1.6

(#5)

$x_1 B_2S_3 + x_2 H_2O \rightarrow x_3 H_3BO_3 + x_4 H_2S$

$$B_2S_3 \begin{pmatrix} 2 \\ 3 \\ 0 \\ 0 \end{pmatrix} \quad H_2O \begin{pmatrix} 0 \\ 0 \\ 2 \\ 1 \end{pmatrix} \quad H_3BO_3 \begin{pmatrix} 1 \\ 0 \\ 3 \\ 3 \end{pmatrix} \quad H_2S \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$x_1 \begin{pmatrix} 2 \\ 3 \\ 0 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \rightarrow x_3 \begin{pmatrix} 1 \\ 0 \\ 3 \\ 3 \end{pmatrix} + x_4 \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{cccc|c} 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 0 & 0 & -3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array}\right) = \left(\begin{array}{cccc|c} 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & -\frac{3}{2} & 1 & 0 \\ 0 & 2 & -\frac{9}{2} & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 2 & 0 & -1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & -\frac{3}{2} & 1 & 0 \\ 0 & 2 & -3 & -2 & 0 \end{array}\right) = \left(\begin{array}{cccc|c} 2 & 0 & -1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & -\frac{3}{2} & 1 & 0 \\ 0 & 0 & -3 & 2 & 0 \end{array}\right)$$

$$= \left(\begin{array}{cccc|c} 2 & 0 & -1 & 0 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & -3 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \qquad \begin{cases} x_1 = \frac{1}{2} x_3 \\ x_2 = 3 x_3 \\ x_3 = \frac{2}{3} x_4 \\ x_4 \text{ is free} \end{cases}$$

$x_4 : 3 \quad x_3 : 2 \quad x_2 : 6 \quad x_1 : 1$

$B_2S_3 + 6H_2O \rightarrow 2H_3BO_3 + 3H_2S$

1.10

#9.

$$x_{k+1} = M x_k \quad \text{for } k = 0, 1, 2 \cdots$$

$$M = \begin{pmatrix} 0.93 & 0.05 \\ 0.07 & 0.95 \end{pmatrix} \text{이고} \quad x_0 = \begin{pmatrix} 800,000 \\ 500,000 \end{pmatrix} \text{이다.}$$

2017년도가 x_0이므로

2015년도 population은 x_2이다.

$$x_1 = \begin{pmatrix} 0.93 & 0.05 \\ 0.07 & 0.95 \end{pmatrix} \begin{pmatrix} 800,000 \\ 500,000 \end{pmatrix} = \begin{pmatrix} 769,000 \\ 531,000 \end{pmatrix}$$

$$x_2 = \begin{pmatrix} 0.93 & 0.05 \\ 0.07 & 0.95 \end{pmatrix} \begin{pmatrix} 769,000 \\ 531,000 \end{pmatrix} = \begin{pmatrix} 741,720 \\ 558,280 \end{pmatrix}$$