CSE 150A/250A. Assignment 6

Out: *Tues Nov 12*
Due: *Mon Nov 18* (by 11:59 PM, Pacific Time, via gradescope)
Grace period: 24 hours

6.1 Viterbi algorithm

In this problem, you will decode an English sentence from a long sequence of non-text observations. To do so, you will implement the same basic algorithm used in most engines for automatic speech recognition. In a speech recognizer, these observations would be derived from real-valued measurements of acoustic waveforms. Here, for simplicity, the observations only take on binary values, but the high-level concepts are the same.

Consider a discrete HMM with $n = 27$ hidden states $S_t \in \{1, 2, \ldots, 27\}$ and binary observations $O_t \in \{0, 1\}$. Download the ASCII data files from the Canvas web site for this assignment. These files contain parameter values for the initial state distribution $\pi_i = P(S_1 = i)$, the transition matrix $a_{ij} = P(S_{t+1} = j | S_t = i)$, and the emission matrix $b_{ik} = P(O_t = k | S_t = i)$, as well as a long bit sequence of $T = 430000$ observations.

Use the Viterbi algorithm to compute the most probable sequence of hidden states conditioned on this particular sequence of observations. As always, you may program in the language of your choice. Turn in the following:

(a) a print-out of your source code

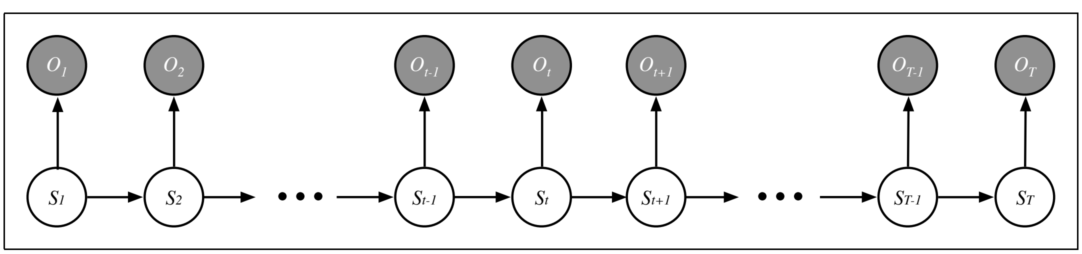
(b) a plot of the most likely sequence of hidden states versus time.

To check your answer: suppose that the hidden states $\{1, 2, \ldots, 26\}$ represent the letters $\{\mathtt{a}, \mathtt{b}, \ldots, \mathtt{z}\}$ of the English alphabet, and suppose that hidden state 27 encodes a space between words. If you have implemented the Viterbi algorithm correctly, the most probable sequence of hidden states *(ignoring repeated elements)* will reveal a famous quotation.

6.2 Conditional independence

Consider the hidden Markov model (HMM) shown below, with hidden states S_t and observations O_t for times $t \in \{1, 2, \ldots, T\}$. State whether the following statements of conditional independence are true or false.

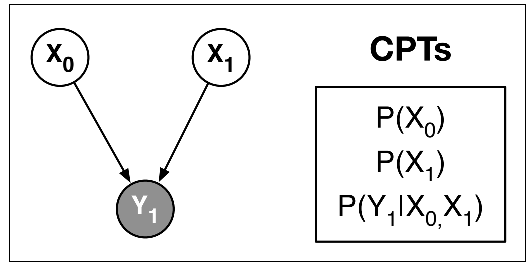
$$\begin{array}{lrcl}
\rule{2cm}{0.4pt} & P(S_t|S_{t-1}) & = & P(S_t|S_{t-1}, S_{t+1}) \\
\rule{2cm}{0.4pt} & P(S_t|S_{t-1}) & = & P(S_t|S_{t-1}, O_{t-1}) \\
\rule{2cm}{0.4pt} & P(S_t|S_{t-1}) & = & P(S_t|S_{t-1}, O_t) \\
\rule{2cm}{0.4pt} & P(S_t|O_{t-1}) & = & P(S_t|O_1, O_2, \ldots, O_{t-1}) \\
\rule{2cm}{0.4pt} & P(O_t|S_{t-1}) & = & P(O_t|S_{t-1}, O_{t-1}) \\
\rule{2cm}{0.4pt} & P(O_t|O_{t-1}) & = & P(O_t|O_1, O_2, \ldots, O_{t-1}) \\
\rule{2cm}{0.4pt} & P(S_2, S_3, \ldots, S_T|S_1) & = & \prod_{t=2}^{T} P(S_t|S_{t-1}) \\
\rule{2cm}{0.4pt} & P(S_1, S_2, \ldots, S_{T-1}|S_T) & = & \prod_{t=1}^{T-1} P(S_t|S_{t+1}) \\
\rule{2cm}{0.4pt} & P(S_1, S_2, \ldots, S_T|O_1, O_2, \ldots, O_T) & = & \prod_{t=1}^{T} P(S_t|O_t) \\
\rule{2cm}{0.4pt} & P(S_1, S_2, \ldots, S_T, O_1, O_2, \ldots, O_T) & = & \prod_{t=1}^{T} P(S_t, O_t) \\
\rule{2cm}{0.4pt} & P(O_1, O_2, \ldots, O_T|S_1, S_2, \ldots, S_T) & = & \prod_{t=1}^{T} P(O_t|S_t) \\
\rule{2cm}{0.4pt} & P(O_1, O_2, \ldots, O_T) & = & \prod_{t=1}^{T} P(O_t|O_1, \ldots, O_{t-1})
\end{array}$$



6.3 Belief updating

Consider the simple belief network on the right with nodes X_0, X_1, and Y_1. To compute the posterior probability $P(X_1|Y_1)$, we can use Bayes rule:

$$P(X_1|Y_1) \;=\; \frac{P(Y_1|X_1)\,P(X_1)}{P(Y_1)}$$



(a) Show how to compute the term $P(Y_1|X_1)$ in the numerator of Bayes rule.

(b) Show how to compute the term $P(Y_1)$ in the denominator of Bayes rule.

Now consider the belief network shown at the bottom of the page. It does not have the same structure as an HMM, but using similar ideas we can derive efficient algorithms for inference. In particular, consider how to compute the posterior probability $P(X_t|Y_1, Y_2, \ldots, Y_t)$ that accounts for evidence up to and including time t. We can derive an efficient recursion from Bayes rule:

$$P(X_t|Y_1, Y_2, \ldots, Y_t) \;=\; \frac{P(Y_t|X_t, Y_1, Y_2, \ldots, Y_{t-1})\,P(X_t|Y_1, Y_2, \ldots, Y_{t-1})}{P(Y_t|Y_1, \ldots, Y_{t-1})}$$

where the nodes $Y_1, Y_2, \ldots, Y_{t-1}$ are treated as background evidence. In parts (c-e) of this problem you will compute the individual terms that appear in this version of Bayes rule. You should express your answers in terms of the CPTs of the belief network and the probabilities $P(X_{t-1}\!=\!x|Y_1, Y_2, \ldots, Y_{t-1})$, *which you may assume have been computed at a previous step of the recursion*. Your answers to parts (a) and (b) may be instructive for parts (d) and (e).

(c) Show how to simplify the term $P(X_t|Y_1, Y_2, \ldots, Y_{t-1})$ in the numerator of Bayes rule.

(d) Show how to compute the term $P(Y_t|X_t, Y_1, Y_2, \ldots, Y_{t-1})$ in the numerator of Bayes rule.

(e) Show how to compute the term $P(Y_t|Y_1, Y_2, \ldots, Y_{t-1})$ in the denominator of Bayes rule.