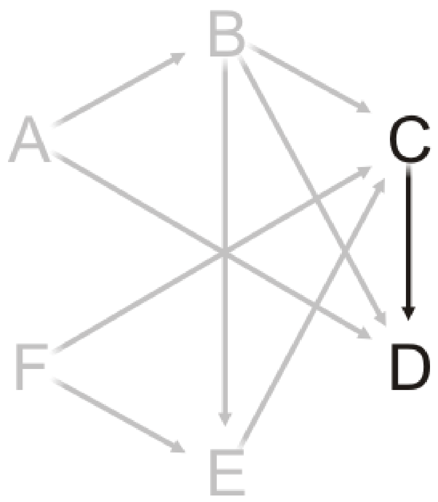


Queue

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	0	4.7	26.6	E
D	1	8.1	11.3	B
E	0	9.5	26.6	F
F	0	17.1	17.1	Ø

Finding the critical path

Pop Task C and update its critical time 26.6 + 4.7 = 31.3



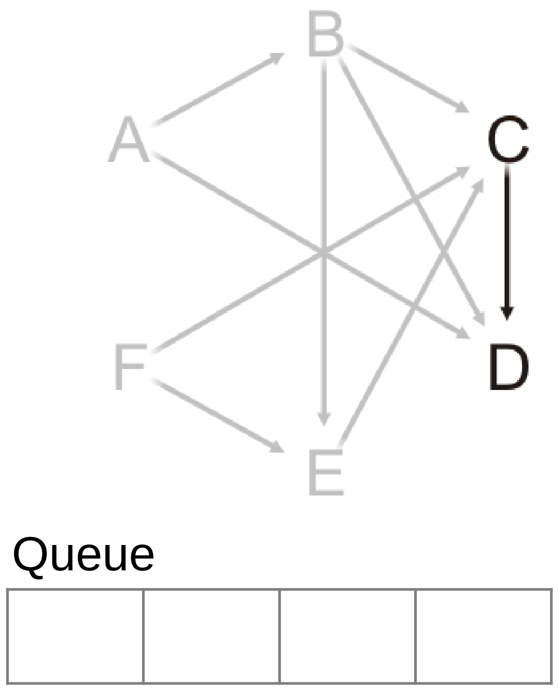
Queue

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	0	4.7	**31.3**	E
D	1	8.1	11.3	B
E	0	9.5	26.6	F
F	0	17.1	17.1	Ø

Finding the critical path

For each neighbor of Task C:

- Decrement the in-degree, push if necessary, and check if we must update the critical time

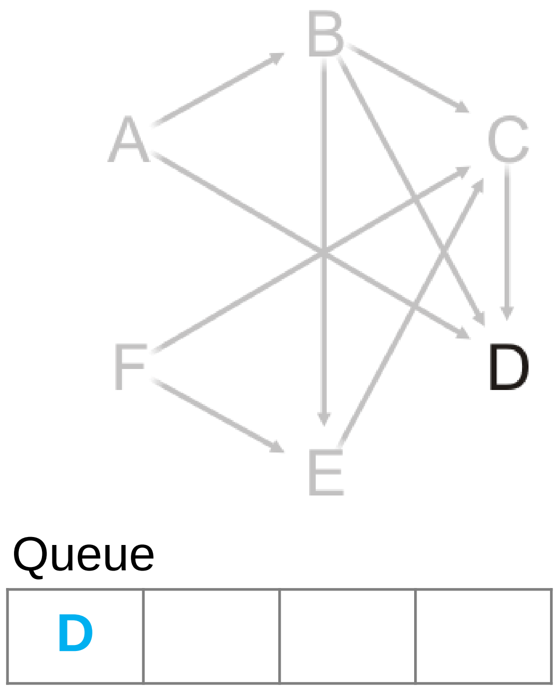


Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	0	4.7	31.3	E
D	1	8.1	11.3	B
E	0	9.5	26.6	F
F	0	17.1	17.1	Ø

Finding the critical path

For each neighbor of Task C:

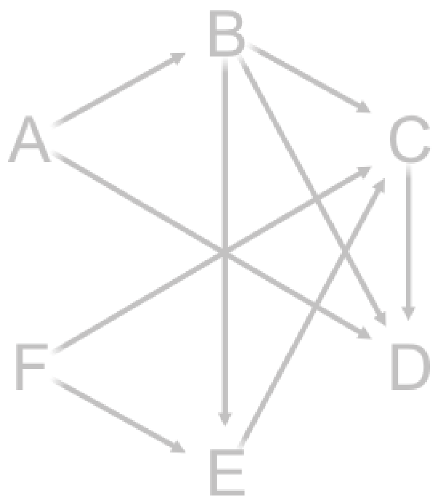
- Decrement the in-degree, push if necessary, and check if we must update the critical time



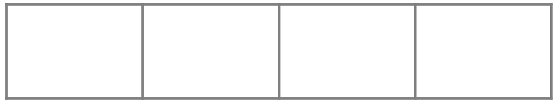
Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	0	4.7	31.3	E
D	**0**	8.1	**31.3**	**C**
E	0	9.5	26.6	F
F	0	17.1	17.1	Ø

Finding the critical path

Pop Task D and update its critical time 31.3 + 8.1 = 39.4



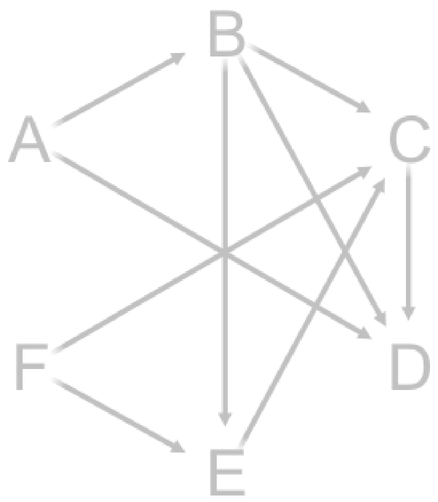
Queue



Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	0	4.7	31.3	E
D	0	8.1	31.3	C
E	0	9.5	26.6	F
F	0	17.1	17.1	Ø

Finding the critical path

Pop Task D and update its critical time 31.3 + 8.1 = 39.4



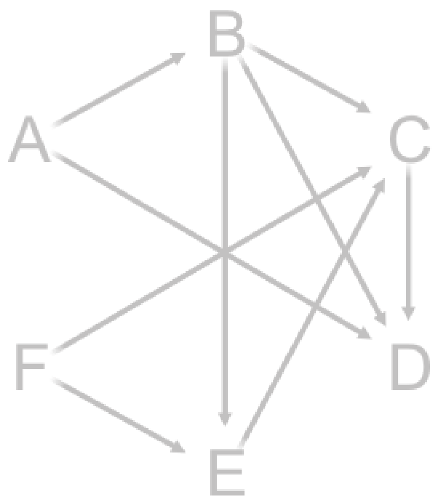
Queue

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	0	4.7	31.3	E
D	0	8.1	**39.4**	C
E	0	9.5	26.6	F
F	0	17.1	17.1	Ø

Finding the critical path

Task D has no neighbors and the queue is empty

- We are done

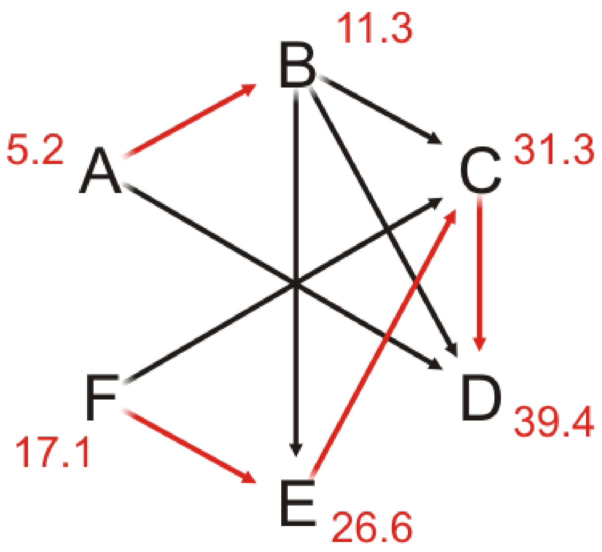


Queue

Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	0	4.7	31.3	E
D	0	8.1	39.4	C
E	0	9.5	26.6	F
F	0	17.1	17.1	Ø

Finding the critical path

Task D has no neighbors and the queue is empty
- We are done



Task	In-degree	Task Time	Critical Time	Previous Task
A	0	5.2	5.2	Ø
B	0	6.1	11.3	A
C	0	4.7	31.3	E
D	0	8.1	39.4	C
E	0	9.5	26.6	F
F	0	17.1	17.1	Ø