2D COLLISION-FREE TRANSFORMATION

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1. ONE IS STATIC AND ONE IS DYNAMIC

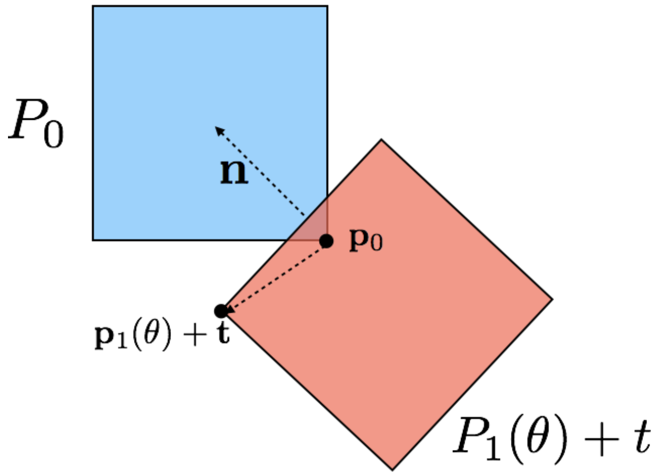

FIGURE 1. blue is fixed and red is movable

The penetration distance between P_0 and P_1 in this figure is $\mathbf{n}^T[\mathbf{p}_1(\theta) + \mathbf{t} - \mathbf{p}_0]$. Even though for different $\theta, \mathbf{t}$ there is a different way of choosing $p_0, p_1, \mathbf{n}$, let us assume that it won't change locally. More specifically:
when $|\theta - \theta_0| < \epsilon$, $|\mathbf{t} - \mathbf{t}_0| < \epsilon$
the penetration distance $d(P_1(\theta, \mathbf{t}), P_0)$ can be linearize:

$$d(P_1(\theta, \mathbf{t}), P_0) \approx d(P_1(\theta_0, \mathbf{t}_0), P_0) + \Delta d(P_1(\theta_0, \mathbf{t}_0), P_0)^T \begin{bmatrix} \theta - \theta_0 \\ t^x - t_0^x \\ t^y - t_0^y \end{bmatrix} \tag{1}$$

Then, using trust region method, the K iteration optimization function is:

$$\min_{\theta^K, \mathbf{t}^K} ||d(P_1(\theta^K, \mathbf{t}^K), P_0)||^2 \tag{2}$$

$$\text{subject: } |\theta^K - \theta^{K-1}| \leq \epsilon^K \tag{3}$$

$$|\mathbf{t}^K - \mathbf{t}^{K-1}| \leq \epsilon^K \tag{4}$$

With:

$$d(P_1(\theta^K,\mathbf{t}^K),P_0)=d(P_1(\theta^{K-1},\mathbf{t}^{K-1}),P_0)+\Delta d(P_1(\theta^{K-1},\mathbf{t}^{K-1}),P_0)^T\begin{bmatrix}\theta^K-\theta^{K-1}\\ t_x^K-t_x^{K-1}\\ t_y^K-t_y^{K-1}\end{bmatrix} \tag{5}$$

Specific d:

$$\begin{aligned}
d(\theta,\mathbf{t}) &= \mathbf{n}^T(\mathbf{q}(\theta,\mathbf{t})-\mathbf{p}) && (6)\\
&= \mathbf{n}^T(R_\theta(\mathbf{q}-\hat{\mathbf{q}})+\mathbf{t}+\hat{\mathbf{q}}-\mathbf{p}) && (7)\\
&= \mathbf{n}^T(R_\theta\bar{\mathbf{q}}+\mathbf{t}+\mathbf{c}_0) && (8)\\
d(\theta,\mathbf{t}) &\approx \mathbf{n}^T(R_{\theta_0}\bar{\mathbf{q}}+\mathbf{t}_0+\mathbf{c}_0)+\mathbf{n}^T(R'_{\theta_0}\bar{\mathbf{q}}(\theta-\theta_0)+(\mathbf{t}-\mathbf{t}_0)) && (9)\\
&= a_0+\mathbf{n}^T(R'_{\theta_0}\bar{\mathbf{q}}(\theta-\theta_0)+(\mathbf{t}-\mathbf{t}_0)) && (10)\\
&= a_0+a_1\theta+a_2x+a_3y && (11)
\end{aligned}$$

$$\begin{aligned}
a_1 &= \mathbf{n}^T R'_{\theta_0}\bar{\mathbf{q}} && (12)\\
a_2 &= \mathbf{n}_0 && (13)\\
a_3 &= \mathbf{n}_1 && (14)\\
a_0 &= \mathbf{n}^T(R_{\theta_0}\bar{\mathbf{q}}+\mathbf{t}_0+\mathbf{c}_0)-\theta_0\mathbf{n}^T R'_{\theta_0}\bar{\mathbf{q}}-\mathbf{n}^T\mathbf{t}_0 && (15)\\
&= \mathbf{n}^T(\mathbf{q}_0-\mathbf{p})-\theta_0\mathbf{n}^T R'_{\theta_0}\bar{\mathbf{q}}-\mathbf{n}^T\mathbf{t}_0 && (16)\\
&= \mathbf{n}^T(\mathbf{q}_0-\mathbf{p})-a_1\theta_0-\mathbf{n}^T\mathbf{t}_0 && (17)
\end{aligned}$$

for two moving points:
Specific d:

$$d(\theta,\mathbf{t})=\mathbf{n}^T(\mathbf{q}(\theta^0,\mathbf{t}^0)-\mathbf{p}(\theta^1,\mathbf{t}^1)) \tag{18}$$

$$= \mathbf{n}^T(R_{\theta^0}(\mathbf{q}-\hat{\mathbf{q}})+\mathbf{t}^0+\hat{\mathbf{q}}-R_{\theta^1}(\mathbf{p}-\hat{\mathbf{p}})-\mathbf{t}^1-\hat{\mathbf{p}}) \tag{19}$$

$$d(\theta,\mathbf{t})\approx\mathbf{n}^T(\mathbf{q}_0-\mathbf{p}_0)+\mathbf{n}^T(R'_{\theta^0_0}\bar{\mathbf{q}}(\theta^0-\theta^0_0)+\mathbf{t}^0-\mathbf{t}^0_0-R'_{\theta^1_0}(\theta^1-\theta^1_0)\bar{\mathbf{p}}-\mathbf{t}^1+\mathbf{t}^1_0) \tag{20}$$

$$= a_0+a_1\theta^0+a_2x^0+a_3y^0+a_4\theta^1+a_5x^1+a_6y^1 \tag{21}$$

$$a_1 = \mathbf{n}^T R'_{\theta_0^0} \bar{\mathbf{q}}_0 \tag{22}$$

$$a_2 = \mathbf{n}_0 \tag{23}$$

$$a_3 = \mathbf{n}_1 \tag{24}$$

$$a_4 = -\mathbf{n}^T R'_{\theta_0^1} \bar{\mathbf{p}}_0 \tag{25}$$

$$a_5 = -\mathbf{n}_0 \tag{26}$$

$$a_6 = -\mathbf{n}_1 \tag{27}$$

$$a_0 = \mathbf{n}^T(\mathbf{q}_0 - \mathbf{p}_0) - a_1\theta_0^0 - \mathbf{n}^T\mathbf{t}_0^0 + a_4\theta_0^1 + \mathbf{n}^T\mathbf{t}_0^1 \tag{28}$$

$$(\mu\mathbf{a} + (1-\mu)\mathbf{b} - \mathbf{q})^T(\mathbf{a} - \mathbf{b}) = 0 \tag{29}$$

$$\mu(\mathbf{a} - \mathbf{b})^T(\mathbf{a} - \mathbf{b}) + (\mathbf{b} - \mathbf{q})^T(\mathbf{a} - \mathbf{b}) = 0 \tag{30}$$

$$\mu = \frac{(\mathbf{q} - \mathbf{b})^T(\mathbf{a} - \mathbf{b})}{(\mathbf{a} - \mathbf{b})^T(\mathbf{a} - \mathbf{b})} \tag{31}$$