M. Sc. Economics & Business Administration: Applied Economics & Finance

Master Thesis

Fall 2013

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Creating Optimal Portfolios of Stocks with Time-Varying Risk

- Using a Multivariate GARCH Approach

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Hand-in: October 1st, 2013

Number of Pages: 117
Number of Characters: 196.953

Copenhagen Business School
Fall 2013

Copenhagen Business School
HANDELSHØJSKOLEN

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Executive Summary

In this thesis we analyze monthly historical stock returns data on 10 industries, to examine if the presence of time-varying risk is seen in real data and how it affects the construction of optimal portfolios, more specifically the calculations of asset-weights in optimal portfolios. We believe that taking this conditional asset-risk into consideration, we might be able to get better results with optimization methods that usually assumes constant correlation and uses a mean-variance approach.

Through testing we find indications of an autoregressive conditional heteroskedastic process in all 10 series of returns. We construct a multivariate generalized autoregressive conditional heteroskedastic model, called multivariate GARCH, in order to model the time-varying risk of the assets as conditional covariance-matrices of our data-series. Over time there are significant variations in the conditional risks, meaning that using generalized historical means as a measure of today's risk and/or expected returns is an over-simplified and possibly flawed approach.

We find that our data can be modeled by a multivariate GARCH(1,1) model, and using statistical software packages we can compute and estimate the parameters of this model, in matrix-form which is necessary for the multivariate model. We will also be able to compute the conditional covariance-matrices and error-terms for every sample-period.

Given the time-varying risk the GARCH model only allows us to do a single-period forecast into the future, so we will apply Monte Carlo Simulation in order to forecast multiple-periods ahead. We will then use the mean-variance optimization techniques from the Modern Portfolio Theory to compute Global Minimum Variance portfolios. We will present investment strategies that do monthly rebalancing of our portfolio, minimizing the conditional-risk in every month as found by Monte Carlo Simulations, and by applying the GARCH-model at ultimo a given month. We will suggest such an investment-strategy for periods of six consecutive months, and compare our results to portfolios constructed with simple mean-variance optimization. We will test the performance of these different types of strategies and find indications that the time-varying risk approach, seems to create better performing portfolios than the mean-variance optimization.

Table of Contents

Part 1 - Introduction

When we started discussing possible topics for our thesis, a great deal of topics within different aspects of our curriculum was taken into consideration. We found that both of us had a keen interest in Capital Market Theory and Investments as well as Econometrics and Corporate Finance in general. We have previously done extensive work on corporate valuations, and felt that this time we would widen our horizons and try to be more original in our choice of topic. We decided to work with portfolio optimization as known from Modern Portfolio Theory, but with the twist that we would implement time-varying risk.

Today's literature in finance is seeing researchers trying to implement time-varying risk to existing models (i.e. the Capital Asset Pricing Model and different index models like the single index or Eugene Fama & Kenneth R. French's three-factor model[1]). The idea that risk is time-varying was not taken into consideration in the literature and teachings of our previous coursework at CBS. This simply made such a topic more appealing, as we would gain knowledge on how time-varying risk can be implemented into financial models. Looking through library-databases online we found that most research papers implementing time-varying risk would do so by simply altering the structures of previously acknowledged regression models. We especially got captivated by Ralitsa Petkova & Lu Zhang's conditional CAPM-model, and by Tim Bollerslev, Robert Engle & Jeffrey Wooldridge's 1988 research paper "A Capital Asset Pricing Model with Time-Varying Covariances". The latter research paper would have an approach to modeling time-varying risk, which we will adopt in this thesis. In order to "think outside of the box" and go beyond the research already found in the literature, we want to do more than just put up some model with time-varying risk. We found that one aspect of investment-theories had not seemingly been covered in the literature we looked through, i.e. optimization of multiple asset portfolios, under an assumption of time-varying risk. Most research papers that incorporated time-varying risk seemed to work with pricing models like the known factor or index-models; mainly CAPM and the multifactor models by Fama & French. The Bollerslev, Engle & Wooldridge research focuses on return data of different types of securities (i.e. Stock, Bonds and

[1] Fama & French, Common Risk Factors in the Returns on Stocks and Bonds, Journal of Financial Economics, Volume

T-Bills). These are asset pricing models, in which the researchers implement some factor of time-varying conditional risk. In our case we wanted to work with portfolio optimization techniques known from the Modern Portfolio Theory. Therefore we would not be working with an asset pricing model. This is different from what we saw in the literature, and so we find it important to try and research if a proper time-varying risk model can improve existing portfolio-optimization models, and help us calculate more efficient portfolio-weights when applying it to real world data.

We decided to work with stocks only, but classified as portfolios of industry stock (see "Data" section for a further discussion on this choice of data). We believe that this can have certain advantages in testing our model, but it might also give an indication, of whether or not some industry-stocks dominate others in the term of risk-return relations. We also look for indications of certain industries performing better or worse in different states of the overall economy. The first thing we need to do is look into the time-varying risk part. We will argue for constructing a multivariate type of time-series regression model which can statistically model time-varying conditional covariance matrices. We will research how multi-period forecasting with such a model works by using Monte Carlo Simulations. We will try to use different "investment strategies" and accordingly compute asset weights of efficient portfolios. Thereafter we want to compare the performance of these strategies, to see if the results of a time-varying risk model seem to be a more appropriate measure of the input data needed to calculate optimal portfolio weights.

With this in mind we want to look into the Modern Portfolio Theory, more specifically Harry Markowitz's mean-variance optimization model. This model is capable of optimizing portfolio weights with portfolios consisting of multiple assets, by using the covariance-matrices of the assets to optimize the return/risk ratio. Thus, it would fit perfectly with the idea of imposing time-varying risk and the results of the econometric model, should be directly transferable to use in this Markowitz optimization. The reason for this is that when the risk - or variance - in an econometric model, is time-varying or conditional - as often seen for time-series of asset returns - we can use a specific econometric model to forecast risk. This is the Autoregressive Conditional

Heteroskedasticity model or the Generalized Autoregressive Conditional Heteroskedasticity model also abbreviated ARCH and GARCH. Both of these models do conditional forecasts of risk and can be extended to be multivariate, which means that they can work with several assets by using matrices and vectors as model parameters (e.g. a covariance matrix for multiple assets, where the single asset version would just model the assets variance). Thus, the result would also be a computed or forecasted matrix of covariances, which can be directly used for modern portfolio optimization. The model imposes time-varying risk and therefore each period's "conditional covariance matrix" will be described as a function of the previous period's conditional covariance and error-terms, as variances, covariances and errors are the measures of risk, both systematic and unsystematic, in an econometric regression model.

We obviously have to impose a number of delimitations to this topic as the idea behind our model can be used to conduct research on pretty much every type of asset and/or investment strategy. Assumptions are also necessary for our model to hold, and as any economist will know, we need to impose various assumptions especially in the use of modern portfolio theory. We will present detailed discussions of delimitations and assumptions.

We want to research if we can improve the Markowitz optimization of portfolio weights under the assumption that the data for this optimization process can be generated/forecasted by an ARCH or GARCH model, because it is subject to time-varying risk. Even the non-financial savvy person can see examples of how risk in the real world appears to be time-varying, simply by looking at the fluctuations of asset-prices shown in graphs for most stock-data of returns in periods of economic recessions versus non-recessions. Thus we decided that we would create a forecasting autoregressive conditional heteroskedasticity model that could take into consideration that changes in the states of the economy and several other factors could affect risk levels of risky assets.

We decided to base our work on researching investments in stock in specific industries within the US Stock Market. We could have chosen other asset-classes as well, but we wanted to use industry-specific data because previous studies do not seem to use this, as they mainly work with diversification across stocks, bonds T-bills etc., or in some cases across countries/markets. This would generate new knowledge and hopefully help indicate how different industries are affected by the state of the economy. Our 10 industries - which we call industry-portfolios as they are comprised portfolios of stocks within certain industries - would be the assets in our overall portfolio to be constructed and tested. This could also have been done like the before-mentioned studies with regards to stocks, bonds, commodities etc. or internationally with investing in different countries, or pretty much with any existing asset or portfolio. That is also why we find it important to research if our method, and our model, seems to be more efficient. By focusing on industries and different time-periods with the economy being in different states, we are hoping to see how our optimization technique will perform in comparison to standard modern portfolio optimization with fixed correlations. And also if one approach will perform significantly better than others in periods of high equity price fluctuations such as economic booms versus decline periods in recessions.

Our hypothesis will be that imposing time-varying risk in the Markowitz optimization, will give us somewhat more efficient solutions to the portfolios we want to construct. In this thesis we will present research and results to discuss and conclude whether or not this hypothesis seems to hold.

Research Question

We aim to use multivariate analysis of time-series and it is our belief that risk in historical stock returns is time-varying. Our analysis will include the testing of this. Thus we want to look at econometric models that estimate conditional variances. We want to use the GARCH model, developed by Bollerslev in 1988 to construct time-varying covariance matrices for different time periods, and we want to do multiple-period forecasting by applying Monte Carlo Simulations. We will then attempt to forecast a series of conditional covariance matrices for selected time-periods of different economic states. These matrices will be used to construct optimal portfolios, using modern portfolio theory, for each of the periods. Thus suggesting, what we have chosen to call a “semi-passive” investment approach that alters investments when the state of the economy changes.

The reasoning behind looking at recessions versus normal states of the economy is to see if there is any significant difference in the way our model performs in the different states. By doing this we will see if our model can outperform normal portfolio optimization in all cases, in recessions only, in booms only or in no cases. As a bit of a bonus we can also get an indication of which industries seem to perform better in different economic states.

To do this, we have articulated a research question as follows:

"How can we construct an econometric financial time-series model with time-varying risk, which can be used to create efficient portfolio choices, using a combination of 10 US industry portfolios? And how does optimal portfolios suggested by this model perform compared to a portfolio constructed from applying normal/simple mean-variance optimization?"

In answering this research question we have developed a series of sub-questions, that we have organized in three categories (time-series analysis, portfolio theory and performance testing) to help us perform our analysis.

<u>Time Series Analysis:</u>
What data do we use and why?
How do we test our data to find motivations for an econometric model?
How do we construct a multivariate econometric model, which estimates the time-varying variance?
How can our model forecast covariance-matrices to be used for mean-variance optimization, in one and more periods ahead?

<u>Portfolio Theory:</u>
What assumptions do we make in applying modern portfolio theory?
What type of efficient portfolio should we aim to create?
How do we determine the asset weights in the optimal portfolios for our time periods?

How would regular modern portfolio theory, with no regards to time-varying risk, suggest the construction of an optimal portfolio?

Performance Testing:
What recent time period do we use for testing our portfolio suggestions versus normal passive Markowitz optimal portfolio.
What level of risk and return does each model perform with? Which model has the highest Sharpe ratio?

The point of these sub-questions will be working as guidelines to help us, step-by-step, getting closer to and eventually reaching a final conclusion. The reader should not consider the order of these research questions as a part of the actual thesis structure. The structure of the thesis is determined and discussed in the section "Structural Method".

Methodology

Scientific Standpoint

What is reality?[2] How is it recognized[3] and how can we examine it[4]? By answering these questions, we also consider which paradigm is going to build the foundation of the project. Egon Guba defines the scientific paradigm as

"A basal set of values, that controls our actions – both the actions that are done every day, and the ones connected to disciplined studies"[5].

We will now discuss and explain the paradigms that we will work with in this thesis.

[2] Ontology
[3] Epistemology
[4] Methodology
[5] Claus Nygaard, Samfundsvidenskabelige analysemetoder, page 23

Before we start the analysis we want to point out which scientific standpoint we take. We are primarily dealing with quantitative empirics collected from the Kenneth French website, more specifically 10 different industry's performances since July 1926. We start by looking at Guba's scientific paradigms[6] to state how we perceive finding of "the truth". We will take a standpoint similar to the positivistic paradigm, meaning that we believe the ontological status on the perception of reality, is that the reality exists somewhere in its true form and is defined by specific laws of nature. This will not be biased or flawed by subjective human perception, as we only work with objective data. We are thus seeking an objectively defined reality, which will be possible as we only use the objective data that are not biased.

However, the reader should keep in mind that working with a completely positivistic standpoint is not very consistent with reality. In our discussion of results and the implications of our findings we will discuss the possibility of violations of the positivistic ontology. We will also discuss potential deviations or any unexpected results in general. This means that we arguably move towards a neo-positivistic paradigm in the latter part of the thesis (more specifically in our discussion), as we need to take into consideration that biases will exist in the real world, and in our case it's almost certain that biases can be created by a subjective human perception of reality. So we want to base our work on the positivistic standpoint, but we will discuss how a neo-positivistic reality can influence our results.

Scientific Method

We have chosen only to work with quantitative analysis. We want to make in-depth research, as well as take a look at "the big picture", meaning we want to reflect on our findings and possibly relate them to other potential research problems. This demands a lot from our delimitations, which we continually will relate reflexively to. We will use the theory of causal thinking to write this project because it is the connection between cause and effect, which is important to reach a conclusion. Only by a realization of the project, an effectual way of thinking will be relevant.

[6] Claus Nygaard, Samfundsvidenskabelige Analysemetoder, page 23

Our paradigm states that the results of our studies will not depend on our interpretations. Thus, it is not necessary to take any critical standpoints to our results, as well as the theories we use, as they will be objective in its nature. The purpose of the methodology is to find the objective truth, which is how positivism differs from neo-positivism, where the human bias is a part of the output. However, we will of course discuss our findings and evaluate possibilities of existing real-world biases.

We have established a hypothesis about the research we want to conduct, in which we believe we can create a model that outperforms modern portfolio theory. We will compare all the findings in favour of or against this hypothesis, and thus we will be the neutral agent, examining the empirics, and describing it from statistics and numbers, according to the positivistic ideology about finding the one objective truth. This is often seen as the approach from the natural sciences, that we believe will be applicable to our material, as it is only empirical data that forms the basis of the knowledge produced in our thesis.

The conclusion of this paradigm is important, as it will be a prerequisite to our ability to explain and predict optimal portfolios in the future.

The Dual Competency

The dual competency is a description of the ability to produce competent knowledge, but also being able to critically evaluate your studies. The ability of relating critical to collecting, and analyzing data, is very important in a project. That is why we, will try to be critical and reflective about our data, method, assumptions etc. An important question to ask is; *are things as they appear to be, or is anything hidden behind the data?* We have been more critical, since there can be doubts about the validity, reliability and adequacy of results and data. The way we relate to the dual competencies is the following:

The knowledge and data we use and primarily the results we produce, will be staged in such a way that it will reflect the considerations we have made, just as second-order learning requires. It will be criticized and its validity and value of truth will be considered[7]. This applies to the theories, knowledge, results and data. This has been appropriated in first-order learning. Though in some aspects, the naive realist will shine through, because it is impossible to relate reflectively to the tiniest detail. We will strive not to take things for granted. It is important to be able to *[…]interpret the perspectives behind the phenomenon of the immediate manifestations*[8]. To be critical becomes an important factor, as there are many things that need to be taken into account. It is important that we understand the situation - and put ourselves in a position where we can deal critically with the knowledge we gather.

Structural Method

After defining the problem, we have created the research question and a series of sub-questions. These research sub-questions are put up in a way that each part of the analysis can lead back to a specific question in the "Research Question" section. The sections of the analysis come in the same order as the categories of the research sub-questions. However, the order of every single sub-question will not necessarily dictate one chronological order in which results are presented. We want this structural design to contribute to the overall structure of the project and give a better overview to the reader. In the conclusion, we will answer the relevant research sub-questions in a somewhat chronological order, by summarizing our findings. This will hopefully make it easy for the reader, to see the "red line" in the presentation of our conclusion. It also has the benefit that a reader who is only interested in one particular sub-question, will be able to go straight to the analysis where it is described, and thereafter directly to the conclusion to see the connection[9].

[7] Søren Barlebo Rasmussen, Forskningsbaseret Læring, page 26
[8] Søren Barlebo Rasmussen, Forskningsbaseret Læring, page 28
[9] Lotte Rienecker, Den Gode Opgave

Part 1 of the thesis consists of an introduction. This includes the research questions, information about data gathering, limitations, and methods used. In Part 2 we describe the theories used in the thesis. Here we also mention the assumptions that we need to take into consideration. The analysis then begins in Part 3, where we will apply all the theories mentioned to our data. Finally in Part 4, we sum up the thesis, by discussing the results and concluding on our research questions.

The analysis will start by focusing on creating the correct GARCH models. This model will be constructed from returns in both recession periods, and non-recession periods, as we have categorized them. In this process we will be estimating the conditional covariance matrices and squared residuals. It will be tested if the models fit best as an ARCH (1,1), or any other form. This is determined by the information criterion from the theory section.
All time-series regressions will be analyzed and statistically tested, to see if we can solidly indicate the presence of an autoregressive conditional heteroskedastic (ARCH) process.

When the model has been created, we can forecast the expected risks of any month in the given data set and use this forecast to compute the weights of optimal portfolios. This will be designed using modern portfolio theory. With the optimal portfolios created, it can be compared to a portfolio created by the standard modern portfolio approach. By conducting a performance test it will then be possible to see if our model will be able to perform better in terms of risk and returns.

The most important part of the analysis will focus on estimating the optimal portfolios, because our hypothesis is that these will be improved as compared to standard modern portfolio theory. This is primarily due to the fact that we believe risk is time varying. And our project will thus focus on creating an overall model that will be able to forecast the best possible portfolio, with an investment horizon of one month. In our case, this will be for 10 industry portfolios on the American market.

The specific steps that we go through to answer the work questions will be described chronologically in the theory section.

Delimitations

By the selection of primary data, we have had a focus on relevance rather than representativeness. This is why we have limited ourselves to 10 industry portfolios within the US equity market.

We would like to find out which industries will be optimal to invest in, under the assumption that risk is time-varying. To best argue in favour of this assumption, we can simply refer to stock-performance measures in periods of recession versus periods of boom. Just as all other aspects of the economy, the volatility of stock-prices are affected by the economic climate and probably also several other factors or unknown variables. We also found it interesting to be able to identify which industries are more valuable to an investor's portfolio in different economic climates. Thus we have found it necessary to delimit ourselves to analyzing industry portfolios, and leaving out different asset types, like bonds and commodities, which of course are also relevant to an investor.

We have chosen 10 industry portfolios because we believe that this gives a good overview of how a portfolio can be constructed and optimized within a market. Our data covers the stock-market very well, as it contains most of the investment opportunities in US stocks (see data section for further description), and this is also the reason why we haven't found it necessary to add any other industry portfolios, as we probably could have used a lot more, but then they would just consist of less investment opportunities accordingly. The important thing to keep in mind is that our analysis is delimited from using other asset-classes such as T-Bills, Bonds, Options etc. – ONLY stock-prices will be part of the analysis.

Another issue we have delimited ourselves from is international diversification. The theories and research in the area of international diversification (i.e. Fama & French research) suggest that it is always a good idea to diversify internationally to other markets. But as our emphasis is on industry specific performance and measurement, we left out the international aspect and are only looking at the American market. Of course we can't generalize and say that the same industries will be good to invest in different markets at different time periods, but we believe it gives a good overall indication and, more importantly, it will be provide us with good input data for setting up and testing our model. This has been the idea behind focussing on industries rather than countries, which would also leave us with way too much data to process.

We will be analyzing monthly return data on stocks and we are delimited to the time period 1927-2012. A model for daily returns could however be in the same way we build our model, and it would simply do daily instead of monthly forecasting.

Finally we would like to mention that our model is theoretically constructed and all calculations of portfolio weights are done by applying the formulas for mean-variance optimization in the Modern Portfolio Theory. This means that our model will suggest short-selling, so we actually refrain from using a realistic delimitation such as "no short-sales". This is something we do because we are writing this thesis as an academic research paper, and not some "how to invest guide" for the non-financial savvy person. Thus the emphasis will be on developing and testing the model and that is why we do not impose a constraint on short sales, which is also mentioned in our analysis.

Data

Data on Historical Returns

Our entire analysis is based on historic data regarding our chosen assets. We have chosen to analyze industries in the US market, and needed a source that could give us significant historical data on returns for different industry-defined asset classes. In previous coursework from a semester abroad we have worked with data from the "Kenneth French Website": http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

The website is a library database managed by Kenneth R. French, the author and professor who is mostly known for his work with Eugene F. Fama on the three-factor model. The database is a part of the library of "Tuck School of Business" at Dartmouth College, USA. The website holds a massive data library which includes a wide selection of return data. The data is put into constructed portfolios on countries, industries, ratios, value/growth, size of market cap, the Fama-French research factors ("market excess return", "small minus big" and "high minus low") and much more. There are naturally also listings of risk-free and market excess returns. Seeing as this library holds returns data on constructed industry portfolios, we believed that this would be a good source of collecting our data. We also find it very reliable as it is part of a university library.

On the construction of the portfolios or asset-classes that we want to analyze, the French website writes the following:

"*We assign each NYSE, AMEX, and NASDAQ stock to an industry portfolio at the end of June of year t based on its four-digit SIC code at that time. (We use Compustat SIC codes for the fiscal year ending in calendar year t-1. Whenever Compustat SIC codes are not available, we use CRSP SIC codes for June of year t.) We then compute returns from July of t to June of t+1.*"

Basically, what we need to take note of is that they classify the stocks in one industry on their industrial code and they use stocks of the biggest US stock market indexes.

The 10 industries that make up our data are as follows:
Consumer Non-Durables (NoDur): Food, Tobacco, Textiles, Apparel, Leather, Toys
Consumer Durables (Durbl): Cars, TV's, Furniture, Household Appliances
Manufacturing (Manuf): Machinery, Trucks, Planes, Chemicals, Office Furniture, Paper, Computer Printing
Energy (Enrgy): Oil, Gas, and Coal Extraction and Products
Business Equipment (HiTec): Computers, Software, and Electronic Equipment. Also includes: industrial controls, computer programming and data processing, computer integrated systems design, computer processing, data prep, information retrieval services, computer facilities

management service, computer rental and leasing, computer maintenance and repair, computer related services, research & development labs, research, development, testing labs.
Telecommunications (Telcm): Telephone and Television Transmission
Shops: Wholesale, Retail, Services including Laundries and Repair Shops
Health (Hlth): Healthcare, Medical Equipment and Drugs
Utilities (Utils): Generators, Energy Network Operators, Energy Traders and Marketers, Energy Service Providers and Energy Retailers
Other: Mines, Construction, Building Maintenance, Transportation, Hotels, Bus Services, Entertainment, Finance

The data on risk-free rates and market excess returns are according to the website; value-weighted return of all CRSP firms incorporated in the US and listed on the NYSE, AMEX, or NASDAQ. And the website comments on the construction of this data as follows:

"Rm-Rf includes all NYSE, AMEX, and NASDAQ firms. SMB and HML for July of year t to June of t+1 include all NYSE, AMEX, and NASDAQ stocks for which we have market equity data for December of t-1 and June of t, and (positive) book equity data for t-1."

Of the latter mentioned data, we will only use risk-free returns in computing Sharpe ratios and in general we refrain from using returns in our analysis as we seek to model and forecast risk.

The data is obviously of the quantitative type of data, which is what we need, as our analysis is completely quantitative, as all the derivations and estimations that we make throughout the thesis, are based on this "raw" historical stock return data. The full period of the return data is July 1926 through December 2012.

We consider the Kenneth French website a primary source of data.

As the data contains 1,038 observations for each of the 10 assets, we have chosen not to put the full data sample into our thesis, not even as an appendix, as this would simply give us hundreds of pages of appendices which would be too unorganized for the reader and not to mention a waste of printing paper. We will however show a cutout of the data, so that it is easy to see what kind of data we are dealing with. The cutout is shown below:

	Durbl	Enrgy	Hlth	HiTec	Manuf	NoDur	Other	Shops	Telcm	Utils
192607	15,61	-1,17	1,64	2,9	4,6	1,43	2,12	-2,81	0,83	7,1
192608	3,64	3,31	4,51	2,66	2,79	4	4,36	-0,54	2,17	-1,74
192609	4,88	-3,37	0,61	-0,38	1,22	1,2	0,29	0,14	2,41	1,96
192610	-8,22	-0,78	-0,73	-4,58	-3,61	-1,34	-2,85	-5,31	-0,11	-2,67
192611	-0,2	-0,01	5,53	4,71	4,14	5,15	2,13	4,05	1,63	3,76
192612	9,91	2,81	0,15	-0,02	3,7	0,84	3,39	2,89	1,99	-0,12
192701	-0,91	1,2	4,91	-1,14	-0,06	-0,64	1,52	-3,19	1,88	-1,78
192702	8,88	1,42	1,65	4,45	5,8	3,37	5,04	4,55	3,97	4,57
192703	1,65	-5,95	0,88	1,45	1,47	2,71	1,26	2,85	5,56	0,44
192704	3,17	-5,11	3,25	5,4	0,78	3,31	0,9	2,21	-2,13	1,65
192705	5,92	4,8	4,01	7,37	5,09	8,17	6,4	0,63	3,35	9,31
…………………………….										
201201	13,62	1,47	3,39	8,45	6,77	0,34	7,22	4,15	2,45	-2,71
201202	5,95	6,66	1,35	6,7	3,89	3,61	4,21	3,16	4,93	1,3
201203	1,39	-3,11	3,9	4,62	0,41	3,92	4,93	4,67	2,35	1,45
201204	-7,31	-1,74	0,62	-2,06	-0,66	0,76	-1,64	0,98	1,76	1,73
201205	-7,97	-10,87	-3,39	-7,79	-7,78	-2,59	-7,72	-3,05	-0,82	-0,91
201206	-5,41	5,91	6,65	2,96	1,16	3,51	4,81	2,04	7,07	3,4
201207	-0,37	2,71	1,69	-0,16	1,14	1,51	-1,21	1,2	4,05	3,63
201208	4,24	2,41	1,08	4,86	3,14	0,06	3,14	2,81	0,84	-3,41
201209	3,4	3,65	4,79	1,14	1,71	0,99	3,75	1,98	4,84	2,12
201210	6,28	-1,81	-1,87	-6,46	-0,62	-1,67	0,96	-1,08	-2,42	1,08
201211	2,51	-1,58	1,45	1,47	2,7	3,15	-0,29	1,41	0,12	-3,83
201212	7,6	1,4	-0,43	0,24	2,68	-2,29	3,55	-1,01	1,26	0,1
St. Dev	7,820212	6,018714	5,706517	7,388486	6,361961	4,657218	6,528753	5,838905	4,639611	5,614411
Mean	1,085973	1,054807	1,07369	1,077389	1,014817	0,975838	0,890674	0,980048	0,84684	0,877553

Figure 1 – Cutout of Data

From the figure above it is easy to see that our raw input data contains monthly percentage returns for every single month in the period July 1926 through December 2012. The historical mean and standard deviations are not part of the raw data, but we chose to just compute them in Microsoft Excel, as this is very simply done by using Excel functions. These constant-means will only be used in the simple case of modern portfolio theory optimization, but not in our forecasting model.

Data on States of the Economy

It is commonly known in macroeconomics, that most countries will classify the economic state as a recession if the country or economy has experienced at least two consecutive quarters of non-positive GDP growth. In fact this is so commonly known, that the oxford dictionary even mentions this in their definition of the word recession[10].

So since we needed data telling us when the US economy was in a recession, we could simply look at the historical Gross Domestic Product growth in the US. This is publically available data and can be found on many online databases under economic indicators. We found that the easiest way for us to do this was to cross-reference a list of US recessions on Wikipedia[11] with the E-Library of the International Monetary Fund[12]. We have done this, as Wikipedia is not a very reliable source of data (thus the data on economic states from Wikipedia is secondary data), whereas the IMF's E-library can be considered a reliable source (we would consider IMF's online library a primary source of data). Looking closer into performance of the US Stock Markets, we have simply used a graph from Google Finance (see the section "Time Periods for Analysis")

Part 2 - Theory

Time Series Econometrics

In this section we will present the econometric theories used in our thesis, as well as the underlying reasoning behind them and how to apply them to our data etc.

Heteroskedasticity

Financial time series often shows a structure with non-constant variance. For example the variations in the time series can be very small in a period, and then very large for a different

[10] http://oxforddictionaries.com/definition/english/recession
[11] http://en.wikipedia.org/wiki/List_of_recessions_in_the_United_States
[12] http://elibrary-data.imf.org/DataExplorer.aspx

number of periods. Ideally the ARCH or GARCH model will incorporate this heteroskedasticity into the model, so it takes the changes of variation into account.[13]

If we depict our returns of the time series on a log scale, and then differentiate it, the differentiated time-series will look like white noise. In the following [Y_t] will denote the log of the price of the underlying assets at any period t, and the model will usually look like:

$$\Delta Yt = Xt = \varepsilon t, \qquad \text{where } \varepsilon t \sim N(0,1), \text{ i.i.d.}$$

This means that the returns of the industry portfolios Xt = Yt – T(t-1) behaves like a Gaussian white noise sequence, which is consistent with a random walk hypothesis.
Some other structures cannot be explained well enough by the equation, but these stylized facts are usually known to be:

[Xt] is heavily tailed, much more than the Gaussian white noise
Although not much structure is revealed in the correlation function of X_t, the series X_t^2 is highly correlated. These correlations can be negative or positive depending on the direction of the correlation. The changes in X_t tend to be clustered. This means that large changes in X_t tend to be followed by large changes, and small changes in X_t tend to be followed by small changes[14]

Especially this last point is important. In financial markets, it is important to try to understand the impact of volatility because investors are seemingly risk-averse and therefore require higher expected returns for taking more risk.

Consequently, time series models with heteroskedastic errors were developed. One of the most popular models for dealing with this heteroskedasticity is the autoregressive conditional heteroskedasticity (ARCH) model, which we will use in this thesis to test for heteroskedasticity in our time series.

[13] Ngai Hang Chan, Time Series Applications to Finance, page 101
[14] Ngai Hang Chan, Time Series Applications to Finance, page 102

ARCH

In its simplest form, ARCH can be expressed as: $X_t = \boldsymbol{\sigma_t \varepsilon_t}$

Usually [15] it is assumed that $\boldsymbol{\varepsilon_t} \sim N(0,1)$, i.i.d (independent and identically distributed), and $\boldsymbol{\sigma_t}$ satisfies:

$$\boldsymbol{\sigma^2(t) = \alpha_0 + \Sigma(p,t=1)\ \alpha_i\ X^2(t-i)}$$ [16]

In this theory section we will start by explaining the ARCH and then proceed with the generalized autoregressive conditional heteroskedasticity model, which needless to say is abbreviated GARCH. However, in the next step we will describe how we test for the presence of ARCH. After that we will go more into detail with the models.

Testing for ARCH

This subject is important because of the forecasting issues. It is necessary to know the volatility because we need to know what to do with the financial planning, and here variability is bad, because it makes financial planning difficult. But the volatility also presents the opportunity for large profits, as well as huge losses. But with volatility comes uncertainty.

A characteristic of these financial time series is that they can appear to be random walks in their level form; meaning they are non-stationary.

On the other hand, in the first-levet differentiated form they are generally stationary.

Therefore, instead of modeling the levels of the financial time series, it can be a good idea to model their first differences. Although these often come with wide volatility, suggesting that the variance of the financial time-series varies over time, the ARCH modeling developed by Robert Engle takes this into account, which is why we have chosen to use that method to test our time series.[17]

[15] Ngai Hang Chan, Time Series Applications to Finance, page 102
[16] Ngai Hang Chan, Time Series Applications to Finance, page 103
[17] Engle, Robert – Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation

Heteroskedasticity may have an auto-regressive structure, meaning that heteroskedasticity observed throughout various periods may be correlated. This method developed by Engle allows us to statistically measure the volatility.

Y_t = time series returns
Y_t^* = log of Y_t
$dY_t^* = Y_t^* - Y^*_{t-1}$ = relative change in returns
$d\cancel{Y}_t^*$ = mean of dY_t^*
$X_t = dY_t^* - d\cancel{Y}_t^*$

Thus X_t is the relative change in returns and it is adjusted for the mean. Then we can use X_t^2 as a measure of volatility. Since it is a squared quantity, its value will be relatively high in periods when there are big changes in the return, and its value will be comparatively small when there are humble changes.[18]

Now that we use X_t^2 as a measure of volatility, we need to know if and how it changes over time. First we will consider an ARCH(1) model:

$$X_t^2 = ß_0 + ß_1 X^2_{t-1} + u_t$$

This formula assumes that volatility in a given period is related to its value in the previous period plus a term for the white noise error. If $ß_1$ is positive, it suggests that if volatility was high in the previous period, it will continue to be so in the current period, which indicates volatility clustering. If $ß_1 = 0$ there is no volatility clustering. The statistical significance of the estimated $ß_2$ can be analyzed using a simple t-test.

Second an ARCH(*p*) model of volatility like this can be considered:

$$X_t^2 = ß_0 + ß_1 X^2_{t-1} + ß_2 X^2_{t-2} + \ldots + ß_p X^2_{t-p} + u_t$$

[18] Gujarati and Porter, Basic Econometrics, Example 22.1 page 793

This model suggests that volatility in the current period is related to the volatility in the past *p* periods, where we consider *p* from an empirical point of view from the data. Hence *p* represents the number of autoregressive terms in the model. The empirical problem can be solved by using model selection criteria (e.g. the Akaike- and Schwarz Information measure). The model selection criteria will be explained further in the section "Model Selection Criterions" Again, the significance of any given ß coefficient can be tested with the t-test, and the combined significance of 2 or more coefficients by the F-test.

To test for ARCH effects in our regression model that is based on time series data, we need to consider a *k*-variable linear regression model:

$$Y_t = ß_1 + ß_2X_{2t} + \ldots + ß_kX_{kt} + u_t$$

The disturbance term is distributed as[19]

$$u_t \sim N\,[\,0\,,\,(\alpha_0 + \alpha_1 u^2_{t-1})\,]$$

This indicates that u is normally distributed with a mean of 0 and var $(u_t) = \alpha_0 + \alpha_1 u^2_{t-1}$, so that the variance follows an ARCH(1) process. It is important to notice that the variance of *u* at time *t* is dependent on the squared disturbance at the previous period in time (t-1), thus giving an appearance of a serial correlation. With this said, it does not necessarily only depend on one lagged period of the squared error-term, but can also be a function of several lagged squared terms, is in the following equation:

$$\text{var}\,(u_t) = \sigma_t^2 = \alpha_0 + \alpha_1 u^2_{t-1} + \alpha_2 u^2_{t-2} + \ldots + \alpha_p u^2_{t-p}$$

If there is no autocorrelation in the error variance, then:

[19] Gujarati and Porter, Basic Econometrics, Equation 22.10.5, page 793

$$H_0: \alpha_1 = \alpha_2 = \ldots = \alpha_p = 0$$

If this is the case, then var (u_t) = α_0, and then there is no ARCH effect. According to Engle[20], it can be shown that the following regression easily can test the preceding null hypothesis, since we do not directly observe σ_t^2:

$$\hat{u}_t^2 = \hat{a}_0 + \hat{a}_1\hat{u}^2_{t-1} + \hat{a}^2\hat{u}^2_{t-2} + \ldots + \hat{a}_p\hat{u}^2_{t-p}$$

In this equation α is denoted as a.

$\hat{u}_t^2$ denotes the OLS residuals that are obtained from the original *k*-variable linear regression model.

The null hypothesis can be tested using an *F*-test, or alternatively by calculating nR^2, where R^2 is the coefficient of determination from the auxiliary regression.
It can be shown that

$$nR^2_{asy} \sim \chi^2_p$$

so that nR^2 follows the chi-square distribution for large samples, with degrees of freedom equal to the number of autoregressive terms in the auxiliary regression.[21]

What to do if ARCH is present?
The scope of "Gujarati & Porter" does not include the further process, but we know from the ARCH/GARCH theory section how to construct a usable model. Fortunately, our EViews 7.0 software package can also estimate a multivariate GARCH model, which we will need to do.

GARCH

The ARCH & GARCH models
GARCH is an abbreviation of the actual name of the model, which is Generalized Auto-Regressive Conditional Heteroskedasticity model. It was proposed by Bollerslev in 1988 and is

[20] Gujarati and Porter, Basic Econometrics, Equation 22.10.9, page 794
[21] Gujarati and Porter, Basic Econometrics, Example 22.1, page 794

an extension to the Auto-Regressive Conditional Heteroskedasticity- or ARCH-model developed by Robert Engle in 1982.

The ARCH and GARCH models are widely used to measure and forecast volatility in financial time-series, because these time-series such as asset returns are subject to the phenomenon called "volatility clustering", meaning that returns are more volatile in certain periods of time. This makes sense because returns reflect trading patterns, reaction to news, macroeconomic impacts etc.

This indicates that the volatility or risk is varying over time, thus creating a model with time varying variance. This is in other words heteroskedasticity, as heteroskedasticity means unequal variance. The ARCH-model then suggests the autoregressive structure by assuming autocorrelation in heteroskedasticity observed over different periods of the time-series[22].

Observing this varying variance, Engle proposes the ARCH-model based on the idea that the recent past gives information on the conditional variance, and creating the model as such:

$$\begin{aligned}
&\text{Process: } && u_t = v_t\left(\alpha_0 + \alpha_1 u_{t-1}^2 + \ldots + \alpha_p u_{t-p}^2\right)^{1/2} \\
& && \text{where } v_t \text{ is a white noise series and } \sigma_v^2 = 1 \\
&\text{or: } && \sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \ldots + \alpha_p u_{t-p}^2
\end{aligned}$$

Meaning that the variance at time t is depending on the lagged squared value. With *p* denoting number of lags, thus the model is called an ARCH(*p*) model. The denotation u_t simply represents the disturbance- or error-term from a regression model

For example looking at the ARCH(1) model, the process becomes[23]:

[22] Gujarati & Porter, Basic Econometrics, page 791
[23] Slides from lectures in the course "AE58 - Applied Econometrics"

$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 \quad \text{or} \quad u_t = v_t\left(\alpha_0 + \alpha_1 u_{t-1}^2\right)^{1/2}$

It can be shown that the properties of this process are:

1. $\mathrm{E}(u_t) = 0$
2. Conditional variance: $\mathrm{var}(u_t \mid \text{info at time t-1}) = \alpha_0 + \alpha_1 u_{t-1}^2$
3. Unconditional variance: $\sigma^2 = \dfrac{\alpha_0}{1-\alpha_1}$

   (exist when $\alpha_0 > 0$ and $|\alpha_1| < 1$)
4. $\mathrm{cov}(u_t, u_{t-i}) = 0$ when $i \neq 0$

In 1988 Bollerslev then proposes the GARCH-model which shows that the conditional variance at time t depends not only on the lagged squared error-term, but also on the lagged conditional covariance;

In the GARCH(p,q) model the conditional variance is:

$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \ldots + \alpha_p u_{t-p}^2 + \lambda_1 \sigma_{t-1}^2 + \ldots + \lambda_q \sigma_{t-q}^2$

Often a GARCH(1,1) model is a good model for the return on an exchange rate, just to mention an example.

The GARCH(1,1) model:

$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \lambda_1 \sigma_{t-1}^2$

The GARCH(1,1) is the simplest form of the GARCH-model and is the most used for modeling financial time-series. As shown later in this thesis, we shall also use a GARCH(1,1) model for our analysis, we will argue for this choice with the use of model-selection criteria

Multivariate GARCH

When we need to apply the GARCH-model to multiple financial time-series, as in our case with 10 series of portfolio-returns, we need to expand the simpler univariate-model to include several time-series. We need to include possible interactions (correlations) between returns on the different portfolios, thus needing to model the covariance of the assets. The multivariate GARCH model does exactly this and will in fact make us capable of forecasting a covariance-matrix for our portfolios, based on the lagged squared residuals of the returns and the previous conditional covariances. There are several existing multivariate GARCH- or M-GARCH models and our literature mainly focuses on two of them, that is the VECH model developed by Bollerslev, Engle & Wooldridge in 1988 (it is also extended to the Diagonal VECH-model) and the BEKK-model[24]. We shall now describe them in simple forms of 2 assets in order to describe the theory. As this is quite simple, the model will be extended to fit our data with 10 assets.

The VECH-Model

$VECH(H_t) = C + A*VECH(\Theta_{t-1}\,\Theta'_{t-1}) + B*VECH(H_{t-1})$ where $\Theta|\psi_{t-1} \sim N(0,H_t)$

Where H_t is a 2x2 conditional variance-covariance matrix, Θ is a 2x1 vector that represents the residuals (also called an innovation vector), ψ_{t-1} represents the information set at t-1, C is a 3x1 parameter vector (it is similar to the intersection parameter in a simple regression model) and A and B are 3x3 matrices of the parameters for the model.

VECH() denotes the column stacking operator applied to the upper portion of the symmetric matrix[25].

In order to fully illustrate we can denote the previous explanations like this:

$$H_t = \begin{pmatrix} h_{1,1,t} & h_{1,2,t} \\ h_{2,1,t} & h_{2,2,t} \end{pmatrix} \quad \Theta_t = \begin{pmatrix} u_{1,t} \\ u_{2,t} \end{pmatrix} \quad C = \begin{pmatrix} c_{1,1} \\ c_{2,1} \\ c_{3,1} \end{pmatrix} \quad A = \begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix} \quad B = \begin{pmatrix} b_{1,1} & b_{1,2} & b_{1,3} \\ b_{2,1} & b_{2,2} & b_{2,3} \\ b_{3,1} & b_{3,2} & b_{3,3} \end{pmatrix}$$

Note that this model alone yields 21 parameters in the simplest form of just 2 assets. The meaning of VECH as an operator that stacks the upper triangular (incl. the diagonal) part of the matrix and stacks it into a vector can be illustrated by considering:

[24] Chris Brooks, Introductory Econometrics for Finance, pages 432-436
[25] Chris Brooks, Introductory Econometrics for Finance, page 432

$$\text{VECH}(H_t) = \begin{pmatrix} h_{1,1,t} \\ h_{1,2,t} \\ h_{2,2,t} \end{pmatrix}$$

Where the values in the vector represent the conditional covariance between assets 1 and 2 at time t. Thus the VECH($\Theta_{t-1}\ \Theta'_{t-1}$) part of the equation can be denoted as:

$$\text{VECH}(\Theta_{t-1}\ \Theta'_{t-1}) = \text{VECH}\left(\begin{bmatrix} u_{1,t} \\ u_{2,t} \end{bmatrix} \begin{bmatrix} u_{1,t} & u_{2,t} \end{bmatrix}\right) = \text{VECH}\begin{pmatrix} u_{1,t}^2 & u_{1,t}u_{2,t} \\ u_{1,t}u_{2,t} & u_{2,t}^2 \end{pmatrix} = \begin{bmatrix} u_{1,t}^2 \\ u_{1,t}u_{2,t} \\ u_{2,t}^2 \end{bmatrix}$$

Thus the VECH model in the case of just 2 assets can be expressed as follows[26];

$$\begin{aligned} h_{11t} &= c_{11} + a_{11}u_{1t-1}^2 + a_{12}u_{2t-1}^2 + a_{13}u_{1t-1}u_{2t-1} + b_{11}h_{11t-1} \\ &\quad + b_{12}h_{22t-1} + b_{13}h_{12t-1} \\ h_{22t} &= c_{21} + a_{21}u_{1t-1}^2 + a_{22}u_{2t-1}^2 + a_{23}u_{1t-1}u_{2t-1} + b_{21}h_{11t-1} \\ &\quad + b_{22}h_{22t-1} + b_{23}h_{12t-1} \\ h_{12t} &= c_{31} + a_{31}u_{1t-1}^2 + a_{32}u_{2t-1}^2 + a_{33}u_{1t-1}u_{2t-1} + b_{31}h_{11t-1} \\ &\quad + b_{32}h_{22t-1} + b_{33}h_{12t-1} \end{aligned}$$

This expression explains the relations between the conditional variances, covariances and their lagged values as well as the lagged squared residuals.

The Diagonal VECH-Model

Bollerslev, Engle & Wooldridge (1988) chose to simplify the VECH-model by imposing restrictions, to make the A and B matrices diagonal. This way, they are reducing the number of parameters to be estimated in the model. Imagine the model illustrated above, where A and B matrices now only have 3 elements. The model in this case will then become:

$$h_{i,j,t} = w_{i,j} + \alpha_{i,j}u_{i,t-1}u_{j,t-1} + \beta_{i,j}h_{i,j,t-1} \qquad , \qquad \text{for } i,j = 1,2$$

[26] Chris Brooks, Introductory Econometrics for Finance, page 433

Where $w_{i,j}$ and $\alpha_{i,j}$ and $\beta_{i,j}$ are the parameters of the model, that are to be estimated. The Multivariate Diagonal VECH model is actually an infinite order ARCH model, as the covariance is expressed as a "*geometrically declining weighted average of past cross products of unexpected returns, with recent observations carrying higher weights"*[27]

The BEKK-Model

Developed by Engle & Kroner in 1995, the BEKK model ensures a forecasted variance-covariance matrix that is certain to be positive and definite. The BEKK model can be expressed as:

$$H_t = W'W + A'H_{t-1}A + B'\Theta_{t-1}\,\Theta'_{t-1}B$$

Comparing to the way we illustrated the VECH model, we can consider A and B to be 2x2 matrices of parameters, whereas W is the upper triangular matrix of these. The model ensures the covariance-matrix to be positive and definite by the "*quadratic nature of the terms on the equation's RHS"*[28]

Estimating a Multivariate GARCH-Model

Assuming that the parameters of the model are conditionally normally distributed, we can use optimization of the log-likelihood function to estimate the model. We will not go into further details with optimizing the log-likelihood function, as we will not use this particular calculation in our thesis. We only need a thorough understanding of how to construct and estimate multivariate GARCH-models. Once we know how the models work and how they are estimated and used for forecasts, our software will provide the technicalities needed to make the necessary calculations for our model.

As our approach to modeling financial returns mimics the methods of Bollerslev, Engle & Wooldridge (1988) where they use an M-GARCH model for CAPM with time-varying covariances, we have decided to proceed with the Diagonal VECH model. A disadvantage of this

[27] Chris Brooks, Introductory Econometrics for Finance, page 434
[28] Chris Brooks, Introductory Econometrics for Finance, page 435

model compared to the BEKK is that the VECH model does not guarantee that the variance-covariance matrices will be definite and positive, which they need to be to provide us with valid estimations and forecasts, when we are dealing with financial or economic time-series[29]. Besides, as an example, a non-positive or indefinite covariance matrix will not be effectively inverted and used for mean-variance optimization, thus it is important that we are aware of this when we are working with a VECH model. Luckily our software package EViews, can restrict the Diagonal VECH-model estimation to yield positive "full-rank" matrices and this will give us a satisfying estimation of our model. So we should have no problems in estimating our model.

Model Selection Criterions

When estimating our econometric models, we will need to determine which values of p and q the specific GARCH-model needs. This is done by creating models of all possible combinations for p=1,2,3,4 and q=1,2,3,4. As an example we can refer to Bollerslev, Engle & Wooldridge (1988) where a GARCH(1,1) model is used for modeling the conditional covariance of stocks and bonds.

In order to decide which model fits our data the best, we will look at different information criterions. In the case of a normal univariate regression, one would simply have to look at the value of R^2. But since our models are multivariate and a lot more complex than just a standard regression, the R^2 will not be of much use. The information criterions that we will now describe[30] are what we need to look at instead.

Akaike's Information Criterion (AIC)

The AIC can, among other things, be used to determine lag length *p* in an AR(*p*) model, which makes it quite relevant to our model as well.

[29] Fomby and Terrell, Econometric Analysis of Financial and Economic Time Series – Part A, pages 6-8
[30] Gujarati and Porter, Basic Econometrics, page 494

The AIC is calculated as follows:

$$\text{AIC} = \text{e}^{2\text{k/n}} \frac{\sum \hat{\text{u}}_i^2}{n} = \text{e}^{2\text{k/n}} \frac{RSS}{n}$$

RSS is simply an abbreviation for Residual Sum of Squares, whereas *k* is the number of regressors in the model, including the intercept. The value of *n* is simply the number of observations.

When deciding which model to choose, we will look for the one where the AIC has the lowest value.

Schwarz's Information Criterion (SIC)

The SIC is very closely related to the AIC and just like the AIC it imposes penalties on adding regressors to a model. The SIC penalties are however harsher, as can be seen in the formula for calculating the SIC:

$$\text{SIC} = \text{n}^{\text{k/n}} \frac{\sum \hat{\text{u}}^2}{n} = \text{n}^{\text{k/n}} \frac{RSS}{n}$$

Just like with Akaike's Information Criterion, we will have to look for the lowest SIC to find the best fitted model.

Forecasting with a GARCH Model

When creating our GARCH-model the output data will give us different parameters in form of a constant and a parameter for each explanatory variable. In our model the constant will be denoted M, and simple letter detonations A,B,C and so on for the explanatory variables. Thus the GARCH(1,1) model will look like this:

$$\sigma_t^2 = \text{M} + \text{A}^*\varepsilon_{t-1}^2 + \text{B}^*\ \sigma_{t-1}^2$$

In the multivariate case, we need to keep in mind that the parameters above are matrices. In our case each parameter is a 10x10 matrix.
The model allows us to forecast the variance at time t, by using the GARCH-model estimations to create a statistically significant expression that describes the variance at time t as a function of the lagged variance (i.e. the variance σ^2 at time t-1), the lagged squared residuals at time t-1 and a constant. Thus we are capable of easily using our model and data to forecast a covariance matrix for our industry portfolios, as the model we will be using is multivariate.

But what if we want to forecast multiple periods ahead? This can't be done by only using the forecasted variance in the model. The reason for this is simply that the GARCH-model deals with time-varying risk, thus the error-terms at t are different from those at t-1. Therefore, to be able to forecast beyond time t, we need to do a statistical simulation of the model's residuals. What we are eventually interested in obtaining is a forecasted variance, based on simulated error-terms. If we start by looking at a forecast for the variance at time t+1, we know from the model above that the function will look like this:

$$\sigma_{t+1}^2 = M + A^* \varepsilon_t^2 + B^* \eta_{t+1}^2$$

Where eta $\eta_{t+1} = \varepsilon_{t+1}{}^* \sigma_t$

The eta is, in other words, the deviations from the means in each portfolios return, i.e. $r_t = \mu + \eta_t$, where $\eta_t \mid \vartheta_{t-1} \sim N(0, \sigma_{t-1}^2)$ and $r_t \mid \vartheta_{t-1} \sim N(\mu, \sigma_{t-1}^2)$

In our case these denotations of N are not standardized normal distributions, but instead multivariate distributions, which has a mean vector consisting of ten zeros (0,0,0,0,0,0,0,0,0,0) and a covariance matrix of σ_{t-1}^2, which will be obtained from the results of our multivariate GARCH forecast.

We will then be able to express the returns of our 10 portfolios by:

$$r_{t+1} = \mu + \varepsilon_{t+1} * \sigma_t \Leftrightarrow r_{t+1} = \mu + \eta_{t+1}$$

Where μ is a vector of the historical mean value of our returns, ε_t is a vector of (simulated) residuals at time t and σ_{t-1} is the 10x10 variance-covariance matrix, computed by our GARCH model in the previous period (t-1). This is interesting for us, as we shall use our simulated error terms, to calculate a large number of possible simulated returns for each period. From those returns, we can then calculate the variance.

Since we have already been able to forecast σ_t^2 and M, A, and B remain constant, we will need to find a statistically significant value of the error term at time t. This is where the Monte Carlo simulation begins. To simulate the error term we need to look at its characteristics. We know that if our model is assumed to hold, the error term is normally distributed around a zero mean value and has a standard deviation of 1:

$$\varepsilon_{t+1} \sim \mathrm{N}(0, \sigma_t^2) \text{ IID}$$

IID is an abbreviation of "Independent and Identically Distributed" meaning that the simulated numbers of the error terms all have the same probability distribution but are mutually independent. Again, we are dealing with a multivariate normal distribution, as described above, since our model is multivariate. If we used standardized normal distributions with a mean of zero and standard deviation of 1, which is the case in the univariate model, we would disregard the cross-correlations of the errors in the data series. In other words, we would not get useful results from univariate simulations of each of our portfolios independently.

The simulated error-terms for each industry-portfolio then needs to be multiplied by the standard deviations of the corresponding portfolio at t-1, to compute η_{t+1} which will then be added to the historical mean of returns. Thus, we will simulate a series of returns for our model, and calculate the variance, standard deviation for these simulations.

By following this procedure, we will create the first simulated vectors of r_{t+1}, which we will only use to forecast the variance. We will not go into forecasting of actual returns, only simulations of a large number of possible outcomes (i.e. purely hypothetical returns).

However, since the residuals used in this simulation are just possible values of the actual unknown residuals at time t+1, we will need to repeat this whole process a significant amount of times. This repetitive process is called iterations. To make it statistically significant we will need at least 1,000 iterations. We have chosen to make 3,000 just in case. We could also have done 5,000 or 10,000 or any number above 1,000 actually.

So by doing this repetition 3,000 times, we will get 3,000 possible returns of each portfolio. We will then be able to calculate the variance for each portfolio by using these 3,000 possible returns. The variance will be calculated as follows:

$$Var(r_{t+1}) = \frac{1}{3000}\sum_{i=1}^{3000}(r_{t+1}^{i} - R_{t+1})^2$$

Where, r^i is the estimated return number i of 3,000 and R_{t+1} is the historical mean value of the return series at time t+1.

This method then gives us a vector of variances for our portfolios at time t+1. The next step to forecasting the covariance matrix is then relatively simple. We know that we can compute covariances relatively easy, as they are the products of the two portfolios' standard deviations and their correlation coefficient:

$$Cov(A,B) = \sigma_A\sigma_B correlation(A,B)$$

As we have the vector of variances, we will simply take the square root of each number in the vector, to get the standard deviation of each portfolio. The trickier part is the correlation, but to make this part a bit easier, we will simply use the =CORREL function in MS Excel, to compute correlation-coefficients of our time-series of returns.

By using these calculations, we can simply "fill out" the covariance matrix for t+1, which in our case will be August 2012. This matrix will then be used to calculate the first rebalancing of our GMV-portfolio. We will be doing monthly rebalancing, to make our investment strategy "semi-passive".

This whole process is then repeated for t+2 , t+3 and so on, to forecast multiple periods ahead, the data that we have now created for t+1 will then be used to forecast the covariance matrices of t+2, etc.

Monte Carlo Simulations

The process called Monte Carlo Simulation is a widely recognized and valid technique that allows us to make random sampling within a certain probability distribution. Monte Carlo Simulation can produce hundreds or thousands of scenarios. This scenario creation is also called trials or iterations, and they can be used for statistical analysis. Generating random samples with Monte Carlo Simulation is very useful and valid because it follows an exact distribution[31]. Monte Carlo is particularly useful for random sampling of hypothetical values that mimic those of the population. In order for it to prove useful, we will however have to keep in mind that a very large amount of iterations are required.

The way Monte Carlo Simulation works, is that it takes a cumulative distribution function as $F(x) = P(X \leq x)$, which tells us a probability P of X being less or equal to x, ranging from zero to one. In Monte Carlo, we need to reverse this function which can be expressed as $G(F(x))=x$, and this reversion will be the basis of the random number generation. A random number r ranging

[31] David Vose, Risk Analysis, page 16

between zero and one is then plugged into the function, to determine the value generated for the specific distribution.[32]

In our case, we will need to start by defining the probability distribution and the specific characteristics of this distribution, for example in a normal univariate time-series model. The error-terms will follow a standardized normal distribution with a mean of zero and standard deviation of one. In our case, we need to go beyond this simplicity, and in our analysis we will use the results of our model to describe the distribution and the characteristics needed to compute a Monte Carlo Simulation. Needless to say we will be using software for this simulation as we are dealing with very large amounts of input- and output data.

Passive and Active Investment Strategies

When investing in portfolios, one of the big questions you have to ask yourself is whether you want to passively invest in indexes or funds that follows/mimics the market-portfolio, or if you want to try and predict the market outcome by managing your investments actively. This is of course also relevant to considering choices of potential investment funds with different strategies.

An active strategy is based on the trader's ability to predict market movements. The goal here is to deliver a return that is greater than what the market-portfolio is already yielding. This means that if you invest in an active fund, then there is a person who is tactically managing your money for you. And when a certain industry looks like it might be going up, the fund manager will move your money accordingly to expose you to extra profits, or fewer losses.

If we assume that markets are efficient, then all information has already been considered in the prices. With this in mind it will be very difficult for the active trader to earn back what he/she loses on transaction costs, especially compared to the passive index investments.

[32] David Vose, Risk Analysis, page 57

A passive strategy is based on buying assets and securities that follow an index. This means that it is pretty simple to decide what needs to be bought and sold. There are fewer trades, and thus less transaction costs over time, since you are tracking a market where you a charged less in comparison with the active fund.

There are a lot more options to choose actively managed funds compared to passively managed. The Invesmtent Management Association has more than 2000 actively managed funds on their list, while only 70 passive funds. However, access to passive investment strategies has risen dramatically in the past few years.[33]

Of course there are potentially lots of profits in predictions, but predicting future stock prices is not an easy task. If you however are good at generating predictive knowledge, there can be huge profits in selling your knowledge to investors, not to mention what you can earn from the market if you know what stock prices will rise and fall tomorrow. The only problem is just that this type of knowledge does not really exist. We can only make somewhat qualified guesses. Actively managed portfolios are basically build on this type of guessing, because no matter how talented people are at constructing portfolios, no one can tell what the future looks like. Sometimes your guess is right, and sometimes it is wrong. Studies have shown that not a lot of portfolio managers manage to beat the stock market to which their investment selection is linked[34]. An example is an analysis of the "UK ALL Companies Sector", that showed that only 24% of actively managed funds succeeded in beating the benchmark stock market during the past decade. This means that you basically have a 75% chance of ending up with a riskier fund that will not give you the desired return, which you could get by simply tracking a passive index. Additionally to this, you usually pay much higher fees and/or commissions to the managers of active funds, compared to their passive counterparts. The total average annual cost of an actively managed fund is 1.67%, which means that the active portfolio has to deliver almost 2% extra returns as well as beating the benchmark, to prove that it is valuable for the investors. In most cases this high cost will not

[33] www.finanshus.dk/pssiv-er-bedre-end-aktiv-nar-det-gaelder-investeringsstrategi/
[34] www.which.co.uk/money/savings-and-investments/guides/different-types-of-investment/active-vs-passive-investment/

justify the performance. And you will still have to pay a fee even though the active manager does not outperform the benchmark. So a loss of 2% could easily be doubled to 4%, and you have no choice but to pay it, because no actively managed funds will reduce your standard fees if they underperform, even though many ironically charge commissions.[35]

For passively managed portfolios, the future is not easier predictable, but here there are very low transaction costs compared to the actively managed funds, as they do not try to outperform the market by analyzing and rebalancing the portfolios frequently. This means less salary costs to analysts etc.[36]

To conclude it is always a good idea to spread your risk, keeping the transaction costs low and choosing at least a semi-passive investment strategy. This is also the recommendation of the Danish Consumer Council[37] and Burton Malkiel of Princeton University[38]. In Malkiels study, he presents evidence in favour of passive investment strategies, and examines the major criticisms of the technique. He concludes that the evidence strongly supports passive investment strategies in all markets, whether they are efficient or inefficient.

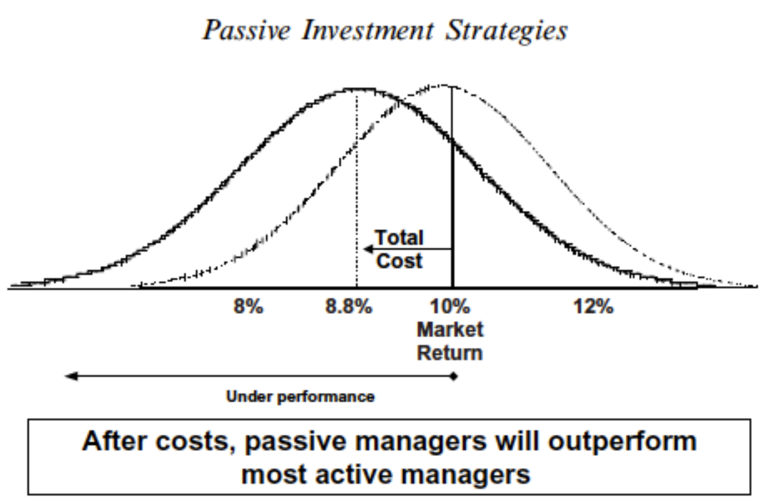
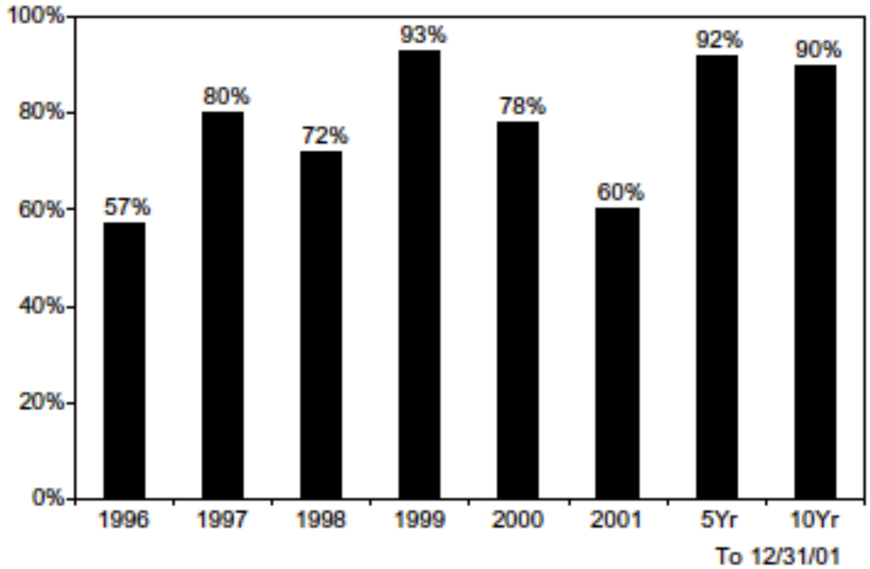


Figure 2 – Passive Investment Return Distribution

[35] www.forbes.com/sites/abrambrown/2013/06/13/taking-the-active-versus-passive-investing-debate-a-step-further/
[36] www.finanshus.dk/passiv-er-bedre-end-aktiv-nar-det-gaelder-investeringsstrategi/
[37] www.optimalpension.dk/index.php/aktiv-forvaltning-vs-indeksforeninger
[38] Passive Investment Strategies and Efficient Markets, European Financial Management, Vol. 9 no. 1, 2003

Passive Investment Strategies



Percentage of global bond funds outperformed by the Salomon World Government Bond index.

Source: The Vanguard Group, based on filter of more than 3,900 funds for portfolios with beta of between 0.9 and 1.0 and R^2 above 0.9 (final universe included 68 portfolios).

Figure 3 – Outperforming Bonds in Passive Investments

Indexing appears to be the best strategy in US markets. In the above shown figure, we see that 69% of the funds invested in US securities were outperformed by the Morgan Stanley Capital International Index in a 10 year period, ending December 31 2001.

The challenge for us is therefore to construct a portfolio with the right weights in the right securities. We want to argue that a passively managed portfolio will perform better than an actively managed, and therefore we follow the same 10 indexes in the US market all the time. We end up doing a semi-passive investment strategy, because we rebalance the weights in the portfolio each month. These rebalances will however be predicted with forecasting techniques, so all in all our strategy is passive. With that said, it is possible to actively manage a passive fund.

Efficient Market Hypothesis

The Efficient Market Hypothesis (abbreviated EMH) is a very commonly discussed and tested hypothesis. The hypothesis states three different types of efficient markets that are defined by how what or how much information is reflected in asset prices.

The underlying thought of an efficient market is very well-expressed by Eugene Fama in the conclusion of his 1965 thesis on market efficiency which states:

"*An efficient market is defined as a market where there are large numbers of rational, profit-maximizers actively competing, with each trying to predict future market values of individual securities, and where important current information is almost freely available to all participants…*"[39]

Note that this is also very consistent with the assumptions used in Modern Portfolio Theory. As mentioned, there are three types of efficiencies according to the EMH; The strong form, the semi-strong form and the weak form. We shall now describe these forms and to see how they apply to our approach.

Strong form: All public and privately available information has been incorporated into the price of the publically traded asset. This will require corporate officers and other potential insiders to have used potential insider-knowledge (of corporate strategies, internal books etc.) so extensively in their trade of securities, that the market prices now reflect that otherwise hidden information. This is only a limiting case because there may be institutional constraints on insider-trading or access to capital constraints put up to avoid unfair exploitation of potential arbitrage opportunities.[40]

[39] Bachelier et al., The Efficient Market Hypothesis, page 104
[40] Bachelier et al., The Efficient Market Hypothesis, page 105

Semi-strong form: Market prices on publically traded securities reflect all publically known information about the overall economy and the particular individual asset. Public knowledge of a certain company strategy, expectations etc. is also reflected in the price of that company's stock. These types of information are commonly published to the public in today's world in corporate annual reports, corporate statements, financial analysis, financial or corporate news-releases, and of course not to forget word of mouth exchange of information. This is a more common form of efficiency.

Weak form: The price of a security incorporates all known information on trading patterns, volume and other technical measures. All technical analysis has been arbitraged into the market price. If measures point to a higher security price, the price will simply instantly adjust due to an increased demand on the particular asset. It is commonly said that stock prices in the weak form are reflections of past trading information, i.e. past stock prices.

These three forms of efficient markets are obviously all very "simplified", and determining if a market is characterized by one or the other type of efficiency is not easy. Several studies conducted in the curricula related to this thesis topic have found indications for and against more than one of these forms, for example "Conrad & Kaul, 1988; Short Horizon Lagged Return Strategies" and several more studies within the literature from authors like Lo & Mackinlay, Fama & French, Campbell & Shiller, Keim & Stambaugh, etc. So basically we cannot with certainty determine which form applies to the US Stock Markets, which are the ones we will be working with, or if the US Stock Markets are even efficient and un-arbitraged.

As we will be working with a time-series model that uses information on past stock-price fluctuations to determine and forecast risk now and in the future, it would be convenient if we could proof or assume that the US Stock markets are efficient in the weak form. However this is not just something that we can do. In our discussion of our analysis-results we will look into the EMH and discuss whether we find indications for or against any of the forms, and how the different EMH forms can affect the results of our model.

The Capital Allocation Line and Sharpe Ratio

The Sharpe ratio is used to measure the reward you get for taking certain risk. The formula denotes the excess return as the difference between the return on a benchmark asset, and the risk free return, and this is divided with the risk, denoted as the standard deviation of the return.

$$S = \frac{E[R - R_f]}{\sqrt{\mathrm{var}[R]}}.$$

The Sharpe ratio indicates how good an investment is, and can be used in the comparison between two investments in order to decide which will be the preferred investment, and how you are compensated for taking on certain risk. The one with a higher Sharpe ratio delivers a better return for the same risk, or lower risk for the same return.
The advantage of the Sharpe ratio is that it is easily computable from any observed series of returns, and compared to the Treynor ratio that only works for the systematic risk of a portfolio, the Sharpe ratio takes both systematic and unsystematic risk into consideration when computing the ratio.[41]

Sharpe ratios are often practically used to rank performances of specific portfolios and mutual funds, and we will use it to compare the performance of our portfolios in the analysis.

Introduction to Modern Portfolio Theory and Risk/Return Relations

The modern portfolio theory (MPT) considers investments as a tradeoff between risk and expected return. Generally, assets with higher expected return are riskier than assets with lower expected return. Therefore, the modern portfolio theory comes out as an investment decision tool that tries to maximize the portfolio expected return for a given amount of portfolio risk, or minimize risk for a given level of expected return.

[41] Bodie, Kane and Marcus, Investment and Portfolio Management, page 161

By considering the relationship between risk and return, this theory tries to construct the optimal portfolio. According to modern portfolio theory, the risk of particular stock should not be looked at individually, but rather at how that particular stock's price varies in relation to the variation in price of the market portfolio. Modern portfolio theory impacts on how investors perceive risk, return and portfolio management. For a well-diversified portfolio, the risk - or average deviation from the mean - of each stock contributes little to portfolio risk. Instead, it is the difference - or covariance - between individual stock's levels of risk that determines overall portfolio risk. As a result, investors benefit from holding diversified portfolios instead of individual stocks.

Assumptions of the Modern Portfolio Theory

- The efficient market theory holds
- Asset returns are (jointly) normally distributed random variables
- Correlations between assets are fixed and constant forever. This assumption will actually not apply to our suggested model, as we propose time-varying covariance and correlations.
- All investors aim to maximize economic utility
- All investors are rational and risk-averse
- All investors have access to the same information at the same time
- Investors have an accurate conception of possible returns
- There are no taxes or transaction costs
- All securities can be divided into parcels of any size

None of these assumptions are entirely true, and each of them compromises the modern portfolio theory to some degree.

The procedure of modern portfolio theory

1. Step one: Data collection

2. Step two: Create a Markowitz efficient frontier
3. Step three: Create the market portfolio
4. Step four: Create the capital market line
5. Step five: The optimal portfolio

Portfolio risk depends on the correlation between the returns of the assets in the portfolio. Covariance and the correlation coefficient provide a measure of the way returns two assets vary

$$E(R_p) = \sum_{i=1}^{n} W_i E(R_i)$$

$$\sigma_p^2 = \sum_{i=1}^{n} W_i^2 \sigma_i^2 + \sum_i \sum_j W_i W_j \, COV_{ij}$$

Modern portfolio theory has some shortcomings in the real world. For starters, it often requires investors to rethink notions of risk. Sometimes it demands that the investor take on a perceived risky investment in order to reduce overall risk. That can be a tough sell to an investor not familiar with the benefits of sophisticated portfolio management techniques. Furthermore, MPT assumes that it is possible to select stocks whose individual performance is independent of other investments in the portfolio. But market historians have shown that there are no such instruments; in times of market stress, seemingly independent investments do, in fact, act as though they are related

Although modern portfolio theory has limitations, it is still accepted. The post modern portfolio theory is a significant advancement of the theory. Post-modern portfolio theory encourages far greater diversification in an investment portfolio.

Optimization of the Multi-Asset Portfolio

As the modern portfolio theory states, we seek to optimize the Sharpe ratio of a portfolio. Thus, we can derive the tangency portfolio, which constitutes the Capital Allocation Line (CAL). When we are dealing with multiple assets we have to construct a covariance matrix of the portfolio. Such a covariance matrix looks like the table below. It includes the weights of the assets and is also called the bordered covariance matrix. w is the weight of the assets:

		Asset 1 w_1	Asset 2 w_2	Asset 3 w_3
Asset 1	w_1	σ_1^2	σ_{12}	σ_{13}
Asset 2	w_2	σ_{21}	σ_2^2	σ_{23}
Asset 3	w_3	σ_{31}	σ_{32}	σ_3^2

Table 1 – Bordered Covariance Matrix

From here, we can calculate the portfolio variance by multiplying the weights with the corresponding covariances, and add up across cells. Mathematically, we can express it:

$$\sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N} w_i w_j \sigma_{ij}$$

Doing the double-summations is just holding i = 1 and then multiplying with j = 1,2,3...n , then holding i=2 and doing the same and so on and so on, until we include all possible covariance combinations.

In the mathematical optimization process, we will also need a matrix of asset weights, expected returns and the unity-matrix. We will not do the actual mathematical optimization that derives the formula, as this is not relevant to our analysis. We will however describe how to implement the mathematical formula. Furthermore, we will not describe certain knowledge of math/calculus and how matrices are multiplied, inversed, added, subtracted etc., but we will demonstrate it in our empiric part. The matrices needed are shown below:

Portfolio weights: $w = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{pmatrix}$ Expected returns: $E(r) = \bar{r} = \begin{pmatrix} \bar{r}_1 \\ \bar{r}_2 \\ \vdots \\ \bar{r}_N \end{pmatrix}$

Covariance matrix: $\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & \vdots & & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_N^2 \end{pmatrix}$ $\mathbf{1} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$

Figure 4 – Matrices used in MPT

1 is the unity matrix, a matrix wherein all the numbers are 1.
It will also be necessary to invert the covariance matrix, deriving the inverse covariance matrix, which is denoted $\sum^{-1}$.

From the above, we can express expected return and variance:

$$\bar{r}_p = w^\top \bar{r}$$
$$\sigma_p^2 = w^\top \Sigma w$$

Where the T-shaped sign denotes multiplication of/into matrices.

As before mentioned, we are seeking to optimize the Sharpe ratio of the portfolio, subject to the constraint that the weight of the individual assets sums up to 1.
Then we can write the optimization problem:

$$\max_w S = \frac{E(r_p) - r_f}{\sigma_p} = \frac{w^\top \bar{r} - r_f}{\sqrt{w^\top \Sigma w}}$$
$$\text{s. t. } w^\top \mathbf{1} = 1$$

By taking the derivative and setting equal to zero the solution is given by:

$$w_{tan} = \frac{\Sigma^{-1}(\bar{r} - r_f \mathbf{1})}{\mathbf{1}^\top \Sigma^{-1}(\bar{r} - r_f \mathbf{1})}$$

Where w_{tan} is the matrix that constitutes the weights of assets in the tangency portfolio, which puts us on P, in the figure below[42]. Note that P' is the global minimum variance portfolio and the optimized Sharpe ratio is the slope of CAL.

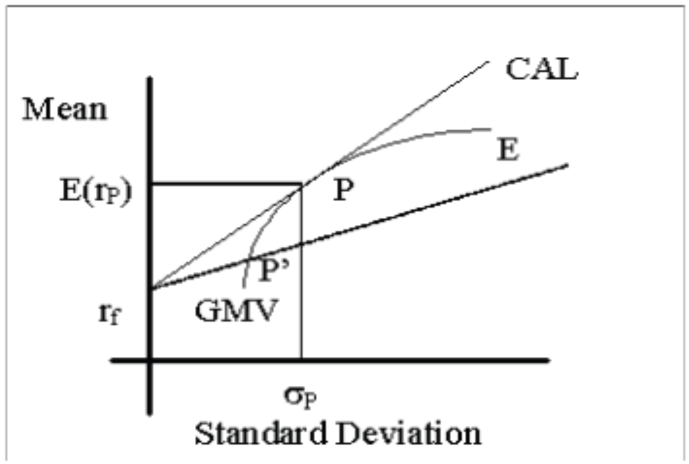


Figure 5 – The Capital Allocation Line

We have now described the principles of the mean-variance optimization as well as derived and provided the mathematical tools to find the tangency portfolio.

The Global Minimum Variance Portfolio

In modern portfolio theory there's also a portfolio referred to as the Global Minimum Variance portfolio (GMV). The GMV is the efficient portfolio that has the lowest risk of all possible efficient portfolios, i.e. it lies on the exact point of Markowitz's efficient frontier that begins the positive slope of the line. The GMV is illustrated in the figure below:

[42] Google Image Search

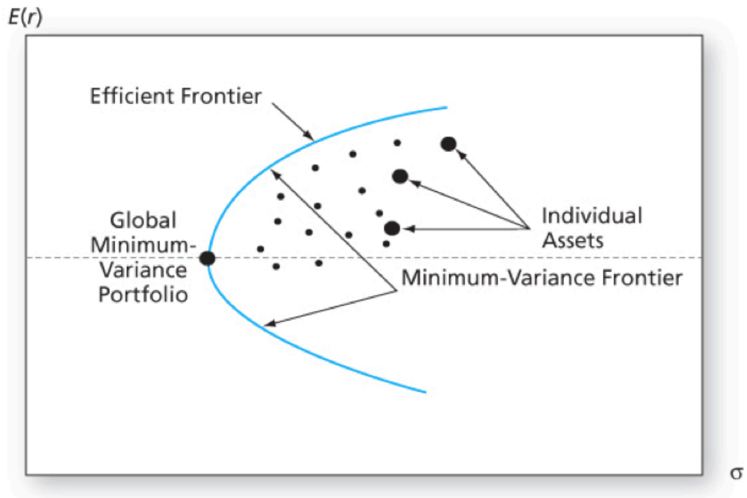


Figure 6 – Markowitz's Efficient Frontier

It is easily seen by this illustration, and the definition of the GMV, that obtaining the asset weights of the GMV can be done by a simple mathematical optimization process. In other words, we need to differentiate the variance with respect to the asset-weights, setting it equal to zero and then solving. The solution is not too different as with the tangency portfolio, as the same mathematical approach is applied. In fact, the GMV solution is simpler as it doesn't take expected returns into consideration, but simply minimizes the variance of the efficient portfolios. The minimization problem to finding the GMV can be expressed as:

$$\min_{w} \sigma_p^2$$
$$\text{s. t:} \quad w^\top \mathbf{1} = 1$$

The solution will then be:

$$w_{GMV} = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}^\top \Sigma^{-1}\mathbf{1}}$$

The denominator is the sum of the weights of the involved assets, which in our case are portfolios. This ensures that the weights sum up to one (i.e. 100%). Thus, the GMV weights are basically just found by plotting $\Sigma^{-1}\mathbf{1}$, and then scaling it by the sum of all the weights.

Applying the Results of Our Model

Under the assumption that risk is time-varying, we have proposed a model that seeks to predict the variance-covariance matrices for 10 industry portfolios. In other words, the model gives us some knowledge of the risk profiles of our assets, which can be used in the optimization processes of the modern portfolio theory. The multivariate GARCH-model can make predictions on the asset risks, but we have to beware of the returns. If we were to assume that the assumptions of the Capital Asset Pricing Model (CAPM) hold, we could argue that the expected returns of our assets would be a function of our predicted risk, or a so-called risk-premium. However, this is not what we are trying to do in this thesis and in reality it is highly unlikely that the CAPM fully holds. In general we don't want to be forecasting returns on our equities, as they have proven quite impossible to predict in the case of stock-price fluctuations. The reason for this is quite simply, that we can never surely identify all explanatory variables that stock prices would be a function of, as well as be sure of the direction of causality in the variables. The CAPM and later the three factor model by Fama & French are simplified suggestions, that can describe returns, but they will not be able to fully predict them.

As there would not be validity in statistical prediction of returns, we want to refrain from using them in the creation of our portfolios. This would mean that instead of using our results to set up the tangency portfolio, we could stick to creating GMV portfolios, as this would only require our predicted time-varying risk. For the sake of the validity of our results, using the GMV would seem to be the obvious choice. But we also need to take into consideration, how the GMV seems to perform with respect to the tangency portfolio. It would not make a lot of sense for us to create a model, which focuses on creating a GMV portfolio, if tangency portfolios in general seem to significantly outperform GMV portfolios. So to present a final argument for using the GMV, we shall refer to Haugen & Baker (2012) "*Low Risk Stocks Outperform within All Observable Markets of the World*". In this research paper, they present results showing that historical stock prices would not be in correspondence with the concepts of the CAPM and the Efficient Market Hypothesis, stating full efficiency in market information. Their results show a general tendency of low-risk stock outperforming the more volatile alternatives, creating a very significant anomaly with regards to modern finance theory. This is showing clear signs, that the results of this theory are highly biased. Haugen & Baker also refer to numerous other studies, showing that minimum-variance or low-volatility portfolios outperform the riskier alternatives. Of course there is no doubt that riskier assets yield higher returns, but in terms of performance, we need to consider the return-to-risk ratio, which is exactly what we do when we compare portfolios on their Sharpe ratio.

What we can take from this research paper, with regards to our results, is that it would make absolute sense for us to create a GMV-portfolio when we suggest an optimal portfolio, as it might outperform the potential tangency portfolio, because our predictions of returns could turn out to be completely off. This means that we will use the GMV and refrain from trying to predict stock-returns, and instead sticking to the risk in terms of variance-covariance matrices with time-varying risk. This way we can present more valid results and hopefully propose a portfolio that could outperform portfolios build on simpler measures of risk.

Part 3 - Analysis

Econometric Analysis

In this first part of the analysis we will apply the econometric theory to conduct the actual analysis.

Testing for ARCH

There are 2 ways we can run the ARCH-test. Either we can do it manually using the Engle procedure, or we can do it with the help of econometric-capable software.
Firstly, we want to demonstrate the theories and formulas behind it by showing how it can be done manually. Secondly, we will run the process using 'SAS Enterprise Guide'. These answers will be the same. For the understanding of the ARCH-testing, we find it relevant to show both methods.

The ARCH-test will be analyzed for our 10 time series of returns from the industry portfolios, via the selected software.

The Manual Approach

From the Engle procedure, we know that as far as an ARCH(p) is considered, the following auxiliary regression can be used to test for a p order ARCH model. This procedure is the same for all 10 time series returns:

$$\hat{u}_t^2 = \hat{\alpha}_0 + \hat{\alpha}_1 \hat{u}_{t-1}^2 + \hat{\alpha}_2 \hat{u}_{t-2}^2 + \ldots + \hat{\alpha}_p \hat{u}_{t-p}^2$$

The null hypothesis (i.e. the data that will support that there is no impact from ARCH and will be kept until alternative hypothesis can be accepted)

$$H_0: \alpha_1 = \alpha_2 = \ldots = \alpha_p = 0$$

and the statistics is nR^2 that follows a chi-squared distribution with degrees of freedom equal to the number of autoregressive terms in the auxiliary regression. The “degree of explanation”-coefficient R^2 is obtained from the estimates of the auxiliary equation.

The value of p is an empirical question, and since we don’t have any specific information about p from previous analysis, we first test for an ARCH(1), that is:

$$\hat{u}_t^2 = \hat{\alpha}_0 + \hat{\alpha}_1 \hat{u}_{t-1}^2$$

Firstly, we build the formula of the needed variables. We already obtained the residuals from $r_t = \beta_1 + u_t$, and squared them. Then we create a new variable, which is the first lag of the squared residuals.

Now we run the ordinary least squares regression (OLS) for the auxiliary model. The squared residuals here are explained by their first lag.

Lastly we run the ARCH/Engle test for H_0: No ARCH.

The SAS Enterprise Guide Approach

Fortunately, SAS runs these econometric forecasts in a user-friendly way. This means we can run our equation of interest in the auto-regressive procedure and create the ARCH test via Enterprise Guide. See Appendix 3 for the process of ARCH testing in SAS Enterprise Guide. The reader can use Appendix 3 as a "how to" guide of running this procedure in SAS.

In SAS, it is important that the data is analyzed using the Time Series function, as our data represents returns from time series. We need to complete a regression analysis with autoregressive errors in the series of returns for each individual industry. Thus, the dependent variable will be one of the time series returns for a given industry-portfolio; this is repeated for every industry. It is important that the regression analysis with autoregressive errors is calculated with maximum likelihood estimates that are fit to the autoregressive model. The results we are interested in are the Q and LM statistics so we can determine whether or not there is absence of ARCH effects.

The results of the 10 time series are presented below:

Durables:

Tests for ARCH Disturbances Based on OLS Residuals				
Order	Q	Pr > Q	LM	Pr > LM
1	26.0554	<.0001	25.9298	<.0001
2	46.4541	<.0001	39.9677	<.0001
3	57.6029	<.0001	44.5586	<.0001
4	68.6329	<.0001	48.8967	<.0001
5	70.8613	<.0001	48.9136	<.0001
6	94.9611	<.0001	64.5035	<.0001
7	109.2202	<.0001	69.0820	<.0001
8	164.1045	<.0001	100.5784	<.0001
9	224.5124	<.0001	128.2247	<.0001
10	233.9457	<.0001	128.3380	<.0001
11	244.5587	<.0001	128.9285	<.0001
12	259.3955	<.0001	130.8086	<.0001

Figure 7 – Durables ARCH Test

Energy:

Tests for ARCH Disturbances Based on OLS Residuals				
Order	Q	Pr > Q	LM	Pr > LM
1	30.7716	<.0001	30.7423	<.0001
2	46.8607	<.0001	40.3892	<.0001
3	81.0077	<.0001	63.1436	<.0001
4	85.3527	<.0001	63.1644	<.0001
5	87.2501	<.0001	63.1663	<.0001
6	96.3666	<.0001	66.5546	<.0001
7	132.1238	<.0001	91.8373	<.0001
8	163.0529	<.0001	105.1353	<.0001
9	243.0227	<.0001	147.2134	<.0001
10	278.7288	<.0001	151.4901	<.0001
11	284.5507	<.0001	152.3491	<.0001
12	295.9072	<.0001	152.4695	<.0001

Figure 8 – Energy ARCH Test

Health:

Tests for ARCH Disturbances Based on OLS Residuals				
Order	Q	Pr > Q	LM	Pr > LM
1	103.4494	<.0001	103.2177	<.0001
2	156.6138	<.0001	121.6935	<.0001
3	252.2938	<.0001	167.5782	<.0001
4	263.1689	<.0001	171.8663	<.0001
5	268.5186	<.0001	172.0574	<.0001
6	276.5471	<.0001	172.1194	<.0001
7	290.7457	<.0001	181.1250	<.0001
8	405.0657	<.0001	269.0889	<.0001
9	475.6477	<.0001	277.3655	<.0001
10	500.7064	<.0001	279.5716	<.0001
11	635.5234	<.0001	304.3315	<.0001
12	672.4200	<.0001	304.5051	<.0001

Figure 9 – Health ARCH Test

Hi-Tech:

Tests for ARCH Disturbances Based on OLS Residuals				
Order	Q	Pr > Q	LM	Pr > LM
1	73.2649	<.0001	73.1164	<.0001
2	122.6088	<.0001	97.3784	<.0001
3	181.3832	<.0001	122.1124	<.0001
4	203.9733	<.0001	123.2267	<.0001
5	219.9427	<.0001	123.9389	<.0001
6	242.9132	<.0001	127.9473	<.0001
7	268.4519	<.0001	133.3860	<.0001
8	376.5611	<.0001	193.2494	<.0001
9	492.1927	<.0001	228.8402	<.0001
10	524.7275	<.0001	229.0885	<.0001
11	606.4542	<.0001	240.3209	<.0001
12	637.1376	<.0001	240.5054	<.0001

Figure 10 – High Tech ARCH Test

Manufacturing:

Tests for ARCH Disturbances Based on OLS Residuals				
Order	Q	Pr > Q	LM	Pr > LM
1	74.5403	<.0001	74.3510	<.0001
2	92.2286	<.0001	78.2126	<.0001
3	124.4176	<.0001	96.0966	<.0001
4	141.0941	<.0001	98.2815	<.0001
5	145.7176	<.0001	98.2913	<.0001
6	158.6625	<.0001	102.9012	<.0001
7	169.4891	<.0001	104.2755	<.0001
8	294.7580	<.0001	202.0458	<.0001
9	406.0605	<.0001	230.9373	<.0001
10	425.0441	<.0001	231.3089	<.0001
11	454.7881	<.0001	233.4044	<.0001
12	475.4388	<.0001	233.4524	<.0001

Figure 11 – Manufacturing ARCH Test

Non-Durables:

Tests for ARCH Disturbances Based on OLS Residuals				
Order	Q	Pr > Q	LM	Pr > LM
1	32.1135	<.0001	32.0631	<.0001
2	54.7329	<.0001	46.6304	<.0001
3	89.1616	<.0001	67.3304	<.0001
4	98.7255	<.0001	68.4151	<.0001
5	106.6281	<.0001	69.4747	<.0001
6	119.2626	<.0001	72.8767	<.0001
7	123.9838	<.0001	73.1255	<.0001
8	183.5106	<.0001	114.1506	<.0001
9	243.9442	<.0001	140.0478	<.0001
10	268.3192	<.0001	142.9336	<.0001
11	342.5786	<.0001	167.7152	<.0001
12	355.6479	<.0001	168.0617	<.0001

Figure 12 – Non-Durables ARCH Test

Others:

Tests for ARCH Disturbances Based on OLS Residuals				
Order	Q	Pr > Q	LM	Pr > LM
1	186.4060	<.0001	185.9198	<.0001
2	218.2859	<.0001	185.9410	<.0001
3	290.2217	<.0001	231.8105	<.0001
4	309.5264	<.0001	235.9928	<.0001
5	315.7100	<.0001	237.2455	<.0001
6	325.3996	<.0001	237.3198	<.0001
7	334.0213	<.0001	238.9072	<.0001
8	384.9835	<.0001	272.2679	<.0001
9	461.2102	<.0001	282.5139	<.0001
10	505.3170	<.0001	284.6291	<.0001
11	532.7791	<.0001	284.8302	<.0001
12	542.2461	<.0001	287.6340	<.0001

Figure 13 –Others ARCH Test

Shops:

Tests for ARCH Disturbances Based on OLS Residuals				
Order	Q	Pr > Q	LM	Pr > LM
1	38.4144	<.0001	38.3324	<.0001
2	84.4805	<.0001	70.7846	<.0001
3	138.4176	<.0001	99.2785	<.0001
4	147.7151	<.0001	99.2790	<.0001
5	153.0365	<.0001	99.4261	<.0001
6	164.0549	<.0001	101.4307	<.0001
7	180.6992	<.0001	109.3260	<.0001
8	283.6198	<.0001	184.6201	<.0001
9	351.9560	<.0001	206.9837	<.0001
10	372.7374	<.0001	207.6718	<.0001
11	475.0096	<.0001	229.1456	<.0001
12	483.5904	<.0001	231.7973	<.0001

Figure 14 – Shops ARCH Test

Telecom:

Tests for ARCH Disturbances Based on OLS Residuals				
Order	Q	Pr > Q	LM	Pr > LM
1	68.1853	<.0001	68.0900	<.0001
2	107.6231	<.0001	86.6656	<.0001
3	163.7493	<.0001	113.5290	<.0001
4	207.9373	<.0001	124.6457	<.0001
5	224.6607	<.0001	124.8381	<.0001
6	261.9870	<.0001	134.5584	<.0001
7	275.7892	<.0001	134.5672	<.0001
8	297.6738	<.0001	138.1815	<.0001
9	337.2721	<.0001	148.1004	<.0001
10	359.0346	<.0001	148.8212	<.0001
11	415.8398	<.0001	169.1209	<.0001
12	421.2138	<.0001	177.4804	<.0001

Figure 15 – Telecommunications ARCH Test

Utilities:

Tests for ARCH Disturbances Based on OLS Residuals				
Order	Q	Pr > Q	LM	Pr > LM
1	65.0150	<.0001	64.8153	<.0001
2	93.8098	<.0001	76.7501	<.0001
3	170.1953	<.0001	124.3753	<.0001
4	210.8566	<.0001	131.5909	<.0001
5	234.0327	<.0001	133.7492	<.0001
6	255.4389	<.0001	134.6930	<.0001
7	307.3416	<.0001	150.9347	<.0001
8	408.1319	<.0001	192.3361	<.0001
9	449.5526	<.0001	195.1107	<.0001
10	510.9398	<.0001	205.5127	<.0001
11	691.9926	<.0001	268.8365	<.0001
12	711.9822	<.0001	273.5859	<.0001

Figure 16 – Utilities ARCH Test

From the results we notice that there are clear indications of very low probabilities in all of the time series. This means that in all specifications, the lagged squared residuals have a significant role. Because of that, we can reject the null-hypothesis of no ARCH and accept the presence of an ARCH process. This result might be due to only an ARCH(1) effect, or maybe an ARCH(q) of a higher order. Notice that the Engle test corresponds to the LM-value and its corresponding p-value in the output from SAS Enterprise Guide.

The statistics Q is distributed as a $\chi^2(p)$, where p is the degrees of freedom being the number of regressors in the auxiliary equation. Since p is equal to 1, the critical value at 95% of significance is 3,84. This means that we can reject the null hypothesis, and that we cannot deny the presence of an ARCH(1)-process. When only 1 lag is involved, as in this case, it would also have been possible to use a simple t-test to check if the coefficient of the lagged residual is equal to 0. This approach actually gives approximately the same significance level as the one we obtained above. We could also test for a higher-order ARCH model, to verify if only the ARCH(1) is statistically significant, for example ARCH(3), but this will not bring any further relevance to our study since we just needed to test for the presence of ARCH, which we now can accept. The same results are showing in the tests of all the time series returns, and we can therefore conclude that ARCH is present in every time series.

Setting Up the Model

As we have suggested earlier, the model we will be using to estimate the time-varying risk of our 10 industry portfolios is a GARCH (1,1) model. The GARCH model can be expressed as:

$$\text{The GARCH(1,1) model}:$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \lambda_1 \sigma_{t-1}^2$$

We have used the same model and procedure as Bollerslev, Engle & Wooldridge (1988), when they use a multivariate GARCH model to estimate a Capital Asset Pricing Model with time-varying risk. In the research paper, Bollerslev, Engle & Wooldridge only use 3 parameters in their multivariate model, returns on T-Bills, Bonds and Stocks, whereas we will be using 10. To illustrate how our model is constructed, we will refer to the model estimates of Bollerslev, Engle & Wooldridge which is estimated by maximum likelihood and looks like this:

$$\left|\begin{array}{c} y_{1t} \\ y_{2t} \\ y_{3t} \end{array}\right| = \left|\begin{array}{r} .070 \\ (.032) \\ -4.342 \\ (1.030) \\ -3.117 \\ (.710) \end{array}\right| + \begin{array}{c} .499 \\ (.160) \end{array} \sum_j \omega_{jt-1} \left|\begin{array}{c} h_{1jt} \\ h_{2jt} \\ h_{3jt} \end{array}\right| + \left|\begin{array}{c} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \end{array}\right|$$

$$\left|\begin{array}{c} h_{11t} \\ h_{12t} \\ h_{22t} \\ h_{13t} \\ h_{23t} \\ h_{33t} \end{array}\right| = \left|\begin{array}{c} .011 \\ (.004) \\ .176 \\ (.062) \\ 13.305 \\ (6.372) \\ .018 \\ (.009) \\ 5.143 \\ (2.820) \\ 2.083 \\ (1.466) \end{array}\right| + \left|\begin{array}{l} \underset{(.105)}{.445} \cdot \epsilon_{1t-1}^2 \\ \underset{(.092)}{.233} \cdot \epsilon_{1t-1}\epsilon_{2t-1} \\ \underset{(.113)}{.188} \cdot \epsilon_{2t-1}^2 \\ \underset{(.132)}{.197} \cdot \epsilon_{1t-1}\epsilon_{3t-1} \\ \underset{(.093)}{.165} \cdot \epsilon_{2t-1}\epsilon_{3t-1} \\ \underset{(.066)}{.078} \cdot \epsilon_{3t-1}^2 \end{array}\right| + \left|\begin{array}{l} \underset{(.056)}{.466} \cdot h_{11t-1} \\ \underset{(.052)}{.598} \cdot h_{12t-1} \\ \underset{(.215)}{.441} \cdot h_{22t-1} \\ \underset{(.361)}{-.362} \cdot h_{13t-1} \\ \underset{(.338)}{-.348} \cdot h_{23t-1} \\ \underset{(.333)}{.469} \cdot h_{33t-1} \end{array}\right|$$

Figure 17 - Bollerslev, Engle & Wooldridge Estimates

The figure above illustrates the model used by Bollerslev, Engle and Wooldridge, and will constitute the frame of our model. We will mainly refer to the section "EViews Results" or appendix 1 for a similar full-image of the multivariate model that we propose using the 10 industry-portfolios.

The multivariate GARCH-model is estimating the time-varying variance-covariance matrix. The covariances h, corresponding to σ_t^2 in the standard GARCH model, are expressed by the bottom part of the model, whereas the top part simply illustrates the CAPM. We will keep our focus on the h-estimates. Just like our model, the estimates for h are a function that is a sum of 3 matrices. The first one is a matrix of constants. The second one is a matrix wherein each number is the product of the two involved assets/portfolios' error-terms at t-1, multiplied by the estimate for α. As an example, h_{12t} is the estimate of the covariance at time t between assets 1 and 2. Thus, the error-term for 1 at time t-1 is multiplied by the error-term for 2 at time t-1 and then multiplied by α. And finally, there is one matrix wherein each number is the product of the one period lagged h, or variance-covariance matrix, and the estimated λ. Simply denoted λ multiplied by h_{t-1}. The sum of these 3 matrices, as seen above, is what the model predicts to be the covariance between the 2 particular assets at time t, i.e. h_{12t} is the covariance at time t between assets 1 and 2.

In our case, we will simply extend the model so that each parameter in the equation is a constructed 10x10 matrix containing the same data on our 10 industry portfolios, as Bollerslev, Engle & Wooldridge used in their model. Giving us covariance matrices like this one:

$$\begin{pmatrix} \mathrm{cov}(M_1,M_1) & \mathrm{cov}(M_1,M_2) & \cdots & \mathrm{cov}(M_1,M_n) \\ \mathrm{cov}(M_2,M_1) & \mathrm{cov}(M_2,M_2) & \cdots & \mathrm{cov}(M_2,M_n) \\ \vdots & \vdots & \ddots & \vdots \\ \mathrm{cov}(M_n,M_1) & \mathrm{cov}(M_n,M_2) & \cdots & \mathrm{cov}(M_n,M_n) \end{pmatrix}_{n\times n}$$

Figure 18 – Illustrative output covariance matrix

Luckily, our software package E-Views allows us to obtain all the data we need to construct the functions of σ_t^2. By doing this, we can simply calculate each matrix just as described above, and then finally summing up the 3 matrices of the GARCH(1,1) model will give us the forecast or estimate of the time-varying variance-covariance matrix at time t.

Eviews 7.0

When analyzing our time-series which contains 10 times 1038 observations, we need a software package that is capable of dealing with a large amount of data and process it into the desired econometric applications that we want to use. We have worked through different software packages and found that the software that was best suited for estimating our GARCH-model and computing the conditional covariance-matrices needed for our forecasts was EViews 7.0. The template in EViews is relatively user-friendly and does not involve any necessities to program advanced statistical and econometric functions. Our coursework on CBS has not included any type of programming and we have previously worked in SAS, which has not required a great deal of programming. Through the book "Introductory Econometrics for Finance" by Chris Brooks, we would learn to navigate EViews and use it for estimating our GARCH model. In Appendix 4 we have attached a step-by-step user-guide explaining how to create the model, and in the following we will go through how the model is set up in EViews.

The first step of our analysis was importing our data into EViews. It was originally downloaded by the Kenneth French Website (see the Data section) as a text-file, which we converted/imported into MS Excel. From MS Excel we could import the data into EViews, where EViews would create separate time series for each of our assets.

When creating our GARCH model in EViews, we will mark all of the return-series for our assets. Next, we open them as "a system" in EViews, creating the platform for a multivariate-model of all the selected data-series. When we have the open system ready in EViews, we will need to ask the software to create the model for us. We simply have to specify the type of model we want. We can click the "Proc" button and choose "Estimate", we will then set "ARCH – Conditional Heteroskedasticity" as our estimation method and choose "Diagonal VECH" as model type.

Recall that we take the same approach as Bollerslev, Engle & Wooldridge, to estimate the model. "Under autoregressive order" we will write a number for "ARCH" which will be equivalent to "p" in the GARCH(p,q) model, where the number written in "GARCH" will be our q. We will estimate all possible models of with p=1,2,3 and q=1,2,3 and use the models information criterions as model selection criteria. GARCH(1,1) is shown to be the best model. As "Restrictions", we choose "Full Rank Matrix"-which ensures that the variance-covariance matrix will be positive and definite. Finally, we choose a multivariate normal error distribution which is a given when we are dealing with this type of multivariate model. By running the model, we will have EViews estimate the desired multivariate GARCH model, and compute parameter estimations, conditional covariances etc. To see an explanation of how the diagonal VECH M-GARCH model-estimation works, we refer to the theory section.

The first output shown in EViews will be the full model and its statistics, such as parameters, information criteria etc., and by scrolling down, we can get a full overview of the 10x10 matrices that constitutes the models parameters. These are the ones we will need for forecasts and that will be shown in the following section. The full output is displayed in Appendix 1.

By clicking "View → Conditional Covariance" and choosing "Matrices" for the whole sample, we can get the conditional covariance matrices for each observed period in our time series. These will also be needed as input in the model, when computing a forecast.

Finally, we need to find the residual to have all the inputs we need for our forecast. By clicking "Proc → Make Residuals" and clicking "OK" with the standard settings, we will get the model residuals for each asset in each observed period.

By going through these steps, EViews will give us the outputs and estimations needed to perform a forecast for the next period. Note that the model can only be used to forecast the conditional covariance of t+1, assuming that the latest observation is at time t. Trying to forecast beyond this period will violate the assumption of conditional normality in the parameters, as they would no longer certainly follow a normal distribution.

Eviews Results

The results from EViews are as follows, where the constant matrix is denoted M, A1 is what we've previously described as α, and B1 corresponds to our previous denotation of λ are shown below. The full estimation output can be found in appendix 1. Here, we will show the computations relevant to make the further calculations of forecasts. Note the Eviews denotation of the GARCH function, not using Greek-letters as the theory has, GARCH = M + A1.*RESID(-1)*RESID(-1)' + B1.*GARCH(-1):

Covariance specification: Diagonal VECH

GARCH = M + A1.*RESID(-1)*RESID(-1)' + B1.*GARCH(-1)

M is a full rank matrix

A1 is an indefinite matrix*

B1 is an indefinite matrix*

	Transformed Variance Coefficients			
	Coefficient	Std. Error	z-Statistic	Prob.
M(1,1)	1.779755	0.314133	5.665612	0.0000
M(1,2)	1.044367	0.225269	4.636090	0.0000
M(1,3)	1.285452	0.251341	5.114384	0.0000
M(1,4)	1.603219	0.259095	6.187776	0.0000
M(1,5)	1.466959	0.212375	6.907415	0.0000
M(1,6)	0.968649	0.149705	6.470387	0.0000
M(1,7)	1.513981	0.226883	6.672948	0.0000
M(1,8)	1.149535	0.188581	6.095726	0.0000
M(1,9)	0.870015	0.169345	5.137534	0.0000
M(1,10)	0.720145	0.142573	5.051057	0.0000
M(2,2)	1.410480	0.344340	4.096185	0.0000
M(2,3)	0.969674	0.222104	4.365861	0.0000

M(2,4)	1.172278	0.222548	5.267542	0.0000
M(2,5)	1.061481	0.194117	5.468241	0.0000
M(2,6)	0.648617	0.140105	4.629497	0.0000
M(2,7)	1.131003	0.207243	5.457382	0.0000
M(2,8)	0.731317	0.171450	4.265492	0.0000
M(2,9)	0.562104	0.123450	4.553286	0.0000
M(2,10)	0.721378	0.142268	5.070554	0.0000
M(3,3)	2.178273	0.390060	5.584454	0.0000
M(3,4)	1.840614	0.264766	6.951848	0.0000
M(3,5)	1.539908	0.221266	6.959530	0.0000
M(3,6)	1.030008	0.170374	6.045585	0.0000
M(3,7)	1.506984	0.235152	6.408556	0.0000
M(3,8)	1.104609	0.184507	5.986817	0.0000
M(3,9)	0.894666	0.164703	5.432006	0.0000
M(3,10)	0.784932	0.149957	5.234371	0.0000
M(4,4)	2.507280	0.331246	7.569233	0.0000
M(4,5)	1.754851	0.212190	8.270207	0.0000
M(4,6)	1.054452	0.143626	7.341674	0.0000
M(4,7)	1.754103	0.224593	7.810157	0.0000
M(4,8)	1.293675	0.181207	7.139217	0.0000
M(4,9)	1.027131	0.159532	6.438399	0.0000
M(4,10)	0.766581	0.126149	6.076801	0.0000
M(5,5)	1.698063	0.209252	8.114907	0.0000
M(5,6)	1.028382	0.138214	7.440524	0.0000
M(5,7)	1.634663	0.206427	7.918852	0.0000
M(5,8)	1.189380	0.162300	7.328271	0.0000
M(5,9)	0.865693	0.136487	6.342660	0.0000
M(5,10)	0.766620	0.121133	6.328755	0.0000
M(6,6)	0.808931	0.130901	6.179723	0.0000
M(6,7)	1.126381	0.155551	7.241227	0.0000

M(6,8)	0.809178	0.125454	6.449979	0.0000
M(6,9)	0.625368	0.107383	5.823708	0.0000
M(6,10)	0.542598	0.089291	6.076733	0.0000
M(7,7)	1.838781	0.265213	6.933226	0.0000
M(7,8)	1.209783	0.170920	7.078082	0.0000
M(7,9)	0.873988	0.143191	6.103634	0.0000
M(7,10)	0.838355	0.126480	6.628350	0.0000
M(8,8)	1.065466	0.186232	5.721177	0.0000
M(8,9)	0.719286	0.125869	5.714543	0.0000
M(8,10)	0.566228	0.101677	5.568871	0.0000
M(9,9)	0.929143	0.157997	5.880775	0.0000
M(9,10)	0.570655	0.106080	5.379482	0.0000
M(10,10)	0.747034	0.140470	5.318104	0.0000
A1(1,1)	0.117998	0.011968	9.859797	0.0000
A1(1,2)	0.087988	0.011038	7.971204	0.0000
A1(1,3)	0.094266	0.011167	8.441316	0.0000
A1(1,4)	0.119004	0.011266	10.56269	0.0000
A1(1,5)	0.119094	0.010121	11.76698	0.0000
A1(1,6)	0.104683	0.009206	11.37137	0.0000
A1(1,7)	0.121965	0.010968	11.11991	0.0000
A1(1,8)	0.107154	0.010398	10.30503	0.0000
A1(1,9)	0.104178	0.013882	7.504357	0.0000
A1(1,10)	0.099192	0.012259	8.091599	0.0000
A1(2,2)	0.095697	0.014945	6.403340	0.0000
A1(2,3)	0.088529	0.011948	7.409358	0.0000
A1(2,4)	0.099001	0.011555	8.567939	0.0000
A1(2,5)	0.102765	0.010746	9.563021	0.0000
A1(2,6)	0.092056	0.009931	9.269824	0.0000
A1(2,7)	0.109058	0.011305	9.646608	0.0000
A1(2,8)	0.090551	0.010552	8.581172	0.0000

A1(2,9)	0.077724	0.012067	6.440851	0.0000
A1(2,10)	0.071975	0.010994	6.546765	0.0000
A1(3,3)	0.124056	0.014552	8.524881	0.0000
A1(3,4)	0.112290	0.012222	9.187580	0.0000
A1(3,5)	0.108241	0.010889	9.940156	0.0000
A1(3,6)	0.104036	0.010087	10.31402	0.0000
A1(3,7)	0.115673	0.011391	10.15460	0.0000
A1(3,8)	0.103764	0.010912	9.509455	0.0000
A1(3,9)	0.101121	0.014049	7.197974	0.0000
A1(3,10)	0.098068	0.012062	8.130259	0.0000
A1(4,4)	0.134337	0.013548	9.915777	0.0000
A1(4,5)	0.125631	0.010446	12.02646	0.0000
A1(4,6)	0.112195	0.009077	12.35999	0.0000
A1(4,7)	0.128634	0.010707	12.01456	0.0000
A1(4,8)	0.111749	0.010871	10.27928	0.0000
A1(4,9)	0.116696	0.012395	9.414413	0.0000
A1(4,10)	0.111659	0.011000	10.15121	0.0000
A1(5,5)	0.131732	0.010804	12.19313	0.0000
A1(5,6)	0.115438	0.009026	12.78917	0.0000
A1(5,7)	0.136325	0.010637	12.81657	0.0000
A1(5,8)	0.115158	0.010242	11.24372	0.0000
A1(5,9)	0.116391	0.011997	9.701455	0.0000
A1(5,10)	0.109457	0.010878	10.06218	0.0000
A1(6,6)	0.107035	0.010410	10.28148	0.0000
A1(6,7)	0.120929	0.010166	11.89536	0.0000
A1(6,8)	0.105854	0.010033	10.55031	0.0000
A1(6,9)	0.098073	0.011179	8.773312	0.0000
A1(6,10)	0.096978	0.010357	9.363781	0.0000
A1(7,7)	0.145597	0.012895	11.29137	0.0000
A1(7,8)	0.119994	0.010764	11.14799	0.0000

A1(7,9)	0.115218	0.012651	9.107611	0.0000
A1(7,10)	0.112555	0.011670	9.644893	0.0000
A1(8,8)	0.110284	0.012742	8.655194	0.0000
A1(8,9)	0.104620	0.012607	8.298338	0.0000
A1(8,10)	0.098222	0.011317	8.679300	0.0000
A1(9,9)	0.147734	0.020083	7.356008	0.0000
A1(9,10)	0.108905	0.013765	7.911707	0.0000
A1(10,10)	0.118909	0.016104	7.384011	0.0000
B1(1,1)	0.864236	0.012175	70.98414	0.0000
B1(1,2)	0.874861	0.014741	59.34885	0.0000
B1(1,3)	0.855839	0.016852	50.78624	0.0000
B1(1,4)	0.850684	0.011467	74.18570	0.0000
B1(1,5)	0.853744	0.010365	82.36889	0.0000
B1(1,6)	0.865024	0.010518	82.24049	0.0000
B1(1,7)	0.845416	0.011228	75.29792	0.0000
B1(1,8)	0.864684	0.010647	81.21493	0.0000
B1(1,9)	0.854428	0.015068	56.70308	0.0000
B1(1,10)	0.871498	0.013750	63.38302	0.0000
B1(2,2)	0.879689	0.017466	50.36657	0.0000
B1(2,3)	0.858478	0.017612	48.74335	0.0000
B1(2,4)	0.856061	0.014676	58.32983	0.0000
B1(2,5)	0.862946	0.012582	68.58511	0.0000
B1(2,6)	0.874497	0.012233	71.48462	0.0000
B1(2,7)	0.856032	0.013220	64.75141	0.0000
B1(2,8)	0.871027	0.014239	61.17043	0.0000
B1(2,9)	0.882309	0.014046	62.81585	0.0000
B1(2,10)	0.888787	0.014972	59.36516	0.0000
B1(3,3)	0.820086	0.018588	44.11926	0.0000
B1(3,4)	0.828977	0.015381	53.89447	0.0000
B1(3,5)	0.836697	0.014186	58.97847	0.0000

B1(3,6)	0.853532	0.012322	69.27049	0.0000
B1(3,7)	0.830262	0.014219	58.38943	0.0000
B1(3,8)	0.853777	0.013590	62.82426	0.0000
B1(3,9)	0.841069	0.017939	46.88603	0.0000
B1(3,10)	0.858003	0.015557	55.15102	0.0000
B1(4,4)	0.830999	0.012808	64.88114	0.0000
B1(4,5)	0.838359	0.010183	82.32735	0.0000
B1(4,6)	0.856173	0.009253	92.53032	0.0000
B1(4,7)	0.832906	0.010320	80.70910	0.0000
B1(4,8)	0.855969	0.010587	80.85218	0.0000
B1(4,9)	0.838714	0.013538	61.95432	0.0000
B1(4,10)	0.860804	0.010357	83.11461	0.0000
B1(5,5)	0.838069	0.010712	78.23909	0.0000
B1(5,6)	0.852704	0.009549	89.29349	0.0000
B1(5,7)	0.830456	0.010304	80.59493	0.0000
B1(5,8)	0.851675	0.010443	81.55095	0.0000
B1(5,9)	0.842669	0.012490	67.46648	0.0000
B1(5,10)	0.858134	0.011470	74.81255	0.0000
B1(6,6)	0.869898	0.010126	85.90834	0.0000
B1(6,7)	0.846208	0.010341	81.82712	0.0000
B1(6,8)	0.869790	0.009948	87.43015	0.0000
B1(6,9)	0.864569	0.012523	69.03905	0.0000
B1(6,10)	0.879160	0.010564	83.22029	0.0000
B1(7,7)	0.827259	0.011997	68.95276	0.0000
B1(7,8)	0.848480	0.009980	85.02111	0.0000
B1(7,9)	0.846136	0.012855	65.82083	0.0000
B1(7,10)	0.857227	0.011338	75.60947	0.0000
B1(8,8)	0.870314	0.011720	74.25882	0.0000
B1(8,9)	0.856972	0.013914	61.59231	0.0000
B1(8,10)	0.876261	0.011976	73.16933	0.0000

B1(9,9)	0.831086	0.018517	44.88113	0.0000
B1(9,10)	0.864181	0.014798	58.39734	0.0000
B1(10,10)	0.870180	0.014718	59.12433	0.0000

Table 2 – Eviews Output

Note that the numbers 1 through 10 represent each of our industry portfolios. As EViews automatically sorts the data-series alphabetically by their names, we find that:

1 = Durables
2 = Energy
3 = Health
4 = HiTec
5 = Manufacturing
6 = Non-Durables
7 = Other
8 = Shops
9 = Telecommunications
10 = Utilities

Residuals and Lagged Conditional Covariance

The conditional covariance matrices of previous periods, as well as the lagged residuals, will also need to be derived in order to calculate the estimate of the covariance matrix at time t. As we start out making forecasts for July 2012 and ahead, we will need the t-1 data from June 2012. First, we can compute the residuals in EViews by pressing “PROC” and selecting “Make Residuals”, we can then get the residual for every observation in our time period.

2012M03	0.375944	-4.235396	2.918776	3.647526	-0.559032	3.024926	3.975914	3.722761	1.507527	0.547824
2012M04	-8.324056	-2.865396	-0.361224	-3.032474	-1.629032	-0.135074	-2.594086	0.032761	0.917527	0.827824
2012M05	-8.984056	-11.99540	-4.371224	-8.762474	-8.749032	-3.485074	-8.674086	-3.997239	-1.662473	-1.812176
2012M06	-6.424056	4.784604	5.668776	1.987526	0.190968	2.614926	3.855914	1.092761	6.227527	2.497824
2012M07	-1.384056	1.584604	0.708776	-1.132474	0.170968	0.614926	-2.164086	0.252761	3.207527	2.727824

Figure 19 – Residuals from EViews Model

To obtain the lagged covariance matrix, we can also have EViews compute a list of the conditional covariance matrices. We will select "View -> Conditional Covariance" and then check off the box that says "Matrices". We will get a list of conditional covariance matrices for each observation. We need to consider the conditional covariance of June 2012:

2012M06	DURBL	ENRGY	HEALTH	HITEC	MANUF	NODUR	OTHERS	SHOPS	TELCM	UTILS
DURBL	91.74166	47.00316	22.90073	50.44344	51.93936	21.26442	52.25552	30.68270	22.73522	12.22446
ENRGY	47.00316	54.33058	16.85920	32.49979	38.40600	16.71489	37.38796	19.46317	17.24101	13.37899
HEALTH	22.90073	16.85920	18.78687	20.59572	19.61474	12.65130	20.55672	14.49218	10.95742	8.513040
HITEC	50.44344	32.49979	20.59572	42.94760	35.85301	17.44806	37.49232	23.55939	16.57538	9.876125
MANUF	51.93936	38.40600	19.61474	35.85301	39.64602	17.59421	37.92106	22.42223	16.99611	10.97175
NODUR	21.26442	16.71489	12.65130	17.44806	17.59421	13.29210	18.85440	13.73115	10.92729	8.685422
OTHERS	52.25552	37.38796	20.55672	37.49232	37.92106	18.85440	43.22284	23.93934	18.21081	11.48815
SHOPS	30.68270	19.46317	14.49218	23.55939	22.42223	13.73115	23.93934	19.48904	12.39668	8.492227
TELCM	22.73522	17.24101	10.95742	16.57538	16.99611	10.92729	18.21081	12.39668	14.87874	8.460777
UTILS	12.22446	13.37899	8.513040	9.876125	10.97175	8.685422	11.48815	8.492227	8.460777	10.54278

Figure 20 – Conditional Covariance Matrix from EViews

Now we have all the input data needed to compute the function;

GARCH = M + A1.*RESID(-1)*RESID(-1)' + B1.*GARCH(-1)

The interesting thing to take into consideration here is that since the variance-covariance matrix is time-varying, the software computes a conditional covariance matrix for each period in the series of observations. This means that we could take a look at specific historical time periods, such as big recessions, to see if the model performs particularly better in recession versus booms, etc.

Organizing the Results

Now, in order to make the actual forecast of the next period's variance-covariance matrix, we need to obtain the model results from our given time-period and organize them in such a way that we can do the actual calculations. We will assume that today is ultimo June 2012, which means that we will forecast the variance for July 2012 and then continue by doing multi-period forecasts with error-term simulations.

We start by looking at the time-series regression model:
GARCH = M + A1.*RESID(-1)*RESID(-1)' + B1.*GARCH(-1)

From the E-Views output we can compute the constant 10x10 matrix M:

M										
	Durbl	Enrgy	Hlth	HiTec	Manuf	NoDur	Other	Shops	Telcm	Utils
Durbl	1,7798	1,0444	1,2855	1,6032	1,4670	0,9686	1,5140	1,1495	0,8700	0,7201
Enrgy	1,0444	1,4105	0,9697	1,1723	1,0615	0,6486	1,1310	0,7313	0,5621	0,7214
Hlth	1,2855	0,9697	2,1783	1,8406	1,5399	1,0300	1,5070	1,1046	0,8947	0,7849
HiTec	1,6032	1,1723	1,8406	2,5073	1,7549	1,0545	1,7541	1,2937	1,0271	0,7666
Manuf	1,4670	1,0615	1,5399	1,7549	1,6981	1,0284	1,6347	1,1894	0,8657	0,7666
NoDur	0,9686	0,6486	1,0300	1,0545	1,0284	0,8089	1,1264	0,8092	0,6254	0,5426
Other	1,5140	1,1310	1,5070	1,7541	1,6347	1,1264	1,8388	1,2098	0,8740	0,8384
Shops	1,1495	0,7313	1,1046	1,2937	1,1894	0,8092	1,2098	1,0655	0,7193	0,5662
Telcm	0,8700	0,5621	0,8947	1,0271	0,8657	0,6254	0,8740	0,7193	0,9291	0,5707
Utils	0,7201	0,7214	0,7849	0,7666	0,7666	0,5426	0,8384	0,5662	0,5707	0,7470

Figure 21 – Matrix of the Constant M

Next, we will look at the A1.*RESID(-1)*RESID(-1)'. As seen in the approach used by Bollerslev, Engle & Wooldridge, we will take the A1 matrix and multiply the residuals of the relevant portfolios into this matrix. We will start by constructing the matrix as follows, with the one-period lagged residuals obtained from EViews output above, listed on the matrix border, in order to easily multiply them into the matrix.

		residuals									
	A	-6,4241	4,7846	5,6688	1,9875	0,1910	2,6149	3,8559	1,0928	6,2275	2,4978
residuals		Durbl	Enrgy	Hlth	HiTec	Manuf	NoDur	Other	Shops	Telcm	Utils
-6,42406	Durbl	0,117998	0,08799	0,094266	0,119004	0,119094	0,10468	0,121965	0,10715	0,104178	0,099192
4,7846	Enrgy	0,087988	0,0957	0,088529	0,099001	0,102765	0,09206	0,109058	0,09055	0,077724	0,071975
5,66878	Hlth	0,094266	0,08853	0,124056	0,11229	0,108241	0,10404	0,115673	0,10376	0,101121	0,098068
1,98753	HiTec	0,119004	0,099	0,11229	0,134337	0,125631	0,1122	0,128634	0,11175	0,116696	0,111659
0,19097	Manuf	0,119094	0,10277	0,108241	0,125631	0,131732	0,11544	0,136325	0,11516	0,116391	0,109457
2,61493	NoDur	0,104683	0,09206	0,104036	0,112195	0,115438	0,10704	0,120929	0,10585	0,098073	0,096978
3,85591	Other	0,121965	0,10906	0,115673	0,128634	0,136325	0,12093	0,145597	0,11999	0,115218	0,112555
1,09276	Shops	0,107154	0,09055	0,103764	0,111749	0,115158	0,10585	0,119994	0,11028	0,10462	0,098222
6,22753	Telcm	0,104178	0,07772	0,101121	0,116696	0,116391	0,09807	0,115218	0,10462	0,147734	0,108905
2,49782	Utils	0,099192	0,07198	0,098068	0,111659	0,109457	0,09698	0,112555	0,09822	0,108905	0,118909

Figure 22 – Matrix of Parameter A1 Bordered With Portfolio Residuals

Now we shall multiply the residuals into A1 for each involved portfolio, i.e. $A1_{durbl,hitec}$ * $RESID_{durbl}$ * $RESID_{hitec}$ and so on. This will give us a full 10x10 matrix for the expression A1.*RESID(-1)*RESID(-1)':

A*RES(t-1)*RES(t-1)										
	Durbl	Enrgy	Hlth	HiTec	Manuf	NoDur	Other	Shops	Telcm	Utils
Durbl	4,8696	-2,70445	-3,43284	-1,51944	-0,1461	-1,75851	-3,02115	-0,75222	-4,16774	-1,59165
Enrgy	-2,70445	2,19074	2,40116	0,94145	0,0939	1,15175	2,01201	0,47344	2,31588	0,86018
Hlth	-3,43284	2,40116	3,98654	1,26515	0,11718	1,54217	2,52842	0,64278	3,56982	1,3886
HiTec	-1,51944	0,94145	1,26515	0,53067	0,04768	0,5831	0,98582	0,24271	1,44439	0,55433
Manuf	-0,1461	0,0939	0,11718	0,04768	0,0048	0,05765	0,10038	0,02403	0,13842	0,05221
NoDur	-1,75851	1,15175	1,54217	0,5831	0,05765	0,73189	1,21932	0,30248	1,59707	0,63342
Other	-3,02115	2,01201	2,52842	0,98582	0,10038	1,21932	2,16475	0,50561	2,76671	1,08406
Shops	-0,75222	0,47344	0,64278	0,24271	0,02403	0,30248	0,50561	0,13169	0,71196	0,2681
Telcm	-4,16774	2,31588	3,56982	1,44439	0,13842	1,59707	2,76671	0,71196	5,72943	1,69405
Utils	-1,59165	0,86018	1,3886	0,55433	0,05221	0,63342	1,08406	0,2681	1,69405	0,74189

Figure 23 – The Matrix of Products of the A1 Matrix and Residuals

Finally, we need to look at the last part of the expression; B1.*GARCH(-1). The B1 matrix is in the model output and we need to multiply each number in this matrix with the corresponding number in the GARCH(-1) matrix, the one-period lagged conditional covariance matrix.

B										
	Durbl	Enrgy	Hlth	HiTec	Manuf	NoDur	Other	Shops	Telcm	Utils
Durbl	0,864236	0,87486	0,855839	0,850684	0,853744	0,86502	0,845416	0,86468	0,854428	0,871498
Enrgy	0,874861	0,87969	0,858478	0,856061	0,862946	0,8745	0,856032	0,87103	0,882309	0,888787
Hlth	0,855839	0,85848	0,820086	0,828977	0,836697	0,85353	0,830262	0,85378	0,841069	0,858003
HiTec	0,850684	0,85606	0,828977	0,830999	0,838359	0,85617	0,832906	0,85597	0,838714	0,860804
Manuf	0,853744	0,86295	0,836697	0,838359	0,838069	0,8527	0,830456	0,85168	0,842669	0,858134
NoDur	0,865024	0,8745	0,853532	0,856173	0,852704	0,8699	0,846208	0,86979	0,864569	0,87916
Other	0,845416	0,85603	0,830262	0,832906	0,830456	0,84621	0,827259	0,84848	0,846136	0,857227
Shops	0,864684	0,87103	0,853777	0,855969	0,851675	0,86979	0,84848	0,87031	0,856972	0,876261
Telcm	0,854428	0,88231	0,841069	0,838714	0,842669	0,86457	0,846136	0,85697	0,831086	0,864181
Utils	0,871498	0,88879	0,858003	0,860804	0,858134	0,87916	0,857227	0,87626	0,864181	0,87018

Figure 24 – Matrix of Parameter B

The conditional covariance matrix is obtained from EViews as described above:

GARCH(t-1)										
	Durbl	Enrgy	Hlth	HiTec	Manuf	NoDur	Other	Shops	Telcm	Utils
Durbl	91,7417	47,0032	22,9007	50,4434	51,9394	21,2644	52,2555	30,6827	22,7352	12,2245
Enrgy	47,0032	54,3306	16,8592	32,4998	38,4060	16,7149	37,3880	19,4632	17,2410	13,3790
Hlth	22,9007	16,8592	18,7869	20,5957	19,6147	12,6513	20,5567	14,4922	10,9574	8,5130
HiTec	50,4434	32,4998	20,5957	42,9476	35,8530	17,4481	37,4923	23,5594	16,5754	9,8761
Manuf	51,9394	38,4060	19,6147	35,8530	39,6460	17,5942	37,9211	22,4222	16,9961	10,9718
NoDur	21,2644	16,7149	12,6513	17,4481	17,5942	13,2921	18,8544	13,7312	10,9273	8,6854
Other	52,2555	37,3880	20,5567	37,4923	37,9211	18,8544	43,2228	23,9393	18,2108	11,4882
Shops	30,6827	19,4632	14,4922	23,5594	22,4222	13,7312	23,9393	19,4890	12,3967	8,4922
Telcm	22,7352	17,2410	10,9574	16,5754	16,9961	10,9273	18,2108	12,3967	14,8787	8,4608
Utils	12,2245	13,3790	8,5130	9,8761	10,9718	8,6854	11,4882	8,4922	8,4608	10,5428

Figure 25 – The Conditional Covariance Matrix of June 2012

As we can't multiply these two matrices together, it is clear, and also seen from the Bollerslev, Engle & Wooldridge approach, that we must multiply each corresponding number/cell in the matrices together and put the results into a final matrix for the B1.*GARCH(-1) part of the model. As an example, we will multiply GARCH(-1)$_{durbl,hitec}$ with B1$_{durbl,hitec}$ and put the results in a corresponding matrix. Multiplying these numbers together we will get the matrix:

B*GARCH(t-1)										
	Durbl	Enrgy	Hlth	HiTec	Manuf	NoDur	Other	Shops	Telcm	Utils
Durbl	79,28644527	41,1212	19,59933786	42,91142731	44,34291696	18,3942	44,1776527	26,5308	19,42561	10,65359
Enrgy	41,12123156	47,794	14,4732523	27,82180273	33,14230408	14,6171	32,00529017	16,9529	15,2119	11,89107
Hlth	19,59933786	14,4733	15,40684907	17,07337818	16,41159411	10,7983	17,06746346	12,3731	9,215946	7,304214
HiTec	42,91142731	27,8218	17,07337818	35,68941265	30,05769361	14,9386	31,22757828	20,1661	13,902	8,501408
Manuf	44,34291696	33,1423	16,41159411	30,05769361	33,22610034	15,0027	31,4917718	19,0965	14,3221	9,415232
NoDur	18,39423365	14,6171	10,79828939	14,93855787	15,00265324	11,5628	15,95474412	11,9432	9,447396	7,635876
Other	44,1776527	32,0053	17,06746346	31,22757828	31,4917718	15,9547	35,7564834	20,3121	15,40882	9,847952
Shops	26,53083977	16,9529	12,37308996	20,1661075	19,09645274	11,9432	20,3120512	16,9616	10,62361	7,441407
Telcm	19,42560855	15,2119	9,215946282	13,90200326	14,32209502	9,4474	15,40882193	10,6236	12,36551	7,311643
Utils	10,65359244	11,8911	7,304213859	8,501407905	9,415231715	7,63588	9,84795236	7,44141	7,311643	9,174116

Figure 26 – The Product of the Conditional Covariance Matrix Values and the Values of the B Matrix

Now we have the 10x10 matrices for M, A1.*RESID(-1)*RESID(-1)' and B1.*GARCH(-1). From here, it is simply a matter of summing up these three 10x10 matrices to get the forecasted variance-covariance matrix for July 2012 - $\text{GARCH}_{07,2012}$ - also denoted H_t or σ_t^2 in the theory section. The denotation of the variance-covariance matrix as GARCH_t is merely adopted from the output of EViews:

GARCH										
	Durbl	Enrgy	Hlth	HiTec	Manuf	NoDur	Other	Shops	Telcm	Utils
Durbl	85,9358	39,4611	17,4519	42,9952	45,6638	17,6044	42,6705	26,9282	16,1279	9,7821
Enrgy	39,4611	51,3952	17,8441	29,9355	34,2977	16,4175	35,1483	18,1577	18,0899	13,4726
Hlth	17,4519	17,8441	21,5717	20,1791	18,0687	13,3705	21,1029	14,1205	13,6804	9,4777
HiTec	42,9952	29,9355	20,1791	38,7274	31,8602	16,5761	33,9675	21,7025	16,3735	9,8223
Manuf	45,6638	34,2977	18,0687	31,8602	34,9290	16,0887	33,2268	20,3099	15,3262	10,2341
NoDur	17,6044	16,4175	13,3705	16,5761	16,0887	13,1036	18,3004	13,0549	11,6698	8,8119
Other	42,6705	35,1483	21,1029	33,9675	33,2268	18,3004	39,7600	22,0274	19,0495	11,7704
Shops	26,9282	18,1577	14,1205	21,7025	20,3099	13,0549	22,0274	18,1587	12,0549	8,2757
Telcm	16,1279	18,0899	13,6804	16,3735	15,3262	11,6698	19,0495	12,0549	19,0241	9,5763
Utils	9,7821	13,4726	9,4777	9,8223	10,2341	8,8119	11,7704	8,2757	9,5763	10,6630

Figure 27 – The Forecasted Covariance Matrix for July 2012

Now that we have done this forecast, we will look into multi-period forecasting with simulation of the residuals in order to suggest monthly rebalancing of our optimal portfolio choices or weights.

Determining the Order of the GARCH-Model

As described in the theory section, we will need to take a look at the order of the GARCH model. In other words, we need to determine *p* and *q* in the GARCH(*p*,*q*) model. To do this, we will need to look at Akaike's Information Criterion (AIC) and/or Schwartz's Information Criterion (SIC). We have decided to look at both of them in our EViews outputs. Unfortunately, there is no simple mathematical formula that can calculate the preferred values of *p* and *q*, so the method of determining them will be simple trial and error. Thus, we will make an M-GARCH model in EViews for every possible combination of p=1,2,3,4 and q=1,2,3,4.

The EViews output automatically computes the SIC and the AIC, so we can simply look into which model has the lowest values of these criterions. Fortunately, it turns out to be the GARCH(1,1) which seem to be the most commonly used model when working with M-GARCH models. It is also the values for *p* and *q* used in Bollerslev, Engle & Wooldridge (1988), which makes it more convenient for us to replicate their method. The information criterions in the M-GARCH(1,1) model look as follows:

Log likelihood	-26251.52	Schwarz criterion	51.75185
Avg. log likelihood	-2.529048	Hannan-Quinn criter.	51.23444
Akaike info criterion	50.91815		

Table 3 – Information Criterions

The process of estimating the EViews model (described in the section "Eviews 7.0") is repeated for the GARCH(1,2), GARCH(2,1), GARCH(2,2), GARCH(1,3), GARCH(3,1), GARCH(2,3), GARCH(3,2), GARCH(3,3), GARCH(1,4), GARCH(4,1), GARCH(4,2), GARCH(2,4), GARCH(4,3), GARCH(3,4) and GARCH(4,4) models. In none of these models we achieved lower values of either the AIC or the SIC. Thus, we will stick to the GARCH(1,1) model.

This approach is adopted from an exercise in the coursework from "Applied Econometrics – AE58", where the structure of the autoregressive model is determined by computing models with up to 4 lags and identifying the best model by its information criterion(s).

Simulations and Forecasts

In this part we use the output of the GARCH-model from EViews, to do the Monte Carlo Simulation.

Monte Carlo Simulation

In the following we will show how we have done our multi-period forecasts with our GARCH-model, as described in the theory section "Forecasting with GARCH". We will show narrowed-down versions of our data in- and output to illustrate how we work on the results we get. The data used in the figures and tables of this section is the data that our GARCH model has generated for July 2012 and then the simulations of August 2012, which can't be forecasted by the GARCH model.

The simulations of error-terms are computed in SAS Enterprise 5.1 by merely writing a program that can generate the errors. We will use the following code:

```
proc iml;
 Mean = {0,0,0,0,0,0,0,0,0,0};
 N = 10;
 Cov = {61.10198 28.70729 28.38950 44.78009 43.39752 27.10954 41.06327 36.16755 22.77330 27.20046,
        28.70729 36.19500 19.38354 27.29603 27.97486 17.42158 27.27217 20.39238 14.27618 20.91217,
        28.38950 19.38354 32.54151 30.50693 27.55056 21.32988 27.54038 24.64128 16.00344 20.09502,
        44.78009 27.29603 30.50693 54.54814 40.55821 25.41373 38.55686 34.01805 23.37239 25.88500,
        43.39752 27.97486 27.55056 40.55821 40.43765 25.20418 37.63547 31.22502 20.15706 24.99122,
        27.10954 17.42158 21.32988 25.41373 25.20418 21.67531 25.70347 23.50303 14.60982 18.49666,
        41.06327 27.27217 27.54038 38.55686 37.63547 25.70347 42.58758 31.40176 21.25333 26.78324,
        36.16755 20.39238 24.64128 34.01805 31.22502 23.50303 31.40176 34.06104 18.25356 21.33982,
        22.77330 14.27618 16.00344 23.37239 20.15706 14.60982 21.25333 18.25356 21.50527 16.59224,
        27.20046 20.91217 20.09502 25.88500 24.99122 18.49666 26.78324 21.33982 16.59224 31.49185};
  NumSamples = 3000;
 call randseed(1);
 X = RandNormal(NumSamples, Mean, Cov);
 print (X[1:3000,])[label="MV Normal Simulated Errors"];
```

Figure 28 – SAS Multivariate Normal Distribution Generation

As we can see in the program, we use a covariance matrix and mean vector, and we ask for 3,000 samples of the simulated error-term. As this is a very large amount of data, we have simply shown an outtake of the first couple of simulations, since putting all the result in the appendices would end up giving us hundreds of pages of appendices. The "RandNormal" command tells SAS to compute the random normal distributed numbers. The "randseed" is set to 1, as suggested by "David Vose – Risk Analysis"[43]. This is the author's personal suggestion, but in general the random number seed should be set between 0 and 1. The random seed is the basis number for the generation of the algorithms, meaning that the generation will start with this value.

The first series of simulated error-terms are shown below. This is a simple cut-out of the first 9 generated vectors out of the 3,000 simulated ones. Note that the vectors are horizontal here.

0,01672	0,68285	0,42834	0,50678	0,66253	0,57134	0,66505	0,68899	0,93155	3,02867
0,19062	-0,08398	0,97816	0,51611	0,30367	0,7705	0,49127	1,29291	2,11497	0,3868
0,38846	1,17042	0,64806	0,17872	0,95723	1,00805	1,13834	0,07526	-0,15157	0,82458
-0,65573	0,37812	1,0849	0,6225	-0,30163	-0,1166	-0,22949	-0,4812	0,04715	-0,99634
-0,71605	0,59941	1,6345	0,05103	-0,05674	-0,05986	0,48104	1,32378	0,89525	2,35208
-0,81321	-1,40884	-2,50955	-2,29139	-1,35778	-2,01483	-2,49479	-1,65921	-2,07687	-4,34468
-1,46077	-1,38168	-0,76869	-1,79337	-2,59587	-2,23059	-2,23294	-1,93256	-0,94282	-1,84626
-1,40942	0,41183	-0,30149	-1,07173	-0,51767	-0,4765	-1,12727	-1,67488	-1,06534	0,29765
-1,58537	-2,11559	-1,77855	-0,99789	-1,47168	-2,70855	-1,25493	-2,26758	-0,68367	-3,88238

Figure 29 – Simulated Error-Terms

Now, the next step is calculating $\eta_{t+1} = \varepsilon_{t+1} * \sigma_t$, so we will multiply with the vector of standard deviations, which is simply generated by taking the square root of the diagonal in the GARCH-forecasted covariance matrix. The first couple of simulated etas are shown below:

[43] David Vose, Risk Analysis, page 64

Eta simulations:									
Durbl	Enrgy	Hlth	HiTec	Manuf	NoDur	Other	Shops	Telcm	Utils
0,15502	4,89536	1,98946	3,15374	3,91561	2,06819	4,1935	2,93599	4,06311	9,88992
1,76704	-0,6021	4,54308	3,2118	1,79473	2,78911	3,09775	5,50947	9,22478	1,26307
3,60112	8,39079	3,00995	1,11217	5,65729	3,64902	7,1779	0,32069	-0,6611	2,6926
-6,0788	2,71075	5,03884	3,87388	-1,7827	-0,4221	-1,4471	-2,0505	0,20567	-3,2535
-6,6379	4,2972	7,59149	0,31756	-0,3353	-0,2167	3,03325	5,64102	3,90479	7,68057
-7,5386	-10,1	-11,656	-14,26	-8,0246	-7,2934	-15,731	-7,0704	-9,0586	-14,187
-13,542	-9,9053	-3,5702	-11,16	-15,342	-8,0745	-14,08	-8,2353	-4,1123	-6,0288
-13,066	2,95242	-1,4003	-6,6695	-3,0594	-1,7249	-7,1081	-7,1372	-4,6466	0,97195
-14,697	-15,167	-8,2605	-6,21	-8,6978	-9,8047	-7,913	-9,6629	-2,9819	-12,678
-1,758	-7,8073	-2,0696	4,81334	-3,3782	-1,1599	-4,7655	0,36765	-1,6173	2,24551

Figure 30 – First Simulated Eta Values

Now that we have the Etas, we can finally compute the return simulations, by simply adding the historical mean returns to each corresponding simulation. The vector of mean returns looks as follows:

Portfolio	Historical mean return
Durbl	1,085973025
Enrgy	1,054807322
Hlth	1,073689788
HiTec	1,07738921
Manuf	1,014816956
NoDur	0,97583815
Other	0,890674374
Shops	0,98004817
Telcm	0,846840077
Utils	0,877552987

Table 4 – Historical Mean Returns

We will then compute the simulated returns as described, the first couple of simulations then become:

5	Simulated Returns									
6	Durbl	Enrgy	Hlth	HiTec	Manuf	NoDur	Other	Shops	Telcm	Utils
7	1,240994	5,950172	3,063148	4,231128	4,930426	3,044026	5,084172	3,916043	4,909954	10,76747
8	2,853014	0,452747	5,61677	4,28919	2,809544	3,76495	3,98842	6,489515	10,07162	2,14062
9	4,687091	9,445595	4,08364	2,189561	6,672104	4,624858	8,06857	1,300738	0,185723	3,570148
10	-4,99278	3,765556	6,112529	4,951268	-0,76786	0,553751	-0,55638	-1,0705	1,052512	-2,37592
11	-5,55189	5,352011	8,665185	1,39495	0,679493	0,759136	3,92392	6,621065	4,751628	8,558121
12	-6,45262	-9,04524	-10,582	-13,1822	-7,00978	-6,31761	-14,8404	-6,09036	-8,21177	-13,3097
13	-12,4556	-8,8505	-2,49652	-10,083	-14,327	-7,09863	-13,1893	-7,2552	-3,26542	-5,15128
14	-11,9796	4,007227	-0,32661	-5,5921	-2,04463	-0,74903	-6,2174	-6,15711	-3,7998	1,849507
15	-13,6106	-14,112	-7,18685	-5,13263	-7,68294	-8,82882	-7,02233	-8,68282	-2,13511	-11,8001
16	-0,67198	-6,75246	-0,99586	5,890727	-2,36336	-0,18407	-3,87484	1,347697	-0,77044	3,123061

Figure 31 – Simulated Returns

Computing the Covariance Matrix

The next step will be to compute our covariance matrix at t+1 (August 2012), denoted σ_{t+1}^2. The process of this calculation is described in the theory section, but basically we will start by computing a correlation matrix of the 3,000 simulated returns. We will use the =CORREL function in Microsoft Excel to do so and for t+1 we will get the following correlation matrix:

	Correlation									
	Durbl	Enrgy	Hlth	HiTec	Manuf	NoDur	Other	Shops	Telcm	Utils
Durbl	1	0,6116	0,6524	0,7858	0,8801	0,7564	0,8142	0,7885	0,6332	0,641
Enrgy	0,6116	1	0,5745	0,6161	0,7307	0,625	0,6893	0,5821	0,5066	0,6142
Hlth	0,6524	0,5745	1	0,7434	0,7732	0,8047	0,7455	0,7497	0,6167	0,6328
HiTec	0,7858	0,6161	0,7434	1	0,8712	0,7571	0,8106	0,7933	0,6862	0,6375
Manuf	0,8801	0,7307	0,7732	0,8712	1	0,8582	0,9056	0,8453	0,6877	0,7085
NoDur	0,7564	0,625	0,8047	0,7571	0,8582	1	0,853	0,8715	0,6828	0,7097
Other	0,8142	0,6893	0,7455	0,8106	0,9056	0,853	1	0,8298	0,7129	0,743
Shops	0,7885	0,5821	0,7497	0,7933	0,8453	0,8715	0,8298	1	0,6781	0,6574
Telcm	0,6332	0,5066	0,6167	0,6862	0,6877	0,6828	0,7129	0,6781	1	0,6411
Utils	0,641	0,6142	0,6328	0,6375	0,7085	0,7097	0,743	0,6574	0,6411	1

Figure 32 – Matrix of Calculated Portfolio Correlations

From here, we simply need to multiply the standard deviations into the matrix to get the covariance, as cov(*i*,*j*) = $\sigma_i \sigma_j * CORREL(i,j)$, so we just place the standard deviations at the borders of the matrix, to make a simple excel calculation of the covariance. The forecasted covariance matrix σ_{t+1}^2 or in this case for August 2012, then becomes:

		Covariance									
St. Devs		7,8431	6,0402	5,8344	7,5238	6,4906	4,7694	6,6547	5,9489	4,7747	5,6476
		Durbl	Enrgy	Hlth	HiTec	Manuf	NoDur	Other	Shops	Telcm	Utils
7,8431	Durbl	61,514	28,975	29,856	46,369	44,802	28,293	42,496	36,788	23,711	28,394
6,040181	Enrgy	28,975	36,484	20,246	27,998	28,646	18,005	27,705	20,916	14,61	20,953
5,834375	Hlth	29,856	20,246	34,04	32,631	29,279	22,392	28,945	26,022	17,18	20,851
7,523836	HiTec	46,369	27,998	32,631	56,608	42,545	27,169	40,586	35,505	24,652	27,086
6,490636	Manuf	44,802	28,646	29,279	42,545	42,128	26,568	39,114	32,637	21,312	25,972
4,769381	NoDur	28,293	18,005	22,392	27,169	26,568	22,747	27,074	24,727	15,549	19,116
6,654701	Other	42,496	27,705	28,945	40,586	39,114	27,074	44,285	32,849	22,653	27,923
5,948875	Shops	36,788	20,916	26,022	35,505	32,637	24,727	32,849	35,389	19,26	22,085
4,774705	Telcm	23,711	14,61	17,18	24,652	21,312	15,549	22,653	19,26	22,798	17,287
5,647595	Utils	28,394	20,953	20,851	27,086	25,972	19,116	27,923	22,085	17,287	31,895

Figure 33 – Covariance Matrix for August 2012

This concludes our forecast using Monte Carlo simulation of returns. The process can be repeated using the Covariance Matrix for August 2012, as an input for the SAS programming code, to generate simulated error-terms for September 2012 or t+2, and then repeating all the steps.

When we are going to apply modern portfolio theory, and use the forecasted covariance matrices to compute Global Minimum Variance portfolios, we will also need the inverse covariance matrix which we simply compute in Microsoft Excel by using the "=MINVERT" function.

The inverse of the covariance matrix above is shown here:

	Inverse Covarianse									
	Durbl	Enrgy	Hlth	HiTec	Manuf	NoDur	Other	Shops	Telcm	Utils
Durbl	0,0779	0,0062	0,0118	-0,0039	-0,0736	0,0108	-0,0057	-0,0196	-0,0027	-0,004
Enrgy	0,0062	0,0634	-0,0016	0,0034	-0,0467	0,0041	-0,0079	0,0088	0,0022	-0,014
Hlth	0,0118	-0,0016	0,0975	-0,0198	-0,0168	-0,0566	0,0026	-0,0055	-0,0025	-0,006
HiTec	-0,0039	0,0034	-0,0198	0,0858	-0,062	0,0296	-0,0039	-0,0203	-0,0182	0,0015
Manuf	-0,0736	-0,0467	-0,0168	-0,062	0,3128	-0,064	-0,0765	-0,0056	0,0123	0,0076
NoDur	0,0108	0,0041	-0,0566	0,0296	-0,064	0,2971	-0,0388	-0,0955	-0,0117	-0,02
Other	-0,0057	-0,0079	0,0026	-0,0039	-0,0765	-0,0388	0,1618	-0,0148	-0,0192	-0,024
Shops	-0,0196	0,0088	-0,0055	-0,0203	-0,0056	-0,0955	-0,0148	0,1555	-0,0088	0,0042
Telcm	-0,0027	0,0022	-0,0025	-0,0182	0,0123	-0,0117	-0,0192	-0,0088	0,1036	-0,018
Utils	-0,0043	-0,0138	-0,0059	0,0015	0,0076	-0,02	-0,0236	0,0042	-0,0183	0,0804

Figure 34 – Inverse Covarianse Matrix for August 2012

Now we will be able to use matrix algebra to compute the GMV weights.

Time Periods for Analysis

We have divided our large data sample into smaller samples that are defined by the economic state of the time periods. These smaller samples will all belong to one of two categories; a recession period or a non-recession period. The idea behind doing this is to research how the optimal allocation of industry portfolios will look like in different economic states.
The definition of a recession, according to NBER (National Bureau of Economic Research), is:

"A significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real gross domestic product (GDP), real income, employment, industrial production, and wholesale-retail sales"[44]

This definition is applied to our data sample, and dividing it into time periods accordingly, we have 29 time periods in total, from July 1926 to December 2012. However, we will not need to analyze all 29 periods and even if we did it would take up far too much space. So we have chosen to look at the 3 most recent time periods, where the recession period will be in the most significantly declining phase of the latest great recession, and the boom period will be the time prior to the sudden decline in the great recession, where markets and most asset prices were at top levels. We will also look at the most recent time-period in our data which includes December 2012 as the latest observation.

Dec 2001 – Nov 2007	Non-recession	
Dec 2007 – Jun 2009	Recession	The subprime mortgage crisis led to a collapse of the US housing bubble. A world financial crisis occurred.

[44] Robert Hall, The NBER's Recession Dating Procedure

Jul 2009 – Dec 2012	Non-recession	

Table 5 – Chosen Periods of Recession and non-Recession

We will be analyzing periods of 6 consecutive months, so we have decided to look into the periods of boom and recession above, and try to identify the most critical periods in terms of highest market index price inclines and declines.

Looking at the graph below[45] we see the historical price fluctuations of the Dow Jones Index, S&P500 Index and Nasdaq.

Figure 35 – Historical US Stock Market Performance

From this graph we have chosen our periods for analysis to be:

[45] Google Finance

- Boom: September 2006 through February 2007
- Recession: July 2008 through December 2008
- Most Recent: July 2012 through December 2012

These will be the periods of time in which we will do our model forecasts as well as suggest optimal portfolio weights with simulated forecast, GARCH Conditional Covariance results and standardized mean-variance optimization and finally use these suggested portfolio weights to compare performance by testing their return and Sharpe ratios in the given periods of time.

Creating Portfolios

We are now ready to use the Markowitz optimization techniques and construct portfolios of our industry-stocks with time-varying risk.

Portfolio Weights

In the last part of the analysis we attempt to test the performance of the difference methods of portfolio optimization. We have earlier described our calculations on how to optimally balance our portfolio weights using GARCH as one method, and using simulations as the other method. These two methods will be compared to each other in our attempt to outperform the optimization using modern portfolio theory. All the portfolios have been created manually in Excel, and therefore we have assumed that short selling is allowed. This means that we will get negative weights in different industries and periods, and every negative result represents a short sale, where you will gain profits from an actual negative return in the industry.

All the portfolios of the different models have been created on the basis of covariance matrices. But these matrices have been achieved through different methods. The covariance matrices of the GARCH model have been created through econometric analysis using EViews. The covariance matrices of the Simulation model are based on GARCH covariance matrices, but then forecasted up to t+5 using Monte Carlo simulations in SAS and Excel. This means that the GARCH-

Portfolio will be equal to the Simulation-Portfolio in the first month of every time period we forecast, but then different in the following 5 months.

The covariance matrices of Modern Portfolio Theory have been created via historical numbers of returns. We set up the matrices manually using Excel, calculating the variances for each industry, and the covariance between two industries, using the Excel formulas for variance and covariance. This method is chosen according to the Harvard Business Case, "Harvard Management Company, asset management case", from the CBS course of Capital Market Theory - CM_AE57. This case describes how to optimize portfolios using modern portfolio theory, and we have used the same approach, with average numbers from time series returns. One of the assumptions of the modern portfolio theory is that correlations between asset returns are forever constant. Our model will remove this assumption as we have found indications of ARCH/GARCH in our tests.
This case also dealt with various assets that had to be optimized into a portfolio, which is very relevant for our approach as we attempt to do the same. The essence of the case is the basis of average numbers in finding means and standard deviations, which differs to our two other methods. This also means that the weights of the portfolio in Modern Portfolio Theory will not change in the 6 month periods that we forecast. Thus, the return of the portfolio will only change due to the changes in realized returns of the industries. For GARCH and the Simulation model, the weights will change each month, and therefore the return of the portfolio will change on behalf of the weights and the returns of the industries.

In all cases, the covariance matrices have been inverted and multiplied with the unity-matrix which basically is a 1x10 vector containing only the number 1 as all 10 numbers. This will create a 1x10 vector wherein all the numbers will be summarized. The sum is then used as the denominator in a fraction where the numerator is the number 1. This calculation gives us a unique number which is then multiplied into the product of the unity matrix and the inverse covariance matrix. This results in the GMV weights for each industry in the portfolio. The calculations can be shown by the following formula, which is recited from our theory section.

$$w_{GMV} = \frac{\Sigma^{-1}\mathbf{1}}{\mathbf{1}^{\top}\Sigma^{-1}\mathbf{1}}$$

An important decision has been which time periods we wanted to compare the performances of the portfolios of each method. We decided to optimize every month in a 6 month interval, in 3 different periods of time; a period of economic growth, a period of recession, and the most recent period of time, with available information. We described the selection of these periods earlier, and we ended up focusing on September 2006 through February 2007, July 2008 through December 2008, and July 2012 through December 2012. The results of the portfolios, including their return, risk and Sharpe ratio are presented below. The Sharpe ratio indicates how good the investment is, comparing the excess return to the risk, where the risk is measured from the historical volatility.

Portfolio Risk

We have measured the risk in the portfolios by standard deviations. Every single assets standard deviation is simply the corresponding weight multiplied by the square root of the assets total variance at the given point in time. The variance is of the single asset is simply found in the diagonal of the covariance matrix.

To compute the standard deviation of the portfolio, we will simply take the square root of the portfolio variance, which you will recall from our theory section is given by:

$$\sigma_p^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N} w_i w_j \sigma_{ij}$$

In order to compute the portfolio variance, which will be found in the bottom right corner, we use this formula and not just a sum of the weighted standard deviations. So please don’t be confused by the bottom left corner denotation “sum”, as this only applies to the sum of weights, to ensure the weights sum to 1 or 100%.

The actual portfolio standard deviation computed in the bottom right corner, have been calculated by multiplications of matrices in order to compute the easiest form of the function above. We multiply the transposed vector of weights with the covariance matrix and multiply that product by the vector of weights, which gives us a single number, the variance of the portfolio. We simply take the square root of that variance and that gives us the standard deviation, which we will use directly as the denominator in the Sharpe ratio expression.

The formula for computing the portfolio standard deviation in Microsoft Excel, looks like this:
=SQRT(MMULT(MMULT(TRANSPOSE(w_t);Cov_t);w_t))
Where w_t is the vector of weights at a given time (t) and Cov_t is the calculated or forecasted covariance matrix at that time.

Portfolios in Economic Growth, September 2006 – February 2007

September 2006

Simultaion			
GMV Weights:		Return	St. Dev
Durbl	-0,194945	-0,836313	-0,943849
Enrgy	0,033775	-0,120916	0,205953
Hlth	-0,066081	-0,100443	-0,26159
HiTec	-0,160735	-0,589897	-0,906362
Manuf	0,154518	0,290493	0,659161
NoDur	0,419358	-0,260002	1,21983
Other	-0,084964	-0,273586	-0,314578
Shops	0,373734	2,152707	1,476493
Telcm	0,346743	1,251744	1,026211
Utils	0,178597	-0,391127	0,632254
Sum	1	1,12266	2,793523
Risk free r	0,41		
Sharpe:	0,255112		

GARCH			
GMV Weights:		Return	St. Dev
Durbl	-0,194945	-0,836316	-0,943851
Enrgy	0,033775	-0,120916	0,205952
Hlth	-0,066081	-0,100443	-0,26159
HiTec	-0,160734	-0,589895	-0,906359
Manuf	0,154518	0,290494	0,659163
NoDur	0,419358	-0,260002	1,219829
Other	-0,084965	-0,273587	-0,31458
Shops	0,373734	2,152707	1,476493
Telcm	0,346744	1,251745	1,026212
Utils	0,178597	-0,391127	0,632255
Sum	1	1,122661	2,793523
Sharpe:	0,255112		

MPT			
GMV Weights:		Return	St. Dev
Durbl	-0,003057	-0,013115	-0,049004
Enrgy	0,276853	-0,991132	1,655557
Hlth	0,041473	0,063039	0,241121
HiTec	-0,103276	-0,379022	-0,773281
Manuf	-0,205236	-0,385843	-1,307652
NoDur	0,723569	-0,448613	3,415785
Other	-0,390079	-1,256054	-2,540945
Shops	0,012383	0,071324	0,073424
Telcm	0,587353	2,120344	2,696319
Utils	0,060017	-0,131438	0,343333
Sum	1	-1,350509	3,754658
Sharpe:	-0,468887		

Figure 36 – Suggested Portfolios for September 2006

In the first month of the portfolio optimization in the economic growth period, it is interesting to see that all the methods agree to which industries need to be bought and which need to be shorted, in 8 out of 10 cases. The only difference is that MPT wants to short Manufacturing while GARCH wants to short Health. Both GARCH and MPT agree to invest a lot in Non-Durables, with respectively 43% and 72%. For this month, that turned out to be a bad idea, since that industry had a negative return. The GARCH-Portfolio collects a significant part of its revenue from Shops and Telecom, while MPT minimize the overall loss mainly from Telecom. Overall, the GARCH model is outperforming the MPT model with a return of 2,79% compared to a 1,35% loss to MPT. The risk of GARCH is also lower, with a standard deviation of 2,79 compared to 3,75 in MPT. This also gives GARCH a higher Sharpe ratio of 0,26, while MPT has -0,47.

October 2006

Simulation			
GMV Weights:		Return	St. Dev
Durbl	-0,17981	-0,92065	-0,87353
Enrgy	0,0315	0,15622	0,19275
Hlth	-0,07222	-0,1235	-0,2918
HiTec	-0,16371	-0,66632	-0,94034
Manuf	0,15288	0,66807	0,66658
NoDur	0,42879	1,20062	1,2727
Other	-0,10843	-0,32636	-0,41231
Shops	0,36984	1,43497	1,4815
Telcm	0,3256	1,71264	0,98956
Utils	0,21557	1,15979	0,75526
Sum	1	4,29549	2,84038
Risk free r	0,41		
Sharpe:	1,36795		

GARCH			
GMV Weights:		Return	St. Dev
Durbl	-0,19199	-0,98297	-0,92682
Enrgy	0,03129	0,15522	0,18838
Hlth	-0,0768	-0,13133	-0,29809
HiTec	-0,15692	-0,63867	-0,85818
Manuf	0,17015	0,74355	0,70275
NoDur	0,4388	1,22864	1,2729
Other	-0,10705	-0,32222	-0,39949
Shops	0,36012	1,39725	1,49366
Telcm	0,3378	1,77682	1,03237
Utils	0,1946	1,04694	0,69593
Sum	1	4,27323	2,90341
Sharpe:	1,00065		

MPT			
GMV Weights:		Return	St. Dev
Durbl	-0,00306	-0,01565	-0,049
Enrgy	0,27685	1,37319	1,65556
Hlth	0,04147	0,07092	0,24112
HiTec	-0,10328	-0,42033	-0,77328
Manuf	-0,20524	-0,89688	-1,30765
NoDur	0,72357	2,02599	3,41579
Other	-0,39008	-1,17414	-2,54095
Shops	0,01238	0,04804	0,07342
Telcm	0,58735	3,08948	2,69632
Utils	0,06002	0,32289	0,34333
Sum	1	4,42351	3,75466
Sharpe:	1,06894		

Figure 37 – Suggested Portfolios for October 2006

For this month, none of the portfolios suggests that you should change the way you invest and short in the portfolio. As seen before, Non-Durables is the preferred industry with respectively 43%, 44% and 72%, and the portfolios gain a significant part of their profits from this industry. For GARCH and the Simulation model Shops, Telecom, and Utilities are important, while MPT gains a lot from Energy and Telecom. Although MPT has the highest return, the Sharpe ratio indicates that the Simulation model will be the better investment due to the lower risk.

November 2006

Simulation			
GMV Weights:		Return	St. Dev
Durbl	-0,163919	-0,299972	-0,79899
Enrgy	0,029482	0,253544	0,181078
Hlth	-0,077757	-0,006998	-0,320812
HiTec	-0,166539	-0,504614	-0,975159
Manuf	0,15026	0,53643	0,669907
NoDur	0,438725	0,618603	1,329969
Other	-0,133799	-0,223445	-0,522914
Shops	0,36611	-0,486926	1,488572
Telcm	0,304422	0,197874	0,950709
Utils	0,253016	0,789409	0,878259
Sum	1	0,873905	2,880619
Risk free r	0,42		
Sharpe:	0,157572		

GARCH			
GMV Weights:		Return	St. Dev
Durbl	-0,201918	-0,369509	-0,987333
Enrgy	0,064866	0,557849	0,382082
Hlth	0,01517	0,001365	0,057962
HiTec	-0,177467	-0,537724	-0,949898
Manuf	0,060627	0,216437	0,253751
NoDur	0,564077	0,795349	1,646245
Other	-0,044652	-0,074569	-0,166925
Shops	0,394792	-0,525074	1,627124
Telcm	0,228062	0,14824	0,775899
Utils	0,096443	0,300901	0,364196
Sum	1	0,513265	3,003103
Sharpe:	0,031056		

MPT			
GMV Weights:		Return	St. Dev
Durbl	-0,003057	-0,005594	-0,049004
Enrgy	0,276853	2,380932	1,655557
Hlth	0,041473	0,003733	0,241121
HiTec	-0,103276	-0,312926	-0,773281
Manuf	-0,205236	-0,732691	-1,307652
NoDur	0,723569	1,020232	3,415785
Other	-0,390079	-0,651432	-2,540945
Shops	0,012383	-0,016469	0,073424
Telcm	0,587353	0,381779	2,696319
Utils	0,060017	0,187253	0,343333
Sum	1	2,254818	3,754658
Sharpe:	0,488678		

Figure 38 – Suggested Portfolios for November 2006

For the GARCH-Portfolio, Health is not shorted anymore, which shows to be a good idea as it brings a positive return. Non-Durables is the favoured one once again, but for MPT Energy adds a high return, which makes it the portfolio with the highest return. It still has the highest risk as well, but as the Sharpe ratio indicates, it will be the better investment for this period.

December 2006

Simulation			
GMV Weights:		Return	St. Dev
Durbl	-0,147273	0,113401	-0,720271
Enrgy	0,027756	-0,05468	0,1711
Hlth	-0,082649	-0,014877	-0,34835
HiTec	-0,16907	0,160616	-1,009941
Manuf	0,146471	0,0747	0,667883
NoDur	0,449183	0,898367	1,391891
Other	-0,16097	-0,508665	-0,646909
Shops	0,362591	0,522131	1,497834
Telcm	0,283469	1,207578	0,910196
Utils	0,290491	0,200439	1,000253
Sum	1	2,59901	2,913686
Risk free r	0,4		
Sharpe:	0,754717		

GARCH			
GMV Weights:		Return	St. Dev
Durbl	-0,204181	0,15722	-0,968994
Enrgy	0,064434	-0,126935	0,393408
Hlth	-0,005424	-0,000976	-0,020475
HiTec	-0,168035	0,159633	-0,871256
Manuf	0,088033	0,044897	0,365848
NoDur	0,564472	1,128943	1,620993
Other	-0,071295	-0,225293	-0,261708
Shops	0,367855	0,529711	1,49065
Telcm	0,257125	1,095352	0,835324
Utils	0,107018	0,073842	0,396703
Sum	1	2,836394	2,980493
Sharpe:	0,817447		

MPT			
GMV Weights:		Return	St. Dev
Durbl	-0,003057	0,002354	-0,049004
Enrgy	0,276853	-0,5454	1,655557
Hlth	0,041473	0,007465	0,241121
HiTec	-0,103276	0,098112	-0,773281
Manuf	-0,205236	-0,10467	-1,307652
NoDur	0,723569	1,447138	3,415785
Other	-0,390079	-1,23265	-2,540945
Shops	0,012383	0,017831	0,073424
Telcm	0,587353	2,502123	2,696319
Utils	0,060017	0,041412	0,343333
Sum	1	2,233716	3,754658
Sharpe:	0,488384		

Figure 39 – Suggested Portfolios for December 2006

For the GARCH-Portfolio, Health is going back to being shorted. The Energy industry that dominated the MPT-Portfolio in the previous period, now gives a negative return to MPT. The highest weights are still in the Non-Durables industry, which brings significant profits to the portfolios. Telecom is also an important asset to the portfolios. Overall, the GARCH-Portfolio is the most successful one with regards to return and Sharpe ratio.

January 2007

Simulation				
GMV Weights:			Return	St. Dev
Durbl	-0,129868		-0,351941	-0,637283
Enrgy	0,026345		-0,037146	0,162988
Hlth	-0,086868		-0,303171	-0,374164
HiTec	-0,171151		-0,354283	-1,043667
Manuf	0,141274		0,454902	0,658991
NoDur	0,460212		0,699522	1,45879
Other	-0,189767		-0,201153	-0,784559
Shops	0,359309		1,271954	1,509376
Telcm	0,262982		0,841542	0,868642
Utils	0,327534		0,144115	1,120181
Sum	1		2,16434	2,939296
Risk free r	0,44			
Sharpe:	0,586651			

GARCH				
GMV Weights:			Return	St. Dev
Durbl	-0,166681		-0,451705	-0,775017
Enrgy	0,071803		-0,101242	0,425525
Hlth	0,021774		0,07599	0,081306
HiTec	-0,126808		-0,262493	-0,638392
Manuf	0,110963		0,357302	0,446617
NoDur	0,573277		0,871382	1,632921
Other	-0,2269		-0,240514	-0,839657
Shops	0,404307		1,431245	1,58577
Telcm	0,177816		0,569012	0,60106
Utils	0,160449		0,070598	0,57201
Sum	1		2,319574	3,092142
Sharpe:	0,607855			

MPT				
GMV Weights:				St. Dev
Durbl	-0,003057		-0,008285	-0,049004
Enrgy	0,276853		-0,390362	1,655557
Hlth	0,041473		0,144741	0,241121
HiTec	-0,103276		-0,213781	-0,773281
Manuf	-0,205236		-0,660859	-1,307652
NoDur	0,723569		1,099825	3,415785
Other	-0,390079		-0,413484	-2,540945
Shops	0,012383		0,043835	0,073424
Telcm	0,587353		1,879529	2,696319
Utils	0,060017		0,026408	0,343333
Sum	1		1,507568	3,754658
Sharpe:	0,284332			

Figure 40 – Suggested Portfolios for January 2007

The Health industry is no longer shorted in the GARCH-Portfolio, which once again shows to be a good investment. Shops, Telecom and Non-Durables bring in the largest returns to all the portfolios. The Sharpe ratio again suggests that the GARCH-Portfolio is the best investment, with the highest return to risk ratio.

February 2007

Simulation				
GMV Weights:			Return	St. Dev
Durbl	-0,111697		0,04803	-0,549962
Enrgy	0,02526		-0,054056	0,156845
Hlth	-0,09041		0,256765	-0,398074
HiTec	-0,172621		0,407385	-1,075153
Manuf	0,134424		0,102162	0,641538
NoDur	0,47178		-0,721823	1,530772
Other	-0,219989		0,514773	-0,935943
Shops	0,356327		-0,220923	1,523421
Telcm	0,24322		-0,642101	0,826797
Utils	0,363706		1,545749	1,237123
Sum	1		1,235961	2,957364
Risk free r	0,38			
Sharpe:	0,289434			

GARCH				
GMV Weights:			Return	St. Dev
Durbl	-0,155293		0,066776	-0,70831
Enrgy	0,088185		-0,188716	0,505979
Hlth	-0,027168		0,077158	-0,103076
HiTec	-0,09323		0,220023	-0,454156
Manuf	0,011963		0,009092	0,047767
NoDur	0,599804		-0,9177	1,686775
Other	-0,155346		0,36351	-0,563737
Shops	0,381882		-0,236767	1,488625
Telcm	0,16065		-0,424116	0,538746
Utils	0,188554		0,801353	0,648582
Sum	1		-0,229387	3,087195
Sharpe:	-0,197392			

MPT				
GMV Weights:			Return	St. Dev
Durbl	-0,003057		0,001315	-0,049004
Enrgy	0,276853		-0,592465	1,655557
Hlth	0,041473		-0,117784	0,241121
HiTec	-0,103276		0,243731	-0,773281
Manuf	-0,205236		-0,155979	-1,307652
NoDur	0,723569		-1,107061	3,415785
Other	-0,390079		0,912785	-2,540945
Shops	0,012383		-0,007677	0,073424
Telcm	0,587353		-1,550612	2,696319
Utils	0,060017		0,255073	0,343333
Sum	1		-2,118674	3,754658
Sharpe:	-0,665486			

Figure 41 – Suggested Portfolios for February 2007

The Health industry is shorted again for the GARCH-Portfolio, and this brings a positive return. Non-Durables brings significant losses to all the portfolios because of the high weights and a negative return in the industry. Utilities and Others seem to be the industries that perform the best. The Simulation model is the only one that ends with a profit for this month, and also a positive Sharpe ratio.

Overall for the time period it is worth to notice that the individual weights of the industries in the portfolios do not change a lot from month to month. The Health industry is the only one that changes from negative to positive, and only for the GARCH-Portfolio.
The Sharpe ratio indicates that the GARCH-Portfolio is the best investment in 3 out of 6 months, the Simulation-Portfolio is the best in 3 months as well, whereas the MPT-Portfolio only is the best in 1 month. The reason this adds up to 7 is that the GARCH-Portfolio is equal to the Simulation-Portfolio in the first month, where this is the preferred portfolio.
In the period where MPT is the preferred portfolio, the others are less risky and still bring in positive revenue, suggesting that they are not bad investments either. This cannot be said for the MPT-Portfolio, as it collects negative returns in 2 out of 6 months, which is not very impressive in a period of economic growth where only 16 of 60 observed industry returns has a monthly negative return[46]. In comparison, the GARCH-Portfolio only has 1 month of 6 with a negative return, and the Simulation-Portfolio has 0. For this 6 month period the average return for the industries have been:

Durables	Energy	Health	HiTec	Manuf	NoDur	Others	Shops	Telecom	Utilities
2,125	0,743	0,692	1,588	2,385	0,93	1,63	2,112	2,39	1,948

Table 6 – Average Returns for Period of Economic Growth

The objective would have been to invest the most in the industries that have performed the best, but of course that is not easily foreseeable. The simulation model is the least risky of the portfolios in all of the months, while MPT is the riskiest portfolio.

[46] Appendix 2

Portfolios in Economic Recession, July 2008 – December 2008

July 2008

GARCH				
GMV Weights:			Return	St. Dev
Durbl	-0,16668		-0,45171	-0,77502
Enrgy	0,0718		-0,10124	0,42552
Hlth	0,02177		0,07599	0,08131
HiTec	-0,12681		-0,26249	-0,63839
Manuf	0,11096		0,3573	0,44662
NoDur	0,57328		0,87138	1,63292
Other	-0,2269		-0,24051	-0,83966
Shops	0,40431		1,43125	1,58577
Telcm	0,17782		0,56901	0,60106
Utils	0,16045		0,0706	0,57201
Sum	1		2,31957	3,09214
Risk free r	0,15			
Sharpe	0,70164			

GARCH				
GMV Weights:			Return	St. Dev
Durbl	-0,16668		-0,45171	-0,77502
Enrgy	0,0718		-0,10124	0,42552
Hlth	0,02177		0,07599	0,08131
HiTec	-0,12681		-0,26249	-0,63839
Manuf	0,11096		0,3573	0,44662
NoDur	0,57328		0,87138	1,63292
Other	-0,2269		-0,24051	-0,83966
Shops	0,40431		1,43125	1,58577
Telcm	0,17782		0,56901	0,60106
Utils	0,16045		0,0706	0,57201
Sum	1		2,31957	3,09214
Sharpe	0,70164			

MPT				
GMV Weights:			Return	St. Dev
Durbl	-0,06349		0,06603	-0,48233
Enrgy	0,25242		-4,59906	1,50517
Hlth	0,03809		0,24834	0,21963
HiTec	-0,11178		0,08942	-0,83155
Manuf	-0,1054		-0,08853	-0,66642
NoDur	0,75004		0,93005	3,51662
Other	-0,35189		-0,91491	-2,28061
Shops	-0,03632		-0,04467	-0,21379
Telcm	0,56433		-2,07673	2,58989
Utils	0,064		-0,14079	0,3636
Sum	1		-6,53087	3,72021
Sharpe	-1,79583			

Figure 42 – Suggested Portfolios for July 2008

All portfolios short the same industries, except MPT that additionally shorts Manufacturing and Shops. These 2 industries had positive returns, which makes a difference in the overall return for GARCH compared to MPT. As in the last time period, the Non-Durables industry is the one with the highest weight in the portfolio. 'Shops' is another industry with high weights in the GARCH-Portfolio. MPT on the other hand puts a lot of weight into Energy and Telecom. Since the period with economic growth the weights have marginally changed. This is reasonable since the periods of time are close to each other and the weights are based on the historical covariance. It is interesting to see that the GARCH-portfolio significantly outperforms the MPT, with positive returns and low risk. MPT on the other hand has -6,5% and a higher risk. Considering the recession, it is surprising that the GARCH-portfolio has a positive return, when the MPT-portfolio has a negative result. In recognition of the results, the Sharpe ratio also suggests that the GARCH-portfolio is the better investment of the 2, with 0,7 compared to -1,8 of MPT.

August 2008

Simulation		Return	St. Dev
GMV Weights:			
Durbl	-0,08113	-0,199581	-0,726363
Enrgy	0,111121	-0,021113	0,675051
Hlth	0,082737	0,148926	0,363521
HiTec	-0,409075	-0,957236	-2,890569
Manuf	0,13048	0,186586	0,569214
NoDur	0,487942	0,800225	2,131652
Other	-0,09544	-0,02386	-0,740409
Shops	0,363738	1,949638	1,814171
Telcm	0,176951	0,707805	0,790759
Utils	0,232676	-0,114011	0,991728
Sum	1	2,47738	2,978756
Risk free r	0,13		
Sharpe	0,78804		

GARCH		Return	St. Dev
GMV Weights:			
Durbl	-0,141124	-0,347165	-1,187429
Enrgy	0,123468	-0,023459	0,964918
Hlth	0,243716	0,43869	1,128326
HiTec	-0,32063	-0,750274	-2,100679
Manuf	-0,038233	-0,054673	-0,205354
NoDur	0,512717	0,840856	2,10492
Other	-0,116489	-0,029122	-0,836621
Shops	0,459339	2,462058	2,16345
Telcm	0,06954	0,278162	0,415969
Utils	0,207695	-0,10177	1,026544
Sum	1	2,713301	3,474044
Sharpe	0,743601		

MPT		Return	St.Dev
GMV Weights:			
Durbl	-0,06349	-0,039364	-0,482326
Enrgy	0,252418	-0,373579	1,505171
Hlth	0,038088	0	0,21963
HiTec	-0,111778	-0,221321	-0,831553
Manuf	-0,105397	-0,203416	-0,666422
NoDur	0,750042	2,445137	3,516618
Other	-0,35189	-1,104934	-2,280608
Shops	-0,03632	-0,193588	-0,213792
Telcm	0,56433	1,484188	2,589889
Utils	0,063997	0,110074	0,363602
Sum	1	1,903198	3,72021
Sharpe	0,476639		

Figure 43 – Suggested Portfolios for August 2008

GARCH now shorts Manufacturing like MPT. But besides this, the weights do not change significantly in either of the portfolios. The Shops industry adds the most to the return of the Simulation and GARCH-portfolios. Non-Durables also adds a large percentage, and for MPT Telecom is another significant part of the positive returns. All portfolios gain profits in this month, but with lower risk, the Sharpe ratio indicates that the Simulation model is the preferred one, even though GARCH has the highest return.

September 2008

Simultaion		Return	St. Dev
GMV Weights:			
Durbl	-0,071057	0,666516	-0,638319
Enrgy	0,113885	-1,433809	0,694126
Hlth	0,091157	-0,543294	0,409453
HiTec	-0,433208	5,406433	-3,113848
Manuf	0,133084	-1,7993	0,591238
NoDur	0,49933	-1,483009	2,235359
Other	-0,089343	0,540524	-0,703954
Shops	0,328983	-1,894941	1,662675
Telcm	0,153313	-1,933277	0,704376
Utils	0,273856	-3,308186	1,167659
Sum	1	-5,782342	3,008766
Risk free r	0,15		
Sharpe	-1,971686		

GARCH		Return	St. Dev
GMV Weights:			
Durbl	-0,147274	1,381429	-1,170912
Enrgy	0,121279	-1,526897	0,901907
Hlth	0,250035	-1,490208	1,11368
HiTec	-0,335736	4,189982	-2,08127
Manuf	-0,017807	0,240744	-0,090627
NoDur	0,52658	-1,563941	2,075153
Other	-0,055453	0,335489	-0,370254
Shops	0,347221	-1,999993	1,647725
Telcm	0,054416	-0,686186	0,30849
Utils	0,256739	-3,101405	1,210627
Sum	1	-4,220986	3,54452
Sharpe	-1,233167		

MPT		Return	St.Dev
GMV Weights:			
Durbl	-0,06349	0,726329	-0,482326
Enrgy	0,252418	-5,729899	1,505171
Hlth	0,038088	-0,496289	0,21963
HiTec	-0,111778	1,612957	-0,831553
Manuf	-0,105397	1,688459	-0,666422
NoDur	0,750042	-4,980279	3,516618
Other	-0,35189	2,135971	-2,280608
Shops	-0,03632	0,344318	-0,213792
Telcm	0,56433	-8,944633	2,589889
Utils	0,063997	-0,540132	0,363602
Sum	1	-14,1832	3,72021
Sharpe	-3,852793		

Figure 44 – Suggested Portfolios for September 2008

This is a severe month to all the portfolios. They all end with losses, and the HiTec industry is the only one that adds significant returns to the GARCH and Simulation-Portfolios. For MPT the Other industry adds, but overall it is not enough. Returns from -4% to -14% show that the recession really has kicked in. The Sharpe ratio is highest for GARCH, which also has the lowest loss, and shows to be the preferred portfolio.

October 2008

Simulation			
GMV Weights:		Return	St. Dev
Durbl	-0,061043	2,005888	-0,550211
Enrgy	0,116446	-2,019177	0,712087
Hlth	0,101256	-1,115843	0,465065
HiTec	-0,458303	8,455692	-3,352394
Manuf	0,137246	-2,854722	0,621352
NoDur	0,510266	-6,664077	2,341144
Other	-0,084029	1,646973	-0,672585
Shops	0,293807	-4,454119	1,50571
Telcm	0,130115	-2,024588	0,615069
Utils	0,314239	-3,481765	1,341991
Sum	1	-10,50574	3,027227
Risk free r	0,08		
Sharpe	-3,496843		

GARCH			
GMV Weights:		Return	St. Dev
Durbl	-0,085469	2,808527	-0,710773
Enrgy	0,174292	-3,022222	1,437909
Hlth	0,185021	-2,038935	0,914407
HiTec	-0,279547	5,157641	-2,142738
Manuf	-0,311348	6,47604	-2,224813
NoDur	0,80469	-10,50925	3,210444
Other	-0,051464	1,00869	-0,348519
Shops	0,340548	-5,162706	1,723919
Telcm	0,034236	-0,532707	0,252451
Utils	0,189042	-2,094582	1,197636
Sum	1	-7,909502	3,309922
Sharpe	-2,413804		

MPT			
GMV Weights:		Return	St.Dev
Durbl	-0,06349	1,996135	-0,482326
Enrgy	0,252418	-8,201075	1,505171
Hlth	0,038088	-0,715296	0,21963
HiTec	-0,111778	2,510535	-0,831553
Manuf	-0,105397	2,680244	-0,666422
NoDur	0,750042	-15,88589	3,516618
Other	-0,35189	6,16159	-2,280608
Shops	-0,03632	0,878591	-0,213792
Telcm	0,56433	-14,84188	2,589889
Utils	0,063997	-0,651486	0,363602
Sum	1	-26,06854	3,72021
Sharpe	-7,02878		

Figure 45 – Suggested Portfolios for October 2008

This month is the worst for the portfolios. Negative returns for all of them, and -8% to -26% is a lot to lose in a single month. The GARCH-Portfolio loses least though, even though its risk is higher than the Simulation-Portfolio. But the Sharpe ratio also indicates that the GARCH-Portfolio is the preferred investment.

November 2008

Simulation			
GMV Weights:		Return	St. Dev
Durbl	-0,051284	0,587205	-0,463805
Enrgy	0,118784	-0,065331	0,728793
Hlth	0,112959	-0,829119	0,530591
HiTec	-0,484034	5,314698	-3,604469
Manuf	0,1428	-1,109558	0,659219
NoDur	0,520702	-2,82741	2,448708
Other	-0,079621	0,985705	-0,647545
Shops	0,258744	-1,992325	1,345487
Telcm	0,1077	-0,169089	0,52418
Utils	0,35325	0,377978	1,512871
Sum	1	0,272753	3,03403
Risk free r	0,03		
Sharpe	0,08001		

GARCH			
GMV Weights:		Return	St. Dev
Durbl	-0,131155	1,501722	-1,840591
Enrgy	0,164521	-0,090487	1,594368
Hlth	0,153491	-1,126622	0,971689
HiTec	-0,211464	2,321874	-2,135756
Manuf	-0,487354	3,786742	-5,039291
NoDur	0,971656	-5,276094	5,789301
Other	-0,119715	1,482068	-1,204836
Shops	0,286425	-2,205475	2,065052
Telcm	0,098536	-0,154702	0,913075
Utils	0,275057	0,294311	1,997617
Sum	1	0,533337	3,110627
Sharpe	0,161812		

MPT			
GMV Weights:		Return	St.Dev
Durbl	-0,06349	1,051399	-0,482326
Enrgy	0,252418	-4,881773	1,505171
Hlth	0,038088	-0,527522	0,21963
HiTec	-0,111778	1,608486	-0,831553
Manuf	-0,105397	1,628383	-0,666422
NoDur	0,750042	-12,39069	3,516618
Other	-0,35189	4,264904	-2,280608
Shops	-0,03632	0,675923	-0,213792
Telcm	0,56433	-6,997694	2,589889
Utils	0,063997	-0,069116	0,363602
Sum	1	-15,6377	3,72021
Sharpe	-4,211511		

Figure 46 – Suggested Portfolios for November 2008

This is another difficult month for MTP, with a negative return of 15%. On the other hand though, it shows that both the Simulation- and GARCH-Portfolio have ended up with positive returns. For the Simulation-Portfolio it is mainly because of HiTec and Others, while GARCH is held up by HiTec, Manufacturing and Others. Again this month, the GARCH-Portfolio is the one with the highest return and Sharpe ratio.

December 2008

Simulation			
GMV Weights:		Return	St. Dev
Durbl	-0,041949	-0,063342	-0,380651
Enrgy	0,120892	-0,385647	0,744188
Hlth	0,126139	0,85144	0,606023
HiTec	-0,510087	-1,117091	-3,86826
Manuf	0,14952	0,215308	0,704183
NoDur	0,530647	0,525341	2,558018
Other	-0,076227	-0,065555	-0,630059
Shops	0,224311	1,09464	1,184311
Telcm	0,086376	0,297998	0,433095
Utils	0,390376	-0,804175	1,678607
Sum	1	0,548917	3,029453
Risk free r	0,09		
Sharpe	0,151485		

GARCH			
GMV Weights:		Return	St. Dev
Durbl	-0,145925	-0,220347	-2,013123
Enrgy	0,15479	-0,493781	1,421163
Hlth	0,131738	0,889229	0,870211
HiTec	-0,174898	-0,383026	-1,804644
Manuf	-0,480802	-0,692355	-4,840674
NoDur	0,98568	0,975823	5,911807
Other	-0,100259	-0,086223	-1,058756
Shops	0,268036	1,308018	1,979685
Telcm	0,088461	0,305192	0,756593
Utils	0,273178	-0,562746	1,86578
Sum	1	1,039784	3,088041
Sharpe	0,307568		

MPT			
GMV Weights:		Return	St.Dev
Durbl	-0,06349	0,182217	-0,482326
Enrgy	0,252418	-0,724441	1,505171
Hlth	0,038088	-0,109313	0,21963
HiTec	-0,111778	0,320803	-0,831553
Manuf	-0,105397	0,302489	-0,666422
NoDur	0,750042	-2,152621	3,516618
Other	-0,35189	1,009924	-2,280608
Shops	-0,03632	0,10424	-0,213792
Telcm	0,56433	-1,619628	2,589889
Utils	0,063997	-0,18367	0,363602
Sum	1	-2,87	3,72021
Sharpe	-0,795654		

Figure 47 – Suggested Portfolios for December 2008

The last month of the recession period shows that MPT is still delivering negative returns. The simulation and GARCH-Portfolio have positive returns again, mainly because of Non-Durables, Shops and Health. This time as well, the GARCH-Portfolio has the highest return and Sharpe ratio.

Overall for the time period, it is worth noticing that the individual weights of the industries in the portfolios do not change a lot from month to month once again. The Manufacturing industry is the only one that changes from negative to positive, and only for the GARCH-Portfolio.
The Sharpe ratio indicates that the GARCH-Portfolio is the best investment in 5 out of 6 months, the Simulation-Portfolio is the best in 2 months, whereas the MPT-Portfolio underperforms compared to the others, as it is never the best. Again, the first period is favored towards the GARCH and Simulations portfolios, and after that the GARCH model completely dominates.
In the month where the Simulation-Portfolio is the preferred one, it almost has the same Sharpe ratio as GARCH, and GARCH even has a higher return. It is only because of lower risk that the Simulation-Portfolio should be chosen, but still the GARCH-Portfolio shows to be a good option.
It is interesting to see that even though we are in the midst of a recession, the GARCH and

Simulation-Portfolios manages to deliver a positive return in 4 out of 6 months, whereas the MPT-Portfolio only achieves this in 1 out of 6 months.

For this 6 month period the average return for the industries have been:

Durables	Energy	Health	HiTec	Manuf	NoDur	Others	Shops	Telecom	Utilities
-8,338	-8,277	-1,507	-6,31	-6,58	-2,745	-5,467	-2,878	-4,245	-5,31

Table 7 – Average Returns of Recession Period

With average returns that are negative for all industries, it is impressive that GARCH can dominate MPT and still create positive returns in 4 out of 6 months. The Simulation-Portfolio is the least risky in all of the months, with the GARCH-Portfolio right after. The riskiest is MPT.

Portfolios in the Most Recent Time Period, July 2012 - December 2012

July 2012

Simulation				
GMV Weights:			Return	St. Dev
Durbl	0,01553		-0,00575	0,144
Enrgy	-0,06368		-0,17258	-0,45654
Hlth	-0,01186		-0,02005	-0,05509
HiTec	-0,00039		6,2E-05	-0,00243
Manuf	0,0989		0,11275	0,5845
NoDur	0,37201		0,56174	1,34665
Other	-0,33738		0,40823	-2,12735
Shops	0,21946		0,26335	0,93519
Telcm	0,10216		0,41376	0,4456
Utils	0,60524		2,19701	1,97636
Sum	1		3,75853	2,79091
Risk free r	0,00			
Sharpe	1,34671			

GARCH				
GMV Weights:			Return	St. Dev
Durbl	0,01553		-0,00575	0,144
Enrgy	-0,06368		-0,17258	-0,45654
Hlth	-0,01186		-0,02004	-0,05508
HiTec	-0,00039		6,2E-05	-0,00242
Manuf	0,0989		0,11275	0,58451
NoDur	0,37201		0,56174	1,34664
Other	-0,33738		0,40823	-2,12735
Shops	0,21946		0,26335	0,9352
Telcm	0,10216		0,41376	0,4456
Utils	0,60524		2,19701	1,97636
Sum	1		3,75853	2,79091
Sharpe	1,34671			

MPT				
GMV Weights:			Return	St. Dev
Durbl	-0,06729		0,0249	-0,52716
Enrgy	0,24222		0,65642	1,46093
Hlth	0,04609		0,07789	0,26355
HiTec	-0,10325		0,01652	-0,76419
Manuf	-0,14274		-0,16273	-0,91018
NoDur	0,77034		1,16321	3,59445
Other	-0,34566		0,41825	-2,26161
Shops	-0,01409		-0,01691	-0,08248
Telcm	0,5425		2,19712	2,52153
Utils	0,07189		0,26095	0,40431
Sum	1		4,63561	3,69914
Sharpe	1,25316			

Figure 48 – Suggested Portfolios for July 2012

In several industries, the GARCH-portfolio and the MPT-portfolio disagree on whether to take a long- or short position in specific assets. Since the time period of recession in 2008, the GARCH model suggests a lower percentage in Non-Durables, while Modern Portfolio Theory suggests a higher percentage. Telecom still has a great weight of the MPT-Portfolio, and has also increased its weight for the GARCH-Portfolio. Additionally, there are a lot of low weights in GARCH, so they do not make a big difference for the total return, except for Utilities that has risen from 27% to 61% weights. Energy also still has a great weight for MPT-Portfolio. Both portfolios perform well this month, with MPT having the highest return of 4,6%. But this comes with a higher risk, and if we consider the Sharpe ratio, it suggests that the GARCH-Portfolio is a better investment.

August 2012

Simulation			
GMV Weights:		Return	St. Dev
Durbl	-0,04506	-0,19107	-0,35344
Enrgy	0,25782	0,621347	1,557281
Hlth	0,04623	0,049928	0,269723
HiTec	-0,11304	-0,54938	-0,85051
Manuf	-0,17669	-0,5548	-1,14681
NoDur	0,785785	0,047147	3,747706
Other	-0,36975	-1,16102	-2,46058
Shops	-0,02032	-0,05709	-0,12086
Telcm	0,525312	0,441262	2,508211
Utils	0,109712	-0,37412	0,619609
Sum	1	-1,72779	3,770335
Risk free r	0,01		
Sharpe	-0,46091		

GARCH			
GMV Weights:		Return	St. Dev
Durbl	0,012731	0,053981	0,111191
Enrgy	-0,08446	-0,20356	-0,57821
Hlth	-0,02639	-0,0285	-0,11781
HiTec	0,002001	0,009723	0,011812
Manuf	0,039149	0,122927	0,217883
NoDur	0,468779	0,028127	1,640607
Other	-0,21709	-0,68167	-1,29188
Shops	0,205074	0,576257	0,842459
Telcm	0,032929	0,027661	0,140712
Utils	0,567282	-1,93443	1,873801
Sum	1	-2,02948	2,850563
Sharpe	-0,72		

MPT			
GMV Weights:		Return	St. Dev
Durbl	-0,06729	-0,28532	-0,52716
Enrgy	0,242222	0,583755	1,460929
Hlth	0,046089	0,049776	0,263549
HiTec	-0,10325	-0,50177	-0,76419
Manuf	-0,14274	-0,44821	-0,91018
NoDur	0,770338	0,04622	3,594454
Other	-0,34566	-1,08536	-2,26161
Shops	-0,01409	-0,03961	-0,08248
Telcm	0,542499	0,455699	2,521529
Utils	0,071886	-0,24513	0,40431
Sum	1	-1,46996	3,699145
Sharpe	-0,40008		

Figure 49 – Suggested Portfolios for August 2012

The Simulation model suggests that 6 of the industries change from short to long, and long to short, but these changes are not so significant. The most significant changes lie within the industries Energy, Shops, Telecom and Utilities. This month the portfolios perform almost equally but MPT has a slightly higher Sharpe ratio than the Simulation-Portfolio.

September 2012:

Simultaion				
GMV Weights:			Return	St. Dev
Durbl	-0,02513		-0,08543	-0,19773
Enrgy	0,273408		0,99794	1,658013
Hlth	0,046796		0,224151	0,279257
HiTec	-0,12		-0,1368	-0,92019
Manuf	-0,21168		-0,36197	-1,40221
NoDur	0,804227		0,796185	3,930014
Other	-0,39789		-1,49208	-2,70098
Shops	-0,02448		-0,04846	-0,1485
Telcm	0,506996		2,453861	2,493594
Utils	0,147747		0,313223	0,840621
Sum	1		2,660611	3,831878
Risk free r	0,01			
Sharpe	0,691726			

GARCH				
GMV Weights:			Return	St. Dev
Durbl	-0,01895		-0,06444	-0,15736
Enrgy	-0,07818		-0,28536	-0,51142
Hlth	-0,04293		-0,20566	-0,18479
HiTec	0,034178		0,038963	0,197844
Manuf	0,088659		0,151607	0,471463
NoDur	0,433704		0,429367	1,473206
Other	-0,24865		-0,93244	-1,40283
Shops	0,298469		0,590969	1,198934
Telcm	0,12867		0,622763	0,516359
Utils	0,405038		0,858681	1,429292
Sum	1		1,204455	3,03069
Sharpe	0,39			

MPT				
GMV Weights:			Return	St. Dev
Durbl	-0,06729		-0,2288	-0,52716
Enrgy	0,242222		0,88411	1,460929
Hlth	0,046089		0,220768	0,263549
HiTec	-0,10325		-0,1177	-0,76419
Manuf	-0,14274		-0,24409	-0,91018
NoDur	0,770338		0,762635	3,594454
Other	-0,34566		-1,29621	-2,26161
Shops	-0,01409		-0,02791	-0,08248
Telcm	0,542499		2,625694	2,521529
Utils	0,071886		0,152399	0,40431
Sum	1		2,730894	3,699145
Sharpe	0,735547			

Figure 50 – Suggested Portfolios for September 2012

Again, we see that GARCH is the least risky of the portfolios. The weights have not changed a lot since the previous month, but the Simulation-Portfolio and MPT are much like each other for risk and return. This almost gives them the same Sharpe ratio, but again for this month, MPT is slightly better and even less than previously.

October 2012:

Simulation				
GMV Weights:			Return	St. Dev
Durbl	-0,00466		-0,02929	-0,03683
Enrgy	0,288986		-0,52307	1,759464
Hlth	0,048094		-0,08994	0,293567
HiTec	-0,12659		0,817791	-0,98974
Manuf	-0,24804		0,153784	-1,67668
NoDur	0,821614		-1,3721	4,114251
Other	-0,42686		0,046955	-2,95673
Shops	-0,02651		0,02863	-0,16411
Telcm	0,488337		-1,18177	2,47511
Utils	0,185638		0,200489	1,065122
Sum	1		-1,94852	3,883428
Risk free r	0,01			
Sharpe	-0,50433			

GARCH				
GMV Weights:			Return	St. Dev
Durbl	-0,02344		-0,14718	-0,18457
Enrgy	-0,07735		0,140012	-0,48718
Hlth	-0,05901		0,110351	-0,25837
HiTec	0,030614		-0,19777	0,168671
Manuf	0,10204		-0,06326	0,514965
NoDur	0,458497		-0,76569	1,51005
Other	-0,25388		-0,24373	-1,37444
Shops	0,286359		-0,30927	1,117401
Telcm	0,132467		-0,32057	0,540916
Utils	0,403708		0,436004	1,384379
Sum	1		-1,3611	2,931816
Sharpe	-0,46766			

MPT				
GMV Weights:			Return	St. Dev
Durbl	-0,06729		-0,4226	-0,52716
Enrgy	0,242222		-0,43842	1,460929
Hlth	0,046089		-0,08619	0,263549
HiTec	-0,10325		0,666966	-0,76419
Manuf	-0,14274		0,088501	-0,91018
NoDur	0,770338		-1,28646	3,594454
Other	-0,34566		-0,33183	-2,26161
Shops	-0,01409		0,015223	-0,08248
Telcm	0,542499		-1,31285	2,521529
Utils	0,071886		0,077637	0,40431
Sum	1		-3,03003	3,699145
Sharpe	-0,82182			

Figure 51 – Suggested Portfolios for October 2012

This month, all of the portfolios deliver a negative return. With GARCH being the least risky, it is not surprising that it has minimized the losses compared to the two other portfolios, and therefore giving it an advantage and a higher Sharpe ratio.

November 2012:

Simulation			
GMV Weights:		Return	St. Dev
Durbl	0,016342	0,041017	0,129468
Enrgy	0,304415	-0,48098	1,86079
Hlth	0,05008	0,072617	0,312684
HiTec	-0,13256	-0,19487	-1,05708
Manuf	-0,28582	-0,77171	-1,97129
NoDur	0,837925	2,639465	4,300028
Other	-0,45642	0,132361	-3,22673
Shops	-0,02644	-0,03729	-0,16713
Telcm	0,469547	0,056346	2,453395
Utils	0,22293	-0,85382	1,291101
Sum	1	0,603147	3,92524
Risk free r	0,01		
Sharpe	0,151111		

GARCH			
GMV Weights:		Return	St. Dev
Durbl	0,012293	0,030856	0,094147
Enrgy	-0,1118	0,176646	-0,68123
Hlth	-0,05885	-0,08534	-0,25592
HiTec	-0,08824	-0,12971	-0,52318
Manuf	0,130567	0,352531	0,63127
NoDur	0,399993	1,259977	1,323545
Other	-0,16974	0,049224	-0,86689
Shops	0,292561	0,41251	1,124377
Telcm	0,121814	0,014618	0,492697
Utils	0,471405	-1,80548	1,56229
Sum	1	0,275829	2,9011
Sharpe	0,09163		

MPT			
GMV Weights:			St. Dev
Durbl	-0,06729	-0,16891	-0,52716
Enrgy	0,242222	-0,38271	1,460929
Hlth	0,046089	0,066829	0,263549
HiTec	-0,10325	-0,15177	-0,76419
Manuf	-0,14274	-0,38541	-0,91018
NoDur	0,770338	2,426565	3,594454
Other	-0,34566	0,100241	-2,26161
Shops	-0,01409	-0,01987	-0,08248
Telcm	0,542499	0,0651	2,521529
Utils	0,071886	-0,27532	0,40431
Sum	1	1,274742	3,699145
Sharpe	0,341901		

Figure 52 – Suggested Portfolios for November 2012

This month we are back at positive returns. The Non-Durables industry is doing well, and therefore pushing up the total returns in all the portfolios. But for MPT and the Simulation-Portfolio this number is much more significant since the weight are 83,8% and 77% respectively, compared to 40 for GARCH. Manufacturing and Utilities give the Simulation-Portfolio a greater loss than MPT whereas the other industries are more similar. This ends up making MPT the most successful for both return and Sharpe ratio.

December 2012:

Simulation			
GMV Weights:		Return	St. Dev
Durbl	0,037928	0,288254	0,301502
Enrgy	0,319561	0,447385	1,961158
Hlth	0,05268	-0,02265	0,336428
HiTec	-0,13767	-0,03304	-1,12002
Manuf	-0,32509	-0,87123	-2,28734
NoDur	0,853193	-1,95381	4,487203
Other	-0,48626	-1,72622	-3,50955
Shops	-0,02437	0,024609	-0,15727
Telcm	0,450832	0,568048	2,429167
Utils	0,259183	0,025918	1,516494
Sum	1	-3,25274	3,957777
Risk free r	0,01		
Sharpe	-0,82439		

GARCH			
GMV Weights:		Return	St. Dev
Durbl	0,015131	0,114993	0,109875
Enrgy	-0,09688	-0,13563	-0,57125
Hlth	-0,05855	0,025177	-0,24641
HiTec	-0,11252	-0,02701	-0,63406
Manuf	0,135535	0,363235	0,631123
NoDur	0,453766	-1,03912	1,496579
Other	-0,19155	-0,68001	-0,9314
Shops	0,286557	-0,28942	1,070049
Telcm	0,164838	0,207696	0,629894
Utils	0,403674	0,040367	1,453636
Sum	1	-1,41972	3,008031
Sharpe	-0,4753		

MPT			
GMV Weights:		Return	St. Dev
Durbl	-0,06729	-0,51143	-0,52716
Enrgy	0,242222	0,339111	1,460929
Hlth	0,046089	-0,01982	0,263549
HiTec	-0,10325	-0,02478	-0,76419
Manuf	-0,14274	-0,38255	-0,91018
NoDur	0,770338	-1,76407	3,594454
Other	-0,34566	-1,22708	-2,26161
Shops	-0,01409	0,014236	-0,08248
Telcm	0,542499	0,683548	2,521529
Utils	0,071886	0,007189	0,40431
Sum	1	-2,88565	3,699145
Sharpe	-0,78279		

Figure 53 – Suggested Portfolios for December 2012

The last month the portfolios deliver negative returns again. GARCH is the least risky, and also has the highest return. The weights have not changed a lot from the previous month and the Sharpe ratio indicated that the GARCH-Portfolio is.

In this period of time, the individual weights change more from month to month than they did in the first 2 periods. The Sharpe ratio indicates that the GARCH-Portfolio is the best investment in 3 out of 6 months, and the same is the case for the MPT-Portfolio. In the 3 months when MPT has the highest Sharpe ratio, it is not a lot better than the Simulation-Portfolio, whereas it seems like the GARCH-Portfolio is performing significantly better in the months it has the highest Sharpe ratio, except for the first month.

For this 6 month period the average return for the industries have been:

Durables	Energy	Health	HiTec	Manuf	NoDur	Others	Shops	Telecom	Utilities
3,943	1,13	1,118	0,182	1,792	0,292	1,65	0,885	1,448	-0,052

Table 8 – Average Returns for Most Recent Period

The average return is positive in 9 out of 10 industries for this time period, but the portfolios deliver a negative return in half of the months. For those 3 months GARCH performs better in 2 of them, while MPT performs best in 1.

The least risky portfolio is GARCH, which has changed from the two first time periods, where the Simulation-Portfolio was the least risky. MPT is the riskiest in 1 month, and the Simulation-Portfolio is the riskiest in 5.

For all of the time periods you could think that the least risky portfolios would be the ones that performed the best in a recession, while the riskiest would perform better in a boom. But this has shown not to be the case. For the recession, the GARCH was completely dominating. It was less risky than MPT, but still a bit riskier than the Simulation-Portfolio. Additionally, for the period of economic growth, we saw that MPT was the riskiest portfolio in all of the 6 month, but only managed to deliver a higher return than the GARCH-Portfolio in 1 month.

In the period of economic growth, the industries that performed the best in the 6 chosen months were Durables, Manufacturing, Shops and Telecom. We find this very reasonable, as theories suggest that people will spend more in these industries, whenever they have more funds available to spend.

In the economic recession, all of the industries delivered negative returns on average. But it is interesting to see that the Health industry had significantly less losses than any of the other industries. Once again we find this very reasonable, as people need to purchase medicine regardless of their economic state. After the Health industry, the ones that came out of the period with the best results were Non-Durables and Shops.

In the most recent time period, the Durables industry had the highest average return, which might indicate that people are starting to spend money again on leisure items.

Overall we see that the GARCH model is outperforming the others in 11 out of 18 cases. The Simulation model is the best 6 times, where 3 of them are because it is sharing the portfolio with GARCH. The MPT-Portfolio only outperforms the others in 4 of 18 months, and when it does, the others are often almost as good in the return, and sometimes even less risky, suggesting that it is very reasonable to use GARCH forecasting to rebalance the weights in your portfolio. If not a better model, then it has shown to be at least as good as the traditional optimization using modern portfolio theory.

If we look at the period of economic recession, the GARCH was absolutely dominating with risk and return. This suggests that the constellation of weights for the GARCH model is comparatively better than MPT in recession than in the other periods. In the other periods it is more equaled out, but still seems like GARCH has a lot to offer for portfolio selections.

Part 4 – Discussion, Conclusion, and Reflective Thoughts

Discussion of Results

After presenting the results of our analysis above, it is crucial for the interpretation of these results that we discuss our findings and use theories to explain some of the indications of the results.

We have seen that our GMV portfolio construction by using our multivariate GARCH results generally outperforms the standard, fixed mean risk, modern portfolio theory mean-variance optimization. That is at least the case in the majority of the months, for which we have presented portfolio-strategies. However, we cannot ignore the fact that there are a few months where the mean-risk optimization outperforms the optimization done with the results of our time-varying risk model. Most notably there are three months in out of our most recent 6 month period, in the second half of 2012, where the conditional variance-covariance model falls short of the simpler alternative. In the following, we will discuss possible explanations and indications of these results. We will go through theoretical and statistical complications that may affect the results we get, or help explain why our model in a few cases seems to have shortfalls.

Does the Model Hold?

A reasonable question to ask when trying to explain those few earlier mentioned shortfalls is if the model actually holds. We will refer to our ARCH/GARCH testing in the early stages of the analysis as well as the selection of a GARCH(1,1) model based on both the Akaike- and Schwarz information criterion. This means that statistically the GARCH(1,1) model should apply to our data and that the GARCH-model holds.

We also know that our historical returns data is accurate. It is the same data used by Fama & French in some of their most widely recognized studies (e.g. the three-factor model), and as historical stock prices are easily accessible through most media, the data completely reflects the correct historical returns.

The GARCH-model will use Monte Carlo Simulations to forecast into the future, which means that we have a certain statistical framework of the conditional risk that we eventually try to forecast. The model also interprets which directions the risk generally moves in based on how the stock prices historically have fluctuated. This means that the model proposes qualified guesses of the variance-covariance matrices we forecast, and it can always be expected that there is some degree of errors in this model, compared to what will actually happen in reality, as the model obviously cannot tell us exactly what is going to happen on the US Stock Markets or the economy in general, which can affect the conditional risks of our industry-portfolios, as well as all other risky securities for that matter. By this reasoning it is only as expected that there are some months, where our models suggestions are off and do not seem to hold. In general though, we have statistically shown that a GARCH(1,1) model will be the appropriate model to use for our forecasting of risk, because the variance-covariance matrices seems to be conditional or time-varying. So the model choice is arguably correct in this type of time-series model, where heteroskedasticity is obvious. But we must keep in mind that an econometric model used for forecasting will produce a result of what is statistically most likely to happen in the forecasting period. But in reality, a lot of things happen that are statistically unlikely. The stock market is no different. The Monte Carlo Simulation method for multi-period forecasting is also just based on the probability distributions and the parameters estimated by the model. If anything, a multi-period forecast using Monte Carlo and GARCH will be as or less statistically deviating from reality. This also explains why the optimization based on the conditional covariance matrices from the GARCH model in EViews, seem to be the best option out of our three methods of constructing Global Minimum Variance portfolios.

So we believe that the model holds, but obviously cannot be taken for more than what it is: a model to produce a statistically significantly qualified guess of how the conditional heteroskedasticity will behave. Keeping this in mind, we actually believe that our model is doing a good job in estimating conditional variance-covariance matrices, as the output used to create

GMV-portfolios actually produces minimum variance portfolios that yields better risk-return ratios than regular and simpler GMVs estimated with the Modern Portfolio Theory mean-variance approach. This is due to the fact that our suggestions of GMVs in 14 out of 18 sample-cases are outperforming the MPT approach. Another important thing to keep in mind is that we are computing GMVs, meaning the portfolio on the efficient frontier that has the lowest risk. In our case, we actually manage to use the GARCH-model to create efficient portfolios with a lower standard deviation in all of our 18 cases, indicating that the model holds, even if the suggestions from the MPT approach gives a better return or risk-return ratio. So in conclusion, we consider the results a successful indication of our model holding.

Discussion on Portfolio Results

All the portfolios are created as Global Minimum Variance portfolios. This means that they represent the least risky efficient portfolios possible.
The returns are changing a lot from month to month, according to how well the industries perform. This is the biggest impact on the overall return of the portfolios, because the weights do not change a lot for the GARCH and Simulation-Portfolios, and for the MPT-Portfolio, they do not change at all. If you consider the average returns from the industry, for the half-year periods, then you could suggest that some industries should have a higher weight in the portfolio. But standing at T=0 it is of course impossible to predict which industries will perform better compared to the others. The optimal way would always be to have the highest weights in the industries that perform the best. But since the models are based on covariance between the industries, it is a suggestion on how you should invest according to how the industries go up and down together, and in the given time period. That is why we believe this is a good method.

We have seen that our suggested models of creating portfolios have outperformed the MPT-Portfolios in 14 out of 18 months. And in all 18 months, the MPT-Portfolio was the riskiest in comparison with GARCH and the Simulation-Portfolio. This means that we are finding portfolios that are efficient and have less risk than the portfolios of Modern Portfolio Theory. In itself this is a great achievement, as the object was to find global minimum variance portfolios. To spread out the risk even more, we could make a portfolio that will consist of 50% weights in the GARCH-Portfolio and 50% in the Simulation-Portfolio. Doing so would eliminate some of the risk that one of the portfolios would underperform in a given month, but also remove some of the return in a month with profits. The good thing is that you would not need to choose between the 2 methods, and it would be a safe way to optimize your portfolio, as both of the methods have shown to give a low risk, and still be efficient.

Even though the results look very promising for the GARCH and Simulation-Portfolios, we still have to be a little critical. The models are created with historical data from 1926 until 2012. This creates certain bias when we forecast for our three selected time periods, because these returns already are calculated into the model. It can be said that we have used future numbers to forecast the past, and then it will be easier to "guess" the outcome. But to support our model, we can argue that these data account for a very small portion of the entire model. And if we had stopped the model and only accepted data until 2005, and then used it to forecast 2006, there is a big probability that the results would not have changed significantly.

Maybe a more realistic result would have been achieved with the help of the solver function in excel, and then not allowing short sales by setting up limits. But for the purpose of this performance test, and to demonstrate how to optimize portfolios using modern portfolio theory and Global Minimum Variance, we believe that we are achieving true and fair portfolios from this method.

Omitted Variable Bias

In econometrics, one of the most common factors that disrupt models and results is the omitted variable bias. Another very common bias is causality or direction of causality, but this will not be discussed here as we will discuss the Efficient Market Hypothesis and thus discuss the indications of causality in historic stock price fluctuations. The omitted variable bias however, is a factor that you should never forget to discuss in a model that tries to forecast stock price fluctuations. In our case we have analyzed time-varying risk in a time-series model, meaning that the explanatory variable in our regression model, is in reality lagged values of conditional systematic and unsystematic risk (covariance and error-terms), which is all derived from our raw input-data of historical returns. We have through statistical testing found strong indications of the presence of ARCH, which serves a proof of time-varying variance/covariance, so the model seem to be statistically valid. However, it is important to keep in mind that our model forecasts today's volatility by looking at the volatility of the previous month, which raises the question; is today's risk merely a function of the lagged values? Needless to say the answer to the question is in pretty much all cases, "NO". There is a ton of explanatory factors that could be taken into consideration when we talk about conditional time-varying risk. The obvious ones are of course the state of the economy, implementations of business or tax laws and also corporate strategies. But also investors' risk-aversion levels which are immeasurable and psychologically determined. Recent theories and what some are calling "Post Modern Portfolio Theory" has a higher emphasis on behavioral finance, which deals with investor psychology and to a great extend more qualitative and less quantitative research. This thesis have been a quantitative study, so we will not go into details with behavioral finance and investor psychology, etc., but it is important when we assess the model and discuss the results to keep in mind that there are several explanatory variables that are not part of the model, and some that are immeasurable and can never be perfectly quantified. There is no doubt that a "better" investment model can be developed, perhaps by including more explanatory variables, but for now we only need to look at the implications of our model, and whether or not it seems to perform better than a more standardized approach.

In the results, we see indications that our model actually outperforms portfolio weights calculated on a mean-return and mean-risk basis, in far most of the periods. As expected there are some examples of the mean-method that performs better, and we believe that the omitted variable bias can be part of an explanation to why this is happening. Of course there are several other factors that need to be taken into consideration, and we will discuss these in the following sub-sections of our discussion.

Efficient Market Hypothesis

Although the model that we have suggested outperforms the simpler version of mean-variance optimization in most of our analyzed months, there is still a recent consecutive period of three months where it does not. There can be several factors that can help explain this, other than the fact that we are dealing with statistical measures and cannot expect our model to outperform simple mean-variance optimization in every period.

One of the things we want to discuss is how the Efficient Market Hypothesis (EMH) can be related to these results. If we look back at the theory section of this thesis, we will see an explanation of the three forms of efficient markets; strong efficient, semi-strong efficient and weak efficient. The weak form of market efficiency is when stock prices fully reflect past trading information on securities. As past stock price fluctuations is basically the raw input data we use in our forecasting, we would in fact need to have weak form efficiency in the US Stock market, in order for our GARCH-model to fully hold. An explanation to the shortfall of our model in a few of the periods, other than statistical outliers or such, could be that the GARCH-model does not fully hold because there is not weak-form market efficiency. However, our results do not present actual proof that the weak-form efficiency does not apply to the US Stock markets, so we can only discuss the indications of our model and potential explanations to why our results turn out as they do. As mentioned in the theory sections, several studies have found indication for and against some of the efficiency forms, so we do not really know how to classify the US Stock market. It is also likely that in some states or periods, efficiency in the stock market does not exist. Looking at conditional covariance matrices, mainly their diagonals, from throughout our sample, we will see several cases where some industry-portfolios are dominated by others, indicating several inefficient stock, which could also indicate that for that period at least, the Capital Asset Pricing Model will not hold. This would also violate Markowitz's assumptions in the Modern Portfolio Theory.

From our results, we cannot fully determine if the US Stock market is really efficient, and in what form such efficiency would exist. By using the modern portfolio theory to optimize the portfolio weights we can only assume that market efficiency exists. In reality, it is most likely that a semi-strong efficiency would exist in the market, but for our GARCH-model to fully hold, we must assume that there to some degree exists a weak-efficiency market form.

Are the Assumptions Realistic?

Although our model removes the assumption that correlation between assets is constant throughout time, there are still several more assumptions that we have listed in the theory section. If these assumptions are too unrealistic and do not apply to today's US stock market, it could create significant biases in all of our results, when we suggest our investment strategies. As we have already discussed the Efficient Market Hypothesis in a separate section of the discussion, we will not focus any further on the first assumption stating that the efficient market theory holds.

The MPT assumes that asset returns are normally distributed random variables, which is a bit of a tricky assumption in our case. Not because the returns aren't seemingly normally distributed, but because classifying them as random variables is not in complete correspondence with creating an econometric model that tries to explain the conditional variance-covariance in these returns. In reality it is highly unlikely that asset returns are completely random and, statistically speaking, there do not seem to be indications of large random-walks in most sample data of historical stock returns (this goes for our model and also for some of the studies to which we have earlier referred).

Investors are risk averse and seek to maximize their economic utility. In the case of the investment in stocks, this will be done by maximizing their profits, possibly at a given level of risk or alternatively minimizing their risk at a given level of returns. We do not feel the need to discuss this assumption further, as it is given that investors will obviously always try to maximize their economic utility.

Another assumption of the MPT is that all investors have equal and simultaneous access to information, which is consistent with the strong-form of market efficiency in the Efficient Market Hypothesis, and could also be consistent with the weak form. However, it is only realistic that insiders have access to information that the public does not have, and even with legislative incentives to prevent insider trading, it is in reality only reasonable that some small bias will occur due to such unequal information.

The last of the MPT assumptions is that investors have correct conceptions of return possibilities, no taxes or transaction costs, and that all securities can be divided into any possible fraction. These three assumptions are unrealistic in the real world, but should not bias the results of our GARCH-model. They might create a bias in the computation of the optimal portfolio weights, but we do not consider them to be specifically notable to our results.

As we are working in a very theoretical spectrum and trying to apply these theories to a real world study, we can only expect that biases will be created from assumptions deviating from realities, which might be perceived as defying the scientific standpoint of a positivistic paradigm, moving towards a more neo-positivistic standpoint. However, we will mainly focus on the positivistic standpoint (as described in the method section), so we disregard that some assumptions might be unrealistic, but simply keep it in mind when we need to analyze and interpret our results.

Forecasting Beyond the Model

The GARCH model is based on data dating up until December 2012. Data on returns for the industry portfolios was recently made available at the Kenneth French Website[47] that included returns of January 2013. This made us capable of constructing portfolios for January 2013 just as we have recently done with our selected periods. This led to the following results for the GARCH forecast for:

[47] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

GMV Weights:			Return	St. Dev
Durbl	0,009461		0,050427679	0,068534
Enrgy	-0,17173		-0,606220013	-0,97157
Hlth	-0,09441		-0,574011005	-0,38869
HiTec	-0,13619		-1,070422084	-0,73296
Manuf	0,300961		0,695219205	1,354514
NoDur	0,314433		1,455826686	1,059656
Other	-0,12524		-0,680032092	-0,59244
Shops	0,188281		1,526954886	0,694946
Telcm	0,274136		1,384384929	0,991849
Utils	0,440294		2,95437413	1,532038
Sum	1		5,136502322	3,015877
Risk free return		0,00		
Sharpe Ratio		1,703154		

Figure 54 – GARCH-Forecasted GMV Portfolio for January 2013

In comparison to the MPT mean-variance constructed portfolio which looks like this:

GMV Weights:			Return	St. Dev
Durbl	-0,07184		-0,382918308	-0,56279
Enrgy	0,238454		0,841742021	1,438201
Hlth	0,05499		0,334339781	0,314445
HiTec	-0,10883		-0,855421069	-0,80554
Manuf	-0,12388		-0,286168683	-0,78992
NoDur	0,731425		3,386498061	3,412883
Other	-0,32839		-1,783141538	-2,14862
Shops	-0,00957		-0,077604993	-0,056
Telcm	0,540317		2,728603336	2,51139
Utils	0,077326		0,518860252	0,434908
Sum	1		4,424788858	3,748964
Risk free return		0,00		
Sharpe Ratio		1,18027		

Figure 55 – Mean-Variance Constructed GMV Portfolio of January 2013

Once again, our model is able to construct a more efficient minimum-variance portfolio that yields a higher return and significantly higher Sharpe ratio. But as we unfortunately only have this one sample, we would have to wait until more data is made available. This should be very soon according to Kenneth French, who actually replied to an e-mail we sent him regarding this matter. We believe that it would be of great interest to do further research on this data, to see if the results we have gotten so far will continue to look as much in favor of the GARCH-predictions as it has in the performance tests previously conducted. The above is merely an addition to our list of results, as the data gave us the opportunity to calculate it. We cannot say from just this one sample, if the GARCH-predictions will outperform regular MPT throughout 2013, but at least it does so for January.

Possible Explanations to why GARCH 2012 Results do not Outperform MPT

As we saw in our most recent time period with available data, the MPT and GARCH were the most preferred portfolios in 3 months each. Compared to the two first periods of economic growth and recession this is a change, because GARCH dominated MPT that only had 1 month as the preferred portfolio. We have tried to discuss whether there are any explanations why this is the case, that the GARCH does not outperform MPT in 2012, although it shows to be just as good, if not better, since the risk is lower. We believe that this can be due to anything, and a lot of it will probably be purely guessing, as there is no way of finding out if it is true. It could be because of the occurrence of unexpected events, but we do not believe that this is the case, even though there was the fiscal cliff of 2012 where the tax rate increased, while a decrease in government spending happened[48]. After all, we have seen positive developments in the American economy in 2012, compared to when the recession was deepest. Whether or not certain events or other factors have had an influence on the 2012 results is not something we can say for sure, as it is not part of our research in this thesis.

Conclusion

We will now start by looking at the findings that present answers to the relevant research sub-questions. By doing this we will lead up to finally concluding on the actual research question.

[48]Page and Reichling, Economic Effects of Policies Contributing to Fiscal Tightening in 2013

Summarizing the Findings

In order to reach a conclusion to our research question, we have gone through our sub-questions and have used the theory- and analysis sections to answer these sub-questions. We will summarize the findings of the previous sections in the thesis and we will then make a final conclusion to our research question and overall problem. We will summarize the previous findings which answer the sub-questions. We will do so in a chronological order, starting with the time-series analysis questions followed by portfolio theory questions and finally the performance testing questions

We have picked out stock-return data on 10 different portfolios comprised by industry stock across the three biggest US Market indexes (Dow Jones, Nasdaq and S&P500) and analyzed it.

We ran ARCH-tests on each of our time-series to find indications of the autoregressive conditional heteroskedasticity process. In each of the series, we find statistical evidence to support that such a process characterizes the returns. In conclusion, we find evidence to support the choice of a GARCH model that incorporates all 10 portfolios' returns, thus making it a multivariate GARCH model. We have chosen to repeat the methods used by Bollerslev, Engle & Wooldridge (1988) where they use a multivariate GARCH-model to suggest a conditional "trivariate" CAPM. We expand their model to our 10 portfolios; hence we can call it a decavariate-model, and use the same approach in constructing a diagonal VECH model. The order of our GARCH-model is determined to be a GARCH(1,1) by comparing information criterions to other possible orders.

We compute the results of our model, conditional covariances for each observed month and corresponding residuals. With these results, we are able to use simple calculations and matrix algebra to construct a one-period forecast for the conditional covariance matrix. In order to forecast several periods ahead, the most convenient approach is just to run the GARCH-optimization again by the end of each month and then rebalance the portfolios. However, if we wanted a more long-term investment-strategy today, and still propose monthly rebalancing, we needed to do a forecast. As the GARCH-model had varying variance-covariance matrices we had to use Monte Carlo simulation of errors from a multivariate normal distribution in order to simulate possible values of returns. From those simulations we derived variances and correlations, which we used along with the model estimation results, to compute the "next periods" covariance matrix.

We wanted to find portfolios that are on the efficient frontier. We decided to work with the Global Minimum Variance portfolio, as this gives us the opportunity to avoid trying to forecast returns, which we would have to do if we were trying to find tangency portfolios. This is basically impossible and not in correspondence with the MPT assumption of returns being randomly normally distributed. This seemed to make us capable of producing a more valid result, and it is also a well-argued choice as we relate to a study by Robert Haugen, showing that minimum-risk portfolios tend to outperform portfolios with higher risk.

We calculate the portfolio weights by optimization techniques from the Modern Portfolio Theory, which uses Lagrange optimization to minimize risk in efficient portfolios and derive a simple function for the individual asset weights of the GMV. As an input for this function we will use the results computed in our GARCH-model.

In regular modern-portfolio optimization problems, we work with mean-variance instead of conditional time-varying variance. So we wanted to compare the model described above to a model that uses historical variance as a constant measure of risk and use the same MPT optimization method to calculate GMV-weights, which will not be subject to monthly rebalances. If they were, the rebalancing would be relatively unnoticeable.

We focus on the most recent time periods because we believe that it would be most valid for testing the model in today's world. We look at periods of 6 consecutive months. We have chosen July 2012 through December 2012 as this is our most recent period in the data, and to test the model performance in periods of high fluctuations we have chosen a severe part of the latest recession, July through December 2008, and the boom leading up to that recession, September 2006 through February 2007.

Our analysis shows that in 14 of 18 months or 15 of 19 if you include January 2013, our model was able to generate higher Sharpe ratios. In most months, we find the highest Sharpe ratio when we simply do a one-period forecast from the conditional covariance matrices in our EViews output, a very close "second place" is the portfolio weights suggested by the Monte Carlo simulation approach, and finally in most cases the worst performing method out of the three, is simple MPT mean-variance optimization. It is also important to keep in mind that we are working with Global Minimum Variance portfolios, and our results show that in every month the GARCH- and Monte Carlo Simulation approaches was able to suggest efficient portfolios with lower risk than the ones suggested by the MPT approach.

Final Conclusion on Research Question

Finally, in extension of these sub-question conclusions, we can present a final conclusion to our research question. Recall the problem definition by the research question:

"How can we construct an econometric financial time-series model with time-varying risk, which can be used to create efficient portfolio choices, using a combination of 10 US industry portfolios? And how does optimal portfolios suggested by this model perform compared to a portfolio constructed from applying normal/simple mean-variance optimization?"

In summarizing our findings we do believe that this has actually already been answered to the reader, so we will simply summarize and put our final words on these conclusions.

We have found evidence supporting an autoregressive conditional heteroskedastic process in our time-series returns data. Based on testing, we construct a multivariate GARCH-model, analyzing the historical returns since 1926, of 10 different industries. We use the diagonal VECH-model and mimic and expand the methodology and approach used by Bollerslev, Engle and Wooldridge

(1988) in construction of our model. We use information criterions to determine the orders of our model to be a GARCH(1,1). We use the software package EViews 7.0 to compute this model and estimate the model parameters as well as conditional covariances and error-terms for each observed month. Using the model results and Monte Carlo Simulations we can compute multi-period forecasts of conditional covariance matrices. When we have these forecasted matrices we can apply Modern Portfolio Theory optimization techniques to construct a series of Global Minimum Variance portfolios for each sample period.

To analyze how our models results perform compared to normal modern portfolio theory results, we have also constructed suggested GMV portfolios using historical mean-variance. We have chosen three sample periods of six consecutive months each, where one is the latest period of our sample, one is the latest period of severe recession (2007-2008 great mortgage-lending recession) and one is the latest period of severe growth (leading up to the 2007-2008 recession). In the recession period, our model significantly outperformed the MPT approach with lower risk and higher returns in all portfolios. In every single month of all three sample periods, we were able to get efficient portfolios with lower risks from using GARCH computations. And in 18 of 19 with the Monte Carlo Simulation approach. This is a good indicator for our model, as we want to minimize risk when creating GMV portfolios, and also as minimized-risk portfolios are the best performing portfolios, according to a study by Robert Haugen, as mentioned earlier. In 14 of 18 (or 15 of 19), our models portfolio suggestions had higher Sharpe ratios than the MPT mean-variance suggestions. In general, we believe that this gives us some good indications, that our approach can actually be used to suggest better portfolios, than the MPT approach would suggest. However, we do believe that further research is needed as we only present 19 samples, which statistically is not that high of a number, and in extension to our results in this thesis, we would wish to apply our model output and optimization approach to the 2013 data, as soon as it becomes available. But for now, we believe that we have presented an indication of how modern portfolio theory optimizations can be improved, by taking into consideration the obvious time-varying risk, which exists in most historical returns data.

Reflective Thoughts

To follow up on our analysis and conclusion, we want to discuss our reflective thoughts on the subject and how we believe others with interest can use it. We would also like to reflect on alternative approaches to implementing time-varying risk in other aspects or studies of financial theory. The purpose here is to take a critical view of what other directions we could have gone in.

This is a financial analysis of time series data using econometric methods to create alternative and possibly more efficient portfolio-optimization tools, to the ones we already know from the Modern Portfolio Theory. And that is what the thesis should be used for. You can use it academically for as a research method to incorporate time-varying risk in other financial studies or practically if you wish to adopt an investment strategy similar to the one we suggest. If you have interest in further studies you can use this thesis as an inspiration. If you believe that this is a valid model for portfolio constructions, you can use it to suggest an efficient low-risk portfolio, to work as a possible basis for your strategy. If you want to use the model practically, one of the prerequisites is that data needs to be published fast in order for people to use it, because it makes no sense wanting to optimize a portfolio for the following month, if you only have data from a year ago. This could be a problem if you only went to Kenneth French's website to gather data (as we did), but luckily it is possible to get these updated data from other sources. Stock exchange websites is needless to say a good place to start. It is important that the data is up to date if you want to use the model, and it is necessary to make sure that the sources you use are reliable.

Another issue we would like to relate to is if we can put the results into a bigger professional and social relevance. Do our results suggest that the Modern Portfolio Theory created by Markowitz is outdated when it comes to optimizing Global Minimum Variance portfolios? We cannot say this for sure, but we believe that there are indications that our model is at least as good, if not better than doing an optimization where the time-varying variance-covariance relations in historical stock return series are ignored, to suggest how to balance the perfect weights in a portfolio. But we need to keep in mind that our model to a large extent actually uses the same method as Modern Portfolio Theory's optimal portfolio, since we create covariance-matrices that are eventually optimized in the same way as the Modern Portfolio Theory would do it. This approach is the exact same for the different models, which is why we don't think that the approach is wrong or outdated. But at the same time we think that using mean-variance is not proper, as we find clear indications of time-varying variance.

To further examine if our model outperforms the one of Modern Portfolio Theory, we could have optimized portfolios for all available months in the data set, if time and space had allowed it. Then it would be easy to statistically determine if, or not, or how much our model would outperform MPT. We could have looked at risk, return and Sharpe ratios for way more extensive periods of time, not only from 1926 through 2012 but possibly further into 2013, as long as valid data is available. This would have given a better idea of how good our model is, in forecasting optimal Global Minimum Variance portfolios.

To improve our study, there are certain things we believe could be done in the future to examine this subject. This model could have been tested with the use of other data, and it could have been analyzed on various asset-classes such as bonds, stocks, T-bills. We could also look into international diversification, comparing market returns in different countries as investment theory and study-results generally suggest international diversification. Furthermore it would be a good idea to refer to other studies, for example Petkova and Zhang "Is Value Riskier than Growth?" (2003). Here it could have been examined the relative risk of value and growth stocks, because they found out that time-varying risk goes in the right direction in explaining the value premium, since value betas tend to covary positively, and growth betas tend to covary negatively with the expected market risk premium, according to Petkova and Zhang[49]. They sort the betas on the expected market risk premium instead of on the realized market excess returns, which would have been interesting to look into. In general there are several studies conducted by Fama & French (i.e. their construction of multifactor models) and several other acknowledged researchers within this field. These studies propose different models, wherein we could try and implement the element of time-varying risk. Luckily this seems to be on the agenda of several studies today, so

[49] Petkova & Zhang, Is Value Riskier Than Growth?

we can hope to learn a lot from this in the near future. Recall in the introduction, we mentioned that this was a big part of our motivation to work with this topic.

Other changes could have been with the selection of theories. We could have looked into index models, which are more equal to what Bollerslev, Engle and Wooldridge did in their study of 1988, as they work with a model that resembles the CAPM in its structure. This could have been done for example with a single index model, or one of the Fama & French multi-index models. We could have adopted their approach, and tried to impose time-varying risk into the model, not unlike Petkova & Zhangs implementation of a "time variable" in their conditional CAPM.

Our critical reflections on methodology as well as our model and the statistics, is mainly what we have tried to cover in the "Discussion" section. In general we could write extensive chapters on self-criticism, especially if we take a look at more realistic scientific paradigms, and direct our analysis into the world of behavioral finance. After all we are dealing with statistics and do not have a crystal ball that can make exact future-predictions.

We believe that our data and theory selections, together with the presented results are reliable and useful for indicating the possibility of improvement in existing optimization techniques. This is obviously a hypothesis that would require further testing and studies, but with the spectrum of this thesis we will stand by these indications.

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Appendix 1 - Entire EViews 7.0 Model Estimate Output

System: SYS01
Estimation Method: ARCH Maximum Likelihood (Marquardt)
Covariance specification: Diagonal VECH
Date: 08/12/13 Time: 14:35
Sample: 1926M07 2012M12
Included observations: 1038
Total system (balanced) observations 10380
Presample covariance: backcast (parameter =0.7)
Convergence achieved after 53 iterations

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	1.014056	0.180213	5.626978	0.0000
C(2)	1.125396	0.171486	6.562604	0.0000
C(3)	0.981224	0.152701	6.425808	0.0000
C(4)	0.972474	0.169337	5.742818	0.0000
C(5)	0.969032	0.142677	6.791813	0.0000
C(6)	0.895074	0.110881	8.072362	0.0000
C(7)	0.954086	0.141535	6.740981	0.0000
C(8)	0.947239	0.137931	6.867469	0.0000
C(9)	0.842473	0.115339	7.304345	0.0000
C(10)	0.902176	0.117761	7.661088	0.0000
	Variance Equation Coefficients			
C(11)	1.334074	0.117734	11.33122	0.0000
C(12)	0.782840	0.145355	5.385727	0.0000
C(13)	0.963554	0.168721	5.710915	0.0000
C(14)	1.201746	0.132926	9.040711	0.0000
C(15)	1.099608	0.094774	11.60237	0.0000
C(16)	0.726083	0.088793	8.177282	0.0000
C(17)	1.134855	0.118310	9.592179	0.0000
C(18)	0.861673	0.107141	8.042396	0.0000
C(19)	0.652149	0.112215	5.811607	0.0000
C(20)	0.539809	0.094713	5.699416	0.0000
C(21)	0.893108	0.154466	5.781904	0.0000
C(22)	0.241142	0.238303	1.011914	0.3116
C(23)	0.259211	0.181558	1.427707	0.1534
C(24)	0.224681	0.119727	1.876607	0.0606
C(25)	0.089810	0.119293	0.752858	0.4515
C(26)	0.271628	0.176185	1.541723	0.1231
C(27)	0.063560	0.142187	0.447015	0.6549
C(28)	0.057749	0.128731	0.448601	0.6537
C(29)	0.334556	0.151820	2.203641	0.0275
C(30)	1.091645	0.150339	7.261215	0.0000
C(31)	0.568097	0.153231	3.707450	0.0002
C(32)	0.390417	0.100880	3.870123	0.0001
C(33)	0.282813	0.108732	2.601014	0.0093
C(34)	0.318776	0.134723	2.366154	0.0180
C(35)	0.237270	0.134651	1.762113	0.0781
C(36)	0.231174	0.143230	1.614007	0.1065
C(37)	0.168664	0.128597	1.311570	0.1897

C(38)	0.820463	0.106803	7.682061	0.0000
C(39)	0.186927	0.102089	1.831015	0.0671
C(40)	-0.002511	0.092869	-0.027041	0.9784
C(41)	0.169161	0.121447	1.392873	0.1637
C(42)	0.130286	0.119799	1.087544	0.2768
C(43)	0.118366	0.118226	1.001185	0.3167
C(44)	-0.078822	0.126889	-0.621191	0.5345
C(45)	0.501075	0.056574	8.856926	0.0000
C(46)	0.199273	0.089361	2.229984	0.0257
C(47)	0.338595	0.123876	2.733336	0.0063
C(48)	0.220743	0.113856	1.938798	0.0525
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C(169)	0.857227	0.011338	75.60947	0.0000
C(170)	0.870314	0.011720	74.25882	0.0000
C(171)	0.856972	0.013914	61.59231	0.0000
C(172)	0.876261	0.011976	73.16933	0.0000
C(173)	0.831086	0.018517	44.88113	0.0000
C(174)	0.864181	0.014798	58.39734	0.0000
C(175)	0.870180	0.014718	59.12433	0.0000

Log likelihood	-26251.52	Schwarz criterion	51.75185
Avg. log likelihood	-2.529048	Hannan-Quinn criter.	51.23444
Akaike info criterion	50.91815		

Equation: DURBL = C(1)

R-squared	-0.000085	Mean dependent var	1.085973
Adjusted R-squared	-0.000085	S.D. dependent var	7.820212
S.E. of regression	7.820543	Sum squared resid	63423.85
Durbin-Watson stat	1.701472		

Equation: ENRGY = C(2)

R-squared	-0.000138	Mean dependent var	1.054807
Adjusted R-squared	-0.000138	S.D. dependent var	6.018714
S.E. of regression	6.019128	Sum squared resid	37570.41
Durbin-Watson stat	1.982214		

Equation: HEALTH = C(3)

R-squared	-0.000263	Mean dependent var	1.073690
Adjusted R-squared	-0.000263	S.D. dependent var	5.706517
S.E. of regression	5.707267	Sum squared resid	33778.09
Durbin-Watson stat	1.850915		

Equation: HITEC = C(4)

R-squared	-0.000202	Mean dependent var	1.077389
Adjusted R-squared	-0.000202	S.D. dependent var	7.388486

S.E. of regression	7.389231	Sum squared resid	56620.97
Durbin-Watson stat	1.781943		

Equation: MANUF = C(5)

R-squared	-0.000052	Mean dependent var	1.014817
Adjusted R-squared	-0.000052	S.D. dependent var	6.361960
S.E. of regression	6.362125	Sum squared resid	41974.28
Durbin-Watson stat	1.764321		

Equation: NODUR = C(6)

R-squared	-0.000301	Mean dependent var	0.975838
Adjusted R-squared	-0.000301	S.D. dependent var	4.657218
S.E. of regression	4.657919	Sum squared resid	22498.97
Durbin-Watson stat	1.764917		

Equation: OTHERS = C(7)

R-squared	-0.000094	Mean dependent var	0.890674
Adjusted R-squared	-0.000094	S.D. dependent var	6.528753
S.E. of regression	6.529062	Sum squared resid	44205.91
Durbin-Watson stat	1.655964		

Equation: SHOPS = C(8)

R-squared	-0.000032	Mean dependent var	0.980048
Adjusted R-squared	-0.000032	S.D. dependent var	5.838905
S.E. of regression	5.838997	Sum squared resid	35355.36
Durbin-Watson stat	1.722362		

Equation: TELCM = C(9)

R-squared	-0.000001	Mean dependent var	0.846840
Adjusted R-squared	-0.000001	S.D. dependent var	4.639611
S.E. of regression	4.639613	Sum squared resid	22322.47
Durbin-Watson stat	1.834238		

Equation: UTILS = C(10)

R-squared	-0.000019	Mean dependent var	0.877553
Adjusted R-squared	-0.000019	S.D. dependent var	5.614411
S.E. of regression	5.614465	Sum squared resid	32688.54
Durbin-Watson stat	1.777945		

Covariance specification: Diagonal VECH
GARCH = M + A1.*RESID(-1)*RESID(-1)' + B1.*GARCH(-1)
M is a full rank matrix
A1 is an indefinite matrix*
B1 is an indefinite matrix*

Transformed Variance Coefficients

	Coefficient	Std. Error	z-Statistic	Prob.
M(1,1)	1.779755	0.314133	5.665612	0.0000
M(1,2)	1.044367	0.225269	4.636090	0.0000
M(1,3)	1.285452	0.251341	5.114384	0.0000
M(1,4)	1.603219	0.259095	6.187776	0.0000

M(1,5)	1.466959	0.212375	6.907415	0.0000
M(1,6)	0.968649	0.149705	6.470387	0.0000
M(1,7)	1.513981	0.226883	6.672948	0.0000
M(1,8)	1.149535	0.188581	6.095726	0.0000
M(1,9)	0.870015	0.169345	5.137534	0.0000
M(1,10)	0.720145	0.142573	5.051057	0.0000
M(2,2)	1.410480	0.344340	4.096185	0.0000
M(2,3)	0.969674	0.222104	4.365861	0.0000
M(2,4)	1.172278	0.222548	5.267542	0.0000
M(2,5)	1.061481	0.194117	5.468241	0.0000
M(2,6)	0.648617	0.140105	4.629497	0.0000
M(2,7)	1.131003	0.207243	5.457382	0.0000
M(2,8)	0.731317	0.171450	4.265492	0.0000
M(2,9)	0.562104	0.123450	4.553286	0.0000
M(2,10)	0.721378	0.142268	5.070554	0.0000
M(3,3)	2.178273	0.390060	5.584454	0.0000
M(3,4)	1.840614	0.264766	6.951848	0.0000
M(3,5)	1.539908	0.221266	6.959530	0.0000
M(3,6)	1.030008	0.170374	6.045585	0.0000
M(3,7)	1.506984	0.235152	6.408556	0.0000
M(3,8)	1.104609	0.184507	5.986817	0.0000
M(3,9)	0.894666	0.164703	5.432006	0.0000
M(3,10)	0.784932	0.149957	5.234371	0.0000
M(4,4)	2.507280	0.331246	7.569233	0.0000
M(4,5)	1.754851	0.212190	8.270207	0.0000
M(4,6)	1.054452	0.143626	7.341674	0.0000
M(4,7)	1.754103	0.224593	7.810157	0.0000
M(4,8)	1.293675	0.181207	7.139217	0.0000
M(4,9)	1.027131	0.159532	6.438399	0.0000
M(4,10)	0.766581	0.126149	6.076801	0.0000
M(5,5)	1.698063	0.209252	8.114907	0.0000
M(5,6)	1.028382	0.138214	7.440524	0.0000
M(5,7)	1.634663	0.206427	7.918852	0.0000
M(5,8)	1.189380	0.162300	7.328271	0.0000
M(5,9)	0.865693	0.136487	6.342660	0.0000
M(5,10)	0.766620	0.121133	6.328755	0.0000
M(6,6)	0.808931	0.130901	6.179723	0.0000
M(6,7)	1.126381	0.155551	7.241227	0.0000
M(6,8)	0.809178	0.125454	6.449979	0.0000
M(6,9)	0.625368	0.107383	5.823708	0.0000
M(6,10)	0.542598	0.089291	6.076733	0.0000
M(7,7)	1.838781	0.265213	6.933226	0.0000
M(7,8)	1.209783	0.170920	7.078082	0.0000
M(7,9)	0.873988	0.143191	6.103634	0.0000
M(7,10)	0.838355	0.126480	6.628350	0.0000
M(8,8)	1.065466	0.186232	5.721177	0.0000
M(8,9)	0.719286	0.125869	5.714543	0.0000
M(8,10)	0.566228	0.101677	5.568871	0.0000
M(9,9)	0.929143	0.157997	5.880775	0.0000
M(9,10)	0.570655	0.106080	5.379482	0.0000
M(10,10)	0.747034	0.140470	5.318104	0.0000
A1(1,1)	0.117998	0.011968	9.859797	0.0000
A1(1,2)	0.087988	0.011038	7.971204	0.0000
A1(1,3)	0.094266	0.011167	8.441316	0.0000
A1(1,4)	0.119004	0.011266	10.56269	0.0000

A1(1,5)	0.119094	0.010121	11.76698	0.0000
A1(1,6)	0.104683	0.009206	11.37137	0.0000
A1(1,7)	0.121965	0.010968	11.11991	0.0000
A1(1,8)	0.107154	0.010398	10.30503	0.0000
A1(1,9)	0.104178	0.013882	7.504357	0.0000
A1(1,10)	0.099192	0.012259	8.091599	0.0000
A1(2,2)	0.095697	0.014945	6.403340	0.0000
A1(2,3)	0.088529	0.011948	7.409358	0.0000
A1(2,4)	0.099001	0.011555	8.567939	0.0000
A1(2,5)	0.102765	0.010746	9.563021	0.0000
A1(2,6)	0.092056	0.009931	9.269824	0.0000
A1(2,7)	0.109058	0.011305	9.646608	0.0000
A1(2,8)	0.090551	0.010552	8.581172	0.0000
A1(2,9)	0.077724	0.012067	6.440851	0.0000
A1(2,10)	0.071975	0.010994	6.546765	0.0000
A1(3,3)	0.124056	0.014552	8.524881	0.0000
A1(3,4)	0.112290	0.012222	9.187580	0.0000
A1(3,5)	0.108241	0.010889	9.940156	0.0000
A1(3,6)	0.104036	0.010087	10.31402	0.0000
A1(3,7)	0.115673	0.011391	10.15460	0.0000
A1(3,8)	0.103764	0.010912	9.509455	0.0000
A1(3,9)	0.101121	0.014049	7.197974	0.0000
A1(3,10)	0.098068	0.012062	8.130259	0.0000
A1(4,4)	0.134337	0.013548	9.915777	0.0000
A1(4,5)	0.125631	0.010446	12.02646	0.0000
A1(4,6)	0.112195	0.009077	12.35999	0.0000
A1(4,7)	0.128634	0.010707	12.01456	0.0000
A1(4,8)	0.111749	0.010871	10.27928	0.0000
A1(4,9)	0.116696	0.012395	9.414413	0.0000
A1(4,10)	0.111659	0.011000	10.15121	0.0000
A1(5,5)	0.131732	0.010804	12.19313	0.0000
A1(5,6)	0.115438	0.009026	12.78917	0.0000
A1(5,7)	0.136325	0.010637	12.81657	0.0000
A1(5,8)	0.115158	0.010242	11.24372	0.0000
A1(5,9)	0.116391	0.011997	9.701455	0.0000
A1(5,10)	0.109457	0.010878	10.06218	0.0000
A1(6,6)	0.107035	0.010410	10.28148	0.0000
A1(6,7)	0.120929	0.010166	11.89536	0.0000
A1(6,8)	0.105854	0.010033	10.55031	0.0000
A1(6,9)	0.098073	0.011179	8.773312	0.0000
A1(6,10)	0.096978	0.010357	9.363781	0.0000
A1(7,7)	0.145597	0.012895	11.29137	0.0000
A1(7,8)	0.119994	0.010764	11.14799	0.0000
A1(7,9)	0.115218	0.012651	9.107611	0.0000
A1(7,10)	0.112555	0.011670	9.644893	0.0000
A1(8,8)	0.110284	0.012742	8.655194	0.0000
A1(8,9)	0.104620	0.012607	8.298338	0.0000
A1(8,10)	0.098222	0.011317	8.679300	0.0000
A1(9,9)	0.147734	0.020083	7.356008	0.0000
A1(9,10)	0.108905	0.013765	7.911707	0.0000
A1(10,10)	0.118909	0.016104	7.384011	0.0000
B1(1,1)	0.864236	0.012175	70.98414	0.0000
B1(1,2)	0.874861	0.014741	59.34885	0.0000
B1(1,3)	0.855839	0.016852	50.78624	0.0000
B1(1,4)	0.850684	0.011467	74.18570	0.0000

B1(1,5)	0.853744	0.010365	82.36889	0.0000
B1(1,6)	0.865024	0.010518	82.24049	0.0000
B1(1,7)	0.845416	0.011228	75.29792	0.0000
B1(1,8)	0.864684	0.010647	81.21493	0.0000
B1(1,9)	0.854428	0.015068	56.70308	0.0000
B1(1,10)	0.871498	0.013750	63.38302	0.0000
B1(2,2)	0.879689	0.017466	50.36657	0.0000
B1(2,3)	0.858478	0.017612	48.74335	0.0000
B1(2,4)	0.856061	0.014676	58.32983	0.0000
B1(2,5)	0.862946	0.012582	68.58511	0.0000
B1(2,6)	0.874497	0.012233	71.48462	0.0000
B1(2,7)	0.856032	0.013220	64.75141	0.0000
B1(2,8)	0.871027	0.014239	61.17043	0.0000
B1(2,9)	0.882309	0.014046	62.81585	0.0000
B1(2,10)	0.888787	0.014972	59.36516	0.0000
B1(3,3)	0.820086	0.018588	44.11926	0.0000
B1(3,4)	0.828977	0.015381	53.89447	0.0000
B1(3,5)	0.836697	0.014186	58.97847	0.0000
B1(3,6)	0.853532	0.012322	69.27049	0.0000
B1(3,7)	0.830262	0.014219	58.38943	0.0000
B1(3,8)	0.853777	0.013590	62.82426	0.0000
B1(3,9)	0.841069	0.017939	46.88603	0.0000
B1(3,10)	0.858003	0.015557	55.15102	0.0000
B1(4,4)	0.830999	0.012808	64.88114	0.0000
B1(4,5)	0.838359	0.010183	82.32735	0.0000
B1(4,6)	0.856173	0.009253	92.53032	0.0000
B1(4,7)	0.832906	0.010320	80.70910	0.0000
B1(4,8)	0.855969	0.010587	80.85218	0.0000
B1(4,9)	0.838714	0.013538	61.95432	0.0000
B1(4,10)	0.860804	0.010357	83.11461	0.0000
B1(5,5)	0.838069	0.010712	78.23909	0.0000
B1(5,6)	0.852704	0.009549	89.29349	0.0000
B1(5,7)	0.830456	0.010304	80.59493	0.0000
B1(5,8)	0.851675	0.010443	81.55095	0.0000
B1(5,9)	0.842669	0.012490	67.46648	0.0000
B1(5,10)	0.858134	0.011470	74.81255	0.0000
B1(6,6)	0.869898	0.010126	85.90834	0.0000
B1(6,7)	0.846208	0.010341	81.82712	0.0000
B1(6,8)	0.869790	0.009948	87.43015	0.0000
B1(6,9)	0.864569	0.012523	69.03905	0.0000
B1(6,10)	0.879160	0.010564	83.22029	0.0000
B1(7,7)	0.827259	0.011997	68.95276	0.0000
B1(7,8)	0.848480	0.009980	85.02111	0.0000
B1(7,9)	0.846136	0.012855	65.82083	0.0000
B1(7,10)	0.857227	0.011338	75.60947	0.0000
B1(8,8)	0.870314	0.011720	74.25882	0.0000
B1(8,9)	0.856972	0.013914	61.59231	0.0000
B1(8,10)	0.876261	0.011976	73.16933	0.0000
B1(9,9)	0.831086	0.018517	44.88113	0.0000
B1(9,10)	0.864181	0.014798	58.39734	0.0000
B1(10,10)	0.870180	0.014718	59.12433	0.0000

Appendix 2 – Industry Returns in 3 Selected Periods

Returns from period 1, September 2006 – February 2007

	Durabl	Energ	Healt	HiTe	Manu	NoDu	Other	Shops	Telecom	Utilities
09/06	4,29	-3,58	1,52	3,67	1,88	-0,62	3,22	5,76	3,61	-2,19
10/06	5,12	4,96	1,71	4,07	4,37	2,8	3,01	3,88	5,26	5,38
11/06	1,83	8,6	0,09	3,03	3,57	1,41	1,67	-1,33	0,65	3,12
12/06	-0,77	-1,97	0,18	-0,95	0,51	2	3,16	1,44	4,26	0,69
01/07	2,71	-1,41	3,49	2,07	3,22	1,52	1,06	3,54	3,2	0,44
02/07	-0,43	-2,14	-2,84	-2,36	0,76	-1,53	-2,34	-0,62	-2,64	4,25

Returns from period 2, July 2008 – December 2008

	Durable	Energ	Healt	HiTec	Manu	NoDu	Other	Shop	Teleco	Utilities
07/08	-0,31	-15,8	6,73	-0,48	-0,26	2,36	4,12	1,11	-3,18	-7,22
08/08	2,46	-0,19	1,8	2,34	1,43	1,64	0,25	5,36	4	-0,49
09/08	-9,38	-12,59	-5,96	-12,48	-13,52	-2,97	-6,05	-5,76	-12,61	-12,08
10/08	-32,86	-17,34	-11,02	-18,45	-20,8	-13,06	-19,6	-15,16	-15,56	-11,08
11/08	-11,45	-0,55	-7,34	-10,98	-7,77	-5,43	-12,38	-7,7	-1,57	1,07
12/08	1,51	-3,19	6,75	2,19	1,44	0,99	0,86	4,88	3,45	-2,06

Returns from period 3, July 2012 – December 2012

	Durable	Energ	Healt	HiTec	Manu	NoDu	Other	Shop	Teleco	Utilities
07/12	-0,37	2,71	1,69	-0,16	1,14	1,51	-1,21	1,2	4,05	3,63
08/12	4,24	2,41	1,08	4,86	3,14	0,06	3,14	2,81	0,84	-3,41
09/12	3,4	3,65	4,79	1,14	1,71	0,99	3,75	1,98	4,84	2,12
10/12	6,28	-1,81	-1,87	-6,46	-0,62	-1,67	0,96	-1,08	-2,42	1,08
11/12	2,51	-1,58	1,45	1,47	2,7	3,15	-0,29	1,41	0,12	-3,83
12/12	7,6	1,4	-0,43	0,24	2,68	-2,29	3,55	-1,01	1,26	0,1

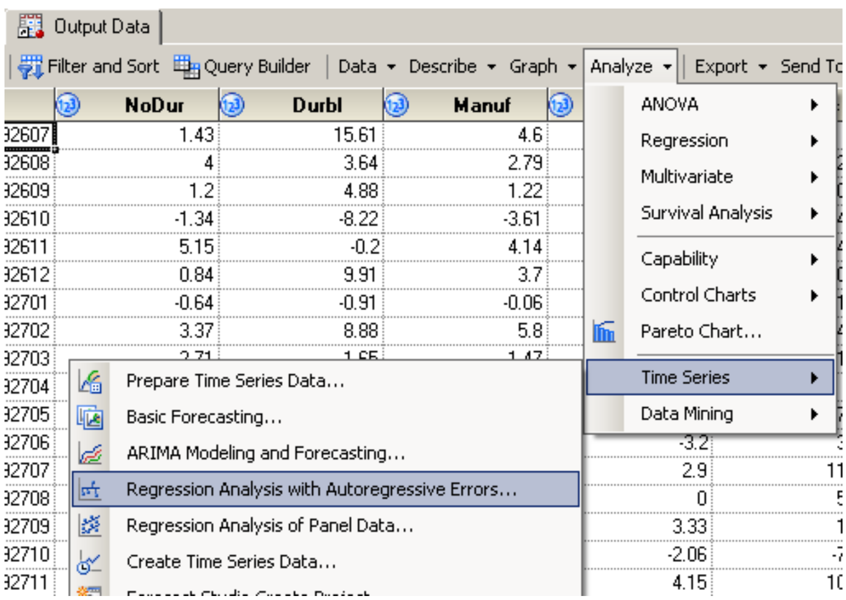
Appendix 3 – Testing for ARCH in SAS.

Open your data of returns in SAS Enterprise Guide.

Loading SAS data

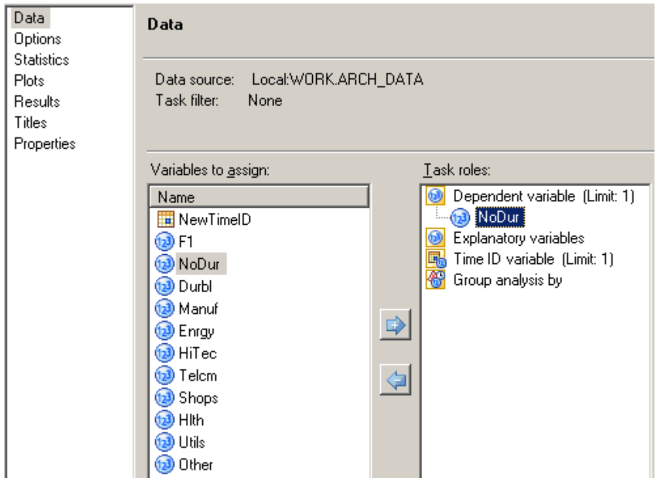
You will see your data appear in the Project Tree.

Double click on the data and chose Analyze, Time Series and Regression with autoregressive errors.



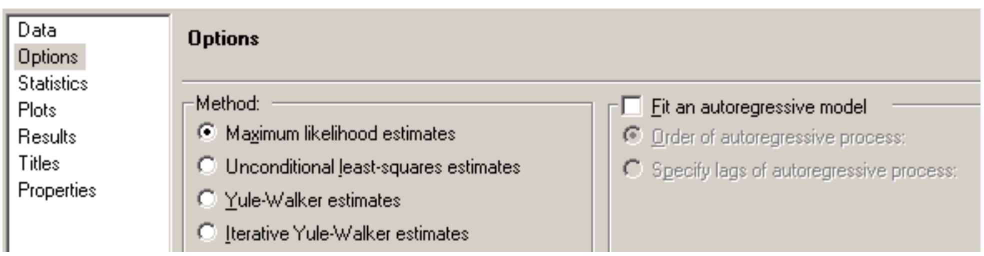
SAS Regression with Autoregressive Errors

Select 'Return' as the dependent variable, and make sure not to select any explanatory variables



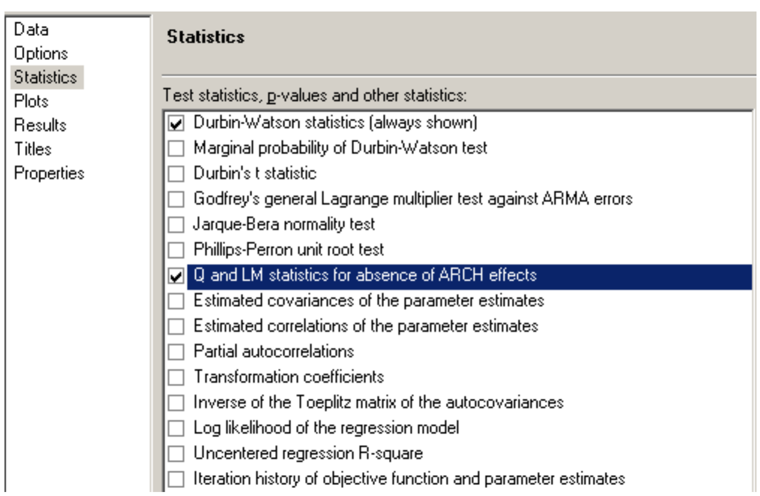
SAS regression "Data" settings

Go to 'Options': Choose "Maximum likelihood" and unmark "Fit autoregressive model"



SAS Regression "Options" settings

Go to 'Statistics': Mark "Q and LM statistics for absence of ARCH effects"

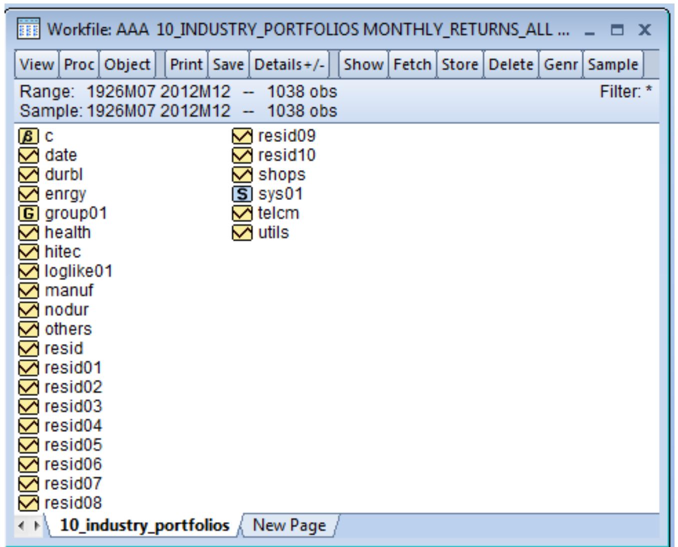


SAS regression "Statistics" settings

You are now ready to run the model and test the first time series for the presence of ARCH.

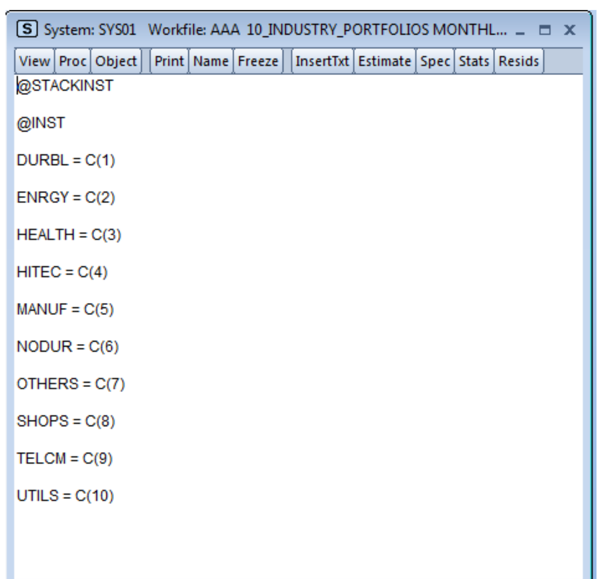
Appendix 4 - Estimating the GARCH-Model in Eviews

We start by loading the data into EViews. This will give us the following picture:



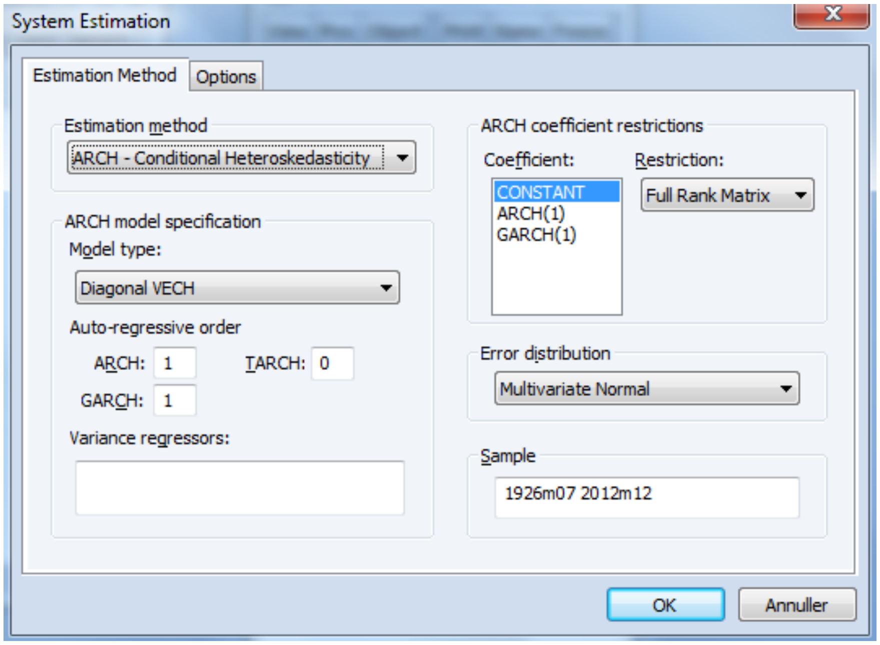
Eviews workfile with time-series returns and their residuals

Right click and choose “Open → as System”, we will get the following:



Eviews System Specifications

Click the "Proc" button and choose "Estimate"



Model specifications in EViews

At this point we will have to set up the model. We chose an ARCH-model and since the system is already set up with multiple variables we simply choose ARCH as our method and "Diagonal VECH" as model type, to ensure a multivariate GARCH-model of the Diagonal VECH type, with a multivariate normal error-distribution and the constraint that our output has to be a full rand matrix.