2901/201
APPLIED MATHEMATICS
Oct./Nov. 2021
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN PETROLEUM GEOSCIENCE

MODULE II

APPLIED MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination.
Mathematical tables / a non programmable scientific calculator (fx-82);
An abridged table of Laplace Transforms;
The Standard Normal Distribution and the X^2-Distribution tables are attached.
*This paper consists of **EIGHT** questions.*
*Answer any **FIVE** questions in the answer booklet provided.*
All questions carry equal marks.
Marks for each part of a question are indicated.
Candidates should answer the questions in English.
Candidates should indicate the questions they have answered in the answer booklet.

This paper consists of 7 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) 10% of the pipes produced by a machine are defective. A pipe line company bought 12 pipes. Determine the probability that:

(i) none;
(ii) exactly two;
(iii) at most three;
(iv) at least four are defective.

(8 marks)

(b) A continuos random variable x has a probability density function defined by:

$$f(x) = \begin{cases} Kxe^{-2x}, & x70 \\ 0 & elsewhere \end{cases}$$

where K is a constant.

Determine the:

(i) value of x;
(ii) mean;
(iii) standard deviation of the distribution.

(12 marks)

2. (a) Forces of magnitude 4, 3 and 2 units in the directions of the vectors $6\underset{\sim}{i}+2\underset{\sim}{j}+3\underset{\sim}{k}$, $3\underset{\sim}{i}-2\underset{\sim}{j}+6\underset{\sim}{k}$ and $2\underset{\sim}{i}+3\underset{\sim}{j}+6\underset{\sim}{k}$, respectively act on a particle which is displaced from the point $(2,-1,-3)$ to $(5,-1,1)$. Determine the work done by the forces.

(7 marks)

(b) Determine the value of the constant λ such that the vectors $2\underset{\sim}{i}-\underset{\sim}{j}+\underset{\sim}{k}$, $\underset{\sim}{i}+2\underset{\sim}{j}-3\underset{\sim}{k}$ and $3\underset{\sim}{i}+\lambda\underset{\sim}{j}+5\underset{\sim}{k}$ are coplanar.

(5 marks)

(c) Given the vector field $\underset{\sim}{F}=2x^2yz\underset{\sim}{i}+4xyz\underset{\sim}{j}+5xy^2z^2\underset{\sim}{k}$, determine at the point $(1,-2,1)$,

(i) $\nabla.\underset{\sim}{F}$;

(ii) $\nabla\times\underset{\sim}{F}$. $\begin{pmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ 2x^2yz & 4xyz & 5xy^2z^2 \end{pmatrix}$

(8 marks)

3. (a) Determine the Laplace transform of $f(t)=t\cos 7t$ from **first** principles. (9 marks)

(b) Use Laplace transforms to solve the differential equation:

$\frac{d^2y}{dt^2}+4\frac{dy}{dt}+5y=4^{e^{-3t}}$ given that when $t=0$, $y=1$ and $\frac{dy}{dt}=3$

(11 marks)

$\begin{pmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \end{pmatrix}$

$= 10xyz^2 - 4xy\underset{\sim}{i} + 5yz^2 - 2x^2y\underset{\sim}{j} + 4xyz - 2x^2z\underset{\sim}{k}$

4. (a) Uranium disintegrates at a rate proportional to the amount present at any instant. If M_1 and M_2 grames of Uranium are present at time t_1 and t_2 respectively, show that the half-life of Uranium is :

$$\frac{(t_1 - t_2)\log_e^2}{\log_e \frac{m1}{m2}}$$

(8 marks)

(b) Use the D-operator method to solve the differential equation:

$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = e^{-2t}$, given that when $t = 0$, $y = 1$ and $\frac{dy}{dt} = 3$

(12 marks)

5. (a) Given the matrix $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$;

(i) Show that : $A^3 - 7A^2 + 16A - 12I = 0$; where I is an identity matrix.

(ii) Hence determine A^{-1}

(8 marks)

(b) Students in a petroleum, geoscience class purchased resistors, capacitors and inductors for projects in groups as shown in table I.

27125
-99
495

Table I

Group	Resistors	Capacitors	Inductors	Cost
A	6	3	4	440
B	4	5	3	380
C	7	4	3	390

0 460 30
-68 -150 -240
12 14 100

Use Cramer's rule to determine the cost of each item. (12 marks)

6. (a) (i) Determine the first **three** non zero terms of the Maclaurin's series expansion of $f(x) = e^{\sin 3x}$;

(ii) hence evaluate the integral $\int_1^2 \frac{e^{\sin 3x}}{x^{2/3}} dx$, correct to **three** decimal places.

(11 marks)

(b) (i) Expand $f(x) = Sin\,x$ in Taylor's series about the point $x = \frac{\pi}{6}$, as far as the **fourth** term.

(ii) Use the result in (i) to determine the approximate value of $Sin\,33°30'$, correct to **five** decimal places.

(9 marks)

7. (a) Table II shows the marks scored by nine students in Mathematics and Earth science.

Table II

Mathematics	10	20	30	40	50	60	70	80	90
Earth science	32	20	24	36	40	42	48	44	60

Taking the assumed mean for Mathematics and Earth Science as 40 and 36 respectively, us the step deviation method to determine the coefficient of correlation between the scores.

(8 marks)

(b) Table III shows the length of pipes sold in an electronic shop.

Table III

Length of pipe (m)	5-9	10-14	15-19	20-24	25-29	30-34	35-39	40-44
No of pipes	6	B	12	10	5	6	2	1

Given that the mode length is 12.68 m, determine the:

(i) value B;
(ii) mean;
(iii) median.

(12 marks)

8. (a) Given that $U = e^{(4x+6)} Sin(4y+3) + 7x + 8y + 9$.

Determine $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$. (5 marks)

(b) The radius of a right circular cone increases at a rate of $1\, cms^{-1}$ while the height decreases of $2\, cms^{-1}$. Use partial differentiation to determine the rate of change of the volume when the radius is 1.2 cm and height is 3.6 cm. (5 marks)

(c) Locate the stationary points of the function

$f(x,y) = \frac{7}{2}x^2 + \frac{11}{2}y^2 + 9xy - 4x - 4y + 11$, and determine their nature. (10 marks)