# A Comparison of GARCH-class Models and MIDAS Regression with Applications in Volatility Prediction and Value at Risk Estimation

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**Abstract**

We use GARCH(1,1), EGARCH and MIDAS regression to forecast weekly and monthly conditional variance of the OMXS30 equity index and USD/SEK exchange rate. Forecasts are compared with realized volatility and accuracy is evaluated using a Quasi-likelihood loss function and Diebold Mariano test. We estimate normal and t-distributed Value at Risk using forecasted conditional variances and evaluate these estimates using Likelihood Ratio tests for unconditional coverage and temporal independence. We show that MIDAS regression outperforms both GARCH-class models in forecast accuracy, while the difference between GARCH(1,1) and EGARCH varies between data and frequency. Findings suggest that GARCH-class models underestimate conditional variance and react slowly to shocks, producing temporally dependent Value at Risk exceptions for some data. The superiority of MIDAS regression in the variance forecasting problem has implications for option pricing and risk management in the financial sector.

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# Part I
# Introduction

Difficulties in application of traditional econometric models for financial data are consequences of often observed volatility clustering and non-normality. Data exhibits time varying error term variance and thus the homoscedasticity assumption of econometric analysis is violated. Besides this stylized fact of financial data, modelling the mean of such data has proven difficult due to lack of serial dependence. The presence of volatility clustering suggests dependencies of higher moments that can be modelled using other approaches than the traditional of conditional mean modelling. Engle (1982) introduced the autoregressive conditional heteroscedasticity (ARCH) model to capture these dependence structures, which was later generalized by Bollerslev (1986). Another approach to modelling financial data is the more recent Mixed Data Sampling (MIDAS) regression, introduced by Ghysels, Santa-Clara and Valkanov (2004). This type of regression aims to make use of dependency structures found in data measured at higher frequencies. A distributed lag polynomial maps the high frequency data to a desired lower frequency.

This paper applies these models on the currency pair USD/SEK between 1994 and 2014 and the Swedish OMXS30 equity index between 1986 and 2014 to forecast weekly and monthly conditional variance or volatility[1]. We also examine the stylized fact that a model utilizing more information produces more accurate forecasts. We will not consider the temporal aggregation schemes for GARCH-class models proposed by Drost and Nijman (1993) or any naive methods such as scaling high frequency predictions or increasing the out-of-sample prediction horizon. Instead we consider GARCH-class models a low frequency approach and MIDAS regression a high frequency approach to the variance forecasting problem.

In earlier research, Hansen and Lunde (2005) compare the out of sample predictive ability of a large variety of GARCH-class models on the D-mark/USD exchange rate and IBM stock data using intraday observations and find no evidence that GARCH(1,1) is outperformed by more sophisticated specifications. Alper, Fendoglu and Saltoglu (2008) compare the predictive ability of GARCH(1,1) and MIDAS regression for weekly stock market volatility of four developed and ten emerging market economies. They conclude that MIDAS significantly outperforms GARCH for four of the emerging markets, with less conclusive results for the developed markets, suggesting that MIDAS is superior for more volatile data.

We evaluate models in two steps, firstly by how accurate they are at forecasting conditional variance. Forecasts are compared with realized volatility (Barndorff-Nielsen and Shepard, 2002) using a Quasi-likelihood loss function (Patton, 2009) and Diebold Mariano test for predictive accuracy, introduced by Diebold and Mariano (1995). Secondly we test validity of confidence intervals for returns, Value at Risk, estimated using forecasted conditional variances with a normal and t-distributional assumption. We use the Kupiec test (Kupiec, 1995) for testing equality of nominal and empirical tail size (unconditional coverage). We also use a likelihood ratio test for temporal independence of Value at Risk exceptions proposed by Christoffersen (1998). A joint hypothesis version of these tests is also presented (Jorion, 2007).

[1] Conditional variance and volatility are used interchangeably in this paper.

# Part II
# Theoretical framework

## 1 Models

### 1.1 ARCH/GARCH

The autoregressive conditional heteroscedasticity (ARCH) model was developed by Engle (1982) for use in time series econometrics where the assumption of homoscedasticity is violated. The approach gained recognition for the application possibilities to financial markets and use for modelling variance of returns where the data typically shows signs of volatility clustering. The presence of volatility clustering means that periods with high variance are typically followed by high variance and vice versa (Mandelbrot, 1967). Because of this, conditional variance can be modelled by fitting weights for lagged values of itself.

Returns are calculated as

$$r_t = ln\Big(\frac{P_t}{P_{t-m}}\Big) \tag{1}$$

where $P_t$ is price at time $t$ and $m = 1$ for daily, $m = 5$ for weekly and $m = 22$ for monthly. We assume that returns can be modelled as

$$r_t = \mu_t + \varepsilon_t \tag{2}$$

where $\mu_t = 0$ for all $t$ and

$$\varepsilon_t = \sigma_t z_t. \tag{3}$$

The conditional variance is then modelled as

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2, \tag{4}$$

where and $\alpha_0$ and $\alpha_1$ are estimated parameters, $\sigma_t$ is conditional standard deviation and $z_t$ is an independent and identically distributed random variable with zero mean and unit variance. In this thesis we consider both a standard normal and t-distributed specification. Using the t-specification, degrees of freedom in a modified t-distribution are also estimated.[2]

A more generalized model was developed by Bollerslev (1986) for modelling conditional variance. It is denoted GARCH(p,q) where q is the ARCH term specifying the number of autoregressive lags and p is the GARCH term specifying the number of moving average lags.

The GARCH(p,q) model is given by

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2, \tag{5}$$

with the following restrictions

$$\alpha_0 > 0$$
$$\alpha_i > 0$$
$$\beta_i > 0$$
$$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1,$$

[2] See Section 3, Value at Risk.

the first three ensuring non-negative conditional variance and the last for covariance stationarity. We can also solve for the unconditional variance by rewriting the expression and obtaining for a GARCH(1,1)

$$\sigma^2 = \frac{\alpha_0}{1-(\alpha_1+\beta_1)}. \tag{6}$$

Model parameters are obtained using maximum likelihood estimation. This is typically done on intra-day, daily or weekly data to capture the volatility clustering in data of such frequency.

## 1.2 Exponential GARCH

In this paper we use a second GARCH model for comparison known as Exponential GARCH (EGARCH) proposed by Nelson (1991). As it models the logarithm of conditional variance we do not have to impose the same restrictions on the parameters to ensure non-negativity. The EGARCH model also allows for reactions from different types of shocks by including a third parameter, $\gamma_i$. Doing so, EGARCH takes into consideration whether past returns are positive or negative.

$$ln(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{q} \alpha_i \left( \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| - E\left( \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| \right) \right) + \sum_{j=1}^{p} \beta_j ln(\sigma_{t-j}^2) + \sum_{i=1}^{q} \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}}. \tag{7}$$

We also need to comment on that $E\left( \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| \right)$ depends on what error distribution we assume. If the distribution is normal we have

$$E\left( \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| \right) = \sqrt{\frac{2}{\pi}}$$

and if t-distributed

$$E\left( \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| \right) = \sqrt{\frac{\nu-2}{\pi}} \frac{\Gamma\left(\frac{\nu-1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)}$$

with $\nu > 2$ degrees of freedom. $\Gamma(\nu)$ is the gamma function defined by

$$\int_0^\infty x^{\nu-1} e^{-x} dx.$$

## 1.3 MIDAS Regression

A recent competitor to autoregressive models for forecasting conditional variance is the Mixed Data Sampling regression introduced by Ghysels et al. (2004). MIDAS stems from distributed lag models and makes it possible to estimate a model using variables measured at different frequencies. By using lagged conditional variance measured at a higher frequency MIDAS utilizes information which would otherwise be smoothed out when aggregating to the desired forecasting time scale, as is a common approach with any autoregressive model. In order to achieve a parsimonious model while using a large number of high frequency lags a functional form for the lag structure is used. In this paper we use two different lag structures to map the high frequency data to the low frequency data.

Consider the following regression where $t = 1, 2, ..., T$ is the low frequency time index and regressor $x$ is observed $m$ times during each period $t$. In this paper $m = 5$ is used for weekly forecasts and $m = 22$ for monthly forecasts, based on number of trading days during each period.

$$y_t = c + \beta B(L^{1/m}, \theta) x_{t-1}^{(m)} + \varepsilon_t \tag{8}$$

where $\beta$ is a common coefficient for all high frequency lags with the following intuition

$$\hat{y} = \beta_1 x_1 + \beta_2 x_2 + ... + \beta_K x_K = \beta'(w_1 x_1 + w_2 x_2 + ... + w_K x_K). \tag{9}$$

$B(L^{1/m}, \theta)$ is a polynomial lag operator which allows for fractional lags

$$L^{1/m} x_t = x_{t-1/m} \tag{10}$$

and

$$B(L^{1/m}, \theta) = \Sigma_{k=1}^{K} w(k, \theta) L^{k-1/m} \tag{11}$$

where $K$ is the number of lags and $w(\bullet)$ is some weighting function.

As weighting functions we use the Exponential Almon lag polynomial, based on Almon lags (Almon, 1965), and the Beta lag polynomial (Ghysels et al. 2004), based on the beta distribution. The Exponential Almon lag polynomial is constructed in the following way

$$w(k, \theta) = \frac{exp\{\theta_1 k + ... + \theta_Q k^Q\}}{\Sigma_{k=1}^{K} exp\{\theta_1 k + ... + \theta_Q k^Q\}}, \tag{12}$$

where $Q$ is the degree of the lag polynomial and thus the number of parameters that have to be estimated. For this structure to make sense we want $Q$ to be much smaller than $K$ which provides a reduction in the number of estimated parameters by $K - Q$.

The second weighting function, the Beta lag polynomial is constructed as follows

$$B(k, \theta_1, \theta_2) = \frac{f(\frac{k}{K}, \theta_1, \theta_2)}{\Sigma_{k=1}^{K} f(\frac{k}{K}, \theta_1, \theta_2)} \tag{13}$$

where

$$\frac{k}{K} = x$$

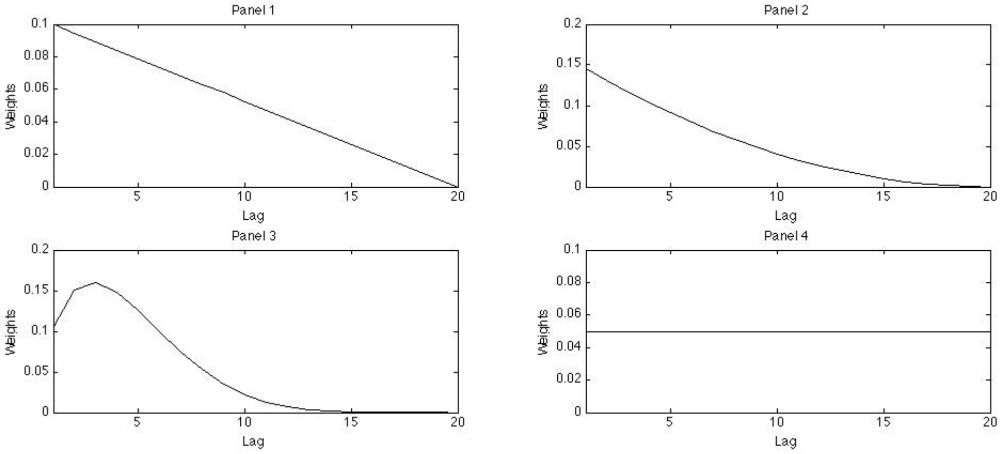
$$0 < x \leq 1$$

and

$$f(x, \theta_1, \theta_2) = \frac{\Gamma(\theta_1 + \theta_2)}{\Gamma(\theta_1)\Gamma(\theta_2)} x^{\theta_1 - 1}(1 - x)^{\theta_2 - 1} \tag{14}$$

Figure 1 shows the flexibility of the beta density in setting weights for 20 lags using only two parameters. Panel 1 shows $\theta_1 = 1$ and $\theta_2 = 2$, producing a linear decline in weight. In Panel 2, $\theta_1 = 1$ and $\theta_2 = 3$, the weight decline is non-linear. The hump shape in Panel 3 is created by $\theta_1 = 2$ and $\theta_2 = 7$. Panel 4 shows the equal weight structure obtained by $\theta_1 = \theta_2 = 1$.

**Figure 1: Examples of Beta Densities**




Notes: The figure shows examples of different beta densities for the lag structure

Choice of functional form for lag structure and number of lags is purely data driven and set as to minimize a common loss function. In this thesis we use between 10 and 24 high frequency lags. In addition to high frequency lags we also use between one and two low frequency lags as regressors. For model estimation and forecasting we are using the MIDAS regression toolbox v1.1 for MatLab (Qian, 2014). Parameters are estimated using non-linear least squares.

# 2 Forecasting Methodology

To produce forecasts we use a rolling window approach where observations between $t-w$ and $t$ are used to estimate parameters for the three competing models. The estimated model gives a single one step ahead forecast for period $t+1$ which is stored as the first entry in a vector of length $N$, the out of sample prediction period. We then iterate the procedure and estimate parameters using observations $t-w+1$ through $t+1$, where the last observation is the realization and not the prediction from the first iteration. This approach enables the capturing of any shifts in volatility regime. We do not make any arguments regarding the feasibility of re-estimation frequency in any practical application. We report mean QL from the best specification of both GARCH-class models, be it normal or t-distributed. Findings suggest that differences in QL are very small between these specifications and reported test results hold for all combinations.

Choosing the size of estimation window, $w$, is a crucial part of obtaining good forecasts. There are two major components to this choice. We want the estimated parameters to have large sample properties for reliable and accurate forecasts. At the same time, a too large $w$ will smooth out any shifts in volatility regime and structure. We also want to keep $w$ small enough to yield out of sample forecasts that have large sample properties. The last issue is purely a matter of data quantity and is solved through the usage of long time series. For evaluation purposes the competing models are estimated using the same calendar period. It is safe to assume that any model produces better forecasts during certain volatility regimes, and by keeping the estimation period equal across models we ensure that no unfair advantage is given to any model. In this thesis we use a rolling window length 500 weeks for the estimation of weekly conditional variance. For monthly OMXS30 we use $w = 90$ months and for USD/SEK $w = 200$ months. It is worth noting however, that the information usage between GARCH-class and MIDAS models is not comparable as the latter uses a higher frequency of input information.

# 3 Value at Risk

Value at risk (VaR) has become a common approach for calculating and assessing the risk of loss of financial positions by an estimation of the worst loss over a time-horizon at a given level of confidence. If for example the daily 5 per cent VaR of a portfolio is 50 million SEK, we expect the losses to exceed that amount no more often than one in 20 days.

Value at risk has its origin from the 1990s when managers in the banking industry had a need for a metric that quantified risk and VaR became the most used method. One of the reasons for the breakthrough is that it allows for measurements of minimal capital requirements for coverage of market risks. This has been widely reinforced by the introduction of the Basel II accord which states the international standard of capital coverage for financial institutions. The idea is that these institutions should use a generally acceptable method for measuring market risk and also the size of the potential loss for exposure of such risk in order to maintain enough capital for negative events.

The definition of VaR can expressed in terms of probabilities as

$$Pr(L > VaR) \leq (1-\alpha) \tag{15}$$

where $\alpha$ is the level of confidence. To find the VaR that satisfies Equation (15) one can either use the empirical unconditional return distribution or make a distributional assumption. In the case of assuming a distribution, literature suggests fat-tails such as the t-distribution (Jorion, 2007). In this paper we use the normal and t-distribution for one-step ahead predictions of VaR at the 5 per cent and 1 per cent level.

We calculate normal distribution VaR as

$$VaR_t = -\sqrt{\hat{\sigma}_t^2} * \pi_\alpha \tag{16}$$

where we forecast $\hat{\sigma}_t^2$, and $\pi_\alpha$ is the number of standard deviations giving a nominal tail size of 5 and 1 per cent, that is 1.645 and 2.329 respectively. If $r_t < VaR_t$ we have a VaR exception at time $t$.

When modelling conditional variance with a t-distributional assumption and GARCH-class models a conditional return distribution is estimated. It is a modified t-distribution with two parameters, $\nu$ and $\sigma_t^2$. In the regular t-distribution the variance is a function of the degrees of freedom

$$\sigma^2 = \frac{\nu}{\nu-2},$$

but the reparameterization introduced by Bollerslev (1987) gives a conditional return distribution, $f_\nu(r_t|\Psi_{t-1})$, with degrees of freedom equal to $\nu$, variance $\sigma_t^2$, and $\Psi_{t-1}$ being the information set available at time $t-1$. The density function is

$$\Gamma\Big(\frac{\nu+1}{2}\Big)\Gamma\Big(\frac{\nu}{2}\Big)^{-1}\Big((\nu-2)\pi\sigma_t^2\Big)^{-1/2}\Big(1+r_t^2\sigma_t^{-2}(\nu-2)^{-1}\Big)^{-(\nu+1)/2} \tag{17}$$

with finite variance for $\nu > 2$.

As $1/\nu \to 0$, the density $f_\nu(r_t|\Psi_{t-1})$ converges to a normal distribution with variance $\sigma_t^2$ but for $1/\nu > 0$ it has more probability mass in the tails.

The one step ahead predictions of conditional variances yield equally many conditional densities which we numerically integrate to obtain a vector $\Lambda$ that satisfies Equation (18) for all $t$ for a given nominal tail size $\alpha$, be it 1 or 5 per cent.

$$\int_{-\infty}^{\Lambda_t}\Gamma\Big(\frac{\nu+1}{2}\Big)\Gamma\Big(\frac{\nu}{2}\Big)^{-1}\Big((\nu-2)\pi\sigma_t^2\Big)^{-1/2}\Big(1+r_t^2\sigma_t^{-2}(\nu-2)^{-1}\Big)^{-(\nu+1)/2}dr_t = \alpha. \tag{18}$$

For a correctly specified model, $\Lambda_t$ is the negative return exceeded with probability $P(r_t < \Lambda_t) = \alpha$ at time $t$ and therefore the number of periods where $r_t < \Lambda_t$ must converge to $(\alpha \times T)$ as $T \rightarrow \infty$.

The calculation of $\Lambda$ is performed using a brute force algorithm with an error acceptance of .001, giving VaR-measures that at most deviate from the stated probability by .1 percentage units. When calculating t-distributed VaR using conditional variance forecasts from MIDAS regression the degrees of freedom, $\nu$, are still estimated using a GARCH-t(1,1) model, replacing only $\sigma_t^2$ in Equations (17) and (18) with MIDAS forecasts.

# 4 Forecast Evaluation

As with every forecasting procedure, evaluation methods are essential for further use of the model and the underlying assumptions. This paper evaluates models in two steps, predictive accuracy and Value at Risk validity. In this section we introduce measures and tests used to evaluate and compare predictive accuracy of the models.

## 4.1 Realized Volatility

As conditional variance is unobservable even ex post a realization proxy is necessary. We use realized volatility which is computed as follows.

$$RV_t = \sum_{i=1}^{m} r_i^2 \tag{19}$$

where $RV_t$ is realized volatility for period $t$ and $r_i^2$ is squared log-return measured $m$ times during period $t$. The literature suggests using intraday squared returns as it has been shown that $RV$ is a consistent estimate of conditional variance as $m \rightarrow \infty$. However, using finer time increments for the summation introduces other biases due to market microstructures such as bid-ask spread and discrete pricing (Barndorff-Nielsen and Shepard, 2002). Due to data availability we are using squares of daily log-returns observed 5 times per week and 22 times per month.

## 4.2 Quasi-likelihood loss function

Patton (2009) identifies a class of loss functions for evaluating forecast errors. We use averages of the Quasi-likelihood loss function due to two conducive features.

$$QL_t = \frac{RV_t}{\hat{\sigma}_t^2} - ln\frac{RV_t}{\hat{\sigma}_t^2} - 1 \tag{20}$$

and

$$\overline{QL} = \frac{1}{N}\Sigma_{t=1}^{N} QL_t$$

Firstly, $QL$ produces identically and independently distributed loss series under the assumption that the model is correctly specified. This is useful as model misspecification can be identified by serial correlation in the loss series. It also has the quality of producing consistent rankings of models regardless of what volatility proxy is being used, assuming the proxy is unbiased. When comparing models a simple average of the $QL$-series is computed, $\overline{QL}$. It is worth noting that $\overline{QL}$ is a point estimate of forecast accuracy and the difference between loss series averages need therefore be tested formally.

### 4.3 Diebold Mariano Test

The DM-test for predictive accuracy proposed by Diebold and Mariano (1995) handles a wide range of loss functions and can be used for both model and non-model based predictions, assuming large sample properties of the loss differential. We use a version of the test that assumes that loss differentials, $d$, are serially uncorrelated. The following hypotheses are used

$$H_0 : \overline{QL}_{model1} = \overline{QL}_{model2}$$
$$H_a : \overline{QL}_{model1} \neq \overline{QL}_{model2}$$

and the test statistic

$$DM = \frac{\bar{d}}{\sqrt{\frac{\hat{\sigma}_d^2}{N}}} \sim N(0,1) \tag{21}$$

where

$$\overline{d} = \overline{QL}_{model1} - \overline{QL}_{model2}$$

rejecting the null for all $|DM| > 1.96$

# 5 Value at Risk Evaluation

One way of assessing the accuracy of a VaR procedure is to count the frequency by which the portfolio losses exceed the VaR estimate. For a correctly specified model the exception rate should converge to the stated probability as the sample size increases. This count can be summarized as a Bernoulli trial that follows a binomial distribution with two outcomes which is expressed by

$$f(x) = \binom{N}{y} p^y (1-p)^{N-y}. \tag{22}$$

When the sample is increased the binomial distribution will converge to a normal distribution by the rules of the central limit theorem and the function gets the following expression

$$z = \frac{y - pN}{\sqrt{p(1-p)N}}. \tag{23}$$

### 5.1 Kupiec Test

Performing the formal Kupiec test (Kupiec, 1995), we state the following hypotheses

$$H_0 : p = p_0$$

$$H_a : p \neq p_0$$

with the test statistic

$$LR_{UC} = -2ln\Big[(1-p_0)^{N-y} p_0^y\Big] + 2ln\Big[\Big(1-(y/N)\Big)^{N-y} (y/N)^y\Big], \tag{24}$$

where $p_0$ is the nominal tail size and $p$ the empirical tail size. $N$ is the prediction sample size and $y$ is the number of tail observations in the sample. With a sufficiently large sample, this test statistic follows a $\chi^2$ distribution with one degree of freedom under the null hypothesis. Given that $LR_{UC}$ is larger than 3.841 we reject the null hypothesis on a 5 per cent significance level. The test assesses whether the empirical tail size is significantly different from the nominal tail size, or lack of unconditional coverage of the VaR estimates. It is worth noting that the test does not distinguish between over and under estimation of tail end cut-off and the statistic should be viewed in combination with the empirical tail size for useful interpretation.

## 5.2 Likelihood Ratio Test for Independence

Further, in order for VaR to be adequately specified and useful, VaR exceptions should be independent over time. The likelihood ratio test for independence tests temporal correlation of exceptions. An indicator vector of length $N$ is established where a VaR exception at time $t$ gives a value 1 and 0 otherwise. The following test statistic then follows a $\chi^2$ distribution with one degree of freedom under the null that VaR exceptions are temporally independent (Christoffersen, 1998),

$$LR_{IND} = -2ln\left[(1-\pi)^{(N_{00}+N_{10})}\pi^{(N_{01}+N_{11})}\right] + 2ln\left[(1-\pi_0)^{N_{00}}\pi_0^{N_{01}}(1-\pi_1)^{N_{10}}\pi_1^{N_{11}}\right], \tag{25}$$

where $N_{ij}$ is the number of observations on which j occurs given that i occurred on previous day.

The proportions are calculated as

$$\pi = \frac{no.of exceptions}{N}$$

$$\pi_1 = \frac{N_{11}}{no.of exceptions}$$

$$\pi_0 = \frac{N_{01}}{N - no.of exceptions}.$$

We combine these these two tests as

$$LR_{CC} = LR_{UC} + LR_{IND} \tag{26}$$

which is $\chi^2$ distributed with two degrees of freedom under the joint null hypothesis that the nominal and empirical tail size are not significantly different and exceptions are temporally independent (Jorion, 2007).

# Part III
# Data

We compare the conditional variance forecasting abilities of the three models using two different sets of data for two periods of time. The OMXS30 is a value weighted equity index of the 30 largest stocks in Sweden and we use daily observations between 1986 and 2014. This gives a total of 7260 daily return observations. We also use the exchange rate of USD/SEK between 1994 and 2014 with a total of 5220 daily return observations. The return series are calculated using Equation (1) with $m = 1$ for daily, $m = 5$ for weekly and $m = 22$ for monthly returns.

Looking at the descriptive statistics in Table 1, it is clear that data are not normal and hence in line with the literature regarding financial return distributions. Excess kurtosis indicates fat tailed distributions with higher tail-probability compared to the normal distribution. Non-zero skewness indicates that the return distributions are skewed. The Jarque-Bera test rejects the null hypothesis of normally distributed returns for both data sets. These results are illustrated in the OMXS30 histogram plots with a normal curve fit in Figure 2.

The plot of the Daily Returns for OMXS30 in Figure 3 shows signs of volatility clustering as is expected for financial return data, suggesting that the homoscedasticity assumption does not hold. The clustering means that periods with high volatility tend to be followed by high volatility and vice versa (Mandelbrot, 1967). This is also the case for the weekly and monthly returns.[3]

The volatility clustering phenomena is also visible in the daily return plots for the USD/SEK series in Figure 5. This is also true for the weekly and monthly return series of the exchange rate.[4] We have a similar issue with the distribution as with the OMXS30 return data. Non-zero skewness indicates that the empirical distribution is skewed and excess kurtosis indicates that a fat tailed distribution would be a better fit.

**Table 1: Descriptive Statistics of Daily Returns**

| Index | Mean | St.Dev | Skewness | Kurtosis | JB-probability |
|---|---|---|---|---|---|
| OMXS30 | 4.561e-04 | 0.015 | 0.087 | 7.763 | 0.000 |
| USD/SEK | -4.996e-05 | 0.007 | -0.169 | 6.561 | 0.000 |

The table shows the mean,standard deviation, skewness, kurtosis and Jarque-Bera test for normality for the daily returns of the OMXS30 and USD/SEK indices. For normally distributed returns, skewness is expected to be 0, kurtosis 3 and JB-probability $> 0.05$.

[3] Appendix, Figure 7 and 8.

[4] Appendix, Figure 12 and 13.

**Figure 2: OMXS30 Histogram of Daily Returns**

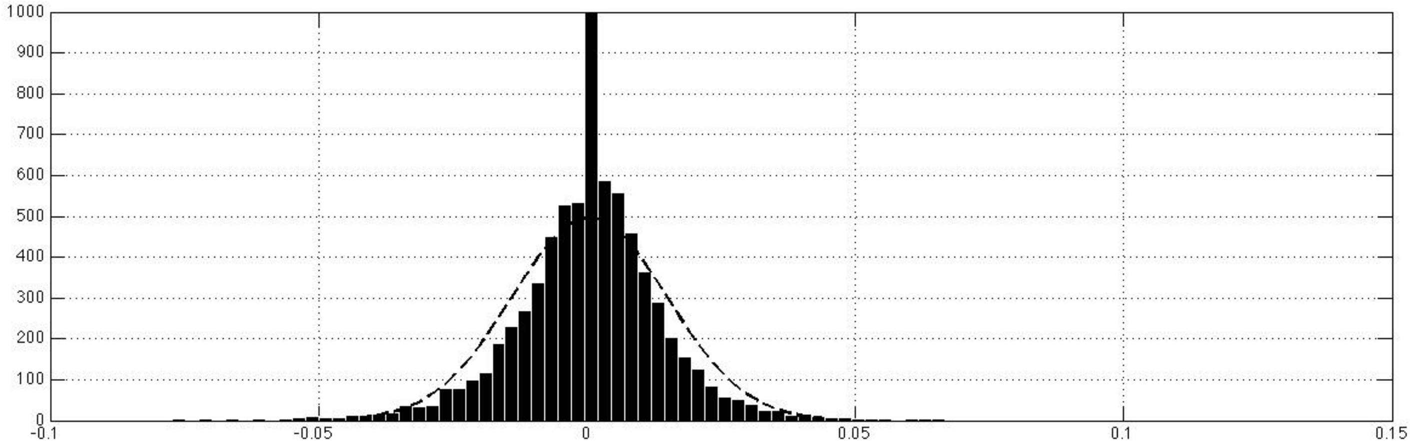



**Figure 3: OMXS30 Daily Returns 1986-2014**

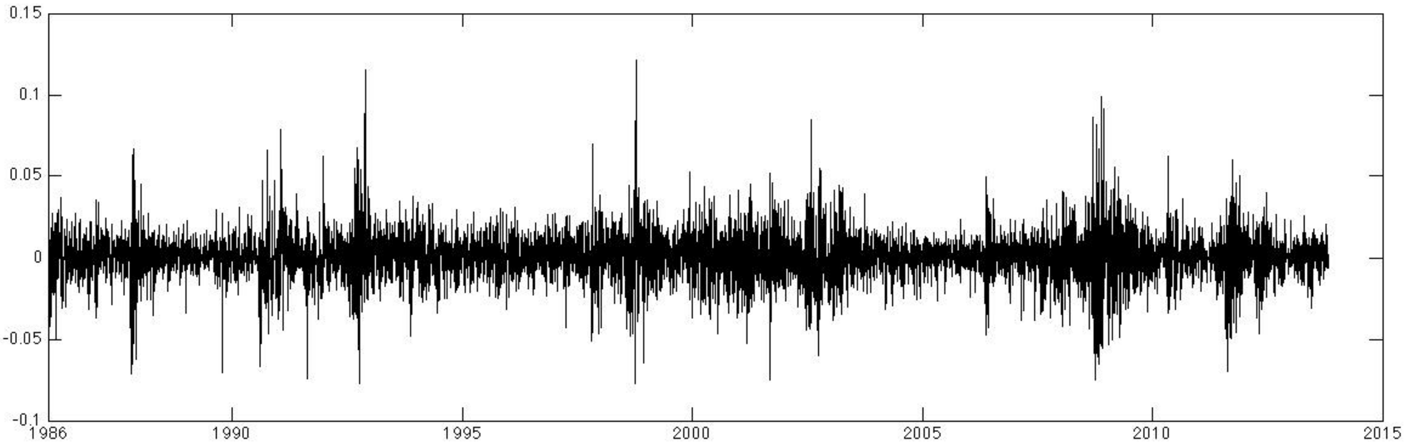


**Figure 4: USD/SEK Histogram of Daily Returns**

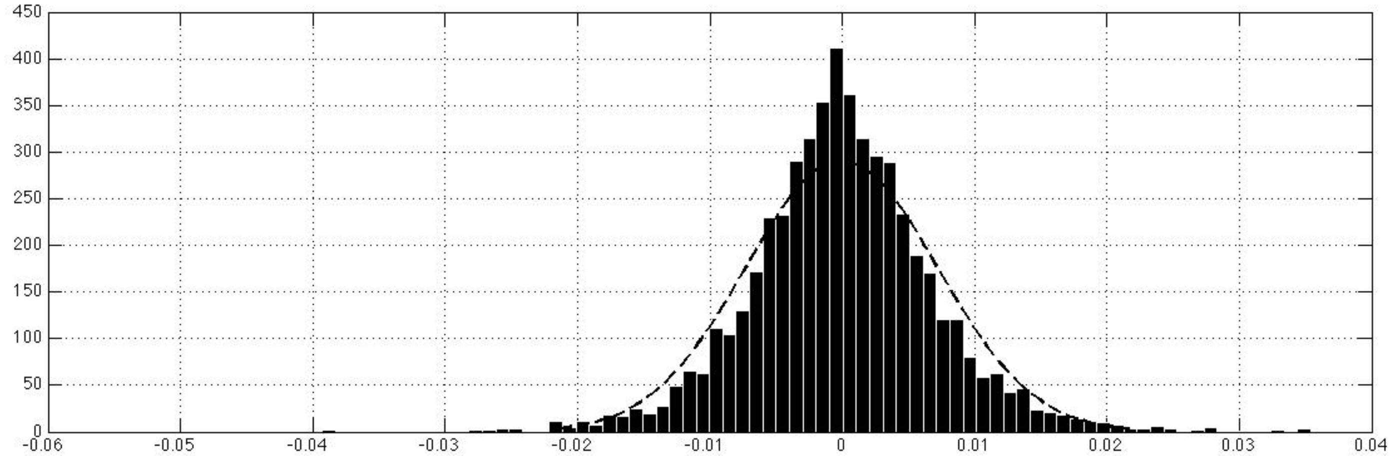
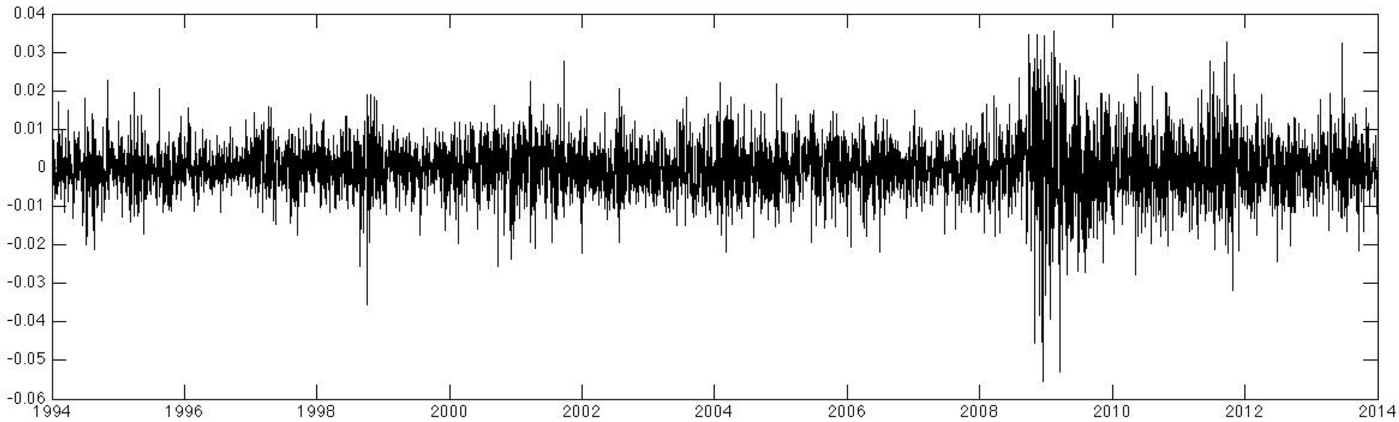



**Figure 5: USD/SEK Daily Returns USD/SEK 1994-2014**



# Part IV
# Results

## 6 OMXS30, Weekly Estimation

Looking at QL in Table 2 we see that for weekly OMXS30 data MIDAS outperforms both GARCH-class models while EGARCH outperforms GARCH.[5] GARCH-class models seem to do an adequate job at forecasting during low volatility periods but fail to quickly adjust to spikes in volatility. MIDAS on the other hand captures spikes in volatility but with a slight delay.[6]

In Table 3 we see that five per cent VaR with conditional variance from the MIDAS regression significantly overestimates tail size of the conditional return distribution for both distributional specifications. Both GARCH-class models are sufficiently close to five per cent for both distributions, however, we reject temporal independence of VaR exceptions.

For one per cent VaR, MIDAS regression performs well with both distributions, while we reject unconditional coverage for all GARCH-class specifications, except EGARCH-t. The EGARCH-t model performs well for 1 per cent VaR but does not forecast conditional variance as well as MIDAS.

**Table 2: VaR exception proportion & QL**

OMXS30 Weekly VaR

| | 5% | | 1% | | QL |
|---|---|---|---|---|---|
| | *Normal* | *Student-t* | *Normal* | *Student-t* | |
| MIDAS | 0.035 | 0.033 | **0.009** | **0.006** | 0.391 |
| GARCH | 0.047 | 0.052 | 0.018 | 0.021 | 0.463 |
| E-GARCH | 0.047 | 0.054 | 0.017 | **0.016** | 0.423 |

The table shows the proportion of exceptions for various specifications of VaR. The results in bold indicate that all three tests for unconditional coverage, independence and the joint hypothesis are not significant.

**Table 3: Likelihood ratio tests for unconditional coverage & independence of VaR**

OMXS30 Weekly

| | | 5% | | | 1% | | |
|---|---|---|---|---|---|---|---|
| | | $LR_{UC}$ | $LR_{IND}$ | $LR_{CC}$ | $LR_{UC}$ | $LR_{IND}$ | $LR_{CC}$ |
| MIDAS | *Normal* | 6.310 (0.012) | 2.880 (0.089) | 9.190 (0.010) | 0.212 (0.645) | 0.175 (0.675) | 0.387 (0.824) |
| | *Student-t* | 6.914 (0.008) | 2.090 (0.148) | 9.004 (0.011) | 1.514 (0.218) | 0.076 (0.782) | 1.590 (0.451) |
| GARCH | *Normal* | 0.152 (0.697) | 4.295 (0.038) | 4.447 (0.108) | 4.813 (0.028) | 0.605 (0.437) | 5.418 (0.066) |
| | *Student-t* | 0.043 (0.836) | 5.129 (0.024) | 5.172 (0.075) | 8.851 (0.003) | 0.859 (0.354) | 9.710 (0.007) |
| E-GARCH | *Normal* | 0.242 (0.623) | 5.319 (0.021) | 5.561 (0.062) | 4.103 (0.043) | 2.712 (0.010) | 6.815 (0.033) |
| | *Student-t* | 0.250 (0.617) | 5.576 (0.018) | 5.826 (0.054) | 0.638 (0.424) | 0.480 (0.488) | 1.118 (0.572) |

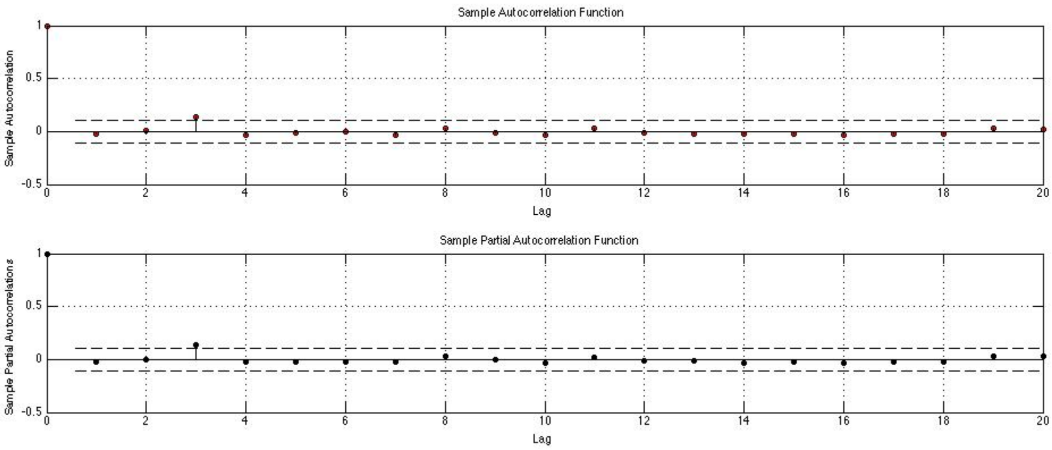
The table presents the tests for unconditional coverage ($LR_{UC}$), tests for independence ($LR_{IND}$) and the joint hypothesis test ($LR_{CC}$). P-values in parentheses.

[5] Appendix, Table 10.
[6] Appendix, Figure 17-19.

# 7 OMXS30, Monthly Estimation

The autocorrelation and partial autocorrelation plot in Figure 6 illustrates the correlation between squared return and lagged squared return for 20 lags, without and with consideration to previous lag correlation respectively. There is a clear lack of serial correlation and as we do not have an ARCH effect, the estimation of GARCH parameters is impossible for this frequency and forecasts converge to the unconditional variance in the estimation window. As MIDAS uses squared daily returns with high serial correlation this is not an issue. However, as we cannot estimate the degrees of freedom in the conditional return distributions we can only evaluate VaR-estimates based on the normal distribution.

**Figure 6: ACF & PACF for OMXS30 monthly squared returns**




Notes: The figure shows autocorrelation and partial autocorrelation of OMXS30 squared monthly returns for 20 lags.

With MIDAS forecasts, both 5 and 1 per cent VaR are significantly different from the stated probabilities, indicating that the normal distribution is not a good fit. The out of sample prediction of 240 is not enough to give a single 1 per cent VaR exception, and only one exception for the 5 per cent VaR. GARCH-class models have low predictive accuracy due to failing to estimate parameters, but Value at Risk estimation using unconditional variance is valid for both tail sizes.

**Table 4: VaR exception proportion & QL**

| OMXS30 Monthly | | | | | |
|---|---|---|---|---|---|
| | 5% | | 1% | | QL |
| | *Normal* | *Student-t* | *Normal* | *Student-t* | |
| MIDAS | 0.0042 | - | 0.000 | - | 0.236 |
| GARCH | **0.033** | - | **0.013** | - | 0.486 |
| E-GARCH | **0.033** | - | **0.013** | - | 0.486 |

The table shows the proportion of exceptions for various specifications of VaR. The results in bold indicate that all three tests for unconditional coverage, independence and the joint hypothesis are not significant.

**Table 5: Likelihood ratio tests for unconditional coverage & independence of VaR**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| OMXS30 Monthly | | | | | | | |
| | | 5% | | | 1% | | |
| | | $LR_{UC}$ | $LR_{IND}$ | $LR_{CC}$ | $LR_{UC}$ | $LR_{IND}$ | $LR_{CC}$ |
| MIDAS | *Normal* | 17.553 (0.000) | 0.008 (0.929) | 17.561 (0.000) | 4.824 (0.028) | - | - |
| | *Student-t* | - | - | - | - | - | - |
| GARCH | *Normal* | 1.582 (0.208) | 0.542 (0.462) | 2.124 (0.346) | 0.140 (0.708) | 0.076 (0.783) | 0.216 (0.898) |
| | *Student-t* | - | - | - | - | - | - |
| E-GARCH | *Normal* | 1.582 (0.208) | 0.542 (0.462) | 2.124 (0.346) | 0.140 (0.708) | 0.076 (0.783) | 0.216 (0.898) |
| | *Student-t* | - | - | - | - | - | - |

The table presents the tests for unconditional coverage ($LR_{UC}$), tests for independence ($LR_{IND}$) and the joint hypothesis test ($LR_{CC}$). P-values in parentheses.

# 8 USD/SEK, Weekly Estimation

For weekly USD/SEK data MIDAS regression outperformes both GARCH and EGARCH in volatility forecasting, while the difference between GARCH-class models is not significant.[7] MIDAS captures the full extent of large shocks in volatility while GARCH-class models barely react.[8] The VaR estimation results show that MIDAS gives a normal VaR exception rate of 4.79 per cent where 5 is expected and 0.74 per cent where 1 is expected. The results for t-distribution VaR are identical due to the t-distribution converging to normal as the degrees of freedom are estimated to 200.

The Kupiec, Christoffersen and joint null hypotheses cannot be rejected, suggesting that we do not have a significant deviation between nominal and empirical tail size. These results favour all models suggesting that there is no over- or under-estimation of the risk.

**Table 6: VaR exception proportion & QL**

| USD/SEK Weekly | | | | | |
|---|---|---|---|---|---|
| | 5% | | 1% | | QL |
| | *Normal* | *Student-t* | *Normal* | *Student-t* | |
| MIDAS | **0.048** | **0.048** | **0.007** | **0.007** | 0.128 |
| GARCH | **0.053** | **0.056** | **0.009** | **0.006** | 0.227 |
| E-GARCH | **0.056** | **0.056** | **0.005** | **0.004** | 0.215 |

The table shows the proportion of exceptions for various specifications of VaR. The results in bold indicate that all three tests for unconditional coverage, independence and the joint hypothesis are not significant.

**Table 7: Likelihood ratio tests for unconditional coverage & independence of VaR**

| USD/SEK Weekly | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | 5% | | | 1% | | |
| | | $LR_{UC}$ | $LR_{IND}$ | $LR_{CC}$ | $LR_{UC}$ | $LR_{IND}$ | $LR_{CC}$ |
| MIDAS | *Normal* | 0.052 (0.820) | 0.052 (0.820) | 2.621 (0.263) | 0.418 (0.518) | 0.060 (0.806) | 0.478 (0.787) |
| | *Student-t* | 0.052 (0.820) | 0.052 (0.820) | 2.621 (0.263) | 0.418 (0.518) | 0.060 (0.806) | 0.478 (0.787) |
| GARCH | *Normal* | 0.130 (0.718) | 3.280 (0.070) | 3.410 (0.182) | 0.035 (0.852) | 0.093 (0.760) | 0.128 (0.938) |
| | *Student-t* | 0.305 (0.581) | 3.518 (0.061) | 3.823 (0.148) | 1.311 (0.252) | 0.033 (0.855) | 1.344 (0.511) |
| E-GARCH | *Normal* | 0.305 (0.581) | 3.518 (0.061) | 3.8213(0.148) | 1.311 (0.252) | 0.033 (0.855) | 1.344 (0.511) |
| | *Student-t* | 0.305 (0.581) | 3.518 (0.061) | 3.8213(0.148) | 2.887 (0.089) | 0.015 (0.903) | 2.902 (0.234) |

The table presents the tests for unconditional coverage ($LR_{UC}$), tests for independence ($LR_{IND}$) and the joint hypothesis test ($LR_{CC}$). P-values in parentheses.

[7] Appendix, Table 11.
[8] Appendix, Figure 21-23.

# 9 USD/SEK, Monthly Estimation

The results from the monthly data for USD/SEK show problems arising from trying to estimate models using too few data points. MIDAS regression clearly outperforms in forecasting volatility, with a mean QL close to half that of the GARCH-class models. Any application in VaR is however difficult to evaluate. The finding that normal and t-distributed VaR perform equally well are due to the fact that GARCH models estimate the degrees of freedom to 200 for all $t$, making the t-distribution converge to normal. We also do not reject any hypotheses regarding independence or unconditional coverage but we keep in mind that the tests have low power due to the small sample size.

**Table 8: VaR exception proportion & QL**

USD/SEK Monthly VaR

| | 5% | | 1% | | QL |
|---|---|---|---|---|---|
| | *Normal* | *Student-t* | *Normal* | *Student-t* | |
| MIDAS | **0.020** | **0.020** | **0.000** | **0.000** | 0.119 |
| GARCH | **0.020** | **0.020** | **0.000** | **0.000** | 0.227 |
| E-GARCH | **0.020** | **0.020** | **0.000** | **0.000** | 0.213 |

The table shows the proportion of exceptions for various specifications of VaR. The results in bold indicate that all three tests for unconditional coverage, independence and the joint hypothesis are not significant.

**Table 9: Likelihood ratio tests for unconditional coverage & independence of VaR**

USD/SEK Monthly

| | | 5% | | | 1% | | |
|---|---|---|---|---|---|---|---|
| | | $LR_{UC}$ | $LR_{IND}$ | $LR_{CC}$ | $LR_{UC}$ | $LR_{IND}$ | $LR_{CC}$ |
| MIDAS | *Normal* | 1.277 (0.258) | 0.041 (0.840) | 1.318 (0.517) | 1.025 (0.311) | - | - |
| | *Student-t* | 1.214 (0.271) | 0.042 (0.838) | 1.256 (0.534) | 1.005 (0.316) | - | - |
| GARCH | *Normal* | 1.214 (0.271) | 0.042 (0.838) | 1.256 (0.534) | 1.005 (0.316) | - | - |
| | *Student-t* | 1.214 (0.271) | 0.042 (0.838) | 1.256 (0.534) | 1.005 (0.316) | - | - |
| E-GARCH | *Normal* | 1.214 (0.271) | 0.042 (0.838) | 1.256 (0.534) | 1.005 (0.316) | - | - |
| | *Student-t* | 1.214 (0.271) | 0.042 (0.838) | 1.256 (0.534) | 1.005 (0.316) | - | - |

The table presents the tests for unconditional coverage ($LR_{UC}$), tests for independence ($LR_{IND}$) and the joint hypothesis test ($LR_{CC}$). P-values in parentheses.

## Part V
# Discussion

This paper assesses the conditional variance forecasting performance of three different models and their usage for VaR. In a short conclusion, EGARCH and GARCH are very similar for all data and frequencies except weekly estimation of OMXS30 conditional variance where EGARCH performs significantly better. MIDAS regression makes use of information contained in higher frequency data as the model is significantly better at forecasting for all examined data and frequencies. MIDAS construction allows for capturing variance shocks to a larger extent which is an advantage for all types of data and especially when data is scarce.

The fact that GARCH-class models perform poorly during high variance periods is further supported by the result of weekly OMXS30 VaR estimates. While the proportion of exceptions are adequately close to the stated 5 per cent, exceptions based on GARCH-class forecasts are temporally dependent, producing a higher frequency of exceptions during high volatility periods. This result indicates a forecasting issue rather than erroneous conditional distribution assumptions. MIDAS on the other hand significantly overestimates tail size, even with the assumption of normally distributed conditional returns. From a risk management perspective however, this is not as bad as an underestimation of tail size. For 1 per cent VaR MIDAS performs well, while we reject unconditional coverage for all GARCH-class models except EGARCH-t.

As for the results of the monthly forecast and VaR of OMXS30 we find that GARCH-class model estimation fails due to lack of serial correlation in aggregated data. This might also be a sample size issue. One could argue that the model comparison becomes unfair and that the GARCH aggregation scheme (Drost and Nijman, 1993) is a more appropriate approach. Some problems arise when evaluating VaR based on MIDAS due to the small out of sample prediction period. With only 220 predictions we expect few tail observations and tests regarding these have low power. MIDAS does however forecast conditional variance well, even a relatively small estimation window of 90 months. The results for VaR are the opposite, as the unconditional variance from GARCH models gives more accurate estimates, indicating a discrepancy between accurate variance forecasts and accurate VaR estimation.

The result that all models and specification are valid for USD/SEK weekly VaR is surprising. The unconditional distribution of daily returns is clearly not normal. GARCH-class models estimate the degrees of freedom for the conditional distributions giving t-distributions that converge to normal in terms of VaR exception rate. Conditional return distributions seem close to normal for this data and frequency, given that conditional variance forecasts are accurate. Evaluation of monthly VaR for USD/SEK suffers from small sample issues, the 5 per cent VaR is valid for all models but results from 1 per cent VaR is inconclusive.

The hypothesis that MIDAS regression is more accurate in forecasting conditional variance is supported by both data sets and frequencies while results from VaR estimation are not as clear. For further studies it would be interesting to make the same comparison using a temporal aggregation scheme for GARCH-class models as proposed by Drost and Nijman (1993) to avoid the smoothing of dependencies found in daily data. This could also solve the small sample issues for monthly data in this paper.

# Part VI
# References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*http://www.mathworks.com/matlabcentral/fileexchange/45150-midas-regression*

## Part VII
# Appendix

**Figure 7: OMXS30 Weekly Returns**

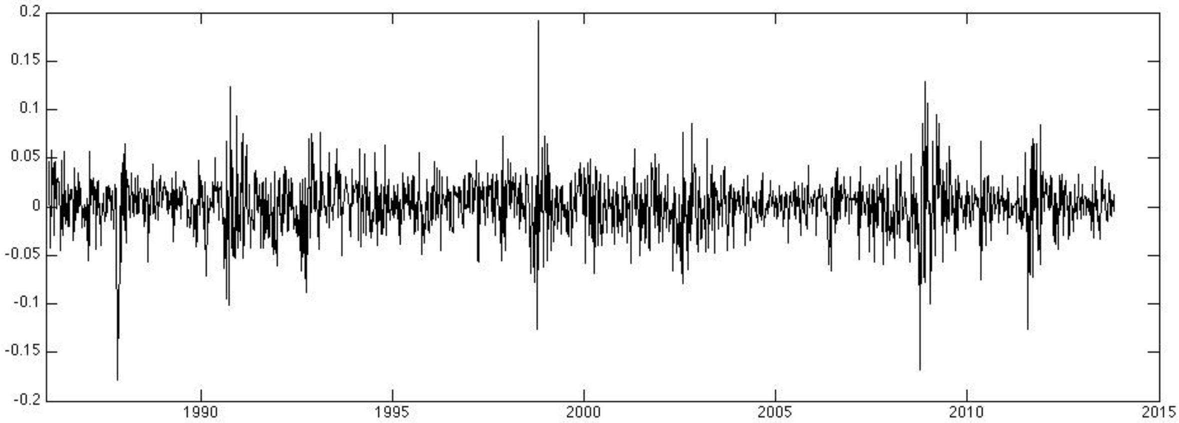



**Figure 8: OMXS30 Monthly Returns**

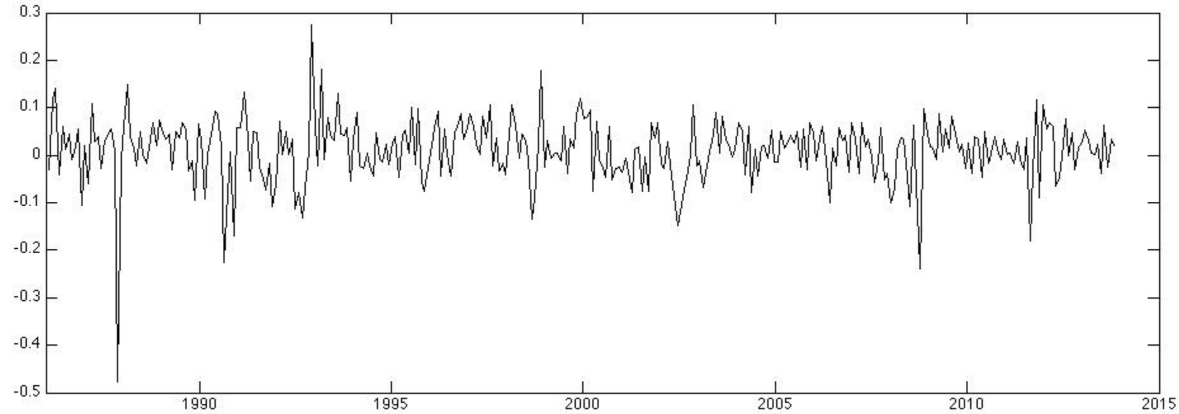


**Figure 9: OMXS30 Weekly Realized Volatiliy**

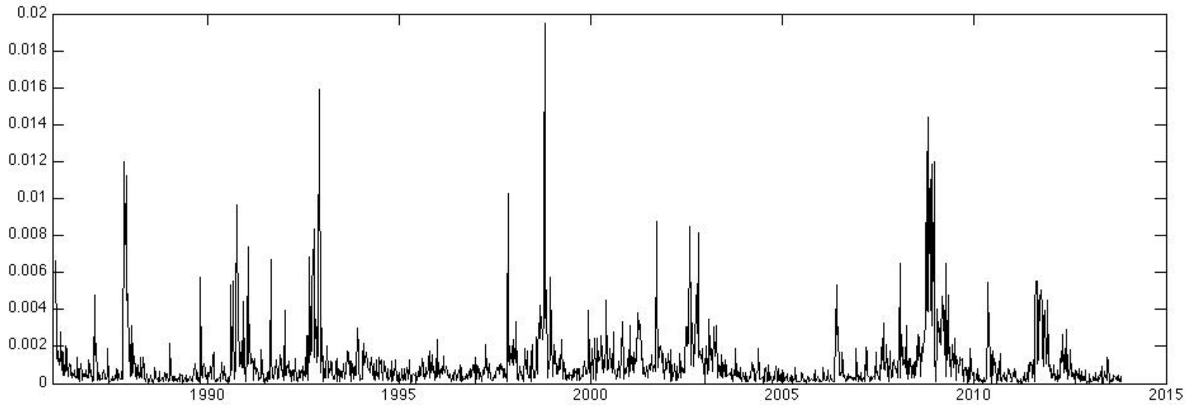



**Figure 10: OMXS30 Monthly Realized Volatiliy**

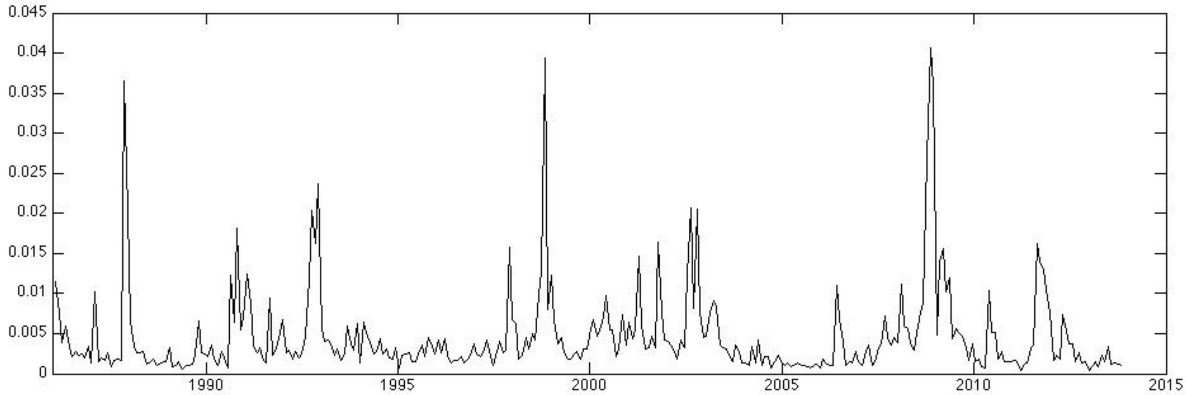


**Figure 11: OMXS30 Histogram Weekly Returns**

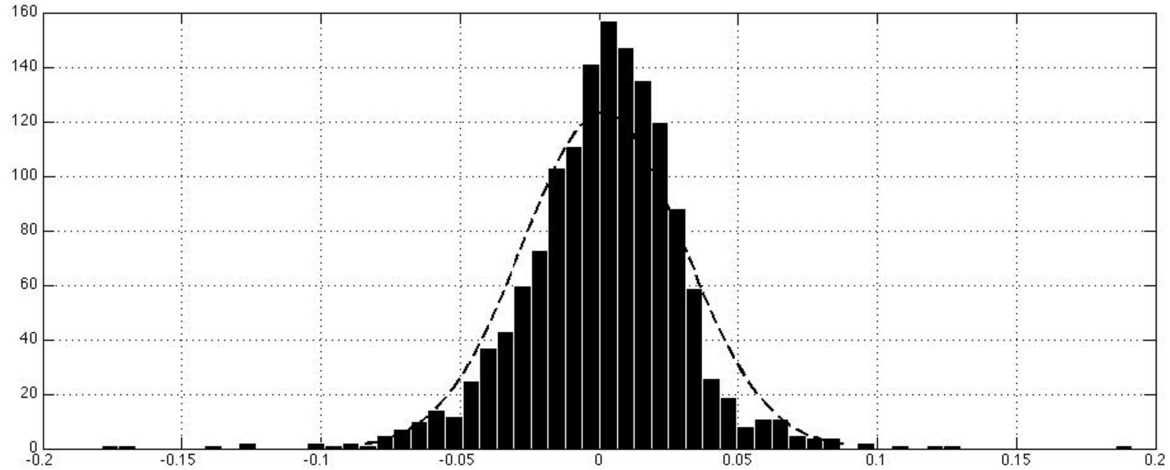



**Figure 12: USD/SEK Weekly Returns**

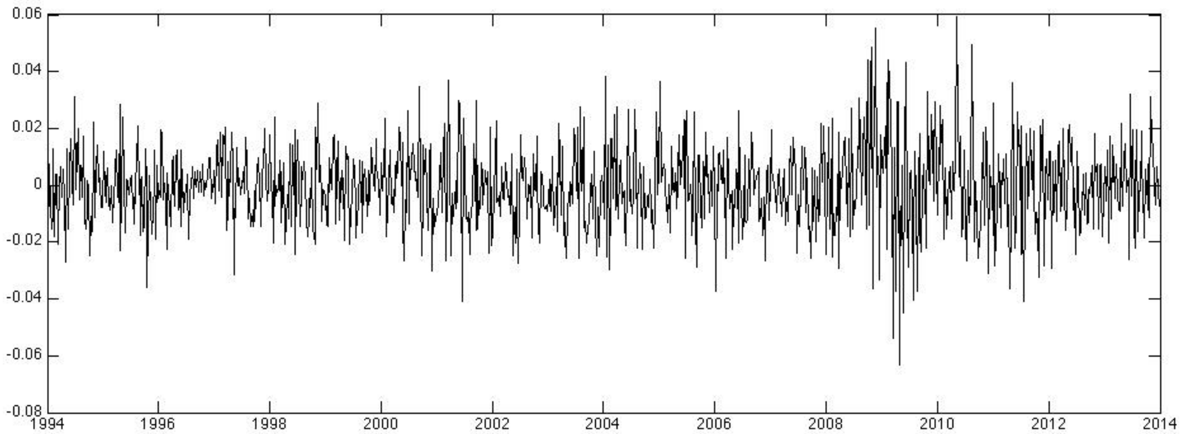


**Figure 13: USD/SEK Monthly Returns**

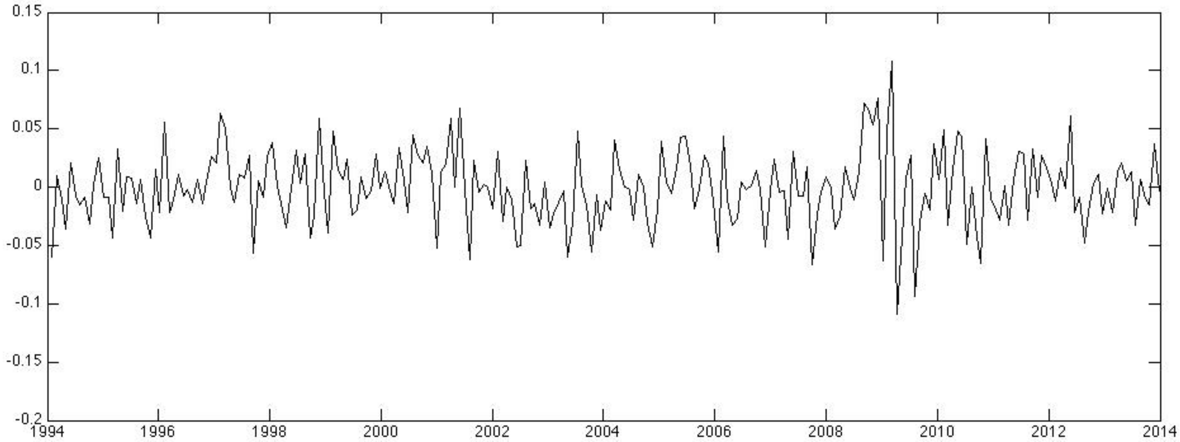



**Figure 14: USD/SEK Weekly Realized Volatility**

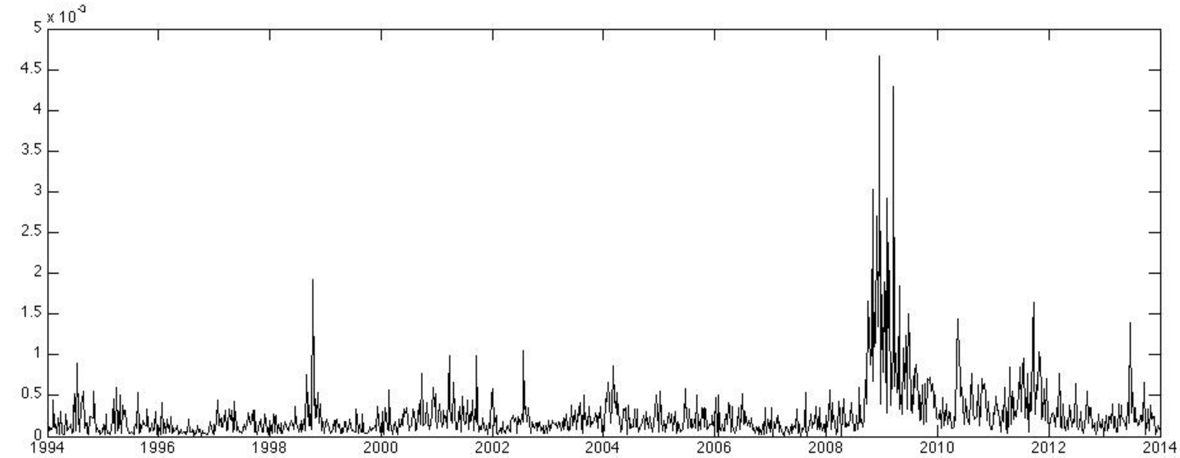


**Figure 15: USD/SEK Monthly Realized Volatility**

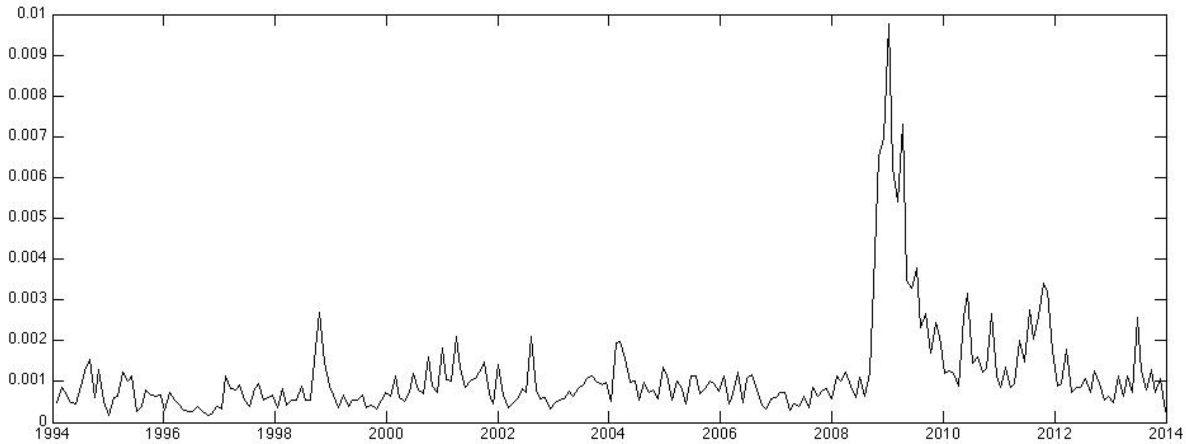
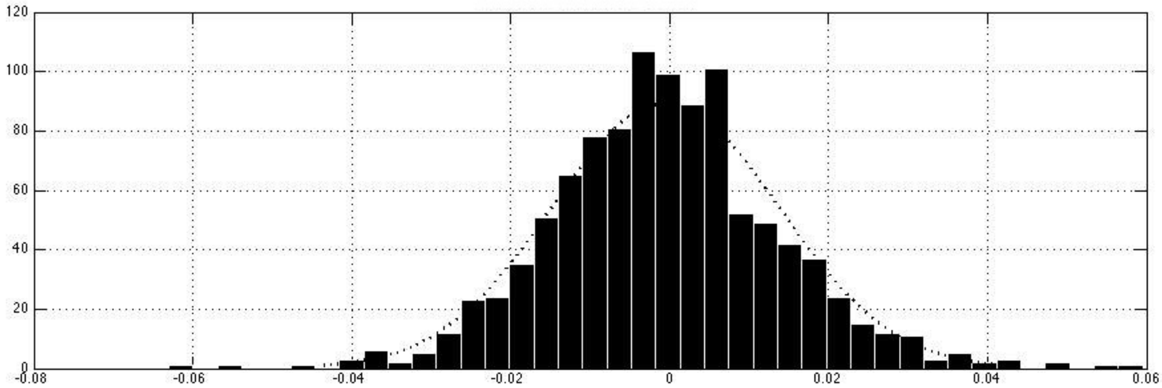
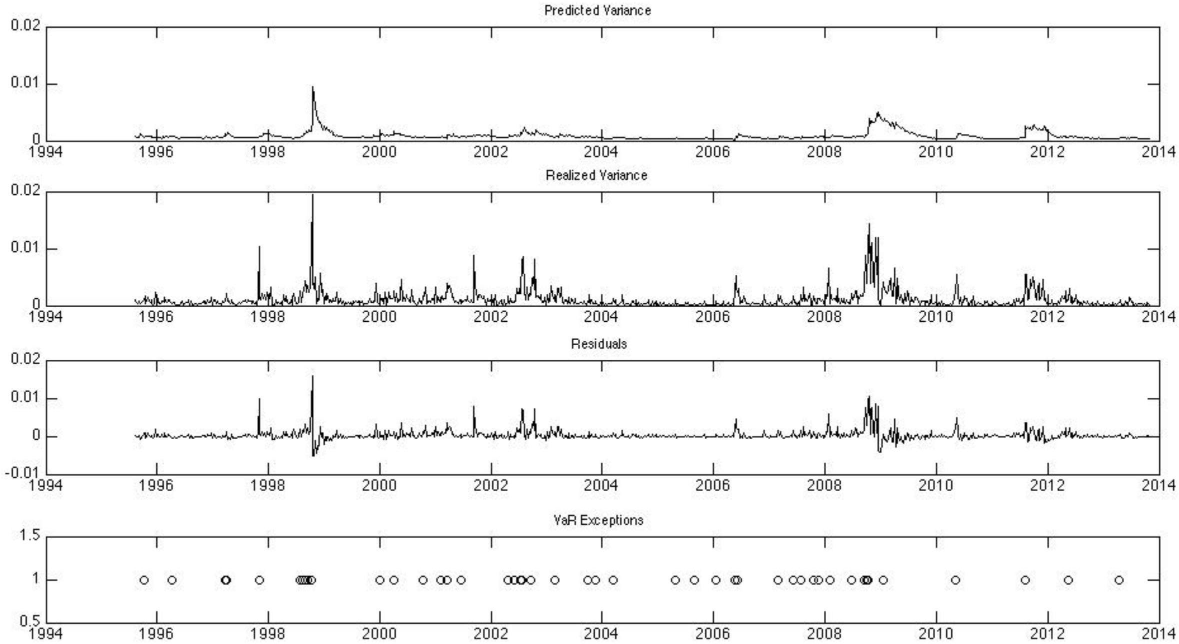



**Figure 16: USD/SEK Histogram of Weekly Returns**



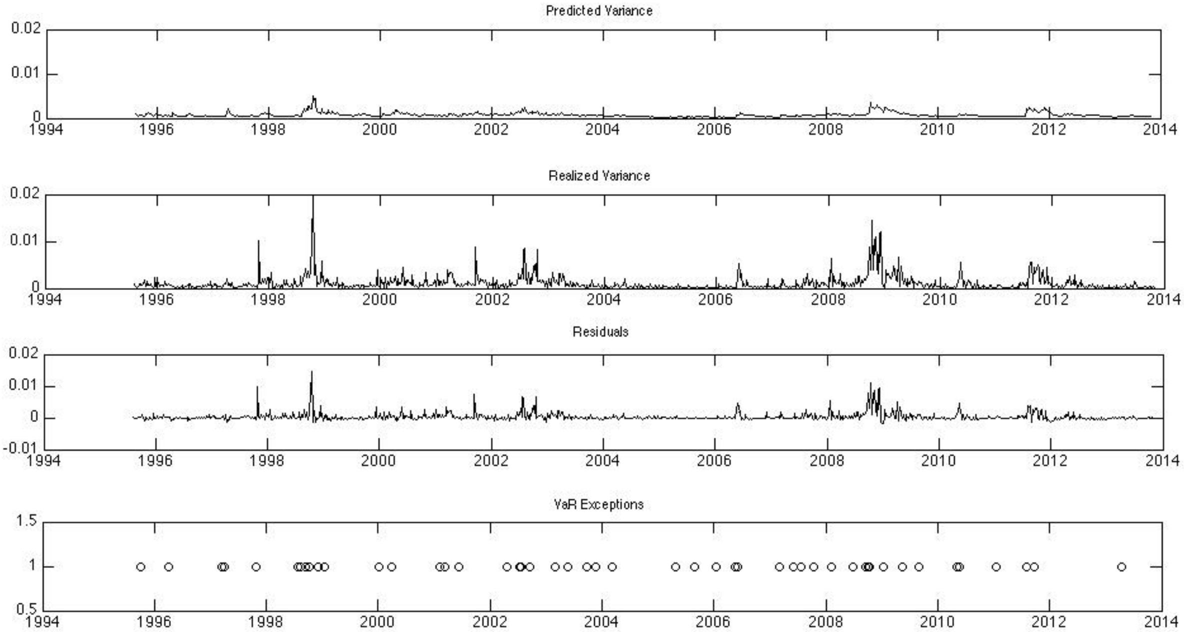
# 11 Forecasts, realizations and residuals

**Figure 17: OMXS30 weekly variance estimated with GARCH(1,1)**



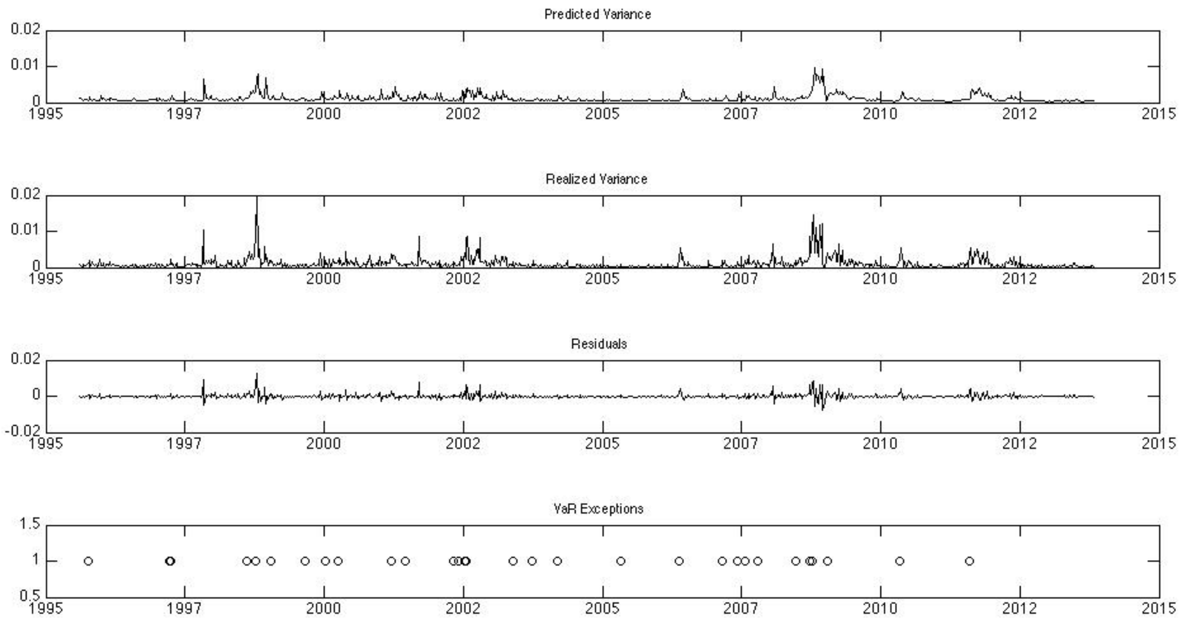

Notes: The plot visually compares predicted variance with the realized for a GARCH(1,1) on the OMXS30 Weekly estimated variance. The residuals between the predicted variance and the realized are also plotted. The plot of VaR exceptions show during which periods we get exceptions and suggests that exceptions are clustered at periods with large variance.

**Figure 18: OMXS30 weekly variance estimated with EGARCH(1,1)**



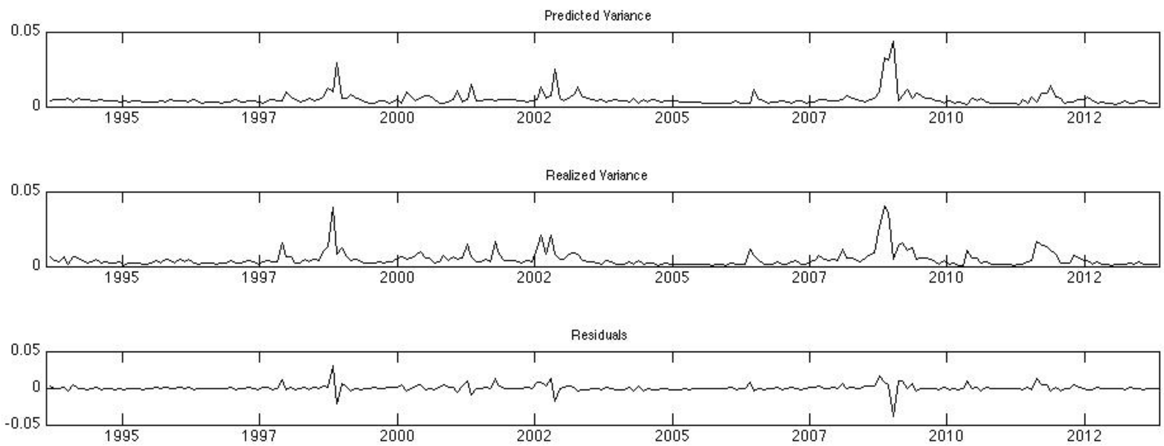

Notes: The plot visually compares predicted variance with the realized for a EGARCH(1,1) on the OMXS30 Weekly estimated variance. The residuals between the predicted variance and the realized are also plotted. The plot of VaR exceptions show during which periods we get exceptions and suggests that exceptions are clustered at periods with large variance.

**Figure 19: OMXS30 weekly variance estimated with beta lag MIDAS regression**



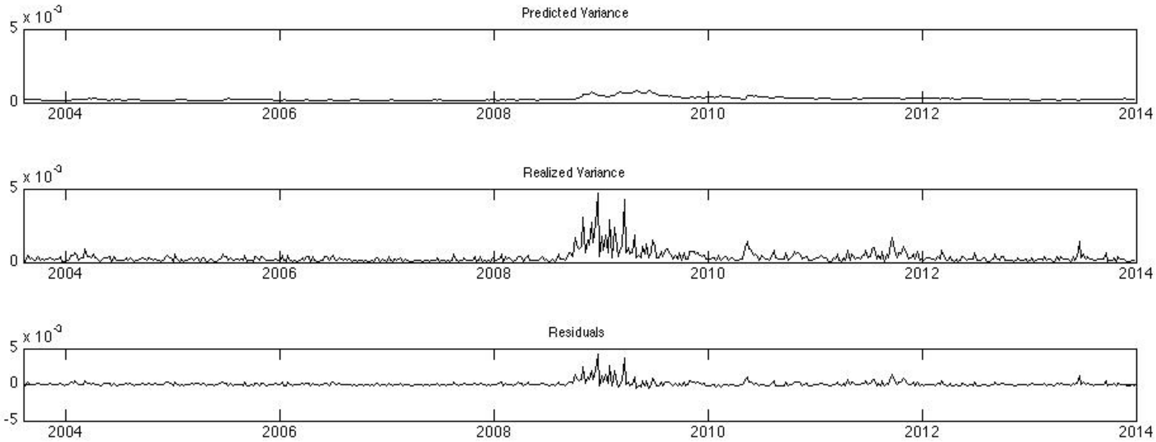

Notes: The plot visually compares predicted variance with the realised for a MIDAS on the OMXS30 Weekly estimated variance. The residuals between the predicted variance and the realized are also plotted. The plot of VaR exceptions show during which periods we get exceptions and suggests that exceptions are clustered at periods with large variance.

**Figure 20: OMXS30 monthly variance estimated with beta lag MIDAS regression**



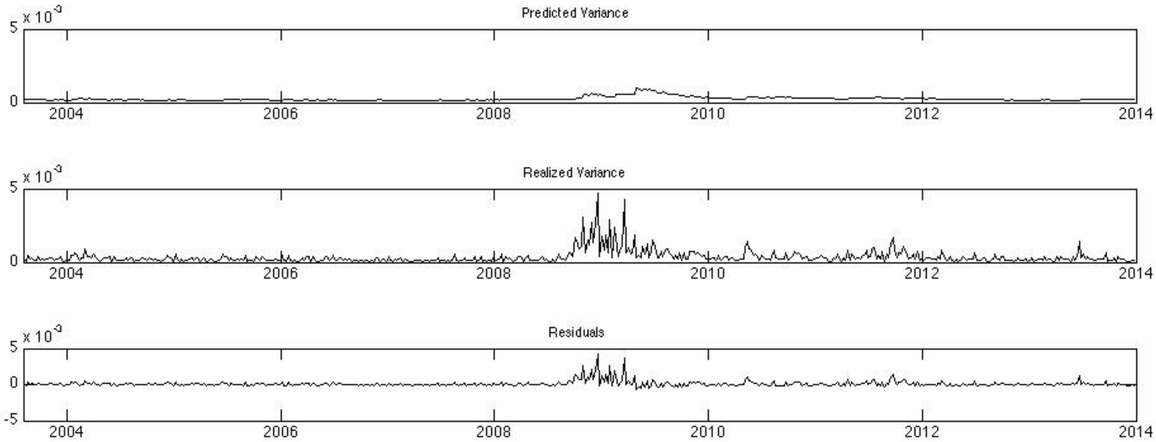

Notes: The plot visually compares predicted variance with the realised for a MIDAS on the OMXS30 Monthly estimated variance. The residuals between the predicted variance and the realized are also plotted.

**Figure 21: USD/SEK weekly variance estimated with EGARCH(1,1)**



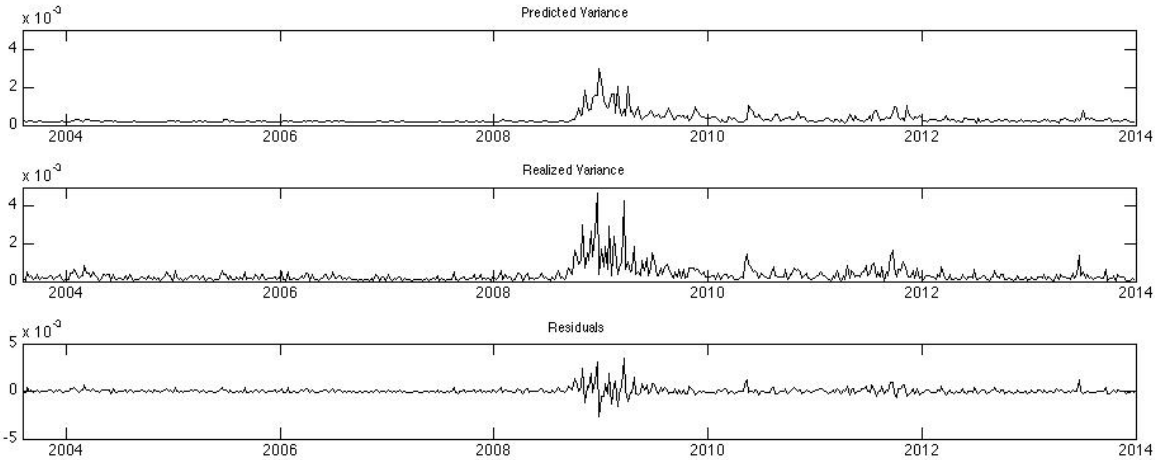

Notes: The plot visually compares predicted variance with the realised for a EGARCH(1,1) on the USD/SEK Weekly estimated variance. The residuals between the predicted variance and the realized are also plotted suggesting that the prediction works best when the volatility is quite small.

**Figure 22: USD/SEK weekly variance estimated with GARCH(1,1)**



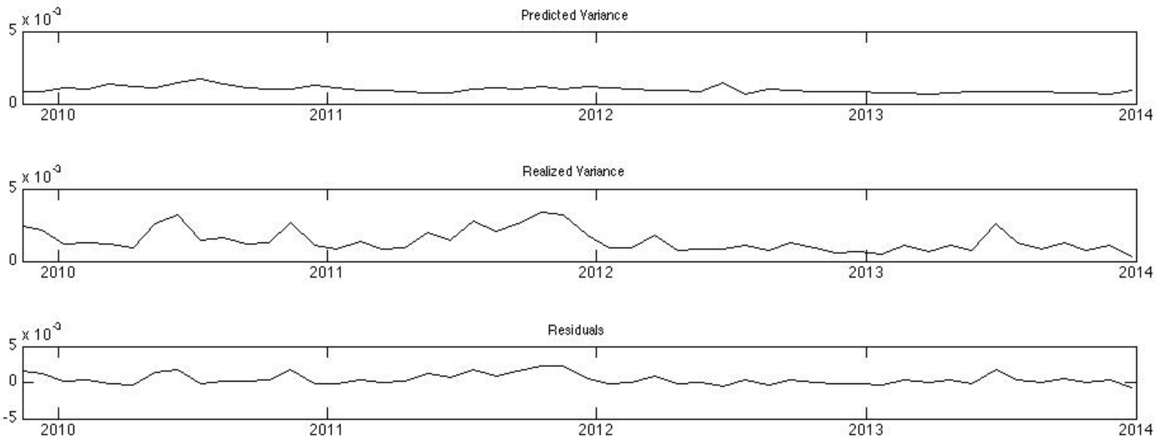

Notes: The plot visually compares predicted variance with the realised for a GARCH(1,1) on the USD/SEK Weekly estimated variance. The residuals between the predicted variance and the realized are also plotted suggesting that the prediction works best when the volatility is quite small.

**Figure 23: USD/SEK weekly variance estimated with beta lag MIDAS regression**



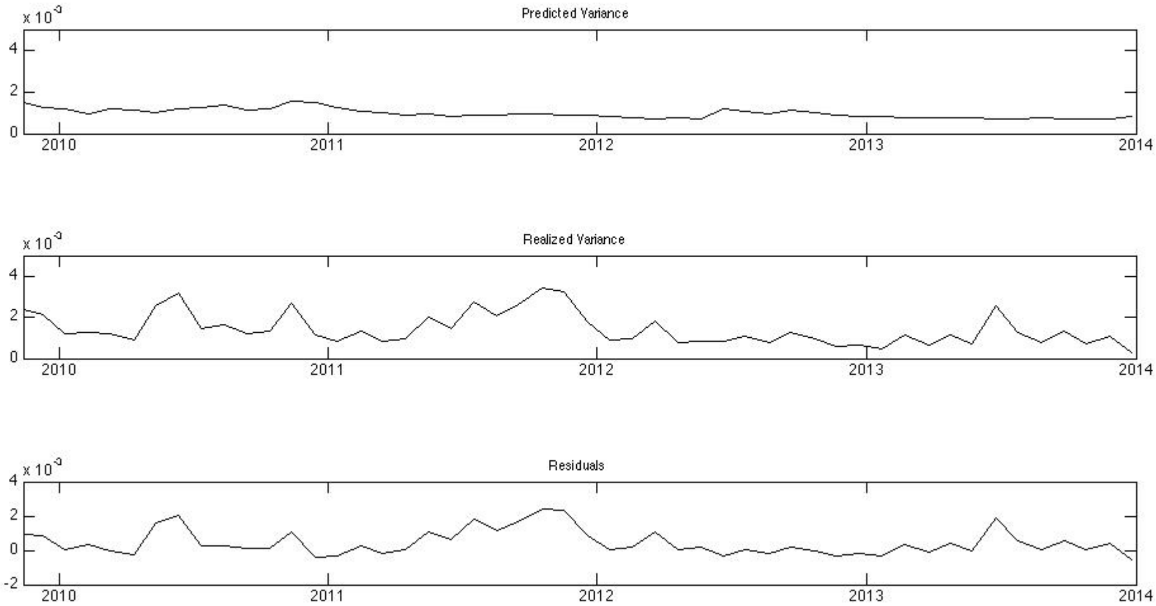

Notes: The plot visually compares predicted variance with the realised for a MIDAS on the USD/SEK Weekly estimated variance. The residuals between the predicted variance and the realized are also plotted.

**Figure 24: USD/SEK monthly variance estimated with EGARCH(1,1)**



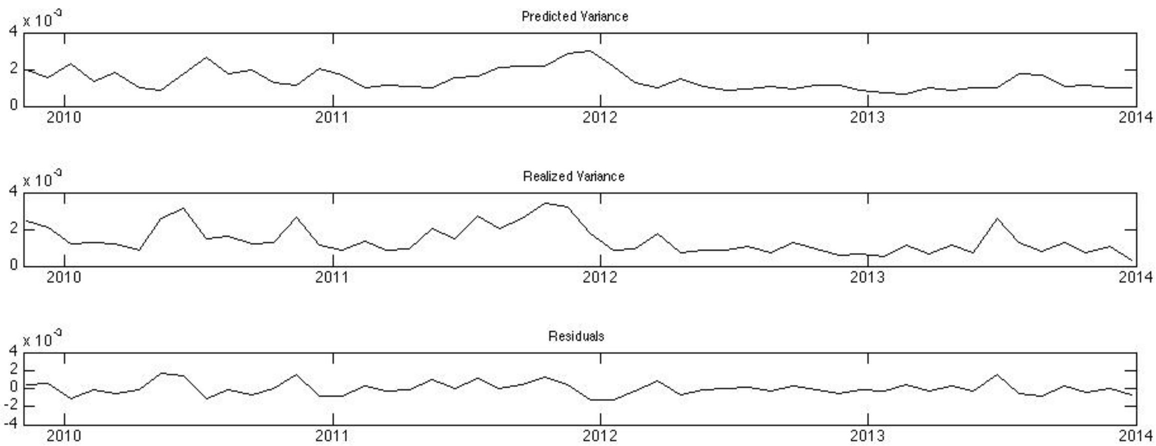

Notes: The plot visually compares predicted variance with the realised for a EGARCH(1,1) on the USD/SEK Monthly estimated variance. The residuals between the predicted variance and the realized are also plotted.

**Figure 25: USD/SEK monthly variance estimated with GARCH(1,1)**




Notes: The plot visually compares predicted variance with the realised for a GARCH(1,1) on the USD/SEK Monthly estimated variance. The residuals between the predicted variance and the realized are also plotted.

**Figure 26: USD/SEK monthly variance estimated with exponential almon lag MIDAS regression**




Notes: The plot visually compares predicted variance with the realised for a MIDAS on the USD/SEK Monthly estimated variance. The residuals between the predicted variance and the realized are also plotted.

# 12 Diebold Mariano Tests

Reject equal predictive power for $|DM| > 1.96$.

**Table 10: Pairwise Diebold Mariano test**

| OMXS30 Weekly | | | |
|---|---|---|---|
| | MIDAS | GARCH | EGARCH |
| MIDAS | - | 3.227 (0.001) | 2.001 (0.045) |
| GARCH | 3.227 (0.001) | - | 3.169 (0.002) |
| EGARCH | 2.001 (0.045) | 3.169 (0.002) | - |

The table shows pairwise Diebold Mariano test statistics. We reject $H_0$ of equal predictive power on the 5 % significance level for $|DM| > 1.96$. P-values in parentheses.

**Table 11: Pairwise Diebold Mariano test**

| USD/SEK Weekly | | | |
|---|---|---|---|
| | MIDAS | GARCH | EGARCH |
| MIDAS | - | 2.045 (0.041) | 2.260 (0.024) |
| GARCH | 2.045 (0.041) | - | 0.407 (0.684) |
| EGARCH | 2.260 (0.024) | 0.407 (0.684) | - |

The table shows pairwise Diebold Mariano test statistics. We reject $H_0$ of equal predictive power on the 5 % significance level for $|DM| > 1.96$. P-values in parentheses.

## Table 12: Pairwise Diebold Mariano test

| USD/SEK Monthly | | | |
|---|---|---|---|
| | MIDAS | GARCH | EGARCH |
| MIDAS | - | 2.045 (0.041) | 2.200 (0.028) |
| GARCH | 2.045 (0.041) | - | -0.486 (0.627)) |
| EGARCH | 2.200 (0.028) | -0.486 (0.627) | - |

The table shows pairwise Diebold Mariano test statistics. We reject $H_0$ of equal predictive power on the 5 % significance level for $|DM| > 1.96$. P-values in parentheses.