Лабораторная работа 6
Элементы вариационного исчисления

Пример решения задачи разобран на лекции

Найти допустимые экстремали функционала $J(y)$, удовлетворяющие граничным условиям $y(a)=A, y(b)=B.$ Что можно сказать об экстремумах функционала?

1. $J(y)=\int_1^e\left(\frac{2y(x)}{x^2}-\left(y'(x)\right)^2\right)dx, y(1)=0, y(e)=1.$
2. $J(y)=\int_0^{\pi/4}\left(\left(y'(x)\right)^2-4y^2(x)\right)dx, y(0)=0, y(\frac{\pi}{4})=1.$
3. $J(y)=\int_{-1}^0\left(12x\cdot y(x)-\left(y'(x)\right)^2\right)dx, y(-1)=1, y(0)=0.$
4. $J(y)=\int_{-1}^1\left(\left(8-24x^2\right)y(x)-\left(y'(x)\right)^2\right)dx, y(-1)=y(1)=-1.$
5. $J(y)=\int_0^{\pi}\left(\frac{y(x)}{2}\sin\frac{x}{2}-\left(y'(x)\right)^2\right)dx, y(0)=0, y(\pi)=1.$
6. $J(y)=\int_0^{\pi/2}\left(2\left(\sin x+\cos x\right)y(x)-\left(y'(x)\right)^2\right)dx, y(0)=1, y(\frac{\pi}{2})=1.$
7. $J(y)=\int_0^1\left(8e^{2x}y(x)+\left(y'(x)\right)^2\right)dx, y(0)=1, y(1)=e^2-1.$
8. $J(y)=\int_0^1\left(\frac{2y(x)}{\left(x+1\right)^2}-\left(y'(x)\right)^2\right)dx, y(0)=0, y(1)=\ln 2.$
9. $J(y)=\int_1^2\left(\frac{8y(x)}{x^3}+\left(y'(x)\right)^2\right)dx, y(1)=2, y(2)=1.$
10. $J(y)=\int_0^{\pi}\left(6x\cdot y(x)-\left(y'(x)\right)^2-4\cos 2x\cdot y'(x)\right)dx, y(0)=0, y(\pi)=\pi^3.$
11. $J(y)=\int_0^3\left(\frac{y(x)}{2\sqrt{\left(x+1\right)^3}}-\left(y'(x)\right)^2+8x^3\cdot y'(x)\right)dx, y(0)=1, y(3)=83.$
12. $J(y)=\int_0^2\left(\left(y'(x)\right)^2+x\cdot y'(x)\right)dx, y(0)=y(2)=0.$
13. $J(y)=\int_0^{\pi}\left(\left(y'(x)\right)^2-4\cos 2x\cdot y'(x)\right)dx, y(0)=0, y(\pi)=0.$
14. $J(y)=\int_0^{\pi}\left(\left(y'(x)\right)^2+6\sin 3x\cdot y'(x)\right)dx, y(0)=1, y(\pi)=-1.$
15. $J(y)=\int_1^e\left(\left(y'(x)\right)^2-\frac{2y'(x)}{x}\right)dx, y(1)=0, y(e)=1.$
16. $J(y)=\int_1^e\left(\left(y'(x)\right)^2-\frac{4x\cdot y'(x)}{x^2+1}\right)dx, y(-1)=\ln 2, y(1)=\ln 2.$

17. $J(y)=\int_{2}^{6}\left(\left(y'(x)\right)^{2}-\dfrac{y'(x)}{\sqrt{x-2}}\right)dx,\ y(2)=0, y(6)=2.$

18. $J(y)=\int_{0}^{2}\left(\left(y'(x)\right)^{4}+\left(y'(x)\right)^{3}\right)dx,\ y(0)=0,\ y(2)=4.$

19. $J(y)=\int_{-1}^{0}\left(x\cdot y(x)-\left(y'(x)\right)^{2}\right)dx,\ y(0)=1, y(1)=\dfrac{1}{4}.$

20. $J(y)=\int_{0}^{\pi}\left(4\cos 2x\cdot y'(x)-\left(y'(x)\right)^{2}\right)dx,\ y(0)=-2, y(\pi)=-2.$

21. $J(y)=\int_{0}^{1}\left(\dfrac{2y'(x)}{x+1}-\left(y'(x)\right)^{2}\right)dx,\ y(0)=3, y(1)=3+\ln 2.$

22. $J(y)=\int_{0}^{3}\left(\dfrac{y'(x)}{\sqrt{x+1}}+\left(y'(x)\right)^{2}\right)dx,\ y(0)=1, y(3)=0.$

23. $J(y)=\int_{1}^{2}\left(\dfrac{4y'(x)}{x^{3}}+\left(y'(x)\right)^{2}\right)dx,\ y(1)=1, y(2)=\dfrac{1}{4}.$

24. $J(y)=\int_{1}^{2}\left(\left(4x+\dfrac{2}{x^{2}}\right)y'(x)-\left(y'(x)\right)^{2}\right)dx,\ y(1)=0,\ y(2)=3{,}5.$

25. $J(y)=\int_{0}^{1}\left(\left(8x^{3}-4\right)y'(x)-\left(y'(x)\right)^{2}\right)dx,\ y(0)=1, y(1)=0.$

26. $J(y)=\int_{0}^{1}\left(y(x)+2x\cdot y'(x)+\left(y'(x)\right)^{2}\right)dx,\ y(0)=y(1)=0.$

27. $J(y)=\int_{0}^{\ln 2}\left(\left(y'(x)\right)^{2}+3y^{2}(x)\right)e^{2x}dx,\ y(0)=0, y(\ln 2)=\dfrac{15}{8}.$

28. $J(y)=\int_{0}^{1}x^{2}\left(y'(x)\right)^{2}dx,\ y(0)=3,\ y(1)=1.$

29. $J(y)=\int_{0}^{6}\left(2x\cdot y(x)-\left(y'(x)\right)^{2}\right)dx,\ y(0)=y(6)=1.$

30. $J(y)=\int_{0}^{1}\left(\left(y'(x)\right)^{4}-6\left(y'(x)\right)^{2}\right)dx,\ y(0)=y(1)=0.$

31. $J(y)=\int_{0}^{\pi}\left(4\cos x\cdot y(x)+\left(y'(x)\right)^{2}-y^{2}(x)\right)dx,\ y(0)=y(\pi)=0.$

32. $J(y)=\int_{0}^{1}\left(e^{x+y(x)}y(x)-y'(x)-\sin x\right)dx,\ y(0)=2, y(1)=0.$