9주차 회귀분석 심화 강의 코드

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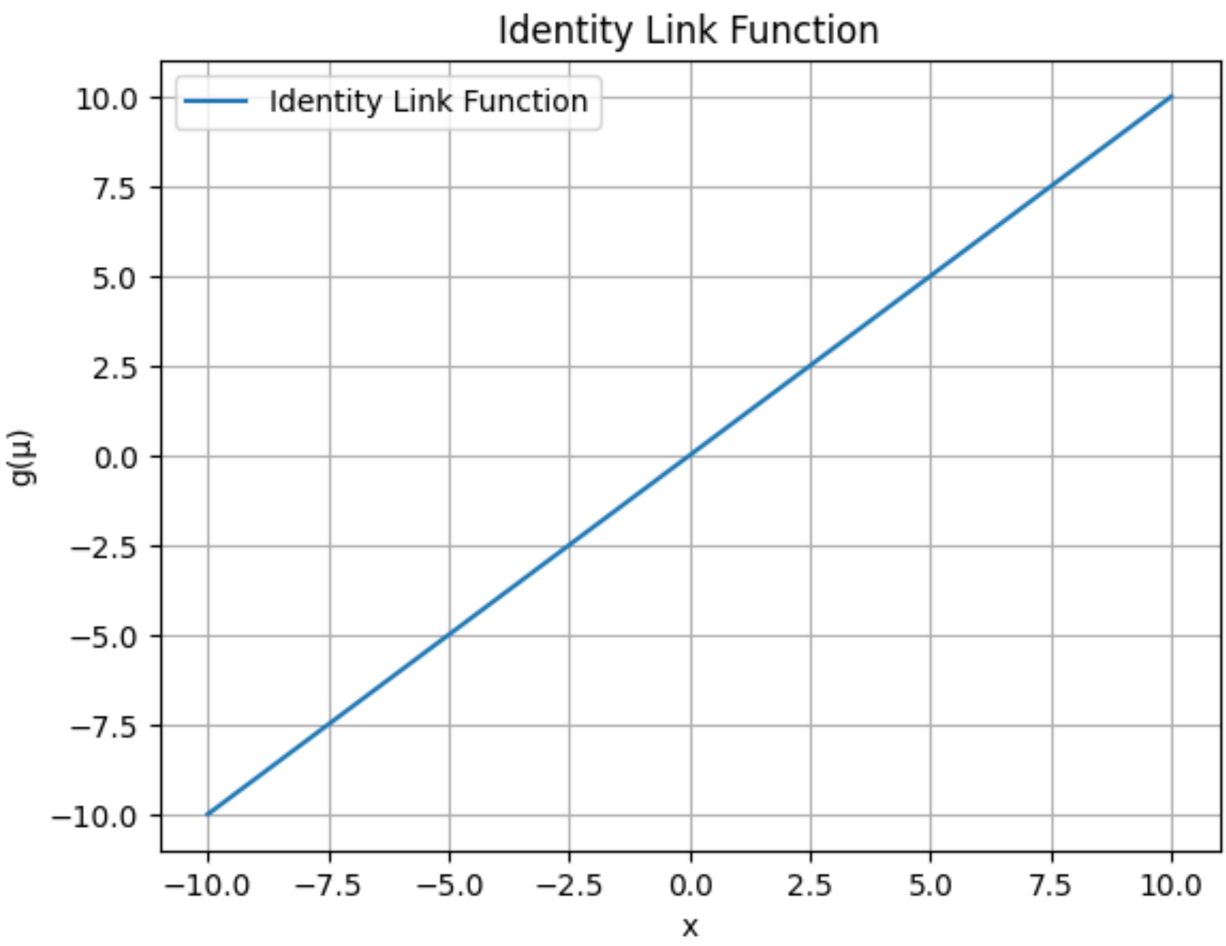
In [24]:
```
import warnings

# 워닝 메시지 필터링
warnings.filterwarnings("ignore")
```

링크함수 소개

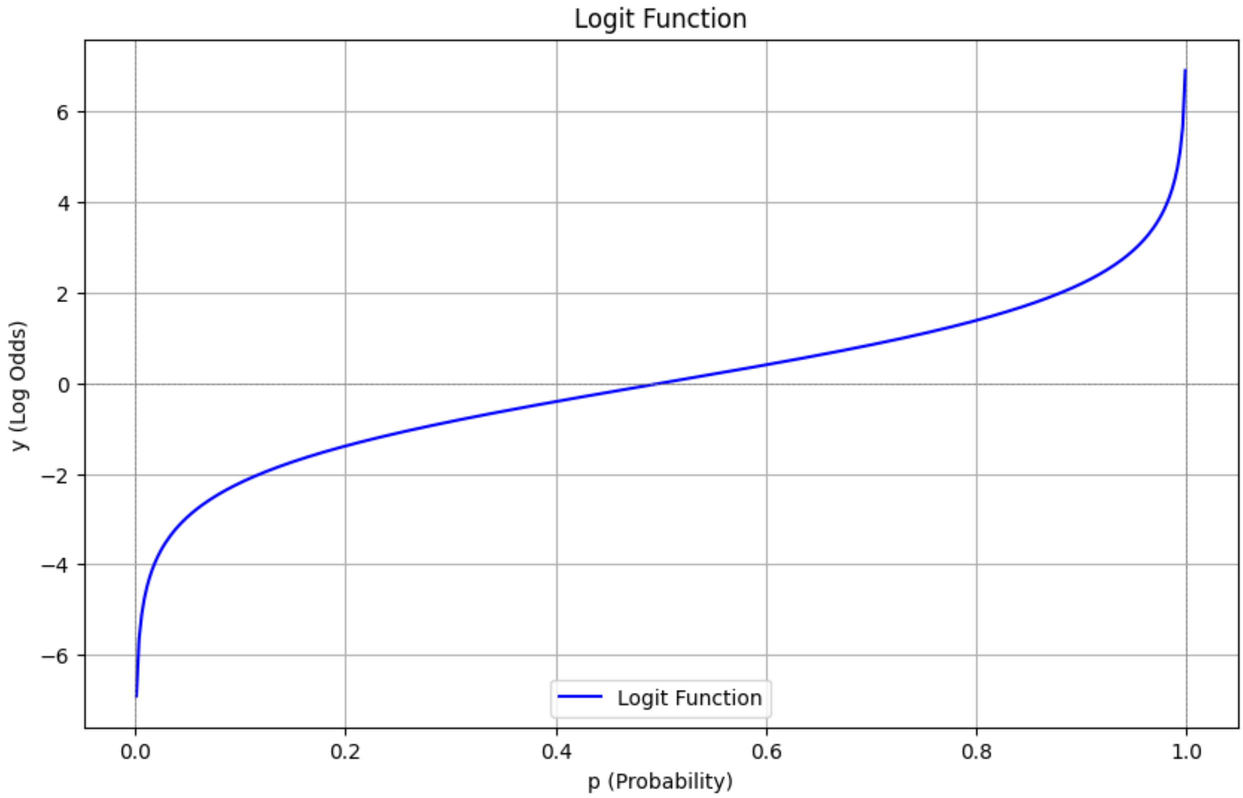
- Identity 링크

In [2]:
```
import matplotlib.pyplot as plt
import numpy as np

x = np.linspace(-10, 10, 100)
y = x

plt.plot(x, y, label="Identity Link Function")
plt.xlabel("x")
plt.ylabel("g(μ)")
plt.title("Identity Link Function")
plt.legend()
plt.grid(True)
plt.show()
```



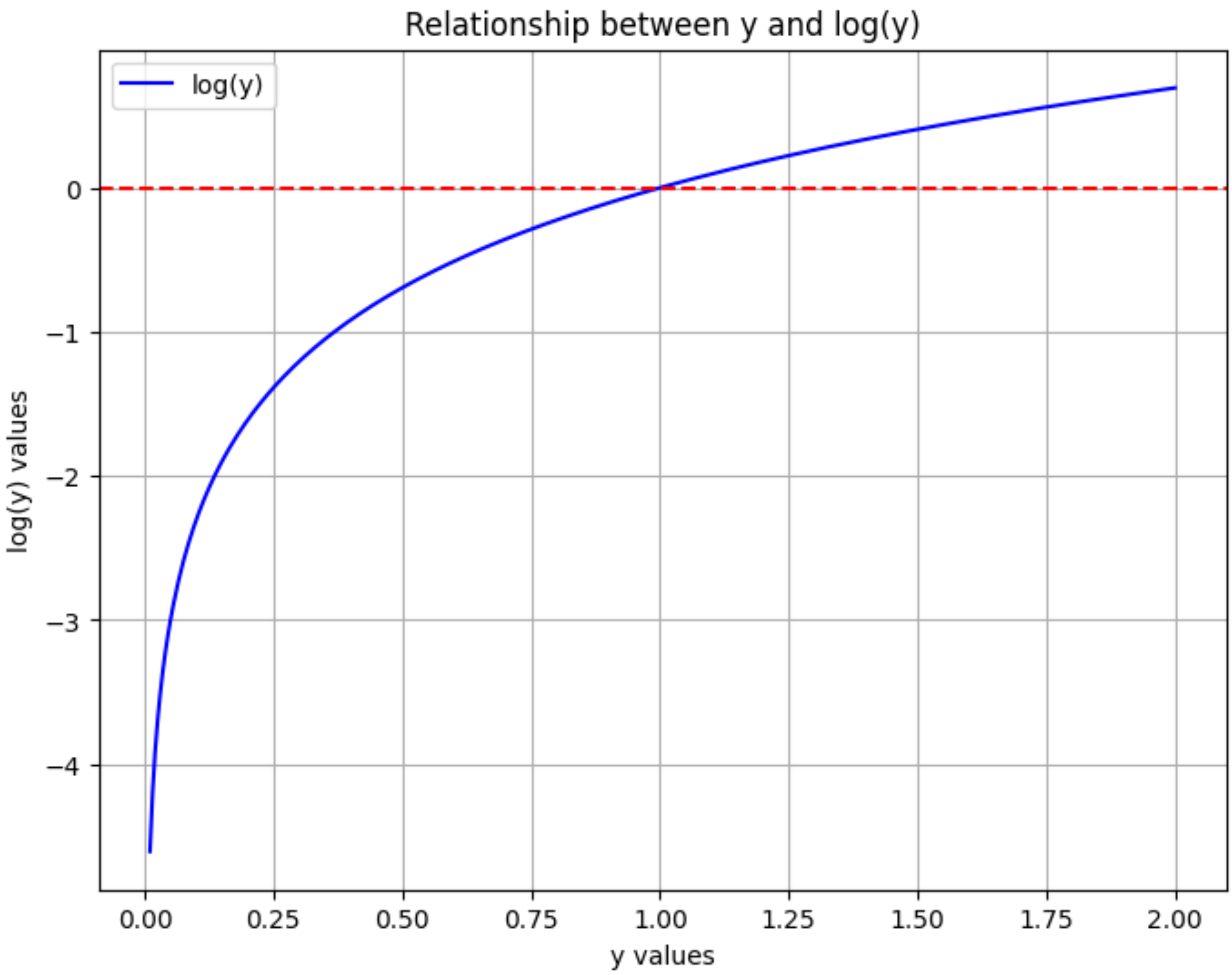
- Logit 링크

In [8]:
```python
import matplotlib.pyplot as plt
import numpy as np

# p는 0에 가깝게부터 1에 가깝게까지의 범위로 설정
p = np.linspace(0.001, 0.999, 400)

# 로짓 함수 계산
logit = np.log(p / (1 - p))

plt.figure(figsize=(10, 6))

# 그래프 그리기
plt.plot(p, logit, label="Logit Function", color='blue')
plt.axhline(0, color='grey', linestyle='--', linewidth=0.5)
plt.axvline(0, color='grey', linestyle='--', linewidth=0.5)
plt.axvline(1, color='grey', linestyle='--', linewidth=0.5)
plt.xlabel("p (Probability)")
plt.ylabel("y (Log Odds)")
plt.title("Logit Function")
plt.legend()
plt.grid(True)
plt.show()
```



- log 링크

In [10]:
```
import numpy as np
import matplotlib.pyplot as plt

# y 값을 0에서 2까지의 범위로 설정
y_values = np.linspace(0.01, 2, 400) # 0에 가까운 값부터 시작해서 log(0)의 정의되
log_y_values = np.log(y_values)

plt.figure(figsize=(8, 6))
plt.plot(y_values, log_y_values, label='log(y)', color='blue')
plt.axhline(0, color='red', linestyle='--')  # y=0 선
plt.xlabel('y values')
plt.ylabel('log(y) values')
plt.title('Relationship between y and log(y)')
plt.legend()
plt.grid(True)
plt.show()
```



일반화 선형모형(GLM)

- 로지스틱

In [12]:

```
import numpy as np
import statsmodels.api as sm
import pandas as pd

# 가상의 데이터 생성
np.random.seed(123)
n = 100
x1 = np.random.randn(n)
x2 = np.random.randn(n)
y_continuous = 1 + 2*x1 + 3*x2 + np.random.randn(n)
y = (y_continuous > 0.6).astype(int)

# 데이터프레임으로 변환
data = pd.DataFrame({'x1': x1, 'x2': x2, 'y': y})
data
```

Out[12]:

	x1	x2	y
0	-1.085631	0.642055	1
1	0.997345	-1.977888	0
2	0.282978	0.712265	1
3	-1.506295	2.598304	1
4	-0.578600	-0.024626	0
...	...	...	...
95	1.031114	-3.231055	0
96	-1.084568	-0.269293	0
97	-1.363472	-0.110851	0
98	0.379401	-0.341262	1
99	-0.379176	-0.217946	0

100 rows × 3 columns

In [13]:

```
# 로지스틱 회귀모형 적합
X = sm.add_constant(data[['x1', 'x2']])
model = sm.GLM(data['y'], X, family=sm.families.Binomial())
result = model.fit()

# 결과 출력
print(result.summary())

# 잔차 검정
print("\nResiduals:")
print(result.resid_deviance.head())
```

```
                 Generalized Linear Model Regression Results
==============================================================================
====
Dep. Variable:                      y   No. Observations:
100
Model:                            GLM   Df Residuals:
97
Model Family:                Binomial   Df Model:
2
Link Function:                  Logit   Scale:                          1.
0000
Method:                          IRLS   Log-Likelihood:                -17
.165
Date:                Fri, 27 Oct 2023   Deviance:                       34
.331
Time:                        13:03:15   Pearson chi2:
40.2
No. Iterations:                     8   Pseudo R-squ. (CS):             0.
6453
Covariance Type:            nonrobust
==============================================================================
====
                 coef    std err          z      P>|z|      [0.025      0.
975]
------------------------------------------------------------------------------
----
const          0.8742      0.492      1.777      0.076      -0.090       1
.839
x1             3.8204      0.958      3.989      0.000       1.943       5
.697
x2             6.1775      1.648      3.748      0.000       2.947       9
.408
==============================================================================
====

Residuals:
0    0.900588
1   -0.032693
2    0.058925
3    0.005307
4   -0.638016
dtype: float64
```

- 포아송 GLM

In [16]:
```
# 필요한 라이브러리 임포트
import numpy as np
import pandas as pd
import statsmodels.api as sm

# 데이터 생성
np.random.seed(123)  # 재현성을 위한 시드 설정

n_samples = 1000
X = np.linspace(0, 10, n_samples)
# 실제 모델: log(기대값) = intercept + coef * X
```

```python
intercept = 1.0
coef = 0.3

# 포아송 분포에서 랜덤하게 샘플 추출
y = np.random.poisson(np.exp(intercept + coef * X))

# 데이터프레임 형태로 변환
data = pd.DataFrame({"X": X, "y": y})

data
```

Out[16]:

	X	y
0	0.00000	2
1	0.01001	5
2	0.02002	3
3	0.03003	0
4	0.04004	2
...	...	...
995	9.95996	47
996	9.96997	46
997	9.97998	57
998	9.98999	48
999	10.00000	47

1000 rows × 2 columns

In [15]:

```python
# 포아송 GLM 모델 적합
exog = sm.add_constant(data["X"])  # 상수항 추가
poisson_model = sm.GLM(data["y"], exog, family=sm.families.Poisson())
result = poisson_model.fit()

# 결과 출력
print(result.summary())
```

```
                 Generalized Linear Model Regression Results
==========================================================================
====
Dep. Variable:                      y   No. Observations:
1000
Model:                            GLM   Df Residuals:
998
Model Family:                 Poisson   Df Model:
1
Link Function:                    Log   Scale:                          1.
0000
Method:                          IRLS   Log-Likelihood:                -26
44.7
Date:                Fri, 27 Oct 2023   Deviance:                       10
09.8
Time:                        13:07:54   Pearson chi2:
971.
No. Iterations:                     5   Pseudo R-squ. (CS):              1
.000
Covariance Type:            nonrobust
==========================================================================
====
                 coef    std err          z      P>|z|      [0.025      0.
975]
--------------------------------------------------------------------------
----
const          0.9866      0.024     40.374      0.000       0.939       1
.034
X              0.3011      0.003     93.386      0.000       0.295       0
.307
==========================================================================
====
```

정규화 회귀모형

- 라쏘

In [22]:

```
import numpy as np
import statsmodels.api as sm
from sklearn.datasets import make_regression

# 데이터 생성
X, y = make_regression(n_samples=1000, n_features=20, noise=0.1, random_s

# statsmodels를 위한 상수항 추가
X_const = sm.add_constant(X)

pd.DataFrame(X_const)
```

Out[22]:

	0	1	2	3	4	5	6	
0	1.0	0.225842	1.551378	-0.107347	0.859695	-0.942963	-1.096625	-1.1!
1	1.0	0.110836	-1.454615	0.263888	-1.654510	0.818549	0.482849	0.35
2	1.0	0.458600	-0.081280	-0.698474	0.737528	0.860085	0.275249	0.33
3	1.0	-1.795643	-0.453414	-0.423760	0.155325	0.487775	0.398147	0.73
4	1.0	-1.180626	0.339530	0.328010	-0.224555	0.963951	-1.058450	0.94
...	...	...	...	...	...	...	...	
995	1.0	1.726964	-0.372833	0.722381	1.024063	-1.760809	0.592527	0.22
996	1.0	0.919229	-1.438278	0.113270	2.062525	1.281016	-1.067533	1.87
997	1.0	-0.512589	1.124777	0.898360	0.906544	-2.301472	0.072252	1.39
998	1.0	-2.968368	-0.929848	0.055208	1.366747	0.427677	0.313143	0.72
999	1.0	-0.487167	2.801373	-1.088635	0.288150	0.321653	0.358848	-0.84

1000 rows × 21 columns

In [25]:

```
# Lasso 회귀 적합 (statsmodels의 GLM 사용)
alpha = 0.5  # 정규화 파라미터
lasso_model = sm.GLM(y, X_const, family=sm.families.Gaussian(), link=sm.g
lasso_results = lasso_model.fit_regularized(method='elastic_net', L1_wt=1

# 결과 출력
print(lasso_results.summary())

# 변수 선택 결과
coef = lasso_results.params
selected_features = np.where(coef != 0)[0]

print("\nSelected features:")
for index in selected_features:
    if index == 0:
        print("Intercept")
    else:
        print(f"Feature {index-1}")
```

```
                 Generalized Linear Model Regression Results
===========================================================================
====
Dep. Variable:                      y   No. Observations:
1000
Model:                            GLM   Df Residuals:
990
Model Family:                Gaussian   Df Model:
10
Link Function:               Identity   Scale:                         0.01
0377
Method:                   elastic_net   Log-Likelihood:                  87
0.15
Date:                Fri, 27 Oct 2023   Deviance:                        10
```

```
.274
Time:                        13:23:26   Pearson chi2:                     
10.3
No. Iterations:                    10   Pseudo R-squ. (CS):              1
.000
Covariance Type:            nonrobust                                         
==============================================================================
====
                 coef    std err          z      P>|z|      [0.025      0.
975]
------------------------------------------------------------------------------
----
const               0          0        nan        nan           0        
0
x1            80.0020      0.003    2.5e+04      0.000      79.996      80
.008
x2            98.5792      0.003    2.9e+04      0.000      98.573      98
.586
x3             5.5671      0.003   1748.798      0.000       5.561       5
.573
x4                  0          0        nan        nan           0        
0
x5            86.4662      0.003   2.67e+04      0.000      86.460      86
.473
x6                  0          0        nan        nan           0        
0
x7            69.4306      0.003   2.18e+04      0.000      69.424      69
.437
x8                  0          0        nan        nan           0        
0
x9                  0          0        nan        nan           0        
0
x10                 0          0        nan        nan           0        
0
x11           18.6065      0.003   5928.834      0.000      18.600      18
.613
x12           39.6357      0.003   1.23e+04      0.000      39.629      39
.642
x13                 0          0        nan        nan           0        
0
x14            3.1033      0.003    950.599      0.000       3.097       3
.110
x15                 0          0        nan        nan           0        
0
x16           26.3863      0.003   7891.026      0.000      26.380      26
.393
x17                 0          0        nan        nan           0        
0
x18           86.8828      0.003   2.66e+04      0.000      86.876      86
.889
x19                 0          0        nan        nan           0        
0
x20                 0          0        nan        nan           0        
0
==============================================================================
====
```

```
Selected features:
Feature 0
Feature 1
Feature 2
Feature 4
Feature 6
Feature 10
Feature 11
Feature 13
Feature 15
Feature 17
```

비선형 회귀모형 소개

- 다항 회귀(Poly Regression)

In [32]:
```
import numpy as np
import pandas as pd
import statsmodels.api as sm
from sklearn.datasets import make_regression
from sklearn.preprocessing import PolynomialFeatures

# 데이터 생성 (2개의 설명변수)
X, y = make_regression(n_samples=1000, n_features=2, noise=0.1, random_st

# 다항식으로 데이터 변환
poly_transformer = PolynomialFeatures(degree=2, include_bias=False, inter
X_poly = poly_transformer.fit_transform(X)

# X1^2와 X1*X2 항 제거
X_poly = X_poly[:, [0, 1, 3]]  # X1, X2, X2^2 항만 선택

# 변수명 지정
column_names = ['X1', 'X2', 'X2^2']

# DataFrame으로 변환
X_df = pd.DataFrame(X_poly, columns=column_names)

# statsmodels를 위한 상수항 추가
X_poly_const = sm.add_constant(X_df)

# OLS 회귀 모델 적합
poly_model = sm.OLS(y, X_poly_const).fit()

# 결과 출력
print(poly_model.summary())
```

```
                            OLS Regression Results
==============================================================================
====
Dep. Variable:                      y   R-squared:                       1
.000
Model:                            OLS   Adj. R-squared:                  1
.000
Method:                 Least Squares   F-statistic:                 5.407
e+07
Date:                Fri, 27 Oct 2023   Prob (F-statistic):
0.00
Time:                        13:30:58   Log-Likelihood:                 89
9.63
No. Observations:                1000   AIC:                            -1
791.
Df Residuals:                     996   BIC:                            -1
772.
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
====
                 coef    std err          t      P>|t|      [0.025      0.
975]
------------------------------------------------------------------------------
----
const          0.0002      0.003      0.050      0.960      -0.006       0
.006
X1            40.7142      0.003   1.25e+04      0.000      40.708      40
.721
X2             6.6030      0.003   2148.023      0.000       6.597       6
.609
X2^2        9.439e-05      0.003      0.029      0.977      -0.006       0
.006
==============================================================================
====
Omnibus:                        1.906   Durbin-Watson:                   2
.013
Prob(Omnibus):                  0.386   Jarque-Bera (JB):                1
.791
Skew:                           0.066   Prob(JB):                        0
.408
Kurtosis:                       3.160   Cond. No.
1.12
==============================================================================
====

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is cor
rectly specified.
```

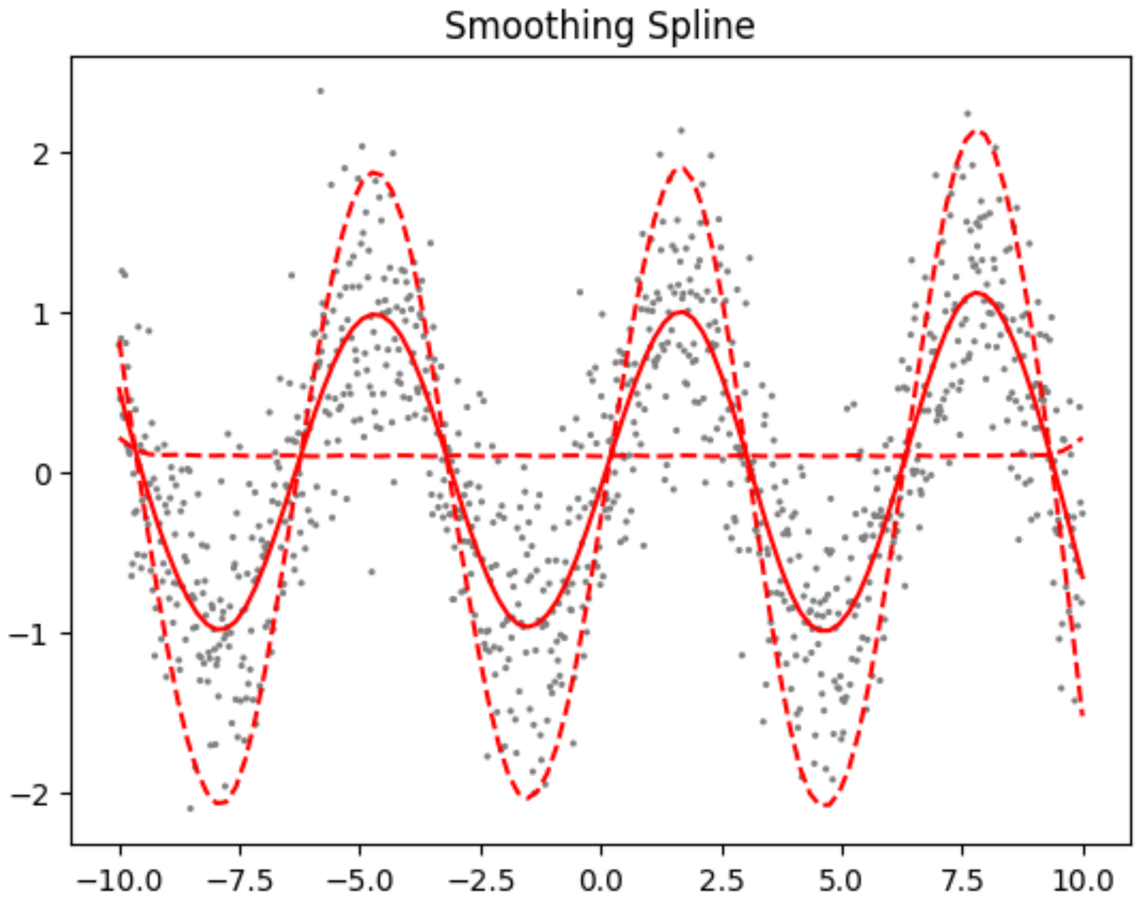
- 스무딩 스플라인 (Smoothing Spline)

In [35]:

```
import numpy as np
from pygam import LinearGAM, s
import matplotlib.pyplot as plt
```

```
# 간단한 비선형 데이터 생성
np.random.seed(42)
X = np.linspace(-10, 10, 1000)
y = np.sin(X) + 0.5 * np.random.normal(size=1000)

# 데이터 형태 변환 (2D array로)
X = X[:, np.newaxis]

# 스무딩 스플라인 적합
gam = LinearGAM(s(0)).fit(X, y)

# 결과 출력
print(gam.summary())

# 예측값 및 신뢰구간 플롯
XX = gam.generate_X_grid(term=0)
plt.plot(XX, gam.predict(XX), 'r-')
plt.plot(XX, gam.predict(XX) + gam.confidence_intervals(XX, width=0.95)[:
plt.plot(XX, gam.predict(XX) - gam.confidence_intervals(XX, width=0.95)[:
plt.scatter(X, y, facecolor='gray', edgecolors='none', s=5)
plt.title("Smoothing Spline")
plt.show()
```

```
LinearGAM
=============================================== ==========================
================================
Distribution:                        NormalDist Effective DoF:
14.3348
Link Function:                     IdentityLink Log Likelihood:
-1548.9948
Number of Samples:                         1000 AIC:
3128.6592
                                                AICc:

3129.1685
                                                GCV:

0.2455
                                                Scale:

0.2392
                                                Pseudo R-Squared:

0.6836
==========================================================================
================================
Feature Function                  Lambda               Rank         EDoF
P > x        Sig. Code
================================= ==================== ============ ======
====== ============ ============
s(0)                              [0.6]                20           14.3
1.11e-16     ***
intercept                                              1            0.0
9.51e-01
==========================================================================
================================
Significance codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

WARNING: Fitting splines and a linear function to a feature introduces a m
odel identifiability problem
         which can cause p-values to appear significant when they are not.

WARNING: p-values calculated in this manner behave correctly for un-penali
zed models or models with
         known smoothing parameters, but when smoothing parameters have be
en estimated, the p-values
         are typically lower than they should be, meaning that the tests r
eject the null too readily.
None
```




```
In [ ]:
```