The Logic Evaluation Engine (LEE)

Toroidal Phase Geometry, Phase Tensors, and Stress-Aware Inference *independently developed, later recognized as convergent with neuroscience*

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Abstract

The Logic Evaluation Engine (LEE) is a symbolic inference system whose discrete phase dynamics (ALIVE, JAM, MEM) produce closed cyclic flows on a low-dimensional manifold. The toroidal structure observed in LEE predates—and was *not* inspired by—neuroscience reports of low-dimensional toroidal activity [**? ?**]; the similarity was noted ex post. We present a constructive account of why a two-torus $\mathbb{T}^2$ (and higher tori) arise from material-implication rotations and contradiction archival, define a Global Logical Memory Space (GLMS), and introduce operational metrics (Winding, Resistance, and a StressIndex) that quantify manifold distortion and system "health." We give falsifiable predictions, an executable instrumentation plan, and show how LEE can model configurations *beyond presently known biological constraints*, offering hypotheses testable in neuroscience and medicine.

1 Introduction: independence, then convergence

LEE was built from first principles in logic and phase geometry. Only later did external readers point out its resemblance to toroidal neural manifolds reported in *Nature* (2011; 2024). Our claim is therefore *constructive*: toroidal structure is a necessity of LEE's operators, not an import from biology. That such manifolds also appear in cortical data suggests convergent constraints across information-processing substrates.

Contributions. (i) A minimal discrete-time model that yields $\mathbb{T}^2$ from logical first principles; (ii) a tensorial memory field (GLMS) that explains curvature and loop persistence; (iii) stress-aware metrics with precise measurement recipes and provenance hooks; (iv) a protocol to compare LEE manifolds against neural-manifold datasets without tuning to biology.

2 Formal model

Definition 1 (Phase alphabet and angles)**.** *Let* $\mathcal{S} = \{\mathsf{ALIVE}, \mathsf{JAM}, \mathsf{MEM}\}$ *with angle map* $\varphi : \mathcal{S} \to \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\}$*. A run produces a sequence* $(s_t)_{t=0}^{T}$*,* $s_t \in \mathcal{S}$*. The rotation at step t is* $\Delta\theta_t = (\varphi(s_t) - \varphi(s_{t-1})) \bmod 2\pi$*, constrained to* $\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\}$*.*

Definition 2 (Global Logical Memory Space (GLMS))**.** *Each transition emits a local tensor* $\boldsymbol{g}_t \in \mathbb{R}^{k\times k}$ *encoding contradiction strain (e.g., unmatched premises, unresolved constraints). The* phase tensor *of a run is* $G = \sum_{t=1}^{T} \boldsymbol{g}_t$*; archival in* MEM *accumulates* G*, replay exposes* G *for audit.*

Proposition 1 (Two independent cycles $\Rightarrow \mathbb{T}^2$)**.** *Suppose (i) evaluation rotates on* $\theta \in S^1$ *via* ALIVE → JAM → MEM → ALIVE*; (ii) memory accrual rotates on an independent angle* $\psi \in S^1$ *via archival/replay. Then the attractor manifold generically factorizes as* $\mathbb{T}^2 = S^1_\theta \times S^1_\psi$ *up to gluing along identification sets where contradictions are resolved.*

Remark 1 (Beyond $\mathbb{T}^2$). *Adapters or side-cycles add independent S^1 factors (yielding $\mathbb{T}^k$); nontrivial gluing changes genus—LEE can thus exceed cortical constraints while preserving discrete stability.*

3 Manifold distortion and operational metrics

Let $\mathcal{M}_{\text{ideal}}$ denote the ideal torus with major/minor radii (R, r). For an embedded cloud $\{\mathbf{x}_i(t)\}$ obtained from LEE state vectors (or phase traces embedded via spectral/PH methods), define:

Radial distortion.

$$D_r(t) = \frac{1}{N} \sum_{i=1}^{N} \left(\left| \|\Pi_{\text{maj}} \mathbf{x}_i(t)\| - R \right| + \left| \|\Pi_{\text{min}} \mathbf{x}_i(t)\| - r \right| \right). \tag{1}$$

Curvature concentration. Let $\kappa(\cdot)$ be an estimator of geodesic curvature along trajectories on $\mathcal{M}$. Define $D_\kappa(t) = \text{IQR}\{\kappa(\gamma_i(t))\}$ over run segments γ_i.

Winding, Resistance, StressIndex.

$$\text{Winding}(t) = \sum_{\tau \le t} \Delta\theta_\tau, \tag{2}$$

$$\text{Resistance}(t) = \alpha\, \mathbb{E}|\Delta\theta_\tau|/\pi + \beta\, \lambda_t, \quad \lambda_t = \text{returns to prior phases up to } t, \tag{3}$$

$$\text{StressIndex}(t) = \frac{1}{t} \int_0^t w(\tau) \Big(D_r(\tau) + \gamma D_\kappa(\tau) \Big)\, d\tau, \tag{4}$$

with $w(\tau)$ a recency weight and γ a curvature weight. In absence of durations, use a count-based proxy $\rho_{\mathsf{JAM}} = \frac{\#\{\tau : s_\tau = \mathsf{JAM}\}}{t}$ and the simplified StressIndex $\approx \frac{\text{Winding}}{2\pi} \rho_{\mathsf{JAM}}$.

4 Executable instrumentation (provenance hooks)

Record in provenance (`.prov.jsonl`) per step: (i) phase after transition; (ii) timestamp; (iii) local tensor $\boldsymbol{g}_t$ hash or summary; (iv) loop counter λ_t. Emit end-of-run:

```
{"event":"stress_index",
 "value": S,
 "winding_deg": int(Wind * 180/pi),
 "jam_ratio": rho_JAM,
 "resistance": Resist}
```

For time-weighted ρ_{JAM}, log entry/exit timestamps for each phase.

5 Predictions and falsifiable tests

P1 (monotonicity). Increasing contradiction density raises Winding and ρ_{JAM}; hence StressIndex rises monotonically.
P2 (collapse indicator). Beyond a threshold in Resistance, orbits degenerate near JAM; manifold Betti-1 persistence shortens (PH signature).
P3 (universality). The $\mathbb{T}^2$ attractor persists across domains (legal, medical, defense) without retuning operators.
P4 (beyond biology). LEE realizes $\mathbb{T}^k$ and altered-genus surfaces; some may later be measured biologically.

6 Relation to neural-manifold reports (post hoc)

The resemblance to low-dimensional neural tori [**? ?**] was observed *after* LEE had an explicit toroidal phase space. We emphasize convergence, not inspiration: LEE's torus follows from discrete implication rotations and contradiction archival; any biological parallel is supportive but not formative.

7 Applications and implications

Diagnostics. Stress-aware runs separate resolvable contradictions from degenerative loops.
Governance. Provenance + manifold health provides auditable decision trails.
Biomedical hypotheses. LEE can propose manifolds beyond current cortical constraints for empirical probing.

8 Limitations

We presently approximate time weights from state counts when phase durations are absent; future engine versions will log exact durations. Distortion estimators depend on embedding choice (spectral, isomap, PH); we mitigate via method agreement and reporting.

9 Conclusion

LEE's toroidal geometry is a law of its logic, not an import from biology. With GLMS and stress-aware metrics, the engine not only performs inference but also exposes a measurable manifold of reasoning—usable for engineering stability and cross-domain science.

Data and code. The engine emits JSON, PROV, SVG and (optionally) StressIndex metrics; figures in the arXiv version will be generated directly from these artifacts.