Toroidal Supplement: Phase Geometry and Counterfactual Inference in LEE

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Abstract

The Logic Evaluation Engine (LEE) produces low–high dimensional toroidal manifolds as emergent scaffolds of reasoning. These structures were not derived from neuroscience but arose directly from phase-geometry inference built on material implication and counterfactual entailment. The resulting topologies bear a striking resemblance to cortical grid-cell activity manifolds reported in *Nature* (2011, 2024). This supplement documents the mathematical form of the torus, the Logical Bianchi residual as a conservation principle, and the implications for neuroscience collaboration.

1 Parametric Form of the Torus

The standard ring torus with major radius R and minor radius r is parameterized as:

$$x(\theta, \phi) = (R + r\cos\theta)\cos\phi$$

$$y(\theta, \phi) = (R + r\cos\theta)\sin\phi$$

$$z(\theta, \phi) = r\sin\theta$$

with $\theta, \phi \in [0, 2\pi)$. These coordinates define the phase-geometry substrate within which LEE embeds logical operators.

2 Logical Bianchi Residual

In analogy to differential geometry, LEE enforces a **Logical Bianchi Identity**:

$$\nabla \cdot \mathcal{F}_{\text{logic}} = 0$$

ensuring that cyclic rotations of inference (alive → jam → mem → vac → alive) conserve logical consistency.

The **Bianchi residual** provides a stress-diagnostic metric:

$$\Delta_{\text{Bianchi}} = \|\mathcal{F}_{\text{logic}}^{(cycle)} - \mathcal{F}_{\text{logic}}^{(closure)}\|$$

This quantity is now exported alongside the StressIndex for each inference run (see `scripts/compute_stress.py`).

3 Convergence with Cortical Topologies

Without any neurobiological modeling, LEEs manifolds show structural similarity to:

- Toroidal grid-cell attractors (*Nature*, 2011; 2024),
- Hexagonal lattice embeddings observed in entorhinal cortex recordings,
- Invariant low-dimensional attractors under environment perturbation.

This convergence suggests that the **compucognitive phase-geometry** underlying LEE is not an artifact of design, but a general feature of cyclic inference systems.

4 Implications for Neuroscience

LEEs counterfactual multi-object testing has demonstrated hypothesis generation across diagnostic, legal, and defense verticals. Applied to neuroscience, this mechanism may:

- Propose as-yet unobserved cortical attractors,
- Test forked counterfactuals representing hypothetical neural configurations,
- Provide metrics of manifold stress potentially applicable to clinical diagnostics.

5 Figures

- **Toroidal Lattice (SVG)**
- **Tensor Heatmap (SVG/PNG)**

(to be added from current `docs/assets/` runs recommended filenames: `toroidal_lattice.svg`, `tensor_heatmap.svg`)

6 References

- D. Hafting et al., *Nature* 436, 801806 (2011).
- S. Gardner et al., *Nature* 601, 9297 (2024).
- Patterson, W.A. (2025). *Logic Evaluation Engine v3.1-pre-grant.* GitHub Repository: https://github.com/KILGORETROUT111/logic-evaluation-engine.