#25)

Since x is vector in $\mathbb{C}$.

So, x can be expressed as $x = Re\,x + i\,Im\,x$

$$Ax = A(Re\,x + i\,Im\,x)$$
$$= A(Re\,x) + iA(Im\,x)$$

Since A is real, $A(Re\,x)$ and $A(Im\,x)$ are also real

Therefore, $A(Re\,x)$ is the real part of Ax

$A(Im)$ is the imaginary part of Ax.

#26

a. $$Av = (a-bi)v$$
$$= (a-bi)(Re\,v + i\,Im\,v)$$
$$= \underbrace{(a\,Re\,v + b\,Im\,v)}_{Re\,Av} + i\,\underbrace{(a\,Im\,v - b\,Re\,v)}_{Im\,Av}$$

$$\therefore A(Re\,v) = Re\,Av = a\,Re\,v + b\,Im\,v$$
$$A(Im\,v) = Im\,Av = -b\,Re\,v + a\,Im\,v.$$

b. $P = [Re\,v \;\; Im\,v]$ $\quad C = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$

$$A(Re\,v) = P\begin{pmatrix} a \\ b \end{pmatrix}$$
$$A(Im\,v) = P\begin{pmatrix} -b \\ a \end{pmatrix}$$

$$\therefore AP = [A(Re\,v) \;\; A(Im\,v)]$$
$$= \left[P\begin{pmatrix} a \\ b \end{pmatrix} \;\; P\begin{pmatrix} -b \\ a \end{pmatrix}\right] = P\begin{bmatrix} a & -b \\ b & a \end{bmatrix} = PC$$