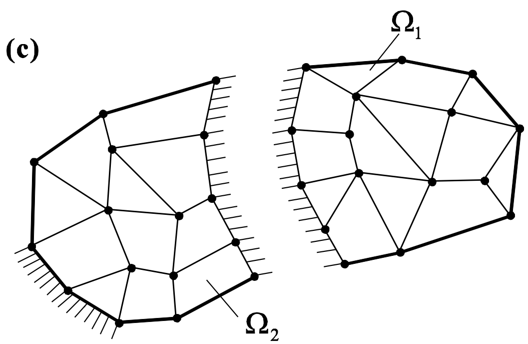
Craig-Bampton component mode synthesis

Original Craig-Bampton formulation



TEXAS
The University of Texas at Austin

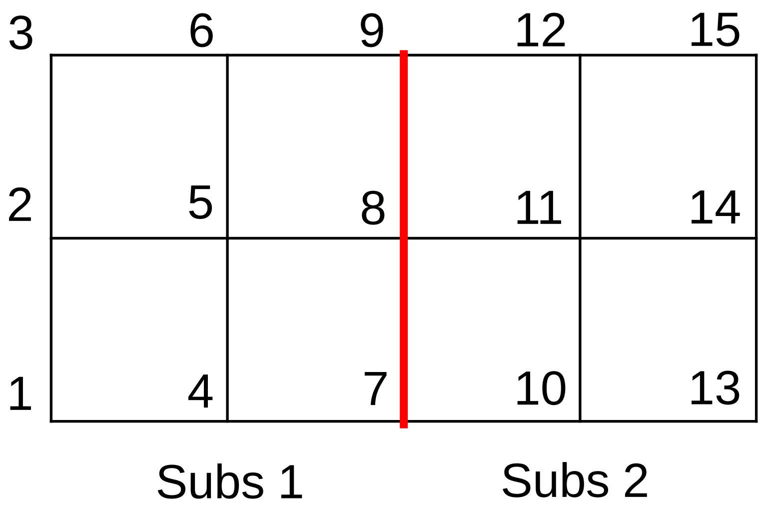
ANSYS

ABAQUS

Bampton, M. C., & CRAIG, JR, R. R. (1968). Coupling of substructures for dynamic analyses. *AIAA Journal*, *6*(7), 1313-1319.

Craig-Bampton component mode synthesis

1. Partitioned formulation



$$\mathbf{M}_g \ddot{\mathbf{u}}_g + \mathbf{K}_g \mathbf{u}_g = \mathbf{f}_g,$$

$$\mathbf{M}_g = \begin{bmatrix} \mathbf{M}_s & \mathbf{M}_c \\ \mathbf{M}_c^T & \mathbf{M}_b \end{bmatrix}, \quad \mathbf{K}_g = \begin{bmatrix} \mathbf{K}_s & \mathbf{K}_c \\ \mathbf{K}_c^T & \mathbf{K}_b \end{bmatrix},$$

$$\mathbf{u}_g = \begin{bmatrix} \mathbf{u}_s \\ \mathbf{u}_b \end{bmatrix}, \quad \mathbf{f}_g = \begin{bmatrix} \mathbf{f}_s \\ \mathbf{f}_b \end{bmatrix},$$

$$\begin{array}{cc} & \begin{array}{ccc} 1\sim6 & 10\sim15 & 7,8,9 \end{array} \\ \begin{array}{c} 1\sim6 \\ 10\sim15 \\ 7,8,9 \end{array} & \begin{bmatrix} \mathbf{M}_s^{(1)} & \mathbf{0} & \mathbf{M}_c^{(1)} \\ \mathbf{0} & \mathbf{M}_s^{(2)} & \mathbf{M}_c^{(2)} \\ \mathbf{M}_c^{(1)^T} & \mathbf{M}_c^{(2)^T} & \mathbf{M}_b \end{bmatrix} \end{array}$$

$\mathbf{u}_g$ could be expressed by $\mathbf{T}_0$:

$$\mathbf{u}_g = \begin{bmatrix} \mathbf{u}_s \\ \mathbf{u}_b \end{bmatrix} = \mathbf{T}_0 \begin{bmatrix} \mathbf{q}_s \\ \mathbf{u}_b \end{bmatrix},$$

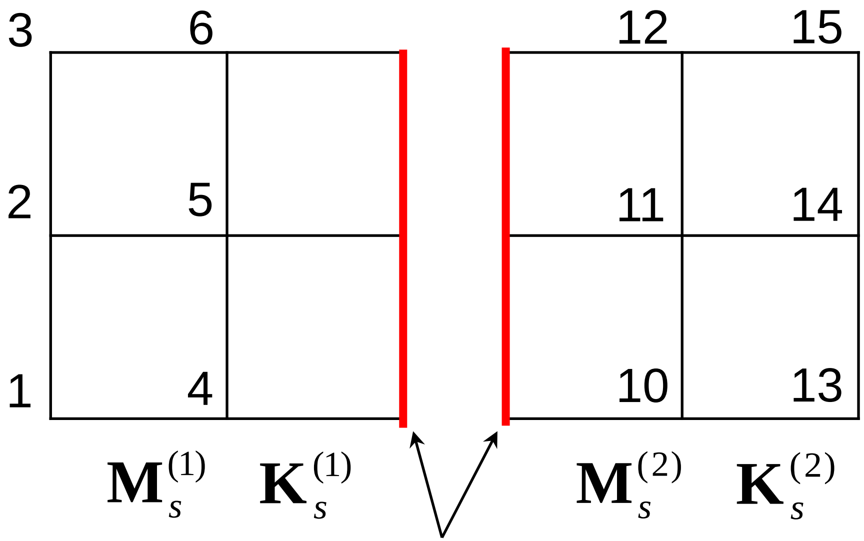
$$\mathbf{T}_0 = \begin{bmatrix} \boldsymbol{\Phi}_s & -\mathbf{K}_s^{-1}\mathbf{K}_c \\ \mathbf{0} & \mathbf{I}_b \end{bmatrix}$$

Substructural eigenvector matrix

Interface constraint mode (=super-element)

Craig-Bampton component mode synthesis

2. Transformation matrix and reducing the system



Fixed boundary condition

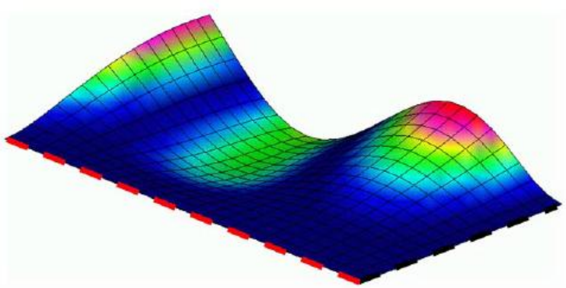
$$\mathbf{u}_g = \begin{bmatrix} \mathbf{u}_s \\ \mathbf{u}_b \end{bmatrix} = \mathbf{T}_0 \begin{bmatrix} \mathbf{q}_s \\ \mathbf{u}_b \end{bmatrix},$$

$$\left[\mathbf{K}_s^{(k)} - \lambda_i^{(k)} \mathbf{M}_s^{(k)}\right] (\boldsymbol{\phi}^{(k)})_i = \mathbf{0},$$

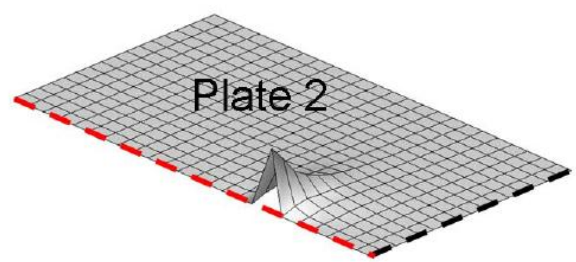
$$\mathbf{T}_0 = \begin{bmatrix} \boldsymbol{\Phi}_s & -\mathbf{K}_s^{-1}\mathbf{K}_c \\ \mathbf{0} & \mathbf{I}_b \end{bmatrix}$$

e.g.)

Fixed interface normal mode



Interface constraint mode



Craig-Bampton component mode synthesis

2. Transformation matrix and reducing the system

$$\mathbf{u}_s = \boxed{\boldsymbol{\Phi}_s \mathbf{q}_s} \boxed{- \mathbf{K}_s^{-1} \mathbf{K}_c \mathbf{u}_b}$$

$$= \begin{bmatrix} \boldsymbol{\Phi}_d & \cancel{\boldsymbol{\Phi}_r} \end{bmatrix} \begin{bmatrix} \mathbf{q}_d \\ \cancel{\mathbf{q}_r} \end{bmatrix} - \mathbf{K}_s^{-1} \mathbf{K}_c \mathbf{u}_b ,$$

- ✓ Phi_s has entire fixed interface normal modes.
- ✓ For ROM, the residual normal modes should be removed.

<u>Rule of thumb: frequency cut-off rule in mode superposition</u>

$$\mathbf{u}_s = \boldsymbol{\Phi}_d \mathbf{q}_d - \mathbf{K}_s^{-1} \mathbf{K}_c \mathbf{u}_b$$

$\mathbf{u}_b$ remains in physical coordinates.

Thus, we can get the following matrix using the dominant modes only:

$$\bar{\mathbf{T}}_0 = \begin{bmatrix} \boldsymbol{\Phi}_d & -\mathbf{K}_s^{-1} \mathbf{K}_c \\ \mathbf{0} & \mathbf{I}_b \end{bmatrix}$$

Craig-Bampton component mode synthesis

3. Reduced system

$$N_g \times N_g \qquad\qquad N_g \times \overline{N}_p$$

$$\mathbf{T}_0 = \begin{bmatrix} \mathbf{\Phi}_d & \mathbf{\Phi}_r & -\mathbf{K}_s^{-1}\mathbf{K}_c \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_b \end{bmatrix} \longrightarrow \bar{\mathbf{T}}_0 = \begin{bmatrix} \mathbf{\Phi}_d & -\mathbf{K}_s^{-1}\mathbf{K}_c \\ \mathbf{0} & \mathbf{I}_b \end{bmatrix}$$

$$\mathbf{u}_g = \begin{bmatrix} \mathbf{u}_s \\ \mathbf{u}_b \end{bmatrix} = \mathbf{T}_0 \begin{bmatrix} \mathbf{q}_s \\ \mathbf{u}_b \end{bmatrix} \longrightarrow \mathbf{u}_g \approx \bar{\mathbf{u}}_g = \bar{\mathbf{T}}_0 \begin{bmatrix} \mathbf{q}_d \\ \mathbf{u}_b \end{bmatrix}$$

$$\boxed{\begin{gathered} \bar{\mathbf{M}}_p \ddot{\bar{\mathbf{u}}}_p + \bar{\mathbf{K}}_p \bar{\mathbf{u}}_p = \bar{\mathbf{f}}_p, \\ \bar{\mathbf{M}}_p = \bar{\mathbf{T}}_0^T \mathbf{M}_g \bar{\mathbf{T}}_0, \quad \bar{\mathbf{K}}_p = \bar{\mathbf{T}}_0^T \mathbf{K}_g \bar{\mathbf{T}}_0, \\ \bar{\mathbf{u}}_p = \begin{bmatrix} \mathbf{q}_d \\ \mathbf{u}_b \end{bmatrix}, \quad \bar{\mathbf{f}}_p = \bar{\mathbf{T}}_0^T \begin{bmatrix} \mathbf{f}_s \\ \mathbf{f}_b \end{bmatrix}, \end{gathered}}$$

Enhanced Craig-Bampton method

✓ Original CB transformation matrix

$$\mathbf{u}_s = \boxed{\boldsymbol{\Phi}_s \mathbf{q}_s} \boxed{- \mathbf{K}_s^{-1} \mathbf{K}_c \mathbf{u}_b}$$
$$= \begin{bmatrix} \boldsymbol{\Phi}_d & \cancel{\boldsymbol{\Phi}_r} \end{bmatrix} \begin{bmatrix} \mathbf{q}_d \\ \cancel{\mathbf{q}_r} \end{bmatrix} - \mathbf{K}_s^{-1} \mathbf{K}_c \mathbf{u}_b ,$$

$$\bar{\mathbf{T}}_0 = \begin{bmatrix} \boldsymbol{\Phi}_d & -\mathbf{K}_s^{-1}\mathbf{K}_c \\ \mathbf{0} & \mathbf{I}_b \end{bmatrix}$$

: Residual normal modes were simply removed based on <u>Rule of thumb, which is frequency cut-off rule in mode superposition</u>

Let's consider residual modal effect for more precise reduced-order modeling without increasing the system size!

Enhanced Craig-Bampton method

1. Fully partitioned system

$$\mathbf{u}_g = \begin{bmatrix} \mathbf{u}_s \\ \mathbf{u}_b \end{bmatrix} = \mathbf{T}_0 \begin{bmatrix} \mathbf{q}_d \\ \mathbf{q}_r \\ \mathbf{u}_b \end{bmatrix}, \quad \mathbf{T}_0 = \begin{bmatrix} \mathbf{\Phi}_d & \mathbf{\Phi}_r & -\mathbf{K}_s^{-1}\mathbf{K}_c \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_b \end{bmatrix}.$$

$$\left[\frac{d^2}{dt^2}\mathbf{M}_p + \mathbf{K}_p\right]\mathbf{u}_p = \mathbf{f}_p,$$

$$\mathbf{M}_p = \mathbf{T}_0^T \mathbf{M}_g \mathbf{T}_0, \ \mathbf{K}_p = \mathbf{T}_0^T \mathbf{K}_g \mathbf{T}_0,$$

$$\Rightarrow \quad \frac{d^2}{dt^2}\mathbf{M}_p + \mathbf{K}_p = \begin{bmatrix} \hat{\mathbf{\Lambda}}_d & \mathbf{0} & \frac{d^2}{dt^2}\bar{\mathbf{M}}_c \\ \mathbf{0}^T & \hat{\mathbf{\Lambda}}_r & \frac{d^2}{dt^2}\mathbf{D} \\ \frac{d^2}{dt^2}\bar{\mathbf{M}}_c^T & \frac{d^2}{dt^2}\mathbf{D}^T & \hat{\mathbf{K}}_b + \frac{d^2}{dt^2}\hat{\mathbf{M}}_b \end{bmatrix}$$

$$\mathbf{u}_p = \begin{bmatrix} \mathbf{q}_d \\ \mathbf{q}_r \\ \mathbf{u}_b \end{bmatrix}, \ \mathbf{f}_p = \mathbf{T}_0^T \begin{bmatrix} \mathbf{f}_s \\ \mathbf{f}_b \end{bmatrix},$$

<u>This makes same solution with the original FE model.</u>

Enhanced Craig-Bampton method

1. Fully partitioned system

$$\mathbf{u}_g = \begin{bmatrix} \mathbf{u}_s \\ \mathbf{u}_b \end{bmatrix} = \mathbf{T}_0 \begin{bmatrix} \mathbf{q}_d \\ \mathbf{q}_r \\ \mathbf{u}_b \end{bmatrix}, \quad \mathbf{T}_0 = \begin{bmatrix} \mathbf{\Phi}_d & \mathbf{\Phi}_r & -\mathbf{K}_s^{-1}\mathbf{K}_c \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_b \end{bmatrix}.$$

$$\left[\frac{d^2}{dt^2}\mathbf{M}_p + \mathbf{K}_p\right]\mathbf{u}_p = \mathbf{f}_p,$$

$$\mathbf{M}_p = \mathbf{T}_0^T\mathbf{M}_g\mathbf{T}_0, \; \mathbf{K}_p = \mathbf{T}_0^T\mathbf{K}_g\mathbf{T}_0,$$

$$\Rightarrow \quad \frac{d^2}{dt^2}\mathbf{M}_p + \mathbf{K}_p = \begin{bmatrix} \hat{\mathbf{\Lambda}}_d & \mathbf{0} & \frac{d^2}{dt^2}\bar{\mathbf{M}}_c \\ \mathbf{0}^T & \hat{\mathbf{\Lambda}}_r & \frac{d^2}{dt^2}\mathbf{D} \\ \frac{d^2}{dt^2}\bar{\mathbf{M}}_c^T & \frac{d^2}{dt^2}\mathbf{D}^T & \hat{\mathbf{K}}_b + \frac{d^2}{dt^2}\hat{\mathbf{M}}_b \end{bmatrix} \Rightarrow \boxed{\mathbf{q}_r = -\hat{\mathbf{\Lambda}}_r^{-1}\left[\frac{d^2}{dt^2}\mathbf{D}\mathbf{u}_b\right]}$$

$$\mathbf{u}_p = \begin{bmatrix} \mathbf{q}_d \\ \mathbf{q}_r \\ \mathbf{u}_b \end{bmatrix}, \; \mathbf{f}_p = \mathbf{T}_0^T \begin{bmatrix} \mathbf{f}_s \\ \mathbf{f}_b \end{bmatrix},$$

<u>This makes same solution with the original FE model.</u>

Enhanced Craig-Bampton method

2. Reducing the system

- From the second row in the fully partitioned system:

$$\Rightarrow \quad \mathbf{q}_r = -\hat{\boldsymbol{\Lambda}}_r^{-1}\left[\frac{d^2}{dt^2}\mathbf{D}\mathbf{u}_b\right]$$

No approximation!

Then,

$$\mathbf{u}_s = \boxed{\boldsymbol{\Phi}_d\mathbf{q}_d - \mathbf{K}_s^{-1}\mathbf{K}_c\mathbf{u}_b} - \boxed{\frac{d^2}{dt^2}\hat{\mathbf{F}}_r\left[-\mathbf{M}_s\mathbf{K}_s^{-1}\mathbf{K}_c + \mathbf{M}_c\right]\mathbf{u}_b},$$

Original CB method (Static)

Additional terms (dynamic)

Residual flexibility: $\hat{\mathbf{F}}_r = \boldsymbol{\Phi}_r\left[\boldsymbol{\Lambda}_r - \lambda\mathbf{I}_r\right]^{-1}\boldsymbol{\Phi}_r^T$

$$\approx \boldsymbol{\Phi}_r\boldsymbol{\Lambda}_r^{-1}\boldsymbol{\Phi}_r^T + \lambda\boldsymbol{\Phi}_r\boldsymbol{\Lambda}_r^{-2}\boldsymbol{\Phi}_r^T + \ldots = \mathbf{F}_{rs} + \lambda\mathbf{F}_{rm} + \ldots$$

- Neglecting higher order terms:

$$\mathbf{u}_s \approx \bar{\mathbf{u}}_s = \boxed{\boldsymbol{\Phi}_d\mathbf{q}_d - \mathbf{K}_s^{-1}\mathbf{K}_c\mathbf{u}_b} + \boxed{\lambda\mathbf{F}_{rs}\left[-\mathbf{M}_s\mathbf{K}_s^{-1}\mathbf{K}_c + \mathbf{M}_c\right]\mathbf{u}_b},$$

$$\mathbf{F}_{rs} = \mathbf{K}_s^{-1} - \boldsymbol{\Phi}_d\boldsymbol{\Lambda}_d^{-1}\boldsymbol{\Phi}_d^T.$$

Enhanced Craig-Bampton method

3. Enhanced transformation matrix

$$\mathbf{T}_0 = \begin{bmatrix} \boldsymbol{\Phi}_d & \boldsymbol{\Phi}_r & -\mathbf{K}_s^{-1}\mathbf{K}_c \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_b \end{bmatrix} \longrightarrow \bar{\mathbf{T}}_1 = \bar{\mathbf{T}}_0 + \bar{\mathbf{T}}_r,$$

Substructural eigenvector matrix

Interface constraint mode

$$\bar{\mathbf{T}}_r = \begin{bmatrix} \mathbf{0} & \lambda\mathbf{F}_{rs}\left[-\mathbf{M}_s\mathbf{K}_s^{-1}\mathbf{K}_c + \mathbf{M}_c\right] \\ \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\mathbf{F}_{rs} = \mathbf{K}_s^{-1} - \boldsymbol{\Phi}_d\boldsymbol{\Lambda}_d^{-1}\boldsymbol{\Phi}_d^T$$

✓ No calculation of residual modes

$$\mathbf{u}_g \approx \bar{\mathbf{u}}_g = \bar{\mathbf{T}}_1\bar{\mathbf{u}}_p$$

✓ More precisely approximated global eigenvector

Since λ is unknown, it might be handle to employ it for the model reduction method.

Enhanced Craig-Bampton method

Let's see the original CB reduced matrices again

$$\mathbf{u}_g = \begin{bmatrix} \mathbf{u}_s \\ \mathbf{u}_b \end{bmatrix} = \mathbf{T}_0 \begin{bmatrix} \mathbf{q}_s \\ \mathbf{u}_b \end{bmatrix} \longrightarrow \mathbf{u}_g \approx \bar{\mathbf{u}}_g = \bar{\mathbf{T}}_0 \begin{bmatrix} \mathbf{q}_d \\ \mathbf{u}_b \end{bmatrix}$$

$$\boxed{\begin{gathered} \bar{\mathbf{M}}_p \ddot{\bar{\mathbf{u}}}_p + \bar{\mathbf{K}}_p \bar{\mathbf{u}}_p = \bar{\mathbf{f}}_p, \\ \bar{\mathbf{M}}_p = \bar{\mathbf{T}}_0^T \mathbf{M}_g \bar{\mathbf{T}}_0, \quad \bar{\mathbf{K}}_p = \bar{\mathbf{T}}_0^T \mathbf{K}_g \bar{\mathbf{T}}_0, \\ \bar{\mathbf{u}}_p = \begin{bmatrix} \mathbf{q}_d \\ \mathbf{u}_b \end{bmatrix}, \quad \bar{\mathbf{f}}_p = \bar{\mathbf{T}}_0^T \begin{bmatrix} \mathbf{f}_s \\ \mathbf{f}_b \end{bmatrix}, \end{gathered}}$$

$$\Rightarrow \quad \boxed{\lambda \bar{\mathbf{u}}_p = \bar{\mathbf{M}}_p^{-1} \bar{\mathbf{K}}_p \bar{\mathbf{u}}_p}$$

Enhanced Craig-Bampton method

4. Reduced system

- **Handling technique of λ**

$$\bar{\mathbf{T}}_r = \begin{bmatrix} \mathbf{0} & \lambda \mathbf{F}_{rs}\left[-\mathbf{M}_s\mathbf{K}_s^{-1}\mathbf{K}_c + \mathbf{M}_c\right] \\ \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\boxed{\lambda \bar{\mathbf{u}}_p = \bar{\mathbf{M}}_p^{-1}\bar{\mathbf{K}}_p\bar{\mathbf{u}}_p} \Rightarrow \bar{\mathbf{T}}_r = \begin{bmatrix} \mathbf{0} & \mathbf{F}_{rs}\left[-\mathbf{M}_s\mathbf{K}_s^{-1}\mathbf{K}_c + \mathbf{M}_c\right] \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \bar{\mathbf{M}}_p^{-1}\bar{\mathbf{K}}_p$$

<u>**redefined without unknown**</u>

O'Callahan J. A procedure for an improved reduced system (IRS) model. *Proceeding the 7th International Modal Analysis Conference*, CT, Bethel, 1989; 17–21.

- **Reduced model using** $\bar{\mathbf{T}}_1 = \bar{\mathbf{T}}_0 + \bar{\mathbf{T}}_r,$

$$\Rightarrow \tilde{\mathbf{M}}_p = \bar{\mathbf{T}}_1^T\mathbf{M}_g\bar{\mathbf{T}}_1 \quad \tilde{\mathbf{K}}_p = \bar{\mathbf{T}}_1^T\mathbf{K}_g\bar{\mathbf{T}}_1$$

Enhanced Craig-Bampton method

4. Reduced system

Original CB transformation matrix

$$\bar{\mathbf{T}}_0 = \begin{bmatrix} \boldsymbol{\Phi}_d & -\mathbf{K}_s^{-1}\mathbf{K}_c \\ \mathbf{0} & \mathbf{I}_b \end{bmatrix}$$

Enhanced transformation matrix

$$\bar{\mathbf{T}}_1 = \bar{\mathbf{T}}_0 + \bar{\mathbf{T}}_r,$$

$$\bar{\mathbf{T}}_r = \begin{bmatrix} \mathbf{0} & \mathbf{F}_{rs}\left[-\mathbf{M}_s\mathbf{K}_s^{-1}\mathbf{K}_c + \mathbf{M}_c\right] \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \bar{\mathbf{M}}_p^{-1}\bar{\mathbf{K}}_p$$

$$\mathbf{F}_{rs} = \mathbf{K}_s^{-1} - \boldsymbol{\Phi}_d\boldsymbol{\Lambda}_d^{-1}\boldsymbol{\Phi}_d^T$$

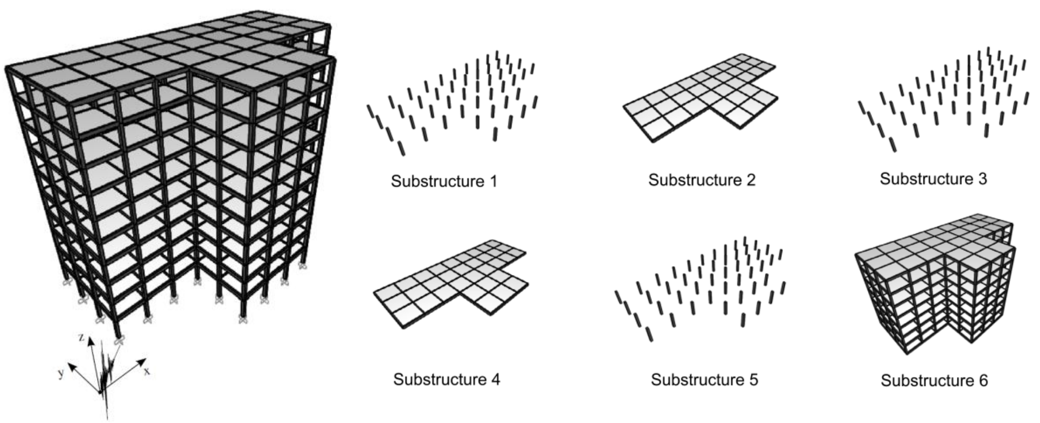
Original CB — Additional terms in E-CB

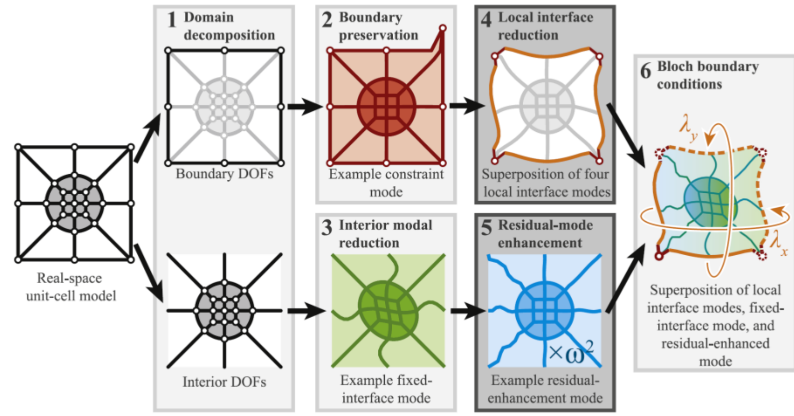
$$\tilde{\mathbf{M}}_p = \bar{\mathbf{T}}_1^T\mathbf{M}_g\bar{\mathbf{T}}_1 = \underbrace{\bar{\mathbf{T}}_0^T\mathbf{M}_g\bar{\mathbf{T}}_0}_{\text{Original CB}} + \underbrace{\bar{\mathbf{T}}_r^T\mathbf{M}_g\bar{\mathbf{T}}_0 + \bar{\mathbf{T}}_0^T\mathbf{M}_g\bar{\mathbf{T}}_r + \bar{\mathbf{T}}_r^T\mathbf{M}_g\bar{\mathbf{T}}_r}_{\text{Additional terms in E-CB}},$$
$$\tilde{\mathbf{K}}_p = \bar{\mathbf{T}}_1^T\mathbf{K}_g\bar{\mathbf{T}}_1 = \bar{\mathbf{T}}_0^T\mathbf{K}_g\bar{\mathbf{T}}_0 + \bar{\mathbf{T}}_r^T\mathbf{K}_g\bar{\mathbf{T}}_0 + \bar{\mathbf{T}}_0^T\mathbf{K}_g\bar{\mathbf{T}}_r + \bar{\mathbf{T}}_r^T\mathbf{K}_g\bar{\mathbf{T}}_r.$$

Better approximation! Same matrix size!

JG Kim, PS Lee. An enhanced Craig-Bampton method, International Journal for Numerical Methods in Engineering, 103, 79-93, 2015.

Resent extensions of ECB

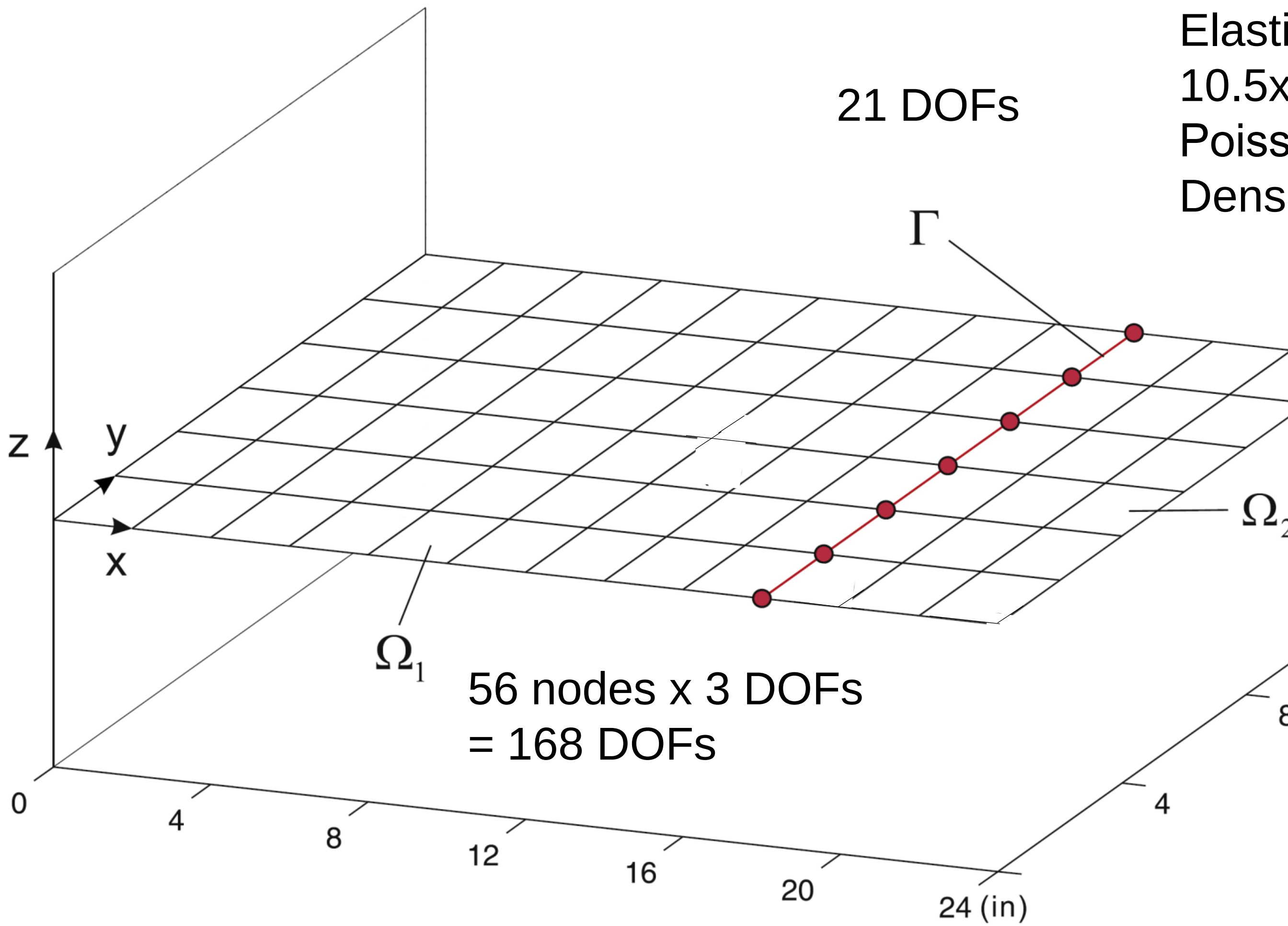




1. Jensen, H.A., Muñoz, A., Papadimitriou, C. and Vergara, C., 2016. An enhanced substructure coupling technique for dynamic re-analyses: application to simulation-based problems. *Computer Methods in Applied Mechanics and Engineering*, *307*, pp.215-234.
2. Jensen, H.A., Araya, V.A., Muñoz, A.D. and Valdebenito, M.A., 2017. A physical domain-based substructuring as a framework for dynamic modeling and reanalysis of systems. *Computer Methods in Applied Mechanics and Engineering*, *326*, pp.656-678.
3. Cui, J., Xing, J., Wang, X., Wang, Y., Zhu, S. and Zheng, G., 2017. A Simultaneous Iterative Scheme for the Craig-Bampton Reduction Based Substructuring. In *Dynamics of Coupled Structures, Volume 4* (pp. 103-114). Springer, Cham.
4. Krattiger, D. and Hussein, M.I., 2018. Generalized Bloch mode synthesis for accelerated calculation of elastic band structures. *Journal of Computational Physics*, *357*, pp.183-205.
5. Boo, S.H., Kim, J.H. and Lee, P.S., 2018. Towards improving the enhanced Craig-Bampton method. *Computers & Structures*, *196*, pp.63-75.
6. Kim, J., Boo, S.H. and Lee, P.S., 2017. Considering the Higher-Order Effect of Residual Modes in the Craig–Bampton Method. *AIAA Journal*, pp.1-10.

A MATLAB example code

A cantilever plate example



t = 0.125 in
Elastic modulus = 10.5×10^6 psi
Poisson's ratio = 0.33
Density = 0.101 lb/in^3

A MATLAB example code

A cantilever plate example

```matlab
% Original system ---------------------------------------------------------------
MM = load('cbexample_Mt.txt');
KK = load('cbexample_Kt.txt');
```

Two input txt: No boundary condition (BC)

```matlab
size_free = length(MM);
node_dof = 3;
K = KK(22:size_free,22:size_free);
M = MM(22:size_free,22:size_free);
```

BC as cantilever

```matlab
sdof = length(K);

% Point load
P1 = zeros(sdof - 7*node_dof);
P = [diag(P1);0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;0;-1;0;0];

% Substructuring
K22_1 = K([1:168],[1:168]);
K22_2 = K([190:252],[190:252]);
K21_1 = K([1:168],[169:189]);
K21_2 = K([190:252],[169:189]);
K22 = blkdiag(K22_1, K22_2);
K21 = [K21_1; K21_2];

M22_1 = M([1:168],[1:168]);
M22_2 = M([190:252],[190:252]);
M21_1 = M([1:168],[169:189]);
M21_2 = M([190:252],[169:189]);
M22 = blkdiag(M22_1, M22_2);
M21 = [M21_1; M21_2];

K11 = K([169:189],[169:189]);
M11 = M([169:189],[169:189]);
```

Partitioning (or substructuring)

```matlab
K12 = K21';
M12 = M21';
```

A MATLAB example code

A cantilever plate example

```
% CB system -----------------------------------------------------------------
nmode1 = 2;  % Number of retained modes in subs1
nmode2 = 1;  % Number of retained modes in subs2
nmode = nmode1 + nmode2;

[Ms1,Ks1,Cs1,Ps1,phis1]= ModalAnalysis(M22_1,K22_1,C22_1,P22_1,nmode1);
[Ms2,Ks2,Cs2,Ps2,phis2]= ModalAnalysis(M22_2,K22_2,C22_2,P22_2,nmode2);

phis = blkdiag(phis1, phis2);
Ks = blkdiag(Ks1, Ks2);

inv_K22 = inv(K22);
Psi = -inv_K22*K21;
T_cb = [phis Psi; zeros(size(K12,1), size(phis, 2)) eye(size(K11))];
M_cb = T_cb'*M*T_cb;
K_cb = T_cb'*K*T_cb;
C_cb = T_cb'*C*T_cb;
P_cb = T_cb'*P;

% Transforming the System Matrices to modal coordinates
lambda_r0 = length(K_cb);  % Number of modes to be considered
[M_r0,K_r0,C_r0,P_r0,phi_r0]= ModalAnalysis(M_cb,K_cb,C_cb,P_cb,lambda_r0);

% Newmark Method for time integration
[depl_r0,vel_r0,accl_r0,U_r0,t] = NewmarkMethod(M_r0,K_r0,C_r0,P_r0,phi_r0,lambda_r0,acceleration);
```

Numbers of substructural mode
(Freq. cut-off rule)

Substructural eigenvalue problem

Interface constraint modes

Transformation matrix

Projection

A MATLAB example code

A cantilever plate example

Residual flexibility matrix

Additional transformation matrix

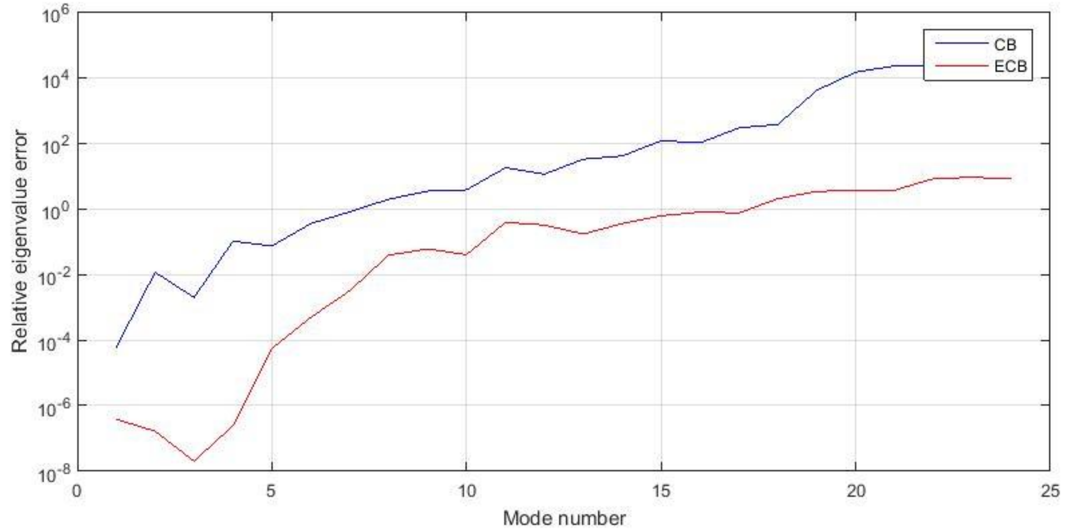
```
% Enhanced CB system -----------------------------------------------------------------------------------
F_rs = inv_K22 - phis*inv(Ks)*phis';
t_r = F_rs*(M22*Psi + M21);
T_r = [zeros(size(phis,1), size(phis,2)) t_r; zeros(size(K12,1), size(phis, 2)) zeros(size(K11,1), size(K11, 2))]*inv(M_cb)*K_cb;
T_ecb = T_cb + T_r;
M_ecb = T_ecb'*M*T_ecb;
K_ecb = T_ecb'*K*T_ecb;
C_ecb = T_ecb'*C*T_ecb;
P_ecb = T_ecb'*P;

% Transforming the System Matrices to modal coordinates
[M_r1,K_r1,C_r1,P_r1,phi_r1]= ModalAnalysis(M_ecb,K_ecb,C_ecb,P_ecb,lambda_r0);

% Newmark Method for time integration
[depl_r1,vel_r1,accl_r1,U_r1,t] = NewmarkMethod(M_r1,K_r1,C_r1,P_r1,phi_r1,lambda_r0,acceleration);
```

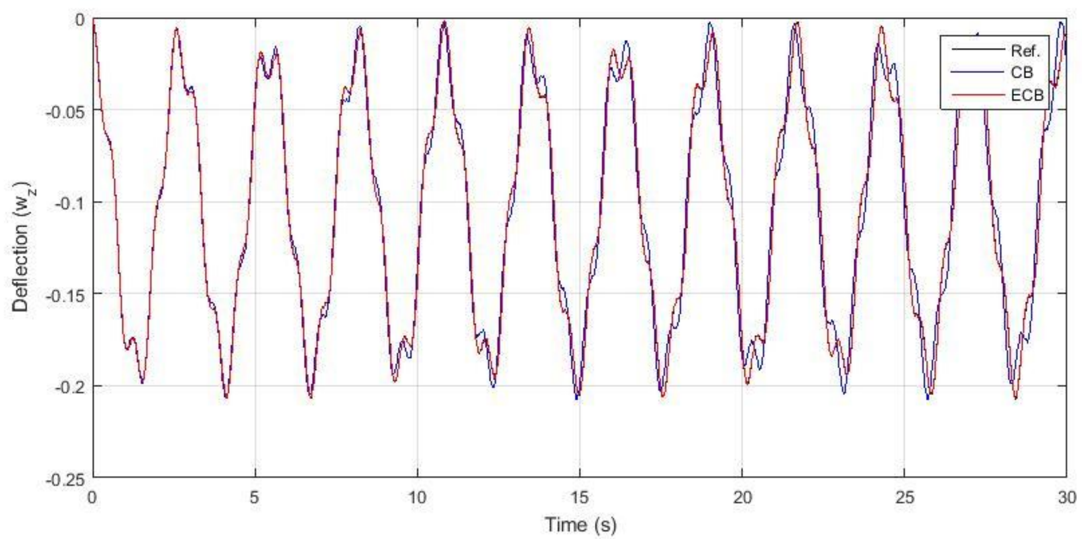
Projection

A MATLAB example code

A cantilever plate example

Subs1: 2 modes
Subs2: 1 modes





A MATLAB example code

A cantilever plate example

Subs1: 5 modes
Subs2: 3 modes