Grieg: A Four-Fold Phase Semantics on an Ontic Logical Manifold

William A. Patterson

September 10, 2025

Abstract

Grieg augments classical truth with an ontic phase over $\{\mathsf{ALIVE}, \mathsf{JAM}, \mathsf{MEM}, \mathsf{VAC}\}$ capturing boundary, transport, and witness on a logical manifold. We give a sequent calculus, prove conservativity over classical propositional logic on phase-free fragments, show an embedding of Belnap–Dunn with a separation (no phase-erasing compositional translation), establish NP/coNP bounds, and present a trace semantics whose invariants (sink on $\rightarrow$, π rotation on $\neg$, etc.) are observational and do not alter results.

1 Syntax

Atoms $p \in \mathcal{P}$; connectives $\neg, \wedge, \vee, \rightarrow$; phase operators @$\mathsf{mem}(\cdot)$, @$\mathsf{jam}(\cdot)$, @$\mathsf{vac}(x)$, @$\mathsf{alive}(\cdot)$.

2 Semantics

Truth values $V = \{0, 1, \bot\}$; phases $P = \{\mathsf{ALIVE}, \mathsf{JAM}, \mathsf{MEM}, \mathsf{VAC}\}$. A model $\mathcal{M} = \langle W, \nu, \phi \rangle$ with boolean evaluation $[\![\cdot]\!]_w$: Form $\rightarrow V$ (classical on defined points) and phase map ϕ : Form $\times$ $W \rightarrow P$ satisfying invariants: negation rotates on the factual sheet, disjunction is centrifugal with boundary short-circuit, implication has a monotone radial sink under modus ponens, @mem transports between sheets, @vac projects to counterfactual where witness is absent.

2.1 Sheets (F:C), Transport, and Sinks

Each sheet is a solid torus $\Sigma = S^1 \times D^2$ with angles (θ, φ) and radial potential $\rho \in [0, 1]$. We evaluate on two ontic sheets: **F** (factual) and **C** (counterfactual).

Judgments:

$$\langle s, \phi \rangle \ \vdash \ E \Downarrow (v, \phi', s')$$

with $s \in \{\mathbf{F}, \mathbf{C}\}$, $\phi \in \{\mathsf{ALIVE}, \mathsf{JAM}, \mathsf{MEM}, \mathsf{VAC}\}$, and $v \in \{\mathsf{true}, \mathsf{false}, \bot\}$ ($\bot$ = no total boolean).

VAC projection.

$$\frac{}{\langle s, \phi \rangle \vdash @vac(x) \Downarrow (\bot, \mathsf{VAC}, \mathbf{C})}.$$

MEM transport (truth-preserving). Let $\overline{s}$ be the opposite sheet. Then

$$\frac{\langle \overline{s}, \mathsf{MEM} \rangle \vdash E \Downarrow (v, \phi_1, \overline{s})}{\langle s, \phi \rangle \vdash @mem(E) \Downarrow (v, \mathsf{MEM}, s)}.$$

Implication and F-local sinks. On **F**, if the antecedent is established, implication chains flow toward the core ($\rho \downarrow 0$) and may mark a *sink* (absorbing point). On **C**, sinks are never registered.

Conservativity. Restricted to **F** without phase operators, the boolean projection agrees with CPL; sheet/phase are observational structure.

3 Proof System Gr

Classical propositional rules; admissible phase rules enforcing invariants without changing boolean projection.

Theorem 1 (Conservativity over CPL). *For every phase-free formula F, F is valid in Grieg iff F is valid in classical propositional logic.*

Sketch. Define projection $\pi : (v, p) \mapsto v$. By structural induction, truth-clauses coincide with CPL and no phase operator is present to produce non-ALIVE on closed formulas; hence $\pi(\llbracket F \rrbracket) = \llbracket F \rrbracket_{\mathrm{CPL}}$. □

Theorem 2 (Embedding + Separation vs Belnap–Dunn). *There exists a translation $E : \mathsf{FOUR} \to$ Grieg preserving boolean projection. However, there is no compositional translation T from Grieg to* FOUR *that preserves satisfaction and context-equivalence while erasing phases.*

Idea. Map BD's 'both/neither' strata into $\mathsf{JAM}/\mathsf{VAC}$ with matching boolean projection. For separation, build contexts $C[\_]$ where Grieg distinguishes formulas that differ only by an *operational sink* (licensed modus ponens) while BD assigns the same four-value because it is extensional and phase-blind. Compositionality of T would force $C[T(F_1)] = C[T(F_2)]$, contradicting Grieg's distinction. □

4 Trace Semantics

Define per-step records with $(\mathrm{op}, \mathrm{pre}, \mathrm{post}, \mathrm{sink}, \mathrm{jam}, \theta, \rho)$. Prove observational adequacy (I1I5) by induction on evaluation.

5 Complexity

SAT(Grieg) is NP-complete and VAL(Grieg) is coNP-complete by reduction from/to CPL and polynomial verification of local phase constraints.

6 Implementation & Tests

Reference Rust engine; property tests for I1I5; traces are feature-gated and semantics-preserving.