Search-and-Matching-Theoretic Models and the Explanation of the Main Empirical Facts of Housing Markets

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Due to relevant trading frictions, the search-and-matching models have become the new economic approach to the analysis of real estate markets. Really, *search theory* is an influential economic theory also in the housing market where, as in the labor market, the central problem facing the seeker (in both sides of the market) is how to determine the optimal strategy in choosing among a series of options whose arrival and offering price are both random.[1] Search and matching, therefore, are two fundamental characteristics of the trading process in the housing market and, thus, the effect of search efforts of seekers on selling price should not be overlooked.[2]

Trading frictions are noticeable in the housing market, as it takes weeks or months to buy or sell a house.[3] Search-and-matching frictions arise because finding a good trading partner takes time, in that housing markets are characterized by a strong heterogeneity of both goods and people. Hence, the market functioning can be described accurately by a search process in which the parties look for a trade (match) in several decentralized markets rather than in one centralized market. The search-and-matching models, therefore, provide a realistic description of the costly and time-consuming process through which buyers and sellers are brought together. Consequently, a great amount of search-and-matching models exist in the real estate literature. The first search model of the housing market is perhaps Yinger's.[4] Since then, several papers have developed models to explain the behavior of non-frictionless housing markets.[5]

Unlike a large amount of the related literature, this theoretical article closely follows the standard matching framework *à la* Pissarides without any significant deviation from the baseline model.[6] For example, Diaz and Jerez, Novy-Marx, Genesove and Han, Leung and Zhang, and Peterson define the market tightness from a buyer perspective, *i.e.*, housing market tightness is the ratio of buyers to sellers. Instead, we prefer to use the standard definition of tightness, thus considering the ratio of vacant houses to home seekers (the buyers).[7] Also, only some search and matching models of the housing market adopt an aggregate matching function and focus on the key role of market tightness in determining the selling price and the probability of matching between the parties.

Indeed, important differences between the labor and the housing markets seem to make a straightforward adaptation of the basic matching model to the study of the real estate market impossible. Exactly, the crucial free entry or zero-profit condition for job creation in the labor market seems to have no counterpart in the housing market. In fact, the assumption that vacancies in the housing market are created until the value of a vacant house is equal to zero may make sense if houses are supplied perfectly elastically by competitive house builders, in addition to being supplied by owners who no longer need them for occupation. Nevertheless, the zero-profit condition can be reformulated easily to take the distinctive feature of the housing market into account. Concisely, in the housing market the distinction between buyers and sellers is very fleeting and the zero-profit condition can be used to find the shares of buyers and sellers in equilibrium. Precisely, since the value of a vacant house is closely linked to the value of being a seller, the zero-profit condition allows us to obtain the equilibrium of the model where the transition process from seller (buyer) to buyer (seller) comes to an end. When the value of a vacant house is equal to zero, in fact, no one will be willing to become a seller and thus the matching no longer occurs. In equilibrium, in fact, all the profit opportunities derived from buying/selling houses have been exploited.

This article shows that a slightly modified version of the baseline model *à la* Pissarides is capable of explaining the main empirical facts of housing markets. There are two key mechanisms that can be used to achieve this goal: (1) a simple formalization of the (reasonable) assumption that buyers today are potential sellers tomorrow (and vice versa); and (2) the direct relationship between market tightness and house price, derived by the standard matching model and underestimated by the related literature. Precisely, sellers are assumed to hold a number of houses equal to or higher than two; thus, when a seller (with two houses) manages to sell one house, s/he becomes a buyer, and when a buyer buys a second home, s/he becomes a seller. Hence, the proposed work takes the key distinctive feature of the housing market into account, since buyers today are potential sellers tomorrow,[8] and most houses are bought by those who already own one, and most houses are sold by those wanting to buy another house.[9]

LITERATURE REVIEW

Search-and-Matching Models and Basic Facts of Housing Markets

In the housing markets, three basic facts have been repeatedly reported by empirical studies, viz.: (1) the trade-off between housing price and time-on-the-market;[10] (2) the positive correlation between housing price and trading volume;[11] and (3) the existence of price dispersion.[12] Recently, also, Ortego-Marti and Gabrovski[13] reported another important stylized fact of the housing market, namely buyers and vacant houses are positively correlated.

The time it takes to sell a property or "the expected time until the asset is sold when following the optimal policy"[14]—commonly known as the time-on-market (ToM)—measures the degree of illiquidity of the real estate asset and is a fundamental characteristic differentiating real estate from financial assets. Although this concept generally is regarded as valid, it is in the real estate field that the Lippman-McCall liquidity definition has been most widely accepted as operational measure of liquidity. In search theory, the ToM measures the search effort of participants in the housing market, and the latter affects selling price.[15] This implies a necessary trade-off for sellers: To either get a lower price for a relatively quick sale, or spend additional time searching for a higher price (of course, the additional time must be of a reasonable duration, otherwise the benefit of search does not justify the cost). It follows that even in a declining market, when prices are falling, there is a "benefit of search," namely the effect of the search (the impact of ToM) on house prices is still positive, albeit smaller.[16] In an efficient market, instead, the relation between house price and ToM is straightforward (and much less interesting). Indeed, in a good market when prices are rising, there is a positive price–ToM correlation. Likewise, in a bad market where prices are falling, there is a negative price–ToM correlation. An excellent summary of the "house price and ToM" relation in various contexts can be found in Sirmans *et al.*,[17] where conflicting findings emerge. Within the search framework, however, theoretical and empirical studies confirm a positive relation between price and ToM.[18]

Three recent works enrich the debate on the house price/ToM relation. Cheng, Lin, and Liu[19] find that the price/ToM relation can be positive, negative, or insignificant, depending on the market conditions under which the transactions occur. Basically, they find that the price/ToM relation is mostly positive except in the rapidly declining markets where the relation turns negative. Seiler *et al.*,[20] instead, reveal an asymmetric price/ToM relation, namely, an asymmetric effect of search effort on price. In a growing market, more search effort (longer ToM) is strongly correlated with higher selling prices. But more interestingly, even in a declining market, the effect of the search (or the impact of ToM) on price is still positive, albeit smaller, suggesting the benefit of search is significant in offsetting negative market impact. In an empirical analysis using a large sample of home sales during an extended period of time, Seiler *et al.*,[21] demonstrate that the true relationship between selling price and time-on-the-market should be nonlinear and characterized by an inverted *U*-shaped curve where the selling price increases with ToM up to a certain threshold (due to a positive "exposure effect") and decreases thereafter (due to a negative "stigma effect").

We show that the standard search-and-matching model adapted to the study of housing markets is able to mimic—in a straightforward manner—the trade-off between housing price and the speed of sale for the seller. In short, the house with a higher selling price has a longer time on the market but, the longer the ToM, the lower the sale price. Also, we show that the relation between selling and ToM is non-linear. This theoretical result is consistent with the recent empirical findings.[22]

Price dispersion and the positive price-trading volume relationship are two other well-documented empirical facts in the housing market. Price dispersion (or price volatility) refers to the phenomenon of selling two houses with very similar attributes and in near locations at the same time but at very different prices; whereas, what has been termed the positive price-volume relationship begins with the works by Stein[23] and Ortalo-Magne and Rady.[24] The influential work of Genesove and Mayer[25] uses loss aversion theory to explain the seller behavior. Seiler *et al*.[26] adopt a quasi-hyperbolic discount model where consumers have present-biased preferences.[27] They show that loss aversion is only potentially responsible for explaining seller behavior. Instead, several puzzling behaviors in the housing market, including the positive price-volume relationship and price dispersion effect, could be due to an inter-temporal tradeoff between current and future expected payoffs. Interestingly, they find that the horizon constraint for the seller plays an important role in driving pricing behavior. For example, they show that even with the same amount of net loss, two otherwise identical sellers, but with different time horizons may list at different prices. Recently, Seiler *et al*.[28] built a search model to examine a house seller's pricing decision under a prospect value function. Their model predicts a larger price dispersion in a cold market and reaffirms the positive price-trading volume relation observed in the housing markets.[29]

In this search-and-matching model, heterogeneity of economic agents is enough to consider house price dispersion; whereas, more interestingly, by introducing the seller's choice regarding the optimal number of vacant houses on the market, this simple theoretical model also is able to explain the positive correlation between housing price and the number of contracts traded during a given period (*i.e.*, the trading volume). Indeed, the trading volume of this model is merely represented by the matching function.

Finally, the positive correlation between buyers and vacant houses means that, in the housing market, the counterpart of the empirical relation between job vacancies and unemployment (the famous "Beveridge curve") is upward sloping. Indeed, according to Ortego-Marti and Gabrovski,[30] two shortcomings emerge in the related literature: (1) it has not noted this important stylized fact; (2) any search-theoretic model à la Pissarides *"inherently generates a downward sloping Beveridge Curve"*, thus generating, as in the labor market, a negative relation between vacancies and buyers. In truth, while vacant units in housing markets naturally correspond to job vacancies in the labor market, the concept of unemployment is difficult to translate in the housing market.[31] It follows that some changes are needed in order to use the standard search-and-matching model à la Pissarides also in the housing market. For example, Ioannides and Zabel introduce a novel concept of "unemployment" in the housing market, thus allowing for the definition of Beveridge curve in housing markets.[32]

Anyway, we show that a slightly modified version of the baseline model *à la* Pissarides is capable of generating an upward sloping Beveridge curve.

Price Equation and Hedonic Price Function

The economic theory of hedonic prices[33] is well known and not in question. However, it provides very little theoretical guidance on the appropriate functional relationship between prices and attributes in the hedonic price function.[34]

Several papers have examined the so-called "hedonic pricing equation."[35] These studies almost unanimously underline the intrinsic nonlinearity in the relationship between house prices and housing characteristics, though nothing is known *a priori* about a specific functional form.[36] Nevertheless, while the related literature suggests that the equilibrium price function is nonlinear, most empirical studies make use of linear models, thus relying on an influential simulation study by Cropper, Deck and McConnell.[37] This "puzzle" is indeed due to the absence of theoretical groundwork with regards to the more appropriate functional form to use in the hedonic price models.[38] According to Rosen,[39] there is no reason for the hedonic price function to be linear; in fact, the linearity of the hedonic price function is unlikely as long as the marginal cost of attributes increases for sellers and it is not possible to untie packages. Indeed, Ekeland, Heckman, and Nesheim[40] demonstrate that nonlinearity is a generic property of the hedonic price function. Hence, a linear model would be a special case for the hedonic price function.[41] However, the nonlinearity basically is a general concept and may imply the use of several kinds of empirical models.

In order to build a theoretical foundation for empirical models, we use the standard matching model to show that the hedonic pricing equation is non-linear. In particular, under the realistic assumption of decentralized housing markets with important search and matching frictions,[42] in this model, the equilibrium price function is non-linear with a closed-form solution. Exactly, this article provides empirical evidence for the non-linear effect of housing characteristics on selling price. The house price realistically depends not only on the housing characteristics but also the trading frictions, search costs and bargaining power of the parties. Unlike in an efficient (financial) market where buyers and sellers are mere price-takers, agents in the housing market are able to influence the selling prices through individual search efforts.[43] Indeed, several recent papers have just used search models to study the housing market.[44]

However, none of these existing works of research have considered how to take advantage of the search-and-matching approach to show the nonlinearity of the hedonic price equation and give a precise suggestion for its mathematical functional form. As a result, this article also aims at connecting two important economic theories: (1) the hedonic price theory and (2) the search-and-matching theory.

THE THEORETICAL MODEL

The developed theoretical model is relatively simple. Be that as it may, this search-and-matching model seems capable of explaining the main basic facts of the housing market.[45]

Basic Assumptions

We adopt a standard matching model with random search and prices determined by Nash bargaining. The random matching assumption is absolutely compatible with a market where the formal distinction between the demand and supply side is very subtle; whereas, bargaining is a natural outcome of decentralized markets for heterogeneous goods.[46]

The residential housing market is populated by buyers (b) and sellers (s) who hold a certain number of houses (h). To make our point as simply as possible, we assume an exogenous population in the housing market: $1 < \Phi = s + b$, that is, a person is either a seller or a buyer, but not both, at any point in time.[47]

In the housing market the distinction between buyers and sellers is very fleeting (as discussed in the beginning of this article, in fact, buyers today are potential sellers tomorrow and vice versa). In a nutshell, in this model, the difference between seller and buyer only concerns the number of houses owned. Precisely, sellers hold $h > 1$ houses of which $(h - 1)$ are on the market, namely, the first-home is occupied and all the unoccupied houses (vacant houses) are put on sale. Hence, vacancies (v) or vacant houses are simply given by:[48]

$$v = \sum_{h=2}^{N} (h-1) \cdot s_h \tag{1}$$

where s_h is the number of sellers with h houses. Instead, buyers expend costly search efforts to find a new or better house, in fact, they already hold a house, that is, $h = 1$.[49] In the model, it is therefore possible that a buyer can become a seller and vice versa. Indeed, a seller (with two houses) becomes a buyer after selling one house, while a buyer becomes a seller after buying another house.[50] However, the assumption that all buyers already own a house can be easily replaced. For example, we can assume the existence of first-time buyers (b_0) namely economic agents with $h = 0$. At the time of purchase, therefore, they become buyers with $h = 1$. Nevertheless, the main hypothesis of the model does not change, since one match is still enough to have a "shift" in the position of economic agents, namely, a trade that involves a seller with two houses (that becomes a buyer) and/or a buyer with $h = 1$ that becomes a seller. Note that $s = \sum_{h=2}^{N} s_h$ and $b = (b_0 + s_1)$, where s_1 are buyers with $h = 1$ (that become sellers after a trade).

Furthermore, under the assumption that the houses out-of-the market are owner occupied housing, we can obtain the rate of "fulfilled homeownership" (fh), viz.:

$$fh = \left(\frac{H - v}{H}\right) = \left[\frac{\sum_{h=2}^{N} h \cdot s_h + s_1 - \sum_{h=2}^{N} (h-1) \cdot s_h}{\sum_{h=2}^{N} h \cdot s_h + s_1}\right]$$

where $H = \sum_{h=2}^{N} h \cdot s_h + s_1$ is the total stock of housing (of course, it is the number of buyers that already holds a house that appears in the stock of housing). Hence,

$$uh = 1 - fh$$

can be seen as a measure of "unfulfilled homeownership" (uh) similar to that of Ioannides and Zabel.[51]

As usual in matching-type models, an aggregate matching function is used to summarize the search and matching frictions in the market. Formally, the matching function:

$$m = m(v, b)$$

gives the number of matches formed per unit of time, given the number of vacant houses (v) and the share of home seekers (b) in the housing market. Basically, it gives the number of contracts traded during a given period, that is to say the *trading volume*.

Present Value Equations

The housing market of reference is the homeownership market. In this way, if a contract is legally binding (as hypothesized) it is no longer possible to return to the circumstances preceding the bill of sale, unless a new and distinct contractual relationship is set up. In matching model jargon this means that the destruction rate of a specific buyer-seller match does not exist. As a result, the value of a seller is simply given by the selling price (P). Therefore, the expected values of a vacant house (V) and of a buyer (H) are the following (time is continuous, individuals are risk neutral, live infinitely and discount future payoffs at the exogenous interest rate $r > 0$):

$$r \cdot V = -c + q(\theta) \cdot [P - V + \Gamma] \tag{2}$$

$$r \cdot H = -e + g(\theta) \cdot [x + (V - H) - P] \tag{3}$$

that in the case of first-time buyers becomes: [52]

$$r \cdot H = -e + g(\theta) \cdot [x - H - P] \tag{3'}$$

since a buyer becomes a seller with a vacant house V on the market only when $h = 1$. The term c represents the cost flows sustained by sellers for the advertisement of vacancies; whereas, e represents the effort flows in monetary terms made by buyers to find and visit the largest possible number of houses; x is the buyer's benefit associated with the house.[53] Finally, $\Gamma = 0$ when a seller has more than one house on the market ($h > 2$) and $\Gamma = H$ when a seller has only one house on the market ($h = 2$), since a seller becomes a buyer only when $h = 2$. If the search is successful and a matching takes place, therefore, a trade is realized. Precisely, this occurs at the rates $q(\theta)$ and $g(\theta)$ that are, respectively, the (instantaneous) probability of filling a vacant house and of finding a home (where $\theta \equiv v/b$ is the ratio between vacant houses and buyers). The standard hypothesis of constant returns to scale in the matching function $m = m(v, b)$ is adopted,[54] since it also is used in recent search models of the housing market.[55] Hence, the properties of $q(\theta)$ and $g(\theta)$ are the following:

$$q(\theta) = \frac{m(v, b)}{v} = m\left(1, \theta^{-1}\right); \quad g(\theta) = \frac{m(v, b)}{b} = m(\theta, 1)$$

with $\frac{dg(\theta)}{d\theta} > 0,\ \frac{dq(\theta)}{d\theta} < 0,\ \frac{d^2 g(\theta)}{d\theta^2} < 0,\ \frac{d^2 q(\theta)}{d\theta^2} > 0$ and,[56]

$$\lim_{\theta \to 0} q(\theta) = \lim_{\theta \to \infty} g(\theta) = \infty; \qquad \lim_{\theta \to \infty} q(\theta) = \lim_{\theta \to 0} g(\theta) = 0.$$

Therefore, θ is the housing market tightness from the sellers' standpoint, in the sense that an increase in vacancies, and thus in θ, increases the supply of vacant houses, thus making it more difficult to sell a vacant house (the so-called congestion externalities).[57]

Equilibrium Conditions

In the housing market with search and matching frictions, the endogenous variables that are determined simultaneously at equilibrium are market tightness (θ) and sale price (P). The customary long-term equilibrium condition, namely the "zero-profit" or "free-entry" condition, normally used in matching models[58] can be reformulated to consider the possibility that a buyer today becomes a seller tomorrow (and vice versa). Of course, the condition $V = 0$ does not mean that the value of a vacant house is zero. It means that, at the margin and in equilibrium, all the profit's opportunities should be zero. Eventually, every economic agent stops trying to sell/buy further as s/he is either a seller or a buyer, but not both, at any point in time. It follows that the transition process from seller (buyer) to buyer (seller) must come to an end in a steady state equilibrium and this occurs when the value of a vacant house is equal to zero. In this case, in fact, no one will be willing to become a seller and thus the matching no longer occurs. Thus, the zero-profit condition does not change from a mathematical point of view (it retains its crucial role), but it changes its economic meaning.

In the equilibrium, however, we need to distinguish two cases: $\Gamma = 0$ when $h > 2$, and $\Gamma = H$ when $h = 2$.

Case $\Gamma = 0$ *when* $h > 2$

We start with the case where a seller does not become a buyer after a successful match. By using the condition $V = 0$ in (2), we obtain:

$$\frac{c}{q(\theta)} = P \tag{4}$$

Equation (4) describes the (positive), thus effect of house price on vacancies and, on market tightness, that is, $\frac{\partial\theta}{\partial P} > 0$, since the inverse function of the probability of filling a vacancy $\frac{1}{q(\theta)}$ is increasing in θ. This positive relationship is very intuitive: if the price increases, more vacant houses will be on the market.

Instead, the generalized Nash bargaining solution, usually used for decentralized markets, allows the sale price P to be obtained through the optimal subdivision of surplus deriving from a successful match. The surplus is defined as the sum of the seller's and buyer's value when the trade takes place, net of the respective external options, that is, the value of continuing to search, namely:

$$Surplus = P + (x - H - P) = x - H$$

since $V = 0$ in equilibrium. The selling price is then obtained by solving the following optimization condition:

$$\underbrace{max}_{P}\left\{P^{\gamma} \cdot (x - H - P)^{(1-\gamma)}\right\}$$

$$\overset{yields}{\rightarrow} P = \gamma \cdot (x - H)$$

the selling price is, thus, a share of the total surplus, where $0 < \gamma < 1$ is the share of bargaining power of a seller. Entering into a contractual agreement obviously implies that the surplus is always positive, that is, $x > H$, for all θ. This realistic condition on the buyers' side also ensures that the price is positive. The optimization condition and equation (3) yield the equation for the selling price:

$$P = \gamma \cdot x - \gamma \cdot \left[\frac{-e + g(\theta) \cdot (x - P)}{r + g(\theta)}\right]$$

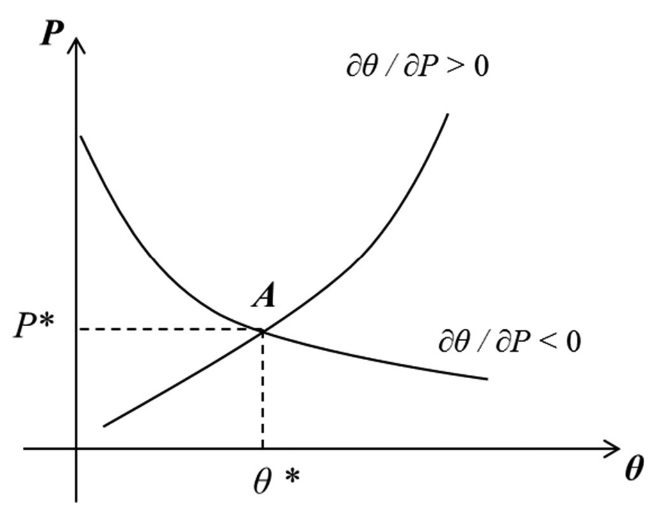
$$P = \frac{\gamma \cdot (r \cdot x + e)}{r + g(\theta) \cdot (1 - \gamma)} \tag{5}$$

Note that when γ tends to zero, P also tends to zero; whereas, when γ tends to 1, P tends to its maximum value, $P = x + \frac{e}{r} > x$, i.e. the price is higher than the buyer's benefit. Equation (5) describes the (negative) effect of market tightness on house price, i.e. $\frac{\partial P}{\partial\theta} < 0$, since $\frac{\partial g(\theta)}{\partial\theta} > 0$ and $\frac{\partial P}{\partial g(\theta)} < 0$. In words, an increase in vacant houses (and thus in market tightness) increases the outside options for buyers (*i.e.*, it will be easier to find a home) and the effect of the well-known congestion externalities on the seller's side[59] will lower the price.

Hence, it is straightforward to obtain from equation (4) that when θ tends to zero (infinity), P tends to zero (infinity), since $q(\theta)$ tends to infinity (zero). Consequently, given the negative slope of equation (5), with positive intercept, that is, $\lim_{\theta\to 0} P = \gamma \cdot \left(x + \frac{e}{r}\right)$, only one equilibrium with positive price exists in the model (*see* point A in Exhibit 1).

The case $\Gamma = 0$ when $h > 2$, therefore, allows to derive a direct relationship between market tightness and price similar to that obtained in a labor market matching model.

EXHIBIT 1—EQUILIBRIUM PRICE AND MARKET TIGHTNESS ($h > 2$)



Optimal Sales Strategy

In addition, in this model it is possible to get the optimal sales strategy. This model borrows the main insight from the seller's optimization problem by Krainer.[60] Our modeling is much simpler because we assume that each seller expects to sell all $(h-1)$ vacant houses on the market. Indeed, this justifies why sellers keep $(h-1)$ vacant houses on the market. Exactly, the optimal number of houses per capita h is obtained by the maximization of the expected overall profit, namely the profit arising from the sale of all vacant houses on the market. Since the value of an occupied home for a seller is simply given by the selling price, the expected overall profit to maximize for each seller is $(h-1)\cdot P(h)$, where $(h-1)$ is the number of houses on the market and P is a function of h, since $\frac{\partial v}{\partial h}>0$, $\frac{\partial \theta}{\partial v}>0$ and $\frac{\partial P}{\partial \theta}<0$.[61] It follows that:

$$
\begin{aligned}
(h-1) &= -\left[\frac{P}{\left(\frac{\partial P}{\partial \theta}\right)}\right] > 0 \\
h &= -\left[\frac{P}{\left(\frac{\partial P}{\partial \theta}\right)}\right] + 1 > 1
\end{aligned}
\tag{6}
$$

Hence, the number of houses on the market is always positive and the number of houses held by sellers, h, is always higher than 1. Note that with $x = x_j$, where j denotes the (preferences of) homeowners that make different the buyer's benefit and thus the selling price, the optimal number of houses on the market is heterogenous among sellers.

Eventually, given the equilibrium value of market tightness and price, we find the optimal number of houses per capita, h^*, and by using the definition of market tightness, we obtain the stock of sellers and buyers in equilibrium. In fact,

$$
\begin{aligned}
\theta^* &\equiv \frac{v}{b} = \frac{\mathrm{v(s)}}{(\Phi - s)} \\
\theta^* \cdot (\Phi - s) &= \mathrm{v(s)} \xrightarrow{yields} s^*
\end{aligned}
\tag{7}
$$

A sufficient condition for the existence of an interior equilibrium is $\Phi \ \cdot \theta > 0$. Of course, $\mathrm{v}(0) = 0$ and $\mathrm{dv/ds} > 0$. Once obtained the total share of sellers, it is straightforward to find the total share of buyers $(b = \Phi - s)$. As a result, the free-entry condition amplifies its crucial role in this housing market matching model, since it also is used to find the share of buyers and sellers in the steady state equilibrium without resorting to the dynamic equations.

Eventually, an exogenous stock of housing H makes it possible to pin down both the number of first-time buyers (b_0) and the number of buyers that already hold a house (s_1), viz.:

$$
b_0 = \sum_{h=2}^{N} h \cdot s_h + b - H
$$

since $s_1 = b - b_0$.

Equilibrium Conditions: Case $\Gamma = H$ *when* $h > 2$

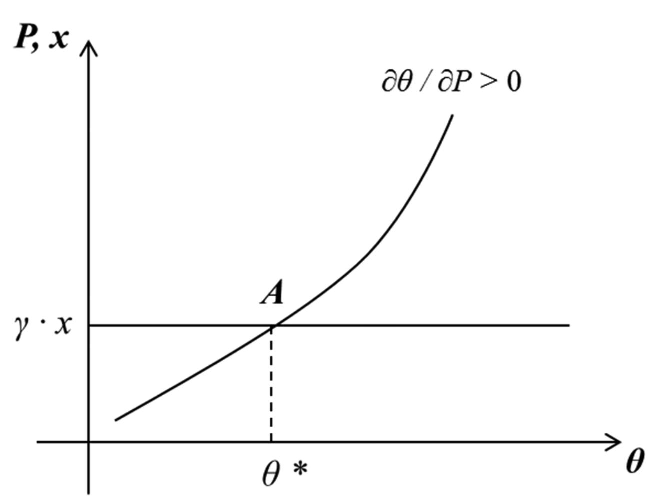
In this case a seller becomes a buyer after a successful match and, thus, Equation (4) becomes: $\frac{c}{q(\theta)} - \Gamma = P$ (4') with $\Gamma = H = \frac{-e+g(\theta)\cdot(x-P)}{r+g(\theta)}$. Essentially, the relation between market tightness and price does not change, that is, it remains positive (*see* Exhibit 2), since P can be higher, lower or equal than x.[62]

More importantly, the surplus deriving from a successful match is merely the buyer's benefit:

$$
\begin{aligned}
Surplus = (P - V + H) + (x + (V - H) - P) = x \\
\xrightarrow{yields} P = \gamma \cdot x
\end{aligned}
\tag{5'}
$$

Equation (5') is very simple and the house price could represent a "take-it-or leave-it" offer that only depends on the buyer's benefit and the bargaining power of seller. Indeed, in this specific case, $P < x$, since $0 < \gamma < 1$ and thus $\frac{\partial \Gamma}{\partial \theta} < 0$. Note that $\Gamma = \frac{-e}{r}$ when market tightness tends to

Exhibit 2—Equilibrium Price and Market Tightness ($h = 2$)



zero; whereas, by using the *Hôpital rule*, $\Gamma = (x - P)$ when market tightness tends to infinity.

THE EMPIRICAL FACTS OF THE HOUSING MARKET

This section shows that the simple theoretical model developed in the previous section is capable of explaining, in a straightforward way, several important empirical facts of the housing market. To do this, we take as a reference the case of the model where the house price is a function of market tightness, namely equation (5).

The Joint Behavior of House Prices and ToM

In the search and matching models, the mathematical definition of ToM is very simple to find out. Really, the expected ToM in the housing market matching model is merely the inverse function of the probability of filling a vacancy $q(\theta)$, viz.:

$$ToM = q(\theta)^{-1}$$

$$\text{with } \frac{dTOM}{d\theta} > 0, \text{ since} \frac{dq(\theta)}{d\theta} < 0.$$

Hence, equation (4) can be rewritten as follows:

$$ToM = c \cdot P$$

A similar reasoning can be made for the price equation (5), viz.:

$$P = f(ToM) = \frac{\gamma \cdot (r \cdot x + e)}{r + \left(\frac{\theta}{ToM}\right) \cdot (1 - \gamma)} \tag{8}$$

since $g(\theta) = \theta \cdot q(\theta)$ by using the properties of the matching function (precisely, the hypothesis of constant return to scale in the matching function), and thus $g(\theta) = \theta \cdot (ToM)^{-1}$. As a result, equation (8) expresses a positive and direct relation between time on-the-market (ToM) and house price (P). Also, the relation is non-linear and, thus, this theoretical result is consistent with recent empirical findings.[63]

Furthermore, the model outlines another and very different relation between time-on-the-market (ToM) and house price (P). Note that ToM depends on market tightness in a non-linear fashion: $\frac{dToM}{d\theta} > 0$ and $\frac{d^2TOM}{d\theta^2} < 0$, since $\frac{dq(\theta)}{d\theta} < 0$ and $\frac{d^2q(\theta)}{d\theta^2} > 0$. Instead, as shown by equation (5) the selling price is decreasing (in a non-linear fashion) in market tightness since $\frac{dg(\theta)}{d\theta} < 0$ and $\frac{d^2q(\theta)}{d\theta^2} > 0$. As a result, an increase in the key variable of the model, namely market tightness, increases ToM but decreases P; Formally, we can express it in the following way:

$$\begin{cases} P = P\left(\underbrace{\theta}_{-}\right) \\ ToM = ToM\left(\underbrace{\theta}_{+}\right) \end{cases} \tag{9}$$

thus, the system of equation (9) expresses a negative and indirect relation between time-on-the-market (ToM) and house price (P).

Eventually, by combining equations (8) and (9), the model is able to reproduce the observed trade-off between the housing price and the speed of sale for the seller.[64] From equation (8), in fact, a higher price requires a longer time to sell a house;[65] instead, the system of equation (9) shows that the longer the ToM, the lower the sale price.[66] Consequently, the standard matching model extended to the housing market is able to reproduce an important stylized fact of the housing market: the trade-off between house prices and the ToM.

The application of search models largely has been limited to quantifying the impact of ToM on property selling price within a static market condition, where the bidding price distribution is static, namely the buyers' valuation of properties does not adjust as market condition changes.[67] In this model the discount rate can take into account a change in market conditions. A change in both the risk free rate and the risk premium (the two major components of the discount rate), in fact, modifies the price equation, viz.:

$$\begin{aligned} \frac{\partial P}{\partial r} &= \frac{\gamma \cdot x \cdot [r + g(\theta) \cdot (1 - \gamma)] - \gamma \cdot (r \cdot x + e)}{[r + g(\theta) \cdot (1 - \gamma)]^2} \\ &= \frac{\gamma \cdot x \cdot g(\theta) \cdot (1 - \gamma) - \gamma \cdot e}{[r + g(\theta) \cdot (1 - \gamma)]^2} \end{aligned}$$

with $\lim_{\theta \to 0} \frac{\partial P}{\partial r} \to \frac{-\gamma \cdot e}{r^2} < 0$, since $g(\theta) \to 0$; instead, $\lim_{\theta \to \infty} \frac{\partial P}{\partial r} \to \frac{\gamma \cdot x}{2 \cdot [r + g(\theta) \cdot (1-\gamma)]} \to 0$, by the *Hôpital rule*, since $g(\theta) \to \infty$. Not surprisingly, the effect of the discount rate on the selling price is negative (or at the limit null). The direct capitalization method defines an inverse relation between price and capitalization rate, and the latter depends positively on the discount rate.

The Positive Correlation between Selling Prices and the Number of Houses for Sale

Note that equation (6) defines a positive relationship between selling price and the number of houses per capita h, viz.:

$$\frac{\partial h}{\partial P} = \frac{(-1) \cdot \left(\frac{\partial P}{\partial \theta}\right) - (-P) \cdot \left(\frac{\partial^2 P}{\partial \theta^2}\right)}{[\partial P / \partial \theta]^2} > 0$$

since $\frac{\partial^2 P}{\partial \theta^2} > 0$.[68]

Indeed, it shows that an increase in selling price increases the optimal number of houses per capita h, which, of course, also increases the number of vacant houses on the market $(h - 1)$:

$$\frac{\partial (h-1)}{\partial h} > 0$$

In turn, this increases vacancies (v) and thus the matching rate (the trading volume for a given period):

$$\frac{\partial v}{\partial (h-1)} > 0 \xrightarrow{yields} \frac{\partial m(v, b)}{\partial v} > 0$$

As a result, the model also can explain the positive relationship between housing price and trading volume, since equation (6) defines a positive correlation between house prices and trading volume. This theoretical result is in line with the empirical works of Fisher *et al.*, Leung, Lau and Leong, Seiler *et al.*[69] Therefore, it can be stated that a matching model of the housing market with the optimal choice of houses on the market is able to explain another important stylized fact of the housing market, namely, the positive correlation between house prices and trading volume.

Price Dispersion

Price dispersion is not a key focus of this analysis, since there are matching models whose equilibrium does not result in price dispersion (especially if there is enough symmetry among buyers or among sellers). Nevertheless, we show that this slightly modified version of the baseline model *à la* Pissarides is able to take into account price dispersion in housing markets in a straightforward way. By assuming different bargaining strengths (γ) and search costs (c and e), in fact, housing prices would be different even for similar houses, that is, houses that have identical or similar housing characteristics and thus give the same buyers' benefit.

However, the conclusion that price dispersion is due to ex-ante heterogeneity and search costs seems obvious; also, this empirical fact already has been explained by assuming the heterogeneity on the seller's side.[70]

Furthermore, if a model is concerned about price dispersion then the relevant variable might not be the selling price but rather the conditional mean $E(P, x)$ or even the variance between selling price and housing characteristics (buyers' benefit), that is, $var(P, x)$. In short, price dispersion is primarily an empirical question.

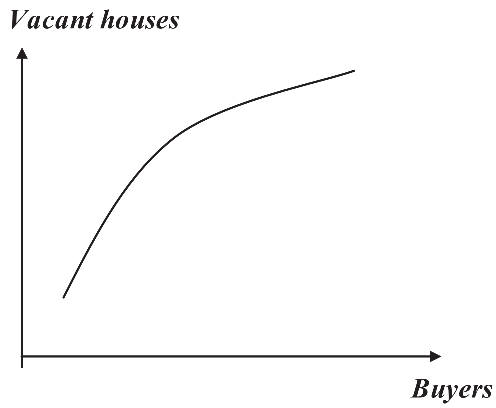
Beveridge Curve in the Housing Market

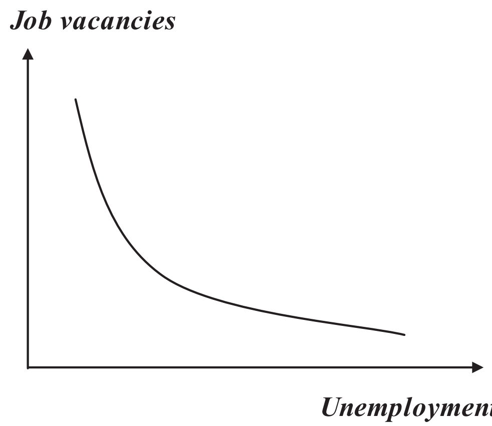
According to Ortego-Marti and Gabrovski,[71] the related literature has not noted another important stylized fact: In the housing market, the counterpart of the empirical relation between job vacancies and unemployment, the so-called Beveridge curve, is upward sloping (*see* Exhibit 3), namely buyers and vacant houses are positively correlated; whereas, any search model à la Pissarides *"inherently generates a downward sloping Beveridge Curve"*, specifically, therefore, a negative relation between vacancies and buyers.

In the search model developed by Ortego-Marti and Gabrovski, when sellers post more vacancies in the market, they make it easier for buyers to find a home. This raises the returns to search and incentivizes buyers to enter the market, which increases the number of buyers. This entry mechanism leads to an upward sloping Beveridge Curve between vacancies and buyers.

However, Ortego-Marti and Gabrovski overlook an important feature of the housing market: Buyers today are (potential) sellers tomorrow and vice versa.[72] Indeed, this is a realistic assumption that makes the standard model capable of generating an upward sloping Beveridge Curve. In effect,

EXHIBIT 3—THE BEVERIDGE CURVE IN THE HOUSING AND LABOR MARKETS





if the number of buyers with $h = 1$ increases, *ceteris paribus*, more people become sellers (*see* Exhibit 4).

In the model, in fact, the expected value of seller' search also depends on the option to become a buyer in the future, viz.:

$$V = \frac{-c + q(\theta) \cdot [P + H]}{r + q(\theta)}$$

with $\frac{\partial V}{\partial H} > 0$ when a seller has only one house on the market ($h = 2$), since only in this case s/he becomes a buyer after a trade. Hence, the value of a vacant house (when $h = 2$), positively depends on the value of buyer. It follows that a positive relation among sellers, vacancies, and buyers emerge from the model. As a result, this search-and-matching model also is able to generate an upward sloping Beveridge Curve in the housing market.

THE HEDONIC PRICE FUNCTION

In order to derive the hedonic price function of a real estate good, we take as a reference the more interesting price equation, namely equation (5). First of all, recall that the better the mix the housing characteristics, the larger is the buyer's benefit x. Thus, the hedonic price function of the model is merely the relation between selling price and buyer's benefit:

$$P = P(x)$$

with, of course, $\frac{\partial P}{\partial x} > 0$.

By using the popular Cobb-Douglas functional form,[73] that is,

$$m = v^{1-a} \cdot b^{a}$$

where a is the elasticity of the matching function with respect to the share of buyers (b), the following system is obtained for selling price:

$$\begin{cases} r \cdot P + \theta^{1-a} \cdot (1 - \gamma) \cdot P = \gamma \cdot (r \cdot x + e) \\ \theta = \left(\frac{P}{c}\right)^{\frac{1}{a}} \end{cases}$$

since $\theta \equiv \frac{v}{b}$, $g(\theta) = \frac{v^{1-a} \cdot b^{a}}{b} = \theta^{1-a}$, $q(\theta) = \frac{v^{1-a} \cdot b^{a}}{v} = \theta^{-a}$.

The above system gives the following implicit function for selling price:

$$\begin{aligned} r \cdot P + \left(\frac{P}{c}\right)^{\frac{1-a}{a}} \cdot (1 - \gamma) \cdot P &= \gamma \cdot (r \cdot x + e) \\ r \cdot P + \left(\frac{1}{c}\right)^{\frac{1-a}{a}} \cdot (1 - \gamma) \cdot P^{\frac{1}{a}} &= \gamma \cdot (r \cdot x + e) \end{aligned} \tag{10}$$

Total differentiation of Equation (10) with respect to P and x thus yields:

$$r \cdot dP + \left[\frac{1}{a} \cdot \left(\frac{1}{c}\right)^{\frac{1-a}{a}} \cdot (1 - \gamma) \cdot P^{\frac{1}{a}-1}\right] \cdot dP = (\gamma \cdot r) \cdot dx$$

EXHIBIT 4—BUYERS AND SELLERS IN THE HOUSING MARKET

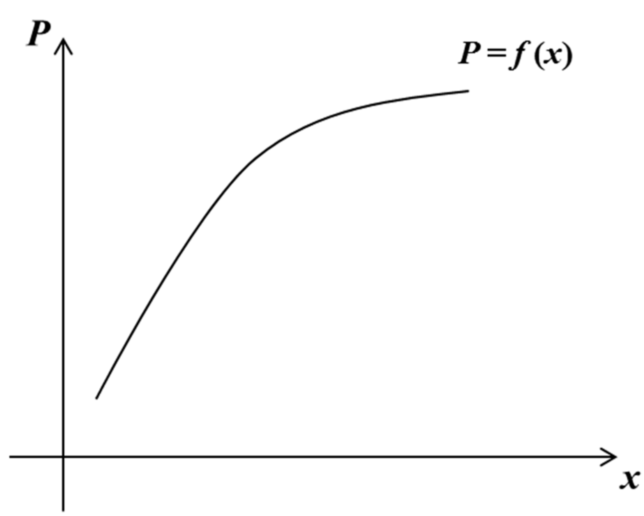
If…	economic agent is a…	that after a trade…
$h = 0$	(first-time) Buyer	remains a buyer
$h = 1$	Buyer	becomes a seller
$h = 2$	Seller	becomes a buyer
$h > 2$	Seller	remains a seller

$$\frac{dP}{dx} = \frac{(\gamma \cdot r)}{r + \left[\frac{1}{a} \cdot \left(\frac{1}{c}\right)^{\frac{1-a}{a}} \cdot (1-\gamma) \cdot P^{\frac{1-a}{a}}\right]} > 0$$

$$\frac{d^2P}{dx^2} < 0$$

since the selling price is increasing in x and thus a further increase in x increases the denominator of $\frac{dP}{dx}$. As a result, the hedonic price function is non-linear even if the buyer is risk neutral and acquires a linear benefit from the property. This is in line with the hedonic price literature that suggests that the equilibrium price function should be nonlinear. Furthermore, this theoretical model gives a precise suggestion about the form of the hedonic price function. Precisely, equation (8) suggests an (intuitive) increasing relationship at decreasing rates between selling price and housing characteristics (*see* Exhibit 5).

EXHIBIT 5—HEDONIC PRICE FUNCTION IN A SEARCH-AND-MATCHING MODEL OF THE HOUSING MARKET



This empirical finding is consistent with a concave hedonic price function. Indeed, from an empirical point of view, an increasing relation at decreasing rates between house prices and housing characteristics could be one of the most appropriate functional forms for the hedonic price function.[74]

Finally, note that equation (5'), instead, conveys a theoretical justification for using a linear function. Indeed, most empirical studies make use of linear models, since they are characterized as being easily interpretable, and the estimated parameters possess a direct economic meaningfulness

CONCLUSIONS

Housing markets are characterized by a decentralized framework of exchange with important search and matching frictions. It has, in fact, been acknowledged that housing markets clear not only through price but also through the time and money that a buyer and a seller spend on the market.

Furthermore, in the housing markets, three basic facts have been reported repeatedly by empirical studies: (1) the empirical anomaly known as "price dispersion" which implies that the variance in house prices cannot be completely attributed to the heterogeneous nature of real estate; (2) the positive relationship between housing price and the number of contracts traded during a given period (the trading volume); and (3) the trade-off between housing price and the speed of sale for the seller.

This theoretical article shows that the behavior of housing markets, reflected in the above empirical findings, can be explained by the standard matching framework without any significant deviation from the baseline model. The key mechanism to explain the *basic facts of the housing market* is the direct relationship between market tightness and house price, derived by the standard theoretical model and underestimated by the related literature. Also, the *matching function* gives the number of contracts traded during a given period, namely it represents the trading volume for a given period.

Also, this article shows that the *zero-profit condition* can be easily reformulated to take the distinctive features of the housing market into account. Although the model used is rather general and simple, the zero-profit condition has a clear economic meaning since it is used to ensure that the transition process from seller (buyer) to buyer (seller) comes to an end. Also, the free-entry or zero-profit condition allows to derive a direct relationship between house price and market tightness. Policy implications and the comparative statistics thus become straightforward since a change in the selling price (market tightness) immediately affects market tightness (selling price).

Eventually, the proposed model can be used to provide theoretical guidance on the appropriate functional relationship between prices and attributes in the *hedonic price function*. In line with the hedonic price literature, this model suggests that the equilibrium price function is nonlinear. Moreover, it gives a precise suggestion about the form of the hedonic price function, since it suggests an increasing relationship at decreasing rates between selling price and housing characteristics.

NOTES

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6. See Pissarides, Christopher A., Equilibrium Unemployment Theory (2nd edition), MIT Press Books (2000).
7. In the labor market, in fact, market tightness is the ratio of job vacancies to job seekers.
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11. Stein, J., "Prices and Trading Volume in the Housing Market: A Model with Down-Payment Constraints," *Quarterly Journal of Economics*, 110, 379–406 (1995); Ortalo-Magne, F. and S. Rady, "Housing Market Fluctuations in a Life-Cycle Economy with Credit Constraints," *Research Paper No. 1501* (1998), Graduate School of Business, Stanford University; Leung, Charles Ka Yui, Garion C. K. Lau, and Youngman Chun Fai Leong, "Testing alternative theories of the property price-trading volume correlation," *Journal of Real Estate Research*, 23(3), 253-264 (2002); Fisher *et al.*, *supra* n.5; Sun Hua, and Michael J. Seiler, "Hyperbolic Discounting, Reference Dependence and its Implications for the Housing Market," *Journal of Real Estate Research*, 35, 1, 1-23 (2013); Zhaohui Li, Michael J. Seiler, and Hua Sun, "Prospect Theory and Two-way Disposition Effect: Theory and Evidence from the Housing Market," *Working paper* (October, 25, 2018).
12. Leung, Leong and Wong, *supra* n.8; Seiler *et al.*, (2013), *supra* n.11; Seiler *et al.*, (2018), *supra* n.11. Although price dispersion research is more commonly found in studies of non-durable consumption goods (*see* Baye, Michael, John Morgan, and Patrick Scholten, "Information, Search, and Price Dispersion," in Handbook on Economics and Information Systems, (T. Hendershott, ed.), Elsevier Press, Amsterdam (2006)), price dispersion studies on durable and re-saleable goods such as real estate also are growing rapidly.
13. Ortego-Marti V., and Gabrovski M., "Housing Market Dynamics with Search Frictions," Working Papers 201804, University of California at Riverside, Department of Economics (2018).
14. Lippman, S. and J. McCall, "An Operational Measure of Liquidity," *American Economic Review*, 76:1, 43–55 (1986).
15. Unlike in an efficient market, in fact, where individuals are price-takers, in the housing market, buyers and sellers are able to affect selling prices through individual search efforts. Seiler *et al.*, (2015), *supra* n.1.
16. Seiler *et al.*, (2015), *supra* n.1.
17. Sirmans, S., D. Macpherson, and E. Zietz, "The Composition of Hedonic Pricing Models," *Journal of Real Estate Literature*, 13:1, 3–43 (2015).
18. See the very interesting work by Seiler *et al.*, (2015), *supra* n.1 and the references therein. The exact nature of the relationship between the selling price of a house and its time-on-the-market (ToM) remains an open question to date, despite almost 40 years of empirical studies and analyses conducted on various data sources; *see*, *e.g.*, Seiler *et al.* (2017), *supra* n.5.
19. An, Z., P. Cheng, Z. Lin, and Y. Liu, "How do Market Conditions Impact Price–TOM Relationship? Evidence from Real Estate Owned (REO) Sales," *Journal of Housing Economics*, 22, 3, 250–63 (2013).
20. Seiler *et al.*, (2015), *supra* n.1.
21. Seiler *et al.* (2017), *supra* n.5.
22. Seiler *et al.*, (2015), *supra* n.1; Seiler *et al.* (2017), *supra* n.5.
23. Stein, *supra* n.11.
24. Ortalo-Magne and Rady, *supra* n.11.
25. Genesove, D. and C. Mayer, "Loss Aversion and Seller Behavior: Evidence from the Housing Market," *Quarterly Journal of Economics*, 116, 1233–60 (2001).
26. Seiler *et al.* (2013), *supra* n.11.
27. The preferences are dynamically inconsistent in the way choices are made between two payoffs in the future, since people will depart from their original choice if they are allowed to revise their consumption plan in the future. Seiler *et al.* (2013), *supra* n.11.
28. Seiler *et al.* (2018), *supra* n.11.

29. Price dispersion in a cold market is greater than in a hot one. This is because in a cold market there will be more home sellers that are subject to potential losses, and loss aversion will come into play and magnify the disposition effect, leading to more heterogeneous asking prices among sellers of similar houses. Finally, as more sellers ask for extreme prices in a distressed market, the selling hazard rate declines, which further lowers the transaction volume. Seiler *et al.* (2018), *supra* n.11.
30. Ortego-Marti and Gabrovski, *supra* n.13.
31. Ioannides M.Y., and Zabel J. E., "Housing and Labor Market Vacancies and Beveridge Curves: Theoretical Framework and Illustrative Statistics," forthcoming in: Ioannides, Yannis M., Ed. Recent Developments in the Economics of Housing, Edward Elgar (2018).
32. Precisely, Ioannides and Zabel (2018) propose the concept of the "unfulfilled homeownership", namely the unfulfilled demand for owner occupied housing, as the counterpart of the unemployment rate (the unfilled demand for employment).
33. Born with the seminal works by Lancaster K.J., "A New Approach to Consumer Theory," *Journal of Political Economy*, 74, 2, 132-157 (1966); Rosen S., "Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition," *Journal of Political Economy*, 82, 1, 34-55 (1974); Epple D.R., "Hedonic Prices and Implicit Markets: Estimating Demand and Supply Functions for Differentiated Products," *The Journal of Political Economy*, 95, 1, 59-80 (1987).
34. *See*, *e.g.*, the influential surveys by Sheppard S., "Hedonic Analysis of Housing Markets," in *Handbook of Regional and Urban Economics Volume 3: Applied Urban Economics*, edited by Paul Cheshire and Edwin Mills, Amsterdam: North Holland, Chapter 41, 1595- 1635 (1999) and Malpezzi, 2003.
35. *See* for *e.g.*, Epple, *supra* n.33; Bartik T.J., "The Estimation of Demand Parameters in Hedonic Price Models," *Journal of Political Economy*, 95, 1, 81-88 (1987); Kahn S., and Lang K., "Efficient Estimation of Structural Hedonic Systems," *International Economic Review,* 29, 1, 157-166 (1988); Palmquist R.B., "Land as a Differentiated Factor of Production: A Hedonic Model and Its Implications for Welfare Measurement," *Land Economics*, 65, 1, 23-28 (1988); Coulson E.N., "Semiparametric Estimates of the Marginal Price of Floorspace," *The Journal of Real Estate Finance and Economics*, 5, 1, 73-83 (1992); Coulson E.N., "Semiparametric Estimates of the Marginal Price of Floorspace: Reply," *The Journal of Real Estate Finance and Economics*, 7, 1, 77-78 (1993); Colwell P.F., "Comment: Semiparametric Estimates of the Marginal Price of Floorspace," *The Journal of Real Estate Finance and Economics*, 7, 1, 73-75 (1993); Brachinger H.W., "Statistical Theory of Hedonic Price Indices," *DQE working paper*, 1 (2003); Ekeland I., Heckman J.J., and Nesheim L., "Identifying Hedonic Models," *American Economic Review*, 92, 2, 304-309 (2002); Ekeland I., Heckman, J.J. and Nesheim L., "Identification and Estimation of Hedonic Models," *Journal of Political Economy*, 112, S1, S60- S109 (2004).
36. *See, e.g.*, Anglin P.M., and Gençay, R., "Semiparametric Estimation of a Hedonic Price Function," *Journal of Applied Econometrics*, 11, 6, 633-648 (1996); Ekeland, Heckman and Nesheim (2002), (2004), *supra* n.35; Henderson, Kumbhakar and Parmeter, 2007; Henderson D.J., Kumbhakar S.C., and Parmeter C.F., "Nonparametric Estimation of a Hedonic Price Function," *Journal of Applied Econometrics*, 22, 3, 695-699 (2007); Haupt H., Schnurbus J., Tschcmig R., "On Nonparametric Estimation of a Hedonic Price Function," *Journal of Applied Econometrics*, 25, 5, 894-901 (2010).
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    These models are characterized as being easily interpretable, and the estimated parameters possess a direct economic meaningfulness, namely, they give absolute prices for the unit of the attributes. Maurer R., Pitzer M., Sebastian S., "Hedonic Price Indices for the Paris Housing Market," *Allgemeines Statistisches Archiv (Journal of the German Statistical Society)*, 88, 3, 303-326 (2004).
38. *See, e.g.*, Anglin and Gençay, *supra* n.36; Malpezzi, 2003
39. Rosen, *supra* n.33.
40. Ekeland, Heckman, and Nesheim (2002), (2004), *supra* n.35.
41. *See* Kuminoff, N.V., Parmeter, C.F. and Pope, J.C., "Specification of Hedonic Price Functions: Guidance for Cross-Sectional and Panel Data Applications," *Virginia Tech Working Paper, No. 2009-02*, (January 2009).
42. *See, e.g.*, Leung, *et al., supra* n.5.
43. Seiler *et al.*, (2015), *supra* n.1.
44. Among others, Diaz and Jerez, *supra* n.5; Novy-Marx, *supra* n.5; Piazzesi *et al., supra* n.5; Genesove and Han, *supra* n.5; Leung, *et al., supra* n.5; Peterson, *supra* n.5; Seiler *et al.*, (2015), *supra* n.1; Seiler *et al.* (2017), *supra* n.5.
45. By definition, an economic model should be a simplification of the real world, *see, e.g.*, Romer D., Advanced Macroeconomics, McGraw Hill (3rd edition) (2006). Simplicity, therefore, is an added value of an economic model. Of course, the economic model must be formally correct, from a mathematical point of view. As a result, between two mathematically correct and empirically effective economic models, it should be favorite the simplest one.
46. The residential real estate transaction process is typically characterized by a sequential search and random match between buyers and sellers (Seiler *et al.*, (2015), *supra* n.1). Search theory typically assumes *random search*, that is, agents meet bilaterally and at random, although whether a meeting results in matching (trading) often is endogenous. In *directed search*, instead, agents have information to target their search towards particular types in the market (individuals or submarkets); *see* Wright R., Kircher P., Julien B., and Guerrieri V., "Directed Search: A Guided Tour," *NBER Working Paper No. w23884* (2017).
47. There can certainly be households who are neither buyers nor sellers at a given point in time but this case does not change the main results of the analysis.
48. As usual in matching-type models, the analysis is restricted to the stationary state in which the values of the variables are not subject to further changes over time.
49. Homelessness is in fact irrelevant in the housing market analysis, since buyers (the home seekers) generally are not homeless, they are tenants or have other housing arrangements, *e.g.*, young people living with their parents.
50. Buyers get utility from the house. Hence, on the one hand, buyers may have incentive for buying a second home; on the other hand, however, it is not optimal for the buyer to sell before buying (second home).
51. Ioannides and Zabel, *supra* n.31.
52. Equations (2) and (3) are known as "asset equations," since there is a clear analogy with the pricing of financial assets, which yield periodic dividends (not necessarily positive) and whose value may change over time. Consider, for example, Eq. (3), the value of searching for a home. While searching, a buyer bears, at every instant, a search cost that must be discounted back to the beginning of the period by the instantaneous discount factor $(1 + r \cdot t)^{-1}$, where r is the interest rate and t is the length of a time period (precisely, a short interval). In any small interval of time t, a buyer searches for a house and, if the search is successful, s/he becomes a seller. This event is modeled using a Poisson process. The probability of finding a home during the time interval t is equal to $e^{-g(\theta)\cdot t}$ where $g(\theta)$ is the instantaneous probability of finding a home; thus, by complementarity, the probability of not searching for a home during the same time interval is $1 - e^{-g(\theta)\cdot t}$. Therefore, the utility of a buyer is the following:

    $$H(t) = \left[\frac{1}{(1+r\cdot t)}\right] \cdot \left\{(-e) \cdot t + \left(1 - e^{-g(\theta)\cdot t}\right) \cdot [x + V(t) - P] + e^{-g(\theta)\cdot t} \cdot H(t)\right\}$$

    $$r \cdot H(t) = -e + \frac{\left(1 - e^{-g(\theta)\cdot t}\right)}{t} \cdot [x + V(t) - H(t) - P]$$

    It is straightforward to show that $\lim_{t \to 0} r \cdot H(t) = r \cdot H$, by using the *Hopital's rule.* Thus, Eq. (3) is the utility of a buyer when the short interval of time tends to zero. A similar reasoning applies to asset equation (2).
53. Intuitively, the better the mix of housing characteristics, the higher the buyer's benefit. It could also be assumed that $x = x_j$, where j denotes the (preferences of) homeowners that make different the buyer's benefit.
54. *See* Pissarides, *supra* n.6; Petrongolo, Barbara, and Christopher A. Pissarides, "Looking into the Black Box: A Survey of the Matching Function," *Journal of Economic Literature*, 39(2), 390-431 (2001).
55. *See* Diaz and Jerez, *supra* n.5; Novy-Marx, *supra* n.5; Piazzesi, *et al., supra* n.5; Genesove and Han, *supra* n.5; Leung, *et al., supra* n.5; Peterson, *supra* n.5.
56. Actually, they are "instantaneous" probabilities which can vary, theoretically, from zero to infinity.
57. By definition, markets with frictions require positive and finite tightness, *i.e.*, $0 < \theta < \infty$, since for $\theta = 0$ the vacancies are always filled, whereas for $\theta = \infty$ the home-seekers immediately find a vacant house.
58. Pissarides, *supra* n.6.
59. *Id.*
60. Krainer, *supra* n.5.
61. Precisely, $\frac{\partial P}{\partial \theta} = (-1) \cdot \frac{\gamma \cdot (r \cdot x + e)}{\left[r + g(\theta) \cdot (1 - \gamma)\right]^2} \cdot \frac{\partial g(\theta)}{\partial \theta} \cdot (1 - \gamma) < 0.$
62. *See* equation (5).
63. *See* Seiler *et al.*, (2015), *supra* n.1; Seiler *et al.* (2017), *supra* n.5.
64. *Id.*
65. As pointed out by Leung, Leong and Chan, *supra* n.10; Anglin *et al., supra* n.10; Merlo and Ortalo-Magne, *supra* n.10; Leung, *et al., supra* n.5; Genesove and Mayer, *supra* n.25 find that owners of properties with higher loan-to-value ratios tend to wait longer and receive higher prices than properties with lower loan-to-value ratios.
66. *See, e.g.*, Krainer, *supra* n.5; Merlo and Ortalo-Magne, *supra* n.10; Leung, *et al., supra* n.5; Diaz and Jerez, *supra* n.5.
67. Seiler *et al.*, (2015), *supra* n.1.

68. Precisely,

$$\frac{\partial^2 P}{\partial \theta^2} = \frac{1}{\left[r+g(\theta)\cdot(1-\gamma)\right]^4} \cdot \left\{ \begin{array}{c} (-1)\cdot\gamma\cdot(r\cdot x+e)\cdot\frac{\partial^2 g(\theta)}{\partial\theta^2}\cdot(1-\gamma)\cdot\left[r+g(\theta)\cdot(1-\gamma)\right]^2 - \ldots \\ \ldots - (-1)\cdot\gamma\cdot(r\cdot x+e)\cdot\frac{\partial g(\theta)}{\partial\theta}\cdot(1-\gamma)\cdot 2\cdot\left[r+g(\theta)\cdot(1-\gamma)\right] \\ \cdot\frac{\partial g(\theta)}{\partial\theta}\cdot(1-\gamma) \end{array} \right\} > 0\cdot$$

69. Fisher *et al., supra* n.5; Leung, Lau and Leong, *supra* n.11; Seiler *et al.* (2013), *supra* n.11; Seiler *et al.* (2018), *supra* n.11.

70. Leung, *et al., supra* n.5.

71. Ortego-Marti and Gabrovski, *supra* n.13.

72. *See, e.g.*, Janssen *et al., supra* n.9; Leung, Leong and Wong, *supra* n.8.

73. Also used by Novy-Marx, *supra* n.5; Piazzesi, *et al., supra* n.5; Peterson, *supra* n.5.

74. *See, e.g.*, the introductory discussion in Lisi, G., and Iacobini, M., "Estimating Adjustment Factors for the Sales Comparison Approach in the Presence of Heterogeneous Housing and Thin Markets," *The Journal of Real Estate Research*, 40, 1, 89-119 (2018).