```
let rec impl = fun n a ->
    match n with 0 -> a | _ -> impl (n-1) (a·n·n)
  and bar = fun n -> impl n 1
```

T_{impl} = fun n a → match n with 0 → a | _ → impl (n-1) (a · n · n)

$$\Sigma_{bar} = GD\ \frac{bar = fun\ n \to impl\ n\ 1 \quad fun\ n \to impl\ n\ 1 \to fun\ n \to impl\ n\ 1}{bar \to fun \to impl\ n\ 1}$$

$$\Sigma_{impl} = GD\ \frac{impl = fun\ n\ a\ \cancel{with}\ match\ n\ with\ 0 \to a \mid \_ \to impl\ (n-1)\ (a \cdot n \cdot n) \quad fun\ n\ a \to match\ n\ with\ 0 \to a \mid \_ \to impl\ (n-1)\ (a \cdot n \cdot n) \to T_{impl}}{impl \to fun\ n\ a \to match\ n\ with\ 0 \to a \mid \_ \to impl\ (n-1)\ (a \cdot n \cdot n)}$$

Now we have to prove by induction that impl n a equals to $n! \cdot n! \cdot a$ for all $n \geq 0$

base case: $n = 0$

$$\frac{0 \to 0 \quad a \to a}{}$$

$$APP\ \frac{\Sigma_{impl} \quad 0 \to 0 \quad a \to a \quad match\ 0\ with\ 0 \to a \mid \_ \to impl\ (0-1)\ (a \cdot 0 \cdot 0) \to a}{impl\ 0\ a \to a}$$

Induction hypothesis: impl n a $= n! \cdot n! \cdot a$

For induction step: we have to prove that same goes for
impl (n+1) (a)

I.H

PM $\underbrace{\text{impl } (n+1-1)(a(n+1)\cdot(n+1))}_{\text{I.H}} \to \cancel{((n+1)!\cdot(n+1)!}\ a)\ (n!\cdot n!\cdot a(n+1)\cdot(n+1) \to (n+1)!\cdot(n+1)!\cdot a$

APP 5c impl $(n+1) \to (n+1)\ a \to a$ match $(n+1)$ with $n \to a!.$ $\to$ impl $(n+1-1)\,(a\cdot(n+1)\cdot(n+1)) \to (n+1)!\cdot(n+1)!\cdot a$

impl $(n+1)\ a \to ((n+1)!\cdot(n+1)!\cdot a)$

our assumption was proved and impl n a $= n!\cdot n!$ for all $n \geq 0$

APP with our prove (impl n 1) $\to n!\cdot n!\cdot 1 \Rightarrow n!\cdot n!$

APP 5c bar $n \to n$ impl n 1 $\to n!\cdot n!$
bar $n \to n!\cdot n!$