Grieg: A Four-Fold Phase Semantics on an Ontic Logical Manifold

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Abstract

Grieg augments classical truth with an ontic phase over $\{\mathsf{ALIVE}, \mathsf{JAM}, \mathsf{MEM}, \mathsf{VAC}\}$ capturing boundary, transport, and witness on a logical manifold. We give a sequent calculus, prove conservativity over classical propositional logic on phase-free fragments, show an embedding of Belnap–Dunn with a separation (no phase-erasing compositional translation), establish NP/coNP bounds, and present a trace semantics whose invariants (sink on $\rightarrow$, π rotation on $\neg$, etc.) are observational and do not alter results.

1 Syntax

Atoms $p \in \mathcal{P}$; connectives $\neg, \wedge, \vee, \rightarrow$; phase operators @$\mathsf{mem}(\cdot)$, @$\mathsf{jam}(\cdot)$, @$\mathsf{vac}(x)$, @$\mathsf{alive}(\cdot)$.

2 Semantics

Truth values $V = \{0, 1, \perp\}$; phases $P = \{\mathsf{ALIVE}, \mathsf{JAM}, \mathsf{MEM}, \mathsf{VAC}\}$. A model $\mathcal{M} = \langle W, \nu, \phi \rangle$ with boolean evaluation $[\![\cdot]\!]_w : \text{Form} \rightarrow V$ (classical on defined points) and phase map $\phi : \text{Form} \times W \rightarrow P$ satisfying invariants: negation rotates on the factual sheet, disjunction is centrifugal with boundary short-circuit, implication has a monotone radial sink under modus ponens, @mem transports between sheets, @vac projects to counterfactual where witness is absent.

3 Proof System $\mathbb{G}\mathrm{r}$

Classical propositional rules; admissible phase rules enforcing invariants without changing boolean projection.

Theorem 1 (Conservativity over CPL)**.** *For every phase-free formula F, F is valid in Grieg iff F is valid in classical propositional logic.*

Sketch. Define projection $\pi : (v, p) \mapsto v$. By structural induction, truth-clauses coincide with CPL and no phase operator is present to produce non-ALIVE on closed formulas; hence $\pi([\![F]\!]) = [\![F]\!]_{\text{CPL}}$. $\square$

Theorem 2 (Embedding + Separation vs Belnap–Dunn)**.** *There exists a translation $E : \mathsf{FOUR} \rightarrow$ Grieg preserving boolean projection. However, there is no compositional translation T from Grieg to FOUR that preserves satisfaction and context-equivalence while erasing phases.*

Idea. Map BD's 'both/neither' strata into $\mathsf{JAM}/\mathsf{VAC}$ with matching boolean projection. For separation, build contexts $C[\_]$ where Grieg distinguishes formulas that differ only by an *operational sink* (licensed modus ponens) while BD assigns the same four-value because it is extensional and phase-blind. Compositionality of T would force $C[T(F_1)] = C[T(F_2)]$, contradicting Grieg's distinction. $\square$

4 Trace Semantics

Define per-step records with $(\text{op}, \text{pre}, \text{post}, \text{sink}, \text{jam}, \theta, \rho)$. Prove observational adequacy (I1I5) by induction on evaluation.

5 Complexity

SAT(Grieg) is NP-complete and VAL(Grieg) is coNP-complete by reduction from/to CPL and polynomial verification of local phase constraints.

6 Implementation & Tests

Reference Rust engine; property tests for I1I5; traces are feature-gated and semantics-preserving.