Welfare Implications of a Discretized Sequential Hotelling Firm Location Choice Model *

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Abstract

We investigate the implications of introducing sequential firm entry into Hotelling's location choice model. We computationally solve firms' location decisions in a discrete $[0, 1]$ space. We then examine how consumer welfare can be affected by the firms' sequential location choices. Using this framework, we can consider the effects of government interventions on consumer welfare.

keywords: Hotelling's Model, Location Choice

1 Introduction

We consider a market with sequential firm entry. Firms choose where to operate from a finite set of open locations. In this type of market, multiple factors can affect both where firms locate and how many firms choose to enter the market. These involve fixed costs and the number of locations available, among other factors. We investigate how these two factors, as well as the number of firms in the market, affect firm location choices, prices, and profits. With a better understanding of these dynamics, we then ask how they affect consumer welfare. Finally, we consider how governments can intervene to increase consumer welfare.

We find that as fixed costs decrease, firms strategically relocate to deter future firm entry. This decreases firm profits but increases consumer welfare. As the number of firms increases, firm profits decrease and consumer welfare increases. Notably, while firms locate relatively equidistantly, early entrants have a distinct profit advantage over late entrants. Finally, as the number of locations increases, early entrants benefit from more location choice and have higher profits, while effects are unclear for late entrants. This decreases consumer welfare dramatically enough to result in a net decrease in total surplus. Finally, we discuss the implications of government policies to improve consumer welfare through fixed cost subsidies that incentivize firm entry.

2 Model

We consider a sequential version of Hotelling's location choice model. Firms do not enter simultaneously: instead, firms sequentially choose to open in unoccupied locations along a discrete $[0, 1]$ space. Once all stores are open, firms set prices. Consumers then choose where to purchase based on both price and distance. The algorithm used to solve the model computationally is explained in Section 3.

The firm's problem can be summarized into two stages. In the first stage, firms enter sequentially. Each firm chooses from the finite set of available locations to maximize expected profits. When making this decision, firms account for the best responses of future firms that may enter the market. In the second stage, firms take locations as given and simultaneously set prices.

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Each firm incurs some fixed (sunk) entry cost f and marginal cost c. Firm j solves the following profit maximization problem:

$$\max_{p_j} \pi_j = (p_j - c)q_j - f$$

where q_j is firm j's residual demand. q_j is defined by considering the location of consumers who are indifferent between purchasing from firm j and the firm(s) next to firm j. We specify how to calculate q_j for a three firm market in Section 3.1.

We assume homogenous consumers who are uniformly distributed along the measure one space $[0, 1]$. Each consumer i at location x_i receives the following surplus net of travel cost when purchasing a product from firm j:

$$U_i = V - p_j - t(x_i - i_j)^2$$

where V is the valuation of the product, p_j is the price of the product sold by firm j, and i_j is the location of firm j. Consumers have unit inelastic demand where V is sufficiently large such that they will always buy from some firm. A consumer decides to buy from the firm that gives her the highest surplus.

3 Algorithm

The algorithm used to solve the model is described below. Code to implement this algorithm in Python can be found in Appendix **??**.

a. Set exogenous variables N_f (number of firms), N_l (number of locations), c (variable cost), t (travel cost), and f (fixed cost). In the case with a fixed number of firms, set $f = 0$ without loss of generality.[1]

b. For *each possible set* of firm locations, solve *each firm's* analytical profit function.[2]

   (a) Given firm locations, analytically solve each consumer's preferred firm (as a function of consumer location and firm prices). In particular, this is accomplished by solving the locations of the $N_f - 1$ indifferent consumers.

   (b) Firms choose price to maximize profits, accounting for other firms' profit maximizing prices and the locations of indifferent consumers. In particular, take first order conditions of each firm's profit function to get a system of N_f equations.

   (c) Analytically solve the system of N_f equations to solve optimal p_i^*, and consequently optimal π_i^*, for each firm $i \in \{1, \ldots, N_f\}$.

c. Using backward induction, solve each firm's best location response function:

   (a) Start with firm N_f: set $i = N_f$.

   (b) Consider all firm location combinations for firms $j < i$.

   (c) For each combination, place firm i in each open location.

   (d) For each location, best response functions for firms $j > i$ are known.

   (e) Using each location combination for firms $j < i$; each location choice for firm i; and best response functions for firms $j > i$; calculate profits for firm i using the results from *b*. It is possible firms $j > i$ have multiple best responses: if this occurs, calculate expected profits taking all indifferent location decisions as equally likely.

   (f) For each location combination for firms $j < i$, take the location choice (choices) that maximizes (maximize) profits for firm i, setting this (these) as its best response (responses). If the maximum profit is negative, we assume firm i chooses not to enter the market (this cannot occur for the fixed number of firms case, as $f = 0$).

[1] For the case in which a fixed number of firms enter, we imagine that we choose a fixed cost f such that only the specified number of firms enter. Moreover, the fixed cost is such that the firms that do enter do not have to worry about deterring potential new firm entry beyond the specified fixed number of firms. We elaborate on how we can find such a fixed cost f in Section 4.2. Since including a non-zero fixed cost only shifts the profits generated by each firm by some finite, equal amount, we can consider $f = 0$ without changing the core dynamics of the problem.

[2] Section 3.1 contains a detailed analysis for the three firm case.

(g) Once firm i's best response is known for each location combination for firms $j < i$, if $i > 1$ then decrease i by 1 and go back to step (b). If $i = 1$, then the process is complete.

d. Take firm 1's best response function and determine which location (or locations) maximize its expected profits. Iterate through each possible combination of best responses for firms $i > 1$ to get all possible equilibrium outcomes.

3.1 Optimal Prices and Profits in a Market with Three Firms

We explain how prices (and consequently profits) are determined for a given set of location combinations in a three firm market. We consider a three firm case for simplicity. Consider three firms a, b, c located at i_a, i_b, and i_c, in this order along a discretized $[0, 1]$ space. Each firm produces an identical good.

Consider x_1 as the location of the consumer who is indifferent between purchasing the good from firm a or firm b. For such a consumer, the surplus gained by purchasing from either firm a or firm b will be equivalent:

$$\begin{aligned} V - p_a - t(x_1 - i_a)^2 &= V - p_b - t(x_1 - i_b)^2 \\ \Leftrightarrow p_a + t(x_1 - i_a)^2 &= p_b + t(x_1 - i_b)^2 \\ \Leftrightarrow x_1 &= \frac{p_b - p_a}{2t(i_b - i_a)} + \frac{i_a + i_b}{2} \end{aligned}$$

Similarly, for the indifferent consumer between firms b and c, whose location we denote by x_2, we have the following:

$$\begin{aligned} p_b + t(x_2 - i_b)^2 &= p_c + t(x_2 - i_c)^2 \\ \Leftrightarrow x_2 &= \frac{p_c - p_b}{2t(i_c - i_b)} + \frac{i_b + i_c}{2} \end{aligned}$$

Therefore, firm a's profit maximization problem can be represented as the following:

$$\begin{aligned} \max_{p_a} \pi_a &= x_1(p_a - c) - f \\ &= \left(\frac{p_b - p_a}{2t(i_b - i_a)} + \frac{i_a + i_b}{2} \right) (p_a - c) - f \end{aligned}$$

We have the following first order condition:

$$[p_a]: \quad \frac{p_b - p_a}{2t(i_b - i_a)} + \frac{i_a + i_b}{2} - \frac{1}{2t(i_b - i_a)}(p_a - c) = 0 \tag{3.1.1}$$

Similarly, firm b's profit maximization problem can be represented as the following:

$$\begin{aligned} \max_{p_b} \pi_b &= (x_2 - x_1)(p_b - c) - f \\ &= \left[\left(\frac{p_c - p_b}{2t(i_c - i_b)} + \frac{i_b + i_c}{2} \right) - \left(\frac{p_b - p_a}{2t(i_b - i_a)} + \frac{i_a + i_b}{2} \right) \right] (p_b - c) - f \end{aligned}$$

We have the following first order condition:

$$[p_b]: \quad \left(\frac{p_c - p_b}{2t(i_c - i_b)} + \frac{i_b + i_c}{2} \right) - \left(\frac{p_b - p_a}{2t(i_b - i_a)} + \frac{i_a + i_b}{2} \right) - \frac{1}{2t(i_c - i_b)}(p_b - c) - \frac{1}{2t(i_b - i_a)}(p_b - c) = 0 \tag{3.1.2}$$

Finally, firm c's profit maximization problem can be represented as the following:

$$\begin{aligned}\max_{p_c} \pi_c &= (1 - x_2)(p_c - c) - f \\ &= \left(1 - \left(\frac{p_c - p_b}{2t(i_c - i_b)} + \frac{i_b + i_c}{2}\right)\right)(p_c - i_c) - f\end{aligned}$$

We have the following first order condition:

$$[p_c]: \quad 1 - \frac{p_c - p_b}{2t(i_c - i_b)} - \frac{i_b + i_c}{2} - \frac{1}{2t(i_c - i_b)}(p_c - c) = 0 \tag{3.1.3}$$

We can use the first order conditions (3.1.1), (3.1.2), and (3.1.3) to solve for p_a^*, p_b^*, p_c^*, and thus profits π_a^*, π_b^*, π_c^* as the following:

$$\begin{aligned}\pi_a^* &= x_1(p_a^* - c) - f \\ \pi_b^* &= (x_2 - x_1)(p_b^* - c) - f \\ \pi_c^* &= (1 - x_2)(p_c^* - c) - f\end{aligned}$$

4 Results

To investigate the consequences of a market with sequential location choice, we consider the following three cases of our model:

1) Fixed costs are positive and firms enter if profits will be non-negative. We consider the effects of changing fixed costs. Results can be found in Section 4.1.

2) Fixed costs are zero (without loss of generality)[3] and the number of firms entering the market is fixed. We consider the effects of changing the number of firms. Results can be found in Section 4.2.

3) Fixed costs are zero (without loss of generality)[4] and the number of firms entering the market is fixed. We consider the effects of changing the number of locations. Results can be found in Section 4.3.

For each case, beyond determining each firm's optimal location and profit, we also analyze consumer welfare. We measure an individual's consumer welfare as the following:

$$CW = \int_0^1 U_i = \int_0^1 V - p_j - t(x_i - i_j)^2 dx_i$$

where U_i is calculated using price and location for the firm that maximizes consumer i's individual welfare. We calculate consumer welfare generated by each firm j based on the following expression:

$$CW_j = \int_{x_{j,l}}^{x_{j,r}} U_i = \int_{x_{j,l}}^{x_{j,r}} V - p_j - t(x_i - i_j)^2 dx_i$$

where $x_{j,l}$ and $x_{j,r}$ represent the location of the indifferent consumer on the left and right of firm j, respectively. For the leftmost firm along the $[0, 1]$ space, the left indifferent consumer will be zero. Similarly, for the rightmost firm along the $[0, 1]$ space, the right indifferent consumer will be one. Aggregating each firm's contribution to consumer welfare, we can represent total consumer welfare as the following:

$$CW = \sum_j CW_j$$

In all cases, variable costs are set to $c = 1$, travel costs are set to $t = \frac{1}{2}$, and consumers' valuations for the good are set to $V = 100$.

[3] Please refer to Footnote 1.
[4] Please refer to Footnote 1.

4.1 Increasing Fixed Cost for Endogenous Firm Entry

The following figures give results for endogenous firm entry with increasing fixed costs. The first set of figures gives results for $f = \frac{1}{50}$, where each sub-figure is equivalent by symmetry. The second set of figures gives results for $f = \frac{2}{50}$. The third set of figures gives results for $f = \frac{3}{50}$. Finally, the fourth set of figures gives results for $f = \frac{4}{50}$. For all four cases, we see that only two firms decide to enter the market. While we computationally solve the problem assuming at most three firms can enter, we know that if the third firm chooses not to enter then no other firm will enter. This is because all firms are identical: if the next firm is not profitable, no future firm will be.

Figure 1: $f = \frac{1}{50}$ (11 Locations), $CW = 98.7836$, $PS = 0.1609$, $TS = 98.9409$

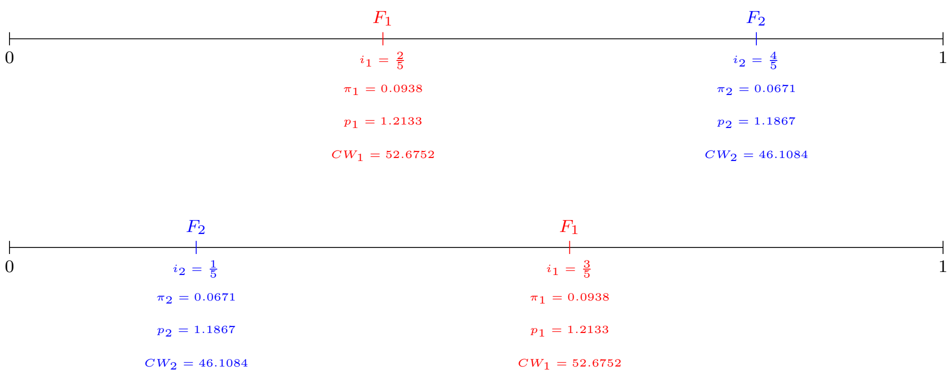


Figure 2: $f = \frac{2}{50}$ (11 Locations), $CW = 98.6262$, $PS = 0.2735$, $TS = 98.8935$

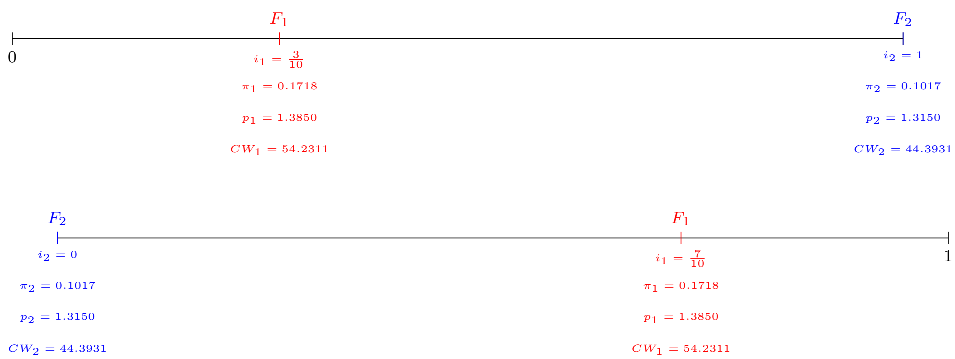


Figure 3: $f = \frac{3}{50}$ (11 Locations), $CW = 98.5184$, $PS = 0.3304$, $TS = 98.8504$

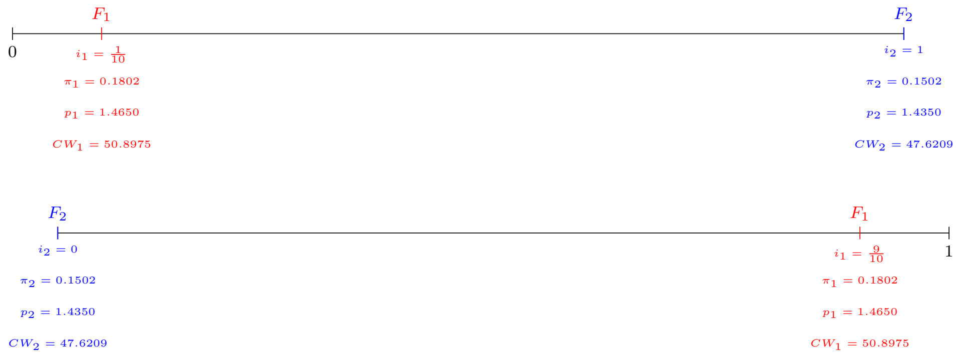
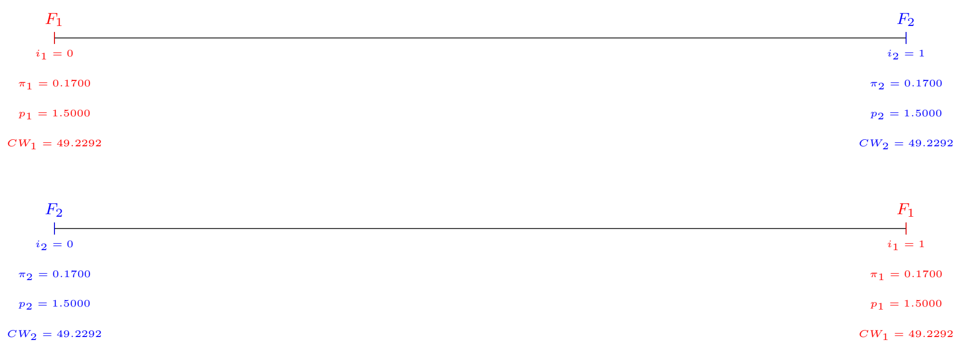


Figure 4: $f = \frac{4}{50}$ (11 Locations), $CW = 98.4584$, $PS = 0.3400$, $TS = 98.7984$



From the figures we can understand how firms react to changes in fixed costs. Having fixed costs means that firms can deter future firm entry: if future firms' expected profits are below the fixed cost, they will not enter. Therefore, early entrants may choose locations to deter future firm entry. Deterrence occurs through location choice. Firms will choose to locate to be more equally spaced in order to reduce the potential market size that new entrants can acquire. With low fixed costs, firms' locations become closer to equidistant. However, as fixed costs increase, early entrants can become less concerned about potential future entrants. Firms locate further apart while still ensuring future firms will not enter.

We can see from the figures above that lower fixed costs are also generally associated with higher consumer welfare. This effect can be understood through the deterrence behavior of firms. Firms' decisions to become more equidistant in order to deter new potential firms from entering causes total consumer welfare to increase through two channels. First, consumers will be closer to some firm on average, meaning the average consumer will have less disutility from traveling. This is particularly important because travel costs are quadratic in distance - the increase in costs from being farther from some firm are far greater than the decrease in costs from being closer to that firm. Second, closer firms compete more on price, meaning individuals pay less for consumption. The competition effect on prices can be observed in the figures above.

4.2 Increasing Number of Fixed Firm Entry

The following figures show firms' optimal location decisions when each firm knows exactly how many firms will enter the market given some fixed cost. From Section 4.1, we can see that as f increases, the firms appear to be moving their locations towards the optimal locations when there are a fixed number of firms with $N_f = 2$. We can argue why this must occur: with positive fixed costs $f > 0$, firms can begin to deter future firms from entering by repositioning themselves. They will reposition themselves to be more equidistant to reduce potential market share (and thus potential profits) for future firms. However, this repositioning must be sub-optimal from the firms' perspectives. If these positions were optimal, the firms would already be choosing (or be indifferent about choosing) these locations. However, as f increases, firms require less repositioning in order to deter future firm entry. There is a critical value of f at which the firms can behave optimally as if no more firms will enter, and future firms will still not enter.

Let's investigate how we can find such a critical value of f in the three firm case. From the third firm's perspective, the best situation is when the two firms optimize as if only two firms will enter and do not consider deterrence. This maximizes the third firm's potential market share and profits (otherwise, they are deterring which reduces the third firm's optimal profits). A fixed cost set to exactly this profit is the critical value at which the first two firms no longer need to consider deterrence. For fixed costs below this profit, the third firm may consider entering, and so (up to a point) the first two firms will locate strategically to deter entry. For fixed costs above this profit, the first two firms can locate as if the number of firms is fixed at two, and the third firm will still choose not to enter. We know that such an f exists and is less than the profits of any firm already in the market. This is because the third firm's maximum potential profits must be strictly less than the profits of either of the other two firms in the two firm market (because profits are strictly decreasing in the number of firms, so firms in the three firm market will have lower profits than firms in the two firm market).

Given that such an f exists and we can find it, in this section we assume for simplicity that a fixed number of firms enters the market with $f = 0$. This allows us to estimate the effects of changing the number of firms in the market without explicitly calculating the threshold value of f, as explained in Footnote 1.

Figure 5: 2 Firms (11 Locations), $CW = 98.4584$, $PS = 0.5000$, $TS = 98.9584$

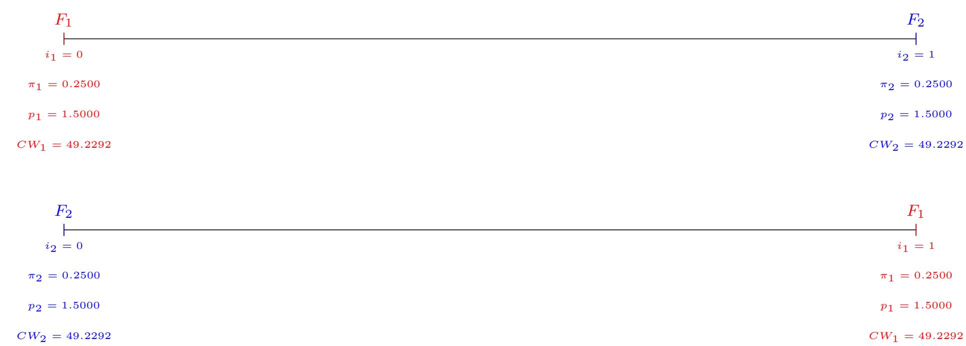


Figure 6: 3 Firms (11 Locations), $CW = 98.8934$, $PS = 0.1004$, $TS = 98.9938$

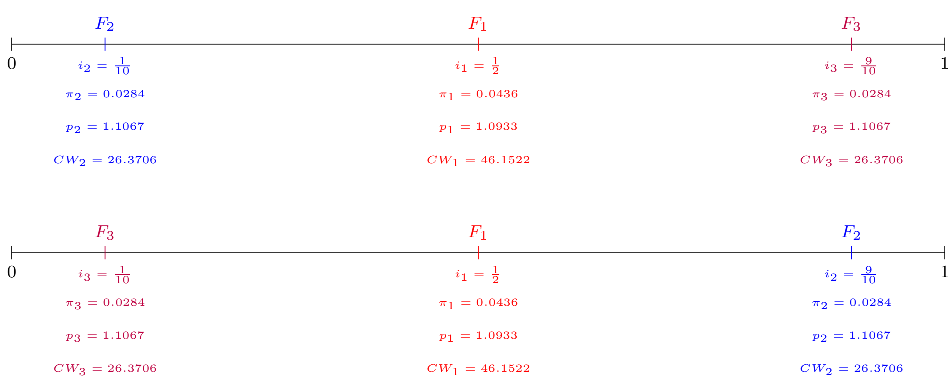
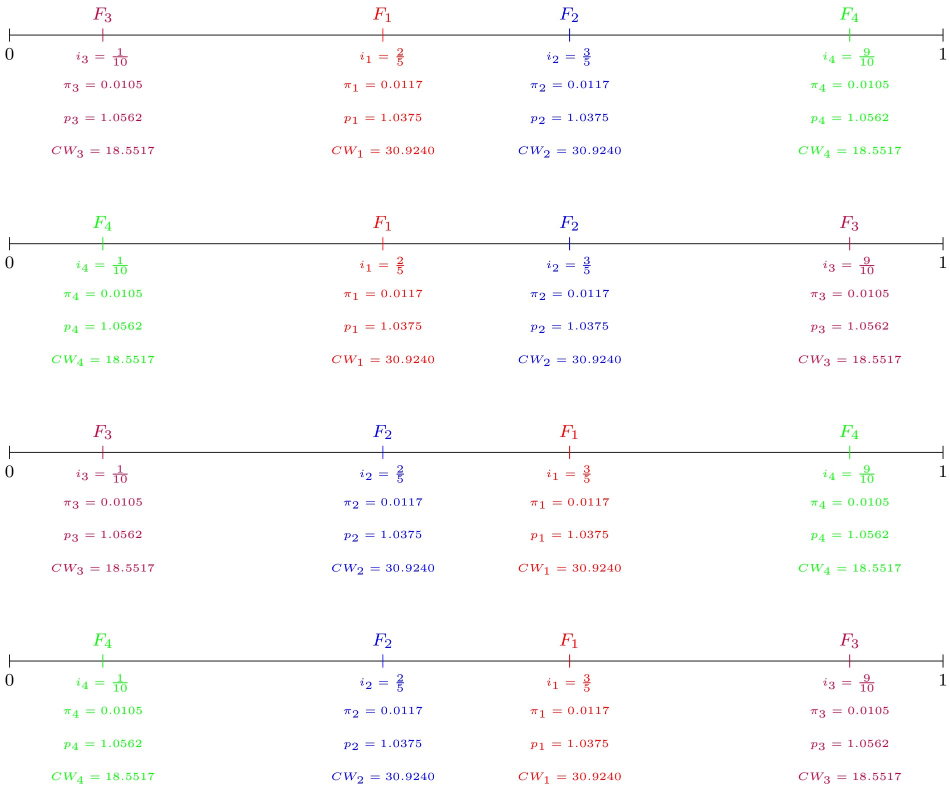


Figure 7: 4 Firms (11 Locations), $CW = 98.9514$, $PS = 0.0444$, $TS = 98.9958$



For the two firm case in Figure 5, we have that the firms locate at 0 and 1. These are also the optimal location choices when two firms choose location simultaneously. We expect these results because firms want to maximize differentiation if they are not considering deterrence. Being closer to each other leads to higher competition. This lowers prices and profits. Similarly, for the case when we have more than 2 firms enter, we see that firms try to locate (relatively) equidistantly. This is similar to results we see in the case of Salop's Circle, where firms are equally spread out. However, in this case we are considering the linear space $[0, 1]$, which prevents firms from locating exactly equidistantly as in the case of a circle.

In this sense, we note that the location decisions for a sequential and simultaneous problem are very similar. Moreover, discretizing the possible locations does not alter the core solutions to the decision problem. However, we note that there is an advantage to being an early entrant: firms that enter the market earlier have a higher profit than firms that enter the market later. We also note that firms exploit the symmetry of the model. When firms choose location, they iteratively reflect their location with respect to the center. For instance, in the four firm case we see that firm 1 and firm 2 choose opposite sides, and similarly, firm 3 and firm 4 choose opposite sides.

Increasing the fixed number of firms that enter leads to more competition and thus price decreases. Further, firms end up locating more equidistantly over the $[0, 1]$ space. As described in Section 4.1, these two effects increase consumer welfare. This can be seen in the figures above. Thus, increasing the number of firms that enter the market is strictly beneficial for consumer welfare.

4.3 Increasing Number of Available Locations

The following figures display the consequences of changing the number of possible locations for a fixed number of firms. As discussed in Section 4.2, we expect similar results to the simultaneous problem. Therefore, we do not expect the results to change dramatically as we alter the number of possible locations.

Figure 8: 3 Firms (11 Locations), $CW = 98.8934$, $PS = 0.1004$, $TS = 98.9904$

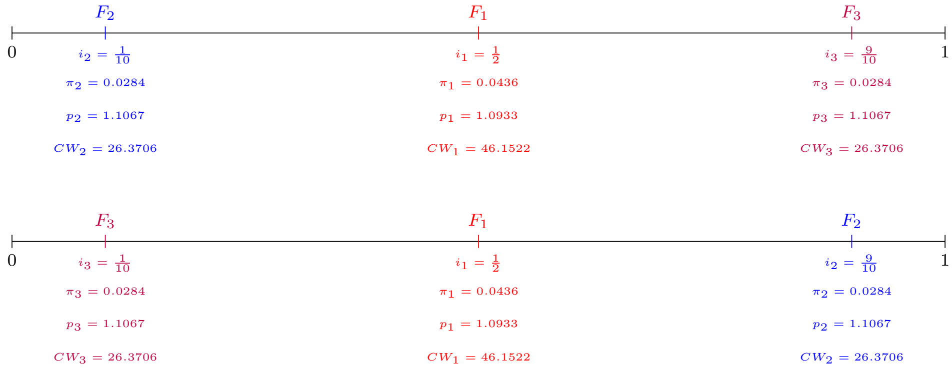


Figure 9: 3 Firms (16 Locations), $CW = 98.8844$, $PS = 0.1083$, $TS = 98.9883$

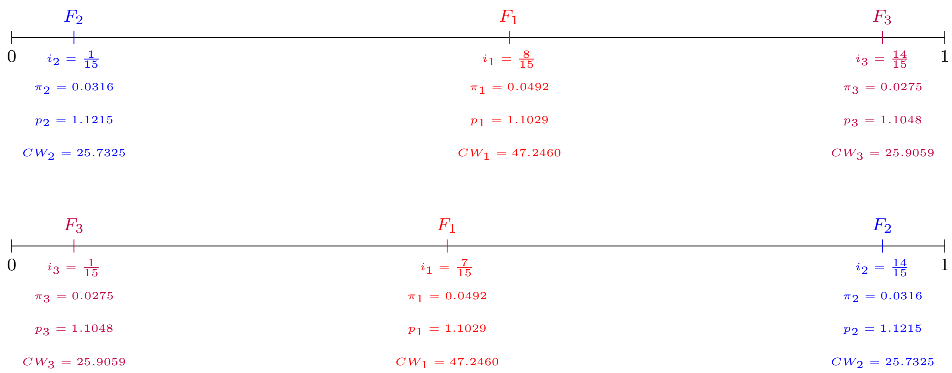
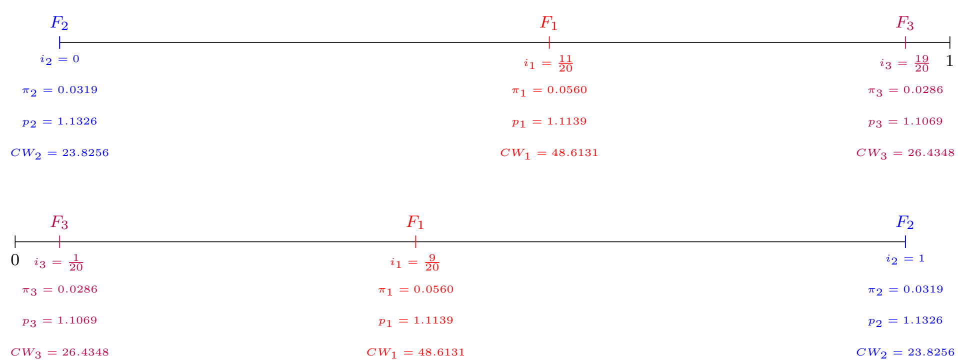


Figure 10: 3 Firms (21 Locations), $CW = 98.8735$, $PS = 0.1165$, $TS = 98.9865$



Similar to the base case with 11 locations, we expect firms to be relatively equally spaced out as seen in Section 4.2. We also see that as in Section 4.2, the first firm chooses a location very close to the center. However, the first firm does not always choose exactly the center. We believe this result arises from asymmetry. For this case, increasing the number of available locations seems to decrease consumer welfare. This is because firms end up less equidistant from each other.

These results are likely a consequence of considering a finite number of locations. As the number of available locations increases, location choices will converge to the optimal when considering a continuum of location availability. However, with a finite number of locations, firms must compromise by selecting locations that may slightly deviate from their theoretically most optimal location. This explains why the first firm would select the exact middle with a sufficiently small number of available locations, even if it eventually converges to a spot slightly off from the center as the number of locations increases. As the number of locations increases, firms have more options about where to locate and early entrants should be weakly better off. While it may be preferable for firms to increase the number of available locations, this comes at the expense of consumers: we see both an increase in profits for early entrants and a decrease in consumer welfare as the number of available locations increases.

5 Discussion and Conclusion

In this paper we consider a sequential entry variation to Hotelling's location choice model. We consider the effects of the model through three case studies. First, we consider the effects of changes in fixed costs with endogenous firm entry. We find that decreasing fixed costs causes firms to become more equidistant across the $[0, 1]$ space, decreasing profits but increasing consumer welfare. This shift reflects strategic decisions by each firm to deter future firm entry by reducing new entrants' potential market share. Second, we consider the effects of changes in the number of firms when a fixed number of firms enter the market. We find that in one sense, these results are similar to the results from the simultaneous entry setting in Salop's circle - firms are relatively equidistant. However, unlike the simultaneous model, there is asymmetry in profits - early entrants have higher profits than late entrants. Unsurprisingly, as more firms enter the market, profits decrease and consumer welfare increases. Third, we consider the effects of changes in the number of locations when a fixed number of firms enter the market. We find that increasing the number of locations gives early entrants more location choice, increasing their profits. This comes at the expense of consumers, decreasing consumer welfare.

Governments interested in improving consumer welfare should consider encouraging firm entry through fixed cost subsidies. While the direct benefits through new firm entry are obvious, there can be benefits even when no new firms choose to enter. Because firms strategically locate to deter future firm entry, government subsidies that induce (or enhance) this deterrence behavior may increase consumer welfare. This can help consumers by making firms more equally spaced, decreasing the maximum distance consumers need to travel and decreasing prices through increased price competition. Therefore, government subsidies can be effective at increasing consumer welfare in markets where firms enter sequentially even if there is no direct effect on the number of firms in the market.

While this model helps answer preliminary questions regarding the Hotelling model with sequential entry, there are several potentially relevant extensions. Future research could consider the effects of including more firms and more locations, different location densities, or having some locations already be filled. These locations could be filled by firms in an unrelated market or in the same market. Future research should also consider optimal location choice and implications in a sequential model with two dimensions. This could provide information about location decisions over larger areas, such as cities. Regardless, our analysis sets a foundation for evaluating the welfare implications of a very general setting of sequential firm location choice with a finite set of locations.