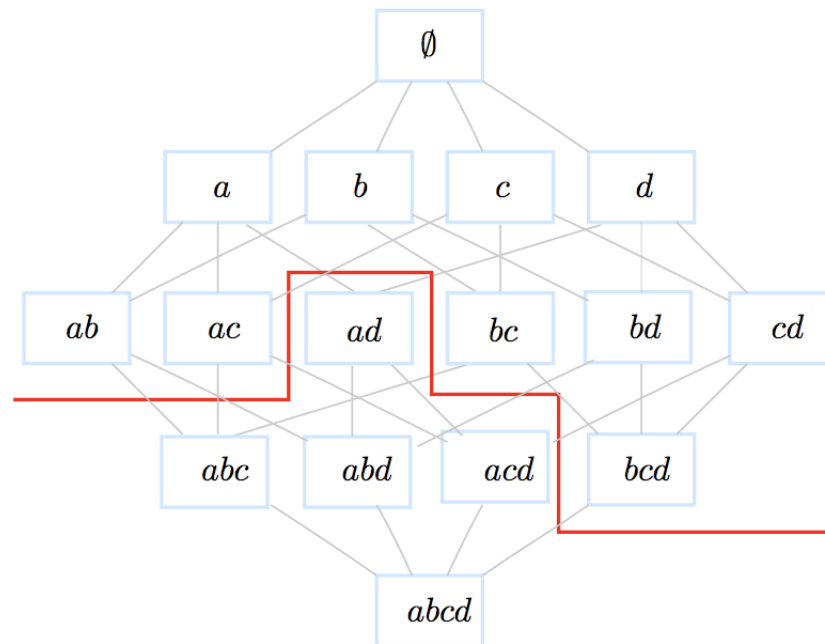


# Artificial Intelligence

# Machine Learning

# Association Rules



# Outline

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1. Introduction
2. A two-step process
3. Applications
4. Definitions and examples
5. Frequent patterns: Apriori algorithm
6. Example
7. Representation of  $\mathcal{D}$
8. Definitions cont'd
9. Association rules algorithm
10. Example
11. A probabilistic framework Association Rules
12. Support-confidence cons
13. Quantitative association rules

# Introduction

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- Unsupervised task.
- R. Agrawal, T. Imielinski and A.N. Swami Mining Association Rules between sets of items in large databases. Proceedings of SIGMOD 1993.
- Highly cited work because of its wide applicability.

# Applications

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- Market Basket Analysis: cross-selling (ex. Amazon), product placement, affinity promotion, customer behavior analysis
- Collaborative filtering
- Web organization
- Symptoms-diseases associations
- Supervised classification

# A two-step process

---

Given a transaction dataset  $\mathcal{D}$

1. Mining **frequent** patterns in  $\mathcal{D}$
2. Generation of **strong** association rules

**Example:**

$\{Bread, Butter\}$  is a frequent pattern (itemset)

$Bread \rightarrow Butter$  is a strong rule

# Definitions

---

- **Item**: an object belonging to  $\mathcal{I} = \{x_1, x_2, \dots, x_m\}$ .
- **Itemset**: any subset of  $\mathcal{I}$ .
- **$k$ -itemset**: an itemset of cardinality  $k$ .
- We define a total order  $(\mathcal{I}, <)$  on the items.
- $\mathcal{P}(\mathcal{I})$  is a **lattice** with  $\perp = \emptyset$  and  $\top = \mathcal{I}$ .
- **Transaction**: itemset identified by a unique identifier **tid**.
- $\mathcal{T}$ : the set of all transactions ids. **Tidset**: a subset of  $\mathcal{T}$ .
- **Transaction dataset**:  $\mathcal{D} = \{(tid, X_{tid}) \mid tid \in \mathcal{T}, X_{tid} \subseteq \mathcal{I}\}$

# Example

---

item	name
a	coffee
b	milk
c	butter
d	bread

$\mathcal{D}$	
tid	transaction
1	$a\ b$
2	$a\ c$
3	$c\ d$
4	$b\ c\ d$
5	$a\ b\ c\ d$

$\mathcal{I} =$

$\mathcal{T} =$

$\mathcal{D} =$

# Example

---

item	name
a	coffee
b	milk
c	butter
d	bread

$\mathcal{D}$	
tid	transaction
1	<i>a b</i>
2	<i>a c</i>
3	<i>c d</i>
4	<i>b c d</i>
5	<i>a b c d</i>

$$\mathcal{I} = \{a, b, c, d\}$$

$$\mathcal{T} = \{1, 2, 3, 4, 5\}$$

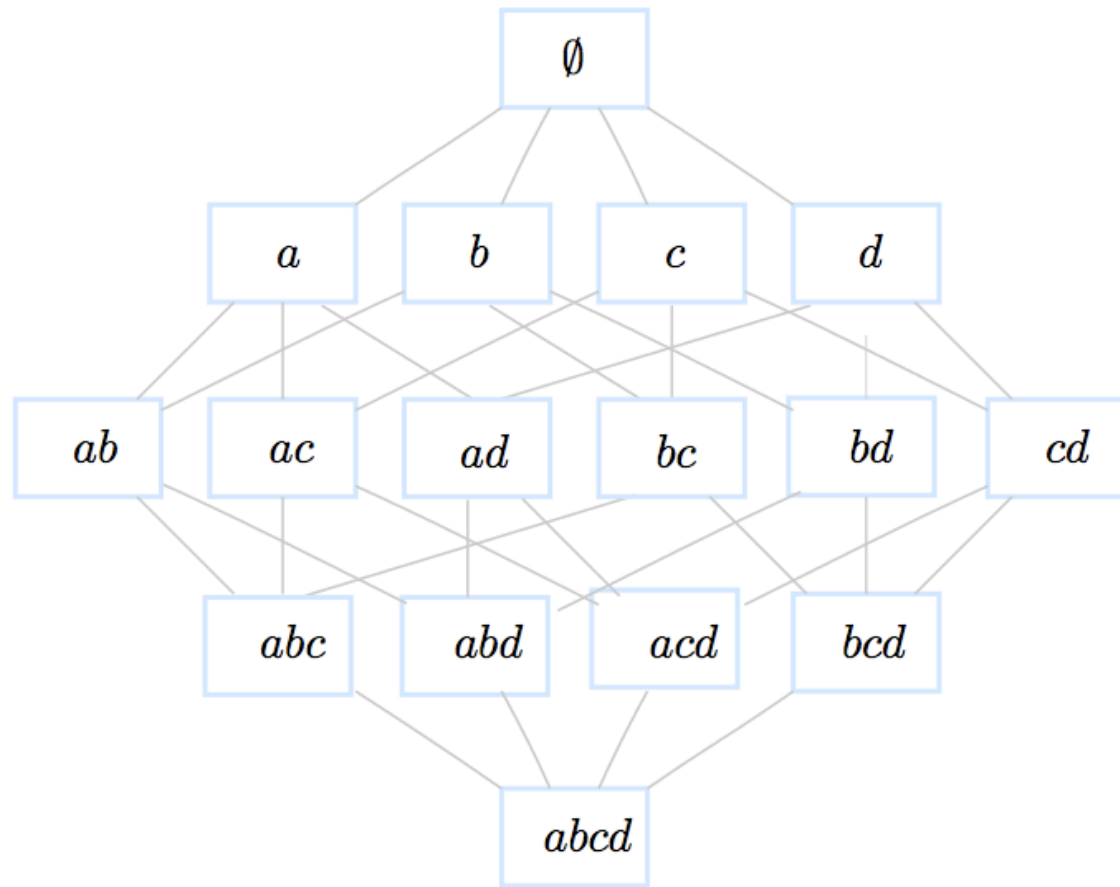
$$\mathcal{D} = \{(1, ab), (2, ac), (3, cd), (4, bcd), (5, abcd)\}$$

E.g.,  $\{b, c\}$  is a 2-itemset, for writing simplification we will give up the braces and write  $bc$ .  $\{3, 4, 5\}$  is a tidset similarly let's abandon the braces here too and write 345.



# Example

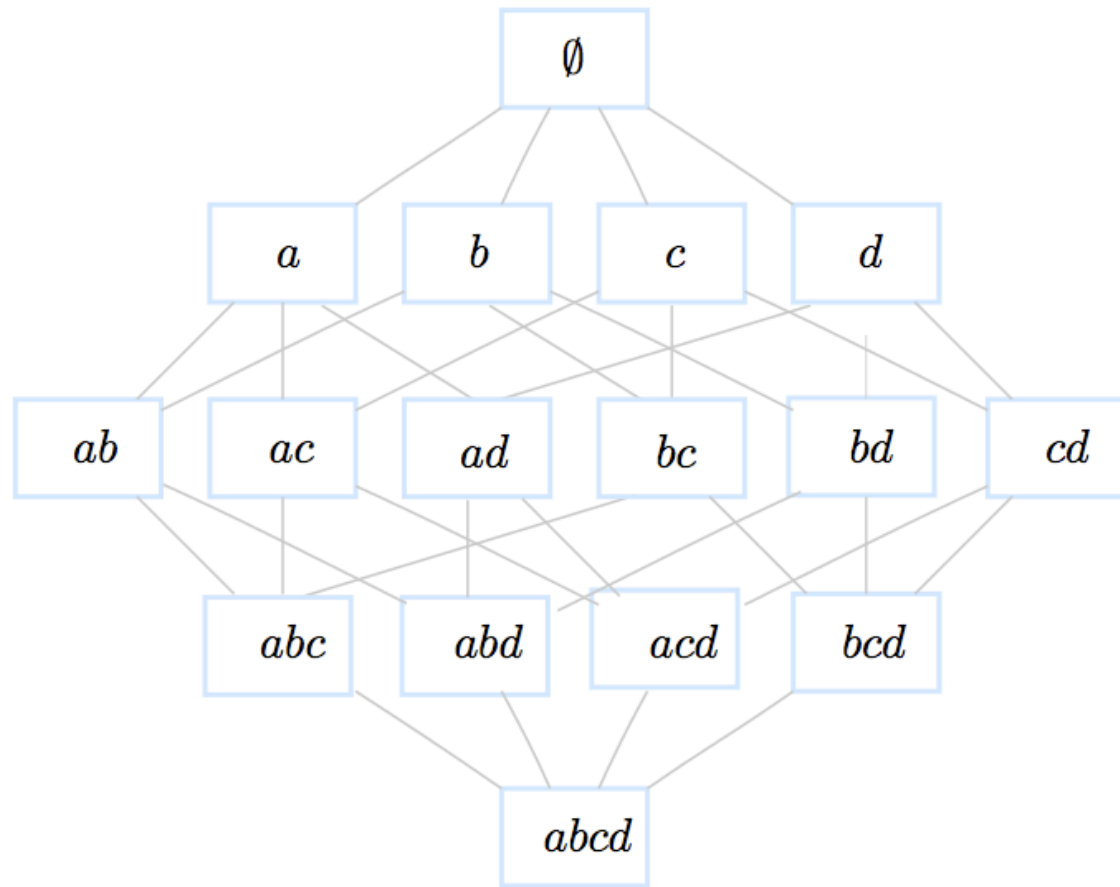
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Lattice of itemsets of size ...

# Example

---



Lattice of itemsets of size  $2^{|I|} = 16$ .

# Definitions cont'd

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- Mapping  $t$ :

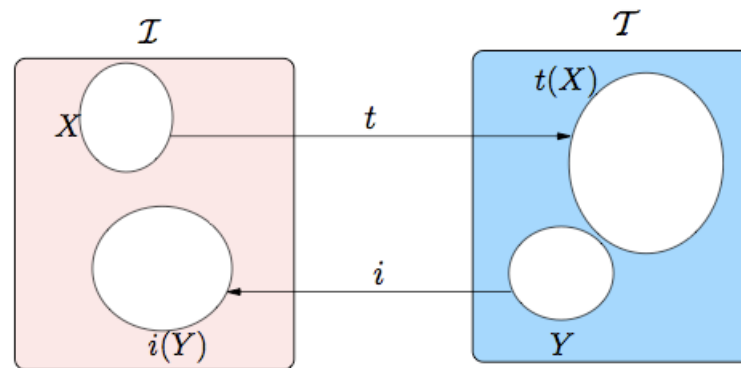
$$t : \mathcal{P}(\mathcal{I}) \rightarrow \mathcal{P}(\mathcal{T})$$

$$X \mapsto t(X) = \{tid \in \mathcal{T} \mid \exists X_{tid}, (tid, X_{tid}) \in \mathcal{D} \wedge X \subseteq X_{tid}\}$$

- Mapping  $i$ :

$$i : \mathcal{P}(\mathcal{T}) \rightarrow \mathcal{P}(\mathcal{I})$$

$$Y \mapsto i(Y) = \{x \in \mathcal{I} \mid \forall (tid, X_{tid}) \in \mathcal{D}, tid \in Y \Rightarrow x \in X_{tid}\}$$



# Definitions cont'd

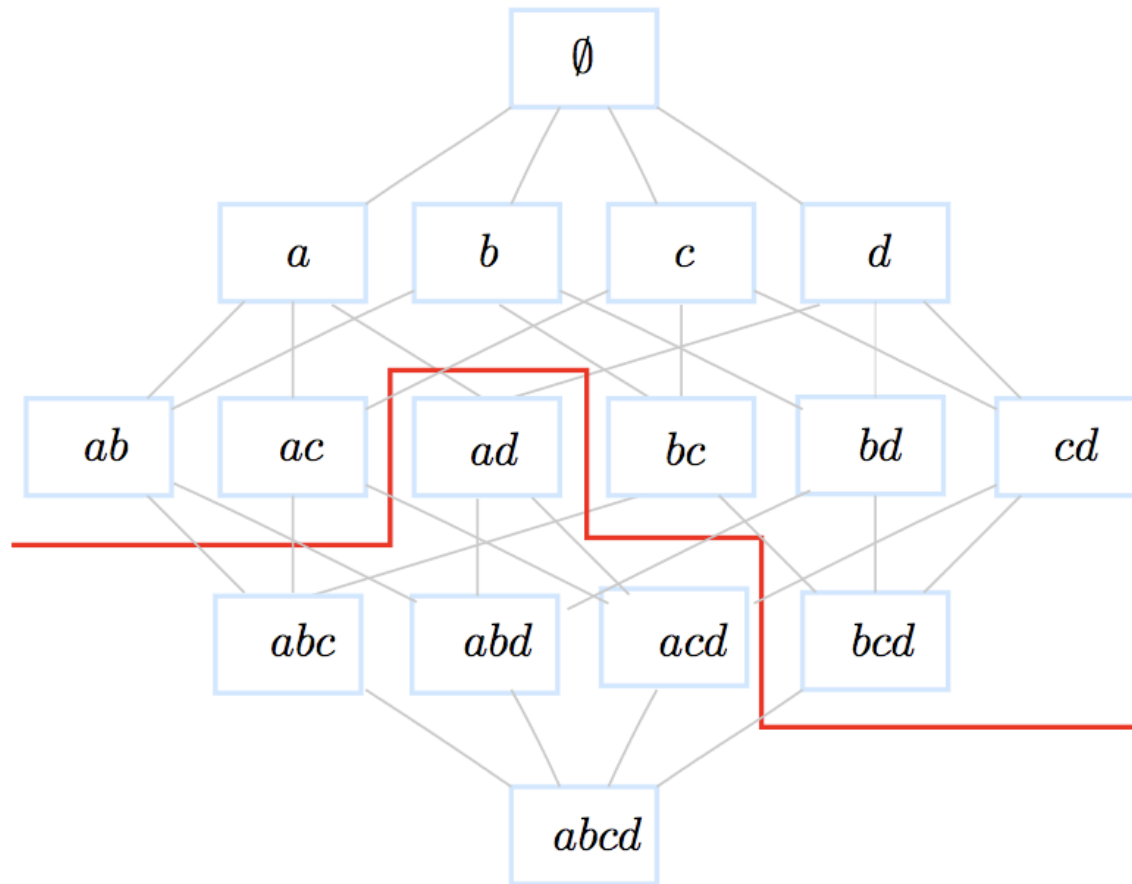
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- **Frequency:**  $freq(X) = |\{(tid, X_{tid}) \in \mathcal{D} / X \subseteq X_{tid}\}| = |t(X)|$
- **Support:**  $supp(X) = \frac{|t(X)|}{|\mathcal{D}|}$
- **Frequent itemset:**  $X$  is frequent iff  $supp(X) \geq MinSupp$
- **Property (Support downward closure) :** if an itemset is frequent then all its subsets also are frequent.
- **Mining Frequent Itemsets:**

$$\mathcal{F} = \{ X \subseteq \mathcal{I} \mid supp(X) \geq MinSupp \}$$

# Example

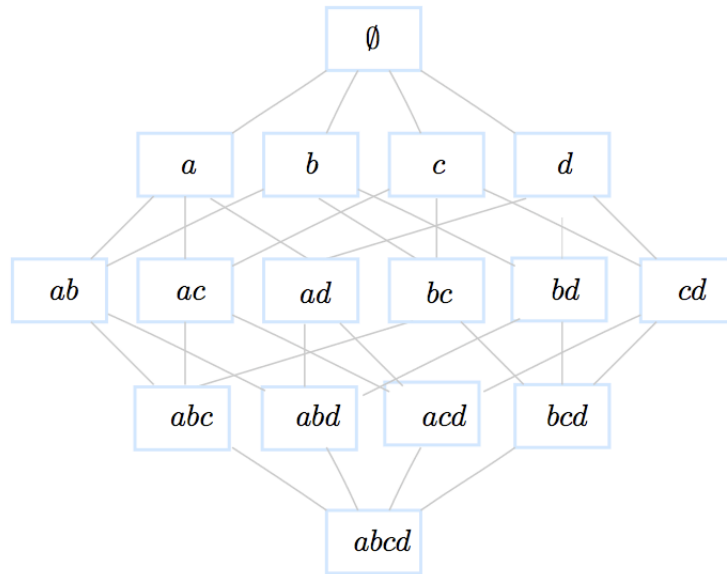
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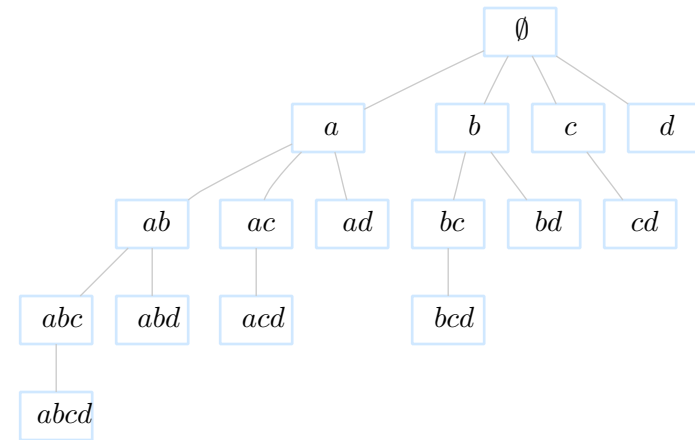
MinSupp=40%

# BFS and DFS

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Breadth First Search



Depth First Search

# Apriori pseudo-algorithm

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Level-wise algorithm – lattice explored with a Breath First Search approach (BFS). Start at level 1 in the lattice:  $k = 1$

- Generate candidates of size  $k$

$$\mathcal{C}_k = \{(c_k, \text{supp}(c_k)) \mid \forall X \subset c_k, X \neq \emptyset, \text{support}(X) \geq \text{MinSupp}\}$$

- Scan the dataset to compute the support of each candidate and keep the frequent ones

$$\mathcal{F}_k = \{(l_k, \text{supp}(l_k)) \mid \text{supp}(l_k) \geq \text{Minsupp}\}$$

- Go to the next level  $k = k + 1$  and redo the process.

# Example

Minsupp=2/5 (40%)

$\mathcal{D}$	
tid	transaction
1	a b
2	a c
3	c d
4	b c d
5	a b c d

$\mathcal{C}_1$
Itemset
a
b
c
d

$\xrightarrow[\text{of } \mathcal{D}]{\text{Scan}}$

$\mathcal{C}_1$	
Itemset	Support
a	3/5
b	3/5
c	4/5
d	3/5

→

$\mathcal{F}_1$	
Itemset	Support
a	3/5
b	3/5
c	4/5
d	3/5

$\mathcal{C}_2$
Itemset
ab
ac
ad
bc
bd
cd

$\xrightarrow[\text{of } \mathcal{D}]{\text{Scan}}$

$\mathcal{C}_2$	
Itemset	Support
ab	2/5
ac	2/5
ad	1/5
bc	2/5
bd	2/5
cd	3/5

→

$\mathcal{F}_2$	
Itemset	Support
ab	2/5
ac	2/5
bc	2/5
bd	2/5
cd	3/5

$\mathcal{C}_3$
Itemset
abc
bcd

$\xrightarrow[\text{of } \mathcal{D}]{\text{Scan}}$

$\mathcal{C}_3$	
Itemset	Support
abc	1/5
bcd	2/5

→

$\mathcal{F}_3$	
Itemset	Support
bcd	2/5



# Apriori bottleneck

---

Characteristics of real-life datasets:

1. Billions of transactions,
2. Tens of thousands of items,
3. Tera-bytes of data.

This leads to:

1. Multiple scans of the dataset residing in the disk (costly I/O operations)
2. A **HUGE** number of candidates sets.

# Representation of $\mathcal{D}$

---

## Row-wise

1	<table><tr><td><math>a</math></td><td><math>b</math></td></tr></table>	$a$	$b$		
$a$	$b$				
2	<table><tr><td><math>a</math></td><td><math>c</math></td></tr></table>	$a$	$c$		
$a$	$c$				
3	<table><tr><td><math>c</math></td><td><math>d</math></td></tr></table>	$c$	$d$		
$c$	$d$				
4	<table><tr><td><math>b</math></td><td><math>c</math></td><td><math>d</math></td></tr></table>	$b$	$c$	$d$	
$b$	$c$	$d$			
5	<table><tr><td><math>a</math></td><td><math>b</math></td><td><math>c</math></td><td><math>d</math></td></tr></table>	$a$	$b$	$c$	$d$
$a$	$b$	$c$	$d$		

## Column-wise

$a$	$b$	$c$	$d$
1	1	2	3
2	4	3	4
5	5	4	5
		5	

## Boolean

	$a$	$b$	$c$	$d$
1	1	1	0	0
2	1	0	1	0
3	0	0	1	1
4	0	1	1	1
5	1	1	1	1

# Definitions cont'd

---

- Given  $\mathcal{F}$  and a Minimum confidence threshold  $MinConf$
- Generate rules:

$$(l - C) \rightarrow C$$

$$conf((l - C) \rightarrow C) = \frac{supp(l)}{supp(l - C)} \geq MinConf$$

- From a  $k - itemset$  ( $k > 1$ ), one can generate  $2^k - 1$  rules.

## Property

Let  $l$  be a large (frequent) itemset:

$$\forall C \subset l, C \neq \emptyset, [(l - C) \rightarrow C] \text{ is strong} \Rightarrow \forall \tilde{C} \subset C, \tilde{C} \neq \emptyset, [(l - \tilde{C}) \rightarrow \tilde{C}] \text{ is strong}$$

# Example

---

Minconf=60%

Itemset	Rule#	Rule	Confidence	Strong?
<i>ab</i>	1	$a \rightarrow b$	$2/3 = 66.66\%$	yes
	2	$b \rightarrow a$	$2/3 = 66.66\%$	yes
<i>ac</i>	3	$a \rightarrow c$	$2/3 = 66.66\%$	yes
	4	$c \rightarrow a$	$2/4 = 50.00\%$	no
<i>bc</i>	5	$b \rightarrow c$	$2/3 = 66.66\%$	yes
	6	$c \rightarrow b$	$2/4 = 50.00\%$	no
<i>bd</i>	7	$b \rightarrow d$	$2/3 = 66.66\%$	yes
	8	$d \rightarrow b$	$2/3 = 66.66\%$	yes
<i>cd</i>	9	$c \rightarrow d$	$3/4 = 75.00\%$	yes
	10	$d \rightarrow c$	$3/3 = 100.00\%$	yes

# Example

---

Minconf=60%

Itemset	Rule#	Rule	Confidence	Strong?
<i>bcd</i>	11	$cd \rightarrow b$	$2/3 = 66.66\%$	yes
	12	$bd \rightarrow c$	$2/2 = 100.00\%$	yes
	13	$bc \rightarrow d$	$2/2 = 100.00\%$	yes

Itemset	Rule#	Rule	Confidence	Strong?
<i>bcd</i>	14	$d \rightarrow bc$	$2/3 = 66.66\%$	yes
	15	$c \rightarrow bd$	$2/4 = 50.00\%$	no
	16	$b \rightarrow cd$	$2/3 = 66.66\%$	yes

# Probabilistic Interpretation

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Brin et al. 97

$$R : A \longrightarrow C$$

- $R$  measures the distribution of  $A$  and  $C$  in the finite space  $\mathcal{D}$ .
- The sets  $A$  and  $C$  are 2 events
- $P(A)$  and  $P(C)$  the probabilities that events  $A$  and  $C$  happen resp. estimated by the the frequency of  $A$  and  $C$  resp. in  $\mathcal{D}$

$$\text{supp}(A \rightarrow C) = \text{supp}(A \cup C) = P(A \wedge C)$$

$$\text{conf}(A \rightarrow C) = P(C|A) = \frac{P(A \wedge C)}{P(A)}$$

# Support-Confidence: cons

---

- Example (Brin et al. 97)

	<i>coffee</i>	$\overline{coffee}$	$\sum rows$
<i>tea</i>	20	5	25
$\overline{tea}$	70	5	75
$\sum columns$	90	10	100

$tea \rightarrow coffee$  ( $supp = 20\%$ ,  $conf = 80\%$ )

Strong rule?

# Support-Confidence: cons

---

- Example (Brin et al. 97)

	<i>coffee</i>	$\overline{coffee}$	$\sum rows$
<i>tea</i>	20	5	25
$\overline{tea}$	70	5	75
$\sum columns$	90	10	100

$$tea \rightarrow coffee \quad (supp = 20\%, conf = 80\%)$$

Strong rule? Yes but a misleading one!

$Support(coffee) = 90\%$  is a bias that the confidence cannot detect because it ignores  $support(coffee)$ .



# Other evaluation Measures

---

- Interest (Piatetsky-Shapiro 91) or Lift (Bayardo et al. 99)

$$Interest(A \rightarrow C) = \frac{P(A \wedge C)}{P(A) \times P(C)} = \frac{supp(A \cup C)}{supp(A) \times supp(C)}$$

Interest is between 0 and  $+\infty$ :

1. If  $Interest(\mathcal{R}) = 1$  then  $A$  and  $C$  are independent;
2. If  $Interest(\mathcal{R}) > 1$  then  $A$  and  $C$  are positively dependent;
3. If  $Interest(\mathcal{R}) < 1$  then  $A$  and  $C$  are negatively dependent.

$$Interest(A \rightarrow C) = \frac{conf(A \rightarrow C)}{supp(C)} = \frac{conf(C \rightarrow A)}{supp(A)}$$

# Other evaluation Measures

---

	<i>coffee</i>	$\overline{coffee}$	$\sum rows$
<i>tea</i>	20	5	25
$\overline{tea}$	70	5	75
$\sum columns$	90	10	100

$$Interest(tea \rightarrow coffee) = \frac{P(tea \wedge coffee)}{P(tea) \times P(coffee)} = \frac{0.2}{0.25 * 0.9} = 0.89 < 1$$

	<i>coffee</i>	$\overline{coffee}$
<i>tea</i>	0.89	2
$\overline{tea}$	1.03	0.66

# Multi-dimensional rules

---

- One-dimensional rules:

$$\textit{buy}(x, \textit{"Bread"}) \longrightarrow \textit{buy}(x, \textit{"Butter"})$$

- Multi-dimensional rules:

$$\textit{buy}(x, \textit{"Pizza"}) \wedge \textit{age}(x, \textit{"Young"}) \longrightarrow \textit{buy}(x, \textit{"Coke"})$$

- Construct k-predicatesets instead of k-itemsets
- How about numerical features?

$$\textit{buy}(x, \textit{"Pizza"}) \wedge \textit{age}(x, \textit{"18 - 22"}) \longrightarrow \textit{buy}(x, \textit{"Coke"})$$

# Post-processing of AR

---

- AR framework may lead to a large number of rules.
- How one can reduce the number of rules?
  1. Use many evaluation measures
  2. Increase minimum support
  3. Increase minimum confidence
  4. use rule templates (define constraints on max rule length, exclude some items, include in the rules specific items)  
(Agrawal et al. 1995, Salleb et al. 2007)

# Implementations

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- **FIMI** Frequent Itemset Mining Implementations Repository  
<http://fimi.cs.helsinki.fi/> FIMI'03 and FIMI'04 workshop,  
Bayardo, Goethals & Zaki
- **Apriori** <http://www.borgelt.net/apriori.html> developed by  
Borgelt
- **Weka** <http://www.cs.waikato.ac.nz/ml/weka/> by Witten &  
Frank
- **ARMADA** Data Mining Tool version 1.3.2 in matlab available  
at Mathworks, by Malone

# FP algorithms

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According to the strategy to traverse the search space:

- Breadth First Search (ex: Apriori, AprioriTid, Partition, DIC)
- Depth First Search (ex: Eclat, Clique, Depth project)
- Hybrid (ex: AprioriHybrid, Hybrid, Viper, Kdci)
- Pattern growth, i.e. no candidate generation (ex: Fpgrowth, HMine, Cofi)

# Uniform notion of item

---

- Apriori has been initially designed for **boolean tables** (transactional datasets) thus propositional logic was sufficient to express: items, itemsets and rules.

$$milk \rightarrow cereals$$

- For **relational tables**, one need to extend the notion of items to literals:

$$item \equiv (attribute, value)$$

An attribute could be:

1. categorical, for ex.  $(color, blue)$ ,
2. quantitativ with a few numerical values, for ex.  $(\#cars, 2)$ ,
3. quantitativ with a large domain values, for ex.  $(age, [20, 40])$ .

# Example

---

$\mathcal{D}$ : people			
id	age	married?	#cars
1	23	no	1
2	25	yes	1
3	29	no	0
4	34	yes	2
5	38	yes	2

Examples of frequent itemsets	
itemset	support
(age, 20..29)	3
(age, 30..39)	2
(married?, yes)	3
(married?, no)	2
(#cars, 1)	2
(#cars, 2)	2
(age, 30..39),(married?, yes)	2

Examples of rules		
rule	support	confidence
(age, 30..39) et (married?, yes) $\longrightarrow$ (#cars, 2)	40%	100%
(age, 20..29) $\longrightarrow$ (#cars, 1)	60%	66.6%



# Quantitative AR

---

**Question:** Mining Quantitative AR is not a simple extension of mining categorical AR. why?

- **Infinite search space:** In Boolean AR, the Apriori property allows to prune the search space efficiently, but we do explore the whole space of hypothesis (lattice of itemsets), which is IMPOSSIBLE for Quantitative AR.
- **The support-confidence tradeoff:** Choosing intervals is quite sensitive to support and confidence.
  - intervals too small, not enough support;
  - intervals too large, not enough confidence.
- What is the difference between supervised and **unsupervised discretization**?

# Approaches to mine QARs

---

- Discretization-based approaches
- Distribution-based approaches
- Optimization-based approaches

# Approaches to mine QARs

---

## Discretization-based approaches

- A pre-processing step
- Use equi-depth, equi-width, domain-knowledge
- Lent et al., 1997; Miller and Yang, 1997; Srikant and Agrawal, 1996; Wang et al., 1998
- Discretization combined with clustering or interval merging.
- Problems: univariate, sensitive to outliers, loss of information.

# Approaches to mine QARs

---

## Distribution-based approaches

*Sex = female  $\rightarrow$  Height : mean = 168  $\wedge$  Weight : mean = 68*

- Aumann and Lindell, 1999, Webb 2001.
- Restricted form of rules:
  1. A set of categorical attributes on the left-hand side and several distributions on the right-hand side,
  2. A single discretized numeric attribute on the left-hand side and a single distribution on the right-hand side.

# Approaches to mine QARs

---

## Optimization-based approaches

- Numerical attributes are optimized during the mining process
- Fukuda et al., 96, Rastogi and Shim 99, Brin et al. 2003. Techniques inspired from image segmentation.

$$Gain(A \rightarrow B) = Supp(AB) - MinConf * Supp(A)$$

Form of the rules restricted to 1 or 2 numerical attributes.

- Mata et al. 2002 Use genetic algorithms to optimize the support of itemsets with non instantiated intervals.

$$Fitness = cov - (\psi * ampl) - (\omega * mark) + (\mu * nAtr)$$

Apriori-like algorithm to mine association rules.

# Approaches to mine QARs

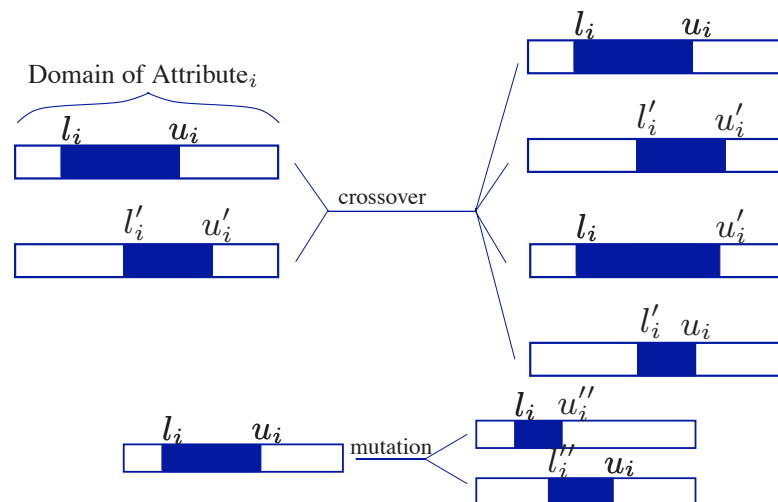
## Optimization-based approaches

- Ruckert et al. 2004 use half-spaces to mine such rules like:

$$x_1 > 20 \rightarrow 0.5x_3 + 2.3x_6 \geq 100$$

Cannot handle categorical attributes.

- Salleb et al 2007: QuantMiner Optimize the *Gain* of rules templates using a genetic algorithm.



# Approaches to mine QARs

---

**Optimization-based approaches:** QuantMiner cont'd.

Example UCI Iris dataset:

$$\begin{array}{l} \text{Species=} \\ \text{value} \end{array} \Rightarrow \left\{ \begin{array}{ll} \text{PW} \in [l_1, u_1] & \text{SW} \in [l_2, u_2] \\ \text{PL} \in [l_3, u_3] & \text{SL} \in [l_4, u_4] \end{array} \right\} \begin{array}{l} \text{supp\%} \\ \text{conf\%} \end{array}$$

$$\begin{array}{l} \text{Species=} \\ \text{setosa} \end{array} \Rightarrow \left\{ \begin{array}{ll} \text{PW} \in [1, 6] & \text{SW} \in [31, 39] \\ \text{PL} \in [10, 19] & \text{SL} \in [46, 54] \end{array} \right\} \begin{array}{l} 23\% \\ 70\% \end{array}$$

$$\begin{array}{l} \text{Species=} \\ \text{versicolor} \end{array} \Rightarrow \left\{ \begin{array}{ll} \text{PW} \in [10, 15] & \text{SW} \in [22, 30] \\ \text{PL} \in [35, 47] & \text{SL} \in [55, 66] \end{array} \right\} \begin{array}{l} 21\% \\ 64\% \end{array}$$

$$\begin{array}{l} \text{Species=} \\ \text{virginica} \end{array} \Rightarrow \left\{ \begin{array}{ll} \text{PW} \in [18, 25] & \text{SW} \in [27, 33] \\ \text{PL} \in [48, 60] & \text{SL} \in [58, 72] \end{array} \right\} \begin{array}{l} 20\% \\ 60\% \end{array}$$

# QuantMiner

---

<http://quantminer.github.io/QuantMiner/>



# QuantMiner

Attributes

Data point / example

Numerical value

sepal_length	sepal_width	petal_length	petal_width	Iris_class
5	2	3.5	1	versicolor
6	2.2	4	1	versicolor
6.2	2.2	4.5	1.5	versicolor
6	2.2	5	1.5	virginica
4.5	2.3	1.3	0.3	setosa
5.5	2.3	4	1.3	versicolor
6.3	2.3	4.4	1.3	versicolor
5	2.3	3.3	1	versicolor
4.9	2.4	3.3	1	versicolor
5.5	2.4	3.8	1.1	versicolor
5.5	2.4	3.7	1	versicolor
5.6	2.5	3.9	1.1	versicolor
6.3	2.5	4.9	1.5	versicolor
5.5	2.5	4	1.3	versicolor
5.1	2.5	3	1.1	versicolor
4.9	2.5	4.5	1.7	virginica
6.7	2.5	5.8	1.8	virginica
5.7	2.5	5	1	versicolor
6.3	2.5	5	1	versicolor
5.7	2.6	3.5	1	versicolor
5.5	2.6	4.4	1	versicolor
5.8	2.6	4	1	versicolor

UCI IRIS dataset

The image displays the QuantMiner software interface, which is used for mining rules from datasets. The interface is divided into several windows and panels:

- Attributes:** A table showing the attributes of the UCI IRIS dataset: sepal\_length, sepal\_width, petal\_length, petal\_width, and Iris\_class.
- Choosing rule templates:** A window where users can select rule templates. It includes buttons for "Select all" and "Select none".
- Mining rules using a genetic algorithm:** A window showing the progress of the mining process. It includes a progress bar and a "Next" button.
- Results:** A window displaying the results of the mining process. It includes a "Save in a file" button, a "Visualize the extraction context" button, and a "Display filter" button. The results are sorted by confidence, and the user can choose to exclude rules with consequent support exceeding a certain percentage (75.0%).
- Rule 6/6 (total : 6):** A detailed view of a specific rule. The rule is:  $A \rightarrow B$ , where  $A$  is "Class = Iris-versicolor" and  $B$  is "petal\_length in [3.3; 4.7]". The rule has a support of 41 (27.33%) and a confidence of 82.0%.
- SUPPORTS:** A table showing the support of the rule and its components. The table has columns for the rule, the support, and the percentage. The data is as follows:

Rule	Support	Percentage
A and B	41	27.33 %
A	50	33.33 %
B	41	27.33 %
A and (~B)	9	6.0 %
(~A) and B	0	0.0 %
(~A) and (~B)	100	66.67 %
- CONFIDENCES:** A table showing the confidence of the rule and its components. The table has columns for the rule, the confidence, and the percentage. The data is as follows:

Rule	Confidence	Percentage
A $\rightarrow$ B	82.0	82.0 %
(~A) $\rightarrow$ B	0.0	0.0 %
B $\rightarrow$ A	100.0	100.0 %
(~B) $\rightarrow$ A	8.26	8.26 %
A $\leftrightarrow$ B	94.0	94.0 %

# References

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- R. Agrawal, T. Imielinski and A.N. Swami “Mining Association Rules between sets of items in large databases” . SIGMOD 1993.
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