

The Use of GARCH Models in VaR Estimation

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Abstract

We evaluate the performance of an extensive family of ARCH models in modeling daily Value-at-Risk (VaR) of perfectly diversified portfolios in five stock indices, using a number of distributional assumptions and sample sizes. We find, first, that leptokurtic distributions are able to produce better one-step-ahead VaR forecasts; the choice of sample size is important for the accuracy of the forecast, whereas the specification of the conditional mean is indifferent. Finally, despite the claims for the contrary, a different and specific structure of ARCH model produces the most accurate VaR forecast for each index portfolio.

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1 Introduction

Following the increase in financial uncertainty in the 90's, resulting to famous financial disasters, (see Jorion (1997), Dowd (1998) and Crouhy et al. (2001) for more information), there has been intensive research from financial institutions, regulators and academics to better develop sophisticated models for market risk estimation. The most well known risk measure is Value-at-Risk (VaR), which refers to a portfolio's worst outcome that is expected to occur over a predetermined period and at a given confidence level.

In general, VaR is an estimation of the tails of the empirical distribution. Many applications presume that asset returns are normally distributed, while it is widely documented that they exhibit skewness and excess kurtosis, resulting in an underestimation or overestimation of the true VaR. Venkataraman (1996) and Zangari (1996) proposed the use of a mixture of normal distributions, which is fatted tailed and able to capture the extreme events compared to the "classical" approaches more easily. Billio and Pelizzon (2000) introduced a multivariate switching regime model in order to calculate the VaR for 10 Italian stocks and for several portfolios that have been generated by them. They contrasted their model under two backtesting measures and they concluded that switching regimes are more accurate than the other known methods (RiskMetricsTM and Garch (1,1) under Normal and Student-t distribution). Gurmat and Harris (2002) estimated an exponentially weighted maximum likelihood model for three equity portfolios and its forecasts improved the VaR numbers at higher confidence levels. Giot and Laurent (2002) estimated daily VaR for stock index returns by using a skewed student distribution and pointed out that it performed better than the pure symmetric one, as it reproduced the characteristics of the empirical distribution more precise. One year later, Giot and Laurent (2003) used the same distributional assumption for six commodities and proposed to market practitioners to use the Skewed Student-t ARCH model because its estimation is easily implemented even in a spreadsheet-like environment and the predicted VaR numbers were close to the expected ones. However, they stated that the most complex model (APARCH) performed better overall. Based on the influence of asymmetric effects in the accuracy of the VaR estimates, Brooks and Persaud (2003) concluded that models, which do not allow for asymmetries either in the unconditional return distribution or in the volatility specification, underestimate the true VaR.

Many researchers prefer to conduct simulations in order to calculate VaR, than to explicitly use a specific parametric distribution. Lambadiaris et al. (2003) performed historical and Monte Carlo simulations in Greek stocks and bonds market, by using two different sample sizes. They concluded that the Monte Carlo method was more appropriate for the stock market, while for the bonds the results were depending on the backtesting procedure and the given confidence level. Similarly, Cabedo and Moya (2003) developed an ARMA historical simulation method, which improved the simple historical VaR estimation. They used eight years of daily oil prices in order to estimate the coefficients of their model and one year to conduct the out of sample forecasts. Both works used different sample sizes as the importance of them has been examined and pointed out by several authors. Jackson et al. (1998) provided evidence that at higher confidence levels, historical simulation based methods work better than other methods. On the contrary, Hendricks (1996) and Vlaar (2000) supported that the increase of the sample size tends to generate more accurate VaR estimations. Hoppe (1998) also examined the issue of the sample size and argued that the use of smaller sample size would lead to more accurate VaR estimates than the longer one. Frey and Michaud (1997) in their study supported the use of short sample sizes in order to capture the

structural changes over time due to changes in trading behaviour. To sum up, the choice of an appropriate historical sample size as well as an adequate model for forecasting volatility should be considered far from resolved.

The purpose of our paper is twofold. First to implement several volatility models under three distributional assumptions and four historical sample sizes in order to estimate the 95% and 99% one-day VaR for five completely diversified index portfolios (S&P 500, Nikkei 225, FTSE 100, CAC 40 and DAX 30). The different distributions (Normal, Student-t and Generalized Error Distribution) will allow the selection of a more flexible model for the return tails, while the four sample sizes (500, 1000, 1500 and 2000 observations) will reveal the importance of past data. For robustness purposes, we have used five different stock indices, to avoid results dependent on a specific financial market. Combined with three different interpretations of the ARCH family (GARCH, TARARCH and EGARCH) and more than 1800 one-step-ahead VaR estimate, it brings the total of estimated models to more than 4 millions ! Despite the enormous set of estimated models (484 models for each index), we did not include all the ARCH specifications that are available in the literature. However, we estimated models that are able to capture the most important characteristics of financial markets.

Second, to evaluate the predictive accuracy of various models under a risk management framework. We employ a two stage procedure to investigate the forecasting power of each volatility forecasting technique. Specifically, in the first stage, two backtesting criteria are implemented to test the statistical accuracy of the models. In the second stage, we employ standard forecast evaluation methods in order to examine whether the differences between the models, which have been converged sufficiently, are statistically significant. We focus on out-sample evaluation criteria because we believe that an inadequate model, according to in-sample evaluation criteria, can yield better forecasts than a correctly specified one.

Our study shows that more flexible GARCH models are more adequate in volatility forecasting than parsimonious models for all indices, distributional assumptions and both confidence levels. The asymmetric models fare better than the simpler ones, as they capture the characteristics of the underlying series more efficiently. As concerns the choice of the distribution, the leptokurtic ones provide better estimators of VaR since they perform better in the low probability regions that Value at Risk tries to measure. Moreover, although the use of all available data in forecasting volatility is common practice, we find out that, at least for some cases, a restricted sample size could generate more accurate one-step-ahead VaR forecasts as it could incorporate changes in trading behaviour more efficiently.

The rest of the paper is organized as follows. Section 2 provides a description of the ARCH models, while the third section describes the evaluation framework for VaR estimates. Section 4 presents preliminary statistics for the dataset, explains the estimation procedure and presents the results of the empirical investigation of the estimated models for the five equity indices. Section 5 concludes the paper.

2 Volatility Models

Let $y_t = \ln(S_t/S_{t-1})$ denote the continuously compound rate of return from time $t - 1$ to t , where S_t is the asset price at time t . We assume that the time series of interest, y_t , is decomposed into two parts, the predictable and unpredictable component, $y_t = \mathbb{E}(y_t|I_{t-1}) + \varepsilon_t$, where I_{t-1} is the information set at time $t - 1$, \mathbb{E} is the conditional mean operator and ε_t is the unpredictable part,

or innovation process. The conditional mean return is considered as a k -th order autoregressive process, $AR(k)$:

$$\mathbb{E}(y_t | I_{t-1}) \equiv c_0 + \sum_{i=1}^k c_i y_{t-i}.$$

The autoregressive process allows for the autocorrelation induced by discontinuous (or non-synchronous) trading in the stocks making up an index (Scholes and Williams (1977), Lo and MacKinlay (1988)). The unpredictable component, ε_t , can be expressed as an ARCH process in the following form:

$$\varepsilon_t = z_t \sigma_t,$$

where z_t is a sequence of independently and identically distributed random variables with zero mean and unit variance. The conditional variance of ε_t is σ_t , is a time-varying, positive and measurable function of the information set at time $t - 1$. Note that, even though the innovation process for the conditional mean is serially uncorrelated, it is not time independent.

Engle (1982) introduced the ARCH(q) model and expressed the conditional variance as a linear function of the past q squared innovations

$$\sigma_t^2 = a_0 + \sum_{i=1}^q a_i \varepsilon_{t-i}^2.$$

For the conditional variance to be positive, the parameters must satisfy $a_0 > 0$ and $a_i \geq 0$ for $i = 1, \dots, q$. Empirical evidence has shown that a high q must be selected in order to estimate the conditional variance properly. To circumvent this problem, Bollerslev (1986) proposed the generalized ARCH, or GARCH(p, q), model:

$$\sigma_t^2 = a_0 + \sum_{i=1}^q a_i \varepsilon_{t-i}^2 + \sum_{j=1}^p b_j \sigma_{t-j}^2, \quad (1)$$

where $a_0 > 0$, $a_i \geq 0$ for $i = 1, \dots, q$, and $b_j \geq 0$ for $j = 1, \dots, p$. If $\sum^q a_i + \sum^p b_j < 1$, then the process ε_t is covariance stationary and its unconditional variance is equal to

$$\sigma^2 = \frac{a_0}{1 - \sum^q a_i - \sum^p b_j}.$$

A special case of the GARCH family is the Exponentially Weighted Moving Average (EWMA) alternative, used by the company RiskMetricsTM, when they introduced their analytic VaR methodology. The volatility forecast is the weighted average of the previous period's forecast and the current squared return. They calculate return variance as an exponentially declining process $\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda) \varepsilon_{t-1}^2$, a special case of GARCH(1,1) with a zero intercept and the two remaining parameters summing to one. RiskMetricsTM use $\lambda = 0.94$ for daily data and go 75 data points backwards in their estimation horizon.

The GARCH(p, q) model successfully captures several characteristics of financial time series, such as thick tailed returns and volatility clustering, as noted by Mandelbrot (1963): "... large changes tend to be followed by large changes of either sign, and small changes tend to be followed by small changes...". On the other hand, the GARCH structure present some drawbacks on implementation, since it (a) requires large numbers of observations to produce reliable estimates,

and (b) may be unstable out of sample. It also imposes important limitations since variance depends on the magnitude and not on the sign of ε_t , which is somewhat at odds with the empirical behaviour of stock market prices where a leverage effect may be present.¹ The term leverage effect, first noted by Black (1976), refers to the tendency for changes in stock returns to be negatively correlated with changes in returns volatility, i.e. volatility tends to rise in response to bad news, ($\varepsilon_t < 0$), and to fall in response to good news ($\varepsilon_t > 0$). Moreover, Brooks and Persaud (2003a) stated that a VaR model which does not account for asymmetries in the volatility specification, it is most likely to generate inaccurate forecasts.

In order to capture the asymmetry exhibited in the data, a new class of models was introduced, termed the asymmetric ARCH models. The most popular model proposed to capture the asymmetric effects is Nelson's (1991) exponential GARCH, or EGARCH(p,q), model:

$$\ln(\sigma_t^2) = a_0 + \sum_{i=1}^q \left(a_i \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| + \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right) + \sum_{j=1}^p (b_j \ln(\sigma_{t-j}^2)). \quad (2)$$

In contrast to the GARCH model, no restrictions need to be imposed on the model estimation, since the logarithmic transformation ensures that the forecasts of the variance are non-negative. The parameters γ_i allow for the asymmetric effect. If $\gamma_1 = 0$ then a positive surprise, $\varepsilon_t > 0$, has the same effect on volatility as a negative surprise, $\varepsilon_t < 0$. The presence of leverage effect can be investigated by testing the hypothesis that $\gamma_1 < 0$.

The number of possible conditional volatility formulations is vast. The threshold GARCH, or TARCH(p,q), model is one of the widely used models:

$$\sigma_t^2 = a_0 + \sum_{i=1}^q a_i \varepsilon_{t-i}^2 + \gamma_1 \varepsilon_{t-1}^2 d_{t-1} + \sum_{j=1}^p b_j \sigma_{t-j}^2, \quad (3)$$

where $d_t = 1$ if $\varepsilon_t < 0$, and $d_t = 0$ otherwise and hence it allows a response of volatility to news with different coefficients for good and bad news.

Although, there are various ways to parameterise the asymmetry effect, the EGARCH and TARCH models are the most widely known models (see Engle and Patton (2001), Brooks and Persaud (2003a) and Brooks and Persaud (2003b) among others.). Furthermore, the most econometric packages include routines for the estimation of these two asymmetric specifications of the conditional variance and therefore they can be used by a risk manager effortlessly.

A wide range of ARCH models proposed in the literature has been reviewed by Bollerslev et al. (1992), Bera and Higgins (1993), Bollerslev et al. (1994), Hamilton (1994), and Degiannakis and Xekalaki (2003).

As concerns the distribution of z_t , Engle (1982) who introduced the ARCH process assumed that they are normally distributed. Bollerslev (1987), however, proposed the standardized t-distribution with $v > 2$ degrees of freedom

$$D(z_t; v) = \frac{\Gamma((v+1)/2)}{\Gamma(v/2)\sqrt{\pi(v-2)}} \left(1 + \frac{z_t^2}{v-2}\right)^{-\frac{v+1}{2}}, \quad (4)$$

¹In such a case, a stochastic volatility model may be more appropriate. There is some evidence that these models may perform better than multivariate GARCH models (Danielsson (1996)) but they are also more difficult to estimate.

where $\Gamma(v) = \int_0^\infty e^{-x} x^{v-1} dx$ is the gamma function and v is the degree of freedom. The t-distribution is symmetric around zero and, for $v > 4$, the conditional kurtosis equals $3(v-2)(v-4)^{-1}$, which exceeds the normal value of three, but for $v \rightarrow \infty$, the density function of the standardized t-distribution converges to the density function of the standard normal distribution. Nelson (1991), on the other hand, suggested the use of the generalized error distribution, or GED:

$$D(z_t; v) = \frac{v \exp(-0.5 |z_t/\lambda|^v)}{2^{(1+1/v)} \Gamma(v^{-1}) \lambda}, \quad v > 0, \quad (5)$$

where v is the tail-thickness parameter and $\lambda \equiv \sqrt{2^{-2/v} \Gamma(v^{-1}) \Gamma(3v^{-1})}$. When $v = 2$, z_t is standard normally distributed. For $v < 2$, the distribution of z_t has thicker tails than the normal distribution (e.g., for $v = 1$, z_t has a double exponential distribution) while for $v > 2$, the distribution of z_t has thinner tails than the normal distribution (e.g., for $v = \infty$, z_t has a uniform distribution on the interval $(-\sqrt{3}, \sqrt{3})$). Guermat and Harris (2002) applied the Student t-distribution and Longerstay (1996) used a mixture of normal distributions. The Gram-Charlier type distribution (Lee and Tse (1991)), the generalized t-distribution (Bollerslev et al. (1994)), the skewed t-distribution (Lambert and Laurent (2000, 2001)), the normal Poisson mixture distribution (Jorion (1988)) and the normal lognormal mixture (Hsieh (1989)) among others are distributions that have been employed.

Maximum likelihood estimation is commonly used to estimate ARCH models. Under the assumption of independently and identically distributed standardized innovations, z_t , and for $D(z_t; v)$ denoting their density function, the log-likelihood function of $\{y_t(\theta)\}$ for a sample of T observations is given by:

$$\mathbb{L}_T(\{y_t\}; \theta) = \sum_{t=1}^T \left[\ln [D(z_t(\theta); v)] - \frac{1}{2} \ln (\sigma_t^2(\theta)) \right], \quad (6)$$

where θ is the vector of the parameters that have to be estimated for the conditional mean, conditional variance and density function. The maximum likelihood estimator $\hat{\theta}$ for the true parameter vector is found by maximizing (6). Hence, we have the following results for the log-likelihood function applied to a sample of T observations:

1. For normally distributed standardized innovations:

$$\mathbb{L}_T(\{y_t\}; \theta) = -\frac{1}{2} \left[T \ln(2\pi) + \sum_{t=1}^T z_t^2 + \sum_{t=1}^T \ln(\sigma_t^2) \right]. \quad (7)$$

2. For standardized t-distributed innovations:

$$\begin{aligned} \mathbb{L}_T(\{y_t\}; \theta) = & T \left[\ln \Gamma \left(\frac{v+1}{2} \right) - \ln \Gamma \left(\frac{v}{2} \right) - \frac{1}{2} \ln[\pi(v-2)] \right] - \\ & \frac{1}{2} \sum_{t=1}^T \left[\ln(\sigma_t^2) + (1+v) \ln \left(1 + \frac{z_t^2}{v-2} \right) \right]. \end{aligned} \quad (8)$$

3. For GED distributed innovations:

$$\mathbb{L}_T(\{y_t\}; \theta) = \sum_{t=1}^T \left[\ln \left(\frac{v}{\lambda} \right) - \frac{1}{2} \left| \frac{z_t}{\lambda} \right|^v - \right.$$

$$(1 + v^{-1}) \ln(2) - \ln \Gamma \left(\frac{1}{v} \right) - \frac{1}{2} \ln(\sigma_t^2) \Big], \quad (9)$$

Maximum likelihood estimates of the parameters are obtained by numerical maximization of the log-likelihood function using the Marquardt algorithm (Marquardt (1963)). We use the quasi-maximum likelihood estimator (QMLE) since, according to Bollerslev and Wooldridge (1992), it is generally consistent, has a normal limiting distribution and provides asymptotic standard errors that are valid under non-normality.

For the GARCH(p,q) model the one-step-ahead conditional variance forecast, $\hat{\sigma}_{t+1|t}^2$, is:

$$\hat{\sigma}_{t+1|t}^2 = a_0^{(t)} + \sum_{i=1}^q a_i^{(t)} \varepsilon_{t-i+1}^2 + \sum_{j=1}^p b_j^{(t)} \sigma_{t-j+1}^2. \quad (10)$$

For the EGARCH(p,q) model, we get instead:

$$\hat{\sigma}_{t+1|t}^2 = \exp \left(a_0^{(t)} + \sum_{i=1}^q \left[a_i^{(t)} \left| \frac{\varepsilon_{t-i+1}}{\sigma_{t-i+1}} \right| + \gamma_i^{(t)} \left(\frac{\varepsilon_{t-i+1}}{\sigma_{t-i+1}} \right) \right] + \sum_{j=1}^p b_j^{(t)} \ln(\sigma_{t-j+1}^2) \right). \quad (11)$$

Notice that the value of $\mathbb{E}|z_t|$ depends on the density function of z_t . For example, for the standard normal distribution, $\mathbb{E}|z_t| = \sqrt{2/\pi}$; for the Student t-distribution,

$$\mathbb{E}|z_t| = \frac{2\Gamma[(1+v)/2]\sqrt{v-2}}{1+(v-1)\Gamma(v/2)\sqrt{\pi}},$$

and for the GED distribution,

$$\mathbb{E}|z_t| = \lambda 2^{1/v} \frac{\Gamma(2/v)}{\Gamma(1/v)}.$$

However, the quantity $\varepsilon_{t-i+1} \sigma_{t-i+1}^{-1}$, both with and without the absolute value operator, for $i \geq 1$, can be straightforwardly computed by the model, as the values of both the innovation and its conditional standard deviation are available. Finally, the corresponding one-step-ahead conditional variance forecast in the case of the TARCH(p,q) model is:

$$\hat{\sigma}_{t+1|t}^2 = a_0^{(t)} + \sum_{i=1}^q \left[a_i^{(t)} \varepsilon_{t-i+1}^2 \right] + \gamma^{(t)} \varepsilon_t^2 d_t + \sum_{j=1}^p \left[b_j^{(t)} \sigma_{t-j+1}^2 \right]. \quad (12)$$

Therefore, it is straightforward to compute the one-step-ahead VaR forecast, since under all distributions, we can compute the corresponding quantiles, which we then multiply by our conditional standard deviation forecast:

$$VaR_{t+1|t} = F(\alpha) \hat{\sigma}_{t+1|t}, \quad (13)$$

given that $F(\alpha)$ is the corresponding quantile of the assumed distribution, and $\hat{\sigma}_{t+1|t}$ is the forecast of conditional standard deviation at time t .

3 Evaluating the different approaches

Our objective is to test these different volatility forecasting techniques in a risk management environment. The quality of our VaR forecast and, naturally, anything else depending on these forecasts, depends on the quality of our volatility forecasts. We must, nevertheless, first choose a metric by which we will judge the "quality" of VaR forecasts. It is well known that there are many sources of error in VaR figures: sampling errors, data problems, inappropriate models or specifications, etc. All these factors will make our VaR estimate often to be biased. We must also tackle the fact that we never actually observe any realized VaR value after the event, since our model forecasts an inherently unobservable variable. We, therefore, have to monitor VaR forecasts and, afterwards, judge volatility models, not by checking whether our forecasts are realized, but whether they are consistent with subsequently realized returns given the confidence interval on which the forecasts were constructed in the first place.

Moreover, an adequate model must not only generate statistically accurate VaR forecasts, but it has to be preferred over the others. The statistical adequacy will be tested based on Kupiec's (1995) and Christoffersen's (1998) backtesting measures, while the comparison between models of equivalent ability, in terms of statistical adequacy, will be performed via a loss function.

3.1 Unconditional Coverage

Let $N = \sum_{t=1}^T I_t$ be the number of days over a T period that the portfolio loss was larger than the VaR estimate, where

$$I_{t+1} = \begin{cases} 1, & \text{if } y_{t+1} < \text{VaR}_{t+1|t} \\ 0, & \text{if } y_{t+1} \geq \text{VaR}_{t+1|t}. \end{cases}$$

Hence, N is the observed number of exceptions in the sample. As argued in Kupiec (1995), the failure number follows a binomial distribution, $N \sim B(T, p)$, and consequently the appropriate likelihood ratio statistic, under the null hypothesis that the expected exception frequency $N/T = p$, is:

$$2 \ln[(1 - \frac{N}{T})^{T-N} (\frac{N}{T})^N] - 2 \ln[(1 - p)^{T-N} p^N].$$

Asymptotically, this test is χ^2 distributed with one degree of freedom, while, in Table 3.1, we present the no rejection regions of N for various sample sizes and confidence levels. This test can reject a model for both high and low failures but, as stated by Kupiec (1995), its power is generally poor. So we turn to a more elaborate criterion.

3.2 Conditional coverage

A more complete test was made by Christoffersen (1998), who developed a likelihood ratio statistic to test the joint assumption of unconditional coverage and independence of failures. Its main advantage over the previous statistic is that it takes account of any conditionality in our forecast: if volatilities are low in some period and high in others, the forecast should respond to this clustering event. The Christoffersen procedure enables us to separate clustering effects from distributional assumption effects. His statistic is computed as:

$$-2 \ln[(1 - p)^{T-N} p^N] + 2 \ln[(1 - \pi_{01})^{n_{00}} \pi_{01}^{n_{01}} (1 - \pi_{11})^{n_{10}} \pi_{11}^{n_{11}}] \sim \chi_2^2, \quad (14)$$

Confidence level	Evaluation sample size			
	250	500	750	1000
5%	$7 \leq N \leq 19$	$17 \leq N \leq 35$	$27 \leq N \leq 49$	$38 \leq N \leq 64$
1%	$1 \leq N \leq 6$	$2 \leq N \leq 9$	$3 \leq N \leq 13$	$5 \leq N \leq 16$
0.5%	$0 \leq N \leq 4$	$1 \leq N \leq 6$	$1 \leq N \leq 8$	$2 \leq N \leq 9$
0.1%	$0 \leq N \leq 1$	$0 \leq N \leq 2$	$0 \leq N \leq 3$	$0 \leq N \leq 3$
0.01%	$0 \leq N \leq 0$	$0 \leq N \leq 0$	$0 \leq N \leq 1$	$0 \leq N \leq 1$

Table 1: Kupiec’s (1995) unconditional coverage test. No rejection regions for a 5% test size.

where n_{ij} is the number of observations with value i followed by j , for $i, j = 0, 1$ and

$$\pi_{ij} = \frac{n_{ij}}{\sum_j n_{ij}}$$

are the corresponding probabilities. $i, j = 1$ denotes that an exception has been made, while $i, j = 0$ indicates the opposite. If the sequence of I_t is independent, then the probabilities to observe or not a VaR violation in the next period must be equal, which can be written more formally as $\pi_{01} = \pi_{11} = p$. The main advantage of this test is that it can reject a VaR model that generates either too many or too few clustered violations, although it needs several hundred observations in order for the test to be accurate.

3.3 Loss Functions

The evaluation of the VaR forecasts is obtained by two hypothesis testing procedures. When the null hypothesis is not rejected, the model is characterized as an adequate model for volatility forecasting. Although the backtesting measures are used to check the adequacy of the models, they cannot conclude if a model is more accurate than another.

Lopez (1998) suggested measuring the accuracy of VaR forecasts on the basis of the distance between the observed returns and the forecasted VaR value:

$$\Psi_{t+1} = \begin{cases} 1 + (y_{t+1} - \text{VaR}_{t+1|t})^2, & \text{if } y_{t+1} < \text{VaR}_{t+1|t} \\ 0, & \text{if } y_{t+1} \geq \text{VaR}_{t+1|t}. \end{cases}$$

According to Lopez (1998), a VaR model is penalized when an exception takes place and is preferred when it minimizes the total loss, $\Psi = \sum_{t=1}^T \Psi_t$. The loss function, Ψ , incorporates both the cumulative number of the exceptions and the magnitude of them. Compared to Kupiec’s (1995) binomial loss function, Ψ adds up the magnitude term, so the larger the failure the more the penalty that is added. The VaR forecast must not overestimate or underestimate the ”true” VaR number as in both cases the firm must allocate more or less capital than it is needed. So, a model is considered appropriate if it forecasts accurately the ”true” but unobservable VaR. The ”true” VaR is proxied by the empirical distribution of the future realised returns. For example, if T observations are available for out-of-sample evaluation, then the p -quantile of them will approximate the ”true” VaR. The proposed loss function, named Quantile Loss, or QL, function, has the following form:

$$\Psi_{t+1} = \begin{cases} (y_{t+1} - \text{VaR}_{t+1|t})^2, & \text{if } y_{t+1} < \text{VaR}_{t+1|t} \\ (\text{Percentile}\{y, 100p\}_1^T - \text{VaR}_{t+1|t})^2, & \text{if } y_{t+1} \geq \text{VaR}_{t+1|t}, \end{cases}$$

At each time t a model is penalized according to either a) the distance between the $VaR_{t+1|t}$ forecast and the future 100p percentile of y_t , which remains constant through the out-of-sample period or b) the total loss magnitude $((y_{t+1} - VaR_{t+1})^2)$ term. Under the framework of the QL function, a model is considered adequate when it minimizes the QL function. Moreover, by constructing a test for the differences of the forecast errors, we can provide statistical inference for the forecasting ability of the models. Based on Diebold and Mariano (1995) and Sarma et al. (2001), we derive hypothesis testing of the forecasting ability of the models. Let $z_{t+1} = \Psi_{A_{t+1}} - \Psi_{B_{t+1}}$, where $\Psi_{A_{t+1}}$ and $\Psi_{B_{t+1}}$ are the loss function indicators of models A and B, respectively. Note that, a negative value of z_{t+1} indicates that model A is superior to model B. The Diebold-Mariano (1995) statistic is the "t-statistic" on a regression of z_{t+1} on a constant with HAC standard errors².

4 Data and Results

In order to evaluate the volatility models, we generate out-of-sample VaR forecasts for five equity indices (CAC40, DAX30, FTSE100, NIKKEI225 and S&P500), obtained from DataStream for the period of July 9th, 1987 to October 18th, 2002. For all indices, we compute daily log returns and plot them. Volatility clustering is clearly visible in Figure 1.

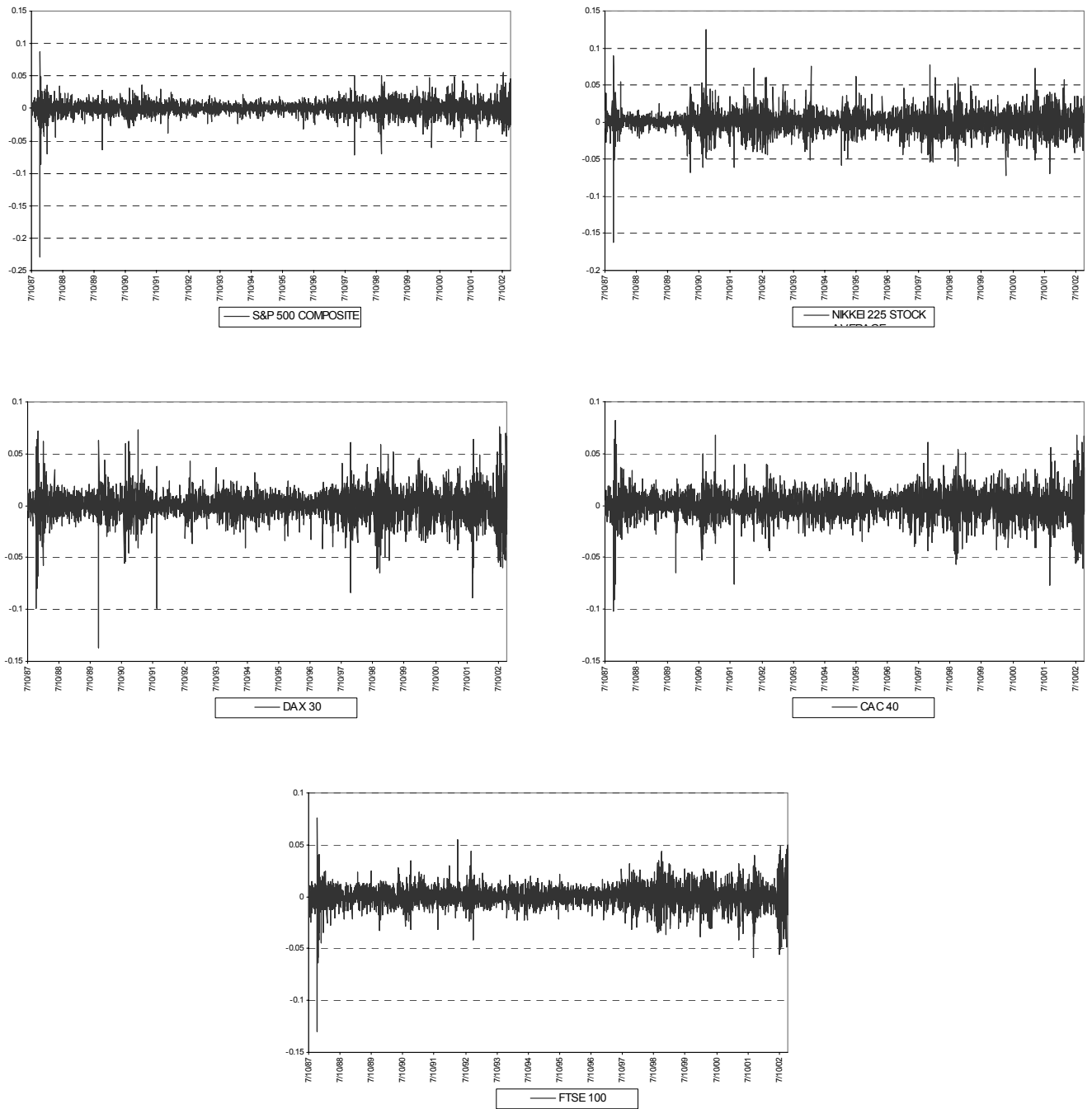
Table 2 provides summary statistics as well as the Jarque–Bera statistic for testing normality. In all cases, the null hypothesis of normality is rejected at any level of significance, as there is evidence of significant excess kurtosis and negative skewness.

	S&P 500	NIKKEI 225	DAX 30	CAC 40	FTSE 100
Mean	0.00027	-0.00026	0.00021	0.00020	0.00014
Median	0.00042	-0.00011	0.00079	0.00034	0.00048
Maximum	0.08709	0.12430	0.07553	0.08225	0.07597
Minimum	-0.22833	-0.16135	-0.13710	-0.10138	-0.13029
Std. Deviation	0.01140	0.01494	0.01455	0.01377	0.01091
Skewness	-2.28112	-0.05997	-0.57695	-0.32182	-0.87756
Kurtosis	49.28424	10.10980	9.78510	7.39398	14.39124
Jarque - Bera	347,258.8	7,936.4	7,569.2	3,143.9	21,348.6
Probability	0.0000	0.0000	0.0000	0.0000	0.0000

Table 2: Descriptive statistics of the daily log returns, for the period of July 9th, 1987 to October 18th, 2002

²For more details about heteroskedastic and autocorrelated consistent standard errors (HAC), see at White (1980) and Newey and West (1987).

Figure 1. Continuously Compounded Daily Returns of Equity Indices (S&P500, NIKKEI225, DAX30, CAC40 and FTSE100) from July 9th, 1987 to October 18th, 2002



We estimate the most frequently applied model to empirical studies, the AR(1) GARCH(1,1) specification:

$$y_t = c_0 + c_1 y_{t-1} + z_t \sigma_t \quad (15)$$

$$z_t \stackrel{iid}{\sim} D(0, 1) \quad (16)$$

$$\sigma_t^2 = a_0 + a_1 \varepsilon_{t-1}^2 + b_1 \sigma_{t-1}^2 \quad (17)$$

assuming three different density functions for z_t : the normal, the Student-t and the Generalized Error Distribution as presented in section 2. The models are estimated using the entire dataset available and results are presented in Table 3. We note, first, that the conditional variance parameters are highly significant; second, that the distribution of the z_t is significantly thicker tailed than the normal distribution and, finally, that the parameter c_1 , which allows for the autocorrelation induced by discontinuous trading, is not significant in all cases.

In what follows, we model the conditional mean as a k -th order autoregressive process and the conditional variance as a GARCH, EGARCH or TARCH process. Using equations (7, 8, 9) presented in Section 2, we will first apply ARCH processes to a GARCH, an EGARCH and a TARCH model, all with parameters (p, q) and assuming a normal underlying distribution. We apply these models to the data using parameter values of $k = 0, \dots, 4$, $p = 0, 1, 2$ and $q = 1, 2$, yielding a total of 85 models.³ Moreover, under the additional assumptions of standardized residuals following a thicker tailed distribution (Student and GED), we reapply the AR(1) model to GARCH(p, q), EGARCH(p, q) and TARCH(p, q) frameworks, yielding another 34 models. These models (a) are able to capture several characteristics of financial markets, such as volatility clustering, non-synchronous trading, leverage effect, thick tailed returns and (b) do not fail to converge frequently (at least in the case of the normal distribution). Cases, for which the numerical maximization of the log-likelihood function failed to converge more than four times⁴, were excluded. On the other hand, for the models that the non-converged cases were less than four, the VaR forecasts were computed based on previous trading day parameter estimations.

For all models and all equity indices, we used a rolling sample of 500, 1000, 1500 and 2000 observations with the same number of VaR forecasts for each sample size. We generated one-day VaR forecasts for both 95% and 99% confidence levels, as recommended by the Basel Committee. The parameters of the models are re-estimated every trading day and all tests are performed using the information that would have been available at the time the estimate was made. Klaassen (2002) and Hansen and Lunde (2003), among others, have estimated the in-sample parameters of the models and based on them, they derived the one-step-ahead volatility forecasts. However, we assume that the estimated parameters incorporate information about the trading behaviour, which alters across time. Thus, their estimation should be based on the most recently available information set.

We did not use classical in-sample model selection criteria, such as the Akaike Information Criterion (Akaike (1973)) and the Schwarz Bayesian Criterion (Schwarz (1978)), which have widely been used in the ARCH literature for model selection, because a good in-sample performance of a model does not necessarily lead to a good out-sample performance. Pagan and Schwert (1990) used

³Numerical maximization of the log-likelihood function, for the EGARCH(2,2) model, frequently failed to converge. The five EGARCH models for these parameters were, therefore, excluded.

⁴Four times were the maximum number that one model failed to converge under the normal distribution assumption.

Parameter	S&P 500	NIKKEI 225	DAX 30	CAC 40	FTSE 100
<i>Normal Distribution</i>					
c_0	0.000557 (0.000132)	0.000549 (0.000192)	0.000614 (0.000199)	0.000432 (0.000197)	0.000395 (0.000150)
c_1	0.028859 (0.017917)	0.015127 (0.019737)	0.037770 (0.019223)	0.047177 (0.017319)	0.058032 (0.017804)
a_0	1.64E-06 (5.53E-07)	4.43E-06 (1.68E-06)	6.76E-06 (3.07E-06)	5.57E-06 (1.95E-06)	3.22E-06 (1.01E-06)
a_1	0.103524 (0.037118)	0.159935 (0.049919)	0.136356 (0.035324)	0.105451 (0.018500)	0.116497 (0.026331)
b_1	0.890270 (0.030393)	0.838772 (0.040506)	0.837132 (0.033176)	0.865466 (0.023878)	0.859395 (0.026836)
Log Likelihood	12468.68	10913.64	11357.62	11397.68	12492.48
<i>Student-t Distribution</i>					
c_0	0.000594 (0.000121)	0.000275 (0.000171)	0.000711 (0.000163)	0.000531 (0.000178)	0.000409 (0.000135)
c_1	0.012507 (0.016490)	-0.006850 (0.016965)	0.017391 (0.017283)	0.042970 (0.017279)	0.049854 (0.017128)
a_0	8.77E-07 2.23E-07	2.07E-06 (5.11E-07)	2.61E-06 (6.01E-07)	3.70E-06 (8.47E-07)	2.21E-06 (4.60E-07)
a_1	0.070246 (0.008139)	0.103547 (0.011004)	0.103558 (0.011609)	0.086050 (0.010117)	0.087884 (0.010318)
b_1	0.925879 (0.007779)	0.895057 (0.009965)	0.886968 (0.011657)	0.894520 (0.012412)	0.891897 (0.011782)
v	5.60013 (0.429782)	6.097681 (0.473549)	7.947894 (0.547896)	10.49132 (1.124427)	11.07864 (0.927738)
Log Likelihood	12638.65	11076.65	11532.06	11445.73	12578.62
<i>Generalized Error Distribution</i>					
c_0	0.000560 (0.000116)	0.000306 (0.000166)	0.000733 (0.000163)	0.000454 (0.000178)	0.000432 (0.000135)
c_1	-0.000820 (0.015669)	-0.006770 (0.016497)	0.017969 (0.017162)	0.038205 (0.017206)	0.046718 (0.017111)
a_0	1.11E-06 (2.60E-07)	3.19E-06 (5.96E-07)	4.13E-06 (8.16E-07)	4.78E-06 (9.61E-07)	2.67E-06 (5.60E-07)
a_1	0.078919 (0.007099)	0.119683 (0.012156)	0.118569 (0.011676)	0.094767 (0.010579)	0.098021 (0.011616)
b_1	0.915449 (0.007706)	0.875434 (0.011438)	0.866873 (0.013691)	0.879949 (0.013724)	0.879750 (0.013507)
v	1.250571 (0.024604)	1.305231 (0.023113)	1.395110 (0.021377)	1.552809 (0.035018)	1.545649 (0.019470)
Log Likelihood	12616.52	11040.99	11471.80	11429.26	12541.29
Observations	3853	3767	3835	3826	3857

Table 3: Parameter estimates of the AR(1) GARCH(1,1) model for the five indices, using the entire dataset (9th July 1987 to 18th October 2002) and assuming three different distributions for the standardized residuals. Standard errors are presented in parentheses.

a collection of parametric and nonparametric models of volatility forecasting (including Kernels, Fourier series and two-stage least squares regressions) and found that although the non-parametric methods did good job as concerns the in-sample forecasting, the parametric models yielded superior out-of-sample forecasting ability. Hansen and Lunde (2003) noted that "a model, which accommodates a significant in-sample relation need not result in better out-of-sample forecasts, compared to the forecasts obtained from a more parsimonious model". Given these facts, we do not restrict our analysis to models that have been pre-selected by any in-sample method of model selection.

Due to the enormous size of the results that have been generated, we will proceed as follows. First, we present the VaR forecasts estimated using the standard normal distribution assumption for all sample sizes and we will examine whether the performance of the models is invariant to the choice of the sample size. We will then go on to present the contributions of alternative distributions, namely the standardized Student-t and the generalized error distribution, to the VaR framework.

4.1 Normal Distribution

The assumption of normality produces very weak results: the vast majority of models, irrespective of sample size chosen, understate the true one-day 95% VaR estimate and the exception rates are higher than the 5% predicted level. The performance of the models is heavily dependent on the stock index, with the Nikkei 225 index faring somewhat better than the others. The p-values for both conditional and unconditional coverage are relatively low for all indices, with the European markets showing the lowest probability values.

More specifically, we observe that, in 2000 and 1500 sample sizes, none out of the 85 models produces an exception rate below the predicted 5%. In very few cases, the estimated exception rate is close enough to the expected one (e.g. for the S&P 500 and the NIKKEI 225 indices, the AR(1) GARCH(1,1) and the AR(1) EGARCH(1,2) models achieve the closest fit, respectively). Moreover, in almost all cases, the different specification of the conditional mean or the introduction of only ARCH terms in the conditional variance does not improve significantly either the unconditional or the conditional coverage of the models. In the case of the S&P 500, the average p-value for the null hypothesis of unconditional coverage of the GARCH(1,1) family is 56%, while the best performing family for the NIKKEI 225 and the CAC 40 indices is the EGARCH(1,2) one, with corresponding average p-values equal to 79,1% and 13,6%, respectively. Similarly, for the DAX 30 and the FTSE 100 indices, we conclude that the null hypothesis of correct unconditional coverage is rejected for all the models at the 10% level of significance, while the hypothesis of conditional coverage is rejected at the 15% level. It is also interesting to point out that, although the EGARCH structure captures better the asymmetric relation between returns and volatility changes and produces better unconditional VaR predictions than the TARARCH family, it does not produce more accurate 95% daily VaR forecasts than the much simpler and less flexible GARCH (p, q) structure.

As for the effect of the sample size, it seems that as the latter gets smaller, the results improve at all levels. This is due to the weight given to the latest observations when using a smaller sample: the *smaller* sample size captures only the *latest* market movements and the produced VaR estimate is, hence, less dependent on the long run volatility trend, in the same sense that a 200-point moving average is smoother than a 50-point one. Consequently, the VaR estimate is less often rejected

	Unconditional Coverage				Conditional Coverage			
	95% daily VaR forecasts							
Index	Size	Model		Pr.	Size	Model		Pr.
S&P 500	2000	AR(1)	GARCH(1,1)	72.25%	2000	AR(1)	GARCH(1,1)	79.48%
NIKKEI 225	1000	AR(0)	EGARCH(1,1)	96.95%	500	AR(3)	EGARCH(1,2)	93.14%
DAX 30	500	AR(3)	TARCH(1,2)	32.92%	500	AR(3)	TARCH(1,2)	57.61%
CAC 40	1000	AR(1)	EGARCH(1,2)	54.44%	1000	AR(1)	EGARCH(1,2)	56.38%
FTSE 100	500	AR(0)	EGARCH(2,1)	33.72%	500	AR(1)	TARCH(2,1)	53.27%
	99% daily VaR forecasts							
Index	Size	Model		Pr.	Size	Model		Pr.
S&P 500	1000	AR(0)	EGARCH(1,2)	1.40%	1000	AR(0)	EGARCH(1,2)	1.19%
NIKKEI 225	2000	AR(0)	EGARCH(2,1)	31.87%	2000	AR(0)	EGARCH(2,1)	45.51%
DAX 30	500	AR(3)	GARCH(1,1)	13.92%	500	AR(3)	GARCH(1,1)	23.40%
CAC 40	500	AR(2)	GARCH(1,2)	8.70%	500	AR(2)	GARCH(1,2)	15.60%
FTSE 100	1000	AR(0)	EGARCH(1,1)	22.56%	1000	AR(0)	EGARCH(1,1)	34.59%

Table 4: Best performed models and the relative probability values of coverage metrics for normally distributed innovations.

at the 95% confidence level, for all models and indices. Such behavioural trend is traced even at the 1000 observations sample, and becomes clearer at the 500 observations level, at least for three out of five indices. The only exception to this "rule of thumb" is the S&P 500, where the highest p-values are achieved for the largest sample size.

For the 99% VaR estimates, the exception rates are all higher than the predicted 1% for all sample sizes, with the sample size having no effect at all in improving results. As for the coverage metrics, they are very low, ranging from 45.51% conditional coverage, for the EGARCH family on the Nikkei 225 index, down to 1.19% for the same metric with the same modeling family applied to the S&P 500 index. Such performance, at least for the latter index, is suggestive of a *hump* in the tail distribution of returns: this means it seriously underestimates risk at a high level of confidence whereas it produces somewhat better results at the lower confidence level, yielding respectable p-values for the 95% VaR metric but extremely low ones for the 99% one.

Table 4 shows the most appropriate⁵ models according to conditional and unconditional coverage, defined as the one with the highest p-value among all the estimated models for all sample sizes⁶. Both coverage measures suggest the same models in most of the cases. For all indices, p-values are deemed relatively low, with the weakest ones for the European indices: we believe this is due to the inadequate description of such data using the normal distribution. The normal distribution performs best with the Nikkei 225 index in both the 95% and the 99% level, since the normality assumption does not significantly increase exception rates, although, through the Jarque - Bera test, it is strictly rejected as a null hypothesis.

The main points of this thorough investigation of ARCH processes can be summarized in the

⁵Brooks and Persaud (2003b) also ranked their competitive models, based on the following rule. They excluded the volatility forecasting techniques that produced exception rate greater than the expected one, while they preferred the models which generated proportion of failures closer to the expected exception frequency.

⁶In case of identical p-values being achieved by more than one model, only one of these models is given. In the appendix we present the results for all models and distribution assumptions.

following points. First of all, the GARCH term in the conditional variance plays an important role since it provides models with a longer memory and a more flexible lag structure. Secondly, the choice of the sample size is important in generating adequate conditional variance forecasts. To our best knowledge, there is no persistent method in the literature for choosing the appropriate sample size. In most studies, researchers make an arbitrary choice of a rolling sample. Nevertheless, Engle et al. (1993) applied three different sample sizes of 300, 1000 and 5000 observations and noted, "that some restrictions on the length of forecasting sample may be profitable". Our study reaches to a similar result, as different sample sizes seem to be the most appropriate for different indices.

Generally speaking, the conditional mean specification is invariant to the VaR forecast accuracy, because the adequacy of the models does not depend on the autoregressive order. However, there are some cases where the mean specification offers an increasing precision in VaR predictions. An example worth noting is the AR(2) GARCH(1,2) model with a 500-point sample size for the CAC 40 index, which produces the most accurate forecasts for the 99% daily VaR, though the rest of the models of the GARCH(1,2) family, as presented in Table 8.A in Appendix, do not produce adequate forecasts.

Lastly, the assumption of normally distributed standardized residuals provides us with adequate forecasts only at the 95% level. For each index, there is at least one family of ARCH processes, producing convenient predictions. On the other hand, normal distribution fails to produce useful estimations at the 99% level. Under the assumption that the residuals are conditionally normal distributed, their unconditional distribution has thicker tails than the normal one. However, the degree of leptokurtosis induced by the ARCH process does not often capture all of the leptokurtosis present in the data. In the next section, we introduce non-normality for the conditional distribution of the residuals in order to produce unconditional distribution with thicker tails.

4.2 Leptokurtic Distributions

As we have already pointed out, the normal distribution assumption does not generate accurately one-step-ahead VaR numbers. In order to model more adequately the thickness of tails, we will use two different distributional assumptions for the standardized residuals: Student's-t and the Generalized Error Distribution (GED). In the previous section, we found out that the precision of forecasts did not depend on the various structures of the conditional mean. We will, therefore, apply only the AR(1) specification for the conditional mean, consistent with the non-synchronous trading effect. Models that failed to converge more than four times are totally excluded from the study.

Turning now to the distribution used for the innovations, it seems that the Student's-t is a better choice overall, based on Tables 5 and 6 which present the models with the highest p-values for the two leptokurtic distributions. Given that the fat tails are observed in all indices, the normal distribution fails prominently if no leptokurtosis is added implicitly to a model. The GED behaves similarly to the normal for the 95% confidence level but yields better results at the 99% confidence level: given that such a distribution exhibits thicker tails than the normal, it seems that the *hump* problem is resolved. Its p-values for both conditional and unconditional coverage are, however, still in the 20-70% range for the S&P 500, DAX 30, CAC 40 and FTSE 100 indices. The use of the Student's-t distribution, improves probability values, as they range from 80% to 95%, for all volatility models and all sample sizes, for these four indices. In the case of the Nikkei 225, it seems

	Unconditional Coverage				Conditional Coverage			
	95% daily VaR forecasts							
Index	Size	Model		Pr.	Size	Model		Pr.
S&P 500	1500	AR(1)	GARCH(0,2)	97.03%	1000	AR(1)	EGARCH(0,1)	93.39%
NIKKEI 225	500	AR(1)	EGARCH(1,2)	6.56%	500	AR(1)	EGARCH(1,2)	14.69%
DAX 30	500	AR(1)	EGARCH(0,2)	97.86%	1500	AR(1)	EGARCH(1,2)	93.21%
CAC 40	1500	AR(1)	EGARCH(2,1)	97.43%	1000	AR(1)	EGARCH(2,1)	93.25%
FTSE 100	1500	AR(1)	GARCH(1,2)	92.78%	1500	AR(1)	TARCH(1,1)	93.61%
	99% daily VaR forecasts							
Index	Size	Model		Pr.	Size	Model		Pr.
S&P 500	1000	AR(1)	EGARCH(2,1)	91.30%	1500	AR(1)	EGARCH(0,2)	82.35%
NIKKEI 225	500	AR(1)	EGARCH(2,1)	36.26%	500	AR(1)	EGARCH(2,1)	58.61%
DAX 30	1500	AR(1)	EGARCH(1,1)	93.43%	1000	AR(1)	EGARCH(1,1)	82.56%
CAC 40	1500	AR(1)	TARCH(1,1)	95.11%	1000	AR(1)	GARCH(1,2)	82.61%
FTSE 100	1000	AR(1)	TARCH(1,1)	92.04%	1000	AR(1)	TARCH(1,1)	80.91%

Table 5: Best performed models and the relative probability values of coverage metrics for t-distributed innovations.

that the choice of the Student-t distribution overcorrects for thick tails and, consequently, either the Normal (at the 95% level), or the GED (at the 99% level), provide better estimates.

For the 95% VaR level, under Student's-t assumption, there is evidence that GARCH or EGARCH models produce better forecasts than the corresponding TARCH model, while for the GED this is the case only for the GARCH family. By increasing the confidence level of VaR, the results are become more mixed because both symmetric and asymmetric models have been selected as adequate. The choice of the sample is turning out to be one of the most important factors in the VaR framework, because the selected model for one sample size is not adequate for the others, while there are extreme cases that produce totally contrary results. For example, the best performing model for the S&P500 index at the 95% VaR level is the AR(1) GARCH(0,2), when assuming the Student-t distribution and using a sample size equal to 2000 observations. Yet, had we used the 500 observations sample size, this model would have been characterized as one of the worst. This conclusion is common for all three distribution assumptions and both confidence levels, revealing the importance of the sample size. Our findings are in line with Brooks and Persaud (2002) who argued that the effect of the sample size on the performance of the models is not clear, since there is no optimal sample length for *all* the volatility forecasting techniques which have been investigated.

Putting together a leptokurtic distribution, to capture fat tails, and a low complexity volatility model, to capture volatility clustering, yields the best combination for the 99% VaR level as the parameter, which models the degree of kurtosis, has the ability to capture the extreme events. As far as the case of 95% VaR level, the excellent performance of the volatility models under the Student-t distribution was a surprise for us, as researchers always reported an overestimation of the risk (see Billio and Pelizzon (2000) and Guermat and Harris (2002) for more information). These results are summarized in Table 7, which demonstrates the best performed models and their corresponding p-values for all distribution assumptions and sample lengths.

	Unconditional Coverage				Conditional Coverage			
	95% daily VaR forecasts							
Index	Size	Model		Pr.	Size	Model		Pr.
S&P 500	2000	AR(1)	GARCH(1,1)	72.25%	2000	AR(1)	GARCH(1,1)	79.48%
NIKKEI 225	500	AR(1)	GARCH(1,2)	61.47%	500	AR(1)	GARCH(1,2)	52.22%
DAX 30	500	AR(1)	GARCH(2,2)	72.92%	500	AR(1)	GARCH(2,2)	57.66%
CAC 40	1500	AR(1)	GARCH(2,1)	21.79%	1500	AR(1)	GARCH(2,1)	38.84%
FTSE 100	500	AR(1)	GARCH(2,2)	20.47%	500	AR(1)	GARCH(2,2)	38.15%
	99% daily VaR forecasts							
Index	Size	Model		Pr.	Size	Model		Pr.
S&P 500	2000	AR(1)	GARCH(1,1)	57.23%	2000	AR(1)	GARCH(1,1)	66.25%
NIKKEI 225	500	AR(1)	GARCH(1,2)	93.73%	1500	AR(1)	GARCH(0,1)	82.87%
DAX 30	2000	AR(1)	TARCH(1,1)	54.33%	2000	AR(1)	TARCH(1,1)	64.44%
CAC 40	2000	AR(1)	TARCH(1,1)	19.78%	2000	AR(1)	TARCH(1,1)	26.41%
FTSE 100	1500	AR(1)	TARCH(1,1)	57.88%	1500	AR(1)	TARCH(1,1)	66.64%

Table 6: Best performed models and the relative probability values of coverage metrics for GED-distributed innovations.

4.3 Model Selection

The two backtesting measures can not compare different VaR models directly, as a greater p-value of a model, does not indicate the superiority of that model among its competitors. Therefore, in order to evaluate the reported differences statistically, for each model that has a p-value for both tests greater than 10% we compute the QL function and carry out the equality test that was described in section 3.3. We preferred a high cut-off point for the p-value, in order to ensure that "successful" models will not statistically over or under estimate the "true" VaR, since a high (low) VaR estimation implies that a firm must allocate more (less) capital than it is actually necessary. Under this framework, the proposed loss function evaluates all the models that converged both conditionally and unconditionally.

Table 8 summarizes the results of the loss function approach. For each index and confidence level we present the six⁷ models with the lowest loss value, while we compare the first one with the rest of them based on the procedure that is explained in section 3.3. For example, for the S&P 500 index and 95% VaR level, the best performed model is the AR(1) EGARCH(0,1) which is statistically different from the other five according to the corresponding t-statistics. In most of the cases, the model with the lowest loss value is significantly better than the others, as the t-statistic indicates, while the preferred distribution is the Student-t. Moreover, the simplest asymmetric volatility specifications seems to be preferred over the most complex ones, while there is no indication which sample length is the optimal. To summarize, the combination of a leptokurtic distribution and a simple asymmetric volatility model yields the best combination.

⁷The results for all the compared models are available from the authors upon request.

	Unconditional Coverage					Conditional Coverage				
	<i>95% daily VaR forecasts</i>					<i>95% daily VaR forecasts</i>				
	Index	Size	Distribution	Model	Pr.	Size	Distribution	Model	Pr.	
	S&P500	2000	T	AR(1) GARCH(0,2)	97.03%	2000	T	AR(1) GARCH(0,1)	93.39%	
	NIKKEI 225	1000	N	AR(0) EGARCH(1,1)	96.95%	500	N	AR(3) EGARCH(1,2)	93.14%	
	DAX 30	500	T	AR(1) EGARCH(0,2)	97.86%	1500	T	AR(1) EGARCH(1,2)	93.21%	
	CAC 40	1500	T	AR(1) EGARCH(1,2)	97.43%	1000	T	AR(1) EGARCH(2,1)	93.25%	
	FTSE 100	1500	T	AR(1) GARCH(1,2)	92.78%	1500	T	AR(1) TARCH(1,1)	93.61%	
	<i>99% daily VaR forecasts</i>					<i>99% daily VaR forecasts</i>				
	Index	Size	Distribution	Model	Pr.	Size	Distribution	Model	Pr.	
	S&P 500	1000	T	AR(1) EGARCH(2,1)	91.30%	1500	T	AR(1) EGARCH(0,2)	82.35%	
	NIKKEI 225	2000	G	AR(1) EGARCH(0,1)	93.73%	2000	G	AR(1) GARCH(0,1)	82.87%	
	DAX 30	2000	T	AR(1) GARCH(0,2)	93.43%	2000	T	AR(1) GARCH(0,2)	82.56%	
	CAC 40	1500	T	AR(1) TARCH(1,1)	95.11%	1000	T	AR(1) GARCH(1,2)	82.61%	
	FTSE 100	1000	T	AR(1) TARCH(1,1)	92.04%	1000	T	AR(1) TARCH(1,1)	80.91%	

Table 7: Best performed models according to probability values of coverage metrics for Normal (N), Student (T) or GED (G) distributed innovations.

S&P 500									
95% daily VaR forecasts					99% daily VaR forecasts				
Model	Size	Distr.	Loss Value	t-stat.	Model	Size	Distr.	Loss Value	t-stat.
AR(1) EGARCH(0,1)	2000	T	5.61%	-	AR(1) GARCH(0,1)	500	T	16.00%	-
AR(1) EGARCH(0,1)	1500	T	6.51%	-1.95***	AR(1) EGARCH(0,1)	500	T	16.04%	-0.08
AR(1) GARCH(0,1)	1500	T	7.10%	-4.13*	AR(1) EGARCH(0,1)	1500	T	17.03%	-0.51
AR(1) GARCH(0,1)	2000	T	7.28%	-3.76*	AR(1) EGARCH(0,1)	1000	T	17.24%	-1.05
AR(1) EGARCH(0,1)	1000	T	7.54%	-4.83	AR(1) GARCH(0,1)	1000	T	18.21%	-1.90***
AR(1) EGARCH(0,1)	500	T	7.54%	-4.63*	AR(1) EGARCH(0,1)	2000	T	18.24%	-1.39
NIKKEI 225									
95% daily VaR forecasts					99% daily VaR forecasts				
Model	Size	Distr.	Loss Value	t-stat.	Model	Size	Distr.	Loss Value	t-stat.
AR(1) GARCH(0,1)	500	GED	4.85%	-	AR(1) EGARCH(0,1)	1500	GED	4.28%	-
AR(1) GARCH(0,2)	1000	GED	5.41%	-2.34**	AR(1) EGARCH(0,1)	1000	GED	5.08%	-3.66*
AR(1) TARCH(0,2)	1000	GED	5.64%	-2.82*	AR(1) EGARCH(0,1)	2000	GED	5.61%	-5.40*
AR(0) GARCH(0,2)	1000	Normal	5.72%	-1.53	AR(1) EGARCH(0,1)	500	GED	6.71%	-4.56*
AR(1) GARCH(0,2)	500	GED	5.95%	-4.11*	AR(1) EGARCH(0,2)	1500	GED	7.31%	-7.75*
AR(1) EGARCH(2,1)	500	GED	6.07%	-3.61*	AR(1) EGARCH(0,2)	1000	GED	7.37%	-8.96*
DAX 30									
95% daily VaR forecasts					99% daily VaR forecasts				
Model	Size	Distr.	Loss Value	t-stat.	Model	Size	Distr.	Loss Value	t-stat.
AR(1) EGARCH(0,1)	2000	T	7.39%	-	AR(1) EGARCH(0,1)	2000	T	22.08%	-
AR(1) EGARCH(0,1)	500	T	11.86%	-8.21*	AR(1) EGARCH(0,1)	1500	T	27.47%	-4.37*
AR(1) GARCH(0,1)	1500	T	13.20%	-7.17*	AR(1) EGARCH(0,2)	2000	T	35.10%	-7.13*
AR(1) GARCH(0,1)	2000	T	14.64%	-4.91*	AR(1) GARCH(0,1)	1500	T	38.26%	-8.47*
AR(1) GARCH(0,1)	1000	T	15.34%	-7.74*	AR(1) EGARCH(0,1)	500	T	40.70%	-7.47*
AR(1) TARCH(0,1)	2000	T	15.65%	-5.71*	AR(1) GARCH(0,1)	2000	T	41.20%	-5.18*
CAC 40									
95% daily VaR forecasts					99% daily VaR forecasts				
Model	Size	Distr.	Loss Value	t-stat.	Model	Size	Distr.	Loss Value	t-stat.
AR(1) EGARCH(0,1)	2000	T	3.96%	-	AR(1) GARCH(0,2)	2000	T	21.66%	-
AR(1) EGARCH(0,1)	1500	T	4.73%	-7.54*	AR(1) GARCH(0,2)	1500	T	26.34%	-12.59*
AR(1) EGARCH(0,2)	2000	T	4.81%	-4.09*	AR(1) EGARCH(1,2)	2000	T	27.85%	-5.48*
AR(1) GARCH(0,1)	2000	T	5.10%	-3.89*	AR(1) EGARCH(1,1)	2000	T	27.95%	-5.86*
AR(1) GARCH(0,1)	1500	T	5.44%	-5.53*	AR(1) EGARCH(2,1)	2000	T	27.96%	-5.68*
AR(1) EGARCH(0,2)	1500	T	5.56%	-6.38*	AR(1) GARCH(1,1)	1500	T	30.37%	-5.48*
FTSE 100									
95% daily VaR forecasts					99% daily VaR forecasts				
Model	Size	Distr.	Loss Value	t-stat.	Model	Size	Distr.	Loss Value	t-stat.
AR(1) GARCH(0,1)	1500	T	6.30%	-	AR(1) EGARCH(2,1)	1500	T	23.19%	-
AR(1) GARCH(0,2)	1500	T	7.66%	-5.14*	AR(1) EGARCH(1,2)	1500	T	23.32%	-0.69
AR(1) EGARCH(1,2)	1500	T	7.70%	-3.66*	AR(1) EGARCH(1,1)	2000	T	23.68%	-0.92
AR(1) EGARCH(1,1)	1500	T	7.72%	-4.12*	AR(1) EGARCH(1,2)	2000	T	23.75%	-1.06
AR(1) EGARCH(1,1)	2000	T	7.79%	-3.76*	AR(1) EGARCH(2,1)	2000	T	23.80%	-1.20
AR(1) EGARCH(1,2)	2000	T	7.85%	-3.65*	AR(1) GARCH(1,1)	1500	T	26.19%	-1.82***

Table 8: Statistical comparison between the six best performed models according to the quantile loss function. *, **, *** indicate significance at the 1%, 5% and 10% levels, respectively. The t-statistics are calculated according to the procedure that is explained in section 3.3.

5 Conclusions

Following the extensive and detailed investigation of a plethora of volatility modeling techniques, briefly presented in the preceding sections, a number of comments are of order, aiming to summarize our results and give, both to the researcher and the practitioner, some fundamental guidelines with which to proceed in VaR estimation.

We have strong indications that the mean process specification plays no important role. Trying to extract autoregressive phenomena from the returns such that only the underlying volatility is left in the residuals, we experimented with a number of AR processes. Our results show that such a methodology does not add anything significant to the VaR framework other than complexity in the estimation procedure. Moreover, using only an ARCH term (without any lagged conditional variances) yields acceptable results only when residuals are modeled under either the Student's-t distribution or the GED; it is never the case for a Normal distribution. Generally speaking, in the VaR framework the leptokurtic distributions and especially the Student's-t, are more appropriate than the Normal assumption, as they generate more accurate forecasts, while there is no volatility model which is clearly superior than the others. However, under the evaluation framework that was developed based on the proposed quantile loss function, there is strong evidence that the combination of the Student-t distribution with the simplest EGARCH models produces the most adequate VaR forecasts for the majority of the markets. Furthermore, the size of the rolling sample used in estimation turns out to be rather important: in simpler models and low confidence levels a sample size smaller than 2000 improves probability values. In more complex models, where leptokurtic distributions are used or where the confidence level chosen is high, a small sample size may lead to lack of convergence in the estimation algorithms. Finally, there is no consistent relation between the sample sizes and the optimal models, as we observe significant differences in the VaR forecasts for the same model under the four sample sizes.

6 References

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Appendix

Table (1.A). Normal Distribution, 2000 sample size, 95%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(0)GARCH(0,1)	8.96%	0.00%	0.00%	6.11%	3.79%	0.96%	8.28%	0.00%	0.00%	7.72%	0.00%	0.00%	7.97%	0.00%	0.00%
AR(1)GARCH(0,1)	8.96%	0.00%	0.00%	6.11%	3.79%	0.96%	8.39%	0.00%	0.00%	7.72%	0.00%	0.00%	7.92%	0.00%	0.00%
AR(2)GARCH(0,1)	8.96%	0.00%	0.00%	6.23%	2.26%	0.51%	8.50%	0.00%	0.00%	7.67%	0.00%	0.00%	8.02%	0.00%	0.00%
AR(3)GARCH(0,1)	9.01%	0.00%	0.00%	6.17%	2.94%	0.70%	8.45%	0.00%	0.00%	7.61%	0.00%	0.00%	8.02%	0.00%	0.00%
AR(4)GARCH(0,1)	8.96%	0.00%	0.00%	6.17%	2.94%	0.70%	8.66%	0.00%	0.00%	7.56%	0.00%	0.00%	8.02%	0.00%	0.00%
AR(0)GARCH(0,2)	8.09%	0.00%	0.00%	5.83%	11.88%	0.86%	7.52%	0.00%	0.00%	7.34%	0.00%	0.01%	7.65%	0.00%	0.00%
AR(1)GARCH(0,2)	8.09%	0.00%	0.00%	5.83%	11.88%	0.86%	7.57%	0.00%	0.00%	7.17%	0.01%	0.03%	7.65%	0.00%	0.00%
AR(2)GARCH(0,2)	8.04%	0.00%	0.00%	5.94%	7.72%	0.49%	7.57%	0.00%	0.00%	7.28%	0.00%	0.01%	7.65%	0.00%	0.00%
AR(3)GARCH(0,2)	8.20%	0.00%	0.00%	5.89%	9.62%	0.65%	7.57%	0.00%	0.00%	7.34%	0.00%	0.01%	7.65%	0.00%	0.00%
AR(4)GARCH(0,2)	8.26%	0.00%	0.00%	5.89%	9.62%	0.65%	7.52%	0.00%	0.00%	7.28%	0.00%	0.01%	7.65%	0.00%	0.00%
AR(0)GARCH(1,1)	5.40%	43.90%	56.23%	6.17%	2.94%	8.36%	5.99%	5.77%	15.04%	6.08%	4.05%	7.80%	6.35%	1.00%	0.24%
AR(1)GARCH(1,1)	5.18%	72.25%	79.48%	6.17%	2.94%	8.36%	6.05%	4.56%	12.18%	6.08%	4.05%	7.80%	6.14%	2.94%	0.80%
AR(2)GARCH(1,1)	5.23%	64.53%	74.32%	6.23%	2.26%	6.55%	6.10%	3.58%	9.75%	6.13%	3.16%	6.60%	6.19%	2.28%	0.72%
AR(3)GARCH(1,1)	5.34%	50.30%	62.51%	6.28%	1.72%	5.07%	6.10%	3.58%	9.75%	6.08%	4.05%	5.25%	6.19%	2.28%	0.72%
AR(4)GARCH(1,1)	5.29%	57.20%	44.47%	6.23%	2.26%	6.55%	5.99%	5.77%	15.04%	6.24%	1.87%	3.26%	6.30%	1.33%	0.56%
AR(0)GARCH(1,2)	5.45%	38.00%	49.95%	6.17%	2.94%	6.66%	5.99%	5.77%	13.23%	6.02%	5.14%	6.00%	5.98%	6.05%	1.00%
AR(1)GARCH(1,2)	5.29%	57.20%	44.47%	6.23%	2.26%	5.11%	6.05%	4.56%	11.17%	6.08%	4.05%	5.25%	6.25%	1.75%	0.30%
AR(2)GARCH(1,2)	5.40%	43.90%	56.23%	6.23%	2.26%	5.11%	6.16%	2.78%	7.67%	6.13%	3.16%	4.53%	6.09%	3.77%	0.87%
AR(3)GARCH(1,2)	5.18%	72.25%	79.48%	6.17%	2.94%	6.66%	6.10%	3.58%	10.34%	6.13%	3.16%	4.53%	6.19%	2.28%	1.40%
AR(4)GARCH(1,2)	5.29%	57.20%	44.47%	6.23%	2.26%	5.11%	6.16%	2.78%	7.67%	6.35%	1.08%	2.23%	6.14%	2.94%	1.58%
AR(0)GARCH(2,1)	5.61%	23.50%	18.48%	6.11%	3.79%	5.59%	6.21%	2.14%	6.65%	5.97%	6.48%	10.49%	6.14%	2.94%	1.58%
AR(1)GARCH(2,1)	5.23%	64.53%	49.27%	6.11%	3.79%	5.59%	6.27%	1.64%	5.02%	6.13%	3.16%	6.60%	6.09%	3.77%	1.77%
AR(2)GARCH(2,1)	5.40%	43.90%	34.87%	6.17%	2.94%	4.25%	6.32%	1.24%	4.08%	6.13%	3.16%	6.60%	6.19%	2.28%	1.40%
AR(3)GARCH(2,1)	5.50%	32.64%	57.55%	6.23%	2.26%	5.11%	6.10%	3.58%	9.75%	6.24%	1.87%	3.26%	6.09%	3.77%	1.77%
AR(4)GARCH(2,1)	5.40%	43.90%	56.23%	6.17%	2.94%	6.66%	6.21%	2.14%	6.65%	6.24%	1.87%	3.26%	6.14%	2.94%	1.58%
AR(0)GARCH(2,2)	5.72%	16.38%	35.74%	6.23%	2.26%	6.55%	6.10%	3.58%	10.34%	6.19%	2.44%	3.87%	6.14%	2.94%	0.80%
AR(1)GARCH(2,2)	5.61%	23.50%	46.52%	6.23%	2.26%	5.11%	6.10%	3.58%	10.34%	6.13%	3.16%	4.53%	6.19%	2.28%	0.72%
AR(2)GARCH(2,2)	5.61%	23.50%	43.44%	6.28%	1.72%	3.87%	6.21%	2.14%	6.24%	6.24%	1.87%	3.26%	6.30%	1.33%	0.27%
AR(3)GARCH(2,2)	5.50%	32.64%	43.82%	6.17%	2.94%	6.66%	6.10%	3.58%	10.34%	6.13%	3.16%	4.53%	6.30%	1.33%	0.27%
AR(4)GARCH(2,2)	5.56%	27.81%	52.07%	6.17%	2.94%	6.66%	6.16%	2.78%	8.34%	6.13%	3.16%	4.53%	6.25%	1.75%	0.64%
AR(0)TARCH(0,1)	9.07%	0.00%	0.00%	6.23%	2.26%	0.10%	8.28%	0.00%	0.00%	7.45%	0.00%	0.00%	7.97%	0.00%	0.00%
AR(1)TARCH(0,1)	9.07%	0.00%	0.00%	6.23%	2.26%	0.10%	8.28%	0.00%	0.00%	7.45%	0.00%	0.00%	7.97%	0.00%	0.00%
AR(2)TARCH(0,1)	9.07%	0.00%	0.00%	6.11%	3.79%	0.19%	8.23%	0.00%	0.00%	7.45%	0.00%	0.00%	8.19%	0.00%	0.00%
AR(3)TARCH(0,1)	8.96%	0.00%	0.00%	6.17%	2.94%	0.14%	8.23%	0.00%	0.00%	7.50%	0.00%	0.00%	8.08%	0.00%	0.00%
AR(4)TARCH(0,1)	9.07%	0.00%	0.00%	6.17%	2.94%	0.14%	8.28%	0.00%	0.00%	7.50%	0.00%	0.00%	8.19%	0.00%	0.00%
AR(0)TARCH(0,2)	8.20%	0.00%	0.00%	5.89%	9.62%	0.04%	7.41%	0.00%	0.00%	7.17%	0.01%	0.03%	7.65%	0.00%	0.00%
AR(1)TARCH(0,2)	8.26%	0.00%	0.00%	5.89%	9.62%	0.04%	7.47%	0.00%	0.00%	7.23%	0.00%	0.02%	7.86%	0.00%	0.00%
AR(2)TARCH(0,2)	8.20%	0.00%	0.00%	5.83%	11.88%	0.05%	7.52%	0.00%	0.00%	7.28%	0.00%	0.01%	7.86%	0.00%	0.00%
AR(3)TARCH(0,2)	8.04%	0.00%	0.00%	5.83%	11.88%	0.05%	7.52%	0.00%	0.00%	7.28%	0.00%	0.01%	7.81%	0.00%	0.00%
AR(4)TARCH(0,2)	8.09%	0.00%	0.00%	5.83%	11.88%	0.05%	7.47%	0.00%	0.00%	7.28%	0.00%	0.01%	7.81%	0.00%	0.00%
AR(0)TARCH(1,1)	6.15%	2.78%	5.81%	5.83%	11.88%	10.06%	5.94%	7.24%	10.16%	5.81%	12.34%	27.00%	5.98%	6.05%	10.47%
AR(1)TARCH(1,1)	6.15%	2.78%	5.81%	5.77%	14.55%	12.56%	5.94%	7.24%	10.16%	5.91%	8.10%	19.88%	6.14%	2.94%	4.56%
AR(2)TARCH(1,1)	6.21%	2.15%	4.46%	5.72%	17.66%	15.48%	5.99%	5.77%	8.00%	5.91%	8.10%	19.88%	6.14%	2.94%	4.56%
AR(3)TARCH(1,1)	6.04%	4.55%	6.06%	5.72%	17.66%	15.48%	6.05%	4.56%	6.22%	5.91%	8.10%	19.88%	6.19%	2.28%	3.87%
AR(4)TARCH(1,1)	6.10%	3.57%	7.49%	5.72%	17.66%	15.48%	5.99%	5.77%	8.00%	5.86%	10.04%	23.31%	6.19%	2.28%	3.87%
AR(0)TARCH(1,2)	6.15%	2.78%	3.54%	5.83%	11.88%	3.87%	5.99%	5.77%	15.31%	5.86%	10.04%	19.03%	5.92%	7.56%	7.74%
AR(1)TARCH(1,2)	5.88%	8.96%	6.38%	5.72%	17.66%	6.21%	6.10%	3.58%	10.34%	6.02%	5.14%	12.11%	6.14%	2.94%	2.84%
AR(2)TARCH(1,2)	6.04%	4.55%	6.06%	5.89%	9.62%	7.97%	6.05%	4.56%	12.66%	5.91%	8.10%	16.58%	6.03%	4.80%	6.09%
AR(3)TARCH(1,2)	6.04%	4.55%	6.06%	5.72%	17.66%	15.48%	5.99%	5.77%	15.31%	6.02%	5.14%	12.11%	6.03%	4.80%	6.09%
AR(4)TARCH(1,2)	6.15%	2.78%	1.65%	5.83%	11.88%	10.06%	6.10%	3.58%	10.34%	5.91%	8.10%	16.58%	6.09%	3.77%	5.30%
AR(0)TARCH(2,1)	6.04%	4.55%	6.06%	5.83%	11.88%	3.87%	6.05%	4.56%	9.72%	6.02%	5.14%	9.10%	6.09%	3.77%	7.70%
AR(1)TARCH(2,1)	5.94%	7.21%	9.90%	5.83%	11.88%	3.87%	6.05%	4.56%	9.72%	5.86%	10.04%	23.31%	6.03%	4.80%	9.04%
AR(2)TARCH(2,1)	5.88%	8.96%	12.43%	5.83%	11.88%	3.87%	6.10%	3.58%	4.78%	5.75%	15.04%	33.50%	5.98%	6.05%	10.47%
AR(3)TARCH(2,1)	6.10%	3.57%	4.66%	5.83%	11.88%	3.87%	6.16%	2.78%	5.91%	5.86%	10.04%	23.31%	6.09%	3.77%	5.30%
AR(4)TARCH(2,1)	5.94%	7.21%	9.90%	5.89%	9.62%	3.00%	6.05%	4.56%	6.22%	5.86%	10.04%	23.31%	6.19%	2.28%	5.40%
AR(0)TARCH(2,2)	5.94%	7.21%	4.98%	5.94%	7.72%	2.29%	5.94%	7.24%	15.25%	5.81%	12.34%	28.68%	5.98%	6.05%	10.47%
AR(1)TARCH(2,2)	5.88%	8.96%	6.38%	5.89%	9.62%	3.00%	6.10%	3.58%	9.75%	5.81%	12.34%	28.68%	5.98%	6.05%	10.47%
AR(2)TARCH(2,2)	5.94%	7.21%	4.98%	5.72%	17.66%	6.21%	5.99%	5.77%	15.04%	5.70%	18.18%	38.65%	5.98%	6.05%	10.47%
AR(3)TARCH(2,2)	5.88%	8.96%	6.38%	5.89%	9.62%	3.00%	6.10%	3.58%	9.75%	5.86%	10.04%	24.24%	5.98%	6.05%	6.91%
AR(4)TARCH(2,2)	5.99%	5.75%	3.84%	5.72%	17.66%	6.21%	6.05%	4.56%	12.18%	5.91%	8.10%	19.88%	6.03%	4.80%	6.09%
AR(0)EGARCH(0,1)	9.23%	0.00%	0.00%	6.06%	4.85%	1.30%	8.50%	0.00%	0.00%	7.56%	0.00%	0.00%	8.35%	0.00%	0.00%
AR(1)EGARCH(0,1)	9.28%	0.00%	0.00%	6.06%	4.85%	1.30%	8.56%	0.00%	0.00%	7.45%	0.00%	0.00%	8.40%	0.00%	0.00%
AR(2)EGARCH(0,1)	9.28%	0.00%	0.00%	6.17%	2.94%	0.70%	8.56%	0.00%	0.00%	7.50%	0.00%	0.00%	8.35%	0.00%	0.00%
AR(3)EGARCH(0,1)	9.34%	0.00%	0.00%	6.11%	3.79%	0.96%	8.50%	0.00%	0.00%	7.50%	0.00%	0.00%	8.35%	0.00%	0.00%
AR(4)EGARCH(0,1)	9.39%	0.00%	0.00%	6.17%	2.94%	0.70%	8.56%	0.00%	0.00%	7.50%	0.00%	0.00%	8.35%	0.00%	0.00%
AR(0)EGARCH(0,2)	8.80%	0.00%	0.00%	6.06%	4.85%	0.27%	8.12%	0.00%	0.00%	7.45%	0.00%	0.00%	8.13%	0.00%	0.00%
AR(1)EGARCH(0,2)	8.85%	0.00%	0.00%	6.11%	3.79%	0.19%	8.01%	0.00%	0.00%	7.45%	0.00%	0.00%	8.13%	0.00%	0.00%
AR(2)EGARCH(0,2)	8.85%	0.00%	0.00%	6.11%	3.79%	0.01%	8.07%	0.00%	0.00%	7.56%	0.00%	0.00%	8.19%	0.00%	0.00%
AR(3)EGARCH(0,2)	8.74%	0.00%	0.00%	6.06%	4.85%	0.01%	8.12%	0.00%	0.00%	7.50%	0.00%	0.00%	8.08%	0.00%	0.00%
AR(4)EGARCH(0,2)	8.69%	0.00%	0.00%	6.11%	3.79%	0.01%	8.07%	0.00%	0.00%	7.39%	0.00%	0.00%	8.13%	0.00%	0.00%
AR(0)EGARCH(1,1)	5.56%	27.81%	49.69%	5.21%	69.22%	57.31%	6.16%	2.78%	7.71%	5.91%	8.10%	16.58%	6.30%	1.33%	1.78%
AR(1)EGARCH(1,1)	5.50%	32.64%	43.82%	5.21%	69.22%	57.31%	6.10%	3.58%	9.75%	5.86%	10.04%	19.03%	6.41%	0.75%	1.23%
AR(2)EGARCH(1,1)	5.61%	23.50%	43.44%	5.21%	69.22%	57.31%	6.32%	1.24%	3.98%	5.86%	10.04%	19.03%	6.41%	0.75%	1.23%
AR(3)EGARCH(1,1)	5.72%	16.38%	31.98%	5.15%	77.34%	62.05%	6.10%	3.58%	9.75%	5.91%	8.10%	11.94%	6.41%	0.75%	1.23%
AR(4)EGARCH(1,1)	5.56%	27.81%	37.96%	5.21%	69.22%	57.31%	6.10%	3.58%	9.75%	5.91%	8.10%	11.94%	6.35%	1.00%	1.49%
AR(0)EGARCH(1,2)	5.61%	23.50%	18.48%	5.15%	77.34%	62.05%	6.32%	1.24%	3.56%	5.81%	12.34%	14.88%	6.30%	1.33%	1.78%
AR(1)EGARCH(1,2)	5.														

Table (2.A). Normal Distribution, 1500 sample size, 95%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(0)GARCH(0,1)	8.31%	0.00%	0.00%	6.23%	2.26%	0.51%	8.45%	0.00%	0.00%	7.12%	0.01%	0.04%	7.38%	0.00%	0.00%
AR(1)GARCH(0,1)	8.31%	0.00%	0.00%	6.23%	2.26%	0.51%	8.56%	0.00%	0.00%	7.06%	0.01%	0.06%	7.54%	0.00%	0.00%
AR(2)GARCH(0,1)	8.36%	0.00%	0.00%	6.40%	0.98%	0.18%	8.50%	0.00%	0.00%	7.12%	0.01%	0.04%	7.65%	0.00%	0.00%
AR(3)GARCH(0,1)	8.36%	0.00%	0.00%	6.28%	1.72%	0.36%	8.45%	0.00%	0.00%	7.12%	0.01%	0.04%	7.54%	0.00%	0.00%
AR(4)GARCH(0,1)	8.42%	0.00%	0.00%	6.40%	0.98%	0.18%	8.45%	0.00%	0.00%	7.12%	0.01%	0.04%	7.59%	0.00%	0.00%
AR(0)GARCH(0,2)	8.09%	0.00%	0.00%	6.28%	1.72%	0.36%	7.41%	0.00%	0.00%	7.06%	0.01%	0.04%	7.38%	0.00%	0.00%
AR(1)GARCH(0,2)	8.20%	0.00%	0.00%	6.17%	2.94%	0.70%	7.36%	0.00%	0.01%	7.23%	0.00%	0.01%	7.49%	0.00%	0.00%
AR(2)GARCH(0,2)	8.15%	0.00%	0.00%	6.06%	4.85%	1.30%	7.36%	0.00%	0.01%	7.23%	0.00%	0.01%	7.43%	0.00%	0.00%
AR(3)GARCH(0,2)	8.09%	0.00%	0.00%	6.06%	4.85%	1.30%	7.41%	0.00%	0.00%	7.34%	0.00%	0.01%	7.43%	0.00%	0.00%
AR(4)GARCH(0,2)	8.15%	0.00%	0.00%	6.06%	4.85%	1.30%	7.36%	0.00%	0.01%	7.39%	0.00%	0.00%	7.49%	0.00%	0.00%
AR(0)GARCH(1,1)	5.34%	50.30%	62.51%	5.89%	9.62%	23.55%	6.10%	3.58%	9.75%	5.86%	10.04%	13.42%	6.14%	2.94%	0.80%
AR(1)GARCH(1,1)	5.34%	50.30%	62.51%	5.94%	7.72%	19.63%	6.16%	2.78%	7.71%	5.81%	12.34%	21.57%	6.14%	2.94%	0.80%
AR(2)GARCH(1,1)	5.45%	38.00%	49.95%	5.94%	7.72%	19.63%	6.16%	2.78%	7.71%	5.91%	8.10%	7.54%	6.09%	3.77%	0.39%
AR(3)GARCH(1,1)	5.40%	43.90%	56.23%	5.94%	7.72%	19.63%	6.16%	2.78%	7.71%	5.91%	8.10%	7.54%	6.09%	3.77%	0.87%
AR(4)GARCH(1,1)	5.40%	43.90%	56.23%	5.89%	9.62%	23.55%	6.16%	2.78%	7.71%	5.86%	10.04%	13.42%	6.19%	2.28%	0.14%
AR(0)GARCH(1,2)	5.40%	43.90%	68.94%	5.66%	21.26%	32.06%	6.05%	4.56%	12.18%	5.91%	8.10%	11.94%	5.92%	7.56%	0.44%
AR(1)GARCH(1,2)	5.56%	27.81%	49.69%	5.60%	25.37%	37.71%	6.16%	2.78%	7.71%	5.91%	8.10%	11.94%	5.98%	6.05%	0.43%
AR(2)GARCH(1,2)	5.45%	38.00%	62.61%	5.60%	25.37%	37.71%	6.10%	3.58%	9.75%	5.86%	10.04%	8.29%	5.92%	7.56%	0.44%
AR(3)GARCH(1,2)	5.45%	38.00%	62.61%	5.66%	21.26%	32.06%	6.10%	3.58%	9.75%	6.02%	5.14%	6.00%	6.03%	4.80%	0.41%
AR(4)GARCH(1,2)	5.34%	50.30%	74.95%	5.60%	25.37%	37.71%	6.10%	3.58%	9.75%	6.08%	4.05%	3.13%	5.98%	6.05%	1.00%
AR(0)GARCH(2,1)	5.61%	23.50%	43.44%	5.77%	14.55%	22.32%	6.27%	1.64%	4.66%	5.86%	10.04%	13.42%	5.98%	6.05%	1.00%
AR(1)GARCH(2,1)	5.40%	43.90%	56.23%	5.77%	14.55%	22.32%	6.27%	1.64%	4.66%	5.91%	8.10%	11.94%	6.09%	3.77%	0.87%
AR(2)GARCH(2,1)	5.40%	43.90%	56.23%	5.77%	14.55%	12.56%	6.32%	1.24%	3.56%	5.91%	8.10%	11.94%	5.92%	7.56%	2.27%
AR(3)GARCH(2,1)	5.34%	50.30%	62.51%	5.72%	17.66%	15.48%	6.49%	0.52%	1.48%	6.02%	5.14%	3.50%	5.98%	6.05%	2.12%
AR(4)GARCH(2,1)	5.50%	32.64%	43.82%	5.72%	17.66%	26.92%	6.10%	3.58%	7.63%	5.97%	6.48%	6.77%	6.09%	3.77%	0.16%
AR(0)GARCH(2,2)	5.77%	13.51%	18.92%	5.77%	14.55%	30.16%	6.27%	1.64%	5.24%	5.81%	12.34%	14.88%	6.03%	4.80%	0.41%
AR(1)GARCH(2,2)	5.61%	23.50%	32.48%	5.60%	25.37%	37.71%	6.16%	2.78%	7.71%	5.75%	15.04%	16.29%	5.98%	6.05%	0.43%
AR(2)GARCH(2,2)	5.67%	19.70%	37.50%	5.72%	17.66%	26.92%	6.43%	0.70%	2.00%	5.86%	10.04%	8.29%	5.92%	7.56%	1.05%
AR(3)GARCH(2,2)	5.72%	16.38%	33.10%	5.72%	17.66%	35.59%	6.54%	0.38%	1.08%	6.02%	5.14%	6.00%	6.19%	2.28%	0.14%
AR(4)GARCH(2,2)	5.61%	23.50%	43.44%	5.72%	17.66%	26.92%	6.16%	2.78%	7.71%	6.02%	5.14%	6.00%	6.09%	3.77%	0.16%
AR(0)TARCH(0,1)	8.36%	0.00%	0.00%	6.34%	1.30%	0.26%	8.45%	0.00%	0.00%	7.17%	0.01%	0.03%	7.65%	0.00%	0.00%
AR(1)TARCH(0,1)	8.42%	0.00%	0.00%	6.34%	1.30%	0.26%	8.39%	0.00%	0.00%	7.17%	0.01%	0.03%	7.70%	0.00%	0.00%
AR(2)TARCH(0,1)	8.42%	0.00%	0.00%	6.40%	0.98%	0.18%	8.45%	0.00%	0.00%	7.23%	0.00%	0.02%	7.75%	0.00%	0.00%
AR(3)TARCH(0,1)	8.36%	0.00%	0.00%	6.34%	1.30%	0.26%	8.45%	0.00%	0.00%	7.17%	0.01%	0.03%	7.75%	0.00%	0.00%
AR(4)TARCH(0,1)	8.47%	0.00%	0.00%	6.34%	1.30%	0.26%	8.50%	0.00%	0.00%	7.17%	0.01%	0.03%	7.75%	0.00%	0.00%
AR(0)TARCH(0,2)	8.31%	0.00%	0.00%	6.17%	2.94%	0.70%	7.25%	0.00%	0.02%	7.12%	0.01%	0.04%	7.54%	0.00%	0.00%
AR(1)TARCH(0,2)	8.31%	0.00%	0.00%	6.17%	2.94%	0.70%	7.25%	0.00%	0.01%	7.28%	0.00%	0.01%	7.75%	0.00%	0.00%
AR(2)TARCH(0,2)	8.26%	0.00%	0.00%	6.06%	4.85%	1.30%	7.30%	0.00%	0.01%	7.23%	0.00%	0.02%	7.70%	0.00%	0.00%
AR(3)TARCH(0,2)	8.15%	0.00%	0.00%	6.06%	4.85%	1.30%	7.30%	0.00%	0.01%	7.23%	0.00%	0.02%	7.65%	0.00%	0.00%
AR(4)TARCH(0,2)	8.15%	0.00%	0.00%	6.11%	3.79%	0.96%	7.30%	0.00%	0.01%	7.23%	0.00%	0.02%	7.59%	0.00%	0.00%
AR(0)TARCH(1,1)	6.31%	1.25%	3.53%	5.66%	21.26%	18.85%	5.89%	9.00%	12.76%	5.75%	15.04%	24.13%	6.03%	4.80%	3.64%
AR(1)TARCH(1,1)	6.31%	1.25%	3.53%	5.77%	14.55%	12.56%	5.89%	9.00%	12.76%	5.81%	12.34%	21.57%	6.14%	2.94%	2.84%
AR(2)TARCH(1,1)	6.37%	0.94%	2.67%	5.66%	21.26%	18.85%	5.89%	9.00%	12.76%	5.70%	18.18%	34.85%	6.46%	0.56%	1.43%
AR(3)TARCH(1,1)	6.53%	0.38%	1.07%	5.72%	17.66%	15.48%	5.89%	9.00%	12.76%	5.81%	12.34%	21.57%	6.25%	1.75%	4.44%
AR(4)TARCH(1,1)	6.37%	0.94%	2.67%	5.60%	25.37%	22.66%	5.89%	9.00%	12.76%	5.81%	12.34%	14.88%	6.19%	2.28%	3.87%
AR(0)TARCH(1,2)	6.04%	4.55%	9.55%	5.43%	40.99%	16.34%	5.94%	7.24%	15.25%	5.59%	25.90%	31.26%	6.03%	4.80%	6.09%
AR(1)TARCH(1,2)	6.04%	4.55%	2.93%	5.49%	35.23%	13.80%	5.94%	7.24%	15.25%	5.59%	25.90%	31.26%	5.98%	6.05%	10.47%
AR(2)TARCH(1,2)	6.26%	1.64%	1.98%	5.43%	40.99%	16.34%	5.94%	7.24%	15.25%	5.64%	21.79%	29.06%	6.19%	2.28%	2.46%
AR(3)TARCH(1,2)	6.21%	2.15%	2.67%	5.49%	35.23%	13.80%	5.94%	7.24%	15.25%	5.75%	15.04%	16.29%	5.87%	9.38%	8.56%
AR(4)TARCH(1,2)	6.26%	1.64%	1.98%	5.60%	25.37%	9.49%	5.89%	9.00%	18.76%	5.75%	15.04%	16.29%	5.92%	7.56%	4.43%
AR(0)TARCH(2,1)	6.26%	1.64%	1.98%	5.60%	25.37%	9.49%	5.78%	13.59%	27.39%	5.81%	12.34%	27.00%	6.09%	3.77%	5.30%
AR(1)TARCH(2,1)	6.37%	0.94%	1.88%	5.49%	35.23%	13.80%	5.89%	9.00%	12.76%	5.81%	12.34%	27.00%	5.98%	6.05%	10.47%
AR(2)TARCH(2,1)	6.37%	0.94%	1.88%	5.49%	35.23%	31.53%	5.94%	7.24%	10.16%	5.70%	18.18%	34.85%	6.03%	4.80%	6.09%
AR(3)TARCH(2,1)	6.26%	1.64%	3.38%	5.49%	35.23%	13.80%	6.05%	4.56%	6.22%	5.64%	21.79%	38.84%	6.03%	4.80%	9.04%
AR(4)TARCH(2,1)	6.31%	1.25%	2.53%	5.55%	30.02%	11.52%	6.05%	4.56%	9.72%	5.81%	12.34%	21.57%	5.98%	6.05%	6.91%
AR(0)TARCH(2,2)	6.21%	2.15%	1.22%	5.60%	25.37%	9.49%	5.94%	7.24%	15.25%	5.81%	12.34%	27.00%	5.98%	6.05%	4.04%
AR(1)TARCH(2,2)	6.26%	1.64%	0.89%	5.55%	30.02%	11.52%	6.10%	3.58%	7.63%	5.81%	12.34%	21.57%	6.03%	4.80%	3.64%
AR(2)TARCH(2,2)	6.21%	2.15%	1.22%	5.60%	25.37%	9.49%	5.99%	5.77%	12.25%	5.81%	12.34%	27.00%	6.03%	4.80%	3.64%
AR(3)TARCH(2,2)	6.31%	1.25%	0.64%	5.43%	40.99%	16.34%	5.99%	5.77%	12.25%	5.81%	12.34%	21.57%	6.03%	4.80%	3.64%
AR(4)TARCH(2,2)	6.42%	0.70%	0.32%	5.43%	40.99%	16.34%	6.05%	4.56%	9.72%	5.64%	21.79%	44.02%	6.03%	4.80%	3.64%
AR(0)EGARCH(0,1)	8.74%	0.00%	0.00%	6.11%	3.79%	0.96%	8.45%	0.00%	0.00%	7.28%	0.00%	0.00%	7.75%	0.00%	0.00%
AR(1)EGARCH(0,1)	8.74%	0.00%	0.00%	6.06%	4.85%	1.30%	8.56%	0.00%	0.00%	7.17%	0.01%	0.01%	7.86%	0.00%	0.00%
AR(2)EGARCH(0,1)	8.74%	0.00%	0.00%	6.11%	3.79%	0.96%	8.50%	0.00%	0.00%	7.28%	0.00%	0.01%	7.92%	0.00%	0.00%
AR(3)EGARCH(0,1)	8.80%	0.00%	0.00%	6.00%	6.14%	1.74%	8.50%	0.00%	0.00%	7.23%	0.00%	0.01%	7.86%	0.00%	0.00%
AR(4)EGARCH(0,1)	8.74%	0.00%	0.00%	6.06%	4.85%	1.30%	8.56%	0.00%	0.00%	7.23%	0.00%	0.01%	7.81%	0.00%	0.00%
AR(0)EGARCH(0,2)	8.20%	0.00%	0.00%	6.00%	6.14%	0.36%	7.85%	0.00%	0.00%	7.12%	0.01%	0.03%	7.75%	0.00%	0.00%
AR(1)EGARCH(0,2)	8.09%	0.00%	0.00%	6.06%	4.85%	0.27%	7.79%	0.00%	0.00%	7.17%	0.01%	0.02%	7.65%	0.00%	0.00%
AR(2)EGARCH(0,2)	8.15%	0.00%	0.00%	6.11%	3.79%	0.19%	7.79%	0.00%	0.00%	7.23%	0.00%	0.01%	7.86%	0.00%	0.00%
AR(3)EGARCH(0,2)	8.09%	0.00%	0.00%	6.00%	6.14%	0.36%	7.79%	0.00%	0.00%	7.23%	0.00%	0.01%	7.75%	0.00%	0.00%
AR(4)EGARCH(0,2)	7.99%	0.00%	0.00%	6.06%	4.85%	0.27%	7.90%	0.00%	0.00%	7.23%	0.00%	0.01%	7.75%	0.00%	0.00%
AR(0)EGARCH(1,1)	5.88%	8.96%	18.44%	5.15%	77.34%	62.05%	6.38%	0.94%	2.69%	5.81%	12.34%	14.88%	5.87%	9.38%	13.53%
AR(1)EGARCH(1,1)	5.83%	11.05%	22.43%	5.15%	77.34%	62.05%	6.43%	0.70%	2.00%	5.64%	21.79%	29.06%	6.19%	2.28%	2.46%
AR(2)EGARCH(1,1)	5.83%	11.05%	22.43%	5.15%	77.34%	62.05%	6.38%	0.94%	2.69%	5.70%	18.18%	26.65%	6.25%	1.75%	2.11%
AR(3)EGARCH(1,1)	5.83%	11.05%	22.43%	5.15%	77.34%	62.05%	6.38%	0.94%	2.69%	5.86%	10.04%	13.42%	6.25%	1.75%	2.11%
AR(4)EGARCH(1,1)	5.88%	8.96%	18.44%	5.15%	77.34%	62.05%	6.32%	1.24%	3.56%	5.86%	10.04%	13.42%	6.25%	1.75%	2.11%
AR(0)EGARCH(1,2)	5.94%	7.21%	4.98%	5.15%	77.34%	62.05%	6.05%	4.56%	9.72%	5.48%	35.72%	34.76%	6.03%	4.80%	3.64%

Table (3.A). Normal Distribution, 1000 sample size, 95%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(0)GARCH(0,1)	7.66%	0.00%	0.00%	5.77%	14.55%	4.93%	7.47%	0.00%	0.00%	6.68%	0.17%	0.53%	6.89%	0.04%	0.04%
AR(1)GARCH(0,1)	7.61%	0.00%	0.00%	5.89%	9.62%	3.00%	7.52%	0.00%	0.00%	6.63%	0.23%	0.70%	7.05%	0.01%	0.01%
AR(2)GARCH(0,1)	7.61%	0.00%	0.00%	6.00%	6.14%	1.74%	7.47%	0.00%	0.00%	6.79%	0.08%	0.30%	6.95%	0.03%	0.02%
AR(3)GARCH(0,1)	7.72%	0.00%	0.00%	5.89%	9.62%	3.00%	7.47%	0.00%	0.00%	6.85%	0.06%	0.22%	7.00%	0.02%	0.01%
AR(4)GARCH(0,1)	7.72%	0.00%	0.00%	6.00%	6.14%	1.74%	7.52%	0.00%	0.00%	6.85%	0.06%	0.22%	6.89%	0.04%	0.04%
AR(0)GARCH(0,2)	7.34%	0.00%	0.00%	5.83%	11.88%	10.06%	6.70%	0.14%	0.36%	6.35%	1.08%	2.99%	6.89%	0.04%	0.02%
AR(1)GARCH(0,2)	7.39%	0.00%	0.00%	5.89%	9.62%	7.97%	6.65%	0.20%	0.46%	6.35%	1.08%	2.99%	6.89%	0.04%	0.02%
AR(2)GARCH(0,2)	7.34%	0.00%	0.00%	5.83%	11.88%	3.87%	6.70%	0.14%	0.36%	6.41%	0.81%	1.22%	7.00%	0.02%	0.01%
AR(3)GARCH(0,2)	7.23%	0.00%	0.01%	5.77%	14.55%	4.93%	6.70%	0.14%	0.36%	6.46%	0.60%	1.45%	6.84%	0.06%	0.03%
AR(4)GARCH(0,2)	7.23%	0.00%	0.01%	6.06%	4.85%	3.67%	6.76%	0.10%	0.27%	6.35%	1.08%	2.23%	6.84%	0.06%	0.03%
AR(0)GARCH(1,1)	5.45%	38.00%	62.61%	5.77%	14.55%	22.32%	5.99%	5.77%	15.04%	5.81%	12.34%	21.57%	5.87%	9.38%	8.56%
AR(1)GARCH(1,1)	5.45%	38.00%	62.61%	5.77%	14.55%	22.32%	5.99%	5.77%	12.25%	6.02%	5.14%	12.11%	6.03%	4.80%	3.64%
AR(2)GARCH(1,1)	5.40%	43.90%	68.94%	5.83%	11.88%	25.23%	6.05%	4.56%	9.72%	5.64%	21.79%	29.06%	6.03%	4.80%	6.09%
AR(3)GARCH(1,1)	5.40%	43.90%	68.94%	5.83%	11.88%	25.23%	5.99%	5.77%	12.25%	5.81%	12.34%	14.88%	6.09%	3.77%	5.30%
AR(4)GARCH(1,1)	5.34%	50.30%	74.95%	5.83%	11.88%	25.23%	5.99%	5.77%	12.25%	5.81%	12.34%	14.88%	6.19%	2.28%	1.40%
AR(0)GARCH(1,2)	5.40%	43.90%	68.94%	6.00%	6.14%	13.70%	5.83%	11.11%	26.32%	5.70%	18.18%	17.60%	5.98%	6.05%	4.04%
AR(1)GARCH(1,2)	5.40%	43.90%	68.94%	6.00%	6.14%	13.70%	5.89%	9.00%	21.59%	5.91%	8.10%	11.94%	5.87%	9.38%	4.80%
AR(2)GARCH(1,2)	5.45%	38.00%	62.61%	6.00%	6.14%	16.15%	5.89%	9.00%	22.11%	5.75%	15.04%	16.29%	5.92%	7.56%	7.74%
AR(3)GARCH(1,2)	5.56%	27.81%	45.19%	6.00%	6.14%	16.15%	5.89%	9.00%	21.59%	5.86%	10.04%	13.42%	5.92%	7.56%	7.74%
AR(4)GARCH(1,2)	5.56%	27.81%	49.69%	5.94%	7.72%	17.00%	5.94%	7.24%	18.29%	5.86%	10.04%	13.42%	5.87%	9.38%	4.80%
AR(0)GARCH(2,1)	5.56%	27.81%	49.69%	5.66%	21.26%	18.85%	5.89%	9.00%	18.76%	5.97%	6.48%	14.26%	5.98%	6.05%	10.47%
AR(1)GARCH(2,1)	5.45%	38.00%	62.61%	5.66%	21.26%	7.72%	5.83%	11.11%	22.81%	5.91%	8.10%	16.58%	6.14%	2.94%	4.56%
AR(2)GARCH(2,1)	5.40%	43.90%	68.94%	5.77%	14.55%	22.32%	5.89%	9.00%	18.76%	6.02%	5.14%	9.10%	5.92%	7.56%	11.97%
AR(3)GARCH(2,1)	5.40%	43.90%	68.94%	5.77%	14.55%	22.32%	5.94%	7.24%	15.25%	5.81%	12.34%	14.88%	6.09%	3.77%	5.30%
AR(4)GARCH(2,1)	5.40%	43.90%	68.94%	5.66%	21.26%	18.85%	5.99%	5.77%	12.25%	6.02%	5.14%	9.10%	6.14%	2.94%	4.56%
AR(0)GARCH(2,2)	5.94%	7.21%	18.32%	5.94%	7.72%	17.00%	5.89%	9.00%	21.59%	5.64%	21.79%	18.77%	5.82%	11.54%	9.35%
AR(1)GARCH(2,2)	6.04%	4.55%	12.66%	5.89%	9.62%	20.84%	5.89%	9.00%	22.11%	5.59%	25.90%	19.74%	5.98%	6.05%	6.91%
AR(2)GARCH(2,2)	5.99%	5.75%	15.32%	5.89%	9.62%	20.84%	5.94%	7.24%	18.29%	5.70%	18.18%	17.60%	5.87%	9.38%	8.56%
AR(3)GARCH(2,2)	5.94%	7.21%	15.67%	5.83%	11.88%	25.23%	5.94%	7.24%	18.29%	5.81%	12.34%	21.57%	6.03%	4.80%	1.95%
AR(4)GARCH(2,2)	5.88%	8.96%	21.64%	5.89%	9.62%	20.84%	5.89%	9.00%	18.76%	5.97%	6.48%	14.26%	6.14%	2.94%	1.58%
AR(0)TARCH(0,1)	7.77%	0.00%	0.00%	5.77%	14.55%	1.12%	7.68%	0.00%	0.00%	6.63%	0.23%	0.85%	7.16%	0.01%	0.01%
AR(1)TARCH(0,1)	7.83%	0.00%	0.00%	5.77%	14.55%	1.12%	7.74%	0.00%	0.00%	6.68%	0.17%	0.64%	7.22%	0.00%	0.01%
AR(2)TARCH(0,1)	7.93%	0.00%	0.00%	5.94%	7.72%	2.29%	7.85%	0.00%	0.00%	6.63%	0.23%	0.85%	7.16%	0.01%	0.01%
AR(3)TARCH(0,1)	7.83%	0.00%	0.00%	5.83%	11.88%	3.87%	7.79%	0.00%	0.00%	6.96%	0.03%	0.13%	7.22%	0.00%	0.01%
AR(4)TARCH(0,1)	7.88%	0.00%	0.00%	5.94%	7.72%	2.29%	7.79%	0.00%	0.00%	6.79%	0.08%	0.35%	7.22%	0.00%	0.01%
AR(0)TARCH(0,2)	7.50%	0.00%	0.00%	5.66%	21.26%	7.72%	6.54%	0.38%	1.41%	6.35%	1.08%	2.99%	7.05%	0.01%	0.02%
AR(1)TARCH(0,2)	7.50%	0.00%	0.00%	5.66%	21.26%	7.72%	6.54%	0.38%	1.41%	6.46%	0.60%	1.87%	7.11%	0.01%	0.01%
AR(2)TARCH(0,2)	7.61%	0.00%	0.00%	5.77%	14.55%	4.93%	6.65%	0.20%	0.74%	6.41%	0.81%	1.81%	7.22%	0.00%	0.01%
AR(3)TARCH(0,2)	7.39%	0.00%	0.00%	5.77%	14.55%	4.93%	6.59%	0.28%	0.99%	6.41%	0.81%	2.38%	7.11%	0.01%	0.01%
AR(4)TARCH(0,2)	7.29%	0.00%	0.00%	5.89%	9.62%	3.00%	6.49%	0.52%	1.80%	6.35%	1.08%	2.99%	7.11%	0.01%	0.01%
AR(0)TARCH(1,1)	6.21%	2.15%	4.46%	5.32%	54.14%	22.01%	5.61%	23.69%	44.06%	5.70%	18.18%	26.65%	6.03%	4.80%	9.04%
AR(1)TARCH(1,1)	6.21%	2.15%	4.46%	5.43%	40.99%	36.48%	5.56%	28.04%	50.35%	5.75%	15.04%	16.29%	5.98%	6.05%	13.90%
AR(2)TARCH(1,1)	6.37%	0.94%	1.88%	5.32%	54.14%	46.95%	5.61%	23.69%	44.06%	5.64%	21.79%	29.06%	6.09%	3.77%	7.70%
AR(3)TARCH(1,1)	6.21%	2.15%	4.46%	5.26%	61.47%	52.22%	5.61%	23.69%	44.06%	5.70%	18.18%	26.65%	6.14%	2.94%	4.56%
AR(4)TARCH(1,1)	6.37%	0.94%	1.88%	5.26%	61.47%	52.22%	5.67%	19.85%	38.07%	5.91%	8.10%	11.94%	6.09%	3.77%	7.70%
AR(0)TARCH(1,2)	6.31%	1.25%	2.53%	5.38%	47.31%	19.08%	5.61%	23.69%	44.06%	5.75%	15.04%	24.13%	6.03%	4.80%	6.09%
AR(1)TARCH(1,2)	6.26%	1.64%	3.38%	5.49%	35.23%	31.53%	5.61%	23.69%	44.06%	5.64%	21.79%	18.77%	6.14%	2.94%	4.56%
AR(2)TARCH(1,2)	6.26%	1.64%	3.38%	5.38%	47.31%	41.66%	5.78%	13.59%	27.39%	5.86%	10.04%	13.42%	5.98%	6.05%	6.91%
AR(3)TARCH(1,2)	6.42%	0.70%	1.38%	5.43%	40.99%	36.48%	5.78%	13.59%	27.39%	5.81%	12.34%	14.88%	6.03%	4.80%	6.09%
AR(4)TARCH(1,2)	6.48%	0.52%	1.00%	5.43%	40.99%	36.48%	5.72%	16.49%	32.49%	5.86%	10.04%	13.42%	5.92%	7.56%	7.74%
AR(0)TARCH(2,1)	6.37%	0.94%	1.88%	5.60%	25.37%	9.49%	5.56%	28.04%	50.35%	5.70%	18.18%	26.65%	5.71%	17.05%	34.04%
AR(1)TARCH(2,1)	6.37%	0.94%	1.88%	5.32%	54.14%	22.01%	5.78%	13.59%	27.39%	5.75%	15.04%	24.13%	5.82%	11.54%	26.10%
AR(2)TARCH(2,1)	6.26%	1.64%	3.38%	5.43%	40.99%	16.34%	5.67%	19.85%	38.07%	5.70%	18.18%	34.85%	5.76%	14.09%	16.62%
AR(3)TARCH(2,1)	6.37%	0.94%	1.88%	5.38%	47.31%	19.08%	5.67%	19.85%	38.07%	5.59%	25.90%	42.72%	5.92%	7.56%	16.24%
AR(4)TARCH(2,1)	6.48%	0.52%	1.00%	5.38%	47.31%	19.08%	5.83%	11.11%	22.81%	5.86%	10.04%	19.03%	6.09%	3.77%	9.82%
AR(0)TARCH(2,2)	6.26%	1.64%	1.98%	5.43%	40.99%	36.48%	5.67%	19.85%	38.07%	5.64%	21.79%	38.84%	5.71%	17.05%	18.07%
AR(1)TARCH(2,2)	6.10%	3.57%	4.66%	5.43%	40.99%	36.48%	5.72%	16.49%	32.49%	5.81%	12.34%	21.57%	5.76%	14.09%	24.02%
AR(2)TARCH(2,2)	6.26%	1.64%	1.98%	5.43%	40.99%	16.34%	5.67%	19.85%	38.07%	5.91%	8.10%	11.94%	5.87%	9.38%	8.56%
AR(3)TARCH(2,2)	6.37%	0.94%	1.06%	5.26%	61.47%	25.06%	5.67%	19.85%	38.07%	5.97%	6.48%	10.49%	5.92%	7.56%	7.74%
AR(4)TARCH(2,2)	6.31%	1.25%	1.46%	5.49%	35.23%	13.80%	5.78%	13.59%	27.39%	5.91%	8.10%	11.94%	5.92%	7.56%	7.74%
AR(0)EGARCH(0,1)	7.93%	0.00%	0.00%	5.66%	21.26%	7.72%	7.63%	0.00%	0.00%	6.52%	0.44%	1.15%	7.22%	0.00%	0.00%
AR(1)EGARCH(0,1)	7.88%	0.00%	0.00%	5.72%	17.66%	6.21%	7.63%	0.00%	0.00%	6.57%	0.32%	0.90%	7.27%	0.00%	0.00%
AR(2)EGARCH(0,1)	7.93%	0.00%	0.00%	5.77%	14.55%	4.93%	7.63%	0.00%	0.00%	6.63%	0.23%	0.70%	7.27%	0.00%	0.00%
AR(3)EGARCH(0,1)	7.88%	0.00%	0.00%	5.89%	9.62%	3.00%	7.68%	0.00%	0.00%	6.74%	0.12%	0.40%	7.32%	0.00%	0.00%
AR(4)EGARCH(0,1)	7.88%	0.00%	0.00%	5.83%	11.88%	3.87%	7.68%	0.00%	0.00%	6.74%	0.12%	0.40%	7.32%	0.00%	0.00%
AR(0)EGARCH(0,2)	7.45%	0.00%	0.00%	5.89%	9.62%	7.97%	6.98%	0.02%	0.04%	6.30%	1.43%	2.71%	7.00%	0.02%	0.01%
AR(1)EGARCH(0,2)	7.56%	0.00%	0.00%	5.89%	9.62%	7.97%	6.92%	0.03%	0.03%	6.35%	1.08%	2.23%	7.00%	0.02%	0.01%
AR(2)EGARCH(0,2)	7.56%	0.00%	0.00%	6.00%	6.14%	4.81%	6.81%	0.07%	0.09%	6.41%	0.81%	1.81%	7.00%	0.02%	0.01%
AR(3)EGARCH(0,2)	7.66%	0.00%	0.00%	5.94%	7.72%	6.23%	6.92%	0.03%	0.03%	6.35%	1.08%	2.23%	7.05%	0.01%	0.01%
AR(4)EGARCH(0,2)	7.61%	0.00%	0.00%	6.06%	4.85%	3.67%	6.92%	0.03%	0.03%	6.35%	1.08%	2.23%	7.11%	0.01%	0.00%
AR(0)EGARCH(1,1)	6.04%	4.55%	9.55%	4.98%	96.95%	72.50%	5.67%	19.85%	38.07%	5.48%	35.72%	34.76%	6.09%	3.77%	5.30%
AR(1)EGARCH(1,1)	6.04%	4.55%	9.55%	5.04%	94.35%	91.91%	5.67%	19.85%	38.07%	5.37%	47.69%	36.59%	6.14%	2.94%	2.84%
AR(2)EGARCH(1,1)	6.04%	4.55%	9.55%	5.21%	69.22%	81.16%	5.72%	16.49%	32.49%	5.48%	35.72%	20.97%	6.14%	2.94%	2.84%
AR(3)EGARCH(1,1)	6.04%	4.55%	9.55%	5.21%	69.22%	81.16%	5.72%	16.49%	32.49%	5.59%	25.90%	19.74%	6.14%	2.94%	2.84%
AR(4)EGARCH(1,1)	6.10%	3.57%	7.49%	5.38%	47.31%	63.30%	5.72%	16.49%	32.49%	5.64%	21.79%	18.77%	6.14%	2.94%	2.84%
AR(0)EGARCH(1,2)	5.83%	11.05%	8.08%	5.32%	54.14%	69.71%	5.99%	5.							

Table (4.A). Normal Distribution, 500 sample size, 95%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(0)GARCH(0,1)	6.26%	1.64%	5.24%	5.83%	11.88%	3.87%	6.54%	0.38%	0.47%	6.52%	0.44%	1.15%	6.62%	0.22%	0.11%
AR(1)GARCH(0,1)	6.26%	1.64%	5.24%	5.89%	9.62%	7.97%	6.70%	0.14%	0.24%	6.57%	0.32%	0.65%	6.73%	0.11%	0.02%
AR(2)GARCH(0,1)	6.42%	0.70%	2.40%	6.00%	6.14%	9.31%	6.81%	0.07%	0.09%	6.52%	0.44%	1.15%	6.73%	0.11%	0.01%
AR(3)GARCH(0,1)	6.48%	0.52%	1.80%	6.00%	6.14%	9.31%	6.87%	0.05%	0.07%	6.63%	0.23%	0.51%	6.84%	0.06%	0.01%
AR(4)GARCH(0,1)	6.42%	0.70%	2.40%	6.00%	6.14%	9.31%	6.92%	0.03%	0.06%	6.63%	0.23%	0.51%	6.95%	0.03%	0.00%
AR(0)GARCH(0,2)	6.31%	1.25%	3.53%	5.66%	21.26%	18.55%	6.27%	1.64%	5.02%	6.30%	1.43%	2.71%	6.35%	1.00%	0.90%
AR(1)GARCH(0,2)	6.26%	1.64%	4.62%	5.77%	14.55%	12.56%	6.27%	1.64%	5.02%	6.24%	1.87%	3.26%	6.35%	1.00%	0.49%
AR(2)GARCH(0,2)	6.31%	1.25%	4.09%	5.72%	17.66%	15.48%	6.54%	0.38%	1.41%	6.24%	1.87%	3.26%	6.52%	0.41%	0.16%
AR(3)GARCH(0,2)	6.37%	0.94%	3.15%	5.77%	14.55%	12.56%	6.43%	0.70%	2.43%	6.30%	1.43%	2.71%	6.52%	0.41%	0.16%
AR(4)GARCH(0,2)	6.42%	0.70%	2.40%	5.89%	9.62%	7.97%	6.49%	0.52%	1.86%	6.35%	1.08%	2.23%	6.52%	0.41%	0.30%
AR(0)GARCH(1,1)	5.56%	27.81%	49.69%	5.43%	40.99%	36.48%	6.05%	4.56%	12.66%	5.97%	6.48%	10.49%	5.71%	17.05%	26.68%
AR(1)GARCH(1,1)	5.50%	32.64%	56.13%	5.38%	47.31%	41.66%	5.99%	5.77%	15.31%	5.97%	6.48%	10.49%	5.76%	14.09%	24.02%
AR(2)GARCH(1,1)	5.56%	27.81%	49.69%	5.43%	40.99%	36.48%	6.10%	3.58%	10.34%	5.64%	21.79%	18.77%	5.65%	20.47%	38.15%
AR(3)GARCH(1,1)	5.50%	32.64%	56.13%	5.38%	47.31%	41.66%	6.16%	2.78%	8.34%	5.91%	8.10%	7.54%	5.71%	17.05%	34.04%
AR(4)GARCH(1,1)	5.50%	32.64%	56.13%	5.38%	47.31%	41.66%	6.05%	4.56%	12.66%	5.86%	10.04%	13.42%	5.82%	11.54%	26.10%
AR(0)GARCH(1,2)	5.61%	23.50%	31.04%	5.72%	17.66%	26.92%	5.94%	7.24%	7.27%	5.81%	12.34%	9.00%	5.76%	14.09%	10.07%
AR(1)GARCH(1,2)	5.72%	16.38%	26.01%	5.77%	14.55%	22.32%	5.99%	5.77%	6.48%	5.81%	12.34%	9.00%	5.76%	14.09%	10.07%
AR(2)GARCH(1,2)	5.72%	16.38%	33.10%	5.72%	17.66%	26.92%	6.10%	3.58%	4.96%	5.91%	8.10%	11.94%	5.82%	11.54%	9.35%
AR(3)GARCH(1,2)	5.67%	19.70%	18.99%	5.66%	21.26%	32.06%	6.16%	2.78%	4.25%	5.64%	21.79%	18.77%	5.87%	9.38%	8.56%
AR(4)GARCH(1,2)	5.67%	19.70%	18.99%	5.60%	25.37%	37.71%	6.10%	3.58%	7.25%	5.75%	15.04%	16.29%	5.87%	9.38%	8.56%
AR(0)GARCH(2,1)	5.40%	43.90%	68.94%	5.49%	35.23%	31.53%	6.05%	4.56%	12.66%	5.86%	10.04%	8.29%	5.60%	24.37%	31.69%
AR(1)GARCH(2,1)	5.61%	23.50%	43.44%	5.43%	40.99%	36.48%	6.05%	4.56%	12.66%	5.86%	10.04%	23.31%	5.82%	11.54%	15.09%
AR(2)GARCH(2,1)	5.56%	27.81%	49.69%	5.43%	40.99%	36.48%	6.27%	1.64%	5.24%	6.19%	2.44%	3.87%	5.76%	14.09%	29.99%
AR(3)GARCH(2,1)	5.67%	19.70%	41.03%	5.38%	47.31%	41.66%	6.21%	2.14%	6.65%	6.30%	1.43%	3.71%	5.82%	11.54%	26.10%
AR(4)GARCH(2,1)	5.50%	32.64%	56.13%	5.49%	35.23%	31.53%	6.10%	3.58%	10.34%	6.19%	2.44%	3.87%	5.87%	9.38%	22.43%
AR(0)GARCH(2,2)	5.61%	23.50%	31.04%	5.66%	21.26%	32.06%	5.83%	11.11%	20.43%	5.91%	8.10%	16.58%	5.60%	24.37%	42.22%
AR(1)GARCH(2,2)	5.72%	16.38%	26.01%	5.66%	21.26%	32.06%	5.78%	13.59%	23.02%	6.02%	5.14%	9.10%	5.98%	6.05%	1.00%
AR(2)GARCH(2,2)	5.77%	13.51%	29.09%	5.66%	21.26%	32.06%	6.16%	2.78%	6.10%	6.02%	5.14%	9.10%	5.98%	6.05%	6.91%
AR(3)GARCH(2,2)	5.77%	13.51%	16.20%	5.66%	21.26%	32.06%	6.10%	3.58%	4.96%	5.97%	6.48%	10.49%	6.09%	3.77%	3.23%
AR(4)GARCH(2,2)	5.83%	11.05%	14.67%	5.83%	11.88%	18.27%	6.10%	3.58%	7.25%	6.13%	3.16%	6.60%	5.82%	11.54%	15.09%
AR(0)TARCH(0,1)	6.69%	0.14%	0.25%	5.89%	9.62%	7.97%	6.70%	0.14%	0.56%	6.41%	0.81%	2.75%	6.73%	0.11%	0.13%
AR(1)TARCH(0,1)	6.64%	0.20%	0.36%	5.94%	7.72%	6.23%	6.81%	0.07%	0.30%	6.52%	0.44%	1.62%	6.84%	0.06%	0.08%
AR(2)TARCH(0,1)	6.64%	0.20%	0.36%	5.94%	7.72%	11.80%	6.87%	0.05%	0.19%	6.63%	0.23%	0.85%	6.89%	0.04%	0.04%
AR(3)TARCH(0,1)	6.75%	0.10%	0.17%	6.00%	6.14%	9.31%	6.87%	0.05%	0.19%	6.68%	0.17%	0.53%	7.00%	0.02%	0.02%
AR(4)TARCH(0,1)	6.64%	0.20%	0.36%	6.06%	4.85%	3.67%	6.81%	0.07%	0.26%	6.68%	0.17%	0.64%	7.00%	0.02%	0.02%
AR(0)TARCH(0,2)	6.69%	0.14%	0.12%	5.72%	17.66%	15.48%	6.16%	2.78%	8.34%	6.19%	2.44%	6.89%	6.52%	0.41%	0.81%
AR(1)TARCH(0,2)	6.42%	0.70%	0.76%	5.72%	17.66%	15.48%	6.32%	1.24%	4.08%	6.30%	1.43%	4.46%	6.62%	0.22%	0.51%
AR(2)TARCH(0,2)	6.48%	0.52%	0.54%	5.77%	14.55%	12.56%	6.32%	1.24%	4.08%	6.30%	1.43%	3.71%	6.79%	0.08%	0.16%
AR(3)TARCH(0,2)	6.53%	0.38%	0.71%	5.89%	9.62%	7.97%	6.43%	0.70%	2.39%	6.35%	1.08%	2.99%	6.73%	0.11%	0.21%
AR(4)TARCH(0,2)	6.64%	0.20%	0.36%	5.89%	9.62%	7.97%	6.54%	0.38%	1.34%	6.24%	1.87%	4.55%	6.79%	0.08%	0.16%
AR(0)TARCH(1,1)	5.77%	13.51%	26.95%	5.32%	54.14%	46.95%	5.67%	19.85%	38.07%	6.08%	4.05%	7.80%	5.55%	28.78%	46.12%
AR(1)TARCH(1,1)	5.77%	13.51%	26.95%	5.21%	69.22%	57.31%	5.78%	13.59%	27.39%	5.86%	10.04%	13.42%	5.71%	17.05%	34.04%
AR(2)TARCH(1,1)	5.61%	23.50%	43.44%	5.21%	69.22%	57.31%	5.94%	7.24%	15.25%	6.19%	2.44%	3.87%	5.60%	24.37%	31.69%
AR(3)TARCH(1,1)	5.77%	13.51%	26.95%	5.04%	94.35%	69.80%	5.78%	13.59%	27.39%	6.30%	1.43%	2.71%	5.82%	11.54%	21.35%
AR(4)TARCH(1,1)	5.94%	7.21%	18.20%	4.92%	88.26%	74.26%	5.83%	11.11%	22.81%	6.35%	1.08%	2.23%	5.76%	14.09%	29.99%
AR(0)TARCH(1,2)	5.94%	7.21%	18.20%	5.55%	30.02%	26.90%	5.61%	23.69%	40.96%	6.57%	0.32%	1.12%	5.87%	9.38%	13.53%
AR(1)TARCH(1,2)	5.88%	8.96%	21.94%	5.66%	21.26%	32.06%	5.67%	19.85%	41.22%	6.30%	1.43%	4.46%	5.71%	17.05%	26.68%
AR(2)TARCH(1,2)	5.94%	7.21%	18.20%	5.60%	25.37%	37.71%	5.56%	28.04%	52.20%	6.30%	1.43%	3.71%	5.65%	20.47%	19.39%
AR(3)TARCH(1,2)	5.83%	11.05%	26.13%	5.72%	17.66%	26.92%	5.50%	32.92%	57.61%	6.35%	1.08%	2.23%	5.87%	9.38%	13.53%
AR(4)TARCH(1,2)	5.94%	7.21%	18.20%	5.60%	25.37%	37.71%	5.67%	19.85%	41.22%	6.35%	1.08%	2.23%	5.76%	14.09%	24.02%
AR(0)TARCH(2,1)	5.72%	16.38%	35.74%	4.98%	96.95%	39.22%	5.78%	13.59%	27.39%	6.08%	4.05%	11.45%	5.55%	28.78%	33.86%
AR(1)TARCH(2,1)	5.99%	5.75%	14.91%	5.09%	85.75%	66.26%	5.78%	13.59%	27.39%	6.30%	1.43%	3.71%	5.55%	28.78%	53.27%
AR(2)TARCH(2,1)	5.94%	7.21%	18.20%	5.09%	85.75%	66.26%	5.94%	7.24%	15.25%	6.57%	0.32%	1.12%	5.82%	11.54%	20.96%
AR(3)TARCH(2,1)	5.99%	5.75%	14.91%	5.32%	54.14%	46.95%	5.89%	9.00%	22.11%	6.57%	0.32%	1.12%	5.82%	11.54%	21.35%
AR(4)TARCH(2,1)	6.04%	4.55%	12.66%	5.26%	61.47%	52.22%	5.83%	11.11%	22.81%	6.57%	0.32%	1.12%	5.87%	9.38%	13.53%
AR(0)TARCH(2,2)	5.72%	16.38%	31.98%	5.32%	54.14%	46.95%	5.72%	16.49%	35.94%	6.13%	3.16%	8.42%	5.65%	20.47%	29.27%
AR(1)TARCH(2,2)	5.94%	7.21%	14.99%	5.38%	47.31%	63.30%	5.83%	11.11%	25.17%	6.13%	3.16%	6.60%	5.60%	24.37%	31.69%
AR(2)TARCH(2,2)	5.99%	5.75%	12.04%	5.38%	47.31%	63.30%	5.56%	28.04%	52.20%	6.41%	0.81%	2.38%	5.76%	14.09%	24.02%
AR(3)TARCH(2,2)	5.88%	8.96%	21.94%	5.26%	61.47%	52.22%	5.61%	23.69%	40.96%	6.02%	5.14%	13.93%	5.76%	14.09%	24.02%
AR(4)TARCH(2,2)	5.94%	7.21%	18.20%	5.55%	30.02%	26.90%	5.56%	28.04%	44.80%	6.41%	0.81%	1.81%	5.76%	14.09%	24.02%
AR(0)EGARCH(0,1)	6.58%	0.28%	0.99%	5.89%	9.62%	14.77%	6.87%	0.05%	0.04%	6.35%	1.08%	2.99%	6.89%	0.04%	0.01%
AR(1)EGARCH(0,1)	6.64%	0.20%	0.72%	6.00%	6.14%	9.31%	7.03%	0.02%	0.02%	6.41%	0.81%	2.38%	7.11%	0.01%	0.00%
AR(2)EGARCH(0,1)	6.64%	0.20%	0.72%	5.94%	7.72%	11.80%	6.98%	0.02%	0.03%	6.52%	0.44%	1.46%	7.16%	0.01%	0.00%
AR(3)EGARCH(0,1)	6.75%	0.10%	0.42%	5.94%	7.72%	11.80%	7.08%	0.01%	0.01%	6.52%	0.44%	1.46%	7.22%	0.00%	0.00%
AR(4)EGARCH(0,1)	6.80%	0.07%	0.30%	5.94%	7.72%	11.80%	7.03%	0.02%	0.01%	6.52%	0.44%	1.46%	7.27%	0.00%	0.00%
AR(0)EGARCH(0,2)	6.42%	0.70%	2.45%	5.94%	7.72%	11.80%	6.38%	0.94%	1.37%	6.24%	1.87%	4.55%	6.79%	0.08%	0.03%
AR(1)EGARCH(0,2)	6.42%	0.70%	2.17%	6.00%	6.14%	9.31%	6.43%	0.70%	1.13%	6.30%	1.43%	3.71%	6.84%	0.06%	0.03%
AR(2)EGARCH(0,2)	6.48%	0.52%	1.70%	6.06%	4.85%	7.25%	6.54%	0.38%	0.74%	6.30%	1.43%	3.71%	6.84%	0.06%	0.03%
AR(3)EGARCH(0,2)	6.64%	0.20%	0.48%	6.06%	4.85%	7.25%	6.54%	0.38%	0.74%	6.46%	0.60%	1.45%	7.00%	0.02%	0.01%
AR(4)EGARCH(0,2)	6.64%	0.20%	0.48%	6.11%	3.79%	5.59%	6.65%	0.20%	0.46%	6.52%	0.44%	1.15%	7.05%	0.01%	0.01%
AR(0)EGARCH(1,1)	5.94%	7.21%	18.20%	5.26%	61.47%	52.22%	6.10%	3.58%	10.34%	6.63%	0.23%	0.20%	5.65%	20.47%	19.39%
AR(1)EGARCH(1,1)	5.99%	5.75%	14.91%	5.38%	47.31%	41.66%	6.16%	2.78%	8.34%	6.52%	0.44%	0.51%	5.76%	14.09%	16.62%
AR(2)EGARCH(1,1)	6.04%	4.55%	12.08%	5.66%	21.26%	18.55%	6.05%	4.56%	12.18%	6.74%	0.12%	0.31%	6.09%	3.77%	3.23%
AR(3)EGARCH(1,1)	6.04%	4.55%	12.08%	5.32%	54.14%	46.95%	6.10%	3.58%	10.34%	6.46%	0.60%	1.00%	5.92%	7.56%	7.74%
AR(4)EGARCH(1,1)	6.04%	4.55%	12.08%	5.38%	47.31%	41.66%	6.10%	3.58%	10.34%	6.46%	0.60%	1.45%	6.03%	4.80%	3.64%
AR(0)EGARCH(1,2)	5.99%	5.75%	15.32%	4.92%											

Table (5.A). Normal Distribution, 2000 sample size, 99%-VaR															
Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(0)GARCH(0,1)	3.94%	0.00%	0.00%	1.53%	3.85%	7.60%	3.38%	0.00%	0.00%	3.45%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(1)GARCH(0,1)	3.99%	0.00%	0.00%	1.53%	3.85%	7.60%	3.43%	0.00%	0.00%	3.45%	0.00%	0.00%	3.50%	0.00%	0.00%
AR(2)GARCH(0,1)	4.10%	0.00%	0.00%	1.53%	3.85%	7.60%	3.49%	0.00%	0.00%	3.45%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(3)GARCH(0,1)	4.10%	0.00%	0.00%	1.58%	2.28%	4.70%	3.49%	0.00%	0.00%	3.45%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(4)GARCH(0,1)	4.05%	0.00%	0.00%	1.64%	1.32%	2.80%	3.54%	0.00%	0.00%	3.40%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(0)GARCH(0,2)	3.40%	0.00%	0.00%	1.64%	1.32%	2.80%	3.11%	0.00%	0.00%	2.90%	0.00%	0.00%	3.12%	0.00%	0.00%
AR(1)GARCH(0,2)	3.29%	0.00%	0.00%	1.70%	0.73%	1.61%	3.11%	0.00%	0.00%	2.96%	0.00%	0.00%	3.12%	0.00%	0.00%
AR(2)GARCH(0,2)	3.29%	0.00%	0.00%	1.58%	2.28%	4.70%	3.11%	0.00%	0.00%	2.96%	0.00%	0.00%	3.18%	0.00%	0.00%
AR(3)GARCH(0,2)	3.29%	0.00%	0.00%	1.64%	1.32%	2.80%	3.16%	0.00%	0.00%	2.85%	0.00%	0.00%	3.39%	0.00%	0.00%
AR(4)GARCH(0,2)	3.24%	0.00%	0.00%	1.64%	1.32%	2.80%	3.22%	0.00%	0.00%	2.85%	0.00%	0.00%	3.45%	0.00%	0.00%
AR(0)GARCH(1,1)	2.16%	0.00%	0.00%	1.64%	1.32%	2.80%	1.58%	2.12%	1.56%	1.81%	0.18%	0.26%	1.72%	0.45%	0.99%
AR(1)GARCH(1,1)	2.10%	0.00%	0.00%	1.70%	0.73%	1.61%	1.53%	3.56%	8.11%	1.86%	0.09%	0.15%	1.67%	0.82%	1.76%
AR(2)GARCH(1,1)	2.16%	0.00%	0.00%	1.70%	0.73%	1.61%	1.58%	2.12%	1.56%	1.81%	0.18%	0.26%	1.67%	0.82%	1.76%
AR(3)GARCH(1,1)	2.10%	0.00%	0.00%	1.70%	0.73%	1.61%	1.63%	1.23%	1.08%	1.81%	0.18%	0.26%	1.72%	0.45%	0.99%
AR(4)GARCH(1,1)	2.16%	0.00%	0.00%	1.64%	1.32%	2.80%	1.58%	2.12%	1.56%	1.81%	0.18%	0.26%	1.67%	0.82%	1.76%
AR(0)GARCH(1,2)	2.10%	0.00%	0.00%	1.64%	1.32%	2.80%	1.69%	0.69%	2.14%	1.75%	0.35%	0.43%	1.62%	1.44%	3.01%
AR(1)GARCH(1,2)	1.94%	0.03%	0.02%	1.64%	1.32%	2.80%	1.74%	0.38%	1.28%	1.64%	1.15%	3.28%	1.62%	1.44%	3.01%
AR(2)GARCH(1,2)	2.10%	0.00%	0.00%	1.64%	1.32%	2.80%	1.74%	0.38%	1.28%	1.70%	0.64%	2.01%	1.67%	0.82%	1.76%
AR(3)GARCH(1,2)	2.21%	0.00%	0.00%	1.70%	0.73%	1.61%	1.69%	0.69%	2.14%	1.75%	0.35%	1.19%	1.67%	0.82%	1.76%
AR(4)GARCH(1,2)	2.16%	0.00%	0.00%	1.64%	1.32%	2.80%	1.69%	0.69%	2.14%	1.75%	0.35%	0.43%	1.67%	0.82%	1.76%
AR(0)GARCH(2,1)	2.21%	0.00%	0.00%	1.64%	1.32%	2.80%	1.69%	0.69%	0.71%	1.81%	0.18%	0.26%	1.72%	0.45%	0.99%
AR(1)GARCH(2,1)	2.16%	0.00%	0.00%	1.64%	1.32%	2.80%	1.80%	0.20%	0.28%	1.81%	0.18%	0.26%	1.67%	0.82%	1.76%
AR(2)GARCH(2,1)	2.10%	0.00%	0.00%	1.64%	1.32%	2.80%	1.74%	0.38%	1.28%	1.81%	0.18%	0.26%	1.67%	0.82%	1.76%
AR(3)GARCH(2,1)	2.16%	0.00%	0.00%	1.64%	1.32%	2.80%	1.58%	2.12%	5.40%	1.75%	0.35%	0.43%	1.62%	1.44%	3.01%
AR(4)GARCH(2,1)	2.16%	0.00%	0.00%	1.64%	1.32%	2.80%	1.69%	0.69%	2.14%	1.81%	0.18%	0.26%	1.67%	0.82%	1.76%
AR(0)GARCH(2,2)	2.10%	0.00%	0.00%	1.64%	1.32%	2.80%	1.69%	0.69%	2.14%	1.75%	0.35%	0.43%	1.62%	1.44%	3.01%
AR(1)GARCH(2,2)	2.10%	0.00%	0.00%	1.64%	1.32%	2.80%	1.74%	0.38%	1.28%	1.81%	0.18%	0.26%	1.56%	2.46%	4.97%
AR(2)GARCH(2,2)	2.16%	0.00%	0.00%	1.64%	1.32%	2.80%	1.69%	0.69%	2.14%	1.75%	0.35%	0.43%	1.62%	1.44%	3.01%
AR(3)GARCH(2,2)	2.05%	0.01%	0.00%	1.64%	1.32%	2.80%	1.74%	0.38%	1.28%	1.75%	0.35%	0.43%	1.62%	1.44%	3.01%
AR(4)GARCH(2,2)	2.10%	0.00%	0.00%	1.64%	1.32%	2.80%	1.69%	0.69%	2.14%	1.75%	0.35%	0.43%	1.72%	0.45%	0.99%
AR(0)TARCH(0,1)	3.83%	0.00%	0.00%	1.53%	3.85%	7.60%	3.43%	0.00%	0.00%	3.34%	0.00%	0.00%	3.45%	0.00%	0.00%
AR(1)TARCH(0,1)	3.83%	0.00%	0.00%	1.53%	3.85%	7.60%	3.43%	0.00%	0.00%	3.40%	0.00%	0.00%	3.50%	0.00%	0.00%
AR(2)TARCH(0,1)	3.78%	0.00%	0.00%	1.58%	2.28%	4.70%	3.49%	0.00%	0.00%	3.34%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(3)TARCH(0,1)	3.72%	0.00%	0.00%	1.58%	2.28%	4.70%	3.54%	0.00%	0.00%	3.29%	0.00%	0.00%	3.50%	0.00%	0.00%
AR(4)TARCH(0,1)	3.72%	0.00%	0.00%	1.58%	2.28%	4.70%	3.54%	0.00%	0.00%	3.29%	0.00%	0.00%	3.50%	0.00%	0.00%
AR(0)TARCH(0,2)	2.97%	0.00%	0.00%	1.64%	1.32%	2.80%	2.89%	0.00%	0.00%	2.85%	0.00%	0.00%	3.28%	0.00%	0.00%
AR(1)TARCH(0,2)	3.08%	0.00%	0.00%	1.64%	1.32%	2.80%	2.89%	0.00%	0.00%	2.85%	0.00%	0.00%	3.07%	0.00%	0.00%
AR(2)TARCH(0,2)	3.02%	0.00%	0.00%	1.64%	1.32%	2.80%	2.89%	0.00%	0.00%	2.85%	0.00%	0.00%	3.12%	0.00%	0.00%
AR(3)TARCH(0,2)	3.13%	0.00%	0.00%	1.64%	1.32%	2.80%	2.89%	0.00%	0.00%	2.79%	0.00%	0.00%	3.28%	0.00%	0.00%
AR(4)TARCH(0,2)	3.13%	0.00%	0.00%	1.64%	1.32%	2.80%	2.94%	0.00%	0.00%	2.79%	0.00%	0.00%	3.28%	0.00%	0.00%
AR(0)TARCH(1,1)	1.94%	0.03%	0.14%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	1.81%	0.18%	0.26%	1.40%	10.22%	17.93%
AR(1)TARCH(1,1)	1.94%	0.03%	0.06%	1.47%	6.27%	11.82%	1.69%	0.69%	1.50%	1.75%	0.35%	0.43%	1.40%	10.22%	17.93%
AR(2)TARCH(1,1)	1.89%	0.06%	0.26%	1.53%	3.85%	7.60%	1.63%	1.23%	2.60%	1.75%	0.35%	0.43%	1.40%	10.22%	17.93%
AR(3)TARCH(1,1)	1.94%	0.03%	0.06%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	1.81%	0.18%	0.26%	1.40%	10.22%	17.93%
AR(4)TARCH(1,1)	1.89%	0.06%	0.26%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	1.81%	0.18%	0.26%	1.40%	10.22%	17.93%
AR(0)TARCH(1,2)	1.94%	0.03%	0.14%	1.53%	3.85%	7.60%	1.80%	0.20%	0.73%	1.75%	0.35%	0.43%	1.45%	6.56%	12.14%
AR(1)TARCH(1,2)	2.00%	0.01%	0.03%	1.53%	3.85%	7.60%	1.80%	0.20%	0.73%	1.75%	0.35%	0.43%	1.51%	4.08%	7.91%
AR(2)TARCH(1,2)	1.94%	0.03%	0.06%	1.53%	3.85%	7.60%	1.80%	0.20%	0.73%	1.81%	0.18%	0.26%	1.45%	6.56%	12.14%
AR(3)TARCH(1,2)	2.05%	0.01%	0.00%	1.53%	3.85%	7.60%	1.80%	0.20%	0.73%	1.81%	0.18%	0.26%	1.51%	4.08%	7.91%
AR(4)TARCH(1,2)	2.00%	0.01%	0.03%	1.58%	2.28%	4.70%	1.80%	0.20%	0.73%	1.81%	0.18%	0.26%	1.51%	4.08%	7.91%
AR(0)TARCH(2,1)	1.78%	0.23%	0.53%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	1.92%	0.05%	0.20%	1.40%	10.22%	17.93%
AR(1)TARCH(2,1)	1.89%	0.06%	0.26%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	1.81%	0.18%	0.68%	1.40%	10.22%	17.93%
AR(2)TARCH(2,1)	1.78%	0.23%	0.84%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	1.81%	0.18%	0.68%	1.40%	10.22%	17.93%
AR(3)TARCH(2,1)	1.78%	0.23%	0.53%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	1.81%	0.18%	0.68%	1.45%	6.56%	12.14%
AR(4)TARCH(2,1)	2.00%	0.01%	0.03%	1.53%	3.85%	7.60%	1.69%	0.69%	2.14%	1.86%	0.09%	0.38%	1.40%	10.22%	17.93%
AR(0)TARCH(2,2)	2.00%	0.01%	0.07%	1.53%	3.85%	7.60%	1.80%	0.20%	0.73%	1.64%	1.15%	3.28%	1.56%	2.46%	4.97%
AR(1)TARCH(2,2)	1.94%	0.03%	0.14%	1.53%	3.85%	7.60%	1.85%	0.10%	0.41%	1.75%	0.35%	1.19%	1.51%	4.08%	7.91%
AR(2)TARCH(2,2)	1.89%	0.06%	0.26%	1.47%	6.27%	11.82%	1.74%	0.38%	1.28%	1.75%	0.35%	0.78%	1.45%	6.56%	12.14%
AR(3)TARCH(2,2)	1.89%	0.06%	0.26%	1.47%	6.27%	11.82%	1.74%	0.38%	1.28%	1.64%	1.15%	2.45%	1.51%	4.08%	7.91%
AR(4)TARCH(2,2)	2.05%	0.01%	0.00%	1.53%	3.85%	7.60%	1.74%	0.38%	1.28%	1.64%	1.15%	3.28%	1.56%	2.46%	4.97%
AR(0)EGARCH(0,1)	4.05%	0.00%	0.00%	1.47%	6.27%	11.82%	3.43%	0.00%	0.00%	3.40%	0.00%	0.00%	3.50%	0.00%	0.00%
AR(1)EGARCH(0,1)	4.05%	0.00%	0.00%	1.47%	6.27%	11.82%	3.54%	0.00%	0.00%	3.40%	0.00%	0.00%	3.50%	0.00%	0.00%
AR(2)EGARCH(0,1)	4.05%	0.00%	0.00%	1.47%	6.27%	11.82%	3.54%	0.00%	0.00%	3.40%	0.00%	0.00%	3.50%	0.00%	0.00%
AR(3)EGARCH(0,1)	4.10%	0.00%	0.00%	1.47%	6.27%	11.82%	3.60%	0.00%	0.00%	3.40%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(4)EGARCH(0,1)	4.05%	0.00%	0.00%	1.64%	1.32%	2.80%	3.60%	0.00%	0.00%	3.40%	0.00%	0.00%	3.61%	0.00%	0.00%
AR(0)EGARCH(0,2)	3.99%	0.00%	0.00%	1.75%	0.40%	0.89%	3.38%	0.00%	0.00%	3.01%	0.00%	0.00%	3.66%	0.00%	0.00%
AR(1)EGARCH(0,2)	3.94%	0.00%	0.00%	1.75%	0.40%	0.89%	3.38%	0.00%	0.00%	3.01%	0.00%	0.00%	3.77%	0.00%	0.00%
AR(2)EGARCH(0,2)	3.99%	0.00%	0.00%	1.75%	0.40%	0.89%	3.43%	0.00%	0.00%	3.01%	0.00%	0.00%	3.77%	0.00%	0.00%
AR(3)EGARCH(0,2)	4.05%	0.00%	0.00%	1.75%	0.40%	0.89%	3.38%	0.00%	0.00%	3.01%	0.00%	0.00%	3.72%	0.00%	0.00%
AR(4)EGARCH(0,2)	4.05%	0.00%	0.00%	1.75%	0.40%	0.89%	3.38%	0.00%	0.00%	3.07%	0.00%	0.00%	3.72%	0.00%	0.00%
AR(0)EGARCH(1,1)	1.94%	0.03%	0.14%	1.36%	15.12%	25.30%	1.74%	0.38%	0.46%	1.86%	0.09%	0.15%	1.45%	6.56%	12.14%
AR(1)EGARCH(1,1)	2.00%	0.01%	0.03%	1.36%	15.12%	25.30%	1.80%	0.20%	0.28%	1.92%	0.05%	0.09%	1.51%	4.08%	7.91%
AR(2)EGARCH(1,1)	1.94%	0.03%	0.06%	1.36%	15.12%	25.30%	1.69%	0.69%	0.71%	1.92%	0.05%	0.09%	1.56%	2.46%	4.97%
AR(3)EGARCH(1,1)	1.94%	0.03%	0.06%	1.36%	15.12%	25.30%	1.74%	0.38%	0.46%	1.81%	0.18%	0.26%	1.51%	4.08%	7.91%
AR(4)EGARCH(1,1)	1.94%	0.03%	0.06%	1.36%	15.12%	25.30%	1.74%	0.38%	0.46%	1.86%	0.09%	0.15%	1.51%	4.08%	7.91%
AR(0)EGARCH(1,2)	1.78%	0.23%	0.84%	1.36%	15.12%	25.30%	1.69%	0.69%	0.71%	2.03%	0.01%	0.03%	1.40%	10.22%	17.93%
AR(1)EGARCH(1,2)	1.83%	0.12%	0.47%	1.36%	15.12%	25.30%	1								

Table (6.A). Normal Distribution, 1500 sample size, 99%-VaR															
Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(0)GARCH(0,1)	3.78%	0.00%	0.00%	1.53%	3.85%	7.60%	3.38%	0.00%	0.00%	3.29%	0.00%	0.00%	3.34%	0.00%	0.00%
AR(1)GARCH(0,1)	3.78%	0.00%	0.00%	1.53%	3.85%	7.60%	3.27%	0.00%	0.00%	3.23%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(2)GARCH(0,1)	3.72%	0.00%	0.00%	1.58%	2.28%	4.70%	3.32%	0.00%	0.00%	3.23%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(3)GARCH(0,1)	3.67%	0.00%	0.00%	1.58%	2.28%	4.70%	3.38%	0.00%	0.00%	3.29%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(4)GARCH(0,1)	3.78%	0.00%	0.00%	1.58%	2.28%	4.70%	3.49%	0.00%	0.00%	3.29%	0.00%	0.00%	3.50%	0.00%	0.00%
AR(0)GARCH(0,2)	3.18%	0.00%	0.00%	1.53%	3.85%	7.60%	3.11%	0.00%	0.00%	2.79%	0.00%	0.00%	3.12%	0.00%	0.00%
AR(1)GARCH(0,2)	3.18%	0.00%	0.00%	1.58%	2.28%	4.70%	3.11%	0.00%	0.00%	2.79%	0.00%	0.00%	3.02%	0.00%	0.00%
AR(2)GARCH(0,2)	3.18%	0.00%	0.00%	1.53%	3.85%	7.60%	3.11%	0.00%	0.00%	2.85%	0.00%	0.00%	3.12%	0.00%	0.00%
AR(3)GARCH(0,2)	3.24%	0.00%	0.00%	1.53%	3.85%	7.60%	3.11%	0.00%	0.00%	2.85%	0.00%	0.00%	3.23%	0.00%	0.00%
AR(4)GARCH(0,2)	3.18%	0.00%	0.00%	1.53%	3.85%	7.60%	3.11%	0.00%	0.00%	2.85%	0.00%	0.00%	3.28%	0.00%	0.00%
AR(0)GARCH(1,1)	2.16%	0.00%	0.00%	1.47%	6.27%	11.82%	1.69%	0.69%	2.14%	1.59%	2.00%	5.14%	1.56%	2.46%	4.97%
AR(1)GARCH(1,1)	2.10%	0.00%	0.00%	1.53%	3.85%	7.60%	1.63%	1.23%	3.46%	1.64%	1.15%	3.28%	1.56%	2.46%	4.97%
AR(2)GARCH(1,1)	2.16%	0.00%	0.00%	1.58%	2.28%	4.70%	1.63%	1.23%	3.46%	1.70%	0.64%	2.01%	1.67%	0.82%	1.76%
AR(3)GARCH(1,1)	2.21%	0.00%	0.00%	1.53%	3.85%	7.60%	1.63%	1.23%	3.46%	1.70%	0.64%	2.01%	1.56%	2.46%	4.97%
AR(4)GARCH(1,1)	2.16%	0.00%	0.00%	1.53%	3.85%	7.60%	1.63%	1.23%	3.46%	1.64%	1.15%	3.28%	1.56%	2.46%	4.97%
AR(0)GARCH(1,2)	2.16%	0.00%	0.00%	1.53%	3.85%	7.60%	1.58%	2.12%	4.35%	1.59%	2.00%	5.14%	1.67%	0.82%	2.48%
AR(1)GARCH(1,2)	2.16%	0.00%	0.00%	1.58%	2.28%	4.70%	1.63%	1.23%	3.46%	1.64%	1.15%	3.28%	1.72%	0.45%	1.49%
AR(2)GARCH(1,2)	2.27%	0.00%	0.00%	1.58%	2.28%	4.70%	1.63%	1.23%	3.46%	1.59%	2.00%	4.11%	1.62%	1.44%	3.96%
AR(3)GARCH(1,2)	2.27%	0.00%	0.00%	1.58%	2.28%	4.70%	1.63%	1.23%	3.46%	1.64%	1.15%	3.28%	1.62%	1.44%	3.01%
AR(4)GARCH(1,2)	2.27%	0.00%	0.00%	1.58%	2.28%	4.70%	1.63%	1.23%	3.46%	1.59%	2.00%	4.11%	1.56%	2.46%	4.97%
AR(0)GARCH(2,1)	2.21%	0.00%	0.00%	1.53%	3.85%	7.60%	1.58%	2.12%	5.40%	1.59%	2.00%	5.14%	1.72%	0.45%	0.99%
AR(1)GARCH(2,1)	2.10%	0.00%	0.00%	1.53%	3.85%	7.60%	1.58%	2.12%	5.40%	1.70%	0.64%	2.01%	1.56%	2.46%	4.97%
AR(2)GARCH(2,1)	2.27%	0.00%	0.00%	1.58%	2.28%	4.70%	1.58%	2.12%	5.40%	1.64%	1.15%	3.28%	1.62%	1.44%	3.01%
AR(3)GARCH(2,1)	2.32%	0.00%	0.00%	1.41%	9.90%	17.66%	1.58%	2.12%	5.40%	1.64%	1.15%	3.28%	1.72%	0.45%	0.99%
AR(4)GARCH(2,1)	2.16%	0.00%	0.00%	1.58%	2.28%	4.70%	1.58%	2.12%	5.40%	1.75%	0.35%	1.19%	1.72%	0.45%	0.99%
AR(0)GARCH(2,2)	2.32%	0.00%	0.00%	1.53%	3.85%	7.60%	1.80%	0.20%	0.73%	1.70%	0.64%	2.01%	1.62%	1.44%	3.96%
AR(1)GARCH(2,2)	2.27%	0.00%	0.00%	1.53%	3.85%	7.60%	1.69%	0.69%	2.14%	1.64%	1.15%	3.28%	1.45%	6.56%	12.14%
AR(2)GARCH(2,2)	2.27%	0.00%	0.00%	1.53%	3.85%	7.60%	1.63%	1.23%	3.46%	1.59%	2.00%	4.11%	1.62%	1.44%	3.01%
AR(3)GARCH(2,2)	2.32%	0.00%	0.00%	1.53%	3.85%	7.60%	1.63%	1.23%	3.46%	1.64%	1.15%	3.28%	1.62%	1.44%	3.01%
AR(4)GARCH(2,2)	2.27%	0.00%	0.00%	1.53%	3.85%	7.60%	1.63%	1.23%	3.46%	1.70%	0.64%	1.40%	1.56%	2.46%	4.97%
AR(0)TARCH(0,1)	3.62%	0.00%	0.00%	1.53%	3.85%	7.60%	3.32%	0.00%	0.00%	3.29%	0.00%	0.00%	3.39%	0.00%	0.00%
AR(1)TARCH(0,1)	3.56%	0.00%	0.00%	1.53%	3.85%	7.60%	3.38%	0.00%	0.00%	3.29%	0.00%	0.00%	3.39%	0.00%	0.00%
AR(2)TARCH(0,1)	3.56%	0.00%	0.00%	1.53%	3.85%	7.60%	3.38%	0.00%	0.00%	3.29%	0.00%	0.00%	3.45%	0.00%	0.00%
AR(3)TARCH(0,1)	3.56%	0.00%	0.00%	1.58%	2.28%	4.70%	3.38%	0.00%	0.00%	3.23%	0.00%	0.00%	3.50%	0.00%	0.00%
AR(4)TARCH(0,1)	3.56%	0.00%	0.00%	1.58%	2.28%	4.70%	3.38%	0.00%	0.00%	3.29%	0.00%	0.00%	3.45%	0.00%	0.00%
AR(0)TARCH(0,2)	3.13%	0.00%	0.00%	1.53%	3.85%	7.60%	2.94%	0.00%	0.00%	2.96%	0.00%	0.00%	3.18%	0.00%	0.00%
AR(1)TARCH(0,2)	3.13%	0.00%	0.00%	1.53%	3.85%	7.60%	3.00%	0.00%	0.00%	2.96%	0.00%	0.00%	3.02%	0.00%	0.00%
AR(2)TARCH(0,2)	3.18%	0.00%	0.00%	1.53%	3.85%	7.60%	3.00%	0.00%	0.00%	2.90%	0.00%	0.00%	3.12%	0.00%	0.00%
AR(3)TARCH(0,2)	3.18%	0.00%	0.00%	1.53%	3.85%	7.60%	3.00%	0.00%	0.00%	2.79%	0.00%	0.00%	3.23%	0.00%	0.00%
AR(4)TARCH(0,2)	3.18%	0.00%	0.00%	1.58%	2.28%	4.70%	3.00%	0.00%	0.00%	2.85%	0.00%	0.00%	3.18%	0.00%	0.00%
AR(0)TARCH(1,1)	2.00%	0.01%	0.07%	1.41%	9.90%	17.66%	1.74%	0.38%	0.84%	1.70%	0.64%	1.40%	1.45%	6.56%	12.14%
AR(1)TARCH(1,1)	2.10%	0.00%	0.01%	1.41%	9.90%	17.66%	1.74%	0.38%	0.84%	1.81%	0.18%	0.42%	1.40%	10.22%	17.93%
AR(2)TARCH(1,1)	2.00%	0.01%	0.07%	1.41%	9.90%	17.66%	1.69%	0.69%	1.50%	1.75%	0.35%	0.78%	1.40%	10.22%	17.93%
AR(3)TARCH(1,1)	2.00%	0.01%	0.07%	1.41%	9.90%	17.66%	1.69%	0.69%	1.50%	1.81%	0.18%	0.68%	1.40%	10.22%	17.93%
AR(4)TARCH(1,1)	2.00%	0.01%	0.07%	1.41%	9.90%	17.66%	1.69%	0.69%	1.50%	1.75%	0.35%	1.19%	1.40%	10.22%	17.93%
AR(0)TARCH(1,2)	2.05%	0.01%	0.02%	1.36%	15.12%	25.30%	1.63%	1.23%	2.60%	1.70%	0.64%	2.01%	1.45%	6.56%	12.14%
AR(1)TARCH(1,2)	2.10%	0.00%	0.01%	1.36%	15.12%	25.30%	1.63%	1.23%	2.60%	1.70%	0.64%	2.01%	1.51%	4.08%	7.91%
AR(2)TARCH(1,2)	2.10%	0.00%	0.01%	1.36%	15.12%	25.30%	1.69%	0.69%	1.50%	1.70%	0.64%	2.01%	1.56%	2.46%	6.10%
AR(3)TARCH(1,2)	2.16%	0.00%	0.00%	1.36%	15.12%	25.30%	1.63%	1.23%	2.60%	1.75%	0.35%	1.19%	1.51%	4.08%	7.91%
AR(4)TARCH(1,2)	2.10%	0.00%	0.01%	1.41%	9.90%	17.66%	1.63%	1.23%	2.60%	1.97%	0.02%	0.05%	1.51%	4.08%	7.91%
AR(0)TARCH(2,1)	2.10%	0.00%	0.02%	1.41%	9.90%	17.66%	1.69%	0.69%	1.50%	1.81%	0.18%	0.68%	1.45%	6.56%	12.14%
AR(1)TARCH(2,1)	2.05%	0.01%	0.03%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	1.81%	0.18%	0.68%	1.40%	10.22%	17.93%
AR(2)TARCH(2,1)	2.00%	0.01%	0.07%	1.41%	9.90%	17.66%	1.74%	0.38%	1.28%	1.75%	0.35%	1.19%	1.40%	10.22%	17.93%
AR(3)TARCH(2,1)	2.05%	0.01%	0.02%	1.41%	9.90%	17.66%	1.74%	0.38%	1.28%	1.81%	0.18%	0.68%	1.40%	10.22%	17.93%
AR(4)TARCH(2,1)	2.05%	0.01%	0.02%	1.41%	9.90%	17.66%	1.74%	0.38%	1.28%	1.75%	0.35%	1.19%	1.40%	10.22%	17.93%
AR(0)TARCH(2,2)	2.10%	0.00%	0.01%	1.36%	15.12%	25.30%	1.63%	1.23%	2.60%	1.64%	1.15%	2.45%	1.45%	6.56%	12.14%
AR(1)TARCH(2,2)	2.10%	0.00%	0.01%	1.36%	15.12%	25.30%	1.69%	0.69%	1.50%	1.75%	0.35%	0.43%	1.45%	6.56%	12.14%
AR(2)TARCH(2,2)	2.05%	0.01%	0.02%	1.41%	9.90%	17.66%	1.69%	0.69%	1.50%	1.75%	0.35%	0.78%	1.45%	6.56%	12.14%
AR(3)TARCH(2,2)	2.05%	0.01%	0.02%	1.36%	15.12%	25.30%	1.69%	0.69%	2.14%	1.64%	1.15%	2.45%	1.45%	6.56%	12.14%
AR(4)TARCH(2,2)	2.00%	0.01%	0.03%	1.41%	9.90%	17.66%	1.63%	1.23%	3.46%	1.75%	0.35%	1.19%	1.51%	4.08%	7.91%
AR(0)EGARCH(0,1)	3.62%	0.00%	0.00%	1.47%	6.27%	12.19%	3.54%	0.00%	0.00%	3.18%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(1)EGARCH(0,1)	3.72%	0.00%	0.00%	1.47%	6.27%	12.19%	3.54%	0.00%	0.00%	3.23%	0.00%	0.00%	3.61%	0.00%	0.00%
AR(2)EGARCH(0,1)	3.72%	0.00%	0.00%	1.53%	3.85%	8.47%	3.54%	0.00%	0.00%	3.18%	0.00%	0.00%	3.66%	0.00%	0.00%
AR(3)EGARCH(0,1)	3.78%	0.00%	0.00%	1.47%	6.27%	11.82%	3.60%	0.00%	0.00%	3.18%	0.00%	0.00%	3.66%	0.00%	0.00%
AR(4)EGARCH(0,1)	3.78%	0.00%	0.00%	1.47%	6.27%	12.19%	3.54%	0.00%	0.00%	3.18%	0.00%	0.00%	3.66%	0.00%	0.00%
AR(0)EGARCH(0,2)	4.05%	0.00%	0.00%	1.81%	0.21%	0.48%	3.38%	0.00%	0.00%	3.01%	0.00%	0.00%	3.61%	0.00%	0.00%
AR(1)EGARCH(0,2)	4.10%	0.00%	0.00%	1.75%	0.40%	0.89%	3.38%	0.00%	0.00%	3.01%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(2)EGARCH(0,2)	4.05%	0.00%	0.00%	1.87%	0.11%	0.25%	3.43%	0.00%	0.00%	3.01%	0.00%	0.00%	3.72%	0.00%	0.00%
AR(3)EGARCH(0,2)	4.10%	0.00%	0.00%	1.87%	0.11%	0.25%	3.38%	0.00%	0.00%	2.96%	0.00%	0.00%	3.55%	0.00%	0.00%
AR(4)EGARCH(0,2)	3.99%	0.00%	0.00%	1.87%	0.11%	0.25%	3.38%	0.00%	0.00%	2.96%	0.00%	0.00%	3.61%	0.00%	0.00%
AR(0)EGARCH(1,1)	2.16%	0.00%	0.00%	1.25%	31.87%	45.51%	1.80%	0.20%	0.73%	1.75%	0.35%	1.19%	1.51%	4.08%	7.91%
AR(1)EGARCH(1,1)	2.21%	0.00%	0.00%	1.25%	31.87%	45.51%	1.80%	0.20%	0.73%	1.86%	0.09%	0.15%	1.56%	2.46%	4.97%
AR(2)EGARCH(1,1)	2.16%	0.00%	0.00%	1.25%	31.87%	45.51%	1.80%	0.20%	0.73%	1.97%	0.02%	0.05%	1.51%	4.08%	7.91%
AR(3)EGARCH(1,1)	2.05%	0.01%	0.02%	1.25%	31.87%	45.51%	1.80%	0.20%	0.73%	1.92%	0.05%	0.09%	1.56%	2.46%	4.97%
AR(4)EGARCH(1,1)	2.16%	0.00%	0.00%	1.30%	22.33%	34.71%	1.74%	0.38%	1.28%	1.86%	0.09%	0.15%	1.51%	4.08%	7.91%
AR(0)EGARCH(1,2)	1.89%	0.06%	0.11%	1.30%	22.33%	34.71%	1.74%	0.38%	1.28%	1.81%	0.18%	0.68%	1.62%	1.44%	3.01%
AR(1)EGARCH(1,2)	2.00%	0.01%	0.03%	1.30%	22.										

Table (7.A). Normal Distribution, 1000 sample size, 99%-VaR															
Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(0)GARCH(0,1)	3.18%	0.00%	0.00%	1.58%	2.28%	5.65%	3.27%	0.00%	0.00%	3.23%	0.00%	0.00%	3.02%	0.00%	0.00%
AR(1)GARCH(0,1)	3.13%	0.00%	0.00%	1.58%	2.28%	5.65%	3.27%	0.00%	0.00%	3.23%	0.00%	0.00%	2.96%	0.00%	0.00%
AR(2)GARCH(0,1)	3.18%	0.00%	0.00%	1.53%	3.85%	8.47%	3.32%	0.00%	0.00%	3.23%	0.00%	0.00%	3.02%	0.00%	0.00%
AR(3)GARCH(0,1)	3.18%	0.00%	0.00%	1.53%	3.85%	8.47%	3.32%	0.00%	0.00%	3.23%	0.00%	0.00%	3.12%	0.00%	0.00%
AR(4)GARCH(0,1)	3.13%	0.00%	0.00%	1.53%	3.85%	8.47%	3.27%	0.00%	0.00%	3.29%	0.00%	0.00%	3.18%	0.00%	0.00%
AR(0)GARCH(0,2)	3.08%	0.00%	0.00%	1.47%	6.27%	11.82%	2.89%	0.00%	0.00%	2.63%	0.00%	0.00%	2.69%	0.00%	0.00%
AR(1)GARCH(0,2)	3.08%	0.00%	0.00%	1.58%	2.28%	4.70%	2.89%	0.00%	0.00%	2.63%	0.00%	0.00%	2.53%	0.00%	0.00%
AR(2)GARCH(0,2)	3.13%	0.00%	0.00%	1.47%	6.27%	11.82%	2.94%	0.00%	0.00%	2.57%	0.00%	0.00%	2.64%	0.00%	0.00%
AR(3)GARCH(0,2)	3.13%	0.00%	0.00%	1.47%	6.27%	11.82%	2.83%	0.00%	0.00%	2.63%	0.00%	0.00%	2.69%	0.00%	0.00%
AR(4)GARCH(0,2)	3.13%	0.00%	0.00%	1.47%	6.27%	11.82%	3.00%	0.00%	0.00%	2.63%	0.00%	0.00%	2.80%	0.00%	0.00%
AR(0)GARCH(1,1)	2.05%	0.01%	0.00%	1.64%	1.32%	3.62%	1.80%	0.20%	0.73%	1.81%	0.18%	0.42%	1.51%	4.08%	7.91%
AR(1)GARCH(1,1)	2.10%	0.00%	0.00%	1.64%	1.32%	3.62%	1.74%	0.38%	1.28%	1.75%	0.35%	1.19%	1.51%	4.08%	7.91%
AR(2)GARCH(1,1)	2.16%	0.00%	0.00%	1.64%	1.32%	3.62%	1.69%	0.69%	2.14%	1.64%	1.15%	2.45%	1.51%	4.08%	7.91%
AR(3)GARCH(1,1)	2.16%	0.00%	0.00%	1.70%	0.73%	2.23%	1.69%	0.69%	2.14%	1.64%	1.15%	2.45%	1.56%	2.46%	4.97%
AR(4)GARCH(1,1)	2.05%	0.01%	0.02%	1.70%	0.73%	2.23%	1.69%	0.69%	2.14%	1.64%	1.15%	2.45%	1.62%	1.44%	3.01%
AR(0)GARCH(1,2)	2.10%	0.00%	0.00%	1.58%	2.28%	5.65%	1.63%	1.23%	2.60%	1.70%	0.64%	1.40%	1.56%	2.46%	4.97%
AR(1)GARCH(1,2)	2.10%	0.00%	0.00%	1.64%	1.32%	3.62%	1.63%	1.23%	2.60%	1.59%	2.00%	4.11%	1.45%	6.56%	12.14%
AR(2)GARCH(1,2)	2.10%	0.00%	0.00%	1.64%	1.32%	3.62%	1.63%	1.23%	2.60%	1.70%	0.64%	1.40%	1.56%	2.46%	4.97%
AR(3)GARCH(1,2)	2.10%	0.00%	0.00%	1.64%	1.32%	3.62%	1.74%	0.38%	1.28%	1.64%	1.15%	2.45%	1.51%	4.08%	7.91%
AR(4)GARCH(1,2)	2.16%	0.00%	0.00%	1.64%	1.32%	3.62%	1.69%	0.69%	1.50%	1.70%	0.64%	1.40%	1.62%	1.44%	3.01%
AR(0)GARCH(2,1)	2.05%	0.01%	0.00%	1.58%	2.28%	4.70%	1.69%	0.69%	2.14%	1.81%	0.18%	0.68%	1.51%	4.08%	7.91%
AR(1)GARCH(2,1)	2.10%	0.00%	0.00%	1.64%	1.32%	3.62%	1.63%	1.23%	3.46%	1.75%	0.35%	0.78%	1.56%	2.46%	4.97%
AR(2)GARCH(2,1)	2.05%	0.01%	0.00%	1.58%	2.28%	5.65%	1.63%	1.23%	3.46%	1.81%	0.18%	0.42%	1.51%	4.08%	7.91%
AR(3)GARCH(2,1)	2.10%	0.00%	0.00%	1.58%	2.28%	5.65%	1.69%	0.69%	2.14%	1.64%	1.15%	2.45%	1.56%	2.46%	4.97%
AR(4)GARCH(2,1)	2.10%	0.00%	0.00%	1.64%	1.32%	3.62%	1.63%	1.23%	3.46%	1.86%	0.09%	0.22%	1.51%	4.08%	7.91%
AR(0)GARCH(2,2)	2.21%	0.00%	0.00%	1.58%	2.28%	5.65%	1.69%	0.69%	1.50%	1.70%	0.64%	1.40%	1.51%	4.08%	7.91%
AR(1)GARCH(2,2)	2.21%	0.00%	0.00%	1.64%	1.32%	3.62%	1.69%	0.69%	1.50%	1.70%	0.64%	1.40%	1.45%	6.56%	12.14%
AR(2)GARCH(2,2)	2.21%	0.00%	0.00%	1.53%	3.85%	8.47%	1.63%	1.23%	2.60%	1.59%	2.00%	4.11%	1.51%	4.08%	7.91%
AR(3)GARCH(2,2)	2.27%	0.00%	0.00%	1.47%	6.27%	12.19%	1.69%	0.69%	1.50%	1.64%	1.15%	2.45%	1.56%	2.46%	4.97%
AR(4)GARCH(2,2)	2.27%	0.00%	0.00%	1.58%	2.28%	5.65%	1.63%	1.23%	2.60%	1.75%	0.35%	0.78%	1.51%	4.08%	7.91%
AR(0)TARCH(0,1)	3.18%	0.00%	0.00%	1.58%	2.28%	5.65%	3.05%	0.00%	0.00%	3.29%	0.00%	0.00%	3.07%	0.00%	0.00%
AR(1)TARCH(0,1)	3.08%	0.00%	0.00%	1.58%	2.28%	4.70%	3.05%	0.00%	0.00%	3.07%	0.00%	0.00%	3.07%	0.00%	0.00%
AR(2)TARCH(0,1)	3.08%	0.00%	0.00%	1.53%	3.85%	7.60%	3.16%	0.00%	0.00%	3.07%	0.00%	0.00%	3.12%	0.00%	0.00%
AR(3)TARCH(0,1)	3.02%	0.00%	0.00%	1.53%	3.85%	7.60%	3.16%	0.00%	0.00%	3.12%	0.00%	0.00%	3.07%	0.00%	0.00%
AR(4)TARCH(0,1)	3.02%	0.00%	0.00%	1.64%	1.32%	3.62%	3.16%	0.00%	0.00%	3.01%	0.00%	0.00%	3.07%	0.00%	0.00%
AR(0)TARCH(0,2)	2.91%	0.00%	0.00%	1.47%	6.27%	11.82%	2.62%	0.00%	0.00%	2.79%	0.00%	0.00%	2.58%	0.00%	0.00%
AR(1)TARCH(0,2)	2.97%	0.00%	0.00%	1.41%	9.90%	17.66%	2.62%	0.00%	0.00%	2.74%	0.00%	0.00%	2.58%	0.00%	0.00%
AR(2)TARCH(0,2)	2.86%	0.00%	0.00%	1.41%	9.90%	17.66%	2.67%	0.00%	0.00%	2.74%	0.00%	0.00%	2.69%	0.00%	0.00%
AR(3)TARCH(0,2)	3.13%	0.00%	0.00%	1.41%	9.90%	17.66%	2.67%	0.00%	0.00%	2.63%	0.00%	0.00%	2.69%	0.00%	0.00%
AR(4)TARCH(0,2)	3.13%	0.00%	0.00%	1.41%	9.90%	17.66%	2.67%	0.00%	0.00%	2.68%	0.00%	0.00%	2.75%	0.00%	0.00%
AR(0)TARCH(1,1)	2.05%	0.01%	0.03%	1.58%	2.28%	4.70%	1.69%	0.69%	1.50%	1.81%	0.18%	0.42%	1.40%	10.22%	17.93%
AR(1)TARCH(1,1)	2.00%	0.01%	0.07%	1.53%	3.85%	7.60%	1.69%	0.69%	1.50%	1.81%	0.18%	0.42%	1.45%	6.56%	12.14%
AR(2)TARCH(1,1)	2.00%	0.01%	0.07%	1.53%	3.85%	7.60%	1.69%	0.69%	1.50%	1.81%	0.18%	0.42%	1.51%	4.08%	7.91%
AR(3)TARCH(1,1)	1.94%	0.03%	0.14%	1.47%	6.27%	11.82%	1.69%	0.69%	1.50%	1.75%	0.35%	0.78%	1.51%	4.08%	7.91%
AR(4)TARCH(1,1)	1.94%	0.03%	0.14%	1.53%	3.85%	7.60%	1.69%	0.69%	1.50%	1.86%	0.09%	0.22%	1.51%	4.08%	7.91%
AR(0)TARCH(1,2)	1.83%	0.12%	0.19%	1.53%	3.85%	7.60%	1.63%	1.23%	2.60%	1.86%	0.09%	0.22%	1.40%	10.22%	17.93%
AR(1)TARCH(1,2)	2.00%	0.01%	0.03%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	1.86%	0.09%	0.22%	1.51%	4.08%	7.91%
AR(2)TARCH(1,2)	1.94%	0.03%	0.06%	1.41%	9.90%	17.66%	1.69%	0.69%	1.50%	1.81%	0.18%	0.42%	1.56%	2.46%	4.97%
AR(3)TARCH(1,2)	1.94%	0.03%	0.06%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	1.70%	0.64%	1.40%	1.56%	2.46%	4.97%
AR(4)TARCH(1,2)	1.94%	0.03%	0.06%	1.41%	9.90%	17.66%	1.69%	0.69%	1.50%	1.81%	0.18%	0.42%	1.56%	2.46%	4.97%
AR(0)TARCH(2,1)	2.00%	0.01%	0.07%	1.58%	2.28%	4.70%	1.69%	0.69%	1.50%	1.75%	0.35%	0.78%	1.35%	15.43%	25.43%
AR(1)TARCH(2,1)	1.94%	0.03%	0.14%	1.53%	3.85%	7.60%	1.74%	0.38%	1.28%	1.92%	0.05%	0.11%	1.45%	6.56%	12.14%
AR(2)TARCH(2,1)	2.00%	0.01%	0.07%	1.58%	2.28%	4.70%	1.63%	1.23%	2.60%	1.86%	0.09%	0.22%	1.56%	2.46%	4.97%
AR(3)TARCH(2,1)	1.94%	0.03%	0.14%	1.58%	2.28%	4.70%	1.63%	1.23%	2.60%	2.03%	0.01%	0.05%	1.56%	2.46%	4.97%
AR(4)TARCH(2,1)	2.00%	0.01%	0.07%	1.53%	3.85%	7.60%	1.74%	0.38%	1.28%	1.86%	0.09%	0.22%	1.62%	1.44%	3.01%
AR(0)TARCH(2,2)	2.05%	0.01%	0.02%	1.41%	9.90%	17.66%	1.63%	1.23%	2.60%	1.81%	0.18%	0.42%	1.51%	4.08%	7.91%
AR(1)TARCH(2,2)	2.00%	0.01%	0.03%	1.41%	9.90%	17.66%	1.58%	2.12%	4.35%	1.86%	0.09%	0.22%	1.51%	4.08%	7.91%
AR(2)TARCH(2,2)	2.00%	0.01%	0.03%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	1.70%	0.64%	1.40%	1.56%	2.46%	4.97%
AR(3)TARCH(2,2)	2.05%	0.01%	0.02%	1.41%	9.90%	17.66%	1.63%	1.23%	2.60%	1.92%	0.05%	0.20%	1.56%	2.46%	4.97%
AR(4)TARCH(2,2)	2.10%	0.00%	0.01%	1.41%	9.90%	17.66%	1.63%	1.23%	2.60%	1.64%	1.15%	2.45%	1.45%	6.56%	12.14%
AR(0)EGARCH(0,1)	3.02%	0.00%	0.00%	1.53%	3.85%	8.47%	3.00%	0.00%	0.00%	3.01%	0.00%	0.00%	3.02%	0.00%	0.00%
AR(1)EGARCH(0,1)	3.08%	0.00%	0.00%	1.47%	6.27%	12.19%	3.00%	0.00%	0.00%	3.07%	0.00%	0.00%	3.02%	0.00%	0.00%
AR(2)EGARCH(0,1)	3.08%	0.00%	0.00%	1.41%	9.90%	16.79%	3.05%	0.00%	0.00%	3.07%	0.00%	0.00%	3.07%	0.00%	0.00%
AR(3)EGARCH(0,1)	3.02%	0.00%	0.00%	1.41%	9.90%	16.79%	3.05%	0.00%	0.00%	3.07%	0.00%	0.00%	3.23%	0.00%	0.00%
AR(4)EGARCH(0,1)	3.02%	0.00%	0.00%	1.41%	9.90%	16.79%	3.05%	0.00%	0.00%	3.07%	0.00%	0.00%	3.28%	0.00%	0.00%
AR(0)EGARCH(0,2)	3.62%	0.00%	0.00%	1.64%	1.32%	2.80%	3.05%	0.00%	0.00%	2.85%	0.00%	0.00%	3.23%	0.00%	0.00%
AR(1)EGARCH(0,2)	3.62%	0.00%	0.00%	1.64%	1.32%	2.80%	2.94%	0.00%	0.00%	2.74%	0.00%	0.00%	3.23%	0.00%	0.00%
AR(2)EGARCH(0,2)	3.67%	0.00%	0.00%	1.64%	1.32%	2.80%	3.00%	0.00%	0.00%	2.74%	0.00%	0.00%	3.34%	0.00%	0.00%
AR(3)EGARCH(0,2)	3.62%	0.00%	0.00%	1.58%	2.28%	4.70%	3.00%	0.00%	0.00%	2.85%	0.00%	0.00%	3.28%	0.00%	0.00%
AR(4)EGARCH(0,2)	3.72%	0.00%	0.00%	1.64%	1.32%	2.80%	2.94%	0.00%	0.00%	2.85%	0.00%	0.00%	3.34%	0.00%	0.00%
AR(0)EGARCH(1,1)	2.10%	0.00%	0.01%	1.36%	15.12%	25.30%	1.69%	0.69%	2.14%	1.86%	0.09%	0.15%	1.29%	22.56%	34.59%
AR(1)EGARCH(1,1)	1.94%	0.03%	0.14%	1.30%	22.33%	34.71%	1.69%	0.69%	2.14%	1.86%	0.09%	0.15%	1.40%	10.22%	17.93%
AR(2)EGARCH(1,1)	1.89%	0.06%	0.26%	1.30%	22.33%	34.71%	1.69%	0.69%	2.14%	1.81%	0.18%	0.26%	1.35%	15.43%	25.43%
AR(3)EGARCH(1,1)	1.89%	0.06%	0.26%	1.30%	22.33%	34.71%	1.74%	0.38%	1.28%	1.92%	0.05%	0.09%	1.35%	15.43%	25.43%
AR(4)EGARCH(1,1)	1.94%	0.03%	0.14%	1.36%	15.12%	25.30%	1.80%	0.20%	0.73%	1.92%	0.05%	0.02%	1.40%	10.22%	17.93%
AR(0)EGARCH(1,2)	1.62%	1.40%	1.19%	1.41%	9.90%	17.66%	1.74%	0.38%	1.28%	1.53%	3.37%	6.67%	1.40%	10.22%	17.93%
AR(1)EGARCH(1,2)	1.62%	1.40%	1.19%	1.53%	3.85%										

Table (8.A). Normal Distribution, 500 sample size, 99%-VaR

Models	S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(0)GARCH(0,1)	2.27%	0.00%	0.00%	1.58%	2.28%	5.65%	2.67%	0.00%	0.00%	2.52%	0.00%	0.00%	2.58%	0.00%	0.00%
AR(1)GARCH(0,1)	2.32%	0.00%	0.00%	1.64%	1.32%	3.62%	2.67%	0.00%	0.00%	2.57%	0.00%	0.00%	2.64%	0.00%	0.00%
AR(2)GARCH(0,1)	2.43%	0.00%	0.00%	1.70%	0.73%	0.72%	2.72%	0.00%	0.00%	2.57%	0.00%	0.00%	2.75%	0.00%	0.00%
AR(3)GARCH(0,1)	2.43%	0.00%	0.00%	1.75%	0.40%	0.46%	2.72%	0.00%	0.00%	2.46%	0.00%	0.00%	2.80%	0.00%	0.00%
AR(4)GARCH(0,1)	2.48%	0.00%	0.00%	1.70%	0.73%	2.23%	2.72%	0.00%	0.00%	2.46%	0.00%	0.00%	2.75%	0.00%	0.00%
AR(0)GARCH(0,2)	2.37%	0.00%	0.00%	1.53%	3.85%	8.47%	2.34%	0.00%	0.00%	2.52%	0.00%	0.00%	2.26%	0.00%	0.00%
AR(1)GARCH(0,2)	2.43%	0.00%	0.00%	1.53%	3.85%	8.47%	2.34%	0.00%	0.00%	2.35%	0.00%	0.00%	2.37%	0.00%	0.00%
AR(2)GARCH(0,2)	2.37%	0.00%	0.00%	1.58%	2.28%	5.65%	2.45%	0.00%	0.00%	2.46%	0.00%	0.00%	2.42%	0.00%	0.00%
AR(3)GARCH(0,2)	2.37%	0.00%	0.00%	1.64%	1.32%	3.62%	2.45%	0.00%	0.00%	2.52%	0.00%	0.00%	2.42%	0.00%	0.00%
AR(4)GARCH(0,2)	2.37%	0.00%	0.00%	1.64%	1.32%	3.62%	2.45%	0.00%	0.00%	2.52%	0.00%	0.00%	2.42%	0.00%	0.00%
AR(0)GARCH(1,1)	1.94%	0.03%	0.06%	1.41%	9.90%	17.66%	1.63%	1.23%	2.60%	1.70%	0.64%	1.40%	1.51%	4.08%	7.91%
AR(1)GARCH(1,1)	1.94%	0.03%	0.06%	1.47%	6.27%	11.82%	1.53%	3.56%	7.01%	1.64%	1.15%	2.45%	1.62%	1.44%	3.01%
AR(2)GARCH(1,1)	1.94%	0.03%	0.06%	1.58%	2.28%	4.70%	1.47%	5.79%	10.90%	1.59%	2.00%	4.11%	1.62%	1.44%	3.01%
AR(3)GARCH(1,1)	2.05%	0.01%	0.02%	1.58%	2.28%	4.70%	1.36%	13.92%	23.40%	1.59%	2.00%	4.11%	1.62%	1.44%	3.01%
AR(4)GARCH(1,1)	2.00%	0.01%	0.03%	1.47%	6.27%	11.82%	1.36%	13.92%	23.40%	1.59%	2.00%	4.11%	1.62%	1.44%	3.01%
AR(0)GARCH(1,2)	2.10%	0.00%	0.01%	1.70%	0.73%	2.23%	1.74%	0.38%	0.84%	1.70%	0.64%	2.01%	1.56%	2.46%	4.97%
AR(1)GARCH(1,2)	2.10%	0.00%	0.01%	1.70%	0.73%	2.23%	1.69%	0.69%	1.50%	1.53%	3.37%	7.75%	1.56%	2.46%	4.97%
AR(2)GARCH(1,2)	2.10%	0.00%	0.01%	1.70%	0.73%	2.23%	1.53%	3.56%	7.01%	1.42%	8.70%	15.60%	1.88%	0.06%	0.15%
AR(3)GARCH(1,2)	2.16%	0.00%	0.00%	1.64%	1.32%	3.62%	1.58%	2.12%	4.35%	1.59%	2.00%	5.14%	1.72%	0.45%	0.99%
AR(4)GARCH(1,2)	2.10%	0.00%	0.01%	1.64%	1.32%	3.62%	1.63%	1.23%	2.60%	1.59%	2.00%	5.14%	1.72%	0.45%	0.99%
AR(0)GARCH(2,1)	2.05%	0.01%	0.02%	1.64%	1.32%	3.62%	1.74%	0.38%	0.84%	1.70%	0.64%	1.40%	1.62%	1.44%	3.01%
AR(1)GARCH(2,1)	2.05%	0.01%	0.02%	1.58%	2.28%	5.65%	1.53%	3.56%	7.01%	1.64%	1.15%	3.28%	1.51%	4.08%	7.91%
AR(2)GARCH(2,1)	2.05%	0.01%	0.02%	1.64%	1.32%	3.62%	1.53%	3.56%	7.01%	1.81%	0.18%	0.68%	1.62%	1.44%	3.01%
AR(3)GARCH(2,1)	2.05%	0.01%	0.02%	1.58%	2.28%	5.65%	1.53%	3.56%	7.01%	1.70%	0.64%	1.40%	1.62%	1.44%	3.01%
AR(4)GARCH(2,1)	2.00%	0.01%	0.03%	1.41%	9.90%	16.79%	1.47%	5.79%	10.90%	1.70%	0.64%	2.01%	1.62%	1.44%	3.01%
AR(0)GARCH(2,2)	2.21%	0.00%	0.00%	1.64%	1.32%	3.62%	1.74%	0.38%	0.84%	1.64%	1.15%	3.28%	1.62%	1.44%	3.01%
AR(1)GARCH(2,2)	2.10%	0.00%	0.00%	1.58%	2.28%	5.65%	1.69%	0.69%	1.50%	1.70%	0.64%	2.01%	1.62%	1.44%	3.01%
AR(2)GARCH(2,2)	2.10%	0.00%	0.01%	1.58%	2.28%	5.65%	1.53%	3.56%	7.01%	1.75%	0.35%	1.19%	1.72%	0.45%	0.99%
AR(3)GARCH(2,2)	2.16%	0.00%	0.00%	1.70%	0.73%	2.23%	1.47%	5.79%	10.90%	1.59%	2.00%	5.14%	1.67%	0.82%	1.76%
AR(4)GARCH(2,2)	2.21%	0.00%	0.00%	1.58%	2.28%	5.65%	1.53%	3.56%	7.01%	1.59%	2.00%	5.14%	1.67%	0.82%	1.76%
AR(0)TARCH(0,1)	2.32%	0.00%	0.00%	1.64%	1.32%	2.80%	2.78%	0.00%	0.00%	2.41%	0.00%	0.00%	2.69%	0.00%	0.00%
AR(1)TARCH(0,1)	2.27%	0.00%	0.00%	1.70%	0.73%	1.61%	2.78%	0.00%	0.00%	2.46%	0.00%	0.00%	2.69%	0.00%	0.00%
AR(2)TARCH(0,1)	2.32%	0.00%	0.00%	1.70%	0.73%	1.61%	2.62%	0.00%	0.00%	2.57%	0.00%	0.00%	2.69%	0.00%	0.00%
AR(3)TARCH(0,1)	2.43%	0.00%	0.00%	1.70%	0.73%	1.61%	2.67%	0.00%	0.00%	2.57%	0.00%	0.00%	2.64%	0.00%	0.00%
AR(4)TARCH(0,1)	2.48%	0.00%	0.00%	1.70%	0.73%	1.61%	2.67%	0.00%	0.00%	2.46%	0.00%	0.00%	2.64%	0.00%	0.00%
AR(0)TARCH(0,2)	2.32%	0.00%	0.00%	1.53%	3.85%	7.60%	2.40%	0.00%	0.00%	2.14%	0.00%	0.01%	2.37%	0.00%	0.00%
AR(1)TARCH(0,2)	2.32%	0.00%	0.00%	1.53%	3.85%	7.60%	2.40%	0.00%	0.00%	2.19%	0.00%	0.00%	2.53%	0.00%	0.00%
AR(2)TARCH(0,2)	2.43%	0.00%	0.00%	1.53%	3.85%	7.60%	2.40%	0.00%	0.00%	2.19%	0.00%	0.00%	2.48%	0.00%	0.00%
AR(3)TARCH(0,2)	2.43%	0.00%	0.00%	1.53%	3.85%	7.60%	2.40%	0.00%	0.00%	2.08%	0.01%	0.03%	2.48%	0.00%	0.00%
AR(4)TARCH(0,2)	2.54%	0.00%	0.00%	1.53%	3.85%	7.60%	2.40%	0.00%	0.00%	2.14%	0.00%	0.01%	2.48%	0.00%	0.00%
AR(0)TARCH(1,1)	1.83%	0.12%	0.28%	1.53%	3.85%	7.60%	1.58%	2.12%	4.35%	2.25%	0.00%	0.00%	1.35%	15.43%	25.43%
AR(1)TARCH(1,1)	1.83%	0.12%	0.28%	1.58%	2.28%	4.70%	1.47%	5.79%	10.90%	2.14%	0.00%	0.01%	1.35%	15.43%	25.43%
AR(2)TARCH(1,1)	1.89%	0.06%	0.14%	1.58%	2.28%	4.70%	1.47%	5.79%	10.90%	2.08%	0.01%	0.03%	1.51%	4.08%	7.91%
AR(3)TARCH(1,1)	1.89%	0.06%	0.14%	1.64%	1.32%	2.80%	1.47%	5.79%	10.90%	2.19%	0.00%	0.01%	1.51%	4.08%	7.91%
AR(4)TARCH(1,1)	1.94%	0.03%	0.07%	1.53%	3.85%	7.60%	1.47%	5.79%	10.90%	2.25%	0.00%	0.00%	1.45%	6.56%	12.14%
AR(0)TARCH(1,2)	2.00%	0.01%	0.03%	1.47%	6.27%	11.82%	1.69%	0.69%	1.50%	2.03%	0.01%	0.03%	1.56%	2.46%	4.97%
AR(1)TARCH(1,2)	2.05%	0.01%	0.02%	1.53%	3.85%	7.60%	1.63%	1.23%	2.60%	2.14%	0.00%	0.01%	1.51%	4.08%	7.91%
AR(2)TARCH(1,2)	2.00%	0.01%	0.03%	1.36%	15.12%	25.30%	1.58%	2.12%	4.35%	2.03%	0.01%	0.03%	1.51%	4.08%	7.91%
AR(3)TARCH(1,2)	2.05%	0.01%	0.02%	1.41%	9.90%	17.66%	1.53%	3.56%	7.01%	2.14%	0.00%	0.01%	1.45%	6.56%	12.14%
AR(4)TARCH(1,2)	1.89%	0.06%	0.11%	1.53%	3.85%	7.60%	1.53%	3.56%	7.01%	2.35%	0.00%	0.00%	1.45%	6.56%	12.14%
AR(0)TARCH(2,1)	2.00%	0.01%	0.03%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	2.14%	0.00%	0.01%	1.40%	10.22%	17.93%
AR(1)TARCH(2,1)	1.94%	0.03%	0.07%	1.58%	2.28%	4.70%	1.53%	3.56%	7.01%	2.14%	0.00%	0.01%	1.51%	4.08%	7.91%
AR(2)TARCH(2,1)	1.94%	0.03%	0.07%	1.41%	9.90%	17.66%	1.47%	5.79%	10.90%	2.25%	0.00%	0.00%	1.56%	2.46%	4.97%
AR(3)TARCH(2,1)	1.94%	0.03%	0.07%	1.47%	6.27%	11.82%	1.63%	1.23%	2.60%	2.30%	0.00%	0.00%	1.51%	4.08%	7.91%
AR(4)TARCH(2,1)	1.89%	0.06%	0.14%	1.53%	3.85%	7.60%	1.69%	0.69%	1.50%	2.19%	0.00%	0.01%	1.40%	10.22%	17.93%
AR(0)TARCH(2,2)	2.00%	0.01%	0.03%	1.47%	6.27%	11.82%	1.53%	3.56%	7.01%	2.19%	0.00%	0.01%	1.40%	10.22%	17.93%
AR(1)TARCH(2,2)	1.89%	0.06%	0.11%	1.47%	6.27%	11.82%	1.53%	3.56%	7.01%	2.14%	0.00%	0.01%	1.51%	4.08%	7.91%
AR(2)TARCH(2,2)	2.00%	0.01%	0.03%	1.36%	15.12%	25.30%	1.47%	5.79%	10.90%	2.14%	0.00%	0.01%	1.45%	6.56%	12.14%
AR(3)TARCH(2,2)	1.94%	0.03%	0.06%	1.47%	6.27%	11.82%	1.53%	3.56%	7.01%	2.25%	0.00%	0.00%	1.45%	6.56%	12.14%
AR(4)TARCH(2,2)	2.00%	0.01%	0.03%	1.75%	0.40%	0.89%	1.53%	3.56%	7.01%	2.30%	0.00%	0.00%	1.45%	6.56%	12.14%
AR(0)EGARCH(0,1)	2.10%	0.00%	0.02%	1.70%	0.73%	2.23%	2.72%	0.00%	0.00%	2.41%	0.00%	0.00%	2.80%	0.00%	0.00%
AR(1)EGARCH(0,1)	2.10%	0.00%	0.02%	1.75%	0.40%	1.32%	2.72%	0.00%	0.00%	2.41%	0.00%	0.00%	2.85%	0.00%	0.00%
AR(2)EGARCH(0,1)	2.05%	0.01%	0.03%	1.81%	0.21%	0.76%	2.62%	0.00%	0.00%	2.41%	0.00%	0.00%	3.07%	0.00%	0.00%
AR(3)EGARCH(0,1)	2.10%	0.00%	0.01%	1.81%	0.21%	0.76%	2.67%	0.00%	0.00%	2.41%	0.00%	0.00%	3.07%	0.00%	0.00%
AR(4)EGARCH(0,1)	2.10%	0.00%	0.01%	1.81%	0.21%	0.76%	2.62%	0.00%	0.00%	2.41%	0.00%	0.00%	3.07%	0.00%	0.00%
AR(0)EGARCH(0,2)	2.70%	0.00%	0.00%	1.75%	0.40%	1.32%	2.51%	0.00%	0.00%	2.41%	0.00%	0.00%	2.75%	0.00%	0.00%
AR(1)EGARCH(0,2)	2.70%	0.00%	0.00%	1.75%	0.40%	1.32%	2.56%	0.00%	0.00%	2.35%	0.00%	0.00%	2.91%	0.00%	0.00%
AR(2)EGARCH(0,2)	2.86%	0.00%	0.00%	1.75%	0.40%	1.32%	2.78%	0.00%	0.00%	2.41%	0.00%	0.00%	3.02%	0.00%	0.00%
AR(3)EGARCH(0,2)	2.91%	0.00%	0.00%	1.70%	0.73%	2.23%	2.78%	0.00%	0.00%	2.30%	0.00%	0.00%	3.07%	0.00%	0.00%
AR(4)EGARCH(0,2)	3.02%	0.00%	0.00%	1.75%	0.40%	1.32%	2.67%	0.00%	0.00%	2.41%	0.00%	0.00%	3.07%	0.00%	0.00%
AR(0)EGARCH(1,1)	1.94%	0.03%	0.14%	1.58%	2.28%	4.70%	1.63%	1.23%	2.60%	1.97%	0.02%	0.05%	1.56%	2.46%	4.97%
AR(1)EGARCH(1,1)	1.94%	0.03%	0.07%	1.64%	1.32%	2.80%	1.63%	1.23%	2.60%	1.86%	0.09%	0.38%	1.56%	2.46%	4.97%
AR(2)EGARCH(1,1)	1.83%	0.12%	0.28%	1.58%	2.28%	4.70%	1.58%	2.12%	4.35%	2.03%	0.01%	0.03%	1.56%	2.46%	4.97%
AR(3)EGARCH(1,1)	1.89%	0.06%	0.14%	1.64%	1.32%	2.80%	1.63%	1.23%	2.60%	2.08%	0.01%	0.01%	1.51%	4.08%	7.91%
AR(4)EGARCH(1,1)	1.94%	0.03%	0.07%	1.70%	0.73%	1.61%	1.58%	2.12%	4.35%	2.08%	0.01%	0.01%	1.51%	4.08%	7.91%
AR(0)EGARCH(1,2)	2.21%	0.00%	0.00%	1.81%	0.21%	0.48%	1.63%	1.23%	2.60%	1.81%	0.18%	0.26%	1.51%	4.08%	7.91%
AR(1)EGARCH(1,2)	2.05%	0.01%	0.02%	1.98%	0.03%	0.12%	1.63%	1.23%	2.60%	1.81%	0.18%	0.26%	1.51%	4.08%	7.91%
AR(2)EGARCH(1,2)	2.16%														

Table (9.1.A). Student-t Distribution, 2000 sample size, 95%-VaR																
		S&P500			NIKKEI225			DAX30			CAC40			FTSE100		
Models	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	
AR(1)GARCH(0,1)	5.07%	88.58%	93.39%	2.49%	0.00%	0.00%	5.45%	38.34%	63.29%	5.48%	35.72%	20.97%	5.76%	14.09%	5.41%	
AR(1)GARCH(0,2)	5.02%	97.03%	65.26%	2.94%	0.00%	0.00%	4.80%	68.60%	81.39%	5.42%	41.44%	35.91%	5.87%	9.38%	4.80%	
AR(1)GARCH(1,1)	3.35%	0.05%	0.20%	3.51%	0.24%	0.97%	4.20%	10.46%	23.39%	4.71%	56.57%	72.07%	4.85%	76.04%	86.48%	
AR(1)GARCH(1,2)	3.35%	0.05%	0.20%	3.51%	0.24%	0.97%	4.14%	8.25%	12.76%	4.71%	56.57%	51.27%	4.90%	84.34%	12.31%	
AR(1)GARCH(2,1)	3.35%	0.05%	0.20%	3.57%	0.36%	1.37%	-	-	-	4.71%	56.57%	72.07%	4.85%	76.04%	86.48%	
AR(1)GARCH(2,2)	3.51%	0.19%	0.69%	3.51%	0.24%	0.97%	4.09%	6.43%	9.85%	4.65%	49.39%	65.60%	4.79%	67.99%	61.72%	
AR(1)TARCH(0,1)	4.70%	54.31%	38.90%	2.38%	0.00%	0.00%	5.40%	44.30%	69.60%	-	-	-	-	-	-	
AR(1)TARCH(0,2)	-	-	-	3.23%	0.03%	0.02%	5.12%	81.03%	84.97%	-	-	-	-	-	-	
AR(1)TARCH(1,1)	3.62%	0.41%	1.46%	3.45%	0.16%	0.08%	4.31%	16.23%	34.13%	4.82%	72.15%	83.22%	4.74%	60.25%	82.85%	
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
AR(1)TARCH(2,1)	3.62%	0.41%	1.46%	3.45%	0.16%	0.08%	4.31%	16.23%	34.13%	4.93%	88.87%	90.64%	4.90%	84.34%	71.39%	
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
AR(1)EGARCH(0,1)	4.91%	86.00%	91.00%	2.32%	0.00%	0.00%	5.40%	44.30%	67.49%	5.48%	35.72%	11.07%	6.19%	2.28%	0.02%	
AR(1)EGARCH(0,2)	5.23%	64.53%	22.75%	3.06%	0.01%	0.01%	4.96%	93.59%	5.58%	5.37%	47.69%	36.59%	6.19%	2.28%	0.33%	
AR(1)EGARCH(1,1)	4.26%	13.59%	30.78%	3.68%	0.76%	0.23%	4.36%	19.86%	40.25%	5.15%	77.29%	56.51%	5.82%	11.54%	26.10%	
AR(1)EGARCH(1,2)	4.43%	24.73%	47.98%	3.51%	0.24%	0.10%	4.69%	53.38%	78.53%	5.15%	77.29%	56.51%	5.65%	20.47%	29.27%	
AR(1)EGARCH(2,1)	4.43%	24.73%	47.98%	3.74%	1.08%	0.29%	4.69%	53.38%	78.53%	5.31%	54.44%	56.38%	5.39%	45.19%	55.60%	

Table (9.2.A). Student-t Distribution, 1500 sample size, 95%-VaR															
Models	t	S&P500		NIKKEI225			DAX30			CAC40			FTSE100		
		p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	5.40%	43.90%	67.63%	2.89%	0.00%	0.01%	5.56%	28.04%	32.60%	5.31%	54.44%	36.78%	5.60%	24.37%	20.54%
AR(1)GARCH(0,2)	5.02%	97.03%	89.77%	3.06%	0.01%	0.01%	5.23%	65.13%	77.44%	5.42%	41.44%	21.17%	5.71%	17.05%	10.72%
AR(1)GARCH(1,1)	3.56%	0.28%	1.02%	3.34%	0.07%	0.30%	4.47%	28.79%	50.62%	4.71%	56.57%	51.27%	5.12%	81.96%	58.05%
AR(1)GARCH(1,2)	3.62%	0.41%	1.46%	3.51%	0.24%	0.97%	4.41%	24.04%	46.74%	4.82%	72.15%	62.33%	4.95%	92.78%	28.59%
AR(1)GARCH(2,1)	3.51%	0.19%	0.69%	3.34%	0.07%	0.30%	-	-	-	4.71%	56.57%	51.27%	5.06%	90.27%	56.17%
AR(1)GARCH(2,2)	3.83%	1.63%	5.29%	3.45%	0.16%	0.66%	4.47%	28.79%	53.44%	4.87%	80.42%	67.20%	4.95%	92.78%	50.39%
AR(1)TARCH(0,1)	-	-	-	2.94%	0.00%	0.00%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	3.17%	0.02%	0.01%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	-	-	-	3.57%	0.36%	0.14%	4.25%	13.10%	22.04%	4.65%	49.39%	45.46%	5.06%	90.27%	93.61%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	-	-	-	3.57%	0.36%	0.14%	4.20%	10.46%	19.14%	4.65%	49.39%	65.60%	5.06%	90.27%	79.77%
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	5.45%	38.00%	49.95%	2.77%	0.00%	0.00%	5.94%	7.24%	4.13%	5.31%	54.44%	20.68%	6.09%	3.77%	0.39%
AR(1)EGARCH(0,2)	5.45%	38.00%	49.95%	3.06%	0.01%	0.01%	5.45%	38.34%	5.24%	5.48%	35.72%	49.70%	5.60%	24.37%	2.62%
AR(1)EGARCH(1,1)	5.07%	88.58%	62.04%	3.51%	0.24%	0.10%	5.18%	72.92%	80.67%	5.04%	94.02%	52.72%	5.76%	14.09%	29.99%
AR(1)EGARCH(1,2)	4.86%	77.66%	70.10%	3.62%	0.53%	1.19%	5.07%	89.37%	93.21%	5.09%	85.56%	7.40%	5.65%	20.47%	38.15%
AR(1)EGARCH(2,1)	4.96%	94.47%	67.74%	3.45%	0.16%	0.08%	5.18%	72.92%	89.24%	4.98%	97.43%	49.83%	5.87%	9.38%	13.53%

Table (9.3.A). Student-t Distribution, 1000 sample size, 95%-VaR															
Models	t	S&P500		NIKKEI225			DAX30			CAC40			FTSE100		
		p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	4.86%	77.66%	87.14%	3.28%	0.04%	0.15%	5.29%	57.73%	37.26%	5.31%	54.44%	36.78%	5.55%	28.78%	2.56%
AR(1)GARCH(0,2)	4.75%	61.73%	83.70%	3.34%	0.07%	0.22%	5.07%	89.37%	78.58%	5.37%	47.69%	21.07%	5.76%	14.09%	2.60%
AR(1)GARCH(1,1)	3.78%	1.18%	3.94%	3.28%	0.04%	0.19%	4.58%	40.00%	66.74%	4.93%	88.87%	90.64%	5.22%	66.08%	59.45%
AR(1)GARCH(1,2)	3.78%	1.18%	2.91%	3.51%	0.24%	0.97%	4.52%	34.11%	49.37%	5.04%	94.02%	30.47%	5.22%	66.08%	21.01%
AR(1)GARCH(2,1)	3.83%	1.63%	5.29%	3.40%	0.10%	0.45%	-	-	-	-	-	-	5.17%	73.87%	59.15%
AR(1)GARCH(2,2)	3.89%	2.22%	6.98%	3.57%	0.36%	1.37%	4.58%	40.00%	56.23%	-	-	-	5.33%	51.69%	38.65%
AR(1)TARCH(0,1)	-	-	-	3.23%	0.03%	0.10%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	3.34%	0.07%	0.04%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	-	-	-	3.28%	0.04%	0.15%	4.36%	19.86%	40.25%	-	-	-	5.28%	58.67%	58.93%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	-	-	-	3.40%	0.10%	0.32%	-	-	-	-	-	-	5.33%	51.69%	57.63%
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	5.07%	88.58%	93.39%	3.11%	0.01%	0.04%	5.23%	65.13%	9.57%	5.31%	54.44%	56.38%	5.76%	14.09%	2.60%
AR(1)EGARCH(0,2)	5.13%	80.30%	83.88%	3.40%	0.10%	0.32%	5.56%	28.04%	5.47%	5.37%	47.69%	21.07%	5.71%	17.05%	1.12%
AR(1)EGARCH(1,1)	4.70%	54.31%	66.57%	3.79%	1.51%	0.36%	5.29%	57.73%	69.86%	-	-	-	5.76%	14.09%	31.79%
AR(1)EGARCH(1,2)	4.64%	47.33%	63.62%	3.79%	1.51%	2.63%	4.96%	93.59%	92.12%	5.09%	85.56%	32.57%	5.76%	14.09%	10.07%
AR(1)EGARCH(2,1)	4.75%	61.73%	68.67%	3.85%	2.08%	3.32%	5.40%	44.30%	69.60%	5.04%	94.02%	93.25%	5.60%	24.37%	20.54%

Table (9.4.A). Student-t Distribution, 500 sample size, 95%-VaR															
Models	t	S&P500		NIKKEI225			DAX30			CAC40			FTSE100		
		p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	4.16%	8.61%	7.46%	3.34%	0.07%	0.30%	4.63%	46.44%	63.08%	-	-	-	-	-	-
AR(1)GARCH(0,2)	4.05%	5.20%	8.10%	3.74%	1.08%	3.55%	5.18%	72.92%	57.66%	-	-	-	-	-	-
AR(1)GARCH(1,1)	3.83%	1.63%	4.03%	3.74%	1.08%	3.52%	-	-	-	-	-	-	-	-	-
AR(1)GARCH(1,2)	3.89%	2.22%	3.25%	-	-	-	4.47%	28.79%	42.68%	-	-	-	-	-	-
AR(1)GARCH(2,1)	3.89%	2.22%	5.49%	3.85%	2.08%	6.47%	-	-	-	-	-	-	-	-	-
AR(1)GARCH(2,2)	3.94%	2.99%	4.48%	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	-	-	-	3.57%	0.36%	0.88%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	4.26%	13.59%	21.56%	3.28%	0.04%	0.15%	5.07%	89.37%	32.24%	-	-	-	-	-	-
AR(1)EGARCH(0,2)	4.91%	86.00%	90.54%	3.79%	1.51%	4.70%	5.01%	97.86%	30.00%	-	-	-	-	-	-
AR(1)EGARCH(1,1)	4.75%	61.73%	77.33%	3.62%	0.53%	1.92%	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(1,2)	5.02%	97.03%	78.16%	4.07%	6.56%	14.69%	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(2,1)	4.75%	61.73%	77.33%	3.62%	0.53%	1.92%	-	-	-	-	-	-	-	-	-

Note: The table reports for each model, sample size and equity index, the exception rate (t) and the p-values of unconditional (p_uc) and conditional coverage (p_cc)

Table (10.1.A). Student-t Distribution, 2000 sample size, 99%-VaR															
Models	t	S&P500		NIKKEI225			DAX30			CAC40			FTSE100		
		p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	0.70%	17.25%	35.72%	0.23%	0.01%	0.04%	1.04%	87.95%	80.19%	1.53%	3.37%	7.75%	1.88%	0.06%	0.15%
AR(1)GARCH(0,2)	0.81%	39.40%	61.02%	0.28%	0.03%	0.16%	0.98%	93.43%	82.56%	1.48%	5.50%	11.22%	1.45%	6.56%	12.14%
AR(1)GARCH(1,1)	0.59%	5.72%	15.25%	0.23%	0.01%	0.04%	0.60%	6.24%	16.40%	0.82%	42.88%	64.05%	0.92%	71.03%	79.03%
AR(1)GARCH(1,2)	0.59%	5.72%	15.25%	0.23%	0.01%	0.04%	0.54%	3.20%	9.45%	0.82%	42.88%	64.05%	0.92%	71.03%	79.03%
AR(1)GARCH(2,1)	0.59%	5.72%	15.25%	0.23%	0.01%	0.04%	-	-	-	0.82%	42.88%	64.05%	0.92%	71.03%	79.03%
AR(1)GARCH(2,2)	0.65%	10.32%	24.35%	0.23%	0.01%	0.04%	0.54%	3.20%	9.45%	0.82%	42.88%	64.05%	0.92%	71.03%	79.03%
AR(1)TARCH(0,1)	0.70%	17.25%	35.72%	0.23%	0.01%	0.04%	0.93%	74.84%	32.98%	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	0.28%	0.03%	0.16%	0.93%	74.84%	80.27%	-	-	-	-	-	-
AR(1)TARCH(1,1)	0.59%	5.72%	15.25%	0.28%	0.03%	0.16%	0.60%	6.24%	16.40%	1.04%	86.27%	79.83%	0.75%	26.52%	47.96%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	0.59%	5.72%	15.25%	0.28%	0.03%	0.16%	0.60%	6.24%	16.40%	1.04%	86.27%	79.83%	0.75%	26.52%	47.96%
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	0.65%	10.32%	24.35%	0.23%	0.01%	0.04%	1.09%	70.28%	42.48%	1.53%	3.37%	7.75%	1.72%	0.45%	0.99%
AR(1)EGARCH(0,2)	0.86%	54.53%	71.82%	0.40%	0.37%	1.42%	1.36%	13.92%	23.40%	1.53%	3.37%	7.75%	1.78%	0.24%	0.54%
AR(1)EGARCH(1,1)	0.76%	26.90%	48.43%	0.11%	0.00%	0.00%	0.71%	18.54%	37.65%	1.26%	28.39%	32.06%	1.24%	31.92%	45.06%
AR(1)EGARCH(1,2)	0.81%	39.40%	61.02%	0.28%	0.03%	0.16%	0.60%	6.24%	16.40%	1.26%	28.39%	32.06%	1.18%	43.70%	56.11%
AR(1)EGARCH(2,1)	0.81%	39.40%	61.02%	0.11%	0.00%	0.00%	0.60%	6.24%	16.40%	1.20%	39.41%	37.05%	1.18%	43.70%	56.11%

Table (10.2.A). Student-t Distribution, 1500 sample size, 99%-VaR															
Models	t	S&P500		NIKKEI225			DAX30			CAC40			FTSE100		
		p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	0.81%	39.40%	61.02%	0.28%	0.03%	0.16%	1.31%	20.56%	27.08%	1.48%	5.50%	11.22%	1.78%	0.24%	0.54%
AR(1)GARCH(0,2)	0.81%	39.40%	61.02%	0.28%	0.03%	0.16%	1.25%	29.39%	42.50%	1.31%	19.78%	26.41%	1.45%	6.56%	12.14%
AR(1)GARCH(1,1)	0.65%	10.32%	24.35%	0.17%	0.00%	0.01%	0.82%	41.70%	63.05%	0.82%	42.88%	64.05%	0.92%	71.03%	79.03%
AR(1)GARCH(1,2)	0.65%	10.32%	24.35%	0.17%	0.00%	0.01%	0.82%	41.70%	63.05%	0.82%	42.88%	64.05%	0.92%	71.03%	79.03%
AR(1)GARCH(2,1)	0.65%	10.32%	24.35%	0.17%	0.00%	0.01%	-	-	-	0.82%	42.88%	64.05%	0.92%	71.03%	79.03%
AR(1)GARCH(2,2)	0.70%	17.25%	35.72%	0.17%	0.00%	0.01%	0.82%	41.70%	63.05%	0.82%	42.88%	64.05%	0.92%	71.03%	79.03%
AR(1)TARCH(0,1)	-	-	-	0.28%	0.03%	0.16%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	0.34%	0.12%	0.52%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	-	-	-	0.17%	0.00%	0.01%	0.82%	41.70%	63.05%	0.99%	95.11%	38.42%	0.81%	38.90%	60.57%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	-	-	-	0.23%	0.01%	0.04%	0.82%	41.70%	63.05%	1.15%	52.91%	40.67%	0.81%	38.90%	60.57%
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	0.76%	26.90%	48.43%	0.28%	0.03%	0.16%	1.25%	29.39%	32.70%	1.53%	3.37%	7.75%	1.78%	0.24%	0.54%
AR(1)EGARCH(0,2)	0.97%	90.11%	82.35%	0.51%	2.21%	6.92%	1.47%	5.79%	10.90%	1.53%	3.37%	7.75%	1.78%	0.24%	0.54%
AR(1)EGARCH(1,1)	1.08%	73.47%	75.08%	0.23%	0.01%	0.04%	1.09%	70.28%	73.77%	1.53%	3.37%	2.09%	1.45%	6.56%	12.14%
AR(1)EGARCH(1,2)	0.92%	71.72%	79.27%	0.17%	0.00%	0.01%	1.09%	70.28%	73.77%	1.53%	3.37%	2.09%	1.40%	10.22%	17.56%
AR(1)EGARCH(2,1)	1.08%	73.47%	42.81%	0.11%	0.00%	0.00%	1.20%	40.63%	53.58%	1.53%	3.37%	2.09%	1.40%	10.22%	17.93%

Table (10.3.A). Student-t Distribution, 1000 sample size, 99%-VaR															
Models	t	S&P500		NIKKEI225			DAX30			CAC40			FTSE100		
		p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	0.97%	90.11%	82.35%	0.45%	0.96%	3.34%	1.31%	20.56%	27.08%	1.64%	1.15%	1.02%	1.62%	1.44%	3.01%
AR(1)GARCH(0,2)	0.97%	90.11%	82.35%	0.34%	0.12%	0.52%	1.42%	9.12%	16.29%	1.37%	13.33%	20.76%	1.51%	4.08%	7.91%
AR(1)GARCH(1,1)	0.92%	71.72%	79.27%	0.17%	0.00%	0.01%	0.82%	41.70%	63.05%	0.88%	58.71%	74.25%	1.18%	43.70%	56.11%
AR(1)GARCH(1,2)	0.86%	54.53%	71.82%	0.17%	0.00%	0.01%	0.93%	74.84%	80.27%	0.99%	95.11%	82.61%	1.13%	57.88%	66.64%
AR(1)GARCH(2,1)	0.86%	54.53%	71.82%	0.17%	0.00%	0.01%	-	-	-	-	-	-	1.18%	43.70%	56.11%
AR(1)GARCH(2,2)	0.97%	90.11%	82.35%	0.17%	0.00%	0.01%	0.93%	74.84%	80.27%	-	-	-	1.13%	57.88%	66.64%
AR(1)TARCH(0,1)	-	-	-	0.45%	0.96%	3.34%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	0.34%	0.12%	0.52%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	-	-	-	0.34%	0.12%	0.52%	0.82%	41.70%	63.05%	-	-	-	1.02%	92.04%	80.91%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	-	-	-	0.34%	0.12%	0.52%	-	-	-	-	-	-	1.02%	92.04%	80.91%
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	0.86%	54.53%	71.82%	0.51%	2.21%	6.92%	1.42%	9.12%	16.15%	1.59%	2.00%	5.14%	1.67%	0.82%	1.76%
AR(1)EGARCH(0,2)	1.13%	57.23%	66.25%	0.51%	2.21%	6.92%	1.63%	1.23%	3.46%	1.70%	0.64%	0.67%	1.56%	2.46%	4.97%
AR(1)EGARCH(1,1)	0.97%	90.11%	82.35%	0.28%	0.03%	0.16%	0.98%	93.43%	82.56%	-	-	-	1.45%	6.56%	12.14%
AR(1)EGARCH(1,2)	0.92%	71.72%	79.27%	0.28%	0.03%	0.16%	1.14%	54.33%	64.44%	1.53%	3.37%	2.09%	1.51%	4.08%	7.91%
AR(1)EGARCH(2,1)	1.03%	91.30%	80.80%	0.34%	0.12%	0.52%	1.09%	70.28%	73.77%	1.42%	8.70%	15.60%	1.67%	0.82%	1.76%

Table (10.4.A). Student-t Distribution, 500 sample size, 99%-VaR															
Models	t	S&P500		NIKKEI225			DAX30			CAC40			FTSE100		
		p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	0.86%	54.53%	71.82%	0.45%	0.96%	3.34%	1.14%	54.33%	64.44%	-	-	-	-	-	-
AR(1)GARCH(0,2)	0.97%	90.11%	82.35%	0.51%	2.21%	6.92%	1.31%	20.56%	32.23%	-	-	-	-	-	-
AR(1)GARCH(1,1)	0.86%	54.53%	71.82%	0.23%	0.01%	0.04%	-	-	-	-	-	-	-	-	-
AR(1)GARCH(1,2)	0.92%	71.72%	79.27%	-	-	-	0.87%	57.30%	73.46%	-	-	-	-	-	-
AR(1)GARCH(2,1)	0.86%	54.53%	71.82%	0.28%	0.03%	0.16%	-	-	-	-	-	-	-	-	-
AR(1)GARCH(2,2)	0.86%	54.53%	71.82%	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	-	-	-	0.51%	2.21%	6.92%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	0.81%	39.40%	61.02%	0.68%	15.01%	32.48%	1.14%	54.33%	41.06%	-	-	-	-	-	-
AR(1)EGARCH(0,2)	1.19%	43.13%	55.65%	0.74%	24.14%	45.38%	1.25%	29.39%	32.70%	-	-	-	-	-	-
AR(1)EGARCH(1,1)	1.03%	91.30%	80.80%	0.74%	24.14%	45.38%	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(1,2)	1.24%	31.45%	44.59%	0.68%	15.01%	32.48%	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(2,1)	1.19%	43.13%	38.66%	0.79%	36.26%	58.61%	-	-	-	-	-	-	-	-	-

Note: The table reports for each model, sample size and equity index, the exception rate (t) and the p-values of unconditional (p_uc) and conditional coverage (p_cc)

Table (11.1.A). GED, 2000 sample size, 95%-VaR															
Models	t	S&P500		NIKKEI225			DAX30			CAC40		FTSE100			
		p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	9.17%	0.00%	0.00%	6.17%	2.94%	0.70%	8.56%	0.00%	0.00%	7.67%	0.00%	0.00%	7.86%	0.00%	0.00%
AR(1)GARCH(0,2)	8.36%	0.00%	0.00%	5.89%	9.62%	0.65%	7.47%	0.00%	0.00%	7.39%	0.00%	0.00%	7.59%	0.00%	0.00%
AR(1)GARCH(1,1)	5.18%	72.25%	79.48%	6.17%	2.94%	8.36%	5.99%	5.77%	12.25%	6.08%	4.05%	11.45%	6.19%	2.28%	0.72%
AR(1)GARCH(1,2)	5.56%	27.81%	49.69%	6.06%	4.85%	10.90%	5.94%	7.24%	18.34%	6.08%	4.05%	7.80%	6.25%	1.75%	0.13%
AR(1)GARCH(2,1)	5.23%	64.53%	74.32%	6.06%	4.85%	7.25%	-	-	-	5.97%	6.48%	16.75%	6.14%	2.94%	1.58%
AR(1)GARCH(2,2)	5.34%	50.30%	74.95%	6.06%	4.85%	7.25%	5.89%	9.00%	18.76%	6.08%	4.05%	10.16%	6.14%	2.94%	0.80%
AR(1)TARCH(0,1)	9.28%	0.00%	0.00%	6.23%	2.26%	0.00%	8.39%	0.00%	0.00%	7.50%	0.00%	0.00%	-	-	-
AR(1)TARCH(0,2)	-	-	-	6.00%	6.14%	0.36%	7.47%	0.00%	0.00%	7.17%	0.01%	0.03%	-	-	-
AR(1)TARCH(1,1)	5.88%	8.96%	12.43%	5.60%	25.37%	9.49%	5.61%	23.69%	33.29%	5.81%	12.34%	28.68%	6.14%	2.94%	8.11%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	5.77%	13.51%	18.92%	5.77%	14.55%	4.93%	5.83%	11.11%	22.81%	5.81%	12.34%	28.68%	6.14%	2.94%	6.49%
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	9.39%	0.00%	0.00%	6.11%	3.79%	0.96%	8.83%	0.00%	0.00%	7.56%	0.00%	0.00%	8.24%	0.00%	0.00%
AR(1)EGARCH(0,2)	8.69%	0.00%	0.00%	6.06%	4.85%	0.27%	8.12%	0.00%	0.00%	7.39%	0.00%	0.00%	8.02%	0.00%	0.00%
AR(1)EGARCH(1,1)	6.75%	0.10%	0.27%	5.66%	21.26%	7.72%	6.65%	0.20%	0.36%	6.52%	0.44%	1.46%	6.79%	0.08%	0.33%
AR(1)EGARCH(1,2)	6.75%	0.10%	0.27%	5.72%	17.66%	26.92%	6.49%	0.52%	1.80%	6.46%	0.60%	1.00%	6.89%	0.04%	0.02%
AR(1)EGARCH(2,1)	6.80%	0.07%	0.19%	5.55%	30.02%	26.90%	6.54%	0.38%	1.29%	6.57%	0.32%	0.90%	6.89%	0.04%	0.06%

Table (11.2.A). GED, 1500 sample size, 95%-VaR															
Models	t	S&P500		NIKKEI225			DAX30			CAC40		FTSE100			
		p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	8.47%	0.00%	0.00%	6.23%	2.26%	0.51%	8.56%	0.00%	0.00%	7.12%	0.01%	0.04%	7.43%	0.00%	0.00%
AR(1)GARCH(0,2)	8.15%	0.00%	0.00%	6.17%	2.94%	0.70%	7.47%	0.00%	0.00%	7.17%	0.01%	0.01%	7.49%	0.00%	0.00%
AR(1)GARCH(1,1)	5.61%	23.50%	32.48%	5.72%	17.66%	26.92%	6.21%	2.14%	6.03%	5.75%	15.04%	24.13%	5.92%	7.56%	4.43%
AR(1)GARCH(1,2)	5.88%	8.96%	21.94%	5.77%	14.55%	22.32%	6.05%	4.56%	12.18%	5.86%	10.04%	8.29%	6.14%	2.94%	0.15%
AR(1)GARCH(2,1)	5.61%	23.50%	32.48%	5.77%	14.55%	22.32%	-	-	-	5.64%	21.79%	38.84%	5.92%	7.56%	4.43%
AR(1)GARCH(2,2)	5.61%	23.50%	32.48%	5.83%	11.88%	18.27%	6.05%	4.56%	9.72%	5.75%	15.04%	24.13%	5.98%	6.05%	4.04%
AR(1)TARCH(0,1)	-	-	-	6.28%	1.72%	0.36%	8.45%	0.00%	0.00%	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	6.17%	2.94%	0.70%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	6.26%	1.64%	3.38%	5.49%	35.23%	13.80%	5.89%	9.00%	12.76%	5.86%	10.04%	23.31%	6.19%	2.28%	2.46%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	6.21%	2.15%	4.46%	5.49%	35.23%	13.80%	-	-	-	-	-	-	6.09%	3.77%	5.30%
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	8.80%	0.00%	0.00%	6.11%	3.79%	0.96%	8.50%	0.00%	0.00%	7.12%	0.01%	0.02%	7.70%	0.00%	0.00%
AR(1)EGARCH(0,2)	7.99%	0.00%	0.00%	6.11%	3.79%	0.96%	7.79%	0.00%	0.00%	7.12%	0.01%	0.03%	7.59%	0.00%	0.00%
AR(1)EGARCH(1,1)	7.02%	0.02%	0.06%	5.55%	30.02%	11.52%	6.65%	0.20%	0.36%	6.24%	1.87%	5.58%	7.00%	0.02%	0.08%
AR(1)EGARCH(1,2)	6.85%	0.05%	0.21%	5.43%	40.99%	36.48%	6.65%	0.20%	0.56%	5.97%	6.48%	6.77%	6.84%	0.06%	0.08%
AR(1)EGARCH(2,1)	7.07%	0.01%	0.05%	5.55%	30.02%	11.52%	6.70%	0.14%	0.52%	6.35%	1.08%	2.23%	6.73%	0.11%	0.30%

Table (11.3.A). GED, 1000 sample size, 95%-VaR															
Models	t	S&P500		NIKKEI225			DAX30			CAC40		FTSE100			
		p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	7.72%	0.00%	0.00%	5.89%	9.62%	3.00%	7.47%	0.00%	0.00%	6.63%	0.23%	0.70%	6.95%	0.03%	0.02%
AR(1)GARCH(0,2)	7.23%	0.00%	0.01%	5.77%	14.55%	12.56%	6.76%	0.10%	0.27%	6.41%	0.81%	2.38%	6.84%	0.06%	0.03%
AR(1)GARCH(1,1)	5.67%	19.70%	37.50%	5.72%	17.66%	26.92%	5.83%	11.11%	26.32%	5.91%	8.10%	16.58%	6.03%	4.80%	9.04%
AR(1)GARCH(1,2)	5.56%	27.81%	49.69%	5.55%	30.02%	43.77%	5.72%	16.49%	35.94%	5.75%	15.04%	16.29%	5.92%	7.56%	4.43%
AR(1)GARCH(2,1)	5.67%	19.70%	37.50%	5.55%	30.02%	11.52%	-	-	-	-	-	-	5.92%	7.56%	16.24%
AR(1)GARCH(2,2)	5.83%	11.05%	22.43%	5.43%	40.99%	36.48%	5.78%	13.59%	30.95%	5.81%	12.34%	14.88%	5.87%	9.38%	4.80%
AR(1)TARCH(0,1)	-	-	-	5.83%	11.88%	3.87%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	5.55%	30.02%	11.52%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	-	-	-	5.38%	47.31%	19.08%	-	-	-	-	-	-	6.14%	2.94%	8.11%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	-	-	-	5.49%	35.23%	13.80%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	7.93%	0.00%	0.00%	5.72%	17.66%	6.21%	7.57%	0.00%	0.00%	6.68%	0.17%	0.40%	7.16%	0.01%	0.01%
AR(1)EGARCH(0,2)	7.61%	0.00%	0.00%	5.89%	9.62%	7.97%	6.98%	0.02%	0.00%	6.30%	1.43%	2.71%	7.00%	0.02%	0.01%
AR(1)EGARCH(1,1)	6.85%	0.05%	0.08%	5.72%	17.66%	6.21%	6.27%	1.64%	3.43%	5.86%	10.04%	13.42%	6.73%	0.11%	0.30%
AR(1)EGARCH(1,2)	7.02%	0.02%	0.07%	5.60%	25.37%	22.66%	6.27%	1.64%	4.66%	5.70%	18.18%	10.18%	6.52%	0.41%	0.52%
AR(1)EGARCH(2,1)	7.02%	0.02%	0.04%	5.66%	21.26%	7.72%	6.21%	2.14%	4.53%	5.86%	10.04%	8.29%	6.25%	1.75%	3.25%

Table (11.4.A). GED, 500 sample size, 95%-VaR															
Models	t	S&P500		NIKKEI225			DAX30			CAC40		FTSE100			
		p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	6.37%	0.94%	3.16%	5.72%	17.66%	26.92%	6.70%	0.14%	0.24%	6.57%	0.32%	0.90%	6.68%	0.16%	0.05%
AR(1)GARCH(0,2)	6.31%	1.25%	3.53%	5.77%	14.55%	12.56%	6.16%	2.78%	7.67%	6.35%	1.08%	2.23%	6.41%	0.75%	0.42%
AR(1)GARCH(1,1)	5.45%	38.00%	62.79%	5.49%	35.23%	31.53%	5.78%	13.59%	28.98%	-	-	-	5.76%	14.09%	29.99%
AR(1)GARCH(1,2)	5.50%	32.64%	57.55%	5.26%	61.47%	52.22%	5.29%	57.73%	57.43%	-	-	-	5.71%	17.05%	26.68%
AR(1)GARCH(2,1)	5.45%	38.00%	62.79%	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)GARCH(2,2)	5.45%	38.00%	62.79%	-	-	-	5.18%	72.92%	57.66%	-	-	-	5.65%	20.47%	38.15%
AR(1)TARCH(0,1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	6.53%	0.38%	1.34%	6.00%	6.14%	9.31%	7.03%	0.02%	0.02%	6.52%	0.44%	1.46%	7.11%	0.01%	0.00%
AR(1)EGARCH(0,2)	6.26%	1.64%	4.23%	5.94%	7.72%	11.80%	6.43%	0.70%	1.13%	6.35%	1.08%	2.99%	6.84%	0.06%	0.03%
AR(1)EGARCH(1,1)	6.10%	3.57%	7.49%	6.06%	4.85%	3.67%	6.54%	0.38%	1.41%	6.24%	1.87%	5.91%	6.73%	0.11%	0.13%
AR(1)EGARCH(1,2)	6.42%	0.70%	2.45%	5.77%	14.55%	32.62%	6.27%	1.64%	5.02%	6.24%	1.87%	5.58%	6.73%	0.11%	0.13%
AR(1)EGARCH(2,1)	6.42%	0.70%	2.40%	5.60%	25.37%	22.66%	6.59%	0.28%	0.81%	6.57%	0.32%	0.90%	6.79%	0.08%	0.10%

Note: The table reports for each model, sample size and equity index, the exception rate (t) and the p-values of unconditional (p_uc) and conditional coverage (p_cc)

Table (12.1.A). GED, 2000 sample size, 99%-VaR															
Models	t	S&P500		NIKKEI225			DAX30			CAC40			FTSE100		
		p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	2.37%	0.00%	0.00%	0.96%	87.19%	82.87%	2.67%	0.00%	0.00%	2.63%	0.00%	0.00%	2.80%	0.00%	0.00%
AR(1)GARCH(0,2)	2.27%	0.00%	0.00%	1.02%	93.73%	81.98%	2.18%	0.00%	0.00%	2.14%	0.00%	0.01%	2.42%	0.00%	0.00%
AR(1)GARCH(1,1)	1.13%	57.23%	66.25%	0.85%	51.22%	70.34%	1.25%	29.39%	42.50%	1.53%	3.37%	7.75%	1.35%	15.43%	25.43%
AR(1)GARCH(1,2)	1.19%	43.13%	55.65%	0.74%	24.14%	45.38%	1.25%	29.39%	42.50%	1.48%	5.50%	11.22%	1.40%	10.22%	17.93%
AR(1)GARCH(2,1)	1.24%	31.45%	44.59%	0.85%	51.22%	70.34%	-	-	-	1.53%	3.37%	7.75%	1.35%	15.43%	25.43%
AR(1)GARCH(2,2)	1.19%	43.13%	55.65%	0.85%	51.22%	70.34%	1.25%	29.39%	42.50%	1.53%	3.37%	7.75%	1.45%	6.56%	12.14%
AR(1)TARCH(0,1)	2.37%	0.00%	0.00%	0.96%	87.19%	82.87%	2.67%	0.00%	0.00%	2.46%	0.00%	0.00%	-	-	-
AR(1)TARCH(0,2)	-	-	-	1.13%	58.54%	67.75%	2.18%	0.00%	0.00%	2.14%	0.00%	0.01%	-	-	-
AR(1)TARCH(1,1)	1.24%	31.45%	44.59%	0.85%	51.22%	70.34%	1.14%	54.33%	64.44%	1.31%	19.78%	26.41%	1.18%	43.70%	56.11%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	1.19%	43.13%	55.65%	0.74%	24.14%	45.38%	1.20%	40.63%	53.58%	1.31%	19.78%	26.41%	1.24%	31.92%	45.06%
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	2.37%	0.00%	0.00%	1.02%	93.73%	81.98%	2.72%	0.00%	0.00%	2.41%	0.00%	0.00%	2.91%	0.00%	0.00%
AR(1)EGARCH(0,2)	2.54%	0.00%	0.00%	1.19%	43.94%	56.92%	2.51%	0.00%	0.00%	2.52%	0.00%	0.00%	2.91%	0.00%	0.00%
AR(1)EGARCH(1,1)	1.67%	0.79%	2.41%	0.91%	68.49%	78.84%	1.53%	3.56%	7.01%	2.03%	0.01%	0.03%	1.83%	0.13%	0.29%
AR(1)EGARCH(1,2)	1.62%	1.40%	3.87%	0.91%	68.49%	78.84%	1.58%	2.12%	5.40%	1.75%	0.35%	0.43%	1.94%	0.03%	0.14%
AR(1)EGARCH(2,1)	1.73%	0.44%	1.45%	0.96%	87.19%	82.87%	1.58%	2.12%	5.40%	1.92%	0.05%	0.09%	1.88%	0.06%	0.15%

Table (12.2.A). GED, 1500 sample size, 99%-VaR															
Models	t	S&P500		NIKKEI225			DAX30			CAC40			FTSE100		
		p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	2.32%	0.00%	0.00%	0.96%	87.19%	82.87%	2.89%	0.00%	0.00%	2.46%	0.00%	0.00%	2.75%	0.00%	0.00%
AR(1)GARCH(0,2)	2.32%	0.00%	0.00%	0.91%	68.49%	78.84%	2.67%	0.00%	0.00%	2.25%	0.00%	0.00%	2.48%	0.00%	0.00%
AR(1)GARCH(1,1)	1.40%	10.02%	17.62%	0.68%	15.01%	32.48%	1.31%	20.56%	32.23%	1.48%	5.50%	11.22%	1.29%	22.56%	34.59%
AR(1)GARCH(1,2)	1.51%	3.98%	8.86%	0.62%	8.65%	21.35%	1.25%	29.39%	42.50%	1.48%	5.50%	11.22%	1.35%	15.43%	25.43%
AR(1)GARCH(2,1)	1.46%	6.41%	11.91%	0.68%	15.01%	32.48%	-	-	-	1.48%	5.50%	10.42%	1.29%	22.56%	34.59%
AR(1)GARCH(2,2)	1.51%	3.98%	8.86%	0.68%	15.01%	32.48%	1.31%	20.56%	27.08%	1.48%	5.50%	11.22%	1.35%	15.43%	25.43%
AR(1)TARCH(0,1)	-	-	-	0.96%	87.19%	82.87%	2.78%	0.00%	0.00%	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	1.02%	93.73%	81.98%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	1.40%	10.02%	17.62%	0.74%	24.14%	45.38%	1.42%	9.12%	16.29%	1.64%	1.15%	3.28%	1.13%	57.88%	66.64%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	1.40%	10.02%	17.62%	0.74%	24.14%	45.38%	-	-	-	-	-	-	1.18%	43.70%	56.11%
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	2.21%	0.00%	0.00%	0.96%	87.19%	82.87%	2.72%	0.00%	0.00%	2.52%	0.00%	0.00%	2.80%	0.00%	0.00%
AR(1)EGARCH(0,2)	2.81%	0.00%	0.00%	1.25%	31.87%	45.51%	2.67%	0.00%	0.00%	2.46%	0.00%	0.00%	2.75%	0.00%	0.00%
AR(1)EGARCH(1,1)	1.78%	0.23%	0.84%	1.02%	93.73%	81.98%	1.85%	0.10%	0.24%	2.25%	0.00%	0.00%	1.99%	0.02%	0.04%
AR(1)EGARCH(1,2)	1.78%	0.23%	0.32%	0.96%	87.19%	82.87%	1.80%	0.20%	0.45%	1.97%	0.02%	0.05%	2.10%	0.00%	0.02%
AR(1)EGARCH(2,1)	1.83%	0.12%	0.47%	0.96%	87.19%	82.87%	1.85%	0.10%	0.24%	2.08%	0.01%	0.01%	2.10%	0.00%	0.02%

Table (12.3.A). GED, 1000 sample size, 99%-VaR															
Models	t	S&P500		NIKKEI225			DAX30			CAC40			FTSE100		
		p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	2.32%	0.00%	0.00%	1.13%	58.54%	67.75%	2.29%	0.00%	0.00%	2.19%	0.00%	0.00%	2.42%	0.00%	0.00%
AR(1)GARCH(0,2)	2.16%	0.00%	0.01%	1.08%	75.34%	76.58%	2.51%	0.00%	0.00%	2.14%	0.00%	0.01%	2.21%	0.00%	0.00%
AR(1)GARCH(1,1)	1.62%	1.40%	3.87%	0.74%	24.14%	45.38%	1.42%	9.12%	16.29%	1.48%	5.50%	10.42%	1.45%	6.56%	12.14%
AR(1)GARCH(1,2)	1.62%	1.40%	1.19%	0.74%	24.14%	45.38%	1.36%	13.92%	23.40%	1.53%	3.37%	6.67%	1.51%	4.08%	7.91%
AR(1)GARCH(2,1)	1.57%	2.39%	5.97%	0.68%	15.01%	32.48%	-	-	-	-	-	-	1.45%	6.56%	12.14%
AR(1)GARCH(2,2)	1.62%	1.40%	1.19%	0.74%	24.14%	45.38%	1.31%	20.56%	32.23%	1.53%	3.37%	6.67%	1.40%	10.22%	17.93%
AR(1)TARCH(0,1)	-	-	-	1.08%	75.34%	76.58%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	1.08%	75.34%	76.58%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	-	-	-	0.79%	36.26%	58.61%	-	-	-	-	-	-	1.18%	43.70%	56.11%
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	-	-	-	0.79%	36.26%	58.61%	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	1.94%	0.03%	0.07%	1.19%	43.94%	37.70%	2.29%	0.00%	0.00%	2.08%	0.01%	0.00%	2.42%	0.00%	0.00%
AR(1)EGARCH(0,2)	2.27%	0.00%	0.00%	1.25%	31.87%	45.51%	2.34%	0.00%	0.00%	2.14%	0.00%	0.01%	2.69%	0.00%	0.00%
AR(1)EGARCH(1,1)	1.94%	0.03%	0.14%	1.08%	75.34%	76.58%	1.69%	0.69%	1.50%	1.97%	0.02%	0.10%	1.94%	0.03%	0.07%
AR(1)EGARCH(1,2)	2.16%	0.00%	0.00%	0.96%	87.19%	82.87%	1.69%	0.69%	1.50%	2.03%	0.01%	0.03%	1.94%	0.03%	0.07%
AR(1)EGARCH(2,1)	2.21%	0.00%	0.00%	1.08%	75.34%	76.58%	1.63%	1.23%	2.60%	2.14%	0.00%	0.01%	2.15%	0.00%	0.00%

Table (12.4.A). GED, 500 sample size, 99%-VaR															
Models	t	S&P500		NIKKEI225			DAX30			CAC40			FTSE100		
		p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc	t	p_uc	p_cc
AR(1)GARCH(0,1)	1.67%	0.79%	2.41%	1.13%	58.54%	67.75%	2.13%	0.00%	0.01%	2.25%	0.00%	0.00%	2.05%	0.01%	0.02%
AR(1)GARCH(0,2)	1.83%	0.12%	0.19%	1.08%	75.34%	76.58%	2.07%	0.01%	0.03%	1.92%	0.05%	0.09%	2.21%	0.00%	0.00%
AR(1)GARCH(1,1)	1.30%	22.19%	28.41%	0.91%	68.49%	78.84%	1.25%	29.39%	42.50%	-	-	-	1.51%	4.08%	7.91%
AR(1)GARCH(1,2)	1.57%	2.39%	1.71%	1.02%	93.73%	81.98%	1.31%	20.56%	32.23%	-	-	-	1.45%	6.56%	12.14%
AR(1)GARCH(2,1)	1.30%	22.19%	28.41%	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)GARCH(2,2)	1.51%	3.98%	2.35%	-	-	-	1.31%	20.56%	32.23%	-	-	-	1.45%	6.56%	12.14%
AR(1)TARCH(0,1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(0,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(1,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,1)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)TARCH(2,2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
AR(1)EGARCH(0,1)	1.67%	0.79%	2.41%	1.30%	22.33%	27.76%	2.13%	0.00%	0.01%	2.14%	0.00%	0.01%	2.42%	0.00%	0.00%
AR(1)EGARCH(0,2)	2.05%	0.01%	0.03%	1.53%	3.85%	7.60%	2.23%	0.00%	0.00%	2.08%	0.01%	0.01%	2.58%	0.00%	0.00%
AR(1)EGARCH(1,1)	1.78%	0.23%	0.32%	1.25%	31.87%	33.19%	1.69%	0.69%	1.50%	2.14%	0.00%	0.01%	2.32%	0.00%	0.00%
AR(1)EGARCH(1,2)	2.16%	0.00%	0.00%	1.19%	43.94%	56.92%	1.96%	0.03%	0.11%	2.30%	0.00%	0.00%	2.32%	0.00%	0.00%
AR(1)EGARCH(2,1)	1.89%	0.06%	0.11%	1.13%	58.54%	40.55%	1.74%	0.38%	0.84%	2.14%	0.00%	0.01%	2.48%	0.00%	0.00%

Note: The table reports for each model, sample size and equity index, the exception rate (t) and the p-values of unconditional (p_uc) and conditional coverage (p_cc)