Lecture 1: Course Introduction

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1 Course Overview

1.1 Learning Goals

- Have a good understanding of what to expect from this course
- Learn about the grading policy, assignments and final project scope
- Begin our exploration of "Gravity 3 ways"
- Take first steps with the gala package.

1.2 Modern Astrophysics

This course is directed at first/second year graduate students interested in astrophysics research. We cover a few of the important physical processes necessary to understand astronomical objects and observations. The course will use python notebooks throughout helping with experiential learning and give the student resources to explore the code/formulas and algorithms behind the plots and animations.

1.3 Course Notes and Schedule

1.3.1 Texts and resources

Useful free textbook for a number of theoretical concepts. - **Kaiser**: *Elements of Astrophysics*: Link to PDF

1.3.2 Syllabus: Calendar, Reading, and Problem Sets

```
Calendar                                          Reading                 Problem Set
Tu  1 Jan 7   Intro/Overview/Gravity              K:285-300               K: 26.7.1 p295 + 1-2
Th  2 Jan 9   Gravity 3 ways | Stellar dynamics
Tu  3 Jan 14  Cosmology + Spherical Collapse      K:303-308,337-340,385-388  Bertschinger Dus
Th  4 Jan 16  Hydrodynamics          I)           K:246-247,251-253
Tu  5 Jan 21  Hydrodynamics         II)
Th  6 Jan 23  Sound Waves, Shocks & Sedov Taylor K:258,22.2,23.2          K:20.10.3 + Bertsch
Tu  7 Jan 28  Radiation                           K:61-69
Th  8 Jan 30  Radiation Transport                 K:81-89                  K:6.12.1-4 +
Tu  9 Feb  4  Radiative Processes   I)
Th 10 Feb  6  Radiative Processes  II)
Tu 11 Feb 11  Radiative Processes III)                                     K:14.9.2-3 +
Th 12 Feb 13  AT   I)  Spectra of Galaxies        Notes
Tu 13 Feb 18  AT  II)  HII regions                Notes
Th 14 Feb 20  AT III)  Press Schechter theory     Notes                    reproduce Mo & White
Tu 15 Feb 25  AT  IV)  Summary Statistics         Notes
Th 16 Feb 27  AT   V)  Plasmas/Acceleration       Notes
Tu 17 Mar  4  Presentations   I
Th 18 Mar  6  Presentations  II
Tu 19 Mar 11  Presentations III
Th 20 Mar 13  Presentations  IV
```

1.3.3 Grading

- **40%** Final project and presentation
- **60%** Problem set (worst dropped)

Testing our python code setup:

```
import sys
sys.path.append('../code')
from astro_utils import hello_astrophysics

hello_astrophysics()
import sys
print(sys.executable)
```

```
Hello astrophysics!
/opt/hostedtoolcache/Python/3.12.8/x64/bin/python3
```

2 Collision-less Systems

Gravity, despite being the weakest of the four fundamental forces, plays a pivotal role on astronomical scales, shaping the structures and dynamics of the universe. Unlike the electromagnetic, strong, and weak forces, which dominate at subatomic and terrestrial scales, gravity's influence becomes profound over vast distances and when acting on large masses. It governs the formation and evolution of globular clusters, galaxies, and planetary systems, orchestrating the cosmic dance of celestial bodies. Through gravitational attraction, stars coalesce into clusters, galaxies bind their billions of stars together, and planets form stable orbits around their host stars.

In this part of the course, we will delve into collective gravitational phenomena that are essential to understanding astrophysical systems. Key topics include dynamical friction, the process by which massive objects lose momentum and energy due to interactions with a background of less massive particles, shaping the motion of stars and satellites in galaxies. We'll explore two-body relaxation, which describes how random gravitational encounters between stars in a cluster lead to a redistribution of energy and angular momentum, driving systems toward thermal equilibrium. Lastly, we will study virialization, a crucial concept that describes how self-gravitating systems reach a state of balance between kinetic and potential energy. These phenomena not only underpin the stability and dynamics of astrophysical systems but also provide a framework for understanding their formation, evolution, and ultimate fate.

First let's recall some basic facts of Newtonian gravity. This chapter is mostly based on Part V of **Kaiser**: *Elements of Astrophysics*: pages 285-300. The other important book on this subject is Binney and Tremaine (2008) which we have online access to through Stanford Libraries. On the Canvas course site you can find a .pdf version in the files directory.

2.1 Newtonian Gravity 3 Ways

Newtonian gravity can be understood from three complementary perspectives, each providing unique insights into the nature of gravitational interactions. In the following sections, we will explore these perspectives: Forces, where we consider how gravitational acceleration arises directly from pairwise interactions or a continuous mass distribution; Potentials, where we describe the gravitational field as a scalar potential that satisfies Poisson's equation and links mass to field strength; and Action, where the motion of self-gravitating systems is derived from a variational principle. Together, these approaches offer a comprehensive view of the dynamics governed by Newtonian gravity. If it has been a while you may appreciate (re-)watch Feynman's Messenger lectures on these topics. At the very least they may help to motivate further study.

2.1.1 Forces

In Newton's theory the acceleration of a particle is the sum over all other particles of G times the mass times the inverse square of the distance.

$$\ddot{\mathbf{x}}_j = \sum_{i \neq j} \frac{G m_i \left(\mathbf{x}_i - \mathbf{x}_j\right)}{\left|\mathbf{x}_i - \mathbf{x}_j\right|^3}$$

where

$$G \simeq 6.67 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2}$$

For a continuous density distribution this is

$$\ddot{\mathbf{x}} = \mathbf{g}(\mathbf{x}) = G \int d^3 x' \rho\left(\mathbf{x}'\right) \frac{\left(\mathbf{x}' - \mathbf{x}\right)}{\left|\mathbf{x}' - \mathbf{x}\right|^3}$$

2.1.2 Potentials

The gravitational acceleration $\mathbf{g}$ can be written as the negative of the gradient of the gravitational potential $\mathbf{g} = -\nabla\Phi$ where

$$\Phi(\mathbf{x}) = -G \int d^3 x' \frac{\rho\left(\mathbf{x}'\right)}{\left|\mathbf{x}' - \mathbf{x}\right|}$$

Taking the gradient of $\Phi(\mathbf{x})$ one recovers In addition to the explicit formula for the potential as an spatial integral (25.4) there is also an equivalent local relationship between the Laplacian of the potential $\nabla^2\Phi$ and the density ρ

$$\nabla^2 \Phi = 4\pi G \rho \tag{1}$$

which is Poisson's equation.

If we divide by $4\pi G$ and integrate both sides of equation (Equation 1) over an arbitrary volume V, and then apply the divergence theorem, we obtain

$$M = \int_V \mathrm{d}^3\mathbf{x} \rho = \frac{1}{4\pi G} \int_V \mathrm{d}^3\mathbf{x} \nabla^2 \Phi = \frac{1}{4\pi G} \oint_{\partial V} \mathrm{d}^2\mathbf{S} \cdot \nabla\Phi$$

This is Gauss's theorem and says that the integral of the normal component of $\nabla\Phi$ over any closed surface, ∂V, divided by $4\pi G$ equals the mass contained within that surface. This says potentials

2.1.3 Action for N Self-Gravitating Bodies

The action for N self-gravitating bodies is the starting point for deriving their equations of motion using the variational principle. In classical mechanics, the action is expressed as the time integral of the Lagrangian. For self-gravitating bodies, the Lagrangian has two components: the kinetic energy of the particles and the gravitational potential energy between them.

2.1.3.1 General Form of the Action

The action S for N self-gravitating bodies is:

$$S = \int \mathcal{L}\,\mathrm{d}t,$$

where the Lagrangian $\mathcal{L}$ is given by:

$$\mathcal{L} = T - U.$$

Here:

- T is the total kinetic energy of the system.
- U is the total gravitational potential energy of the system.

2.1.3.1.1 Kinetic Energy

The kinetic energy is:

$$T = \frac{1}{2}\sum_{i=1}^{N} m_i \|\dot{\mathbf{r}}_i\|^2,$$

where:

- m_i is the mass of the i-th particle.
- $\dot{\mathbf{r}}_i = \frac{\mathrm{d}\mathbf{r}_i}{\mathrm{d}t}$ is the velocity of the i-th particle.

2.1.3.1.2 Gravitational Potential Energy

The gravitational potential energy is:

$$U = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j\neq i}^{N}\frac{Gm_i m_j}{\|\mathbf{r}_i - \mathbf{r}_j\|},$$

where: - G is the gravitational constant. - $\|\mathbf{r}_i - \mathbf{r}_j\|$ is the Euclidean distance between the i-th and j-th particles.

The factor $\frac{1}{2}$ accounts for double-counting each pair of interactions.

2.1.3.2 Full Expression for the Action

Substituting the kinetic and potential energy terms, the action becomes:

$$S = \int\left(\frac{1}{2}\sum_{i=1}^{N} m_i\|\dot{\mathbf{r}}_i\|^2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{j\neq i}^{N}\frac{Gm_i m_j}{\|\mathbf{r}_i - \mathbf{r}_j\|}\right)\mathrm{d}t.$$

This action governs the dynamics of $ N $ self-gravitating bodies under Newtonian gravity.

2.1.3.3 Deriving Equations of Motion

To find the equations of motion, apply the principle of stationary action:

$$\delta S = 0.$$

Varying S with respect to $\mathbf{r}_i$ leads to the Euler-Lagrange equations:

$$m_i\ddot{\mathbf{r}}_i = -\sum_{j\neq i}^{N}\frac{Gm_i m_j}{\|\mathbf{r}_i - \mathbf{r}_j\|^3}(\mathbf{r}_i - \mathbf{r}_j).$$

This is Newton's second law for the i-th particle in the gravitational field created by the other particles.

Binney, James, and Scott Tremaine. 2008. *Second Edition*. Princeton: Princeton University Press. https://doi.org/doi:10.1515/9781400828722.