# Decision Theory Case: Gillette vs. Energizer

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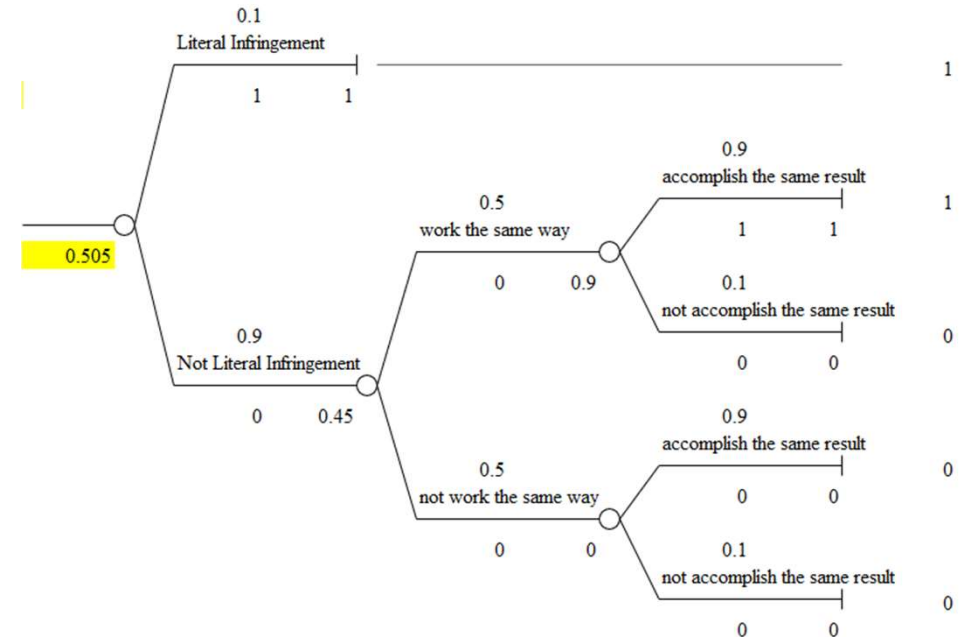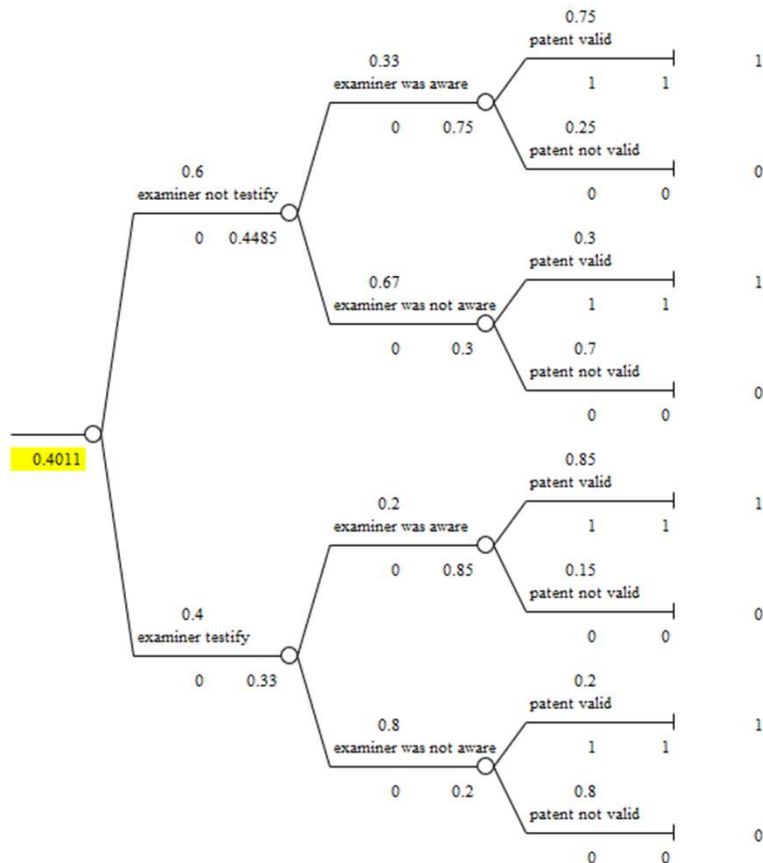
Overview of Decision Tree

- Here is the overview of the whole decision tree. The number in red will be calculated in the next few slides.

- Based on maximizing expectations, we prefer to continue litigation. The expected payoff for litigation is **$4.95** million in this case.
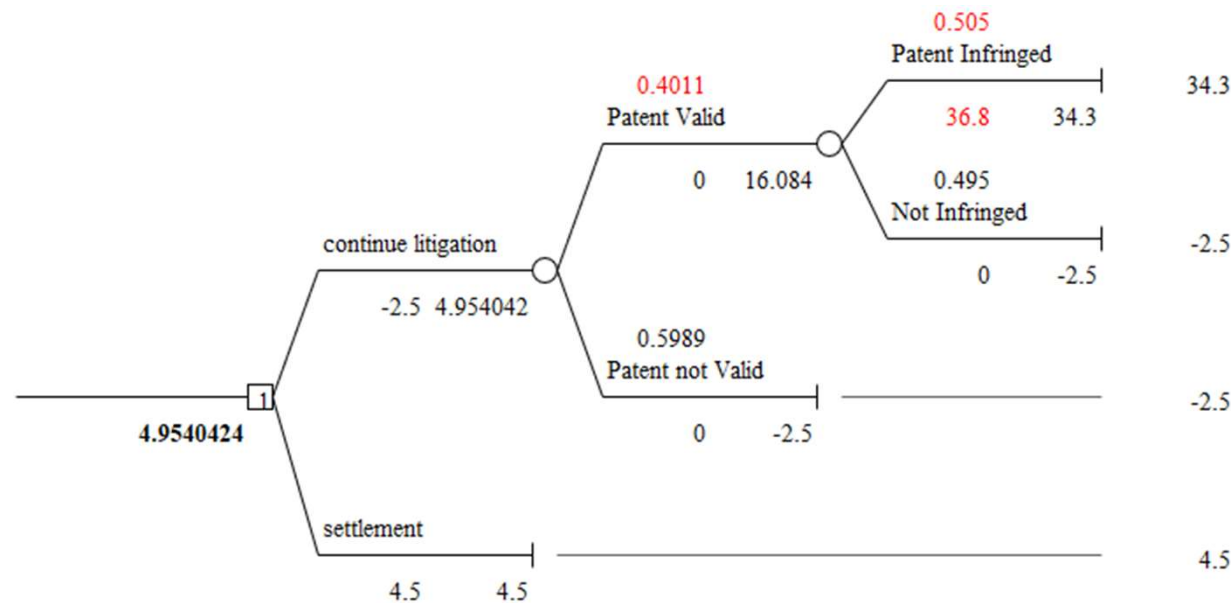
Details in each Branch

- We can use characteristic function to compute the probability. Therefore, if we go to court, the **win probability** is only 0.4011*0.505 = **0.2026**. And therefore, the **loss probability is 0.7974**.

- There are 10 cases overall if we win. The expected damages awarded if we win is $7.3 million. The expected future business gains if we win is $29.5 million.

- The expected additional payoff is 7.3 + 29.5 = **$36.8 million**. (in the overview of tree).
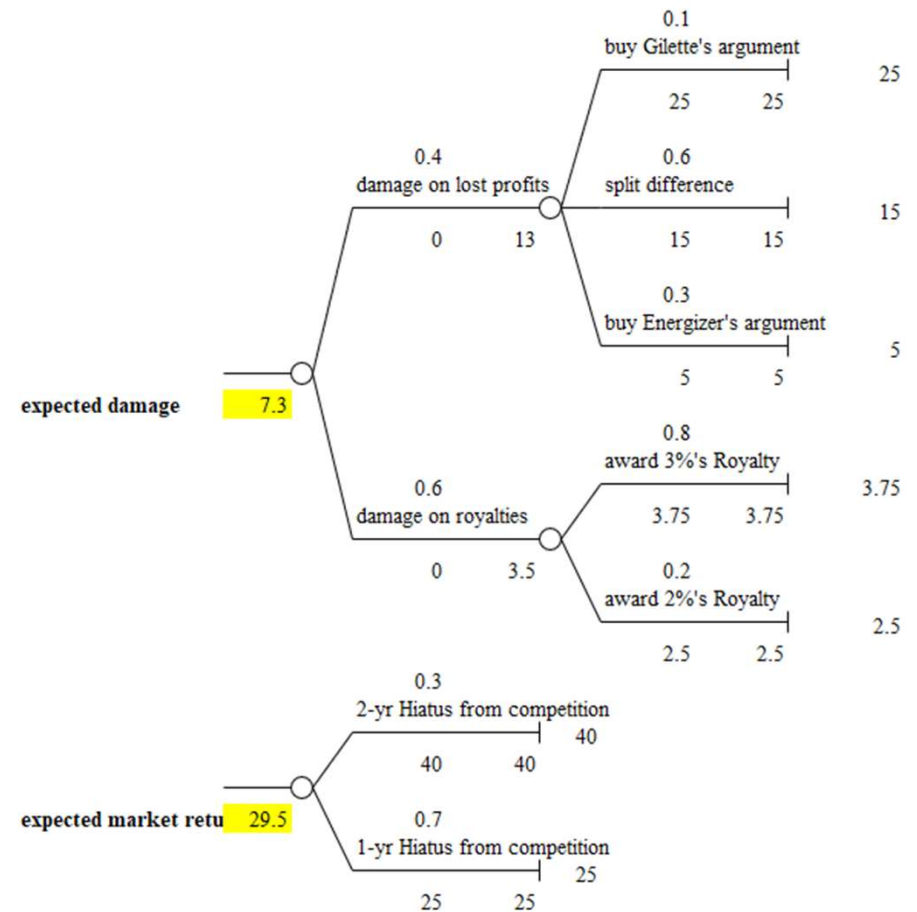
Sensitivity Analysis for Expectations

- This figure and table below shows the sensitivity of expected values of strategies to possibility of 2-yr Hiatus for Business Impact. This relationship between P and Expectations for Continuing Litigation is **linear**.

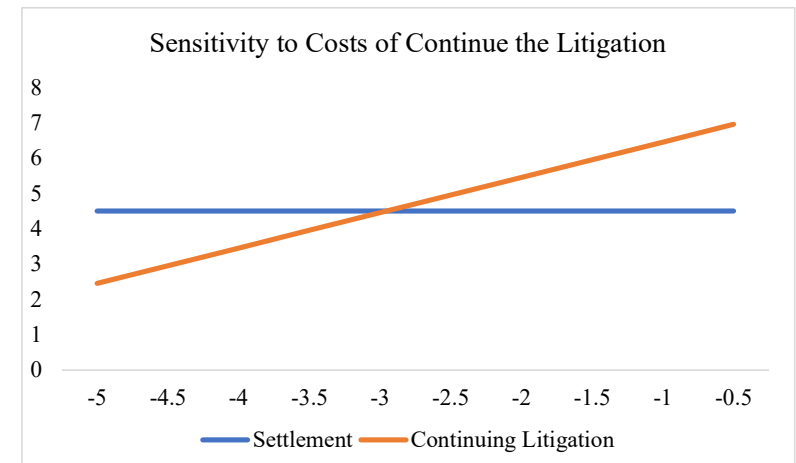| Sensitivity to Possibility of 2-yr Hiatus for Business Impact						
P	Settlement	Continuing Litigation	Best Alternative	Best Return	Perfect Info	Value of Info.
0	4.5	**4.0425**	Settlement	4.5000	4.5000	0.0000
0.1	4.5	**4.3464**	Settlement	4.5000	4.7581	0.2581
0.2	4.5	**4.6502**	Continue Litigation	4.6502	5.0162	0.3660
0.3	4.5	**4.9540**	Continue Litigation	4.9540	5.2743	0.3202
0.4	4.5	**5.2579**	Continue Litigation	5.2579	5.5324	0.2745
0.5	4.5	**5.5617**	Continue Litigation	5.5617	5.7904	0.2287
0.6	4.5	**5.8655**	Continue Litigation	5.8655	6.0485	0.1830
0.7	4.5	**6.1694**	Continue Litigation	6.1694	6.3066	0.1372
0.8	4.5	**6.4732**	Continue Litigation	6.4732	6.5647	0.0915
0.9	4.5	**6.7770**	Continue Litigation	6.7770	6.8228	0.0457
1	4.5	**7.0809**	Continue Litigation	7.0809	7.0809	0.0000

Sensitivity to Possibility of 2-yr Hiatus for Business impact
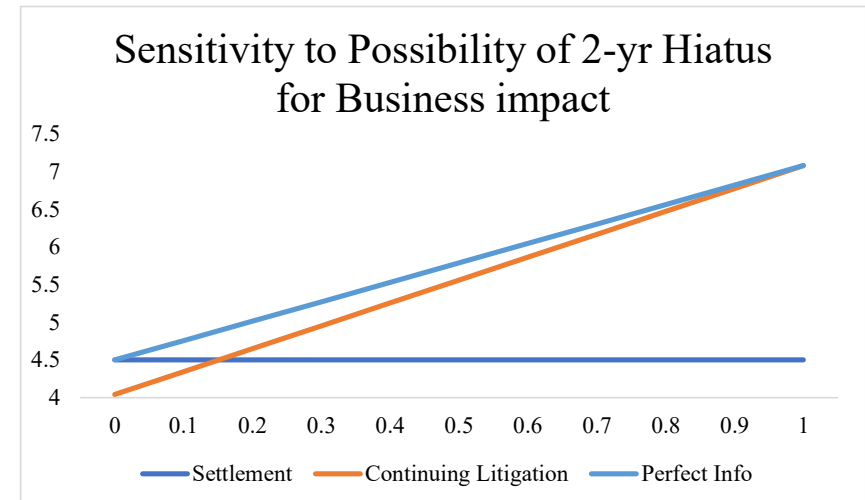
To be Cont'd

- Here are more examples but the relationship is still linear.

Sensitivity to Value of 1-yr Hiatus Business Impact				
Value	Settlement	Continuing Litigation	Best Alternative	Best Return
0	4.5	1.4093	Settlement	4.5
5	4.5	2.1183	Settlement	4.5
10	4.5	2.8272	Settlement	4.5
15	4.5	3.5362	Settlement	4.5
20	4.5	4.2451	Settlement	4.5
25	4.5	4.9540	Continue Litigation	4.9540
30	4.5	5.6630	Continue Litigation	5.6630
35	4.5	6.3719	Continue Litigation	6.3719
40	4.5	7.0809	Continue Litigation	7.0809
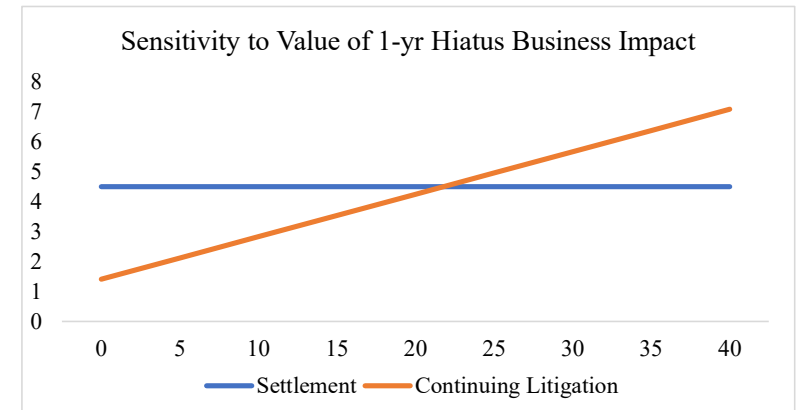
Sensitivity to Cost of Continuing the Litigation				
Cost (negative)	Settlement	Continuing Litigation	Best Alternative	Best Return
-5	4.5	2.4540	Settlement	4.5000
-4.5	4.5	2.9540	Settlement	4.5000
-4	4.5	3.4540	Settlement	4.5000
-3.5	4.5	3.9540	Settlement	4.5000
-3	4.5	4.4540	Settlement	4.5000
-2.5	4.5	4.9540	Continue Litigation	4.9540
-2	4.5	5.4540	Continue Litigation	5.4540
-1.5	4.5	5.9540	Continue Litigation	5.9540
-1	4.5	6.4540	Continue Litigation	6.4540
-0.5	4.5	6.9540	Continue Litigation	6.9540

Sensitivity to Value of 1-yr Hiatus Business Impact

Settlement — Continuing Litigation

Sensitivity to Costs of Continue the Litigation

Settlement — Continuing Litigation

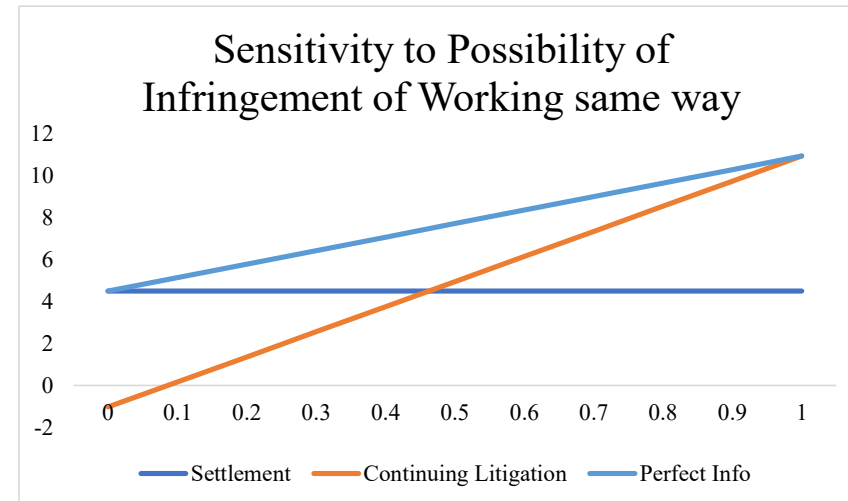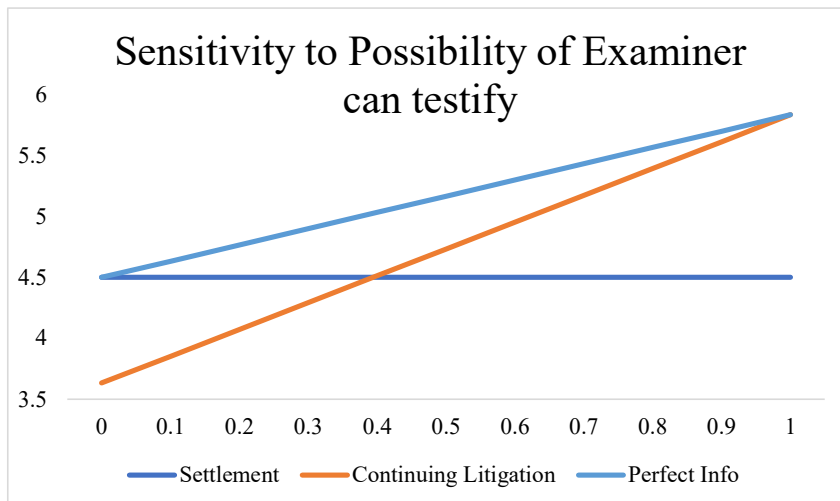# Value of Information Analysis

- For convenience, we only consider the perfect information for each case. We found that the perfect information based on probability is linear. Under such case, the value of whether examiner can testify is **$0.34 million**. The value of whether infringement on working on the same way is **2.76 million** (from the Excel table).

Sensitivity to Possibility of Examiner can testify

Settlement — Continuing Litigation — Perfect Info

Sensitivity to Possibility of Infringement of Working same way

Settlement — Continuing Litigation — Perfect Info

Look at the Payoff Distribution Again…

- Here is the overall payoff distribution for continuing litigation. As we can see, the variance is quite large compared with expectation.

Continue Litigation Risk Profile				
Event on Damage	Event on Business	Return	Probability	Partial Variance
lost profits + buy G	2 yr Hiatus	62.5	0.002430666	3311.537236
lost profits + split difference	2 yr Hiatus	52.5	0.014583996	2260.618084
lost profits + buy E	2 yr Hiatus	42.5	0.007291998	1409.698932
royalties + 3% award	2 yr Hiatus	41.25	0.029167992	1317.396538
royalties + 2% award	2 yr Hiatus	40	0.007291998	1228.219144
lost profits + buy G	1 yr Hiatus	47.5	0.005671554	1810.158508
lost profits + split difference	1 yr Hiatus	37.5	0.034029324	1059.239356
lost profits + buy E	1 yr Hiatus	27.5	0.017014662	508.3202041
royalties + 3% award	1 yr Hiatus	26.25	0.068058648	453.5178101
royalties + 2% award	1 yr Hiatus	25	0.017014662	401.8404161
none	none	-2.5	0.7974445	55.5627481
Statistical Information		E[r]		D[r]
		4.9540424		235.6512942

Risk Profile of Payoff

- Here we can see the pdf and cdf (risk profile) of two alternatives.

Risk Profile for Two Alternatives (pdf)

Risk Profile for Two Alternatives (cdf)
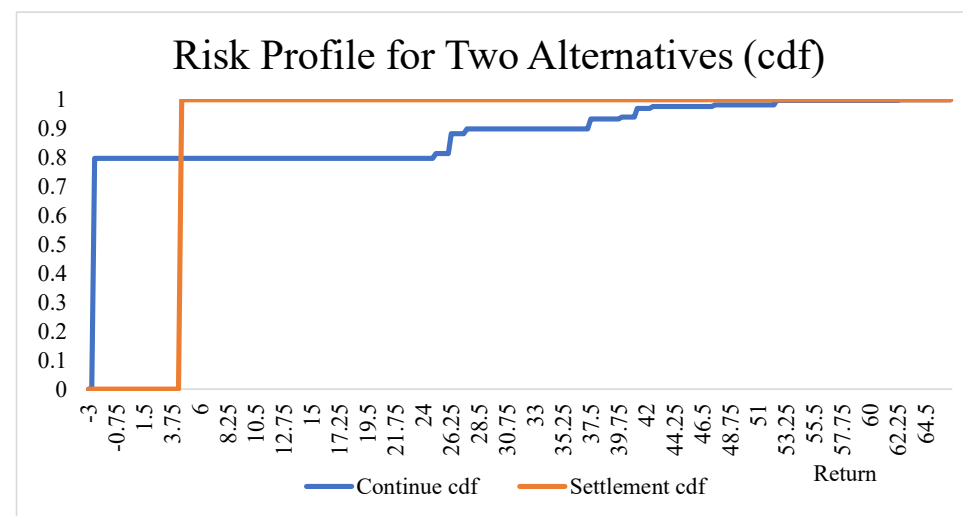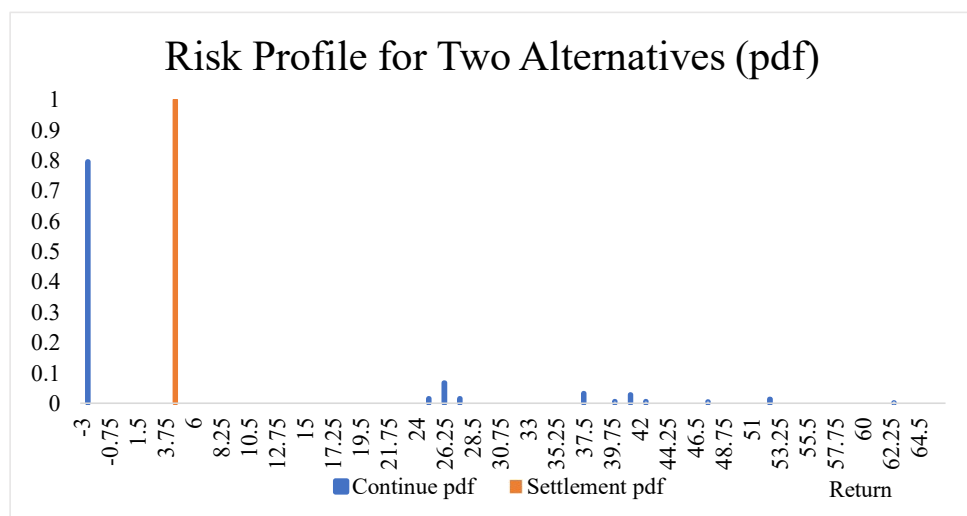
- The continue decision is very risky despite no stochastic dominance.

Downside Risk Analysis

- We can also use VaR (Value at Risk) and CVaR (Conditional Value at Risk) to analyze the **downside risk** for continuing litigation.

$$\mathrm{VaR}_\epsilon[\tilde{r}] \triangleq \inf\left\{v \in \mathbb{R} \mid \mathbb{P}\left[\tilde{r} + v \geq 0\right] \geq 1 - \epsilon\right\}$$

$$\mathrm{CVaR}_\epsilon^*[\tilde{r}] \triangleq \mathbb{E}_\mathbb{P}\left[-\tilde{r} \mid -\tilde{r} \geq \mathrm{VaR}_\epsilon(\tilde{r})\right]$$

$$\mathrm{CVaR}_\epsilon^*[\tilde{r}] \geq \mathrm{VaR}_\epsilon(\tilde{r})$$

- Compared with VaR, CVaR is a coherent risk measure which is widely acknowledged in the areas of portfolio optimization in recent years.
- However, in this case, the probability of loss for continuing litigation is so great.

VaR and CVaR in each case

- From the following table, we know the alternative of continuing litigation is very risky.

	\epsilon Analysis	VaR	CVaR
	5%	2.5	2.5
Downside Risk Analysis for Continuing Litigation	25%	2.5	2.5
	50%	2.5	2.5
	75%	2.5	2.5
	95%	-41.25	-2.50531264
The alternative of settlement, VaR = CVaR = -4.5 for every epsilon.			

Utility Theory

- Motivation: Given the two different streams from expectation and risk profile (and downside risk), we need to incorporate the risk and return together.

- Base case: Simple Exponential Utility Function:
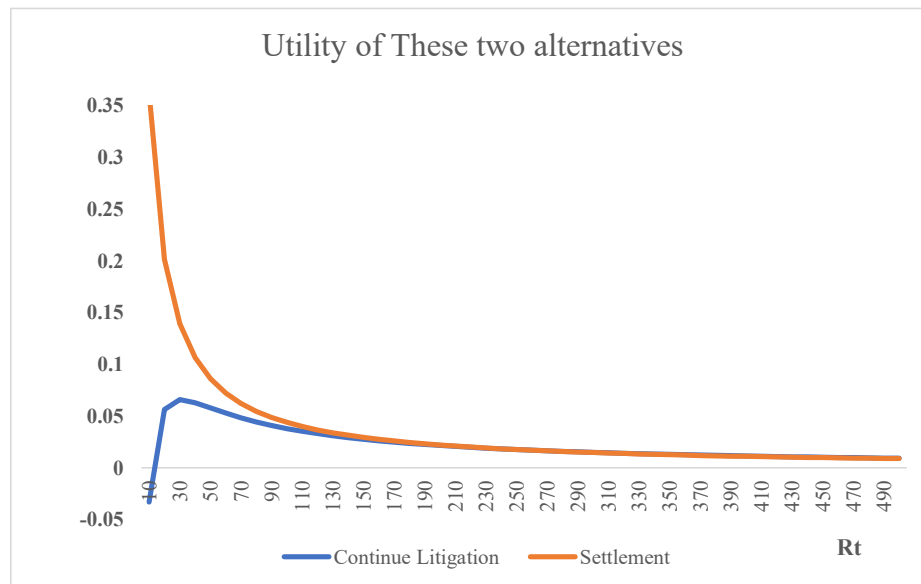
$$u(x) = 1 - exp(-\frac{x}{R}), R > 0$$

- Break-even point for two alternatives to be the same: R* = 250.198.

- If R< R*, choose to settle; Otherwise, choose to continue litigation.

Sensitivity Analysis for Rt

- The figure shows the utility of these two alternatives when R increases. **The speed of decreasing rate for settlement is larger.** Therefore, these two lines have intersections R* = 250.198 million.


From the left figure, when **R* = 30 million**, the utility of continuing litigation attains its maximum. The utility function achieves a balance between loss and payoffs.

Under Different Approximation

- As we see, when R increases, CE, Approx. CE1, Approx. CE2 **increases to** Expected Returns for Continuing Litigation. And the risk premium decreases thus.

Expected Returns under different approximations
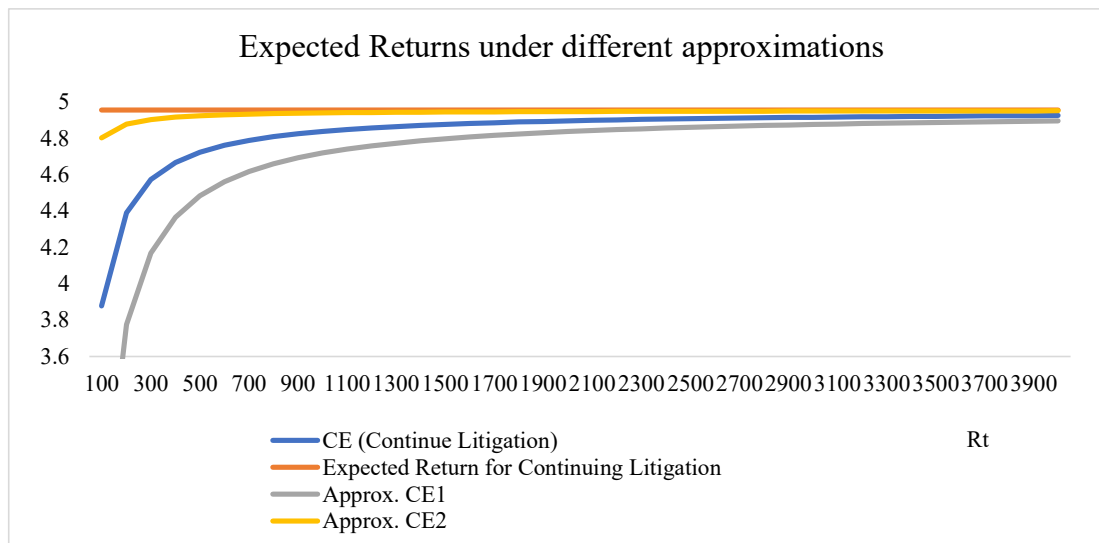
$$Approx.CE_1 = E[r] - \frac{D[r]}{R}, R > 0$$

$$Approx.CE_2 = E[r] - \frac{\sqrt{D[r]}}{R}, R > 0$$

Sensitivity Analysis for Information

- This figure and table below shows the sensitivity of CE of strategies to possibility of 2-yr Hiatus for Business Impact. The relationship between P and CE for Continuing Litigation is **nonlinear** (not obvious).

Sensitivity to Possibility of 2-yr Hiatus for Business Impact						
P	Settlement	Continuing Litigation	Best Alternative	Best Return	Perfect Info	Value of Info.
0	4.5	**2.7175**	Settlement	4.5	4.5	0
0.1	4.5	**2.9101**	Settlement	4.5	4.5160	0.0160
0.2	4.5	**3.1029**	Settlement	4.5	4.5319	0.0319
0.3	4.5	**3.2962**	Settlement	4.5	4.5479	0.0479
0.4	4.5	**3.4898**	Settlement	4.5	4.5638	0.0638
0.5	4.5	**3.6838**	Settlement	4.5	4.5798	0.0798
0.6	4.5	**3.8782**	Settlement	4.5	4.5957	0.0957
0.7	4.5	**4.0730**	Settlement	4.5	4.6117	0.1117
0.8	4.5	**4.2681**	Settlement	4.5	4.6277	0.1277
0.9	4.5	**4.4637**	Settlement	4.5	4.6436	0.1436
1	4.5	**4.6596**	Continue Litigation	4.6596	4.6596	0

CE for Sensitivity to Possibility of 2-yr Hiatus for Business Impact
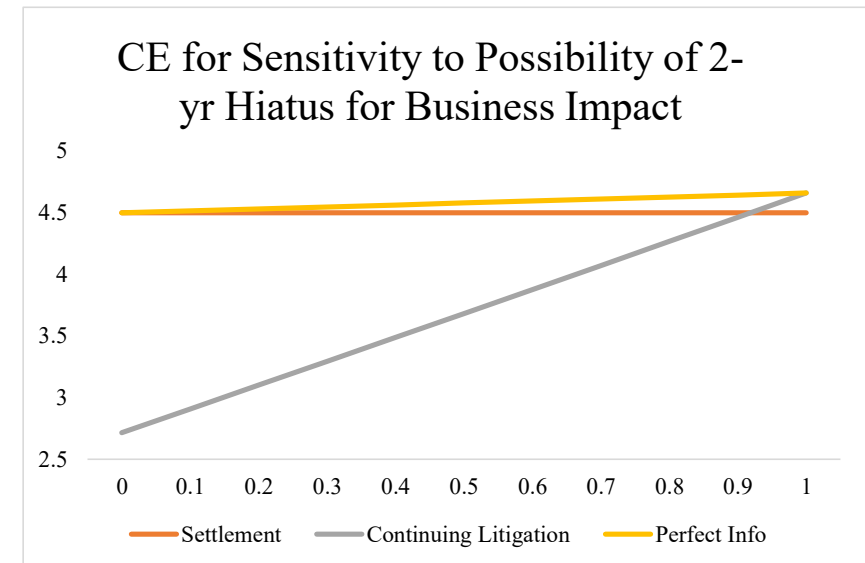
Here we assume Rt = 100.

Another example.

- This figure and table below shows the sensitivity of CE of strategies to value of 1-yr Hiatus for Business Impact. The relationship between Payoff and CE for Continuing Litigation is **nonlinear**.

Sensitivity to Value of 1-yr Hiatus Business Impact				
Payoff.	Settlement	Continuing Litigation	Best Alternative	Best Return
0	4.5	**0.8126**	Settlement	4.5
5	4.5	**1.4805**	Settlement	4.5
10	4.5	**2.1200**	Settlement	4.5
15	4.5	**2.7321**	Settlement	4.5
20	4.5	**3.3178**	Settlement	4.5
25	4.5	**3.8782**	Settlement	4.5
30	4.5	**4.4142**	Settlement	4.5
35	4.5	**4.9267**	Continue Litigation	4.926737829
40	4.5	**5.4167**	Continue Litigation	5.416715127

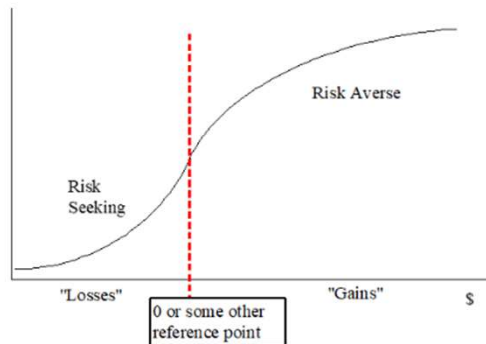
Sensitivity to Value of 1-yr Hiatus Business Impact
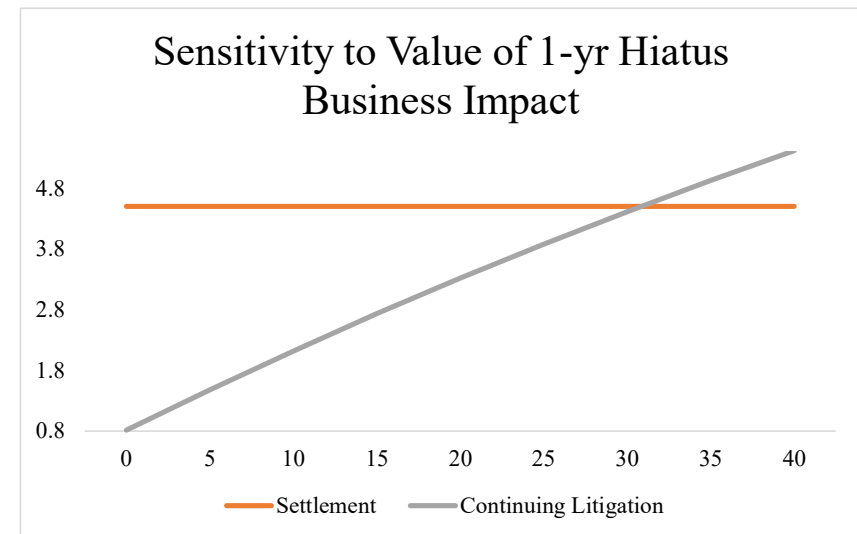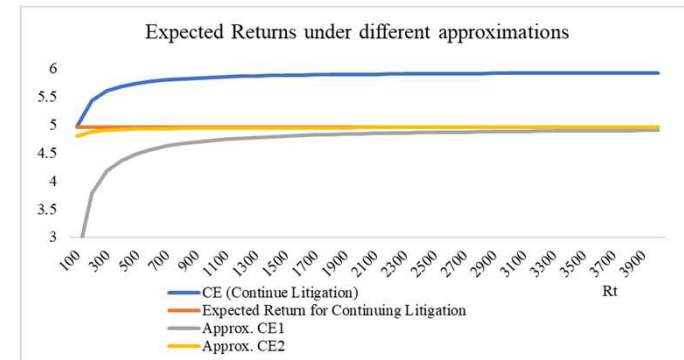
Here we assume Rt = 100.

Further Extensions

- Risk seeking for loss
  - Theoretically: Fewer R to attain equivalence between these alternatives.
  - Empirically: Under the following case, break-even point for R* is **64.23**.

$$u(x) = \begin{cases} 1 - exp(-\frac{x}{R}), & x > 0 \\ exp(\frac{x}{R}) - 1, & x \leq 0 \end{cases}$$

Further Extensions

- Other Utility Function (CARA etc.) to be considered.
  - The more risk seeking is, the higher possibility of choosing to continue litigation.
  - We compare exponential utility function with CRRA and find that CRRA function are more risk averse when R = 100 here.
  - Two CARA Functions:
    $$u(x) = log(x - x_{min} + 1) \qquad u(x) = \sqrt{x - x_{min}}$$

| | Return | Probability | CDF | Different Concave Utility Function | | |
				1-exp(-x/100)	log(x - min + 1)	sqrt(x-min)
	62.5	0.002430666	1	0.464738571	1.819543936	8.062257748
	52.5	0.014583996	0.997569334	0.408444636	1.748188027	7.416198487
	42.5	0.007291998	0.982985338	0.346230215	1.662757832	6.708203932
	41.25	0.029167992	0.97569334	0.338006803	1.65079304	6.614378278
Whole picture of the	40	0.007291998	0.946525348	0.329679954	1.638489257	6.519202405
Return Distribution	47.5	0.005671554	0.93923335	0.378114944	1.707570176	7.071067812
	37.5	0.034029324	0.933561796	0.312710721	1.612783857	6.32455532
	27.5	0.017014662	0.899532472	0.240427877	1.491361694	5.477225575
	26.25	0.068058648	0.88251781	0.230873636	1.47348697	5.361902647
	25	0.017014662	0.814459162	0.221199217	1.45484486	5.244044241
	-2.5	0.7974445	0.7974445	-0.025315121	0	0
Overall Utility			U for continue	0.038039901	0.317120037	1.21980373
			U for Settle	0.044002518	0.903089987	2.645751311

Summary

- Basics: Win Probability, Expected Damages, Costs to Litigation,…
- Sensitivity Analysis & Value of Information:
  - Identify Key Parameters in this problem.
  - Show how they impact expected values with perfect information
- Risk Profile & Downside Risk:
  - The Overall Risk for Decision Makers: Stochastic Dominance or Not.
  - The Percentile Risk (VaR / CVaR) to illustrate downside risk.
- Utility Theory:
  - Utility Function: break-even point, sensitivity to parameters
  - CE: Utility Function / Second-order Moment Approximation.
  - Robustness check and Extensions: other utility functions / strategic behavior

Thank you for Listening!

Remark: the units in slides and spreadsheets are all in million. (For R and expected returns)