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INVESTMENT PORTFOLIO OPTIMIZATION WITH GARCH MODELS

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ABSTRACT

Since the introduction of the Markowitz mean-variance optimization model, several extensions have been made to improve optimality. This study examines the application of two models - the ARMA-GARCH model and the ARMA- DCC GARCH model - for the Mean-VaR optimization of funds managed by HFC Investment Limited. Weekly prices of the above mentioned funds from 2009 to 2012 were examined. The funds analyzed were the Equity Trust Fund, the Future Plan Fund and the Unit Trust Fund. The returns of the funds are modelled with the Autoregressive Moving Average (ARMA) whiles volatility was modelled with the univariate Generalized Autoregressive Conditional Heteroskedasti city (GARCH) as well as the multivariate Dynamic Conditional Correlation GARCH (DCC GARCH). This was based on the assumption of non-constant mean and volatility of fund returns. In this study the risk of a portfolio is measured using the value-at-risk. A single constrained Mean-VaR optimization problem was obtained based on the assumption that investors' preference is solely based on risk and return. The optimization process was performed using the Lagrange Multiplier approach and the solution was obtained by the Kuhn-Tucker theorems. Conclusions which were drawn based on the results pointed to the fact that a more efficient portfolio is obtained when the value-at-risk (VaR) is modelled with a multivariate GARCH.

Keywords: Optimization, Dynamic Conditional Correlation, value-at-risk, multivariate GARCH

INTRODUCTION

A portfolio investment is a passive investment in securities, which does not include an active management or control of the securities by the investor. A portfolio investment can also be seen as an investment made by an investor who is not particularly interested in involvement in the management of a company. The main purpose of the investment is solely for financial gain. Hence a portfolio investment must include investment in

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an assortment or range of securities, or other types of investment vehicles, to spread the risk of possible loss due to below-expectations performance of one or a few of them. Any investor would like to have the highest return possible from an investment.

However, this has to be counterbalanced by some amount of risk the investors are able or be willing to take. The expected return and the risk measured by the variance (or the standard deviation, which is the square-root of the variance) are the two main characteristics of an investment portfolio. Studies show that unfortunately, equities with high returns usually correlate with high risk. The behavior of a portfolio can be quite different from the behavior of individual components of the portfolio.

In recent years, criticism of the basic model has been increasing because of its disregard for individual investors preferences. Observed that most investors do not actually buy efficient portfolios, but rather those behind the efficient frontier. First proposed a compromise programming model for an ‘‘average’’ investor, which was modified to approximate the optimum portfolio of an individual investor. A different approach was described in, who proposed the use of objective and subjective measures for assets. Their idea leads to a simple Linear Programming model. Argued that most models do not incorporate the multidimensional nature of the problem and outline a framework for such a view on portfolio management.

The over reliance of investors and even portfolio managers on cap weighted average in monitoring the benchmark indices only give the investors the general idea about the general market movement. Investments decisions that depend on these benchmarks lead the investors and portfolio managers to underperform since those benchmarks lack the requirement for robust portfolio construction.

The representation of the risk associated with equity through the variance in its returns or of the risk associated with an option through its volatility, takes account of both good and bad risks. A significant variance corresponds to the possibility of seeing returns vastly different from the expected return, i.e. very small values (small profits and even losses) as well as very large values (significant profits). The application of equal weights in the portfolio optimization process is based on the assumption of normality. This assumption is proven not to be always true, hence the need for an optimization model that can produce specific weights for each component of the portfolio.

OBJECTIVES OF THE STUDY

The general objective of this research was to identify alternative approach to portfolio

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optimization to model the various funds of HFC Investment Limited.

Specifically, this study seeks to:

i. evaluate the performance of the various funds managed by HFC Investment Limited;
ii. Employ a recent time series model to forecast future returns of the funds under study;
iii. Identify which portfolio mix is indeed optimized.

LITERATURE REVIEW

The subject of portfolio optimization has raised considerable attentions among scholars and researchers. It has come a long way from [5] seminal work, which introduces return/variance risk management framework. Developments in portfolio optimization are stimulated by two basic requirements: Adequate modelling of utility functions, risks, and constraints; Efficiency, that is, ability to handle large numbers of instruments and scenarios. There have been many efforts to comprehend the concept of portfolio optimization using different theoretical models. The increased attention paid to this has therefore brought significant differences about what it is and perceptions about what precisely it is fundamentally, the selection of assets or equities is not just a problem of finding attractive investments. Designing the correct portfolio of assets cannot be done by human intuition alone and requires modern, powerful and reliable mathematical programs called optimizers [6]. The behavior of a portfolio can be quite different from the behavior of individual components of the portfolio.

2.1. GARCH Models

The challenges faced by econometricians to specify the information which helps to project the mean and variance of the returns is contingent on the previous information. The assumption here is that the variance of the next day's returns is an equally weighted average of the previous days. However, this assumption of equal weights does not look attractive because, the more recent information or events would be much important and hence, must be assigned greater weights. The ARCH model proposed by [7] let these weights be parameters to be estimated. Thus, the model allowed the data to determine the best weights to use in forecasting the variance. A useful generalization of the ARCH model is the GARCH parameterization.

This model is also a weighted average of past squared residuals, but it has declining weights that never go completely to zero. It gives parsimonious models that are easy to estimate and even in its simplest form, has proven surprisingly successful in predicting conditional variances. The most widely used GARCH specification asserts that the best predictor of the variance in the next period is

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a weighted average of the long-run average variance, the variance predicted for this period, and the new information in this period that is captured by the most recent squared residual. Such an updating rule is a simple description of adaptive or learning behavior and can be thought of as Bayesian updating. Consider the trader who knows that the long-run average daily standard deviation of the Standard and Poor's 500 is 1 percent that the forecast he made yesterday was 2 percent and the unexpected return observed today is 3 percent. Obviously, this is a high volatility period, and today is especially volatile, which suggests that the forecast for tomorrow could be even higher.

However, the fact that the long-term average is only 1 percent might lead the forecaster to lower the forecast. The best strategy depends upon the dependence between days. To be precise, we can use h_t to define the variance of the residuals of a regression

$$r_t = m_t + \sqrt{h_t \varepsilon_t} \qquad [2.1]$$

In this definition, the variance of ε is one. The GARCH model for variance looks like this:

$$h_{t+1} = w + \alpha(r_t - m_t)^2 + \beta h_t$$
$$= w + \alpha h_t \varepsilon_t^2 + \beta h_t \qquad [2.2]$$

The econometrician must estimate the constants $w, \alpha,$ and β; updating simply requires knowing the previous forecast h and residual. It should be noted that this only works if $\alpha + \beta < 1$, and it only really makes sense if the weights are positive requiring $\alpha > 0, \beta > 0, w > 0$.

The GARCH model that has been described is typically called the GARCH (1, 1) model. The (1,1) in parentheses is a standard notation in which the first number refers to how many autoregressive lags, or ARCH terms, appear in the equation, while the second number refers to how many moving average lags are specified, which here is often called the number of GARCH terms. Sometimes models with more than one lag are needed to find good variance forecasts.

Although this model is directly set up to forecast for just one period, it turns out that based on the one-period forecast, a two-period forecast can be made. Ultimately, by repeating this step, long-horizon forecasts can be constructed. For the GARCH (1,1), the two-step forecast is a little closer to the long-run average variance than is the one-step forecast, and, ultimately, the distant-horizon forecast is the same for all time periods as long as α + β < 1.

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The GARCH updating formula takes the weighted average of the unconditional variance, the squared residual for the first observation and the starting variance and estimates the variance of the second observation. This is input into the forecast of the third variance, and so forth. Eventually, an entire time series of variance forecasts is constructed. Ideally, this series is large when the residuals are large and small when they are small.

The likelihood function provides a systematic way to adjust the parameters w, α, β to give the best fit. Of course, it is entirely possible that the true variance process is different from the one specified by the econometrician. In order to detect this, a variety of diagnostic tests are available. The simplest is to construct the series of $\{\varepsilon_t\}$, which are supposed to have constant mean and variance if the model is correctly specified.

Various tests such as tests for autocorrelation in the squares are able to detect model failures. Often a "Ljung box test" with 15 lagged autocorrelations is used. The GARCH (1, 1) is the simplest and most robust of the family of volatility models. However, the model can be extended and modified in many ways. Three modifications will be mentioned briefly, although the number of volatility models that can be found in the literature is now quite extraordinary. The GARCH (1, 1) model can be generalized to a GARCH (p, q) model—that is, a model with additional lag terms. Such higher-order models are often useful when a long span of data is used, like several decades of daily data or a year of hourly data. With additional lags, such models allow both fast and slow decay of information. A particular specification of the GARCH (2, 2) by, sometimes called the "component model," is a useful starting point to this approach.

ARCH/GARCH models thus far have ignored information on the direction of returns; only the magnitude matters. However, there is very convincing evidence that the direction does affect volatility. Particularly for broad-based equity indices and bond market indices, it appears that market declines forecast higher volatility than comparable market increases do. There is now a variety of asymmetric GARCH models, including the EGARCH model, the TARCH model threshold ARCH and a collection and comparison by.

The goal of volatility analysis must ultimately be to explain the causes of volatility. While time series structure is valuable for forecasting, it does not satisfy our need to explain volatility. The estimation strategy introduced for ARCH/GARCH models can be directly applied if there are predetermined or exogenous variables. Thus, we can think of the estimation problem for

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the variance just as we do for the mean. We can carry out specification searches and hypothesis tests to find the best formulation. Thus far, attempts to find the ultimate cause of volatility are not very satisfactory.

Obviously, volatility is a response to news, which must be a surprise. However, the timing of the news may not be a surprise and gives rise to predictable components of volatility, such as economic announcements. It is also possible to see how the amplitude of news events is influenced by other news events. For example, the amplitude of return movements on the United States stock market may respond to the volatility observed earlier in the day in Asian markets as well as to the volatility observed in the United States on the previous day. Call these “heat wave” and “meteor shower” effects. A similar issue arises when examining several assets in the same market. Does the volatility of one influence the volatility of another? In particular, the volatility of an individual stock is clearly influenced by the volatility of the market as a whole. This is a natural implication of the capital asset pricing model. It also appears that there is time variation in idiosyncratic volatility.

2.2 VALUE-AT-RISK (VAR) MODEL

Current regulations for finance businesses formulate some of the risk management requirements in terms of percentiles of loss distributions. An upper percentile of the loss distribution is called Value-at-Risk (VaR). For instance, 95% VaR is an upper estimate of losses which is exceeded with 5% probability. The popularity of VaR is mostly related to a simple and easy to understand representation of high losses. VaR can be quite efficiently estimated and managed when underlying risk factors are normally (log-normally) distributed.

However, for non-normal distributions, VaR may have undesirable properties such as lack of sub-additivity, i.e., VaR of a portfolio with two instruments may be greater than the sum of individual VaRs of these two instruments. Also, VaR is difficult to control/optimize for discrete distributions, when it is calculated using scenarios. In this case, VaR is non-convex (definition of convexity in and non-smooth as a function of positions, and has multiple local extrema. Mostly, approaches to calculating VaR rely on linear approximation of the portfolio risks and assume a joint normal (or log-normal) distribution of the underlying market parameters. Also, historical or Monte Carlo simulation-based tools are used when the portfolio contains nonlinear instruments such as options.

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For investors, risk is about the odds of losing money, and VaR is based on that common sense fact. By assuming that investors care about the odds of big losses, VaR can be used to answer the questions, "What is my worst-case scenario?" or "How much could I lose in a really bad month?" The Value at Risk (VaR) also known as expected shortfall is the most widely used and accepted tool for measuring market risk and is now a standard in the financial industry.

Intuitively, VaR measures the maximum loss that the value of an asset (or a portfolio of assets) can suffer with an associated probability within specified time-horizon. In statistical terms the VaR can be thought of as a quantile of the returns distribution [16]. Generally, the calculation methods for being in risk is divided in three categories; variance-covariance, Monte Carlo simulation method, historical simulation method. Most literature on VaR focuses on the computation of the VaR for financial assets such as stocks or bonds, and they usually deal with the modeling of VaR for negative returns.

Applications of the ARCH/GARCH approach are widespread in situations where the volatility of returns is a central issue. Many banks and other financial institutions use the concept of “value at risk” as a way to measure the risks faced by their portfolios. The one (1) per cent value at risk is defined as the number of dollars that one can be 99 percent certain exceeds any losses for the next day. Statisticians call this a one per cent quantile, because one per cent of the outcomes are worse and 99 percent are better.

METHODOLOGY

Research Design

The data analyzed in this study comprises of 178 data points from three funds managed by the HFC Investments Limited. The funds include HFC Equity Trust (ET), HFC Future Plan (FP) and the HFC Unit Trust (UT) respectively. The data points represent the weekly prices of the funds spanning the given time interval.

The weekly return r_{it} on fund *i* at time t and is calculated as

$$r_{it} = ln\left(\frac{P_{it}}{P_{it-1}}\right), \qquad [3.1]$$

Where P_{it} is the price of fund *i* (*i*=1,…,N) with N, the number of funds analysed at time *t* (*t* = 1,…,T) where T is the number of observed data.

3.1 ARCH AND GARCH MODELS

The Autoregressive Conditional Heteroscedasticity (ARCH) models are used to describe and model time series. They are often employed when one suspects that at any point in a series, the terms will have a characteristic size or

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variance. The ARCH models assume the variance of the current error term or innovation to be a function of the error terms of the past time periods. The variance is associated with the squares of the previous innovations.

Assuming one wish to model a time series using an ARCH process with ε_t as the error terms, then ε_t are broken into a stochastic part and a time–dependent standard deviation σ_t. According to Engle (1982), an ARCH model could be described as a discrete time stochastic processes $\{\varepsilon_t\}$ of the form

$$\begin{cases} \varepsilon_t = \sigma_t z_t \\ z_t \sim i.i.d. \end{cases} \quad [3.2]$$

The random variable z_t is a strong White noise process. The series ${\sigma_t}^2$ is modeled as

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_q \varepsilon_{t-q}^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 , \quad [3.3]$$

Where $\alpha_0 > 0$ and $\alpha_i \geq 0, i > 0$.

Engle (1991) has proposed a methodology for testing the lag length of the ARCH errors using Lagrange multiplier test. This is done as follows:

1. Obtain the estimate of the best fitting autoregressive model AR(q)

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \ldots + \alpha_q y_{y-q} + \varepsilon_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i y_{t-i} + \varepsilon_t \quad [3.4]$$

2. Find the estimate of the squares of the errors $\hat{\varepsilon}^2$ and regress them on a constant and q lagged values

$$\hat{\varepsilon}_t^2 = \hat{\alpha}_0 + \sum_{i=1}^{q} \hat{\alpha}_i \hat{\varepsilon}_{t-i}^2 \quad [3.5]$$

where q is the length of ARCH lags.

3. The null hypothesis is that, in the absence of ARCH components, $\alpha_i = 0$ for all $i = 1, \ldots, q$. The alternative hypothesis is that, in the presence of ARCH components, at least one of the estimated α_i coefficients must be significant.

The GARCH (p, q) model is given by

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_q \sigma_{t-q}^2$$
$$= \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2, \quad [3.6]$$

Where p is the order of the GARCH terms σ^2 and q is the order of the ARCH terms ε^2.

3.2 MULTIVARIATE GARCH MODELS

The basic idea to extend univariate GARCH models to multivariate GARCH models is that it is significant to predict the dependence in the co-movements of asset returns in a portfolio. To recognize this feature through a multivariate

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model would generate a more reliable model than separate univariate models.

First, we consider some specifications of an MGARCH model that should be imposed, so as to make it flexible enough to state the dynamics of the conditional variances and covariance's. On the other hand, as the number of parameters in an MGARCH model increases rapidly along with the dimension of the model, the specification should be parsimonious to simplify the model estimation and also reach the purpose of easy interpretation of the model parameters.

Parsimony however, can cause a reduction in the number of parameters, and may also not be able to capture the relevant dynamics in the covariance matrix [17]. It is therefore important to get a balance between the parsimony and the flexibility when a multivariate GARCH model specification is to be fitted. The MGARCH models must satisfy the positive definiteness of the covariance matrix.

3.3 THE DYNAMIC CONDITIONAL CORRELATION (DCC-GARCH) MODEL

The DCC-GARCH assumes that returns of the assets, x_t is normally distributed with zero mean and covariance matrix C_t.

$$(x_t) \sim N(0, C_t) \qquad [3.7]$$

The conditional covariance matrix is obtained by employing the use of conditional standard deviations and dynamic correlation matrix.

Let S_t be a $1 \times n$ vector of conditional standard deviations modeled by univariate GARCH process such that

$$s_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i e_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \qquad [3.8]$$

where $e_{t-i} = \sigma_{t-i}\xi_{t-i}$ and $\xi_{t-i} \sim N(0, 1)$.

In order to find time varying correlation matrix, Engle (2002) proposes a model for the time varying covariance such that

$$K_t = \left(1 - \sum_{i=1}^{p} \alpha_i - \sum_{j=1}^{q} \beta_j\right)\overline{K} + \sum_{i=1}^{p} \alpha_i (e_{t-i}e'_{t-i}) + \sum_{j=1}^{q} \beta_j K_{t-j} \qquad [3.9]$$

Subject to

$\alpha_i \geq 0$,

$\beta_j \geq 0$,

$\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$.

From the above equation, it can be seen that the GARCH process is employed to model time-varying covariance matrices. $\overline{K}$ Represents the unconditional covariance and it is initially obtained by estimating the sample covariance. We forecast K_t by the lagged residuals (e_{t-1}), and they are standardized by conditional standard deviations, and lagged covariance's (K_{t-1}). In estimating conditional covariance matrix, a univariate GARCH is employed to model the

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conditional standard deviations of each asset in the portfolio.

It is important to note that the constraints of GARCH process are still considered in constructing a positive definite covariance matrix. The log likelihood function can be written as below

$$L = -\frac{1}{2}\sum_{t=1}^{T}(klog(2\pi) + log(|S_t M_t S_t|) + r'_t S_t^{-1} M_t^{-1} S_t^{-1} r_t)$$
$$= -\frac{1}{2}\sum_{t=1}^{T}(klog(2\pi) + 2log(|S_t|) + log(|R_t|) + e'_t M_t^{-1} e_t), \quad [3.10]$$

and is used to find estimators of that model.

After finding optimum estimators that maximize the log likelihood function above, it is easy to produce covariance series. But, it is necessary to note that each covariance matrix is not constructed by conditional standard deviations yet. This covariance matrix series is generated by relying on initial unconditional covariance matrix. Then new covariance matrix for next time point is generated by previous one and standardized residuals as in simple univariate GARCH process. Hence, univariate GARCH process is employed to extract time varying positive definite correlation matrices from that covariance matrix series such that

$$M_t = \left(\sqrt{K_t^d}\right)^{-1} K_t \left(\sqrt{K_t^d}\right)^{-1} \quad [3.11]$$

In that equation, K_t^d refers to the diagonal matrix of variances which are obtained from the diagonal items of K_t as following

$$K_t^d = \begin{bmatrix} S_{11}^2 & 0 & \cdots & 0 \\ 0 & S_{22}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & S_{nn}^2 \end{bmatrix} \quad [3.12]$$

The matrix notation for time-varying correlation indicates that the way of calculating correlations such as dividing covariance's by standard deviations extracted from the covariance matrix. The matrix notation can be interpreted algebraically such that

$$\rho_{i,j,t} = \frac{s_{i,j,t}}{\sqrt{S_{ii}^2 S_{jj}^2}} \quad [3.13]$$

Finally, conditional covariance matrix can be found by Hadamard product of the matrix of conditional standard deviations and time varying correlation matrix such that

$$C_t = S'_t S_t \circ M_t$$
[3.14]

This methodology gives the conditional covariance matrix for each data point of the calibration period. To find the conditional covariance matrix that is used to optimize weights of assets in the portfolio, one day conditional standard deviations of assets and their dynamic correlation matrix are forecasted.

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3.4 PORTFOLIO OPTIMIZATION MODEL

Let r_p denote the return of portfolio at the time t, and $w'_i = (w_{1,} w_2, \dots, w_N)$, the weight of fund i. r_p, the portfolio return can be determined using the equation:

$$r_p = \sum_{i=1}^{N} w_i r_{it} \qquad [3.15]$$

Where

$\sum_{i=1}^{N} w_i = 1,\ 0 < w_i < 1, (i = 1, 2, \dots, N)$

r_{it} is the return on fund *i* at time t and is calculated as

$$r_{it} = ln\left(\frac{P_{it}}{P_{it-1}}\right) \qquad [3.16]$$

where

P_{it} is the price of fund *i* (*i=1,…,N*) with *N*, the number of funds analyzed at time t $(t = 1, \dots, T)$, where T is the number of observed data.

Let $\boldsymbol{\mu}' = (\mu_1, \dots, \mu_N)$ be the mean vector transpose, and $\boldsymbol{w}' = (w_1, \dots, w_N)$the weight vector transpose of a portfolio with the weight vector having the property $\boldsymbol{e}'\boldsymbol{w} = 1$, where $\boldsymbol{e}' = (1, \dots, 1)$

The mean of portfolio return can be estimated using the equation

$$\mu_p = \sum_{i=1}^{N} w^2{}_i \mu_{it} = \boldsymbol{w}'\boldsymbol{\mu}, \qquad [3.17]$$

where μ_{it} is the mean of stock i at time t.

The variance of the portfolio return can be expressed as follows:

$$\sigma^2{}_p = \sum_{i=1}^{N} w^2{}_i \sigma^2{}_{it} + \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \quad ; i \neq j$$

$$= \boldsymbol{w}'\boldsymbol{\Omega}\boldsymbol{w}, \qquad [3.18]$$

where $\boldsymbol{\Omega} = \sigma_{ij} = cov(r_{it}, r_{jt})$ denotes the covariance between stock i and stock j .

3.5 VALUE-AT-RISK (VAR)

The Value-at-Risk of an investment portfolio based on standard normal distribution approach is calculated using the equation:

$$VaR_p = -W_o\left\{\boldsymbol{w}'\boldsymbol{\mu} + Z_\alpha(\boldsymbol{w}'\boldsymbol{\Omega}\boldsymbol{w})^{1/2}\right\}, \qquad [3.19]$$

where W_o the number of funds that is allocated in the portfolio, and Z_α , the percentile of standard normal distribution at the significance level α. When it is assumed that $W_o = 1$, the equation reduces to

$$VaR_p = -\left\{\boldsymbol{w}'\boldsymbol{\mu} + Z_\alpha(\boldsymbol{w}'\boldsymbol{\Omega}\boldsymbol{w})^{1/2}\right\} \qquad [3.20]$$

3.6 EFFICIENCY OF A PORTFOLIO

A portfolio weight w^* is called (Mean-VaR) efficient if there is no other portfolio weight w with $\mu_p \geq \mu_p^*$ and $VaR_p \leq VaR^*{}_p$

To obtain the efficient portfolio, we used the objective function:

$$\text{Maximize}\left\{\boldsymbol{2\tau\mu_p - VaR_p}\right\}, \boldsymbol{\tau} \geq 0, \qquad [3.21]$$

where, $\boldsymbol{\tau}$ denotes the investor risk tolerance factor. By substitution we must solve an optimization problem:

$$\max -\left\{(2\boldsymbol{\tau} + 1)\boldsymbol{w}'\boldsymbol{\mu} + Z_\alpha(\mathbf{w}'\boldsymbol{\Omega}\mathbf{w})^{1/2}\right\}$$

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$Subject\ to$

$\boldsymbol{w'e} = 1.$

$\boldsymbol{w'}>0$ [3.22]

The objective function is quadratic concave due to positive semi-definite nature of the covariance matrix. Hence, the optimization problem is quadratic concave with a Lagrangean function which is given by:

$$L(\boldsymbol{w},\lambda) = (2\boldsymbol{\tau}+1)\boldsymbol{w'\mu} + Z_\alpha(\mathbf{w'\Omega w})^{1/2} + \lambda(\boldsymbol{w'e}-1) \quad [3.23]$$

Because of the Kuhn-Tucker theorem, the optimality conditions are:

$\frac{\partial L}{\partial \boldsymbol{w}} = 0$. This implies that:

$$(2\boldsymbol{\tau}+1)\boldsymbol{\mu} + \frac{Z_\alpha \boldsymbol{\Omega w}}{(\boldsymbol{w'}\mathbf{\Omega w})^{1/2}} + \lambda \boldsymbol{e} = 0, \quad [3.24]$$

and $\frac{\partial L}{\partial \lambda} = 0$, which also implies that:

$$\boldsymbol{w'e} - 1 = 0. \quad [3.25]$$

For $\tau \geq 0$, we have an optimum portfolio w^* based on algebra calculations and ,setting

$$\boldsymbol{A} = \boldsymbol{e'\Omega^{-1}e}, \quad [3.26]$$

$$\boldsymbol{B} = (2\boldsymbol{\tau}+1)(\boldsymbol{\mu}'\boldsymbol{\Omega}^{-1}\mathbf{e} + \mathbf{e}'\boldsymbol{\Omega}^{-1}\boldsymbol{\mu}) \text{ and,} \quad [3.27]$$

$$\boldsymbol{C} = (2\boldsymbol{\tau}+1)^2(\boldsymbol{\mu}'\boldsymbol{\Omega}^{-1}\boldsymbol{\mu}) - Z_\alpha, \text{ we have} \quad [3.28]$$

$$\lambda = \frac{-\boldsymbol{B}+(\boldsymbol{B}^2-4\boldsymbol{AC})^{1/2}}{2\boldsymbol{A}} \quad [3.29]$$

By condition, $\boldsymbol{B^2 - 4AC} \geq 0$ and vector of w^* is given by

$$\boldsymbol{w}* = \frac{(2\boldsymbol{\tau}+1)\boldsymbol{\Omega}^{-1}\boldsymbol{\mu}+\lambda\boldsymbol{\Omega}^{-1}\boldsymbol{e}}{(2\boldsymbol{\tau}+1)\boldsymbol{e'\Omega^{-1}\mu}+\lambda\boldsymbol{e'\Omega^{-1}e}} \quad [3.30]$$

The substitution of w^* into μ_p and VaR_p respectively gives the optimum of expected return and optimum Value-at-Risk of portfolios.

4. ANALYSIS AND DISCUSSIONS

4.1 DESCRIPTIVE STATISTICS

Table 4.1 shows a descriptive statistics of the minimum, maximum, mean, variance, standard deviation, skewness and kurtosis of the returns of the various funds. It can be seen from Table 4.1 that Unit Trust had the minimum return of -0.164478 representing an approximate loss of 16.45% in value whiles the Equity Trust had the maximum return of 0.055256 representing an approximate gain of 5.53% in value over the entire period. Also the Future Plan fund had the highest average return coupled with the lowest variability in returns.

However, the Unit trust fund had the lowest average return with the highest variability.

Furthermore, the Unit Trust and the Future Plan returns are negatively skewed and that shows conformity with most financial data.

Finally, all the three funds had positive kurtosis in excess of 3, indicating that the funds have heavier tails as compared with the normal distribution which has a kurtosis of 3.0. The Equity Trust however can be seen to be approximately normal.

(Refer Table 4.3.1)

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4.2 ARMA MODELS FOR THE FUNDS

A number of ARMA and GARCH models are considered for each of the funds. The Akaike Information Criterion (AIC) is calculated to identify the best fit model. In Table 4.2 a below, ARMA (1, 1) was the best fit model for Equity Trust and Future Plan funds. The ARMA (1, 1) model produced the smallest AIC among all the models considered for the two funds. The Unit Trust fund on the other hand had ARMA (1, 2) as the best model. **(Refer Table 4.2 a)**

From Table 4.3, various GARCH models are considered for the three funds. The best GARCH model for each fund is selected according to the AIC value. It could be seen that the Equity trust Fund and the Future Plan Fund follow the GARCH (1, 1) model whilst the Unit Trust Fund follows the GARCH (1, 2) according to the AIC value. **(Refer Table 4.3)**

The respective models for the estimates of the expected means and variances are below:

EQUITY TRUST FUND

$$r_t = 0.0004421 + 0.3902578 \quad r_{t-1} + 0.1331618 \quad r_{t-1} + \varepsilon \qquad [4.2]$$

$$\sigma_t^2 = 1.7220\mathrm{e}-03 + 8.7724\mathrm{e}-01\ \sigma_{t-1}^2 + 1.6740\mathrm{e}-01\ \sigma_{t-1}^2 + \varepsilon \qquad [4.3]$$

FUTURE PLAN FUND

$$r_t = 0.0012658 + 0.5790840\ r_{t-1} - 0.1351939 \quad r_{t-1} + \varepsilon \qquad [4.4]$$

$$\sigma_t^2 = 4.1421\mathrm{e}-04 + \sigma_{t-1}^2 + 4.2340\mathrm{e}-01\ \sigma_{t-1}^2 + \varepsilon \qquad [4.5]$$

UNIT TRUST FUND

$$r_t = 6.582\mathrm{e}-05 + 9.753\mathrm{e}-01\ r_{t-1} - 9.826\mathrm{e}-01\ r_{t-1} - 8.237\mathrm{e}-02\ r_{t-2} + \varepsilon \qquad [4.6]$$

$$\sigma_t^2 = 2.7571\mathrm{e}-03 + \sigma_{t-1}^2 + 1.0000\mathrm{e}-08\ \sigma_{t-1}^2 + 1.0000\mathrm{e}-08\sigma_{t-2}^2 + \varepsilon \qquad [4.7]$$

4.3 ESTIMATES OF THE MEAN AND VARIANCE

The models above were used to predict the mean and variance of the respective funds, and the results are displayed in Table 4.4. The values obtained from the models will be employed in the portfolio optimization process. **(Refer Table 4.4)**

4.4 THE PORTFOLIO OPTIMIZATION PROCESS (ARMA-GARCH)

From the results in the table above, we produce our mean vector

$$\boldsymbol{\mu} = \begin{bmatrix} 0.00271231 \\ 0.001439837 \\ 0.002469056 \end{bmatrix},$$

The unit vector so established is given by

$$\boldsymbol{e} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},$$

The variances in the table above coupled with the covariance between fund *i* and fund *j*, for $i \neq j$ are used to find the covariance matrix, Ω.

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$$\boldsymbol{\Omega} = \begin{bmatrix} 1.909301e-03 & 2.090334e-05 & -3.375212e-07 \\ 2.090334e-05 & 4.885960e-04 & 1.413325e-06 \\ -3.375212e-07 & 1.413325e-06 & 3.011159e-03 \end{bmatrix}$$

whose inverse is calculated to be

$$\boldsymbol{\Omega^{-1}} = \begin{bmatrix} 523.99733540 & -22.4180966 & 0.06925714 \\ -22.41809664 & 2047.6425820 & -0.96359941 \\ 0.06925714 & -0.9635994 & 332.09849937 \end{bmatrix}$$

Table 4.5 provide a list of weights for the various funds at specified risk tolerance level, τ given a percentile of -1.645 at a significance level of 0.05. **(Refer Table 4.5)**

Here, W_1, W_2, and W_3 refer to the weights allocated to Equity Trust Fund, Future Plan Fund and Unit Trust Fund respectively. Tolerance levels from 9.60 and beyond were not used since they produced negative weights. For instance W_1 at tolerance level 9.60 is calculated to be- **0.1755787164.** Thus our objective function would be maximized based on the tolerance level: $0.00 \leq \tau < 9.60$. From Table 4.6, it could be seen that each risk tolerance level of $0 \leq \tau < 9.6$ is associated with different portfolio mean and different portfolio value-at-risk. The results produced a minimum portfolio mean return of 0.0178256289 with a minimum VaR of 0.012949619. This same risk tolerance interval also produced a maximum portfolio mean return of 0.0178256706 and a maximum VaR of 0.012949659. **(Refer Table 4.6)**

A plot of the portfolio mean returns against the value-at-risk gives us the efficiency and our maximum portfolio is expected to lie on this frontier. The plot is displayed in Figures 4.1

After successfully producing a set of efficient portfolios, the main business now is to find out the composition of weights that produced the optimum portfolio. Since investors always require a portfolio that yields maximum returns accompanied with the minimum possible risk, it is assumed for the purpose of this study that investors' preference is based solely on the returns and risk of a portfolio. Based on this assumption, we find our optimum portfolio by finding the ratio of the mean returns to the value-at-risks generated. The result of these ratios is plotted and displayed Figure 4.2. It is established based on our assumption that the portfolio with the maximum ratio is the preferred maximum portfolio. **(Refer Fig. 4.1 or 4.2)**

4.5 THE PORTFOLIO OPTIMIZATION PROCESS (ARMA - DCC GARCH)

The covariance matrix generated from the DCC-GARCH model is given as

$$\boldsymbol{\Omega} = \begin{bmatrix} 0.006970135 & 5.17588e-05 & -6.07616e-06 \\ 5.17588e-05 & 0.00079868 & -3.5122e-05 \\ -6.07616e-06 & -3.5122e-05 & 0.000557583 \end{bmatrix}$$

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The inverse of which is given as

$$\boldsymbol{\Omega}^{-1} = \begin{bmatrix} 143.5388585 & -9.258955654 & 0.980969074 \\ -9.258955654 & 1256.140819 & 79.02296563 \\ 0.980969074 & 79.02296563 & 1798.441997 \end{bmatrix}$$

The mean vector generated from the ARMA model is as below.

$$\boldsymbol{\mu} = \begin{bmatrix} 0.00271231 \\ 0.001439837 \\ 0.002469056 \end{bmatrix}$$

With the same percentile used as in the above, Table 4.10 a gives the generated weights according to their risk tolerance levels.

From Table 4.7, it could be seen that a risk tolerance interval of $0 \le \tau < 18.4$ was realised since tolerance levels from 18.40 and beyond produced negative weights. **(Refer Table 4.7)** The estimated portfolio optimization variables in the objective function are displayed in Table 4.8, together with their respective tolerance levels. From Table 4.8, it could be seen that each risk tolerance level of $0 \le \tau < 18.4$ is associated with different portfolio mean and different portfolio value-at-risk. The results produced a minimum portfolio mean return of 0.020788 with a minimum VaR of 0.002816. This same interval produced a maximum portfolio mean return of 0.024980 and a maximum VAR of 0.003675. **(Refer Table 4.8)** A plot of the portfolio the mean returns against the portfolio value-at-risk gives us the efficient frontier on which the maximum portfolio is expected to lie. The plot is displayed in **(Refer Figure 4.3).** Based on the same assumption stated above the optimum portfolio is obtained by finding the ratio of the mean returns to the value-at-risks generated. The result of these ratios is plotted and displayed in Figure 4.4. The portfolio with the maximum ratio is the preferred maximum portfolio as it is in the first situation. **(Refer Fig. 4.4)**

4.6 DISCUSSION OF FINDINGS

Based on the computations, it was found that the risk tolerance interval of $0 \le \tau < 9.60$ produced a minimum and maximum portfolio return of 0.0178256289 and 0.0178256706 respectively. The table also produced a minimum and maximum value-at-risk of 0.0129496191 and 0.0129496591 respectively. These values were obtained when the VaR was modelled with the univariate GARCH model. When the VaR modelled by the multivariate DCC GARCH was applied to the same optimization process, a risk tolerance interval of $0 \le \tau < 18.4$ was obtained. This interval produced a minimum and maximum value-at-risk of 0.002816 and 0.003675. In addition, a minimum and maximum portfolio return of 0.020788 and 0.024980 respectively were realised. Generally, from the univariate case, it is seen that there is a positive relationship between the tolerance level and the ratio of the mean return to VaR. An increase in the risk tolerance level results in a corresponding increase

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in the mean-to-VaR ratio. A sharp decline is experienced in the efficient frontier after the tolerance level of 7.60 after which slight successive increments are experienced but not to the magnitude at risk level 7.60. It is therefore believed, that the optimum portfolio lies on the tolerance level $\tau = 7.60$, which produced a mean portfolio of 0.0178256706 with an associated risk, measured by the value-at-risk of 0.012949619058910. In the multivariate case, the efficient frontier showed a positive relationship between the mean and the value-at-risk, with a gradual upward increment in trend. The graph of the mean-VaR ratio showed a gradual increase in trend to a peak value of 7.578596663 and then falls gradually over the entire interval of risk tolerance level. This peak ratio corresponds to a portfolio return of 0.021917843with an associated risk, measured by the VaR of 0.002892071. This occurred at a risk tolerance level of $\tau = 6.40$. Based on the calculation in the univariate case, the optimum portfolio produces weights of w_1 = 0.1755793646, w_2 = 0.7084977081, and w_3 = 0.1159229199 respectively. This implies that to obtain an optimum efficient portfolio, an investor must place 0.1755793646, 0.7084977081 and 0.1159229199 as fractions of whatever amount to be invested in the Equity Trust Fund, the Future Plan Fund and the Unit Trust Fund respectively. From the multivariate perspective, the weights generated for the efficient mean-VaR portfolio are $w_1 = 0.055210841$ placed on the Equity Trust Fund, $w_2 = 0.28244908$ on the Future Plan Fund and $w_3 = 0.66234008$ on the Unit Trust Fund respectively.

5. CONCLUSIONS AND RECOMMENDATIONS

CONCLUSION

The research revealed that the financial data deviated from the normal distribution because they had heavy-tailed. The data was modeled with the appropriate ARMA-GARCH and DCC GARCH models to produce estimates for the mean and variance of the funds that were studied. Based on the investor's risk tolerance level appropriate weights are generated to produce an efficient portfolio. The risk tolerance levels ranged from 0 to 9.60 for the univariate GARCH while that of the multivariate GARCH ranged from 0 to 18.40 . Levels from 9.60 and 18.40 respectively and beyond in both situations were not feasible since the produced negative weights. On the efficient frontier generated by the efficient portfolios lied the optimal portfolio and this was attained at risk tolerance level of $\tau = 7.60$ for the univariate and $\tau = 6.40$ for the multivariate. From the two models it is observed that the multivariate DCC GARCH model produced more efficient portfolio, which requires a comparatively less risk aversion constant or risk tolerance level. The

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optimum portfolio therefore is the one that places weights of 0.055210841, 0.28244908 and 0.66234008 as fractions of whatever amount to be invested in the Equity Trust Fund, the Future Plan Fund and the Unit Trust Fund respectively. This portfolio will produce an expected weekly return of approximately 2.192% and a weekly risk level of approximately 0.289% respectively.

RECOMMENDATION

The following are the recommendations made to fund managers, investors, financial analysts, stakeholders as well as policy makers;

It is recommended that:

1. The weekly logarithm returns of funds are modeled with the ARMA-DCC GARCH model.
2. The Mean-VaR should be employed in the optimization process instead of the usual Mean-Variance.
3. So much assumption should not be made on the distribution of the returns of funds and stocks but rather must be explored to know the exact distribution that they follow.

FURTHER STUDIES

The following research areas can be looked:

1. Extensions of the ARMA models as well as other multivariate GARCH models could be employed in the estimation of the mean and in modeling residuals.
2. The distribution of the error terms in the GARCH model could be modelled with stable distributions

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Table 4.3.1: Summary statistics of the funds

	Equity Trust	Future Plan	Unit Trust
Minimum	-0.050249	-0.036753	-0.164478
Maximum	0.055256	0.025880	0.012145
Mean	0.000715	0.002987	0.000518
Variance	0.000175	0.000052	0.000254
Standard deviation	0.013212	0.007228	0.015937
Skewness	0.276766	-0.418791	-7.834369
Kurtosis	3.438914	5.052938	69.837792

Source: HFC Bank (2009 - 2012)

Table 4.2 a: ARMA Models for the funds

	Akaike Information Criterion (AIC)		
	Equity Trust	**Future Plan**	**Unit Trust**
ARMA(1,1)	-1047.48	-1252.82	-945.93
ARMA(1,2)	-958.45	-1251.5	-958.45
ARMA(2,1)	-1044.81	-1251.55	-958.4

Table 4.3: GARCH Models for the funds

	Akaike Information Criterion (AIC)		
	Equity Trust	**Future Plan**	**Unit Trust**
GARCH(1,1)	-6.078459	-7.616262	-8.894389
GARCH (1,2)	-6.066539	-7.568044	-8.895592
GARCH (2,1)	-6.067144	-7.600056	-8.895592
GARCH(2,2)	-6.055584	-7.590923	-8.884031

Table 4.4: Estimates for mean and variance of funds

Fund	Mean	Variance
Equity Fund	0.00271231	0.001909301
Future Plan	0.001439837	0.000488596
Unit Trust	0.002469056	0.003011159

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Table 4.5: Portfolio weights per tolerance level (from univariate GARCH model)

T	**W_1**	**W_2**	**W_3**
0.00	0.1755787760	0.7084985795	0.1159226448
0.20	0.1755787762	0.7084985781	0.1159226448
0.40	0.1755787765	0.7084985780	0.1159226450
...	...	...	...
7.00	0.1755788852	0.7084984194	0.1159226954
7.20	0.1755789275	0.7084983545	0.1159227160
7.40	0.1755790235	0.7084982176	0.1159227589
7.60	0.1755793646	0.7084977081	0.1159229199
7.80	0.1755770257	0.7085010748	0.1159218357
8.00	0.1755784073	0.7084991166	0.1159224760
8.20	0.1755785649	0.7084988702	0.1159225480
8.40	0.1755786266	0.7084987991	0.1159225759
8.60	0.1755786596	0.7084987524	0.1159225906
8.80	0.1755786795	0.7084987168	0.1159226005
9.00	0.1755786935	0.7084986983	0.1159226064
9.20	0.1755787036	0.7084986849	0.1159226115
9.40	0.1755787108	0.7084986737	0.1159226148
9.60	-0.1755787164	-0.7084986621	-0.1159226174

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Table 4.6: Portfolio optimization variables (from univariate GARCH model)

τ	$\sum W_i$	$\hat{\mu}_p$	VaR_p	Max	$\hat{\mu}_p / VaR_p$
0.00	1	0.0178256604	0.012949629506351	-.012949629506351	1.376538255329090
0.20	1	0.0178256604	0.012949629485135	-.005819365326493	1.376538256717440
0.40	1	0.0178256604	0.012949629484432	0.001310898840590	1.376538257539180
7.00	1	0.0178256623	0.012949627590902	0.236609644807578	1.376538606152740
7.20	1	0.0178256630	0.012949626807897	0.243739920901361	1.376538745077570
7.40	1	0.0178256647	0.012949625177186	0.250870212795751	1.376539049122910
7.60	1	0.0178256706	0.012949619058910	0.258000574338763	1.376540154361020
7.80	1	0.0178256289	0.012949659056422	0.265130151566230	1.376532680003410
8.00	1	0.0178256540	0.012949635929093	0.272260827686992	1.376537075915620
8.20	1	0.0178256565	0.012949632905975	0.279391133311974	1.376537590380650
8.40	1	0.0178256578	0.012949632137776	0.286521419220887	1.376537775692310
8.60	1	0.0178256584	0.012949631582023	0.293651692932804	1.376537879793480
8.80	1	0.0178256587	0.012949631133424	0.300781961561382	1.376537948623980
9.00	1	0.0178256589	0.012949630916795	0.307912229907415	1.376537991639580
9.20	1	0.0178256591	0.012949630764611	0.315042497419975	1.376538023708020
9.40	1	0.0178256593	0.012949630628658	0.322172763387081	1.376538047093570
9.60	1	-0.017825659	....	....	....

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Table 4.7: Portfolio weights per tolerance level

τ	W_1	W_2	W_3
0.00	0.041537063	0.388960864	0.569502
0.20	0.041951306	0.385734118	0.572315
0.40	0.042365759	0.382505743	0.575128
...	...	...	...
5.2	0.052537396	0.303273855	0.644189
5.40	0.052977796	0.299843364	0.647179
5.60	0.053420155	0.296397614	0.650182
5.80	0.053864554	0.292935967	0.653199
6.00	0.054311078	0.289457774	0.656231
6.20	0.054759811	0.285962371	0.659278
6.40	0.055210841	0.28244908	0.66234
6.60	0.055664256	0.278917206	0.665419
6.80	0.056120148	0.275366038	0.668514
7.00	0.05657861	0.271794851	0.671627
7.20	0.057039738	0.268202898	0.674757
…..	…..	…..	…..
17.6	0.088610849	0.022279983	0.889109
17.8	0.089496999	0.015377325	0.895126
18	0.090402924	0.008320623	0.901276
18.2	0.091329601	0.001102281	0.907568
18.4	**0.092278072**	**-0.00628582**	**0.914008**

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Table 4.8: Portfolio optimization variables (DCC-GARCH)

τ	$\sum \mathbf{W_i}$	$\hat{\mu}_p$	$\mathbf{VaR_p}$	**Max**	$\hat{\mu}_p / \mathbf{VaR_p}$
0.00	1	0.020788	0.002816	-0.00282	7.381299309
0.20	1	0.020823	0.002816	0.005513	7.393080759
0.40	1	0.020857	0.002817	0.013869	7.404506983
...			...	...	...
5.2	1	0.021697	0.002866	0.222783	7.57054186
5.40	1	0.021733	0.00287	0.231851	7.572843968
5.60	1	0.02177	0.002874	0.240949	7.574764724
5.80	1	0.021807	0.002878	0.250079	7.576302304
6.00	1	0.021844	0.002883	0.25924	7.577454811
6.20	1	0.021881	0.002887	0.268432	7.578220278
6.40	1	0.021918	0.002892	0.277656	7.578596663
6.60	1	0.021955	0.002897	0.286913	7.578581849
6.80	1	0.021993	0.002902	0.296202	7.578173641
7.00	1	0.022031	0.002907	0.305524	7.577369765
7.20	1	0.022069	0.002913	0.314879	7.576167866
…..	…..	…..	…..	…..	…..
17.6	1	0.024677	0.003594	0.865029	6.866178531
17.8	1	0.02475	0.00362	0.87748	6.836957881
18	1	0.024825	0.003647	0.890047	6.806958194
18.2	1	0.024901	0.003675	0.902735	6.776161921
18.4	**1**	0.024980	….	….	……

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Figure 4.1: Efficient frontier (from univariate GARCH model)

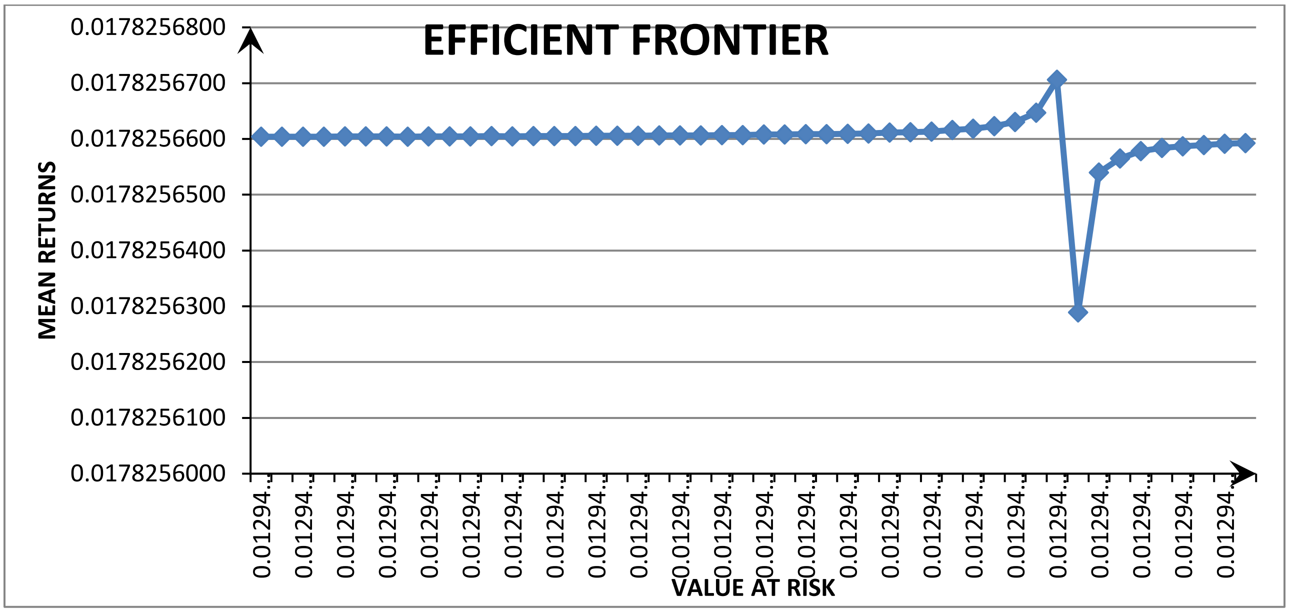

Figure 4.2: Mean-VaR Ratio (from univariate GARCH model)

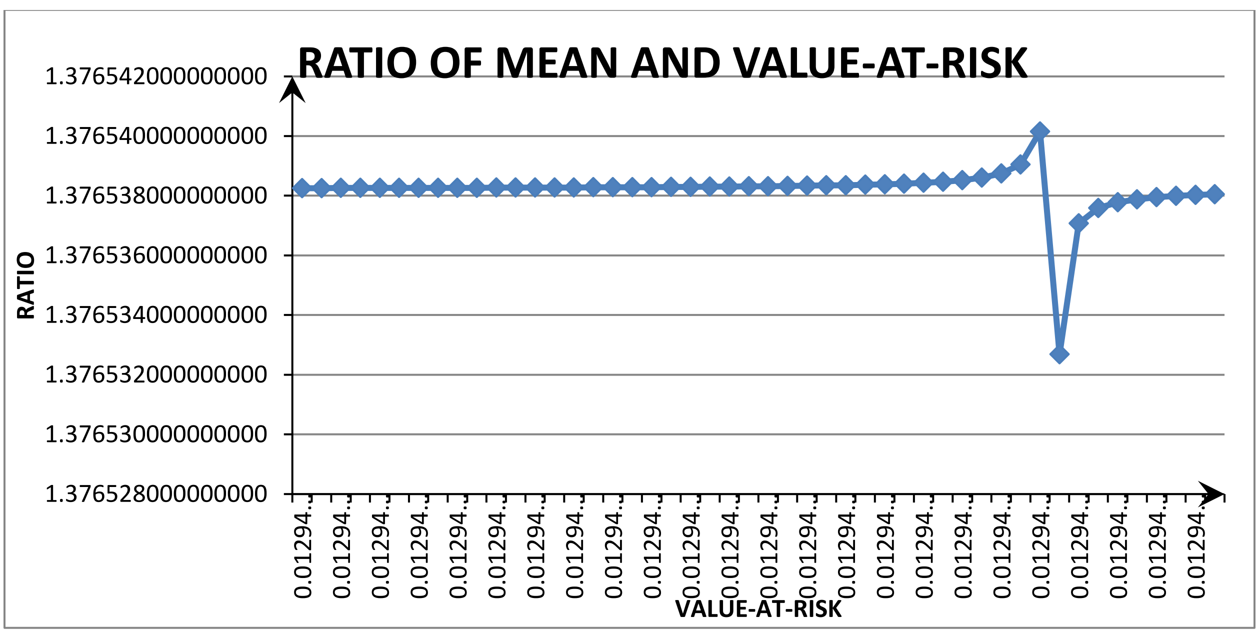

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Figure 4.3: Efficient frontier (from DCC GARCH model)

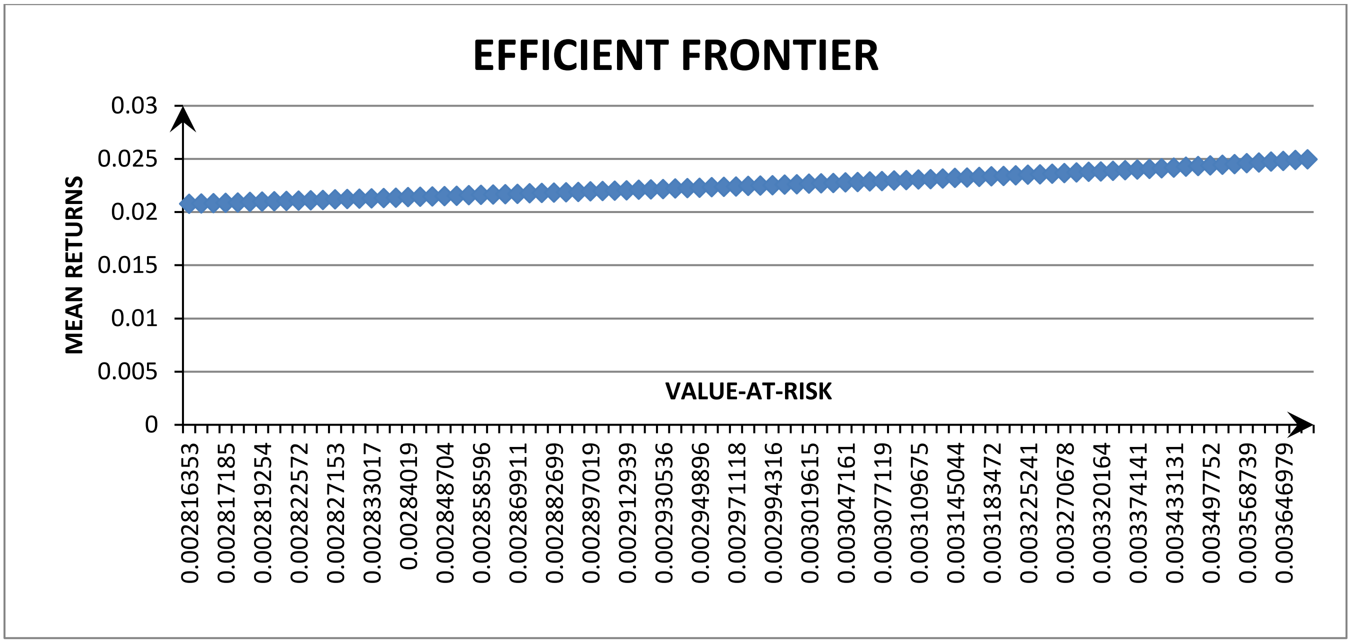

Figure 4.4: Mean-VaR ratio (from DCC GARCH model)