

```
let rec fl f a l = match l with [] -> a
| x::xs -> fl f (f a x) xs;;
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let rec fr f l a = match l with [] -> a
| x::xs -> f x (fr f xs a);;
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```
let rec rev_map f l a = match l with [] -> a
| x::xs -> rev_map f xs (f x :: a);;
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let (+) a b = a + b;;
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fl (+) 0 (rev_map (fun x -> x * 2) l []) = fr (fun x a -> a + 2 * x) l 0
```

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base case: l = []
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fl (+) acc (rev_map (fun x -> x * 2) [] [])
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```
rev_map (fun x -> x * 2) [] [] = match [] with [] -> a
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| x::xs -> rev_map f xs (f x :: a)
```

```
= []
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```
fl (+) acc [] = match [] with [] -> acc
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| x::xs -> fl f (f a x) xs =
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acc =
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```
= fr (fun x a -> a + 2 * x) [] 0 + acc = match [] with [] -> a
```

| x::xs -> f x (fr f xs a)

0 + acc = acc

fr (fun x a -> a + 2 * x) | 0 + acc = fl (+) acc (rev_map (fun x -> x * 2) | [])

fr (fun x a -> a + 2 * x) xs 0 = fl (+) acc (rev_map (fun x -> x * 2) xs []) - acc

for (x::xs)

fl (+) acc (rev_map (fun x -> x * 2) (x::xs) []) =

match x::xs with [] -> a

| x::xs -> fl (+) ((+) a x) xs

= fl (+) (acc + 2x) (rev_map (fun x -> x * 2) xs []) = fr (fun x a -> a + 2 * x) xs 0 + acc + 2x

acc + fr (fun x a -> a + 2 * x) (x::xs) 0 = match (x::xs) with [] -> a

| x::xs -> f x (fr f xs a)

=

acc + (fun x a -> a + 2 * x) x (fr f xs a)

= acc + (fun x a -> a + 2 * x) x (fr (fun x a -> a + 2 * x) xs 0) =

$\text{acc} + (\text{fun } x \ a \rightarrow a + 2 * x) \ x \ (\text{fl } (+) \ \text{acc} \ (\text{rev_map} \ (\text{fun } x \rightarrow x * 2) \ \text{xs} \ [])) - \text{acc} =$

$\text{fl } (+) \ \text{acc} \ (\text{rev_map} \ (\text{fun } x \rightarrow x * 2) \ \text{xs} \ []) + 2x = 2x + \text{acc} + \text{fr} \ (\text{fun } x \ a \rightarrow a + 2 * x) \ \text{xs} \ 0 = \text{fr} \ (\text{fun } x \ a \rightarrow a + 2 * x) \ \text{xs} \ 0 + (2x + \text{acc})$

$\text{fl } (+) \ (\text{acc} + 2x) \ (\text{rev_map} \ (\text{fun } x \rightarrow x * 2) \ \text{xs} \ [])$
 $= 2x + (\text{fl } (+) \ \text{acc} \ (\text{rev_map} \ (\text{fun } x \rightarrow x * 2) \ \text{xs} \ []))$