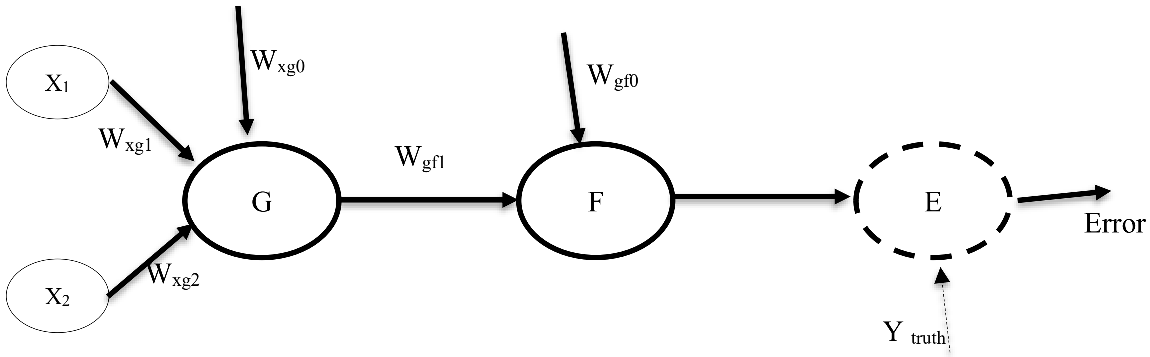

Figure 1: Multilayer neuron network

Q.1 Refer to figure 1 for this question, X_i are inputs, W_{xyi} are weights from layer x to y for sequence number i, G is hidden layer neuron with tanh as activation function, and F as output layer neuron with logistic sigmoid as activation function. The figure is similar to the one we discussed during the lecture on computing partial derivatives. The partial derivative with respect to weight W_{gf0} was calculated as an example for you. Your task is to show the steps and calculations for deriving partial derivatives of this multilayer neuron network for remaining weights, i.e., for the following: **(5 points)**

A. With respect to W_{gf1}
B. With respect to W_{xg0}
C. With respect to W_{xg1}
D. With respect to W_{xg2}

$\tanh'(x) = 1 - \tanh^2(x)$

$s'(x) = s(x) \cdot (1 - s(x))$

$y = a(x),\ z = b(y) \Rightarrow \frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial x}$

neuron error $=$ derivative of err. func. $\cdot$ derivative of act. func.

$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$

$z_g(W_{xg0 \ldots 2}) = W_{xg0} + W_{xg1} \cdot x_1 + W_{xg2} \cdot x_2$

$g(z_g) = \tanh(z_g)$

$z_f(W_{gf0 \ldots 1}, g) = W_{gf0} + W_{gf0} \cdot g$

$f(z_f) = s(z_f),\ e(f) = \frac{1}{2}(y-f)^2$

Partial derivatives:

$$\frac{\partial e}{\partial W_{gf0}} = \frac{\partial e}{\partial f} \cdot \frac{\partial f}{\partial z_f} \cdot \frac{\partial z_f}{\partial W_{gf0}} = (f-y)(s'(z_f))(1)$$
$$= (f-y)((s(z_f) \cdot (1 - s(z_f)))$$

$$\frac{\partial e}{\partial W_{gf1}} = \frac{\partial e}{\partial f} \cdot \frac{\partial f}{\partial z_f} \cdot \frac{\partial z_f}{\partial W_{gf1}} = (f-y)(s'(z_f))(g)$$
$$= (f-y)((s(z_f) \cdot (1 - s(z_f)))(g)$$

$$\frac{\partial e}{\partial W_{xg0}} = \frac{\partial e}{\partial f} \cdot \frac{\partial f}{\partial z_f} \cdot \frac{\partial z_f}{\partial g} \cdot \frac{\partial g}{\partial z_g} \cdot \frac{\partial z_g}{\partial W_{xg0}}$$
$$= (f-y)(s'(z_f))(W_{gf1})(\tanh'(z_g))(1)$$

$$\frac{\partial e}{\partial W_{xg1}} = \frac{\partial e}{\partial f} \cdot \frac{\partial f}{\partial z_f} \cdot \frac{\partial z_f}{\partial g} \cdot \frac{\partial g}{\partial z_g} \cdot \frac{\partial z_g}{\partial W_{xg1}}$$
$$= (f-y)(s'(z_f))(W_{gf1})(\tanh'(z_g))(x_1)$$

$$\frac{\partial e}{\partial W_{xg2}} = \frac{\partial e}{\partial f} \cdot \frac{\partial f}{\partial z_f} \cdot \frac{\partial z_f}{\partial g} \cdot \frac{\partial g}{\partial z_g} \cdot \frac{\partial z_g}{\partial W_{xg2}}$$
$$= (f-y)(s'(z_f))(W_{gf1})(\tanh'(z_g))(x_2)$$

$$\frac{\partial e}{\partial f} = \frac{\partial \frac{1}{2}(y-f)^2}{\partial f} = 2 \cdot \frac{1}{2}(y-f)(0-1)$$
$$= (y-f)(-1) = -(y-f) = f-y$$

$$\frac{\partial f}{\partial z_f} = \frac{\partial s(z_f)}{\partial z_f} = s'(z_f) \cdot 1 = s'(z_f)$$
$$= s(z_f) \cdot (1 - s(z_f))$$

$$\frac{\partial z_f}{W_{gf0}} = \frac{\partial\, W_{gf0} + W_{gf1} \cdot g}{\partial W_{gf0}}$$
$$= 1 + 0 = 1$$

$$\frac{\partial z_f}{W_{gf1}} = \frac{\partial W_{gf0} + W_{gf1} \cdot g}{\partial W_{gf1}} = 0 + 1 \cdot g = g$$

$$\frac{\partial z_f}{\partial g} = \frac{\partial\, W_{gf0} + W_{gf1} \cdot g}{\partial g} = 0 + W_{gf1} \cdot 1 = W_{gf1}$$

$$\frac{\partial g}{\partial z_g} = \frac{\partial \tanh(z_g)}{\partial z_g} = \tanh'(z_g) \cdot 1$$
$$= \tanh'(z_g) = 1 - \tanh^2(z_g)$$

$$\frac{\partial z_g}{\partial W_{xg0}} = \frac{\partial W_{xg0} + W_{xg1} \cdot x_1 + W_{xg2} \cdot x_2}{\partial W_{xg0}}$$
$$= 1 + 0 + 0 = 1$$

$$\frac{\partial z_g}{\partial W_{xg1}} = \frac{\partial W_{xg0} + W_{xg1} \cdot x_1 + W_{xg2} \cdot x_2}{\partial W_{xg1}}$$
$$= 0 + 1 \cdot x_1 + 0 = x_1$$

$$\frac{\partial z_g}{\partial W_{xg2}}$$
$$= \frac{\partial W_{xg0} + W_{xg1} \cdot x_1 + W_{xg2} \cdot x_2}{\partial W_{xg2}}$$
$$= 0 + 0 + 1 \cdot x_2$$
$$= x_2$$

[redacted]

Q.2 Refer to the Figure 1 for this question. You will be working on backpropagation algorithm for this part. You need to show your work (for one iteration) for forward pass, backward pass, and weight adjustment (of all weights) using the following initial values for the network in figure 1.

A. **Initial weights**
   a. $W_{xg0} = 0.4$
   b. $W_{xg1} = 0.5$
   c. $W_{xg2} = -0.2$
   d. $W_{gf0} = -0.1$
   e. $W_{gf1} = 0.45$

B. **Training example**
   a. $X_1 = -0.8$
   b. $X_2 = 0.2$
   c. $Y_{truth} = 1.0$

C. **Learning rate** = 0.2

Grading rubrics for this question:

Forward pass steps: 3 points

Backward pass: 6 points

Weight adjustment: 6 points

You must show the computations involved in forward, backward, and weight adjustments; as failing to show these will result in a penalty of half points for this question.

Submit your work in D2L before the due date.

$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x}-1}{e^{2x}+1}$

$S(x) = \frac{1}{1+e^{-x}} = \frac{e^x}{e^x+1}$

<u>Weights:</u>

$W_{xg0} = 0.4$

$W_{xg1} = 0.5$

$W_{xg2} = -0.2$

$W_{gf0} = -0.1$

$W_{gf1} = 0.45$

<u>Inputs:</u>

$X_1 = -0.8$

$X_2 = 0.2$

$Y_{truth} = 1.0$

$\eta = 0.2$

<u>Forward Pass:</u>

$$Z_g = W_{xg0} + W_{xg1} \cdot X_1 + W_{xg2} \cdot X_2$$
$$= (0.4) + (0.5)(-0.8) + (-0.2)(0.2)$$
$$= -0.04$$

$$y_g = \tanh(Z_g) = \tanh(0.04) \approx -0.04$$

$$Z_f = W_{gf0} + W_{gf1} \cdot y_g$$
$$\approx -0.1 + (0.45)(-0.04) \approx -0.12$$

$$Y_f = S(Z_f) \approx S(-0.12) \approx 0.47$$

$$e = \tfrac{1}{2}(Y_{truth} - Y_f)^2 \approx \tfrac{1}{2}(1.0 - 0.47)^2 \approx 0.14$$

<u>Backward Pass:</u>

$$e' = Y_f - Y_{truth} \approx 0.47 - 1.0 \approx -0.53$$

$$e_f = e' \cdot Y_f' = e' \cdot (Y_f \cdot (1 - Y_f))$$
$$\approx -0.53 \cdot (0.47 \cdot (1 - 0.47)) \approx -0.13$$

$$e_g = e_f \cdot W_{gf1} \cdot Y_g' = e_f \cdot W_{gf1} \cdot (1 - Y_g^2)$$
$$\approx (-0.13)(0.45)(1 - (-0.04)^2) \approx -0.06$$

<u>Weight Adjustment:</u>

$$W_{xg0} = W_{xg0} - (\eta \cdot 1.0 \cdot e_g)$$
$$\approx 0.4 - (0.2 \cdot 1.0 \cdot (-0.06)) \approx 0.41$$

$$W_{xg1} = W_{xg1} - (\eta \cdot X_1 \cdot e_g)$$
$$\approx 0.5 - (0.2 \cdot (-0.8) \cdot (-0.06)) \approx 0.49$$

$$W_{xg2} = W_{xg2} - (\eta \cdot X_2 \cdot e_g)$$
$$\approx (-0.2) - (0.2 \cdot (0.2) \cdot (-0.06)) \approx -0.20$$

$$W_{gf0} = W_{gf0} - (\eta \cdot 1.0 \cdot e_f)$$
$$\approx (-0.1) - (0.2 \cdot 1.0 \cdot (-0.13)) \approx -0.07$$

$$W_{gf1} = W_{gf1} - (\eta \cdot Y_g \cdot e_f)$$
$$\approx 0.45 - (0.2 \cdot (-0.04) \cdot (-0.13)) \approx 0.45$$