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DYNAMIC PORTFOLIO OPTIMIZATION USING GENERALIZED DYNAMIC CONDITIONAL HETEROSKEDASTIC FACTOR MODELS

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We model large panels of financial time series by means of generalized dynamic factor models with multivariate GARCH idiosyncratic components. Such models combine the features of dynamic factors with those of a generalized smooth transition conditional correlation (GSTCC) model, which belongs to the class of time-varying conditional correlation models. The model is applied to dynamic portfolio allocation with Value at Risk constraints on 6.5 years of daily TOPIX Sector Indexes. Results show that the proposed model yields better portfolio performance than other multivariate models proposed in the literature, including the traditional mean-variance approach.

Key words and phrases: GARCH model, generalized dynamic factor model, portfolio optimization, Value-at-Risk.

1. Introduction

Since the 1995 amendment of the Basel Accord, Value at Risk (VaR) has become the standard criterion for assessing risk in the financial industry. Indeed VaR—defined as the worst loss over a target horizon such that there is a pre-specified probability that the actual loss will be larger—has become the cornerstone risk measure for potential losses. The last decade has witnessed a growing academic and professional literature comparing various ways to determine and measure VaR, especially for large portfolios of financial assets.

On the other hand, current practice among institutional investors consists of constructing financial portfolios in two stages. The first stage consists of the asset allocation decision that determines the proportions of the main asset classes, e.g., equity, bonds and cash, in the portfolio. The second stage involves individual asset selection.

Most of the methodological efforts aim at optimizing this second stage. Traditional portfolio management theories include the mean-variance analysis of

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Markowitz (1952) and the capital asset pricing model (CAPM). The Markowitz efficient frontier represents all portfolios that are efficient in the sense that all other portfolios exhibit smaller expected returns for a given level of risk or, equivalently, higher risk for a given level of expected return. However, a number of studies have raised objections to mean-variance efficiency as the appropriate framework for optimal portfolio selection. The major problem arising from the Markowitz model is the choice of an appropriate measure of risk. The use of the variance as such a measure, which implies that investors are giving equal weights to the probabilities of positive and negative returns, which is ad hoc with the empirical evidence of skewed financial return distributions.

Various alternatives to the variance as a risk measure have been proposed. Examples include the mean-semivariance approach by Markowitz (1959), the mean-semideviation models by Konno (1990), and the mean-lower partial moment approach by Bawa and Lindenberg (1977). Often the purpose of the investor is to avoid making large losses at a certain point in the future, that is to say measure risk by the probability that the investment is below the prescribed level at a certain future point. This calls for a mean-VaR approach to portfolio selection. Rockafellar and Uryasev (2000, 2002) proposed the so-called CVaR model, and Campbell *et al.* (2001) applied a mean-VaR model to US stocks and bounds.

Another crucial issue in portfolio management is of a statistical nature, and is related to the size of relevant datasets. The required information is usually scattered through a large or even very large number N of highly interrelated financial time series. Traditional multivariate time series methods, as a rule, are quite helpless in large N. Here also, an alternative methodology is needed. Such a methodology has been developed recently, mainly in empirical macroeconomics, where it has become quite popular, under the name of *dynamic factor models*. Dynamic factor models allow for disentangling commonness (market return) and idiosyncrasy (stock-specific components). These components are mutually orthogonal (at all leads and lags) but unobservable. A possible characterization of commonness/idiosyncrasy is obtained by requiring the common component to account for all cross-sectional correlations, leading to possibly autocorrelated but cross-sectionally orthogonal idiosyncratic components. This yields the so-called "exact factor models" considered, for instance, by Sargent and Sims (1997) and Geweke (1977). These exact models, however, are too restrictive in most real-life applications, and a "weak" or "approximate factor model", allowing for mildly cross-sectionally correlated idiosyncratic components, has been proposed (Chamberlain (1983), Chamberlain and Rothschild (1983)), in which the common and idiosyncratic components are only asymptotically (as $N \to \infty$) identified.

Depending on the assumptions on the dynamics of the common components, two main types of factor models have been considered in the literature. The *static* factor models assume that the common components are driven by q factors that are loaded instantaneously. This static approach is the one adopted by Chamberlain (1983), Chamberlain and Rothschild (1983), Stock and Watson (1989, 2002a, b), Bai and Ng (2002, 2007), and a large number of applied studies. The so-called *general dynamic model* decomposes the common components into q unobservable *common shocks* and are loaded via one-sided linear filters. That "fully dynamic" approach was developed, essentially, in a series of papers by Forni *et al.* (2000, 2003, 2004, 2005), Forni and Lippi (2001), Hallin and Liška (2007).

The static model clearly is a particular case of the general dynamic one. Its main advantage is simplicity, at the expense of a rather severe restriction on the data-generating process, while the dynamic one, as shown by Forni and Lippi (2001), relies on a general representation result and therefore does not place any restriction (beyond the standard assumptions of second-order stationarity) on the data-generating process. In this paper, we consider a conditionally heteroskedastic extension of the Generalized Dynamic Factor Model (GDFM) proposed, under homoskedastic form, in Forni *et al.* (2000).

Both the static and the general dynamic models are receiving increasing attention in finance, where information usually comes under the form of a (very) large number N of interdependent time series. Factor models are at the heart of the extensions proposed by Chamberlain and Rothschild (1983) and Ingersol (1984) of the classical arbitrage pricing theory; they also have been considered in performance evaluation and risk measurement (Chapters 5 and 6 of Campbell *et al.* (1997)), in the statistic analysis of the structure of stock returns (Yao (2008)), and the analysis of commonness in liquidity (Hallin *et al.* (2008)).

All these articles assume constant volatility. However, if factor models are to be used in finance, it is essential that they incorporate conditional heteroskedasticity. Early examples are Diebold and Nerlove (1989) and Engle *et al.* (1990), who study single factor models with conditional heteroskedasticity. These models, however, do not belong to the class of static or dynamic factor models considered here. More recently, Alexander (2001), van der Weide (2002) and Barigozzi *et al.* (2009) consider static and dynamic factor models with conditional heteroskedasticity in the common shocks. Two of the most frequently used multivariate GARCH models are the Constant Conditional Correlation (CCC) and the Dynamic Conditional Correlation (DCC) models of Bollerslev (1990) and Engle (2002) respectively. Silvennoinen and Teräsvirta (2005) propose another way of modeling conditional correlations: the Smooth Transition Conditional Correlation (STCC) model. In this paper we propose a Generalized Smooth Transition Conditional Correlation (GSTCC) model for the idiosyncratic components combined with the GDFM. Therefore, contrary to Barigozzi *et al.* (2009), heteroskedasticity in our approach is in the idiosyncratic, not in the common component.

In practice our procedure is as follows. First we use the GDFM combined with the GSTCC to extract the idiosyncratic components. Second, we compute the VaR of each idiosyncratic component for a given confidence interval (1% or 5%). Third, we construct the portfolio based on the idiosyncratic components and optimize it with respect to the portfolio weights by minimizing the portfolio VaR. Considering the idiosyncratic components instead of the returns for the portfolio optimization is non standard and requires some explanation. The market risk, estimated through the common component, is not diversifiable and the number of assets, though large, is limited. Therefore, minimizing the risk of the portfolio is, in some sense, equivalent to minimizing the risk entailed by the idiosyncratic risks. But since the idiosyncratic components are not observed, they have to be estimated and hence the results may be affected by the choice of the model to disentangle the returns between the market and the stock specific components. It is at this point that GDFM becomes the appropriate tool. GDFM is not, in fact, a model but a canonical representation of the panel under study; this is why it is often called the dynamic factor representation. Contrary to other dynamic factor methods, GDFM methods do not impose any restriction (beyond second-order stationarity) on the actual data-generating process. In other words, if the panel of returns is second-order stationary, then the GDFM is a unique representation.

We apply the above procedure to the dynamic optimization a portfolio formed by all the Sector Indexes of TOPIX (the Tokyo Stock Exchange Index). To measure the performance of all the specifications, we do a one-step-ahead out-of-sample exercise. Among all the conditional covariance specifications, the more general model—the GSTCC with the skewed-t distribution—provides the better performance when compared with other specifications and the classical Markowitz mean-variance approach. We also find that the 5% VaR level constraint produces better portfolio performances than the 1% VaR level, as the later has more variability than the former.

The paper is organized as follows. Section 2 presents the methodology, with emphasis on the GSTCC model and its implications in mean-VaR analysis. Section 3 describes the dataset under study and presents the results. Section 4 concludes.

2. Methodology

Throughout, we consider a N-variate risky asset return series that has been recorded over a time period of length T (we assume that there is no risk-free asset). Let R_{it} be the observation made at time t for stock i, $i = 1, \dots, N$, $t = 1, \dots, T$. This observation is a finite realization of a double-indexed stochastic process $\{R_{it}; i \in \mathbb{N}, t \in \mathbb{Z}\}$. Let $\boldsymbol{\omega} = (\omega_1, \dots, \omega_N)$, $\omega_i \geq 0$ the set of portfolio weights (we do not allow for short selling) such that $\sum_{i=1}^{N} \omega_i = 1$.

Denote by $\boldsymbol{\Sigma}_N(\theta)$ the $N \times N$ spectral density matrix of the N-dimensional vector process $\{\boldsymbol{R}_t := (R_{1t}, \dots, R_{Nt})'; t \in \mathbb{Z}\}$, and assume that, for all $N \in \mathbb{N}$, $k \in \{1, \dots, N\}$ and some $c_k > 0$, $\sup_\theta (\boldsymbol{\Sigma}_N(\theta))_{kk} \leq c_k$. For any $\theta \in [-\pi, \pi]$, let $\lambda_{N,k}(\theta)$ be $\boldsymbol{\Sigma}_N(\theta)$'s k-th eigenvalue in decreasing order of magnitude. Denote by q the number of diverging such eigenvalues, that is, define $q := \min\{k \in \mathbb{N} : \sup_N \|\lambda_{N,k}(\theta)\| < \infty, \theta - \text{a.e.}\} - 1$, and assume that $q < \infty$. Theorem 2 in Forni and Lippi (2001) establishes the existence of a unique decomposition of R_{it} into

$$R_{it} = \mu_i + \chi_{it} + \xi_{it} = \mu_i + b_{i1}(L)u_{1t} + b_{i2}(L)u_{2t} + \cdots + b_{iq}(L)u_{qt} + \xi_{it}, \tag{2.1}$$

where χ_{it} and ξ_{it} are mutually orthogonal at all leads and lags, $\boldsymbol{u}_t := (u_{1t}, \ldots, u_{qt})'$ is q-dimensional orthonormal white noise, $\boldsymbol{b}_i(L) := (b_{i1}(L), \ldots, b_{iq}(L))'$ is a vector of square-summable filters, and μ_i is the mean of R_{it}. This representation is called a *dynamic factor representation* of R_{it}; the χ_{it}'s are the *common*, and the ξ_{it}'s the *idiosyncratic components*, respectively, of R_{it}.

An important problem is to determine the number of common shocks q. To do this, we use the Hallin and Liška (2007) procedure for the identification of the number of dynamic factors. This procedure consists in tuning the penalty term of an information-theoretical criterion by a positive factor c until some stability in the identification is reached.

More precisely, let $\hat{\boldsymbol{\Sigma}}_N(\theta)$ be a periodogram-smoothing estimate of $\boldsymbol{\Sigma}_N(\theta)$. The reader should keep in mind that henceforth the estimates are a function of T. Based on a panel of NT observations, this estimate is defined as

$$\hat{\boldsymbol{\Sigma}}_N(\theta) := \frac{2\pi}{T}\sum_{t=1}^{T-1} \hat{W}\left(\theta - \frac{2\pi T}{T}\right)\hat{\boldsymbol{I}}_N\left(\frac{2\pi t}{T}\right),$$

where

$$\hat{\boldsymbol{I}}_N(\lambda) := \frac{1}{2\pi T}\left[\sum_{t=1}^{T-1}\boldsymbol{R}_t^* \exp(-i\lambda t)\right]\left[\sum_{t=1}^{T-1}\boldsymbol{R}_t^{\prime *} \exp(i\lambda t)\right]$$

and

$$\hat{W}(\lambda) := \sum_{j=-\infty}^{\infty} W(B_T^{-1}(\lambda + 2\pi j))$$

with a positive even weight function $W(\lambda)$ and a bandwidth B_T. Here $\boldsymbol{R}_t^* := \boldsymbol{R}_t - \hat{\boldsymbol{\mu}}_t$ with $\hat{\boldsymbol{\mu}}_t := (\hat{\mu}_{1t}, \ldots, \hat{\mu}_{Nt})'$ and $\hat{\mu}_{it} = t^{-1}\sum_{j=1}^{t} R_{ij}$. The stochastic information criterion is defined in terms of the eigenvalues $\hat{\lambda}_{Nj}(\theta)$ of the estimated spectral density matrices $\hat{\boldsymbol{\Sigma}}_N(\theta)$, as

$$\hat{IC}_N(k;c) := \frac{1}{N}\sum_{j=k+1}^{N}\frac{1}{T-1}\sum_{\ell=1}^{T-1}\hat{\lambda}_{Nj}(\theta_\ell) + ckp(n,T), \qquad 0 \le k \le q_{\max}, \quad c > 0,$$

where $\theta_\ell := 2\pi\ell/T$ for $\ell = 1, \ldots, T-1$, $p(N,T)$ is a penalty function such that

$$p(N,T) = (\min[N, B_T^2, B_T^{1/2}T^{1/2}])^{-1}\log(\min[N, B_T^2, B_T^{-1/2}T^{1/2}])),$$

and $q_{\max}$ is some predetermined upper bound. For given N and T, let

$$\hat{q}_N(c) := \mathrm{argmin}_{0\le k\le q_{\max}} \hat{IC}_N(k;c).$$

A value c_0 of c is then determined for which the sequence $\hat{q}_N(c)$, as a function of N, exhibits a stable behavior, and the number of factors q is identified as $\hat{q}_N(c_0)$.

Once q has been identified, Forni *et al.* (2000) show how the common and idiosyncratic components χ_{ti} and ξ_{ti} can be consistently reconstructed from the observed returns, which we denote by $\hat{\chi}_{ti}$ and $\hat{\xi}_{ti}$.

Experience shows that, even after estimating the common component in the GDFM model appropriately, there still remain significant correlations in the idiosyncratic components. Financial empirical evidence also suggests that the idiosyncratic components show a highly heteroskedastic behavior. Hence we use a multivariate GARCH model for the idiosyncratic part of the panel. The above results on the homoskedastic GDFM and the Hallin and Liška (2007) identification method remain valid under conditional heteroskedasticity as fas as the unconditional second-order stationarity holds.

Once we have estimated the common component in the GDFM model $\hat{\chi}_t$, we obtain an estimated idiosyncratic component $\hat{\boldsymbol{\xi}}_t$. We assume that $\hat{\boldsymbol{\xi}}_t$ is conditionally heteroskedastic and of the form

$$\hat{\boldsymbol{\xi}}_t = \boldsymbol{H}_t^{1/2} \boldsymbol{z}_t,$$

where the $N \times N$ matrix $\boldsymbol{H}_t = [h_{ijt}]$ is the conditional covariance matrix of $\hat{\boldsymbol{\xi}}_t$, and $\boldsymbol{z}_t$ is an iid vector processes such that $E\boldsymbol{z} = \boldsymbol{0}$ and $E\boldsymbol{z}\boldsymbol{z}' = \boldsymbol{I}$. This defines the standard multivariate GARCH framework where each of the univariate idiosyncratic processes has the specification $\hat{\xi_{it}} = h_{it} z_{it}$. The conditional variance h_{it} is specified as three different univariate GARCH-type equations under the appropriate non-negativity and stationarity restrictions: the GARCH$(1,1)$ of Bollerslev (1986), the threshold GARCH$(1,1)$ or GJR$(1,1)$ of Glosten *et al.* (1993), and the APARCH$(1,1)$ of Ding *et al.* (1993):

$$\begin{aligned} h_{it} &= \omega + \alpha \xi_{it-1}^2 + \beta h_{it-1} \\ h_{it} &= \omega + (\alpha + \phi D_{it-1}^-) \xi_{it-1}^2 + \beta h_{it-1} \\ h_{it}^\delta &= \omega + \alpha (|\xi_{it-1}| - \phi \xi_{it-1})^\delta + \beta h_{it-1}^\delta, \end{aligned}$$

where D_{it-1}^- is a dummy variable that equals one if $\hat{\xi}_{it-1}$ is negative, and zero otherwise. The GJR and APARCH models include the leverage effect to account for the well known stylized fact that bad news causes more volatility than good news.

Next we turn to specify the correlations. A class of correlation models is based on the decomposition of the covariance matrix into conditional standard deviations and correlations. The simplest multivariate correlation model is the CCC of Bollerslev (1990):

$$\boldsymbol{H}_t = \boldsymbol{D}_t \boldsymbol{P} \boldsymbol{D}_t = \rho_{ij} \sqrt{h_{it} h_{jt}}, \tag{2.2}$$

where $\boldsymbol{D}_t = \operatorname{diag} \boldsymbol{H}_t^{1/2}$ and $\boldsymbol{P} = [\rho_{ij}]$ is positive definite with $\rho_{ii} = 1, i = 1, \ldots, N$. The off-diagonal elements of $\boldsymbol{P}$ are defined through the constant correlations of z_{it} and z_{jt}. The CCC model is in many respects an attractive parametrization, but empirical studies suggest that the assumption of constant conditional correlations may be too restrictive. Engle (2002) introduced the DCC model:

$$\boldsymbol{H}_t = \boldsymbol{D}_t \boldsymbol{P}_t \boldsymbol{D}_t,$$

where $\boldsymbol{D}_t$ is defined as above. The conditional correlation matrix $\boldsymbol{P}_t$ is

$$\boldsymbol{P}_t = (\operatorname{diag} \boldsymbol{Q}_t)^{-1/2} \boldsymbol{Q}_t (\operatorname{diag} \boldsymbol{Q}_t)^{-1/2}$$

where

$$\boldsymbol{Q}_t = (\boldsymbol{i}\boldsymbol{i}' - \boldsymbol{A} - \boldsymbol{B}) \odot \hat{\boldsymbol{R}} + \boldsymbol{A} \odot (\hat{\boldsymbol{z}}_{t-1} \hat{\boldsymbol{z}}'_{t-1}) + \boldsymbol{B} \odot \boldsymbol{Q}_{t-1},$$

and where $\boldsymbol{i}$ is the vector of ones, $\odot$ is the Hadamard product (elementwise matrix multiplication), $\hat{\boldsymbol{z}}_{1t} = (\hat{z}_t, \ldots, \hat{z}_{Nt})$, and $\hat{z}_{it} = \hat{\xi}_{it}/\hat{h}_{it}$ is the i-th standardized residual. The matrices of parameters $\boldsymbol{A}$ and $\boldsymbol{B}$ are $N \times N$ diagonal with typical elements $\alpha_{ii} \geq 0$ and $\beta_{ii} \geq 0$ and satisfying $\alpha_{ii} + \beta_{ii} < 1$. The matrix $\hat{\boldsymbol{R}}$ is the unconditional correlation matrix that can be estimated as the sample correlation of $\hat{\boldsymbol{z}}_t$. The model becomes the CCC model when $\boldsymbol{A} = \boldsymbol{0}$ and $\boldsymbol{B} = \boldsymbol{0}$.

Another class of models allows the dynamic structure of the correlations to be controlled by an exogenous variable, which may be either observable, latent, or a combination of both. One of these models is the Smooth Transition Conditional Correlation (STCC) model of Silvennoinen and Teräsvirta (2005):

$$\boldsymbol{P}_t = (1 - G(s_t))\boldsymbol{P}_1 + G(s_t)\boldsymbol{P}_2, \tag{2.3}$$

where $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ ($\boldsymbol{P}_1 \neq \boldsymbol{P}_2$) are positive definite correlation matrices that describe the two states of the correlations, and $G(\cdot) : \mathbb{R} \to (0, 1)$ is a monotonic function of an observable transition variable $s_t \in \mathcal{F}_{t-1}$. The most common specification for $G(\cdot)$ is a logistic function

$$G(s_t) = (1 + \exp(-\gamma(s_t - c)))^{-1}.$$

The parameter $\gamma > 0$ determines the velocity of the transition and c its location. The transition variable s_t is chosen by the modeler to suit the application at hand. We use the common factor estimated from the GDFM. A generalization of this model is the generalized smooth transition conditional correlation (GSTCC) model in which the parameters γ and c are vectorized:

$$\boldsymbol{P}_t = \boldsymbol{V}\{(\boldsymbol{I}_N - \boldsymbol{G}(s_t))\boldsymbol{P}_1(\boldsymbol{I}_N - \boldsymbol{G}(s_t)) + \boldsymbol{G}(s_t)\boldsymbol{P}_2\boldsymbol{G}(s_t)\}\boldsymbol{V}, \tag{2.4}$$

where $\boldsymbol{V} = \{(\boldsymbol{I}_N - \boldsymbol{G}(s_t))^2 + \boldsymbol{G}(s_t)^2\}^{-1/2}$, and

$$\boldsymbol{G}(s_t) = \operatorname{diag}((1 + e^{-\gamma_1(s_t - c_1)})^{-1}, \ldots, (1 + e^{-\gamma_N(s_t - c_N)})^{-1}).$$

Note that, if $\boldsymbol{G}(s_t) = \boldsymbol{0}$, i.e. when the model involves no transition, and $\boldsymbol{P}_2 = \boldsymbol{P}_1$, it reduces to the CCC model.

We estimate these models under three multivariate distributional assumptions: Gaussian, Student-t with tail index ν, and the skewed-t of Bauwens and

Laurent (2005), which is based on the univariate skewed-t of Fernandez and Steel (1998), with tail index ν and skewing parameter ζ. The Gaussian distribution is the benchmark widely used in financial practice. The Student-t is often used to account for the fat tails often found in financial returns, and the skewed-t, less used in practice, features both heavy tails and skewness. The three distributions are labelled n, t, and st, respectively. Therefore, the specifications are referred to as GARCH-n, GJR-n, APARCH-n, and so on. Conditional on past information, the log-likelihood function is constructed for each of the three distributions, and maximized with respect to the parameters. The asymptotic theory for such estimators is quite involved and much beyond the scope of this article; we suggest it as an avenue for further investigation.

For the Gaussian distribution, the one-step-ahead $\mathrm{VaR}_t(\tau)$ is given by $h_{it}z(\tau)$ with $z(\tau)$ being the τ%-quantile. Similarly, for the Student-t distribution, the VaR is given by $h_{it}st_\nu(\tau)$, with $st_\nu(\tau)$ being the τ%-quantile for the standardized Student distribution with estimated degrees of freedom ν. Lambert and Laurent (2000) show that the quantile function $skst_{\nu,\zeta}(\tau)$ of a non standardized skewed Student density is

$$skst^*_{\nu\zeta}(\tau) = \begin{cases} \dfrac{1}{\zeta} st_\nu(\tau)\left[\dfrac{\alpha}{2}(1+\xi^2)\right] & \text{if} \quad \alpha < \dfrac{1}{1+\xi^2} \\ -\xi st_\nu(\tau)\left[\dfrac{1-\alpha}{2}(1+\xi^{-2})\right] & \text{if} \quad \alpha \geq \dfrac{1}{1+\xi^2}. \end{cases}$$

The τ%-quantile of a standardized skewed-t distribution is

$$skst_{\nu,\zeta}(\tau) = \frac{skst^*_{\nu,\zeta}(\tau) - m}{s},$$

where m and s^2 are the mean and the variance, which depend on the skewing parameter ζ:

$$m = \frac{\Gamma((\nu-1)/2, \sqrt{\nu-2})}{\sqrt{\pi}\Gamma(\nu/2)}\left(\zeta - \frac{1}{\zeta}\right) \qquad \text{and} \qquad s^2 = \left(\zeta^2 + \frac{1}{\zeta^2} - 1\right) - m^2.$$

The $\mathrm{VaR}_t(\tau)$ for a long position is thus $h_{it}skst_{\nu,\zeta}(\tau)$.

Once the individual VaRs are computed, the portfolio optimization is done by constructing the portfolio VaR, denoted by $\mathrm{VaR}_{Nt}(\tau, \boldsymbol{\omega})$. The one-step ahead VaR for a level τ% of the portfolio is defined as $\mathrm{VaR}_{Nt}(\tau, \boldsymbol{\omega}) = \sqrt{\boldsymbol{\omega}'\boldsymbol{H}_{t+1}\boldsymbol{\omega}}q(\tau)$, where $q(\tau)$ is the τ% quantile of a generic distribution ($z(\tau)$, $st_\nu(\tau)$ or $skst_{\nu,\zeta}(\tau)$). A set of portfolio weights $\boldsymbol{\omega}^*$ belongs to the mean-VaR boundary at the τ% confidence level if and only if $\boldsymbol{\omega}^*$ solves the problem $\min_{\boldsymbol{\omega}} \mathrm{VaR}_{Nt}(\tau, \boldsymbol{\omega})$ subject to $\omega_i \geq 0$, $\sum_{i=1}^N \omega_i = 1$ and a given mean level. A dynamic optimal portfolio is constructed by solving every day during the out-of-sample period.

3. Empirical results

We consider the Tokyo Stock Exchange market indexes and construct daily out-of-sample portfolio allocations. We compare the results among the vari-

Table 1. TOPIX Sector Indexes composition.

No.	Sector	No.	Sector
1	Fishery, Agriculture & Forestry	18	Precision Instruments
2	Mining	19	Other Products
3	Construction	20	Electric Power and Gas
4	Foods	21	Land Transportation
5	Textiles and Apparels	22	Marine Transportation
6	Pulp and Paper	23	Air Transportation
7	Chemicals	24	Warehousing and Harbor Transportation
8	Pharmaceutical	25	Information & Communication
9	Oil and Coal Products	26	Wholesale Trade
10	Rubber Products	27	Retail Trade
11	Glass and Ceremics Products	28	Banks
12	Iron and Steel	29	Securities and Commodities Futures
13	Nonferrous Metal	30	Insurance
14	Metal Products	31	Other Financing Business
15	Machinery	32	Real Estate
16	Electric Appliance	33	Services
17	Transportation Equipment		

ous specifications for the conditional covariance matrix along with the standard Markowitz mean-variance approach.

We use daily log-returns of the TOPIX (Tokyo stock price index) Sector Indexes. They are formed by splitting the constituents of TOPIX into 33 categories. The composition is summarized in Table 1. The sample period goes from January 4, 2001 to June 29, 2007 with $T = 1597$ days and $N = 33$ industries. The in-sample period covers 1097 days, from January 4, 2001 through June 22, 2005. The out-of-sample period covers 500 days, from June 23, 2005 through June 29, 2007.

Descriptive statistics of the full sample are given in Table 2. There are no statistically significant positive or negative average returns and a great deal of heterogeneity in the standard deviations, ranging from 0.887 for the Foods sector and 2.535 for the Securities and Commodities Futures sector. Skewness is also present in most of the sectors, with 5 showing positive asymmetry and 18 negative at the 10% level. The excess kurtosis is always positive, evidencing fat tails. The Ljung-Box tests reject the null hypothesis of no serial correlation of returns at lags 4 for 21 industries at 5% significant level. While Construction, Food, and Metal Products have relatively strong autocorrelations, Mining, Textile and Apparel, and Fishery, Agriculture, and Forestry do not show significant autocorrelations.

To find out the number of shocks in the common component, we apply the Hallin and Liška (2007) information criterion. Identification of $\hat{q}_N$ is based on a visual inspection of a double plot of the type shown in Figure 1. That double plot provides a measure (an empirical variance $S(c)$, dotted line) of the instability of the selection $\hat{q}_N(c)$ associated with various values of the tuning constant c, along

Table 2. Descriptive statistics.

Sector	Mean	SD	Skewness	Excess Kurtosis	$LB(4)$	$LB(8)$
TOPIX	0.020	1.209	-0.256^{***}	4.692^{***}	9.723^{**}	12.363
Fishery, Agriculture & Forestry	0.030	1.218	-0.143^{**}	5.358^{***}	8.722^{*}	13.162
Mining	0.060	1.912	0.222^{***}	4.323^{***}	7.147	11.647
Construction	0.023	1.385	-0.285^{***}	5.071^{***}	22.523^{***}	25.137^{***}
Foods	0.030	0.887	-0.207^{***}	5.820^{***}	30.267^{***}	32.093^{***}
Textiles and Apparels	0.038	1.284	-0.383^{***}	5.557^{***}	5.459	6.708
Pulp and Paper	−0.008	1.494	−0.050	4.109^{***}	7.171	11.711
Chemicals	0.031	1.206	-0.285^{***}	5.121^{***}	8.923^{*}	13.673^{*}
Pharmaceutical	0.010	1.132	0.067	5.503^{***}	17.234^{***}	23.394^{***}
Oil and Coal Products	0.057	1.639	−0.030	4.126^{***}	19.105^{***}	20.326^{***}
Rubber Products	0.060	1.606	-0.209^{***}	6.341^{***}	4.179	4.489
Glass and Ceremics Products	0.034	1.549	-0.167^{***}	4.188^{***}	11.226^{**}	13.531^{*}
Iron and Steel	0.107	1.785	−0.045	4.254^{***}	3.124	6.921
Nonferrous Metals	0.013	1.841	-0.144^{**}	4.036^{***}	17.690^{***}	19.862^{**}
Metal Products	0.044	1.298	-0.213^{***}	5.090^{***}	17.695^{***}	21.612^{***}
Machinery	0.055	1.388	-0.372^{***}	4.619^{***}	13.885^{***}	18.599^{**}
Electric Appliances	0.002	1.584	0.109^{*}	4.311^{***}	11.387^{**}	15.962^{**}
Transportation Equipment	0.048	1.437	−0.063	5.416^{***}	5.283	14.560^{*}
Precision Instruments	0.045	1.428	-0.123^{**}	4.261^{***}	4.139	8.755
Other Products	0.032	1.313	-0.227^{***}	5.509^{***}	4.933	10.743
Electric Power and Gas	0.030	0.929	-0.105^{*}	6.158^{***}	10.106^{**}	22.669^{***}
Land Transportation	0.020	1.117	0.049	4.574^{***}	14.961^{***}	21.012^{***}
Marine Transportation	0.101	1.892	-0.131^{**}	5.001^{***}	5.997	17.880^{**}
Air Transportation	−0.019	1.624	-0.237^{***}	8.654^{***}	15.679^{***}	20.066^{**}
Warehousing	0.052	1.403	−0.025	5.154^{***}	20.339^{***}	24.366^{***}
Information & Communication	−0.022	1.936	0.050	5.336^{***}	10.525^{**}	16.882^{**}
Wholesale Trade	0.056	1.575	-0.287^{***}	4.830^{***}	7.777	8.696
Retail Trade	0.005	1.414	0.106^{*}	5.217^{***}	16.963^{***}	19.205^{**}
Banks	0.009	1.849	0.111^{*}	4.952^{***}	38.147^{***}	38.755^{***}
Securities and Commodities Futures	0.011	2.356	0.036	3.862^{***}	27.268^{***}	33.033^{***}
Insurance	0.050	1.744	0.116^{*}	4.906^{***}	10.583^{**}	14.206^{*}
Other Financing Business	0.007	1.758	-0.104^{*}	4.611^{***}	23.799^{***}	26.667^{***}
Real Estate	0.073	1.888	0.096	4.045^{***}	39.601^{***}	43.074^{***}
Services	−0.028	1.398	-0.122^{**}	5.020^{***}	27.415^{***}	31.439^{***}

Note: The symbols $*, **$, and, $***$ donote statistically significant at 1%, 5%, and 10% level, respectively. SD is the standard deviation of the returns. LB(4) and LB(8) are the Ljung-Box test statistics based on the first 4 and 8 lags of the returns, respectively.

with the final selection $\hat{q}_N(c)$ associated with the same value of c (solid line). The procedure then consists in spotting the second interval (starting from the left) of c values over which the dotted line touches the horizontal axis (hence, the value $(c) = 0$); the number of factors to be selected then is obtained by reading, on the solid line curve, the corresponding value of $\hat{q}_N(c)$.

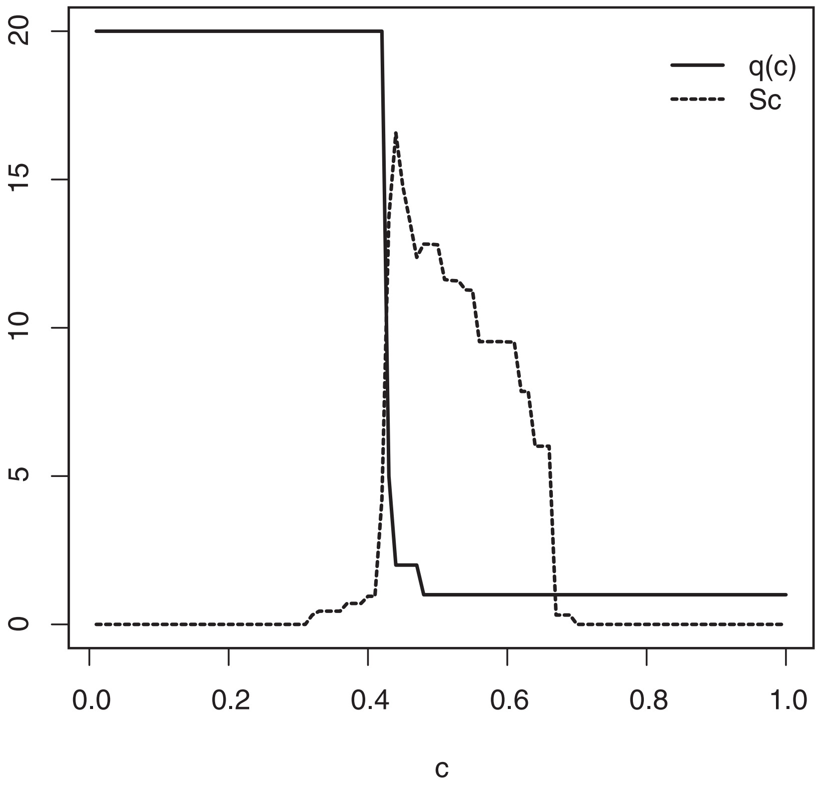

Figure 1. Hallin and Liška plot for determining the number of factors.

Figure 1 leads to a selection of one factor and Figure 2 shows the proportions explained by the common components for each industry. Numbers on the horizontal axis correspond to the 33 sectors according to the classification in Table 1. There is heterogeneity across sectors, with a minimum of explained variance of 20% for Electric Power and Gas, and a maximum of 80% for Machinery. On average the common component explains 65% of the total variation.

The performance of the idiosyncratic VaR is assessed with a specification test. Specification tests in a quantile based framework are based on failure rates, that is the percentage of times that the realization of the idiosyncratic components are below the VaR. If the model is well specified, its failure rate should be equal to the VaR level. The Christoffersen (1998) likelihood ratio test of unconditional coverage is based on this idea. Let I_t be a hit variable that takes value 1 if there is a success, that is if the idiosyncractic component is bigger than the fitted VaR, and 0 otherwise:

$$I_t = \begin{cases} 1, & \text{if} \quad \hat{\xi}_{it} > \mathrm{VaR}_t(\tau) \\ 0, & \text{otherwise.} \end{cases} \tag{3.1}$$

If T is the in-sample size, $T_1 = \sum_{t=1}^{T} I_t$ is the number of successes and $T_0 = T - T_1$ the number of failures. The empirical failure rates can be computed as $\hat{f} = T_0/T$

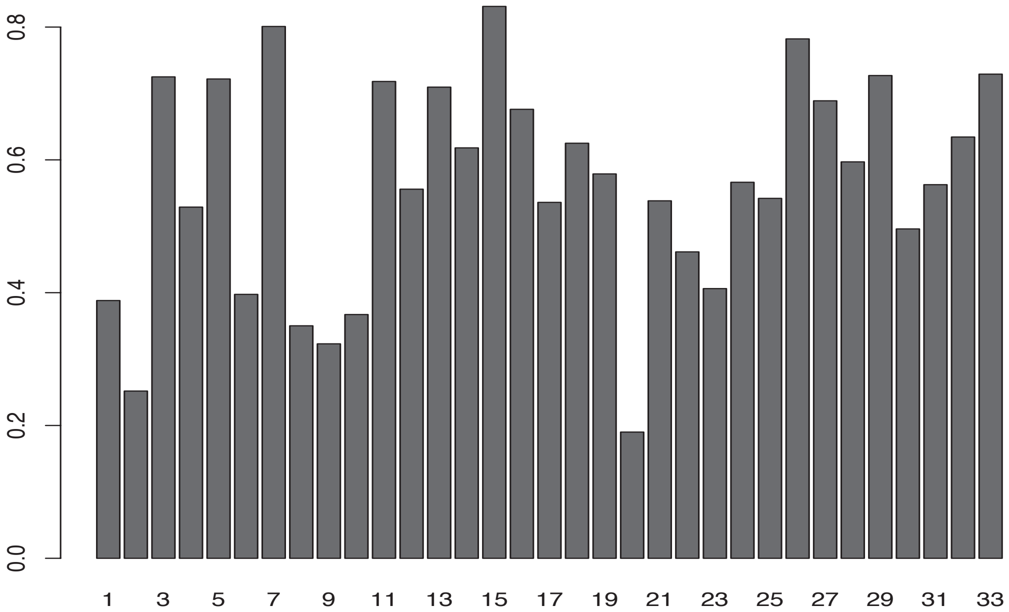

Figure 2. Explained variances by a common factor.

and the test statistic is

$$LR_{uc} = -2 \log \left(\frac{(1-\tau)^{T_0} \tau^{T_1}}{(1-\hat{f})^{T_0} \hat{f}^{T_1}} \right) \sim \chi_1^2, \tag{3.2}$$

where the null of the test is $f = \tau$. Tables 3 and 4 show the specification test results for $\tau = 0.05$ and $\tau = 0.01$, respectively. Figures 3 and 4 provide, for comparison purposes, the box plots among the different models and distributions.

On average, GJR-t produces the most accurate one-step-ahead VaR forecast at the 5% level, followed by APARCH-t. For $\alpha = 1\%$, the horserace winner is again the GJR-t followed by the GJR-st. The Gaussian GARCH models overestimate the VaR at 5% and clearly underestimates it at 1%, as a consequence of the thin tails. The GJR and APARCH models have better performances compared with the GARCH model, which suggests the existence of a leverage effect. This result dovetails with Brownlees *et al.* (2009). They show that, among a battery of the most common used GARCH models, the threshold GARCH is most often the best forecaster.

We construct 500 daily out-of-sample portfolio allocations based on the conditional covariance one-step ahead predictions of the multivariate GARCH models, and we compare the results with those of the Markowitz's mean-variance model. Given the above specification tests of the VaR, we restrict ourselves to the APARCH and GJR models for the conditional variances. Since the parameters of the multivariate GARCH models change slowly from one day to another, they are re-estimated every 10 days (or two weeks).

Table 3. In-sample period 5% VaR results.

Sector	GARCH-n	GJR-n	APARCH-n	GARCH-t	GJR-t	APARCH-t	GARCH-st	GJR-st	APARCH-st
Fishery, Agriculture & Forestry	0.0429	0.0438	0.0447	0.0447	0.0474	**0.0493**	0.0447	0.0474	**0.0493**
Mining	0.0356	0.0365	0.0356	0.0411	0.0420	0.0420	**0.0474**	**0.0474**	**0.0474**
Construction	0.0547	0.0493	0.0465	0.0584	0.0575	0.0547	0.0538	**0.0502**	0.0511
Foods	0.0566	0.0493	0.05657	0.0566	0.0529	0.0502	0.0547	**0.0502**	0.0465
Textiles and Apparels	0.0538	0.0529	0.0520	0.0538	0.0547	0.0557	**0.0502**	0.0493	0.0484
Pulp and Paper	0.0511	0.0529	**0.0493**	0.0538	0.0538	0.0511	0.0474	0.0511	**0.0493**
Chemicals	0.0557	0.0511	0.0520	0.0575	0.0557	0.0566	0.0511	0.0465	**0.0493**
Pharmaceutical	0.0438	0.0438	0.0374	0.0456	**0.0465**	**0.0465**	**0.0456**	0.0456	0.0447
Oil and Coal Products	0.0429	0.0420	0.0420	0.0438	0.0429	0.0438	**0.0447**	**0.0447**	**0.0447**
Rubber Products	0.0493	0.0511	**0.0502**	**0.0502**	0.0511	0.0538	**0.0502**	0.0511	0.0529
Glass and Ceremics Products	0.0566	0.0547	0.0547	0.0575	0.0575	0.0566	**0.0511**	0.0547	0.0538
Iron and Steel	0.0484	0.0474	0.0456	**0.0502**	0.0465	0.0484	0.0538	0.0511	0.0511
Nonferrous Metals	0.0575	**0.0557**	**0.0557**	0.0593	0.0593	0.0593	0.0575	0.0566	**0.0557**
Metal Products	0.0365	0.0365	0.0383	0.0401	0.0401	0.0374	**0.0420**	**0.0420**	0.0383
Machinery	0.0575	0.0602	0.0566	0.0620	0.0620	0.0611	0.0520	**0.0502**	0.0465
Electric Appliances	0.0502	0.0511	**0.0502**	0.0493	0.0511	**0.0502**	0.0557	0.0547	0.0538
Transportation Equipment	0.0456	0.0465	0.0456	0.0511	0.0493	**0.0502**	0.0511	0.0493	0.0493
Precision Instruments	0.0520	0.0484	0.0484	0.0520	0.0484	**0.0502**	0.0520	0.0484	0.0502
Other Products	0.0511	0.0511	0.0520	0.0547	0.0529	0.0538	0.0484	**0.0502**	0.0511
Electric Power and Gas	0.0420	0.0411	0.0411	0.0420	0.0420	0.0392	**0.0429**	**0.0429**	0.0411
Land Transportation	0.0420	0.0429	0.0429	0.0420	0.0420	0.0420	**0.0456**	0.0447	**0.0456**
Marine Transportation	0.0429	0.0456	0.0447	**0.0465**	**0.0465**	0.0447	0.0456	0.0456	0.0438
Air Transportation	0.0383	0.0383	0.0383	0.0429	0.0429	0.0438	0.0447	0.0456	**0.0465**
Warehousing	0.0474	0.0465	0.0484	0.0484	**0.0493**	**0.0493**	0.0511	0.0511	0.0511
Information & Communication	0.0420	0.0356	0.0347	0.0429	0.0401	0.0401	**0.0465**	**0.0456**	0.0420
Wholesale Trade	0.0566	0.0511	0.0511	0.0557	0.0520	0.0520	0.0557	0.0493	**0.0502**
Retail Trade	0.0465	0.0456	0.0456	0.0456	0.0465	0.0456	0.0465	**0.0484**	0.0465
Banks	0.0511	0.0511	0.0465	**0.0502**	.0511	0.0474	0.0557	0.0566	0.0511
Securities and Commodities Futures	0.0429	0.0429	0.0429	0.0429	0.0429	0.0429	0.0456	**0.0484**	**0.0484**
Insurance	0.0493	0.0474	0.0465	0.0529	0.0511	**0.0502**	0.0538	0.0511	0.0529
Other Financing Business	0.0493	**0.0502**	0.0511	0.0529	0.0511	0.0520	0.0529	0.0520	0.0520
Real Estate	0.0538	0.0529	0.0538	0.0520	**0.0502**	0.0529	0.0538	0.0520	0.0566
Services	0.0538	0.0557	0.0575	0.0566	0.0557	0.0575	**0.0520**	0.0538	0.0557

Note: GARCH-n—means the method of GARCH$(1,1)$ models with normal distribution. Respectively other abbreviation are constructed. Model abbreviations are as follows:

1. GARCH- Generalized Autoregressive Conditional Heteroskedasticity;
2. GJR- Glosten, Jagannathan and Runkle;
3. APARCH- Asymmetric Power Autoregressive Conditional Heteroskedasticity.

Table 4. In-sample period 1% VaR results.

Sector	GARCH-n	GJR-n	APARCH-n	GARCH-t	GJR-t	APARCH-t	GARCH-st	GJR-st	APARCH-st
Fishery, Agriculture & Forestry	0.0128	0.0128	0.0128	0.0109	0.0109	**0.0100**	0.0109	0.0109	**0.0100**
Mining	0.0119	0.0119	0.0128	0.0082	0.0082	0.0082	**0.0109**	**0.0109**	**0.0109**
Construction	0.0155	0.0146	0.0146	**0.0100**	**0.0100**	0.0109	0.0073	0.0091	0.0109
Foods	0.0173	0.0192	0.01734	**0.0128**	0.0146	0.0155	**0.0128**	0.0137	0.0137
Textiles and Apparels	0.0164	0.0155	0.0173	0.0137	0.0128	0.0155	0.0119	**0.0109**	**0.0091**
Pulp and Paper	0.0182	0.0164	0.0155	0.0146	0.0137	0.0137	0.0128	**0.0119**	**0.0119**
Chemicals	0.0155	0.0164	0.0173	0.0146	0.0128	0.0128	0.0128	0.0128	**0.0100**
Pharmaceutical	0.0146	0.0146	0.0109	0.0119	0.0109	0.0128	0.0119	**0.0100**	0.0109
Oil and Coal Products	0.0155	0.0155	0.0155	0.0137	**0.0128**	**0.0128**	0.0155	0.0146	0.0146
Rubber Products	0.0128	0.0128	0.0119	0.0109	**0.0109**	0.0119	**0.0109**	**0.0109**	**0.0109**
Glass and Ceremics Products	0.0137	**0.0100**	0.0109	0.0119	**0.0100**	0.0082	0.0100	0.0082	0.0082
Iron and Steel	**0.0109**	**0.0091**	0.0082	0.0073	0.0064	0.0064	**0.0091**	0.0082	0.0082
Nonferrous Metals	0.0146	0.0155	0.0146	0.0137	**0.0119**	0.0128	**0.0119**	**0.0119**	**0.0119**
Metal Products	0.0128	0.0128	0.0128	**0.0119**	**0.0119**	0.0119	0.0128	0.0119	0.0119
Machinery	0.0091	**0.0100**	0.0091	0.0091	0.0082	0.0091	0.0082	0.0064	0.0064
Electric Appliances	**0.0091**	0.0073	0.0073	0.0064	0.0064	0.0064	0.0082	0.0082	0.0082
Transportation Equipment	0.0164	0.0137	0.0146	**0.0100**	0.0109	0.0091	0.0109	0.0109	0.0091
Precision Instruments	0.0164	0.0164	0.0164	**0.0155**	**0.0155**	**0.0155**	**0.0155**	**0.0155**	**0.0155**
Other Products	0.0192	0.0182	0.0173	0.0146	0.0146	0.0128	0.0119	0.0128	**0.0109**
Electric Power and Gas	0.0146	0.0146	0.0128	0.0128	**0.0119**	**0.0119**	0.0128	**0.0119**	**0.0119**
Land Transportation	0.0128	0.0137	0.0137	**0.0100**	**0.0100**	**0.0100**	**0.0100**	**0.0100**	**0.0100**
Marine Transportation	0.0164	0.0164	0.0164	**0.0119**	**0.0119**	0.0128	**0.0119**	**0.0119**	0.0128
Air Transportation	**0.0100**	**0.0100**	0.0109	0.0073	0.0073	0.0073	0.0073	0.0073	0.0073
Warehousing	**0.0100**	0.0109	0.0109	0.0082	0.0082	0.0082	0.0091	0.0082	0.0082
Information & Communication	0.0091	0.0091	**0.0100**	0.0055	0.0055	0.0073	0.0064	0.0073	0.0082
Wholesale Trade	0.0173	0.0155	0.0137	0.0119	0.0128	0.0137	**0.0091**	0.0128	0.0137
Retail Trade	0.0137	0.0137	0.0137	0.0091	**0.0100**	**0.0100**	0.0109	0.0100	0.0109
Banks	0.0091	0.0091	0.0109	0.0064	0.0064	0.0073	0.0073	0.0073	**0.0100**
Securities and Commodities Futures	0.0119	0.0091	0.0091	**0.0100**	0.0064	0.0064	0.0119	**0.0100**	**0.0100**
Insurance	**0.0109**	0.0082	0.0082	0.0064	0.0064	0.0064	0.0064	0.0064	0.0064
Other Financing Business	0.0155	0.0137	0.0137	**0.0100**	**0.0100**	**0.0100**	0.0109	**0.0100**	**0.0100**
Real Estate	**0.0100**	**0.0100**	0.0119	0.0055	0.0055	0.0082	0.0073	0.0064	0.0109
Services	0.0128	**0.0100**	**0.0100**	0.0091	0.0091	**0.0100**	0.0073	0.0082	0.0091

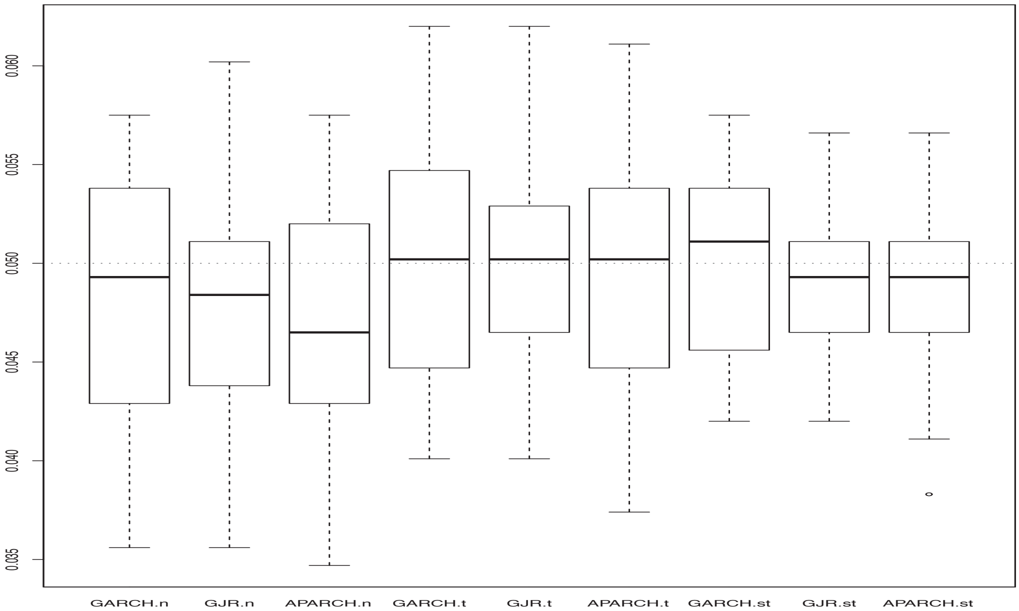

Figure 3. Box plots for in-sample period 5% VaR results.

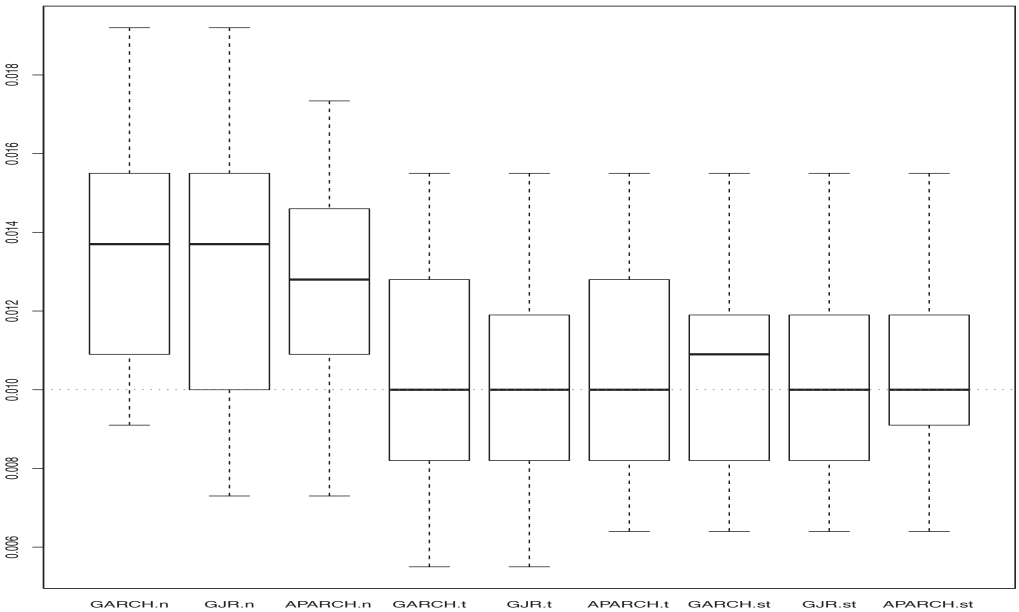

Figure 4. Box plots for in-sample period 1% VaR results.

Table 5. Optimal portfolio results for 5% VaR.

Model	n	t	st
CCC-GJR	0.0824(0.7450)	0.0797(0.7447)	0.0834(0.7466)
CCC-APARCH	0.0823(0.7450)	0.0798(0.7447)	0.0835(0.7467)
DCC-GJR	0.0808(0.7465)	0.0793(0.7451)	0.0815(0.7471)
DCC-APARCH	0.0808(0.7464)	0.0795(0.7454)	0.0812(0.7476)
STCC-GJR	0.0805(0.7459)	0.0786(0.7436)	0.0792(0.7465)
STCC-APARCH	0.0805(0.7453)	0.0788(0.7434)	0.0795(0.7467)
GSTCC-GJR	0.0780(0.7405)	0.0792(0.7426)	0.0791(0.7404)
GSTCC-APARCH	0.0713(0.7426)	0.0772(0.7423)	0.0841(0.7377)

Note: The mean-variance model gives mean return 0.0646 and standard deviation 0.7811. The number in the parenthesis is the standard deviation. CCC-GJR- means the method of using CCC model for conditional correlation matrix and GJR model for univariate GARCH specification. Abbreviations for univariate GARCH models are in the notes of Table 3. Model abbreviations for multivariate GARCH are as follows:

1. CCC- Constant Conditional Correlation;
2. DCC- Dynamic Conditional Correlation;
3. STCC- Smooth Transition Conditional Correlation;
4. GSTCCC- Generarized Smooth Transition Conditional Correlation.

Table 6. Optimal portfolio results for 1% VaR.

Model	n	t	st
CCC-GJR	0.0817(0.7465)	0.0788(0.7461)	0.0816(0.7485)
CCC-APARCH	0.0816(0.7464)	0.0789(0.7462)	0.0817(0.7485)
DCC-GJR	0.0808(0.7459)	0.0780(0.7465)	0.0810(0.7496)
DCC-APARCH	0.0809(0.7458)	0.0778(0.7464)	0.0809(0.7498)
STCC-GJR	0.0808(0.7459)	0.0764(0.7460)	0.0772(0.7495)
STCC-APARCH	0.0809(0.7450)	0.0769(0.7456)	0.0795(0.7467)
GSTCC-GJR	0.0769(0.7440)	0.0749(0.7425)	0.0721(0.7437)
GSTCC-APARCH	0.0789(0.7447)	0.0727(0.7431)	0.0798(0.7429)

Note:The mean-variance model gives mean return 0.0646 and standard deviation 0.7811.

For the estimation of the CCC, the correlation matrix $\boldsymbol{P}$ is calculated using a rolling window of 240 trading days. As for the STCC and GSTCC, the correlation matrices $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$ are estimated with rolling windows of 240 and 60 observations respectively. As a robustness check we used various combinations of the number of observations for $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$. Results, available under request, are very similar in terms of the portfolio performances. The estimated parameters, not reported here, are also available under request.

Tables 5 and 6 show the results for the mean-VaR optimal portfolio for 5% and 1% levels respectively. Numbers are the average returns for the 500 daily out-of-sample observations and the standard deviations in parentheses.

The similarity in the portfolio performance among the different model specifications is clear, but the multivariate GARCH portfolio selection produces significant improvements when compared with mean-variance results. Also, whereas

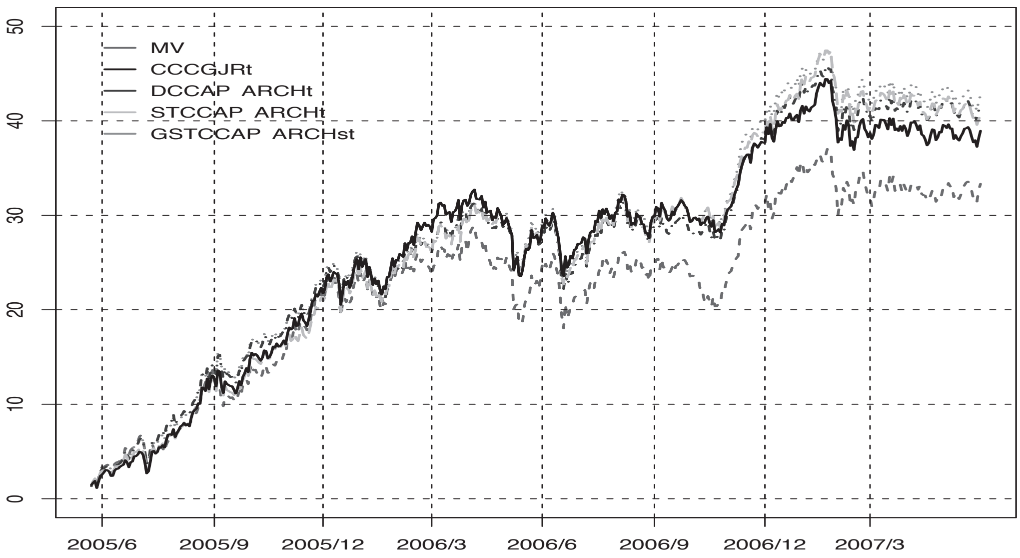

Figure 5. Wealth evolution for 500 out-of-sample 5% VaR forecast with mean-variance portfolio.

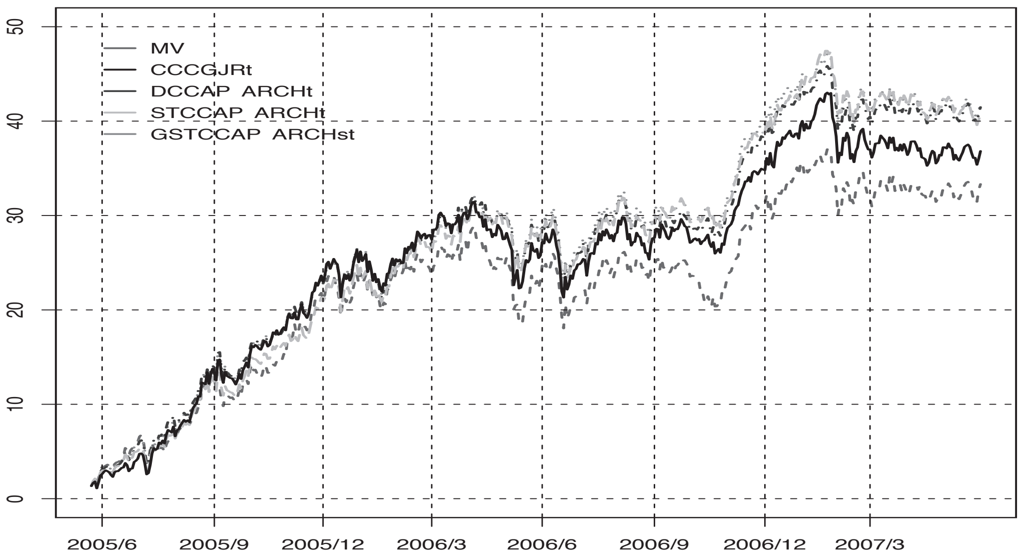

Figure 6. Wealth evolution for 500 out-of-sample 1% VaR forecast with mean-variance portfolio.

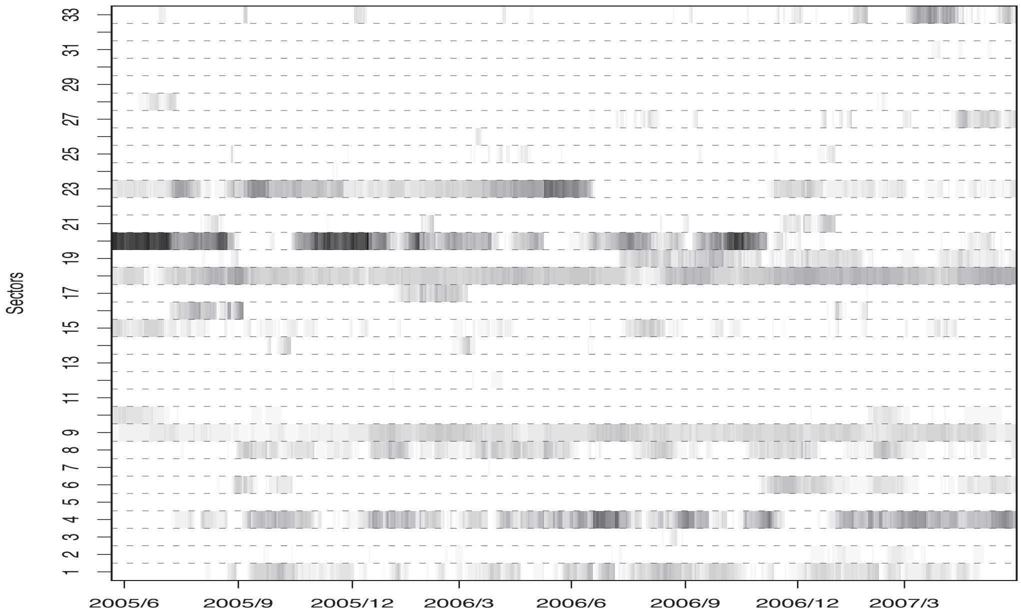

Figure 7. Asset allocation results with GSTCC-APARCH-*st* for 500 out-of-sample 5% VaR.

the portfolio performances among the models are close, the GSTCC-*st* model provides the best performance. Last, the standard deviations based on a 5% VaR level are smaller than those based on a 1% VaR level. To understand this phenomena, we remind that there are some cautions on the use of mean-VaR approach. Basak and Shapiro (2001) found that VaR risk managers often optimally choose a larger exposure to risky assets and consequently incur large losses when they occur. Alexandar and Baptista (2002) argue that an efficient portfolio that globally minimizes the VaR may not exist and it is plausible for certain risk averse agents to end up selecting portfolios with larger standard deviations if they switch from using variance to VaR as a measure of risk.

Figures 5 and 6 present the wealth evolution obtained by applying the GSTCC-GJR-*st*, STCC-APARCH-*t*, DCC-GJR-*t*, and CCC-APARCH-*t* models with the mean-variance model for 5% and 1% levels respectively.

The final wealth obtained by the multivariate GARCH VaR forecast model is not only larger than the wealth attained by the mean-variance model, but also, as pointed out above, it has a lower risk. Moreover, wealth obtained by using the 1% VaR level is slightly lower than those obtained by the 5% VaR level.

Figures 7 and 8 show the one-step-ahead dynamic portfolio allocation using the GSTCC-APARCH-*st* with 5% and 1% VaR levels respectively. We show results for these two models since they are the best performers in terms of portfolio risk. The vertical axis is for the industry identification number (see Table 1) and the horizontal axis is for the out-of-sample. The lighter the rectangles, the smaller the portfolio allocation is in the sector (i.e. white implies 0% allocation), and the darker, the higher the allocation of the sector (i.e. black implies 100%

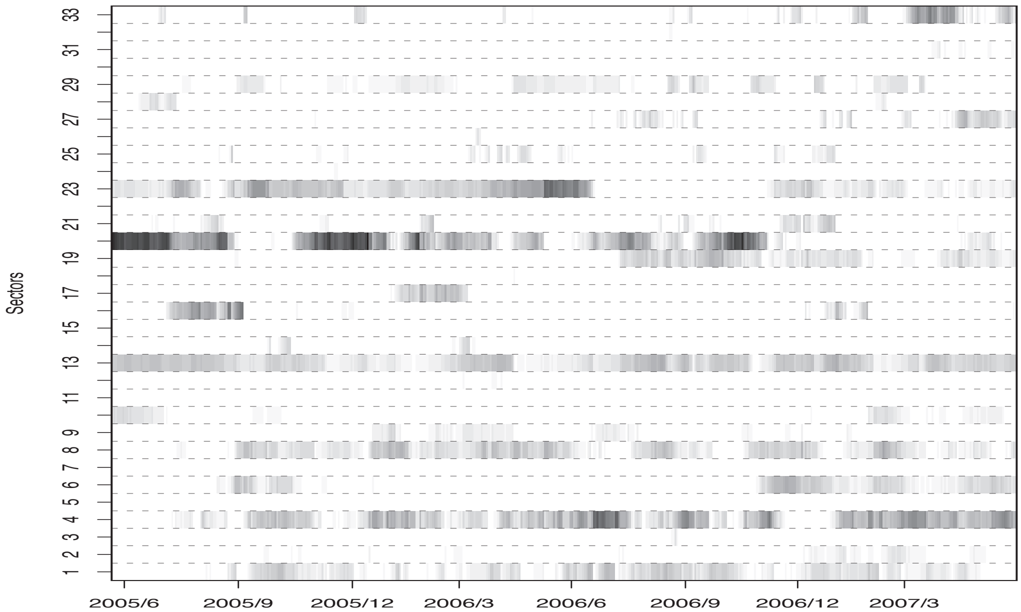

Figure 8. Asset allocation results with GSTCC-APARCH-st for 500 out-of-sample 1% VaR.

allocation).

Recall that the mean and standard deviation for 5% VaR are 0.0841 and 0.7377, and 0.0749 and 0.7425 for 1% VaR. The sectors that receive the higher weights under both 5% and 1% VaR are Fishery, Agriculture, and Forestry (number 1), Food (number 4), Chemicals (number 7), Pharmaceuticals (number 8), Other Products (number 19), Electric Power and Gas (number 20), and Land Transportation (number 21). Among these, Electric Power and Gas (number 20) has a highest allocation at the beginning of the period, around December 2005–October 2006, and then declines dramatically after October 2006 for both 5% and 1% levels.

While the two figures look similar, there are significant differences. Precision Instruments (number 18) and Oil and Coal Products (number 9) receive a high weight for the 5% level, whereas Nonferrous Metal (number 13) obtains a high weight for the 1% level. Last, we compute the variance of the daily portfolio allocations and its mean during the out-of-sample period, which yields 0.0709 and 0.0753 for 5% and 1% VaR levels. We therefore conclude that the 1% VaR restriction for portfolio optimization causes frequent position changes when the variance of the portfolio performance increases. These results are fully in line with those of Basak and Shapiro (2001).

4. Summary and future research agenda

This paper studies the portfolio selection problem based on a generalized dynamic factor model (GDFM) with conditional heteroskedasticity in the idiosyncratic components. We propose a Generalized Smooth Transition Conditional

Correlation (GSTCC) model for the idiosyncratic components combined with the GDFM. Portfolio risk is greatly reduced when compared with the standard Markowitz mean-variance approach. Among all the multivariate GARCH models that we propose, the generalized smooth transition conditional correlation provides the best result. We also find that GARCH models with leverage effects are the most appropriate. Regarding the choice of the distribution for the idiosyncratic components, the skewed-t distribution provides the best results in terms of specification tests based on the failure rates, as it is the most flexible and it captures both asymmetries and fat tails. The one-day ahead portfolio optimizations shows the gains of such approach. Yet, some extensions and further theoretical study is needed. From the statistical side, the study of the asymptotic properties of the estimates is worth looking at. From the finance side, daily rebalancing entails cost (such as transaction cost) and hence it may not be optimal. The inclusion of these costs, as well as multi period rebalancing, is an interesting research avenue.

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