機器學習數學
Section 4.6
(助教課)

助教: 林楷鈞

日期: 2024.10.8

教室:E513

TA HW2 Answer #4.6 –6

6. Consider the bases $B = \{\mathbf{u}_1, \mathbf{u}_2\}$ and $B' = \{\mathbf{u}'_1, \mathbf{u}'_2\}$ for R^2, where

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{u}'_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad \mathbf{u}'_2 = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

11

(a) Find the transition matrix from B' to B.

(b) Find the transition matrix from B to B'.

(c) Compute the coordinate vector $[\mathbf{w}]_B$, where

$$\mathbf{w} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$$

and use (12) to compute $[\mathbf{w}]_{B'}$.

(d) Check your work by computing $[\mathbf{w}]_{B'}$ directly.

6.給定不同bases B和B'之間進行轉換

(a) 找出base B' 到base B 的transition matrix
(b) 找出base B 到 base B' 的transition matrix
(C) 計算座標向量$[w]_B$並使用transition matrix 計算$[w]_{B'}$
(d) 直接計算$[w]_{B'}$，檢查你之前的計算是否正確

TA HW2 Answer #4.6 –6 (a)

6. Consider the bases $B = \{\mathbf{u}_1, \mathbf{u}_2\}$ and $B' = \{\mathbf{u}'_1, \mathbf{u}'_2\}$ for R^2, where

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\quad \mathbf{u}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\quad \mathbf{u}'_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix},\quad \mathbf{u}'_2 = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

(a) Find the transition matrix from B' to B

Explain : 給定一個base B' ， 要求找到base B 的轉換矩陣

In this part. B' is the old basis and B is the new basis :

- B' is old basis ， include vector $u'_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ 和 $u'_2 = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$
- B is new basis, include vector $u_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ 和 $u_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

- **Transition matrix method (Change of basis) => $[I|P_{B'\to B}]$, I denote identify matrix**

$$[new\ basis \mid old\ basis] = \left[\begin{array}{cc|cc} 1 & 0 & 2 & -3 \\ 0 & 1 & 1 & 4 \end{array}\right]$$

Ans : $\boldsymbol{P_{B'\to B}} = \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$

TA HW2 Answer #4.6 –6 (b),(c)

6. Consider the bases $B = \{\mathbf{u}_1, \mathbf{u}_2\}$ and $B' = \{\mathbf{u}'_1, \mathbf{u}'_2\}$ for R^2, where

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \mathbf{u}'_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad \mathbf{u}'_2 = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

(C) Compute the coordinate vector $[w]_B$， where

$w = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$ and use (12) to compute $[w]_{B'}$

Explain : 給定$w = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$ 計算vector w 在basis B 下的座標向量$[w]_{B'}$

Formula 12 : $[\mathbf{v}]_{B'} = \mathrm{P}_{\mathrm{B}\to\mathrm{B}'}[\mathrm{v}]_{\mathrm{B}}$

$\mathrm{P}_{\mathrm{B}\to\mathrm{B}'}$: B is old basis , B' is new basis

[new basis | old basis] = $\left[\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 1 & 4 & 0 & 1 \end{array}\right]$,

做reduce row echelon of matrix 求 $\mathrm{P}_{\mathrm{B}\to\mathrm{B}'}$

$\mathrm{P}_{\mathrm{B}\to\mathrm{B}'} = \begin{bmatrix} \frac{4}{11} & \frac{3}{11} \\ -\frac{1}{11} & \frac{2}{11} \end{bmatrix}$ => Q_B **Answer**

$\mathrm{P}_{\mathrm{B}\to\mathrm{B}'} = \begin{bmatrix} \frac{4}{11} & \frac{3}{11} \\ -\frac{1}{11} & \frac{2}{11} \end{bmatrix}$

$[\mathrm{w}]_{\mathrm{B}} = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$

Q_c **Answer:**

$[\boldsymbol{w}]_{B'} = \begin{bmatrix} -\frac{3}{11} \\ -\frac{13}{11} \end{bmatrix}$

TA HW2 Answer #4.6 –6 (d)

(d) Check your work by computing $[w]_{B'}$ directly.

(d) Check your work by computing $[\mathbf{w}]_{B'}$ directly.

Explain : 表達 $w = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$ 計算vector w 在basis B' 的線性組合，解方程式求出座標向量$[w]_{B'}$
結果是否跟C的部分結果為一致，一致才能證明計算正確

$$\begin{bmatrix} 3 \\ -5 \end{bmatrix} = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

$$\begin{aligned} 2c_1 &- 3c_2 &= 3 \\ c_1 &+ 4c_2 &= -5 \end{aligned}$$

做reduce row echelon of matrix

$$\begin{bmatrix} 1 & 0 & -\frac{3}{11} \\ 0 & 1 & -\frac{13}{11} \end{bmatrix}.$$

$$c_1 = -\frac{3}{11}, c_2 = -\frac{13}{11}$$

Answer:

$$[\mathbf{w}]_{B'} = \begin{bmatrix} -\frac{3}{11} \\ -\frac{13}{11} \end{bmatrix}$$

This matches the result obtained in part (c)