

作业-4

在ND中证明：

$$(1) \quad \vdash_{ND} (\neg A \rightarrow A) \rightarrow A$$

$$(3) \quad \vdash_{ND} ((A \vee B) \rightarrow C) \leftrightarrow (A \rightarrow C) \wedge (B \rightarrow C)$$

$$(5) \quad \vdash_{ND} \neg(A \rightarrow B) \leftrightarrow A \wedge \neg B$$

$$(7) \quad \vdash_{ND} (A \wedge B) \leftrightarrow A \wedge (\neg A \vee B)$$

$$(1) \vdash (\neg A \rightarrow A) \rightarrow A$$

$$1. \neg A \rightarrow A, \neg A \vdash \neg A \quad (E)$$

$$2. \neg A \rightarrow A, \neg A \vdash \neg A \rightarrow A \quad (E)$$

$$3. \neg A \rightarrow A, \neg A \vdash A \quad (1)(2)(\rightarrow -)$$

$$4. \neg A \rightarrow A \vdash \neg \neg A \quad (1)(3)(\neg +)$$

$$5. \neg A \rightarrow A \vdash A \quad (4)(\neg \neg -)$$

$$(3) \vdash ((A \vee B) \rightarrow C) \leftrightarrow (A \rightarrow C) \wedge (B \rightarrow C)$$

$$1) (A \vee B \rightarrow C) \vdash (A \rightarrow C) \wedge (B \rightarrow C)$$

$$(A \vee B \rightarrow C) \vdash A \rightarrow C \Leftrightarrow A \vee B \rightarrow C, A \vdash C$$

$$(A \vee B \rightarrow C) \vdash B \rightarrow C \Leftrightarrow A \vee B \rightarrow C, B \vdash C$$

$$1. A \vee B \rightarrow C, A \vdash A \quad (E)$$

$$2. A \vee B \rightarrow C, A \vdash A \vee B \quad (1)(\vee +)$$

$$3. A \vee B \rightarrow C, A \vdash A \vee B \rightarrow C \quad (E)$$

$$4. A \vee B \rightarrow C, A \vdash C \quad (2)(3)(\rightarrow -)$$

$$5. A \vee B \rightarrow C \vdash A \rightarrow C \quad (4)(\rightarrow +)$$

$$6. A \vee B \rightarrow C, B \vdash B \quad (E)$$

$$7. A \vee B \rightarrow C, B \vdash A \vee B \quad (6)(\vee +)$$

$$8. A \vee B \rightarrow C, B \vdash A \vee B \rightarrow C \quad (E)$$

$$9. A \vee B \rightarrow C, B \vdash C \quad (7)(8)(\rightarrow -)$$

$$10. A \vee B \rightarrow C \vdash B \rightarrow C \quad (9)(\rightarrow +)$$

11. $A \vee B \rightarrow C \vdash (A \rightarrow C) \wedge (B \rightarrow C)$ (5)(10)($\wedge+$)
12. $\vdash (A \vee B \rightarrow C) \rightarrow (A \rightarrow C) \wedge (B \rightarrow C)$ (11)($\rightarrow+$)
- 2) $(A \rightarrow C) \wedge (B \rightarrow C) \rightarrow (A \vee B \rightarrow C)$
 - $(A \rightarrow C) \wedge (B \rightarrow C), A \vee B \vdash C$
 13. $(A \rightarrow C) \wedge (B \rightarrow C), A \vee B, A \vdash A$ (E)
 14. $(A \rightarrow C) \wedge (B \rightarrow C), A \vee B, A \vdash (A \rightarrow C) \wedge (B \rightarrow C)$ (E)
 15. $(A \rightarrow C) \wedge (B \rightarrow C), A \vee B, A \vdash A \rightarrow C$ (14)($\wedge-$)
 16. $(A \rightarrow C) \wedge (B \rightarrow C), A \vee B, A \vdash C$ (13)(15)($\rightarrow-$)
 17. $(A \rightarrow C) \wedge (B \rightarrow C), A \vee B, B \vdash B$ (E)
 18. $(A \rightarrow C) \wedge (B \rightarrow C), A \vee B, B \vdash (A \rightarrow C) \wedge (B \rightarrow C)$ (E)
 19. $(A \rightarrow C) \wedge (B \rightarrow C), A \vee B, B \vdash B \rightarrow C$ (18)($\wedge-$)
 20. $(A \rightarrow C) \wedge (B \rightarrow C), A \vee B, B \vdash C$ (17)(19)($\rightarrow-$)
 21. $(A \rightarrow C) \wedge (B \rightarrow C), A \vee B \vdash A \vee B$ (E)
 22. $(A \rightarrow C) \wedge (B \rightarrow C), A \vee B \vdash C$ (16)(20)(21)
 23. $(A \rightarrow C) \wedge (B \rightarrow C) \vdash A \vee B \rightarrow C$ (22)($\rightarrow+$)
 24. $\vdash (A \rightarrow C) \wedge (B \rightarrow C) \rightarrow (A \vee B \rightarrow C)$ (23)($\rightarrow+$)
 25. $\vdash (A \vee B \rightarrow C) \leftrightarrow (A \rightarrow C) \wedge (B \rightarrow C)$ (12)(24)($\leftrightarrow+$)

$$(5) \vdash \neg(A \rightarrow B) \leftrightarrow A \wedge \neg B$$

- 1) " \rightarrow "
1. $\neg(A \rightarrow B), B \vdash B$ (E)
 2. $\neg(A \rightarrow B), A, B \vdash B$ (1)($+$)

$$3. \neg(A \rightarrow B), B \vdash A \rightarrow B \quad (2)(\rightarrow +)$$

$$4. \neg(A \rightarrow B), B \vdash \neg(A \rightarrow B) \quad (E)$$

$$5. \neg(A \rightarrow B) \vdash \neg B \quad (3)(4)(\neg +)$$

$$6. \neg(A \rightarrow B), \neg A, A \vdash A \quad (E)$$

$$7. \neg(A \rightarrow B), \neg A, A \vdash \neg A \quad (E)$$

$$8. \neg(A \rightarrow B), \neg A, A \vdash B \quad (6)(7)(\neg -)$$

$$9. \neg(A \rightarrow B), \neg A \vdash A \rightarrow B \quad (8)(\rightarrow +)$$

$$10. \neg(A \rightarrow B), \neg A \vdash \neg(A \rightarrow B) \quad (E)$$

$$11. \neg(A \rightarrow B) \vdash \neg \neg A \quad (9)(10)(\neg +)$$

$$12. \neg(A \rightarrow B) \vdash A \quad (11)(\neg \neg -)$$

$$13. \neg(A \rightarrow B) \vdash A \wedge \neg B \quad (12)(5)(\wedge +)$$

$$14. \vdash \neg(A \rightarrow B) \rightarrow A \wedge \neg B \quad (13)(\rightarrow +)$$

$$2) \text{ "}\leftarrow\text{" } A \wedge \neg B \vdash \neg(A \rightarrow B)$$

$$15. A \wedge \neg B, A \rightarrow B \vdash A \wedge \neg B \quad (E)$$

$$16. A \wedge \neg B, A \rightarrow B \vdash A \quad (15)(\wedge -)$$

$$17. A \wedge \neg B, A \rightarrow B \vdash A \rightarrow B \quad (E)$$

$$18. A \wedge \neg B, A \rightarrow B \vdash B \quad (16)(17)(\rightarrow -)$$

$$19. A \wedge \neg B, A \rightarrow B \vdash \neg B \quad (13)(\wedge -)$$

$$20. A \wedge \neg B \vdash \neg(A \rightarrow B) \quad (18)(19)(\neg +)$$

$$21. \vdash A \wedge \neg B \rightarrow \neg(A \rightarrow B) \quad (20)(\rightarrow +)$$

$$22. \vdash \neg(A \rightarrow B) \leftrightarrow A \wedge \neg B \quad (14)(21)(\leftrightarrow +)$$

$$(1) \vdash A \wedge B \leftrightarrow A \wedge (\neg A \vee B)$$

$$1) A \wedge B \vdash A$$

$$A \wedge B \vdash B$$

$$1. A \wedge B \vdash A \wedge B \quad (\epsilon)$$

$$2. A \wedge B \vdash A \quad (1)(\wedge-)$$

$$3. A \wedge B \vdash B \quad (1)(\wedge-)$$

$$4. A \wedge B \vdash \neg A \vee B \quad (3)(\vee+)$$

$$5. A \wedge B \vdash A \wedge (\neg A \vee B) \quad (2)(4)(\wedge+)$$

$$6. \vdash A \wedge B \rightarrow A \wedge (\neg A \vee B) \quad (5)(\rightarrow+)$$

$$2) A \wedge (\neg A \vee B) \rightarrow A \wedge B$$

$$A \wedge (\neg A \vee B) \vdash A$$

$$A \wedge (\neg A \vee B) \vdash B ?$$

$$A \wedge (\neg A \vee B), \neg A \vdash B \quad \text{矛盾}$$

$$A \wedge (\neg A \vee B), B \vdash B \quad \checkmark$$

$$7. A \wedge (\neg A \vee B) \vdash A \wedge (\neg A \vee B) \quad (\epsilon)$$

$$8. A \wedge (\neg A \vee B) \vdash A \quad (7)(\wedge-)$$

$$9. A \wedge (\neg A \vee B) \vdash \neg A \vee B \quad (7)(\wedge-)$$

$$10. A \wedge (\neg A \vee B), \neg A \vdash \neg A \quad (\epsilon)$$

$$11. A \wedge (\neg A \vee B), \neg A \vdash A \quad (8)(+)$$

$$12. A \wedge (\neg A \vee B), \neg A \vdash B \quad (10)(11)(\neg-)$$

$$13. A \wedge (\neg A \vee B), B \vdash B \quad (\epsilon)$$

$$14. A \wedge (\neg A \vee B) \vdash B \quad (12)(13)(9)(\vee-)$$

$$15. A \wedge (\neg A \vee B) \vdash A \wedge B \quad (8)(15)(\wedge+)$$

$$16. \vdash A \wedge (\neg A \vee B) \rightarrow A \wedge B \quad (15)(\rightarrow+)$$

$$17. \vdash A \wedge B \leftrightarrow A \wedge (\neg A \vee B) \quad (6)(16)(\leftrightarrow+)$$