Comparison of BEKK GARCH and DCC GARCH Models: An Empirical Study

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Abstract. Modeling volatility and co-volatility of a few zero-coupon bonds is a fundamental element in the field of fix-income risk evaluation. Multivariate GARCH model (MGARCH), an extension of the well-known univariate GARCH, is one of the most useful tools in modeling the co-movement of multivariate time series with time-varying covariance matrix. Grounded on the review of various formulations of multivariate GARCH model, this paper estimates two MGARCH models, BEKK and DCC form, respectively, based on the data of three AAA-rated Euro zero-coupon bonds with different maturities (6 months/1 year/2 years). Post-model diagnostics indicates satisfying fitting performance of these estimated MGARCH models. Moreover, this paper provides comparison on the goodness of fit and forecasting performances of these forms by adopting the mean absolute error (MAE) criterion. Throughout this application, the conclusion can be drawn that significant fitting and forecasting performances originate from the trade-off between parsimony and flexibility of the MGARCH models.

Keywords: Volatility, Multivariate GARCH Models, BEKK/DCC Form, Quasi – Maximum Likelihood Method, Zero-Coupon Bonds.

1 Introduction

With the increase in the complexity of the instruments in the risk management field, huge demands for the various models which can simulate and reflect the characteristics of the financial time series have expanded. One of the significant features of financial data that has won much attention is the volatility; because it is a numerical measure of the risk faced by individual investors and financial institutions. It is well known that the volatility of financial data often varies over time and tends to cluster in periods, i.e., high volatility is usually followed by high volatility, and low volatility by low volatility. This phenomenon corresponds to the fluctuating volatility. The Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model and its extensions have been proved to be able to capture the volatility clustering and predict volatilities in the future.

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Specifically, when analyzing the co-movements of financial returns, it is always essential to construct, estimate, evaluate, and forecast the co-volatility dynamics of asset returns in a portfolio. This task can be fulfilled by multivariate GARCH (MGARCH) models. The development of MGARCH models could be thought as a great breakthrough against the curse of dimensionality in the financial modeling. Many different formulations have been constructed parsimoniously and still remain necessary flexibility. The application fields that MGARCH models can extend to include asset pricing, portfolio theory, VaR estimation and risk management or diversification, which require the volatilities and co-volatilities of several markets [1].

In this paper, MGARCH models are estimated and evaluated for volatility and co-volatility of three zero coupon bond prices with different maturities. The data is provided by the website of the European Central Bank (ECB) which is the institution of the European Union tasked with administrating the monetary policy of the EU member states taking part in the Euro zone.

A zero coupon bond is a non-coupon-bearing bond that pays face value at the time of maturity even though it is bought at a price lower than its face value. It has no reinvestment risk and is more sensitive to interest rate change than coupon-bearing bonds. Due to these features, zero-coupon bonds can be easily used to create any type of cash flow stream and thus match asset cash flows with liability cash flows (e.g. to provide for college expenses, house-purchase down payment, or other liability funding.), and used by pension funds and insurance companies to offset, or immunize the interest rate risk of these firms' long-term liabilities.

Moreover, the return of zero coupon bond, referred to as zero rate, is a fundamental element in the field of fix-income pricing and risk evaluation. By using cash-flow-mapping method [2], any fixed cash flow can be mapped to a portfolio consisting of a few representative zero coupon bonds, which match the cash flow's return and volatility. This viewpoint exemplifies how to generalize the specific zero coupon bond volatilities into a general case. It also motivates our study to model volatility and co-volatility of three zero-coupon bonds with three conventional maturities of 6 months, 1 year and 2 years.

The reminder of this paper is organized as follows. Section 2 reviews MGARCH models, including its different forms, diagnostics techniques and the forecasting strategy. In section 3 we present the BEKK and DCC MGARCH models of volatility and co-volatility of ECB zero coupon bond data sets. Conclusions are detailed in section 4.

2 Model Specification and Estimation Methodology

At the beginning of reviewing different formulations of MGARCH models, one should consider what specification of an MGARCH model should be imposed in contrast to the univariate case. On the one hand, it should be flexible enough to state the dynamics of the conditional variances and covariances. On the other hand, as the number of parameters in an MGARCH model increases rapidly along with the dimension of the model, the specification should be parsimonious to simplify the model estimation and also reach the purpose of easy interpretation of the model parameters. However, parsimony may reduce the number of parameters, in which situation the relevant dynamics in the covariance matrix cannot be captured. Another feature that