Vacancy, Search, and Prices in a Housing Market Matching Model

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A model of the single-family housing market is proposed in which households that move are both buyers and sellers. Households move when a stochastic process leaves them dissatisfied with their current unit. Household buyers expend costly search effort to find a better house, while sellers hold two units until a buyer is found. The vacancy rate, fixed in the short run, determines the expected length of sale and search, which play a central role in the reservation prices of buyer and seller. Market prices, the result of bargaining, lie between these two. The model yields a strong theoretical relationship (inverse) between vacancy and prices, which with competitive supply explains the existence of longer-run "structural" vacancy.

I. Introduction

It is widely accepted that operation of both housing and commercial real estate markets can be characterized with the following stylized facts: (1) Real estate markets have identifiable normal or "structural" vacancy rates, which differ between areas, and seem related to market growth or levels of activity. (2) Movements in vacancy around a market's "structural" rate, created by changes in either supply or demand, determine the "vacancy duration" for units that are for sale or rent. This seems to trigger significant movement in rents or market prices. (3) The vast majority of market transactions (sales or leases) involve

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turnover or switching by existing market participants. This occurs because structures and occupants have become "mismatched." In contrast to the impact of net demand, the role of such turnover in determining market vacancy and prices has yet to be clearly delineated. (4) The supply of housing reacts relatively slowly to changes in both market prices and vacancy, as in a "stock-flow" model.

The empirical research on which such observations rest has grown rapidly in recent years. Eubank and Sirmans (1979), Rosen and Smith (1983), and Shilling, Sirmans, and Corgel (1987) are among the many to document the relationship of rents to vacancy and the latter's "structural" level. Chinloy (1980), Guasch and Marshall (1985), and Kaserman, Trimble, and Johnson (1989) have examined the duration of vacancy and its relationship to price or rent determination. Finally, Hanushek and Quigley (1979) and Weinberg, Friedman, and Mayo (1981) have studied the gradual "disequilibrium" that evolves between units and occupants, which leads the majority of market transactions to involve turnover or unit switching.

What seems to be missing from this literature is a theoretical perspective that ties together and explains these observations about market behavior. Using the price dispersion, asymmetric information approach to search theory (Kohn and Shavell 1974; Rothschild 1974; Butters 1977; Burdett and Judd 1983), both Stull (1978) and Yinger (1981) have made a start in this direction by describing how uncertainty and costly search can influence the behavior of various market participants. A recent manuscript by Read (1988) advances this literature by fully deriving the steady-state properties of a price-setting equilibrium distribution of rents, with free entry and positive vacancy.

In the labor market, similar behavior has been explained with a family of search and matching models, in which information is symmetric and turnover is explicitly considered. Lucas and Prescott (1974), Burdett and Mortensen (1980), and Diamond (1981) have all developed models in which workers depart from jobs and then find new employment from among the vacancies so created. With uncertainty in the worker-job match, decisions about job acceptance determine vacancy duration, while wages are based on the gains from filled employment.

The search and matching models from the labor market would seem particularly applicable to housing, and this forms the objective of the present paper. The assumption in matching models of fixed jobs and workers, for example, nicely fits the housing market's "stock-flow" character, where prices adjust in the short run to equate demand to a fixed stock. Similarly, the Diamond (1982) and Mortensen (1982) approach to wage determination seems closely parallel to the intense bargaining that surrounds many house price or lease negotia-

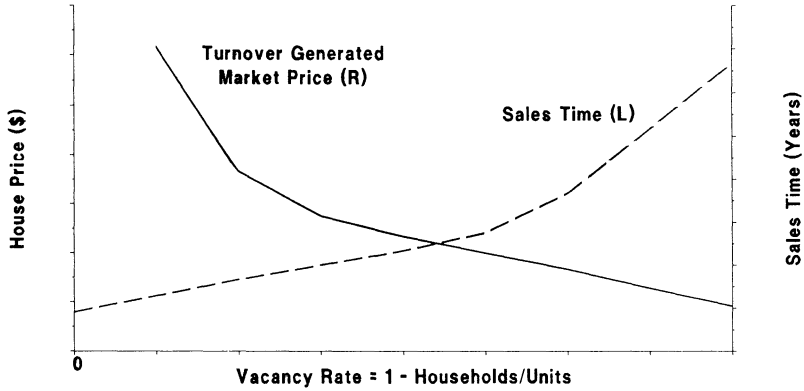

FIG. 1.—Short-run price determination

tions. Despite such similarities, the housing and labor markets are different in several important respects. Unemployment is crucial in the labor market and occurs over a short spell even with voluntary job changes. Just the opposite occurs in the housing market, where there is little homelessness and mobility generally involves a spell in which households own or rent two units. The search and matching model developed in this paper concerns the owner-occupied market and can best be summarized with the aid of a pair of graphs (figs. 1 and 2).

In the short run, the number of units and households is assumed fixed. Households periodically "change" and therefore seek to move from an existing house that no longer suits their needs. The prospect of remaining in such a "mismatched" state determines both the search "effort" and the offer price made by buyers. Sellers are merely buyers who have found a new unit and are seeking to dispose of their old one. Their reservations are determined by expectations about sales time and the costs of holding two units. Given any positive vacancy rate, this matching-bargaining process generates the schedules shown in figure 1. Greater vacancy will increase sales time, lower seller reservations, speed up search time, and lead to lower market prices. Differences in other model parameters (such as a greater rate of household change or turnover) will shift the schedules (in this case upward).

The combination of price and expected sales time determines the "expected price" for a house: market price discounted by expected sales time. In the longer run, given competitive supply, new units are added to the stock (and vacancy adjusts) until the marginal cost of such units equals this "expected price" (fig. 2). The vacancy rate at which this occurs is the market's "structural" rate. The vast bulk of this paper, then, is concerned with the development of figure 1: how with positive vacancy, matching, sales time, and prices are all jointly determined.

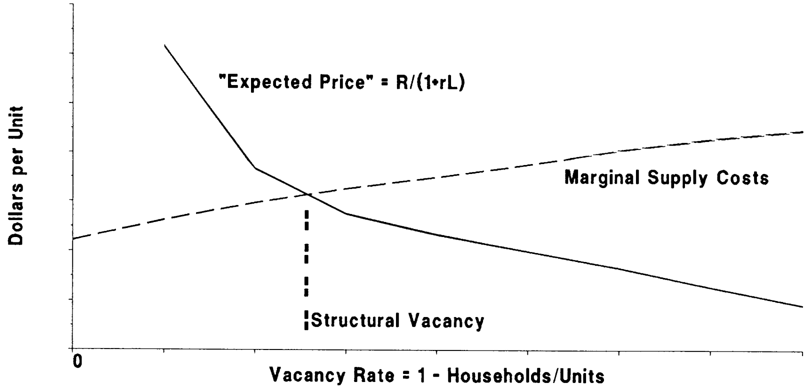

FIG. 2.—Long-run equilibrium

The results of the model provide interesting and consistent explanations for the market behavior so thoroughly documented empirically. Reasonable values are produced for house prices, which are very sensitive to small changes in "demand" or "supply" (i.e., vacancy). House prices also increase significantly with a greater rate of household change or market turnover. This leads (in fig. 2) to a higher long-term structural vacancy rate in markets with more exogenous growth, change, or mobility. Finally, the model evaluates the performance of the private market with respect to a welfare criterion and shows the suboptimality of private search decisions, and the divergence between expected price and the "social value" of additional housing units.

The paper is organized as follows. Section II develops a consistent model of household "change," search, and matching. Section III describes a search technology for finding new (matched) homes, in which costly search effort increases the rate of matching. Section IV deals with individual decision making about search effort and the determination of house prices. Sections V–VII then develop a number of steady-state comparative solutions and evaluate them with respect to a welfare criterion.

II. Household Turnover and Matching

Matching models of the labor market usually assume an exogenous, continuous distribution of matching qualities between workers and jobs. If workers are laid off randomly, then vacant jobs and unemployed workers will find each other with this same distribution. Searching workers must thus select a matching quality to accept as opposed to waiting for a better draw from the distribution of vacant jobs (Diamond 1981). Deere's (1985) variation on this theme has workers always employed, but job productivity is randomly distributed across both vacant and filled jobs. Vacant jobs meet with employed workers, who can be enticed to switch if the vacant job has greater productivity. The existence of setup or transfer costs provides an incentive for each party to "wait" for a more productive match.

While such matching might be applied to the choice of houses by households, the housing market is sufficiently different from the labor market to suggest an alternative approach to modeling turnover. Homelessness (equivalent to unemployment) is relatively inconsequential in the housing market, and so moves are like voluntary "quits" in the labor market. Furthermore, moves involve some spell in which the household owns two units, whereas even voluntary job transitions usually carry some period of unemployment. More important, the causes of housing mobility are usually different from those generating job mobility.

In the housing market, research (Hanushek and Quigley 1979; Weinberg et al. 1981) suggests that households move either when their job changes (by a sufficient commuting distance) or when they undergo some other change (in income, family members, etc.) that makes their current house inadequate. To better reflect this process and keep the analysis as simple as possible, the model developed here assumes that the quality of matching is not continuous, but rather dichotomous: matched or mismatched. This results from having two types of households (e.g., families or singles) that can live in two types of units (e.g., large or small). Households of each type are matched when they reside in the appropriate unit (e.g., families in large units); otherwise they are mismatched. There are no households without a house.[1]

Households move from a state of being matched to being mismatched because they change character (e.g., singles become families) while their unit remains the same. Conversely, they move from being mismatched to matched by finding and purchasing the other (appropriate) unit. In an owner-occupied market, they will then put their old unit up for sale. In a very simple way, this seems to mirror the actual process by which moves occur.

Even though individual households are continually changing in the real world (e.g., becoming larger or smaller), the aggregate distribution of households by type is generally stable over time. Thus a simple model of individual change is needed with steady-state aggregate properties. In the two-household case, the rate of change from type 1

[1] Households may enter or leave the model exogenously, reducing or adding to the number of vacant units. This constitutes a change in aggregate or net demand, as opposed to the demand for different types of units, which can originate purely from change and turnover.

to type 2 can be labeled as β_1, while that from type 2 to type 1 is β_2. With overdots denoting time derivatives, equations (1) and (2) describe the dynamics of this simple model, and equation (3) gives the ratio of households of each type that prevails in the steady state of the model:[2]

$$\dot{H}_1 = \beta_2 H_2 - \beta_1 H_1, \tag{1}$$

$$\dot{H}_2 = \beta_1 H_1 - \beta_2 H_2, \tag{2}$$

$$\frac{H_1}{H_2} = \frac{\beta_2}{\beta_1}, \tag{3}$$

where H_i is the total number of households of type $i = 1, 2$ and β_i is the transition rate between types.

At any time, a household must be in one of three occupancy states. Thus the total number of households of each type (H_i, $i = 1, 2$) equals the sum of those in each of these three occupancy categories: HM_i: matched households (living in a house of type i); HD_i: matched households (living in a house of type i but owning the other type as well); HS_i: mismatched households (living in a house of the other type).

Households that are matched (HM_i) become mismatched because they switch to the other type. Once mismatched (HS_i), a household searches for a vacant house of the alternative type. When found, this house is purchased, and the household immediately moves. At the same time, the household puts its "old" home up for sale and changes to the state of dual ownership, or having two homes (HD_i). On selling its "old" home, the household will return to the single matched state (HM_i). It is important to remember that while a household is mismatched and is searching, there is always the possibility (however remote) that it will switch back to its old type and therefore be matched again. Similarly, while owning two homes, a household may also change back. In this case it withdraws its old house from the market, occupies it, and puts the new (now mismatched) unit up for sale.

In keeping with the stock-flow approach discussed in the Introduction, the stock of housing of each type is fixed in the short run and is greater than the number of households of each type. Since there are no homeless households, vacancy is simply the difference between the number of units and households:

$$V_i = S_i - H_i > 0, \quad i = 1, 2, \tag{4}$$

[2] A demographic interpretation of this model might have young households change into older ones, which then at death become young again (through their offspring). An economic interpretation could involve random upward or downward mobility.

where V_i is vacant units of type $i = 1, 2$ and S_i is the stock of units of type $i = 1, 2$.

The uncertainty in this model arises from the assumption that mismatched households cannot instantly find an appropriate house. While this might seem a small problem with only two types of units, the two-house typology is used only for analytic convenience. The matching between mismatched households and vacant units (of the same type) is assumed to occur with a Poisson process, whose underlying parameters (m_1, m_2) are the rates at which matched homes of that type are "found." Thus m_iHS_i ($i = 1, 2$) are the aggregate flows of new house purchases. The sales of vacant homes will also occur with a Poisson process whose parameters (q_1, q_2) must be determined to equalize the flow of purchases with sales:

$$q_i = \frac{m_iHS_i}{V_i}, \quad i = 1, 2. \tag{5}$$

These matching parameters are here taken as given, while later they will result from buyer decisions about search effort.

Given a fixed number of households and units of each type, the following differential equations characterize how households move and change states in the model:

$$H\dot{S}_i = -m_iHS_i - \beta_iHS_i + \beta_jHM_j, \tag{6}$$

$$H\dot{D}_i = -q_jHD_i + m_iHS_i + \beta_jHD_j - \beta_iHD_i, \tag{7}$$

$$H\dot{M}_i = -H\dot{S}_i - H\dot{D}_i, \quad i = 1, 2, j \neq 1. \tag{8}$$

In effect, these equations provide a logically consistent, stable model of housing market "turnover."[3]

This system of six differential equations can be considerably simplified and greatly reduced in number with one further assumption. If it is assumed that the two types of households are identical in number and behavior, despite having "opposite" preferences, then the system is reduced in half. If $\beta_1 = \beta_2$, $V_1 = V_2$, $H_1 = H_2$, $m_1 = m_2$, then there will be perfect symmetry in the model, with $HS_1 = HS_2$, $HM_1 = HM_2$, and $HD_1 = HD_2$. In this symmetric model, the household type subscripts can be dropped, and with the incorporation of (8), the system reduces to a pair of differential equations that describe the behavior of each type:

$$\dot{HS} = -HS(2\beta + m) + \beta H - \beta HD \tag{9}$$

and

[3] Equations (6)–(8) readily incorporate the identity $H_i = HM_i + HD_i + HS_i$. They also allow for the possibility that mismatched households may become matched in their own unit by rechanging states and that dual-ownership households also can switch types and remain matched.

$$\dot{HD} = mHS\left(1 - \frac{HD}{V}\right). \tag{10}$$

Once HS and HD are determined with (9) and (10), the number of matched households owning only one unit (HM) is found by simply subtracting HS and HD from the total number of households (H). The steady state of the system is

$$HS = \frac{\beta(H - V)}{2\beta + m} \tag{11}$$

and

$$HD = V. \tag{12}$$

Despite its extreme simplicity, the behavior of this housing market is straightforward and intuitively appealing. Consider the impact of changes in aggregate supply or demand. The instant addition of new (vacant and unowned) units to a steady state will first reduce the sale probability (q). This will gradually increase the number of matched households with two units until eventually all the new units are owned ($HD = V$). The tendency to remain longer in the dual-ownership state eventually lowers the steady-state fraction of the population that is mismatched (HS). When a new household enters the system, it does so by instantly acquiring a vacant unit without adding one. Thus HM suddenly increases, while HD and V instantly decrease. This gradually raises HS, and in the new steady state, the *fraction* of households that are mismatched will increase, while the fraction of those owning two units will decrease.

If the rate of matching is exogenously improved (m increases), the flows of matches and sales rise identically, so that the number of dual-ownership households is unaffected. Naturally, the number of mismatched households decreases, while HM increases. In the following sections, where m becomes endogenous, this increase in market efficiency generates a significant "return" from greater search effort. A greater rate of demographic change (larger β) will likewise leave the number of dual-ownership households unaffected but will increase the number of mismatched households. These conclusions can be summarized in the following proposition.

PROPOSITION 1. $\partial HS/\partial m < 0$, $\partial HS/\partial V < 0$, $\partial HS/\partial\beta > 0$, and $\partial(HS/H)/\partial H > 0$.

An interesting calculation that comes out of the model is the expected time to sell a unit.[4] With the sales rate (q) being Poisson, this expected time for a sale is simply $1/q$, which will be defined as L:

[4] With a Poisson distribution, $1/q$ is the expected time (ex ante) that units will take to sell. However, some units put on the market will be withdrawn as their owners change states. The actual distribution of (ex post) sales times thus involves a competing risk model, and its expectation may be slightly different from $1/q$.

$$L = \frac{1}{q} = \frac{V}{mHS} = \frac{V(2\beta + m)}{\beta m(H - V)}. \tag{13}$$

The comparative steady-state properties of the expected sales time can be easily derived from (13) and proposition 1. They are also intuitively reasonable and are summarized in the following proposition.

PROPOSITION 2. $\partial L/\partial m < 0$, $\partial L/\partial H < 0$, $\partial L/\partial \beta < 0$, and $\partial L/\partial V > 0$.

III. Search Technology

In matching models with a continuous distribution of match quality, when buyers and sellers meet, they must decide whether to transact or to gamble on a better match. In this model, households need only "find" any house of the appropriate type. Search is necessary because there is imperfect information, possibly about which units are for sale and certainly about a unit's type. In effect, advertising is assumed to be either nonexistent or imperfect, so that buyers must inspect units to ascertain their status. The decision that searching households must make, then, involves how much effort (such as number of visits) to put into searching. More effort will not yield a "better" match, but rather the more rapid attainment of a known improvement in housing quality. Thus the Poisson parameter (now collapsed to a single m) will be endogenous in this model, determined by the degree of search effort (E) that is undertaken by those households that are mismatched.

As an example, if all units for sale are so advertised but a unit's type cannot be ascertained from the advertisement, then search effort would take the form of number of visits per period. With a ½ likelihood of finding the "right" unit on each visit, m would equal $E/2$.

An alternative, and very imperfect, market would exist if advertising were impossible. In this case a unit's vacancy would have to be identified, as well as type, again through visits or effort (E). Each visit (to any house) would involve a likelihood of $V/2S$ of finding both a vacant unit and one of the correct type. In this case the matching rate m might equal $EV/2S$, and greater vacancy would enhance the productivity of household search.

As discussed in Diamond (1982), it is possible to specify a number of search "technologies." In this paper, the number of matches is always the product of the Poisson parameter m and the number of households searching (HS). The parameter m will then depend on both search effort and possibly the fraction of vacant units, with the following properties:

$$m\left(E, \frac{V}{S}\right); \quad m\left(0, \frac{V}{S}\right) = 0, \frac{\partial m}{\partial E} > 0, \frac{\partial^2 m}{\partial E^2} < 0, \frac{\partial m}{\partial V/S} \geq 0. \tag{14}$$

Search effort is assumed to be costly, in both time and money, and it seems reasonable (particularly with respect to time) to assume further that there are increasing marginal search costs. Thus, in the general case, search costs will have the following properties:

$$c(E);\quad c(0) = 0, \frac{\partial c}{\partial E} > 0, \frac{\partial^2 c}{\partial E^2} > 0. \tag{15}$$

All the mechanics of the model are now in place. Costly search effort must be expended by households that have become mismatched with their current unit. As more effort is engaged, such households will find their appropriate units more rapidly. For the market as a whole, this will reduce the steady-state number of households that are mismatched, yielding an obvious gain in market efficiency. What remains is to determine the value of search effort (in relation to costs) and to determine how this gain from search and matching gets translated into market prices.

IV. Search Behavior, Short-Run Equilibrium, and Market Prices

In determining search behavior, the assumption of household symmetry will be maintained. Thus the decisions made by only one type of household need to be considered, as long as the parameters governing preferences are identical across types. With this in mind, the terms UM and US denote the flow of utility from occupying a matched and mismatched house, respectively. The symmetry assumption implies that type 1 households derive the same utility from being matched or mismatched as type 2 households do.

To determine the gains from search, the present discounted value of being in each of the three occupancy states must be determined. This can be done using the asset market equilibrium condition, as is customary in other search models. Equations (16)–(18) state that the annual return from being in each state must equal the income (utility) flow associated with the state plus any expected capital gains (or losses) from changing states:[5]

$$rWM = UM - \beta(WM - WS), \tag{16}$$

$$rWD = UM + q(WM - WD + R), \tag{17}$$

$$rWS = US - c(E) + \beta(WM - WS) + m(E)(WD - WS - R), \tag{18}$$

[5] To simplify notation in this section, m will be written as an explicit function only of endogenous variables or search effort E. When comparative steady states are examined later, any dependence of m on the exogenous variable V/S will be incorporated.

where UM and US are matched and mismatched utility flows, respectively; WM, WD, and WS are present values of each state (matched one home, matched two homes, and mismatched); r is the discount rate; $c(E)$ is the "cost" of searching with effort E; β, $m(E)$, and q are the transition, matching, and sales rates (Secs. II and III); and R is the market price for a matched house.

Equation (16) says that the annual return from being in a matched state with one house (the discount rate multiplied by the value of the state) equals that utility flow plus the expected loss in welfare due to a demographic transition (which moves the household from a matched state with value WM to a mismatched state with value WS). Similarly, in equation (17), the annual return from being matched with two houses must equal the matched utility flow without the expected loss, because with two houses one can always be matched. With dual ownership, however, there is the prospect of a transition to single ownership, which occurs with the likelihood q (or $1/L$ in Sec. II). This confers a capital gain, with the receipt of the buyer's payment for the second house (R).

Equation (18) says that the return from being mismatched equals its utility flow, plus the expected gain if the household demographically changes back to a matched state, plus the expected gain from search. These gains must consider both the costs of search $c(E)$ and the price that will have to be paid for the matched house (R). While there are two types of houses, the utility flows derived from each are symmetric, and so both units will have the same price in the market (R).

In the spirit of Mortensen (1982) and Diamond (1982), the market price of a house in this model (R) must lie somewhere between the value that makes the seller indifferent to selling ($R = WD - WM$) and the value that makes the buyer indifferent to purchasing ($R = WD - WS$). Since both parties are otherwise identical individuals, it seems reasonable to assume that each has equal bargaining power and that they will split the gains from the transaction:

$$\begin{gathered} WM - WD + R = WD - WS - R, \\ R = \frac{WD - WS + WD - WM}{2}. \end{gathered} \tag{19}$$

It is important to understand that the prices in this model are denominated in present-value dollars, and the model assumes a perfect capital market. Thus the "carrying cost" of owning a second home is the postponed receipt of R. It is easy to reformulate the model into a "rental" market, in which rR becomes the annual carrying cost for each home. As long as tenants are liable for a unit's vacancy, the solutions are identical, but this eliminates an important distinction between the two modes of tenure.

Households of either type will select the level of search effort E so as to maximize the value of being mismatched. This requires that search be conducted until the marginal gain from the last unit of search effort equals the cost of that effort. From equation (18), this is

$$\frac{\partial c}{\partial E} = \frac{\partial m}{\partial E}(WD - WS - R). \tag{20}$$

When equations (16)–(20) are combined with the definition of q or L (eqq. [5] and [13]), there exists a system of six simultaneous relationships in the six unknowns: WM, WD, WS, R, L, and E. The solution will depend on the search technology (as reflected in the c and m functions), as well as the demographic turnover rate (β) and the level of vacancy (V/S).

It is convenient to solve equations (16)–(20) in terms of differences in the state values: $WM - WS$, $WM - WD + R$, and $WD - WS - R$. Subtracting (18) from (16), rearranging, and then incorporating the pricing condition (19) yield

$$\begin{aligned} WM - WS &= \frac{UM - US + c(E) - m(E)(WD - WS - R)}{2\beta + r} \\ &= \frac{UM - US + c(E) - m(E)(WM - WD + R)}{2\beta + r}. \end{aligned} \tag{21}$$

In a similar manner, (17) can be subtracted from (18) and combined with (19) and (21) to yield the net gains to a house purchaser:

$$WD - WS - R = \frac{[UM - US + c(E)](\beta + r)}{Z - \beta m(E)} - \frac{rR(2\beta + r)}{Z - \beta m(E)}, \tag{22}$$

where $Z = [r + m(E) - q](2\beta + r)$.

The same procedure produces the following comparable expression for the net gains to a house seller:

$$WM - WD + R = \frac{-\beta[UM - US + c(E)]}{X - \beta m(E)} + \frac{rR(2\beta + r)}{X - \beta m(E)}, \tag{23}$$

where $X = (r + q)(2\beta + r)$.

Expressions (22) and (23) are both linear in the sales price R, so equating the two yields a straightforward solution for R as a function only of E and the other system parameters:

$$\begin{aligned} R &= [UM - US + c(E)]\left\{\frac{(\beta + r)[X - \beta m(E)] + \beta[Z - \beta m(E)]}{(2\beta + r)r[X + Z - 2\beta m(E)]}\right\} \\ &= [UM - US - c(E)]\left\{\frac{2\beta + r + q}{r[4\beta + 2r + m(E)]}\right\}. \end{aligned} \tag{24}$$

The final step involves incorporating (24) into both (22) and (23) to get the net gains to each party, solely as a function of E and system parameters. This yields

$$WD - WS - R = \frac{UM - US + c(E)}{4\beta + 2r + m(E)}. \tag{25}$$

Expression (25) now permits the determination of the buyer's level of search effort E. Combining (25) with the optimal search condition (20) gives the optimal search rule as an implicit function only of E and the system parameters:

$$\frac{\partial c}{\partial E} = \frac{\partial m}{\partial E}\left[\frac{UM - US + c(E)}{4\beta + 2r + m(E)}\right]. \tag{26}$$

The solution to this system is now recursive. Given a functional form for the c and m functions, equation (26) is first solved for E. With (13), this solution then determines q and L, and together, these determine all prices and present values with (23)–(25). This solution and its comparative steady-state properties are investigated next.

V. Comparative Steady States

A number of comparative steady-state results are obtained quite easily, as long as the technology of search depends only on search effort E and is not influenced by vacancy. In the analysis that follows, it must be remembered that some system parameters may have indirect effects on endogenous variables, in addition to any direct effects. For example, propositions 1 and 2 dealt only with the direct impact of parameter changes on sale length L and mismatching (HS/H), when the matching rate m was fixed. The full steady-state analysis must also incorporate the possible impact of a parameter change on E (and hence m) and then the indirect effects of this on both L and HS. The comparative steady states will focus on the impact of turnover rates (β) and vacancy (V) on housing prices (R), sales length (L), mismatching (HS/H), and search effort (E).

Changes in Vacancy

As vacancy changes, search effort remains unchanged, as long as vacancy is not part of the search technology.[6] Thus any total differentiation of sales price with respect to vacancy need focus only on any direct partial impacts and vacancy's effect on price through the sales rate q.

[6] While prices directly depend on q (and hence V through [13]), the gain from search (eq. [25]) does not. Therefore, the level of vacancy enters the search condition (26) only as it may influence the technology of search.

PROPOSITION 3. If vacancy does not alter the matching technology, then all the vacancy derivatives of propositions 1 and 2 hold and, in addition,

$$\frac{dE}{dV} = 0, \quad \frac{dR}{dV} = \frac{\partial R}{\partial q}\frac{\partial q}{\partial V} < 0.$$

Changes in the Turnover Rate β

As the turnover rate exogenously increases, for example from greater demographic change or economic fluctuation, search effort E will decrease. This is established in the following equation by totally differentiating the search rule (26) and incorporating the characteristics of the search cost and matching functions from Section III:

$$dE\left[\frac{\partial^2 c}{\partial E^2} - \frac{\partial^2 m}{\partial E^2}\left(\frac{\partial c}{\partial E}\Big/\frac{\partial m}{\partial E}\right)\right] = d\beta\left[\frac{-4\partial c/\partial E}{4\beta + 2r + m(E)}\right]. \quad (27)$$

This yields propositions 4 and 5.

PROPOSITION 4. $dE/d\beta < 0$.

PROPOSITION 5. From propositions 4 and 1,

$$\frac{dHS}{d\beta} = \frac{\partial HS}{\partial \beta} + \frac{\partial HS}{\partial m}\left(\frac{\partial m}{\partial E}\frac{dE}{d\beta}\right) > 0.$$

The impact of greater turnover on the length of sale ($1/q$) or on house prices is more complicated and cannot be ascertained unambiguously without explicit parameterization of the m function.

PROPOSITION 6.

$$\frac{dR}{d\beta} = \frac{\partial R}{\partial \beta} + \frac{\partial R}{\partial q}\frac{dq}{d\beta} \gtreqless 0,$$

$$\frac{dq}{d\beta} = \frac{\partial q}{\partial \beta} + \frac{\partial q}{\partial m}\frac{\partial m}{\partial E}\frac{dE}{d\beta} \gtreqless 0.$$

It is possible to prove that for a category of specific m functions (e.g., m is linear in E), $dq/d\beta$ is positive, although $\partial R/\partial\beta$ is negative. The magnitude of $\partial R/\partial q$ is large enough relative to $\partial R/\partial\beta$, however, so that the total effect of turnover on prices, $dR/d\beta$, is positive. For more general specifications of m, this cannot be demonstrated. Thus with the exception of proposition 6, most comparative steady-state derivatives can be signed, as long as the matching rate is independent of the number of vacant units. When this is not the case, the analysis becomes more complicated, as some numerical examples will demonstrate. Before I examine these, it is useful to examine several normative properties of this model.

VI. Welfare Issues

In labor market matching models, the private search decisions by firms or households generally turn out *not* to be welfare maximizing. In the model developed here, this is also the case.

Social welfare in this model is defined as the present discounted value of aggregate net (of search cost) utility. The most direct welfare analysis would involve a comparison of steady-state utility levels: can the steady-state discounted value of utility be improved by adjusting search effort from that level privately chosen? Diamond (1980) has proposed a more realistic welfare test, asking if there is a level of search effort that will increase the discounted value of utility when utility moves along an adjustment path from the market steady state to this welfare-improving steady state.[7] This formulation, while slightly more complicated, tends to yield more powerful results. Welfare is thus the integral

$$W = \int e^{-rt}[HS(US - c) + (H - HS)UM]dt \tag{28}$$

when HS starts at time 0 in the market steady state and moves according to the system of differential equations (9) and (10). For notational ease, the direct dependence of c and m on E will not be depicted in this section.

The Diamond welfare test asks whether (28) can be increased by adjusting search effort from its private level, allowing all endogenous variables to respond according to (9) and (10). This requires merely evaluating the derivative of (28) with respect to E, incorporating (9) and (10). From Diamond (1980), this yields

$$\frac{dW}{dE} = -\frac{HS}{r}\frac{\partial c}{\partial E} - \frac{1}{r}[UM - US + c, 0][HH]^{-1} \times \begin{bmatrix} -\frac{\partial m}{\partial E}HS \\ \frac{\partial m}{\partial E}HS\left(1 - \frac{HD}{V}\right) \end{bmatrix}, \tag{29}$$

where $[HH]$ is the matrix

$$\begin{bmatrix} r + 2\beta + m & \beta \\ -m\left(1 - \frac{HD}{V}\right) & r + \frac{mHS}{V} \end{bmatrix}.$$

[7] The fully optimal policy is to select a time trajectory of search effort that maximizes welfare, starting from the market steady state. Diamond's proposal is a compromise

When all derivatives are evaluated at the initial market steady-state solution (where $HD = V$), the derivative (29) is simplified to

$$\frac{dW}{dE} = \frac{HS}{r}\left(-\frac{\partial c}{\partial E} + \frac{\partial m}{\partial E}\,\frac{UM - US + c}{2\beta + r + m}\right). \tag{30}$$

In expression (30), the term in parentheses is identical to the private search condition (26) except that the denominator on the right-hand side is $2\beta + r + m$ instead of $4\beta + 2r + m$. Thus at the initial market steady-state solution, where E is privately optimal (and [26] holds), welfare is improved with further search ($dW/dE > 0$). Further search is optimal since effort is expended only by the buyer; yet benefits are created for the seller that are not received by the searcher.

The second normative issue with this model involves the question of whether in the long run the number of vacant units is optimal. As discussed in the Introduction, the model to this point has focused on the short-run determination of matching, prices, and expected length of sale. These market conditions combine to determine an "expected price" for a new unit. This expected price must be such that r times that value equals the probability of sale times the gains from sale (market price minus expected price). This gives an expected price equal to $R/[1 + (r/q)]$, which is essentially the market price discounted by the expected length of sale.

In the long run, with competitive, risk-neutral housing producers, new units will be supplied until the expected market price equals the marginal cost of such units. The optimality of this private supply decision can be ascertained by comparing the expected market price to the marginal "social" value of an extra housing unit. To determine the social value of additional units, (28) is differentiated this time with respect to V, again using equations (9) and (10) to govern the transition path of HS. The derivative takes the same form as (29), but with dc/dE replaced by dc/dV ($= [\partial c/\partial E][dE/dV]$) and dm/dE replaced by dm/dV ($= [\partial m/\partial V] + [\partial m/\partial E][dE/dV]$). When this derivative is evaluated at the market steady-state solution (where $HD = V$ and $q = mHS/V$), the following expression is obtained:

$$\frac{dW}{dV} = \frac{HS}{r}\left\{\frac{dE}{dV}\left[-\frac{\partial c}{\partial E} + \frac{\partial m}{\partial E}\left(\frac{UM - US + c}{2\beta + r + m}\right)\right] + \frac{\partial m}{\partial V}\left(\frac{UM - US + c}{2\beta + r + m}\right)\right\} + \frac{(UM - US + c)\beta}{r(2\beta + r + m)[1 + (r/q)]}. \tag{31}$$

formulation between simple comparative steady states and full optimization, in which the control variable (search effort) is restricted to be a scalar, but the system is not presumed to switch steady states instantaneously.

When vacancy plays no role in the search technology ($\partial m/\partial V = dE/dV = 0$), then all the terms in (31) vanish except for the last. Thus the derivative dW/dV will be positive without any "productive" effect by vacancy in the search process. When vacancy facilitates search, the first term within the braces will be positive since it is merely the optimal search expression (30). When (31) is evaluated at the market steady state, the social gains from search are positive. The second term within the braces is positive by definition, and thus so will be the overall sign of dW/dV.

The question, of course, is how expression (31) compares with the expected market price that will determine private supply. Comparing $R/[1 + (r/q)]$ to expression (31) is simple in the case in which vacancy does not facilitate housing search.

PROPOSITION 7. When $dE/dV = \partial m/\partial V = 0$, then from (31), (13), and (24),

$$\frac{dW}{dV} = \frac{UM - US + c}{r(2\beta + r + m)} \frac{\beta}{1 + (r/q)} < \frac{R}{1 + (r/q)}.$$

At any vacancy rate, then, the marginal social value of additional units is less than the market expected price. Thus private behavior will involve not only too little search but excess supply as well, again in the case in which vacancy does not influence the matching process.

In labor search models, it is possible under certain conditions to correct both of these resource misallocations by giving all the gains from search (which here accrue to both buyer and seller) to just that party undertaking the search (here the buyer) (Diamond 1982). This turns out to be true in this model as well *if* vacancy does not facilitate search. If price is determined so that all the gains go to the searcher, then the seller is indifferent, and we get the following proposition.

PROPOSITION 8. If $R = WD - WS$ replaces (19) and (15)–(20) are resolved,

$$R = \frac{(UM - US + c)\beta}{r(2\beta + r + m)} = \frac{dW}{dV}\left(1 + \frac{r}{q}\right) \quad \text{(efficient supply)}$$

and

$$\frac{\partial c}{\partial E} = \frac{\partial m}{\partial E} \frac{UM - US + c}{2\beta + r + m} \quad \text{(efficient search effort).}$$

In the more general case in which vacancy plays some role in the search process, giving all the gains of search to the buyer will still lead to the correct level of search effort but will not necessarily produce the correct level of supply. More important, the practical consequences of this line of argument are not clear. In a market economy, it is not easy to conceive of an actual policy instrument that could be used to reallocate the gains from trade. Thus generally, with the market determining trade shares and prices through bargaining, an efficient allocation of resources will necessitate a specific policy intervention both for supply and for search.

VII. Specific Examples

The two examples of search technology discussed in Section III can serve to illustrate the broad range of solutions that are likely to occur in the model above.

The first example assumes that the vacancy rate plays no role at all in the matching process. In this formulation, the matching rate (m) is simply equal to $E/2$, where E is search effort (e.g., the number of visits to vacant units) and ½ is the likelihood of finding the "right" unit on each visit. Search costs will equal c_0E^2.

All the remaining model parameters are straightforward, with flows expressed in annual rates. The base values used are a 5 percent real interest rate, a 10 percent turnover rate, and a 5 percent vacancy rate. The annual utility derived from a matched (mismatched) house is assumed equal to $10,000 ($5,000). Since the two-house typology creates a very high probability of finding the "right" house on any visit, the simulation can be made more realistic by setting the cost of search high. A value of c_0 = $1,000 results in low search effort but an empirically reasonable matching rate. Table 1 gives the simulation results under the base case, along with variations as the turnover rate (β) changes. Also shown is the optimal level of search effort, calculated by setting (30) to zero, using base parameter values. Figure 3 shows the relationship among vacancy, house prices, and the expected length of house sale.

Perhaps the most interesting result in table 1 and figure 3 is the pronounced effect that vacancy has on house prices and expected sales time. As vacancy ranges from 2 percent to 10 percent, which roughly corresponds to the maximum range observed across "tight" and "loose" metropolitan markets, prices vary by almost fourfold.

TABLE 1

MODEL SOLUTIONS (Nonproductive Vacancy)

Case	E	m	HS/H	L	R
Base	1.44	.722	.103	.67	$201,000
β = .2	1.06	.530	.203	.46	$224,000
β = .05	1.71	.857	.050	1.17	$137,000
Optimal	1.79	.895	.087	.64	$212,000

NOTE.—Base parameters are V/S = .05, β = .1, r = .05, UM = 10,000, US = 5,000, and c_0 = 1,000.

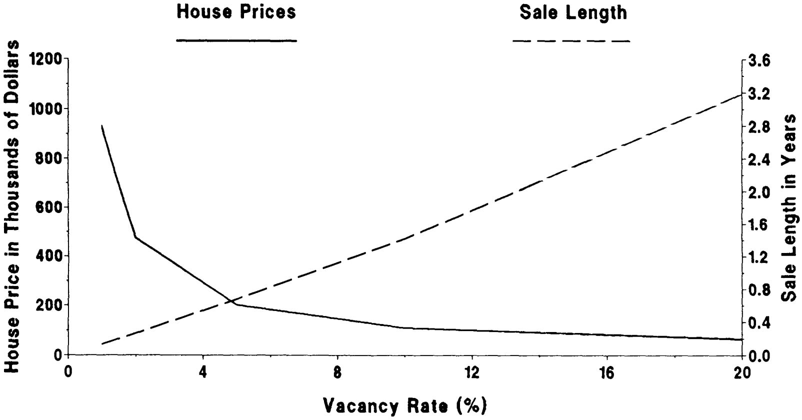

FIG. 3.—Comparative steady states: nonproductive vacancy

The solutions also suggest that the exogenous factors that create household change and mismatching (here modeled as the turnover parameter β) can play an equally important role in determining house prices. A doubling of the turnover rate (from .05 to .10) creates a 40 percent increase in prices, while an increase from .10 to .20 creates a smaller, 15 percent, increase. Finally, the optimal level of search effort is roughly 30 percent higher than that chosen privately. This results in less mismatching and higher house prices.

In the second example, matching is substantially facilitated with greater vacancy since vacant units must be found or identified before they can be inspected. Thus visits or search effort E is "blind" with a likelihood (V/S) of finding a vacant unit and then a probability of ½ that the unit is the "right" one. In this second example, the costs of search will still be c_0E^2, but the matching rate m will equal $(V/S)E/2$.

This second model yields a base solution identical to the first when all parameters are the same as those in table 1, with the exception of search costs. The cost function is the same, but c_0 is set to only \$2.50 rather than \$1,000. This yields enough additional search effort to counteract the effect of being forced to find units that are for sale and produces matching rates identical to those in the first example. Thus in the base solution to this second model, search effort E winds up at 28.9 (rather than 1.44), but matching, sales length, and prices are all the same as in table 1. The impact of a change in the turnover rate β in this model also produces values identical to those in the first example.

This second example of a search technology, however, produces a striking difference in the relationship between vacancy and market prices or sales time (fig. 4). Since the number of matches is now

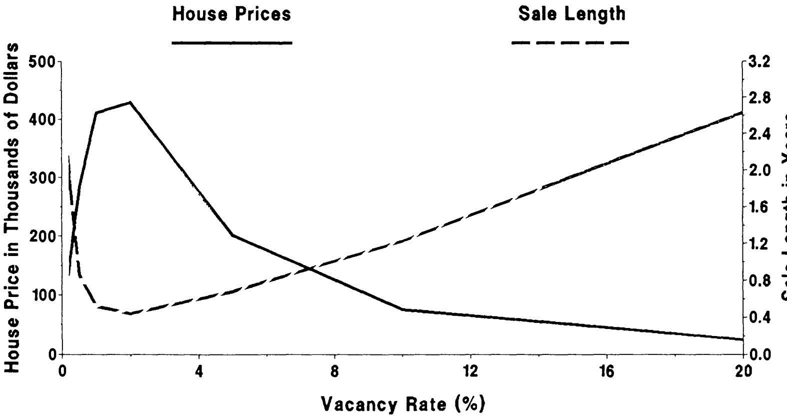

FIG. 4.—Comparative steady states: vacancy alters search success

directly proportional to the vacancy rate, matching becomes impossible without some vacant units. At very low vacancy rates, the ability to match is so poor that even with few units, the length of a sale is long and prices are low. As vacancy rises from zero, there is a dramatic increase in the likelihood of successful search, which in turn encourages search effort. Even with greater vacancy, the increased matching that results is sufficient to lower the sales length of houses and thereby raise prices. In effect, initial increases in vacancy help to "unlock" a "frozen" market, and this generates faster sales and higher prices. Beyond a 1 or 2 percent vacancy rate, this effect diminishes, and greater vacancy increases sales time and reduces prices, as in the first example.

There are also interesting differences between the two models when the expected market price of additional housing is compared to the "social" value of such supply. In table 2, this calculation is performed for each model under varying vacancy rates. In the first example, vacancy plays no "productive" role in search, and from proposition 7, the returns from greater vacancy are significantly below the expected market price. In fact, there is no reason for any vacancy with this search technology. This is mathematically possible and, from (9), would result in no *HD* households, instantaneous sales, and infinite prices.

With the second model, vacancy has a strong positive effect on the matching rate, and this greatly increases its social value. At vacancy levels of less than about 10 percent, the social value of an extra unit exceeds the expected market price from development, while at higher vacancy levels, the social value is less than the expected price. Thus with this example, the market could under- or overprovide housing,

TABLE 2

PRIVATE AND SOCIAL HOUSING VALUES

	First Example		Second Example	
Vacancy	Expected Price	Social Value	Expected Price	Social Value
.002	4,553,000	14,551	134,000	329,000
.005	1,831,000	14,523	266,000	696,000
.01	928,000	14,770	401,000	983,000
.02	474,000	14,383	419,000	812,000
.05	195,000	14,097	195,000	249,000
.10	103,000	13,606	72,000	74,000
.20	56,000	12,566	23,000	19,000

NOTE.—All parameter values are the base values in table 1.

depending on where the industry cost schedule intersects the price/vacancy relationship that emerges from short-run market clearing.

VIII. Conclusions

This paper has developed a simple matching model that deals with housing turnover, search, and pricing in an owner-occupied market. Despite its simplicity, as a positive tool, the model provides realistic explanations for the empirically observed behavior of housing markets. The model explains how prices and vacancy can coexist and suggests that small changes in supply or demand, as they alter vacancy, can have very profound impacts on market prices. This results because in a single-family market, buyers are also sellers, and moving or changing houses involves only transactions "costs." In this context, high prices do little to dampen demand. The model also explains how, given fixed vacancy, greater market turnover can generate higher housing prices. Thus in the long run, with prices ultimately determined by production costs, market activity or turnover can explain differences in permanent or "structural" vacancy.

The model raises several normative issues as well, which call into question the efficiency of housing markets. Consistent with labor market search models, private search effort is socially suboptimal. The model also suggests that the supply of housing may not be optimal because market prices do not necessarily reflect the correct social value of additional units.

Even in its current, quite simple, form, the model has the potential to address a number of additional economic issues. The comparative dynamics of housing markets could be studied, as well as the properties of a wider range of search technologies. With only minor revision, the model could be reformulated to reflect a rental market and might be used to analyze the role that leases play in dividing the risk of vacancy between tenant and landlord. The model is also nicely set up to examine the impact of transaction or turnover-type taxes on this kind of market. The likely impact of such taxes would be to lower transaction gains, reduce search, and inevitably increase the degree of mismatching. These and related questions offer some interesting directions for future research.

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