# Stock market mathematical models

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**2. Description of the program**

1. Introduction

Scientific adviser: Durmagambetov Aset Askhatbekovich, Ph.D., associate professor. His scientific results are of practical importance and are used in mathematical modeling of many natural phenomena and technical problems.

2. The purpose of the program

*Creating a basic platform for a modular system for analyzing the dynamics of the stock market.* The base platform includes its own and existing software, trained personnel, and necessary technical means. If necessary, all of these components can be supplemented in a modular fashion.

3. Tasks of the program

*1) Development of mathematical and software for technical analysis of the stock market based on a dynamic model.*

Development of a mathematical model for analyzing break points of price functions based on the wave model of the Navier-Stokes and Schrödinger equations. This model uses representations not previously used in the theoretical justification of methods of technical analysis of the stock market.

*2) Development of mathematical and software for technical analysis of the stock market based on a probabilistic model.*

Development of a mathematical model based on methods of analysis of phase symmetries arising in the stock market. To develop the model, some results related to the Riemann hypothesis were used. Here, the formation of new symmetries is associated with the appearance of a kink (rupture of the second kind) in the trend.

*3) Development of computational algorithms for assessing the localization of break points.*

The developed mathematical model is presented in a form that allows its computational algorithmization. After that, computational algorithms are compiled. Convergence and completeness of computational algorithms are being studied.

*4) Development of a software implementation of a computational algorithm.*

Development of a software module that provides the implementation of a computational algorithm, an interface with the user and existing software tools for technical analysis from other manufacturers. The layout of own and acquired software modules into a single set of software and hardware tools for organizing work in the stock market.

*5) Preparation of a group of traders.*

Training personnel to work with the software and hardware complex. Providing training courses on the basics of exchange trading and the acquisition of knowledge and skills with the software and hardware complex.

4. Scientific novelty and relevance of the program

4.1 Background to the project

The State Program “Digital Kazakhstan”, in addition to accelerating the pace of development of the economy of the Republic of Kazakhstan and improving the quality of life of the population through the use of digital technologies, is also aimed at creating the conditions for the transition of the economy of Kazakhstan to a fundamentally new development path that will create a digital economy for the future.

One of the segments of this policy is the widespread use and development of components of a modern automated environment for fundamental and technical market analysis. This work is aimed at realizing these tasks of the national program for the development of the digital economy, dedicated to improving the mathematical methods of technical analysis of the stock market.

The legal basis for the implementation of the project is the "Constitutional Law

Of the Republic of Kazakhstan "On the International Financial Center "Astana" dated December 7, 2015" No. 438-V SAM (as amended on December 22, 2017).

In particular, this document establishes the main field for the implementation of the results in the form of a stock exchange, defined as: “The stock exchange is a legal entity in the organizational and legal form of a joint stock company that provides organizational and technical support with financial instruments for trading at the Center”, and also names the priorities of the Center :

- licensing - a set of measures related to the issuance of a permit to carry out activities on the territory of the Center requiring a license;
- development of the securities market of the Republic of Kazakhstan, ensuring its integration with international capital markets;
- development in the Republic of Kazakhstan of the market of insurance, banking services, Islamic finance, financial technologies, e-commerce and innovative projects;
- development of financial and professional services based on best international practices.

The relevance of the project in line with the above assumptions lies in combining the development of theoretical models with their practical application, simultaneously with the deployment of a training system using modern software designed for analysis and management of stock assets.

The project includes creation and training of a group of traders for work with innovative and existing mathematical and software technical analysis of the stock market. This group will become the first professionally trained group of traders in Kazakhstan equipped with its own software. A close combination of the research and practical parts of the project will contribute to the growth of the quality of training.

4.2 The main differences and advantages of the proposed approach.

Existing analogues

Despite the saturation of market analytics with mathematical and software, permanent interest remains in the development of technical analysis tools.

Historically, one of the first generally accepted theoretical paradigms for market modeling is the Elliot Wave Model. This model is based on the Fibonacci numbers and leads to waves repeating in shape. The main drawback of this theory is the artificial nature of the hypothesis underlying the theory. For this reason, this model is often quite reasonably criticized by financial authorities (W. Buffett, 1987).

A more general approach is developed in the Dow phase theory, subsequently formalized by Jones, which builds on the visible reaction of market participants. This reaction allows you to structure the trend into phases of accumulation, participation and implementation. Certain superpositions of actions of traders in different phases can create a wave character of a trend. The wave nature of the trend, with certain statistical processing, in turn, can serve as the basis for making forecasts.

However, as an example, it is better to consider the activities of the Renaissance Technology Corp hedge fund, founded by D. Simons. The activities of this company bring the most tangible and sustainable results in the stock market. The approaches used differ significantly from the empirical formalism of Elliot and the behaviorism of Dow and Jones.

A direct analysis of the methods and algorithms used by the company is excluded since this information has been kept in secret for several decades. At the same time, you can get some idea of the fund based on a number of indirect information.

The initial Medallion hedge fund made a profit of 39.4% in 1991, 34% in 1992 and 39.1% in 1993. After raising the US Federal Reserve federal funds rate from 3 to 5.5 percent in 1994, Medallion brought 71% of annual profit. By this time, Medallion had become Renaissance's internal and most classified division, managing only the assets of Renaissance employees.

In 1995, Renaissance began implementing a program to increase computing power. Between 1994 and 2000, the total power of Renaissance processors grew 50 times. As a result, in the year 2000, during which the Standard & Poor's 500 index fell 10.1%, Medallion earned 98.5% profit. By the end of that year, Renaissance had 148 employees and the fund had 43.6 percent of the average annual profit for 11 years.

In August 2005, the Renaissance Institutional Equities Fund (RIEF) was opened, which invested in American stocks. According to the press, by October 2007, its average annual income was 12.8 percent. At the same time, RIEF held its positions for months and longer, while in contrast, Medallion wraps its funds many times a year.

During the mortgage crisis of 2007, the Renaissance and Medallion fund, based on a strategy, which was founded on its own algorithms and computer calculations in large databases, brought 50% in the first three quarters of the year, the value of assets then amounted to $ 6 billion.

To characterize the dynamics of the company, it should be noted that the founder of the company, James Simons, began his financial activities in 1974, investing money received as bonuses for mathematical work in a Colombian company producing commodity products. This investment brought profit, which allowed to invest 600 thousand dollars in investment. Thanks to the active participation of the mathematician C. Frefield in 7 months, these investments grew to $ 6 million, which allowed D. Simons to start an independent activity. By 2014, the fortune of the company's founder D. Simons was estimated at $ 14 billion.

According to various sources, the main investment in their own company at that time was to attract highly qualified specialists in the field of mathematical statistics, probability theory, programmers, crypto-analysts, pattern recognition and speech, as well as astrophysicists.

It is this approach to personnel policy, which excludes the admission of graduates of economic schools and MBA, as well as people with experience working on Wall Street, according to experts, is the first component of the company's success.

This hedge company, consisting of only scientists who receive all the necessary conditions for research and equipped with computing resources, has developed over the years into a unique tool for activities not only in the financial and stock markets. According to financial observers, Medallion's investment sector also includes commodity production.

The foregoing can be illustrated by the following example. Leonard Baum - one of the co-authors of the Baum-Welch algorithm used to determine probabilities in biology, automatic speech recognition and statistical calculations, on the instructions of the company adapted this algorithm for currency trading (L. Baum, a former decryptor from IDA - NSA division).

It is known that Baum began to make big profits on fundamental trading in the late seventies and early eighties. At that time, D. Ax was acquainted with Baum's algorithms (James Ax, professor of mathematics, a specialist in probability theory, and a prize winner in mathematics), who noted that the developed Baum models can work not only in the foreign exchange market, but can be applied to any commodity futures - wheat, crude oil, etc.

According to observers, at that time, professionals were extremely incredulous in their perception of the systematic approach to trading that Ax carried out, based on existing models and algorithms. The achievement of high financial indicators was interpreted by market players as a manifestation of Ax’s rare intuition rather than the ability of model algorithms to identify and recognize previously invisible structures in the market information field.

After a sufficiently long time, this point of view has changed. It is known that during the crisis of 2007, a number of competing companies (AQR Capital Management LLC) tried to copy the investment strategy of Renaissance, which had a subsidiary of Johnson & Johnson, Lockheed Martin Corp., International Business Machines Corp. and Chevron Corp. As a result, cloning of investment models played against competitors who underestimated the full scale and quality of Renaissance coverage of all the influencing factors in the market and in time.

The length of time interval required to assess the role of model and system approaches is explained by the high degree of conservatism inherent in most professional market players. In contrast, Renaissance demonstrated the opposite approach, based on the widespread adoption of scientific methods.

According to analysts, the success of Renaissance is accompanied by a synergistic approach, supported by a high scientific level of specialists, each of whom was a prominent figure in his previous field of activity. Their developed scientific intuition allows us to reveal structures in the variety of trading information that are not visible to ordinary traders. Their professional qualifications are a prerequisite for the transfer of mathematical methods developed in other areas to analytical market models. A characteristic feature of these models, as the analysis of the company's successful investments shows, is the extremely wide (compared to competitors) volume of data being analyzed. Moreover, the results of theorists are immediately implemented by a team of professional programmers. According to former employees of the company, investments in computing power in the company are made on the basis of necessity, and not for budgetary reasons.

Proposed innovation

A summary of the proposed approaches is summarized below. A more detailed discussion is given in Section 3. The interest and need for the continuous development of mathematical methods for analyzing the trading market is due to several factors.

A) Market development often poses challenges ahead of the capabilities of existing analysis tools. The very structure of market processes has a fundamentally high variability, which cannot be exhaustively covered by existing theories.

B) The observed progress in computing, which made it possible to implement algorithms not previously available due to the large amount of computation. At the same time, the ability of the algorithm to calculations in real time remains important for traders.

C) Concealment of effective mathematical methods and algorithms by companies with their own development and research groups.

The theoretical part of the project is devoted to the study of the problem of identifying and recognizing trends, predicting trend break points. This problem arises from the urgent problem of price spikes (see Fig. 1), which appears in the technical analysis of the stock market.


Figure 1. Graph of the EUR / CHF price functions

In the technical analysis of most stock market models, it is assumed that pricing takes into account all factors and the price curve continuously depends on these parameters. This is supported by the fact that most of the graph to the right and left of the gap is more or less adequately approximated by a continuous function. In other words, in most of the process, a continuous dependence on the parameters affecting the price is maintained. Only in a finite number of points can the price function experience a gap of the 1st, and its derivative of the 2nd kind (“gradient catastrophe”).

The basis of the mathematical approach to solving the described problems in this project proposed two models developed by the authors earlier. The first model is based on the consideration of the well-known Riemann problem, which showed the significance and role of symmetries of trend dynamics on the distribution of break points (breaks) of trends, which is a new result.

Another methodology began in the classical theory of scattering, in particular in approaches to finding a solution to the inverse problem. Here, the mechanism of the occurrence of gradient discontinuities was used, obtained for a model of an elastic medium described by the set of Navier – Stokes – Schrödinger equations, see [1]. The main novelty is the obtained relations [1] and estimates of the scattering amplitudes and phase changes.

Unlike existing wave models of the stock market, for which the amplitude and period of the function under study are significant, the influence of the wave phase on the dynamics of the price function is taken into account to a greater extent.

These approaches lead to a new interpretation of price dynamics, where a change in the phase and symmetry of the wave, in some cases, induces the formation and growth of a gap in the price function.

In addition to this, the availability of our own computational algorithm and software will allow:

- make a wide modification of the methodology;
- supplement the theoretical model due to the results obtained.
- expand the functionality of the software by modular extension of the software package.

Finally, this approach is comprehensive when, along with the development and implementation of new methods of analysis, personnel are trained in the basics of technical analysis of the stock market.

4.3 The novelty of the proposed approach.

Currently, the most famous paradigm of the wave model of the market is the Elliott model. This model comes from the Fibonacci numbers, which leads to waves repeating in shape. This circumstance indicates the rather artificial nature of the hypothesis underlying the theory. A more general approach is developed in the Dow phase theory, which builds on the visible reaction of market participants. This reaction allows you to structure the trend into phases of accumulation, participation and implementation. Certain superpositions of actions of traders in different phases can create a wave character of a trend.

The approach used, based on the Navier-Stokes equation, proceeds from a fairly general qualitative analogy assuming the existence of a “restoring force”, i.e. the presence of a combination of factors seeking to stabilize the trend, along with “disturbing power”, seeking to raise or lower the trend. If we add to this the inertia of the market as an analogue of mass, then you can see the full qualitative analogy with the mechanical oscillatory circuit, where the resulting wave motion provides competition between the external disturbing and internal stabilizing forces in the presence of inertia.

A similar analogy with mechanics at one time made it possible to formulate a mathematical model that accurately described the anomalous duration of student unrest in 1968 in France (Weidlich V., 1972; Haken G., 1978).

With this point of view, market behavior is similar to some mechanical system with vibrational degrees of freedom. Each point on the trend is characterized by a set of parameters $\{a_1, a_2 \dots a_n\}$. Then the trend can be represented as a line drawn in the space formed by the direct topological product of all admissible values of all parameters: $A = a_1 \times a_2 \times \dots \times a_n$. In a direct problem, a trend is a solution to an equation formally similar to the equation of oscillations. For a distributed continuous medium $A$, the equation of motion is the equation of motion of an elastic medium in the Navier-Stokes form.

Since in practice it is more important to determine the break points, the inverse problem is considered. Here, an approach to solving the inverse wave scattering problem is implemented, based on the accompanying Schrödinger equation.

The results obtained in [1] and based on the expanded use of the phase of functions and their images can serve as a starting point for the development of a group of analysis tools that have no analogues in the technical analysis of the stock market.

Another approach, which follows from the consideration of symmetries in the Riemann problem, calls the change of symmetry the cause of jumps in price functions. Both approaches are related to each other because they consider the phase of the process as a key characteristic.

In these works, which investigated the Riemann hypothesis, the fundamental problem of constructing solutions of equations with mirror symmetry was solved, which has not yet been solved.

From the point of view of modeling financial markets, this means that using the stable symmetry of Forex instruments - eur / usd, usd / chf leads to the Monty and Hall paradox. The rejection of these tools creates a sustainable opportunity to use the Bayes formula, which allows you to build forecasts with a stable increased probability of winning with getting into the forecasting zone with a probabilistic measure of 0.8-0.9, which allows you to cost reliable winning strategies

In parallel with the theoretical part of the project, it is planned to develop its technical part, which proposes a comprehensive and systematic approach to organizing fundamental and technical analysis of the stock market in the following structure:

- software analysis modules;
- personnel training system;
- an analytical group developing its own modules.

This structure can serve as a basic platform for creating a trading company.

4.4 National and international significance.

Currently, the republic lacks a unified view and a systematic approach to the analysis and management of stock markets. The project will lay the foundations for systematic training and qualifications of analysts and stock market traders.

Since the global stock market does not have national restrictions, any positive result of the application of innovative methodology will be evaluated at the international level.

In this regard, an analysis method that takes into account the fine structure of phase changes and associates these changes with the parameters of the price function is promising.

4.5 The main technological task to be solved.

The main technological tasks in the project are the development of software tools for technical analysis of the stock market and the prediction of break points in price trends. Creating a common interface with the user and with other software modules developed by third-party manufacturers.

4.6 Socio-economic effect.

Within the framework of the project, it is planned to create a group of traders and analysts who are familiar with theoretical and software methods of market analysis. A group of trained specialists can form the core of a company engaged in fundamental and technical analysis of the stock market.

This group can both perform custom work on the analysis of the state of the customer’s assets, assess risks and perform services for managing the client’s stock assets.

## 3. RESEARCH METHODS AND ETHICAL ISSUES

**1. Description of scientific methods used in the program as a justification of ways to achieve the goals, justification of the chosen approach.**

Symmetry Assessment Method.

This method uses the observed symmetry between different assets in the market. It is known that assets can behave in phase with other assets, and vice versa change in antiphase.

From long-term observation data, it was found that the change in trends is associated with the loss of common mode of some assets and the acquisition of common mode by other assets. One can formally consider a change in trends as a change in symmetries, and a change in symmetry as a change in conservation laws from the point of view of Noether's theorems.

In this case, the relationship between the phase of the trend and the conservation law is clearly traced. According to Noether, the conservation law is determined by a certain symmetry in the market. The emergence of new major players changes the balance in the market, which leads to new symmetries and, as a result, new conservation laws in the market. The new symmetry leads to a break in the trend.

From the point of view of the Schrödinger equations, the phase property corresponds to the appearance of a new discrete eigenvalue, which automatically changes the scattering phase. This qualitative description of the stock market does not directly lead to a result, since the scattering potential is not known.

As a dissipation potential, a situation was proposed and studied when the asset price function served as a dissipation potential. It was found that the Fourier transform modulus of a price asset remains virtually unchanged over time. The reason for the significant change in trends is the change in the phase of the Fourier transform.

The situation is fundamentally changing if we use the mirror symmetry of assets in the market and their phase matching. Equations with mirror symmetry turned out to be a time-consuming task and have not yet been studied. For further discussion, the Riemann zeta function is required. The Riemann zeta function defined by the Dirichlet series:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \quad s = \sigma + it, \tag{1}$$

absolutely and uniformly converging in any bounded region of the complex z-plane for which $\sigma \geq 1 + \varepsilon, \varepsilon > 0$.If$\sigma > 1$, then the $\zeta$ function is represented by the following Euler formula:

$$\zeta(s) = \prod_p \left[1 - \frac{1}{p^s}\right]^{-1}, \tag{2}$$

where p is all the primes $\zeta(s)$ the function was first introduced by Euler, who also obtained formula (2). Where p is an increasing sequence of all primes. For the zeta of the Riemann function, the following functional equation holds:

$$\pi^{-\frac{s}{2}}\Gamma\left(\frac{s}{2}\right)\zeta(s) = \pi^{-\frac{1-s}{2}}\Gamma\left(\frac{1-s}{2}\right)\zeta(1-s).$$

In our work, we studied the functional Riemann equations and solved the fundamental problem of constructing solutions of equations with mirror symmetry. To date, the solution to this problem was not known.

From the point of view of modeling financial markets, this means abandoning the use of stable symmetry of forex instruments, such as $\frac{eur}{usd}, \frac{usd}{chf}$, as illustrated by the following example:

Let $(t) = \frac{eur}{usd}$, $x(t) = \frac{usd}{chf}$ , $z(t) = \frac{eur}{chf}$. For these tools, we have the following equations for cross rates:

$$x(t)y(t) = z(t).$$

Using mirror symmetry with respect to the top of the trend, which we position at $t_0$, we obtain the relation:

$$x(t) = z(t)x(t_0), \tag{3}$$

which we rewrite them in the following form:

$$ln(x(t) - ln(x(t_0)) = ln(z(t)). \tag{4}$$

The last equation actually coincides with the equation for the Riemann zeta function after the logarithm, so all the results on the Riemann problem are fully carried over to the case of market modeling.

In fact, there is equivalence with problems in the Riemann hypothesis. The problem arises of arranging zeros or singularities for the function x (t), due to the fact that the function $x(t_0)$ also has the same mirror singularity and they are leveled during the logarithm.

This gives the opportunity to consider mirror-symmetric features, moreover, to place the axis of symmetry at the top of the trend break and, moreover, to consider functions with given break points. But as indicated above, the break points are associated with the phases of the processes of price changes. Summarizing all of the above, we can describe a parametric description of price movements with a given number of kinks.

As a result, we can obtain a parametric system for describing asset prices using the inverse problem methods for the Schrödinger equation, as well as asset price symmetry, using them as a parameterization of the potential description.

As mentioned above. It was experimentally discovered that the Fourier transform module is an invariant of the description of stock market asset prices. In this case, a finite number of points appears in the Fourier transform module, which significantly changes the pattern of trend movement. These points can be used as a finite number of parameters for locking the model already in the functional equations for the Riemann equations for zeta functions. Numerical modeling revealed the reason for the correction correction in asset price movements, the number of correctional waves, the essential mirror symmetry of the price movement process. Mirror symmetry of prices leads to the imposition of waves, and we come to the classical concepts of interference and diffraction, which are manifested in the movement of prices as correction waves. The prerequisites for this project are a deep qualitative understanding of the Eliot wave theory, which in our terms is embedded in the mirror-symmetric movement of asset prices.

The gradient catastrophe method in the Navier-Stokes equation.

The mathematical models proposed for the technical analysis of the stock market were tested on the problem of turbulence instability arising in numerical modeling. For these applications, it was shown how to avoid a gradient catastrophe or, in the case of a gradient catastrophe irreversibility, how to go through such processes stably.

The results show that, within the framework of the chosen model, the main analogue reflecting the behavior of markets is the movement of waves. Waves are generated by market participants and depend on the mood of the participants. Moreover, in this model it was possible to measure the mood of participants and predict the direction of the price jump.

During the functioning of the world market, stable groups of market participants have developed that are almost in equilibrium. This equilibrium is destroyed by spasmodic price changes. Quantitative methods that implement these methods are based on publications.

In contrast to other wave theories, mathematical models were built here and waves creating trends were modeled. Additionally, analytical methods were developed for analyzing the appearance of new waves in the appearance of a gradient catastrophe. Accordingly, a strategy is generated in the market - the allocation of a trend and the expectation of a break in a trend when raising a break, a change in strategy to the opposite.

At the moment, there is a practical implementation of this strategy, while the strategy is implemented without a marginal leverage, that is, it is technologically stable and has only risks of force majeure nature.

In the following figure, below are graphs of the price functions EUR / CHF, USD / CHF, EUR / USD with characteristic large price jumps. it can be seen that USD / CHF is undergoing a gap in the region of December 22, 2016.

Figure 2. Graph of USD / CHF price functions

The graph of the EUR / CHF function undergoes a jump in the region of January 16-17, 2015.

Figure 3. Graph of the EUR / USD price functions

Actual data show that existing price functions cannot be solutions of differential equations, due to the presence of such gaps. For this reason, it is necessary to use a different mathematical apparatus to describe these phenomena.

For a rigorous presentation of the above, we consider the Schrödinger equations together with the Navier-Stokes equations. According to the completeness of the solutions of the equation, for V we have the representation:

$$V = PV + PD,$$

where PV is the projection of the velocity onto the continuous part of the spectrum of the Schrödinger equation, and PD is the projection onto the discrete part of the spectrum.

As it was found out in our works, gradient catastrophes and, as a result, the loss of reliability of the WRF model occurs in PD. Therefore, we have constructed a numerical implementation of PD is the basis for overcoming gradient catastrophes in the Navier-Stokes equations and, therefore, in the WRF model.

For the numerical implementation of PD, the numerical implementation of PV is required, which is also part of our current work.

In addition, when numerically implementing PD, numerical criteria for finding PD are needed. Therefore, a large amount of theoretical and numerical research fell on the construction of the criterion for finding PD.

After finding the PD, the Navier-Stokes equations are rewritten using the form:

V = W + PD,

W = V-PD.

And for Q we have the following equations:

$$W_t - v\Delta W + (W,\nabla W) + (W,\nabla PD) + (PD,\nabla W) = -\nabla p + f(x,t), \quad divW = 0 \tag{1}$$
$$W\big|_{t=0} = W_0(x)$$

In the area of $Q_T = R^3 \times (0,T)$ where:

$$div\, W = 0,$$

the refined nonlinear equations for W remain, which is a smooth function, since the poor behavior of the velocity is in PD.

Unlike the turbulence model, when applied to the market, the gradient catastrophe has a different interpretation, where the break points are the desired break points of the trends. Thus, the mathematical model tested in the WRF model passes into an adequate description of market dynamics.

It was shown that the classical methods of the theory of function estimates in the framework of the theory of Sobolev - Schwartz spaces are not suitable for the problem of studying gradient catastrophe. The results show that the embedding theorems cannot give the opportunity to study the process of the occurrence of gradient catastrophe. An alternative method is proposed for studying gradient catastrophe through studying the Fourier transform of a function and isolating the features of a function through the features of the phase of the Fourier transform of a given function.

The proposed approach is based on the fact that modern mathematical methods of function theory, devoted to the estimation of functions, have ignored such an important component of the Fourier transform as its phase.

In this vein, the following theorem was proved:

**Theorem.** There are functions from $\boldsymbol{W_2^1(R)}$ with an unchanged norm for a gradient catastrophe, which are sufficient to change the phase of its Fourier transform.

The next goal is to maximize the class of functions in which the phase is important. Here it is necessary to use the phase that appears in the inverse scattering problem, and the phase generated by the discrete spectrum of the Liouville equation is mainly of interest. Thus, a transition to the main subject of research is made - the occurrence of discontinuities, fronts, and other unstable in numerical modeling, but physically stable objects.

To build a deeper analysis, the results of scattering theory should be applied to this problem. Consider the spectral problem for the Liouville equations with potential q belonging to the space of functions M with the following norm.

$$||q||_M = \int_{-\infty}^{+\infty} |q(x)|\,(1+|x|)dx.$$

For these, it is known from the results that one can find solutions of the Liouville equation:

$$-\Psi'' + q\Psi = |k|^2\Psi, k \in C.$$

with the following asymptotics:

$$\lim_{x\to-\infty} \Psi_1(k,x) = e^{ikx} + s_{12}(k)exp(-ikx), \lim_{x\to+\infty} \Psi_1(k,x) = s_{11}(k)\exp(ikx),$$

$$\lim_{x\to-\infty} \Psi_{2,}(k,x) = s_{22}(k)\exp(-ikx), \lim_{x\to+\infty} \Psi_2(k,x) = \exp(-ikx) +$$
$$+ s_{11}(k)\exp(ikx).$$

It is known from the theory of Liouville equations that any solution is a combination of some fundamental solutions satisfying certain boundary conditions.

$$\lim_{x\to\infty} f_{+,}(k,x)\exp(-ikx) = 1,\ \lim_{x\to-\infty} f_{-}(k,x)\exp(ikx) = 1.$$

It is known [2] that they satisfy the following equations:

$$f_{+}(k,x) = \exp(ikx) - \int_{-\infty}^{+\infty} \frac{\sin\big(k(x-t)\big)}{k} q(t) f_{+}(k,t)dt,$$

$$f_{-}(k,x) = \exp(-ikx) + \int_{-\infty}^{x} \frac{\sin\big(k(x-t)\big)}{k} q(t) f_{+}(k,t)dt.$$

Theorem. For fundamental solutions, the following equalities hold.

$$s_{11}f_{+}(k,x) = s_{12}f_{-}(k,x) + f_{-}(-k,x),$$
$$s_{22}f_{-}(k,x) = s_{21}f_{+}(k,x) + f_{+}(-k,x).$$

After that, the issue of gradient disaster for a more general class of functions is considered. To do this, we consider the Liouville equation and as a potential we consider a function from $W_2^1(R)$we give a characterization of these functions through the inverse scattering problem.

For the Fourier transform, the following estimates are valid:

$$|\boldsymbol{U}| \le \boldsymbol{C}(|\boldsymbol{R_{12}}| + |\boldsymbol{R_{21}}| + |\nabla\boldsymbol{R_{12}}| + |\nabla\boldsymbol{R_{21}}|),$$
$$|\boldsymbol{V}| \le \boldsymbol{C}(|\boldsymbol{R_{12}}| + |\boldsymbol{R_{21}}| + |\nabla\boldsymbol{R_{12}}| + |\nabla\boldsymbol{R_{21}}|),$$
$$|\overline{\boldsymbol{q}}| \le \boldsymbol{C}(|\boldsymbol{R_{12}}| + |\boldsymbol{R_{21}}| + |\nabla\boldsymbol{R_{12}}| + |\nabla\boldsymbol{R_{21}}|).$$

The proof follows from the presentation of U, V; here this result stands out in a separate theorem.

Theorem. To assess the maximum potential, the following estimates are valid:

$$|\boldsymbol{q}| \le \boldsymbol{C} \int_{-\infty}^{+\infty} \big(\boldsymbol{C}(|\boldsymbol{I_{12}}| + |\boldsymbol{I_{21}}| + |\nabla\mathbf{I_{12}}| + |\nabla\boldsymbol{I_{21}}|)\big)\boldsymbol{dk}.$$

The proof follows from the estimates of U, V and the use of $\boldsymbol{R_{12}R_{21}}$, these arguments highlight the theorem here to emphasize the significance of this result.

By analyzing the last formula, we can see the effect of the phase on the behavior of the function. Moreover, the finiteness of the discrete spectrum is the main requirement for the absence of a gradient disaster.

Studying now the behavior of the gradient of q, we conclude that its unlimited growth will be due to the point of accumulation of the phase, which in turn will be due to the accumulation of a discrete spectrum. From here we get the most important information about the disaster with discrete jumps. This theorem shows that we have achieved fundamentally new nonlinear integral relations that allow us to take a fundamentally new look at the problem of estimating functions.

**Compliance with the principles and norms of scientific ethics**

Responsibility for observing the principles and norms of scientific ethics during the implementation of the Project lies with the Project Manager. In addition, participants in the research group are personally responsible for the accuracy and originality of published materials.

**The mechanism for conducting studies involving humans and animals.**

The work will not use hazardous substances, experiments with animals and the special conditions of the experiments associated with risk. The conditions and safety precautions for assembling a test bench and conducting experiments are regulated by the internal regulatory documents of the enterprise in whose territory the tests will be conducted, as well as the Labor Code of the Republic of Kazakhstan.

**Terms of registration and separation of intellectual property rights.**

Intellectual property rights and the priority of the results of work are drawn up through the protection documents and authorship in publications. Intellectual rights belong to the authors of scientific articles or reports at conferences, as well as to persons included in the list of authors of a title of protection.

**2. Critical points, alternative ways of implementing the program**

When executing the program, a risk management procedure is provided based on an analysis of expert assessments that will identify critical points in the implementation of the program and carry out operational adjustment, up to the application of an alternative option.

**3. The methods used in the program to ensure compliance with the principles and norms of scientific ethics**

The participants in the research group undertake to comply with the principles of scientific ethics, in particular, to prevent the fabrication of scientific data, falsification, leading to distortion of research data, plagiarism and false co-authorship.

**4. A detailed procedure and mechanism for conducting studies conducted with the** participation of people and animals, a description of the compliance of the planned studies with the legislation of the Republic of Kazakhstan. During the implementation of the program, studies and experiments with the participation of people and animals will not be carried out;

**5. Terms of registration and separation of intellectual property rights to research results**

The separation of intellectual property rights is carried out according to the real participation of each employee in obtaining a scientific result, which is established on the basis of materials published in the open press. Terms of registration of intellectual property rights are carried out in the prescribed manner in the Committee on Intellectual Property Rights of the Republic of Kazakhstan.

## 4. Bibliography


1. K. Prachar, Primzahl Verteilung.-Berlin, 1957, p. 511 (K. Prahar, Distribution of prime numbers, Mir Publishing House, Moscow, 1967.511 pp.)
2. E.C. Titchmarsh, TheTheoryofTheRiemann Zeta-Function, Oxford, 1951,407p.
3. A.L. Karatsuba, Fundamentals of the analytical theory. - M.: Nauka. 1983.
4. S.C. Coutinho, TheMathematicsofCiphers.NumberTheoryandRSACryptography.-AKPeter, Ltd. AllRightsReserved, 1999, p. 323.
5. I.M. Vinogradov, Theory of the Theory.-M.: Science, 1981
6. Don Tsagir, The First 50 Million Simple Numbers, Uspekhimat.Nauk, 1984, vol. 39, issue 6 (240), pp. 175-190.
7. G.H. Hardy, E. M. Wright, AnIntroductiontotheTheoryofNumbers., OxfordScience Publication, 1979.
8. A.E.Ingam, Distribution of prime numbers. –M.: EditorialSURSS, 2005,268 p.
9. Introduction to cryptography. Subdiv.red. Yashchenko, M.: MCNMO-CheRo, 2000,316s.
10. N. Koblits, Kursteyorichiselikriptografii.M.: TVP, 2001, p. 101.
11. D.G. Wells, Primenumbers: themostmysteriousfiguresinmath. New Jersey, 2005.
12. V.I. Zenkin, Distribution of prime numbers. Kaliningrad, 2008, 1858 p.