Geodatenanalyse 2

Termin: Big Data 2 - Modul 1

Frequenzanalyse von Zeitreihen: Motivation und Grundlage

Ca. 20-30 Minuten

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In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
```

Hinweis: In dieser Vorlesung wird auf das wunderbare Online-Material von Jack Schaedler zur digitalen Signalverarbeitung hingewiesen.

Beispiel: Eine Meeresspiegel Zeitreihe

Daten sind von Palau, gemessen vom Sea Level Center der University of Hawaii

In [2]:
```
sea_level = pd.read_csv('data/Palau_sea-level.csv')
sea_level['Datetime[GMT]'] = pd.to_datetime(sea_level['Datetime[GMT]'], dayfirst
sea_level.set_index('Datetime[GMT]', inplace=True)

sea_level = sea_level[sea_level.index > '2018-01-01']
sea_level
```

Out[2]:

	Sea level [mm]
Datetime[GMT]	
2018-01-01 01:00:00	1506
2018-01-01 02:00:00	1283
2018-01-01 03:00:00	1166
2018-01-01 04:00:00	1207
2018-01-01 05:00:00	1425
...	...
2018-12-31 19:00:00	1566
2018-12-31 20:00:00	1496
2018-12-31 21:00:00	1372
2018-12-31 22:00:00	1218
2018-12-31 23:00:00	1062

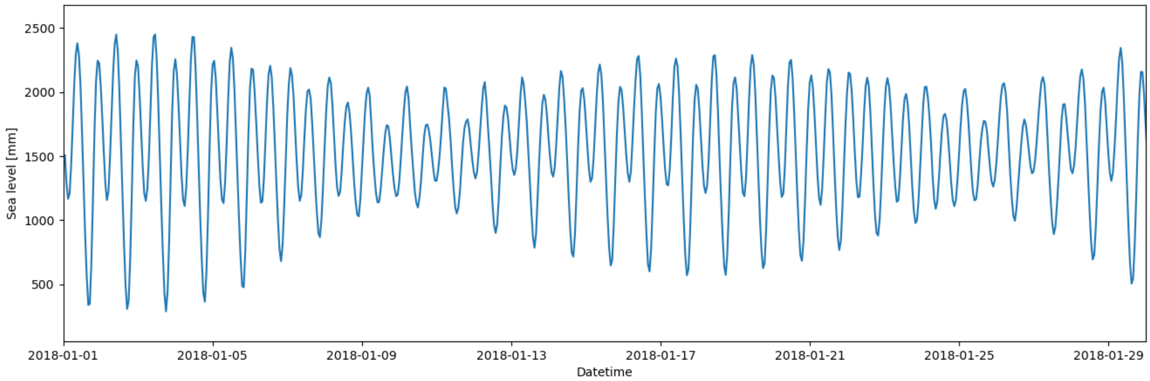
8759 rows × 1 columns

In [3]:

```
fig, ax = plt.subplots(figsize=(16, 5))

ax.plot(sea_level.index, sea_level['Sea level [mm]'])
ax.set_xlim(np.datetime64('2018-01-01'), np.datetime64('2018-01-30'))
ax.set_xlabel('Datetime')
ax.set_ylabel('Sea level [mm]')

plt.show()
```



Beobachtungen

- Viele Zeitreihen enthalten zyklische Schwingungen mit unterschiedlichen Perioden
- Wir können Zeitreihen in ihre Komponenten zerlegen
- Die bekannteste Methode ist die schnelle Fourier-Transformation

Fourier Analyse

- In der Mathematik ist eine Fourier-Reihe eine periodische Funktion, die sich aus harmonisch zusammenhängenden Sinuskurven zusammensetzt, die durch eine gewichtete Summation kombiniert werden.
- Mit geeigneten Gewichten kann ein Zyklus (oder eine Periode) der Summation dazu gebracht werden, eine beliebige Funktion in diesem Intervall zu approximieren (oder die gesamte Funktion, wenn sie ebenfalls periodisch ist).
- Als solche ist die Summation eine Synthese einer anderen Funktion.

Diskrete Fourier-Transformation (DFT)

Kann man auf Zeitreihen anwenden, welche in regelmäßigen Abständen gemessen wurden:

$DFT[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-\phi i} = x[n] \cdot (cos(\phi) - sin(\phi)i)$

wo

$\phi = k\frac{n}{N}2\pi$

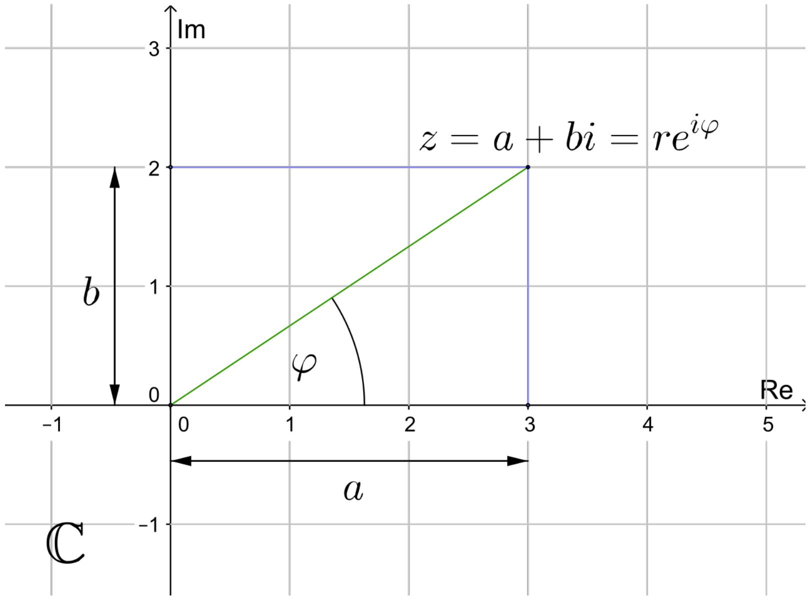
Frage: Was ist hier i?

Umweg: Komplexe Zahlen

Die komplexen Zahlen erweitern den Zahlenbereich auf dass die Gleichung $x^2 = -1$ lösbar wird:

- $i = -1$ ist definiert als imaginäre Zahl
- Kartesische Beschreibung: $z = a + bi$, a der reele Anteil und b der imaginäre Anteil
- Polare Beschreibung: $r = e^{\phi i}$, r der Radius und ϕ der Winkel ist
- Euler's Formel: $r = e^{\phi i} = cos(\phi) + sin(\phi)i$

Quelle: Wikipedia



Komplexe Zahlen werden in *NumPy* unterstützt:

In [5]:
```
compl_number = 3 + 2j
compl_number
```

Out[5]:
```
(3+2j)
```

In [6]:
```
np.real(compl_number)
```

Out[6]:
```
3.0
```

In [7]:
```
np.imag(compl_number)
```

Out[7]:
```
2.0
```

In [8]:
```
np.abs(compl_number)
```

Out[8]:
```
3.605551275463989
```

In [9]:
```
print('Angle in radians: {:.2f}'.format(np.angle(compl_number)))
print('Angle in degrees: {:.2f}°'.format(np.degrees(np.angle(compl_number))))
```

```
Angle in radians: 0.59
Angle in degrees: 33.69°
```

Verbindung zur Sinus- und Cosinus-Funktion

Siehe Beispiel von Jack Schaedler

Beispiel: Zusammengesetzte Komponenten

Als Beispiel "basteln" wir ein paar Zeitreihen-Komponenten:

In [11]:
```
# a linear time vector
time = np.linspace(0, 10, 10*96, endpoint=True)
time
```

```
Out[11]: array([ 0.        ,  0.01042753,  0.02085506,  0.03128259,  0.04171011,
        0.05213764,  0.06256517,  0.0729927 ,  0.08342023,  0.09384776,
        0.10427529,  0.11470282,  0.12513034,  0.13555787,  0.1459854 ,
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       9.80187696,  9.81230448,  9.82273201,  9.83315954,  9.84358707,
```

```
        9.8540146 ,  9.86444213,  9.87486966,  9.88529718,  9.89572471,
        9.90615224,  9.91657977,  9.9270073 ,  9.93743483,  9.94786236,
        9.95828989,  9.96871741,  9.97914494,  9.98957247, 10.        ])
```

In [12]:
```
# component 1
freq1 = 1
amp1 = 1
comp1 = amp1 * np.cos(time*2*np.pi*freq1)

# component 2
freq2 = 0.5
amp2 = 3
comp2 = amp2 * np.cos(time*2*np.pi*freq2)

# component 3
freq3 = 2
amp3 = 2
comp3 = amp3 * np.cos(time*2*np.pi*freq3)

# all combined ...
comp_sum = comp1 + comp2 + comp3
```

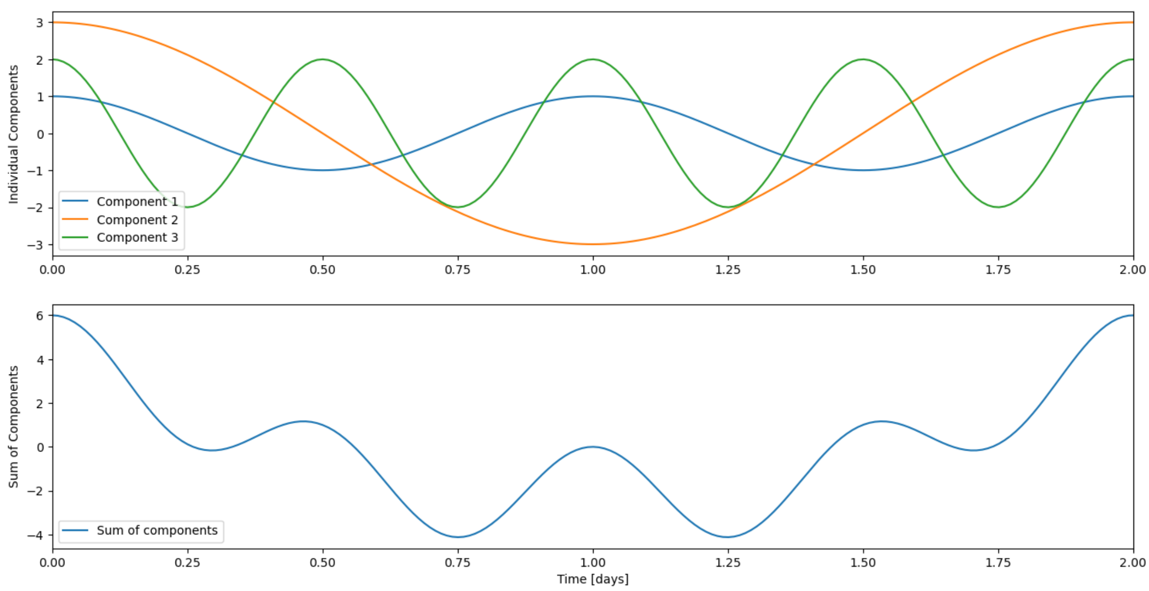
Wie sieht das aus?

In [13]:
```
fig, ax = plt.subplots(2, 1, figsize=(16, 8))

ax[0].plot(time, comp1, label='Component 1')
ax[0].plot(time, comp2, label='Component 2')
ax[0].plot(time, comp3, label='Component 3')
ax[0].set_xlim(0, 2)
ax[0].set_ylabel('Individual Components')
ax[0].legend()

ax[1].plot(time, comp_sum, label='Sum of components')
ax[1].set_xlim(0, 2)
ax[1].set_xlabel('Time [days]')
ax[1].set_ylabel('Sum of Components')
ax[1].legend()

plt.show()
```



HINWEIS: Mit der DFT kann man diesen Summationsprozess umkehren, d.h. eine Zeitreihe in Komponenten bestimmter Frequenzen zerlegen. Allerdings muss man hier einige Dinge beachten!

Fast Fourier Transform (FFT)

- Eine schnelle Fourier-Transformation (FFT) ist ein Algorithmus, der die diskrete Fourier-Transformation (DFT) einer Sequenz oder deren Inverse (IDFT) berechnet
- Die Fourier-Analyse wandelt ein Signal von seiner ursprünglichen Domäne (oft Zeit oder Raum) in eine Darstellung im Frequenzbereich um, und umgekehrt für die inverse Transformation
- Die DFT erhält man, indem man eine Folge von Werten in Komponenten unterschiedlicher Frequenzen zerlegt

NumPy's *fft()* Funktion

Nun wird über *NumPy* die *FFT* funktion angewendet.

Achtung: Das Ergebnis ist komplex, hat zwei Seiten (!) und muss deshalb angepasst werden

In [14]:
```
# Anzahl der Datenwerte
N = len(comp_sum)

# Anzahl der Werte pro Frequenzeinheit
T = 96
```

In [15]:
```
# calculate the FFT
data_fft = np.fft.fft(comp_sum)

# only use half the data
data_fft = data_fft[0:int(N/2)]
data_fft
```

Out[15]:
```
array([ 6.00000000e+00+0.00000000e+00j,  6.13976690e+00+2.00924119e-02j,
        6.63155911e+00+4.34040726e-02j,  7.82612588e+00+7.68352797e-02j,
        1.15744336e+01+1.51517636e-01j,  1.44395808e+03+2.36288175e+01j,
       -3.10614353e+00-6.09968246e-02j,  1.09668408e+00+2.51266270e-02j,
        3.21640112e+00+8.42244274e-02j,  6.37634002e+00+1.87853036e-01j,
        4.81568178e+02+1.57649098e+01j, -2.73587407e+00-9.85269577e-02j,
        2.00972898e-01+7.89624667e-03j,  1.47733918e+00+6.28874986e-02j,
        2.42458442e+00+1.11159861e-01j,  3.37490669e+00+1.65798534e-01j,
        4.55807659e+00+2.38878672e-01j,  6.33971707e+00+3.53057793e-01j,
        9.71201605e+00+5.72747558e-01j,  1.94667061e+01+1.21195050e+00j,
        9.57226446e+02+6.27399360e+01j, -2.03696744e+01-1.40205956e+00j,
       -1.00295763e+01-7.23327823e-01j, -6.61275114e+00-4.98666115e-01j,
       -4.90344337e+00-3.85909362e-01j, -3.87511161e+00-3.17741051e-01j,
       -3.18746175e+00-2.71860905e-01j, -2.69493049e+00-2.38738365e-01j,
       -2.32473436e+00-2.13613080e-01j, -2.03638806e+00-1.93840152e-01j,
       -1.80555045e+00-1.77831197e-01j, -1.61669125e+00-1.64573863e-01j,
       -1.45942605e+00-1.53391859e-01j, -1.32654254e+00-1.43815821e-01j,
       -1.21287193e+00-1.35509537e-01j, -1.11461099e+00-1.28225671e-01j,
       -1.02889784e+00-1.21778084e-01j, -9.53536840e-01-1.16023915e-01j,
       -8.86814993e-01-1.10851617e-01j, -8.27375914e-01-1.06172772e-01j,
       -7.74131491e-01-1.01916344e-01j, -7.26198606e-01-9.80245803e-02j,
       -6.82852997e-01-9.44500141e-02j, -6.43495080e-01-9.11532545e-02j,
       -6.07624239e-01-8.81013218e-02j, -5.74819246e-01-8.52663805e-02j,
       -5.44723148e-01-8.26247633e-02j, -5.17031495e-01-8.01562103e-02j,
       -4.91483061e-01-7.78432696e-02j, -4.67852464e-01-7.56708210e-02j,
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       -2.52909866e-01-5.37576516e-02j, -2.44262701e-01-5.27556870e-02j,
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       -1.54008433e-01-4.12664354e-02j, -1.49840677e-01-4.06757097e-02j,
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```

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-5.54739558e-03-7.63533498e-03j, -5.47134053e-03-7.58271334e-03j,
-5.39616804e-03-7.53034378e-03j, -5.32186653e-03-7.47822333e-03j,
-5.24842463e-03-7.42634909e-03j, -5.17583118e-03-7.37471818e-03j,
-5.10407519e-03-7.32332778e-03j, -5.03314588e-03-7.27217508e-03j,
-4.96303266e-03-7.22125733e-03j, -4.89372511e-03-7.17057180e-03j,
-4.82521301e-03-7.12011581e-03j, -4.75748630e-03-7.06988670e-03j,
-4.69053509e-03-7.01988184e-03j, -4.62434967e-03-6.97009866e-03j,
-4.55892050e-03-6.92053460e-03j, -4.49423819e-03-6.87118713e-03j,
-4.43029352e-03-6.82205377e-03j, -4.36707742e-03-6.77313205e-03j,
-4.30458098e-03-6.72441956e-03j, -4.24279543e-03-6.67591388e-03j,
-4.18171216e-03-6.62761265e-03j, -4.12132270e-03-6.57951353e-03j,
-4.06161872e-03-6.53161421e-03j, -4.00259204e-03-6.48391241e-03j,
-3.94423459e-03-6.43640586e-03j, -3.88653846e-03-6.38909235e-03j,
-3.82949587e-03-6.34196966e-03j, -3.77309915e-03-6.29503562e-03j,
-3.71734078e-03-6.24828809e-03j, -3.66221335e-03-6.20172493e-03j,
-3.60770958e-03-6.15534405e-03j, -3.55382229e-03-6.10914337e-03j,
-3.50054445e-03-6.06312084e-03j, -3.44786912e-03-6.01727443e-03j,
-3.39578948e-03-5.97160214e-03j, -3.34429883e-03-5.92610199e-03j,
-3.29339057e-03-5.88077201e-03j, -3.24305820e-03-5.83561028e-03j,
-3.19329535e-03-5.79061486e-03j, -3.14409572e-03-5.74578389e-03j,
-3.09545314e-03-5.70111547e-03j, -3.04736152e-03-5.65660775e-03j,
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-2.90633312e-03-5.52403065e-03j, -2.86038649e-03-5.48014765e-03j,
-2.81496186e-03-5.43641641e-03j, -2.77005370e-03-5.39283518e-03j,
-2.72565658e-03-5.34940224e-03j, -2.68176517e-03-5.30611591e-03j,
-2.63837420e-03-5.26297450e-03j, -2.59547849e-03-5.21997635e-03j,
-2.55307296e-03-5.17711982e-03j, -2.51115259e-03-5.13440327e-03j,
-2.46971244e-03-5.09182511e-03j, -2.42874766e-03-5.04938372e-03j,
-2.38825348e-03-5.00707754e-03j, -2.34822519e-03-4.96490501e-03j,
-2.30865815e-03-4.92286457e-03j, -2.26954782e-03-4.88095470e-03j,
-2.23088972e-03-4.83917388e-03j, -2.19267942e-03-4.79752062e-03j,
-2.15491258e-03-4.75599342e-03j, -2.11758493e-03-4.71459081e-03j,
-2.08069226e-03-4.67331134e-03j, -2.04423043e-03-4.63215357e-03j,
-2.00819536e-03-4.59111606e-03j, -1.97258303e-03-4.55019740e-03j,
-1.93738950e-03-4.50939620e-03j, -1.90261088e-03-4.46871105e-03j,
-1.86824333e-03-4.42814059e-03j, -1.83428310e-03-4.38768345e-03j,
-1.80072648e-03-4.34733828e-03j, -1.76756981e-03-4.30710375e-03j,
-1.73480950e-03-4.26697853e-03j, -1.70244201e-03-4.22696130e-03j,
-1.67046387e-03-4.18705077e-03j, -1.63887165e-03-4.14724564e-03j,
-1.60766198e-03-4.10754463e-03j, -1.57683153e-03-4.06794648e-03j,
-1.54637705e-03-4.02844994e-03j, -1.51629531e-03-3.98905374e-03j,
-1.48658315e-03-3.94975667e-03j, -1.45723745e-03-3.91055750e-03j,
-1.42825516e-03-3.87145500e-03j, -1.39963324e-03-3.83244799e-03j,
-1.37136874e-03-3.79353526e-03j, -1.34345874e-03-3.75471563e-03j,
-1.31590034e-03-3.71598794e-03j, -1.28869074e-03-3.67735100e-03j,
```

```
       -1.26182713e-03-3.63880368e-03j, -1.23530678e-03-3.60034482e-03j,
       -1.20912700e-03-3.56197328e-03j, -1.18328513e-03-3.52368795e-03j,
       -1.15777855e-03-3.48548769e-03j, -1.13260471e-03-3.44737141e-03j,
       -1.10776107e-03-3.40933800e-03j, -1.08324514e-03-3.37138637e-03j,
       -1.05905449e-03-3.33351542e-03j, -1.03518670e-03-3.29572410e-03j,
       -1.01163941e-03-3.25801132e-03j, -9.88410280e-04-3.22037603e-03j,
       -9.65497034e-04-3.18281717e-03j, -9.42897415e-04-3.14533371e-03j,
       -9.20609206e-04-3.10792459e-03j, -8.98630229e-04-3.07058880e-03j,
       -8.76958344e-04-3.03332530e-03j, -8.55591442e-04-2.99613309e-03j,
       -8.34527453e-04-2.95901115e-03j, -8.13764343e-04-2.92195848e-03j,
       -7.93300107e-04-2.88497409e-03j, -7.73132780e-04-2.84805698e-03j,
       -7.53260427e-04-2.81120619e-03j, -7.33681148e-04-2.77442072e-03j,
       -7.14393076e-04-2.73769961e-03j, -6.95394374e-04-2.70104190e-03j,
       -6.76683239e-04-2.66444663e-03j, -6.58257901e-04-2.62791285e-03j,
       -6.40116618e-04-2.59143961e-03j, -6.22257682e-04-2.55502598e-03j,
       -6.04679414e-04-2.51867102e-03j, -5.87380165e-04-2.48237381e-03j,
       -5.70358316e-04-2.44613341e-03j, -5.53612280e-04-2.40994892e-03j,
       -5.37140496e-04-2.37381942e-03j, -5.20941434e-04-2.33774401e-03j,
       -5.05013591e-04-2.30172178e-03j, -4.89355494e-04-2.26575184e-03j,
       -4.73965697e-04-2.22983329e-03j, -4.58842782e-04-2.19396525e-03j,
       -4.43985359e-04-2.15814683e-03j, -4.29392064e-04-2.12237717e-03j,
       -4.15061561e-04-2.08665538e-03j, -4.00992540e-04-2.05098059e-03j,
       -3.87183717e-04-2.01535195e-03j, -3.73633837e-04-1.97976858e-03j,
       -3.60341666e-04-1.94422965e-03j, -3.47306000e-04-1.90873429e-03j,
       -3.34525658e-04-1.87328165e-03j, -3.21999485e-04-1.83787090e-03j,
       -3.09726350e-04-1.80250118e-03j, -2.97705149e-04-1.76717168e-03j,
       -2.85934800e-04-1.73188154e-03j, -2.74414246e-04-1.69662995e-03j,
       -2.63142456e-04-1.66141608e-03j, -2.52118420e-04-1.62623910e-03j,
       -2.41341152e-04-1.59109820e-03j, -2.30809693e-04-1.55599256e-03j,
       -2.20523104e-04-1.52092137e-03j, -2.10480469e-04-1.48588382e-03j,
       -2.00680896e-04-1.45087910e-03j, -1.91123518e-04-1.41590641e-03j,
       -1.81807485e-04-1.38096496e-03j, -1.72731976e-04-1.34605393e-03j,
       -1.63896187e-04-1.31117254e-03j, -1.55299339e-04-1.27631999e-03j,
       -1.46940675e-04-1.24149549e-03j, -1.38819458e-04-1.20669827e-03j,
       -1.30934976e-04-1.17192752e-03j, -1.23286534e-04-1.13718248e-03j,
       -1.15873463e-04-1.10246235e-03j, -1.08695112e-04-1.06776637e-03j,
       -1.01750853e-04-1.03309375e-03j, -9.50400796e-05-9.98443724e-04j,
       -8.85622049e-05-9.63815517e-04j, -8.23166627e-05-9.29208359e-04j,
       -7.63029096e-05-8.94621484e-04j, -7.05204208e-05-8.60054125e-04j,
       -6.49686929e-05-8.25505516e-04j, -5.96472437e-05-7.90974896e-04j,
       -5.45556094e-05-7.56461502e-04j, -4.96933493e-05-7.21964576e-04j,
       -4.50600400e-05-6.87483359e-04j, -4.06552804e-05-6.53017094e-04j,
       -3.64786884e-05-6.18565025e-04j, -3.25299022e-05-5.84126397e-04j,
       -2.88085803e-05-5.49700459e-04j, -2.53144004e-05-5.15286457e-04j,
       -2.20470602e-05-4.80883640e-04j, -1.90062776e-05-4.46491259e-04j,
       -1.61917895e-05-4.12108564e-04j, -1.36033532e-05-3.77734807e-04j,
       -1.12407450e-05-3.43369240e-04j, -9.10376095e-06-3.09011116e-04j,
       -7.19221728e-06-2.74659690e-04j, -5.50594782e-06-2.40314215e-04j,
       -4.04480898e-06-2.05973947e-04j, -2.80867342e-06-1.71638141e-04j,
       -1.79743488e-06-1.37306052e-04j, -1.01100616e-06-1.02976938e-04j,
       -4.49319972e-07-6.86500530e-05j, -1.12327557e-07-3.43246548e-05j])
```

NumPy's *fftfreq()* Funktion

Diese Funktion berechnet aus der diskreten Abtastung in der Zeitdomäne die äquivalente Abtastung in der Frequenzdomäne.

Achtung: Die Einheit der Frequenzdomäne ist hier 1/Zeit, z.B. Hertz (1/Sekunde) oder Zyklen pro Tag (1/Tag) (Engl. "cycles per day" cpd)

In [16]:
```
# calculate the frequency spacing
freq = np.fft.fftfreq(N, d=1/T)

# only use half the data
freq = freq[0:int(N/2)]
freq
```

Out[16]:
```
array([ 0. ,  0.1,  0.2,  0.3,  0.4,  0.5,  0.6,  0.7,  0.8,  0.9,  1. ,
        1.1,  1.2,  1.3,  1.4,  1.5,  1.6,  1.7,  1.8,  1.9,  2. ,  2.1,
        2.2,  2.3,  2.4,  2.5,  2.6,  2.7,  2.8,  2.9,  3. ,  3.1,  3.2,
        3.3,  3.4,  3.5,  3.6,  3.7,  3.8,  3.9,  4. ,  4.1,  4.2,  4.3,
        4.4,  4.5,  4.6,  4.7,  4.8,  4.9,  5. ,  5.1,  5.2,  5.3,  5.4,
        5.5,  5.6,  5.7,  5.8,  5.9,  6. ,  6.1,  6.2,  6.3,  6.4,  6.5,
        6.6,  6.7,  6.8,  6.9,  7. ,  7.1,  7.2,  7.3,  7.4,  7.5,  7.6,
        7.7,  7.8,  7.9,  8. ,  8.1,  8.2,  8.3,  8.4,  8.5,  8.6,  8.7,
        8.8,  8.9,  9. ,  9.1,  9.2,  9.3,  9.4,  9.5,  9.6,  9.7,  9.8,
        9.9, 10. , 10.1, 10.2, 10.3, 10.4, 10.5, 10.6, 10.7, 10.8, 10.9,
       11. , 11.1, 11.2, 11.3, 11.4, 11.5, 11.6, 11.7, 11.8, 11.9, 12. ,
       12.1, 12.2, 12.3, 12.4, 12.5, 12.6, 12.7, 12.8, 12.9, 13. , 13.1,
       13.2, 13.3, 13.4, 13.5, 13.6, 13.7, 13.8, 13.9, 14. , 14.1, 14.2,
       14.3, 14.4, 14.5, 14.6, 14.7, 14.8, 14.9, 15. , 15.1, 15.2, 15.3,
       15.4, 15.5, 15.6, 15.7, 15.8, 15.9, 16. , 16.1, 16.2, 16.3, 16.4,
       16.5, 16.6, 16.7, 16.8, 16.9, 17. , 17.1, 17.2, 17.3, 17.4, 17.5,
       17.6, 17.7, 17.8, 17.9, 18. , 18.1, 18.2, 18.3, 18.4, 18.5, 18.6,
       18.7, 18.8, 18.9, 19. , 19.1, 19.2, 19.3, 19.4, 19.5, 19.6, 19.7,
       19.8, 19.9, 20. , 20.1, 20.2, 20.3, 20.4, 20.5, 20.6, 20.7, 20.8,
       20.9, 21. , 21.1, 21.2, 21.3, 21.4, 21.5, 21.6, 21.7, 21.8, 21.9,
       22. , 22.1, 22.2, 22.3, 22.4, 22.5, 22.6, 22.7, 22.8, 22.9, 23. ,
       23.1, 23.2, 23.3, 23.4, 23.5, 23.6, 23.7, 23.8, 23.9, 24. , 24.1,
       24.2, 24.3, 24.4, 24.5, 24.6, 24.7, 24.8, 24.9, 25. , 25.1, 25.2,
       25.3, 25.4, 25.5, 25.6, 25.7, 25.8, 25.9, 26. , 26.1, 26.2, 26.3,
       26.4, 26.5, 26.6, 26.7, 26.8, 26.9, 27. , 27.1, 27.2, 27.3, 27.4,
       27.5, 27.6, 27.7, 27.8, 27.9, 28. , 28.1, 28.2, 28.3, 28.4, 28.5,
       28.6, 28.7, 28.8, 28.9, 29. , 29.1, 29.2, 29.3, 29.4, 29.5, 29.6,
       29.7, 29.8, 29.9, 30. , 30.1, 30.2, 30.3, 30.4, 30.5, 30.6, 30.7,
       30.8, 30.9, 31. , 31.1, 31.2, 31.3, 31.4, 31.5, 31.6, 31.7, 31.8,
       31.9, 32. , 32.1, 32.2, 32.3, 32.4, 32.5, 32.6, 32.7, 32.8, 32.9,
       33. , 33.1, 33.2, 33.3, 33.4, 33.5, 33.6, 33.7, 33.8, 33.9, 34. ,
       34.1, 34.2, 34.3, 34.4, 34.5, 34.6, 34.7, 34.8, 34.9, 35. , 35.1,
       35.2, 35.3, 35.4, 35.5, 35.6, 35.7, 35.8, 35.9, 36. , 36.1, 36.2,
       36.3, 36.4, 36.5, 36.6, 36.7, 36.8, 36.9, 37. , 37.1, 37.2, 37.3,
       37.4, 37.5, 37.6, 37.7, 37.8, 37.9, 38. , 38.1, 38.2, 38.3, 38.4,
       38.5, 38.6, 38.7, 38.8, 38.9, 39. , 39.1, 39.2, 39.3, 39.4, 39.5,
       39.6, 39.7, 39.8, 39.9, 40. , 40.1, 40.2, 40.3, 40.4, 40.5, 40.6,
       40.7, 40.8, 40.9, 41. , 41.1, 41.2, 41.3, 41.4, 41.5, 41.6, 41.7,
       41.8, 41.9, 42. , 42.1, 42.2, 42.3, 42.4, 42.5, 42.6, 42.7, 42.8,
       42.9, 43. , 43.1, 43.2, 43.3, 43.4, 43.5, 43.6, 43.7, 43.8, 43.9,
       44. , 44.1, 44.2, 44.3, 44.4, 44.5, 44.6, 44.7, 44.8, 44.9, 45. ,
       45.1, 45.2, 45.3, 45.4, 45.5, 45.6, 45.7, 45.8, 45.9, 46. , 46.1,
       46.2, 46.3, 46.4, 46.5, 46.6, 46.7, 46.8, 46.9, 47. , 47.1, 47.2,
       47.3, 47.4, 47.5, 47.6, 47.7, 47.8, 47.9])
```

In [15]:
```
len(data_fft)
```

Out[15]:
```
480
```

Das Amplitudenspektrum

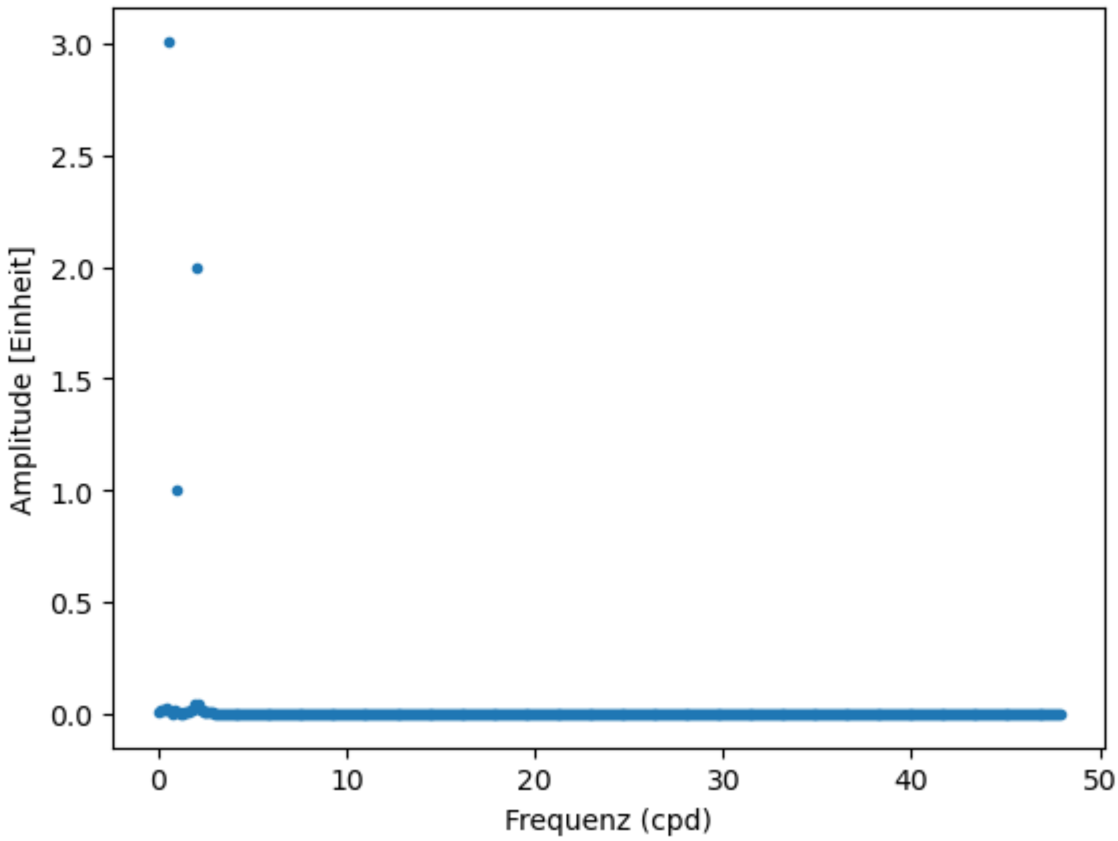
Für das Amplitudenspektrum muss die Energie angepasst werden an:

- Das halbseite Spektrum (Faktor 2)
- Die Anzahl der Werte im Datensatz (Faktor N)

In [17]:
```
# calculate the amplitude spektrum
amplitude = (2/N)*np.abs(data_fft)

plt.plot(freq, amplitude, '.')
plt.xlabel('Frequenz (cpd)')
plt.ylabel('Amplitude [Einheit]')
```

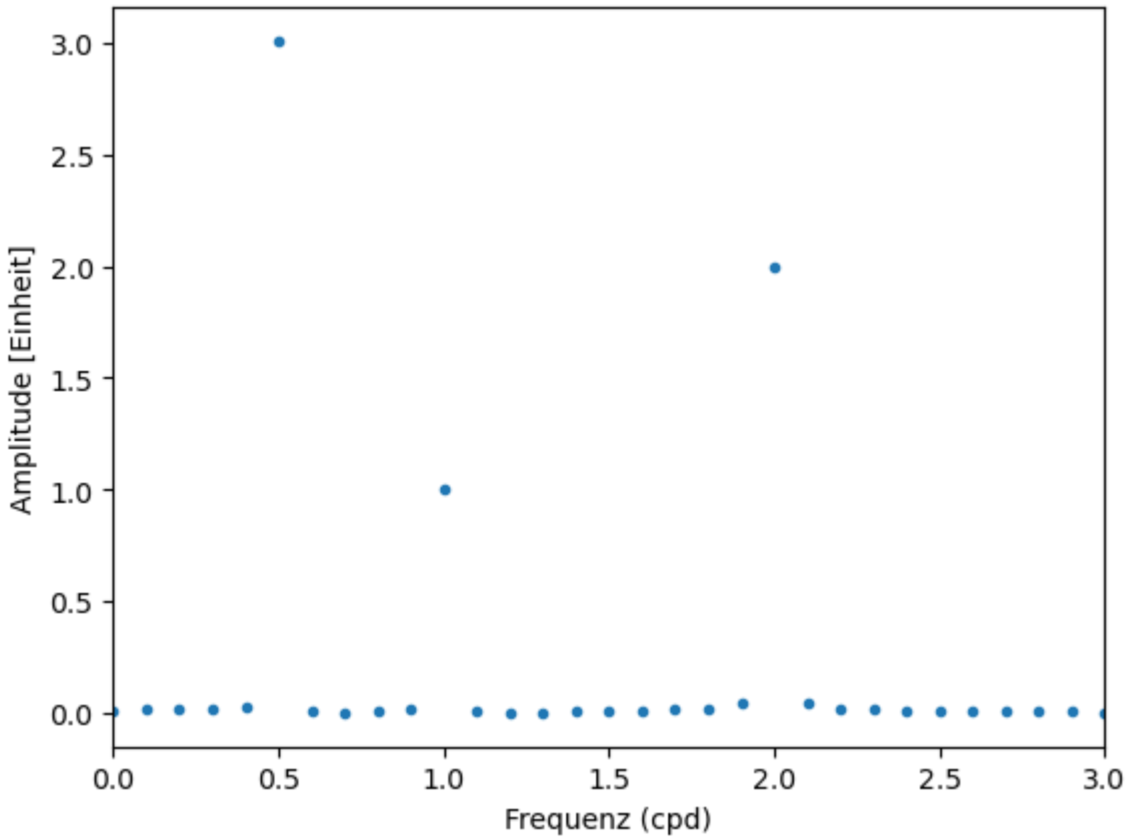
Out[17]: `Text(0, 0.5, 'Amplitude [Einheit]')`



In [18]:
```
# the amplitude scaled correctly
amplitude = (2/N)*np.abs(data_fft)

plt.plot(freq, amplitude, '.')
plt.xlabel('Frequenz (cpd)')
plt.ylabel('Amplitude [Einheit]')
plt.xlim(0,3)
```

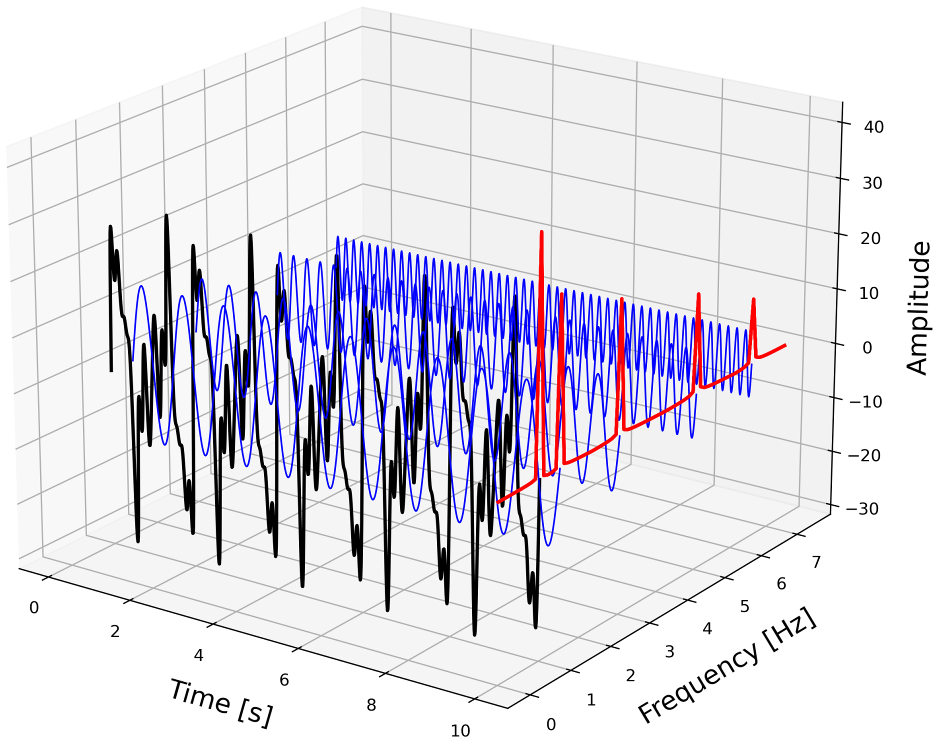
Out[18]: `(0.0, 3.0)`



Frequenz- und Zeitdomäne

Man kann die DFT als ein mehrdimensionales Objekt verstehen:

- Eine Dimension beschreibt die **Zeitdomäne**
- Eine Dimension beschreibt die **Frequenzdomäne**



Anwendung auf die Meeresspiegel Zeitreihe

Das Amplitudenspektrum

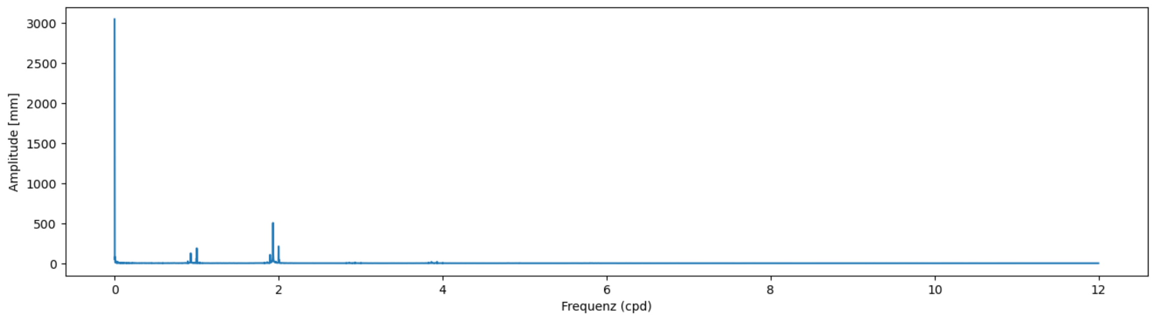
In [19]:
```
N = len(sea_level)
T = 24

data_fft = np.fft.fft(sea_level['Sea level [mm]'])
data_fft = data_fft[0:int(N/2)]

freq = np.fft.fftfreq(N, d=1/T)
freq = freq[0:int(N/2)]

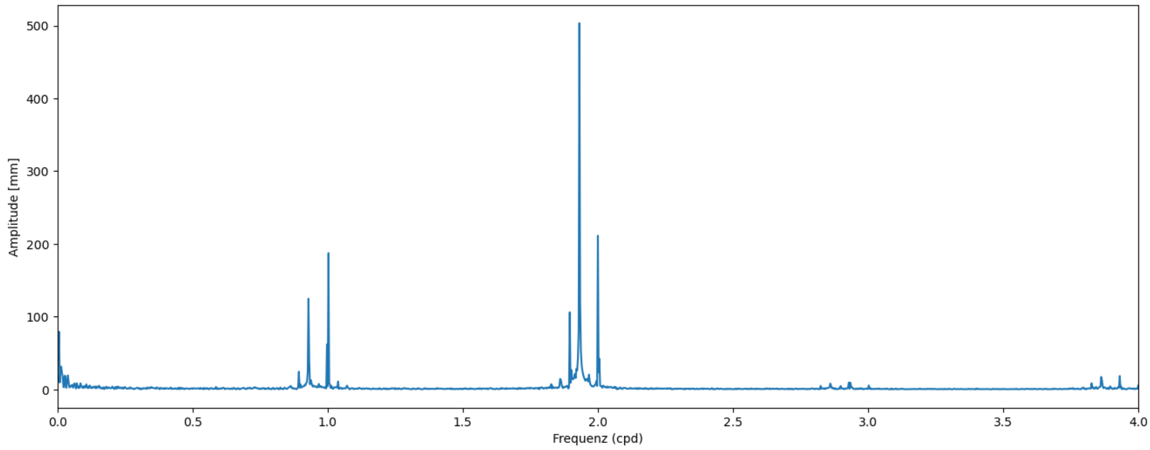
amplitude = (2/N)*np.abs(data_fft)

fig, ax = plt.subplots(figsize=(16,4))
ax.plot(freq, amplitude)
ax.set_xlabel('Frequenz (cpd)')
ax.set_ylabel('Amplitude [mm]')
plt.show()
```



Hinweis: Der Nullte Eintrag der FFT ist die durchschnittliche Abweichung von Null

In [20]:
```
fig, ax = plt.subplots(figsize=(16, 6))
ax.plot(freq[1:], amplitude[1:])
ax.set_xlabel('Frequenz (cpd)')
ax.set_ylabel('Amplitude [mm]')
ax.set_xlim(0, 4)
plt.show()
```



In [21]:
```
result = pd.DataFrame({'Frequency': freq[1:], 'Amplitude': amplitude[1:]})
result.sort_values('Amplitude', ascending=False, inplace=True)
result
```

Out[21]:

	Frequency	Amplitude
704	1.931727	503.494963
729	2.000228	211.264371
365	1.002854	187.401555
338	0.928873	124.666260
705	1.934467	119.012559
...	...	...
4292	11.762987	0.006969
3714	10.179244	0.005686
3946	10.814933	0.005304
3883	10.642311	0.003813
4005	10.976596	0.001969

4378 rows × 2 columns

Zusammenfassung

- Die "Peaks" im Amplitudenspektrum zeigen die Stärke (=Amplituden) der Frequenzkomponenten, welche in diesem Datensatz enthalten sind
- In diesem Beispiel sind die typischen Komponenten der Gezeiten zu sehen

Hinweis: Mehr Informationen zu Gezeitenkomponenten sind von der NOAA Tides & Currents zusammengefasst.

ENDE