Solar Oven Written Report

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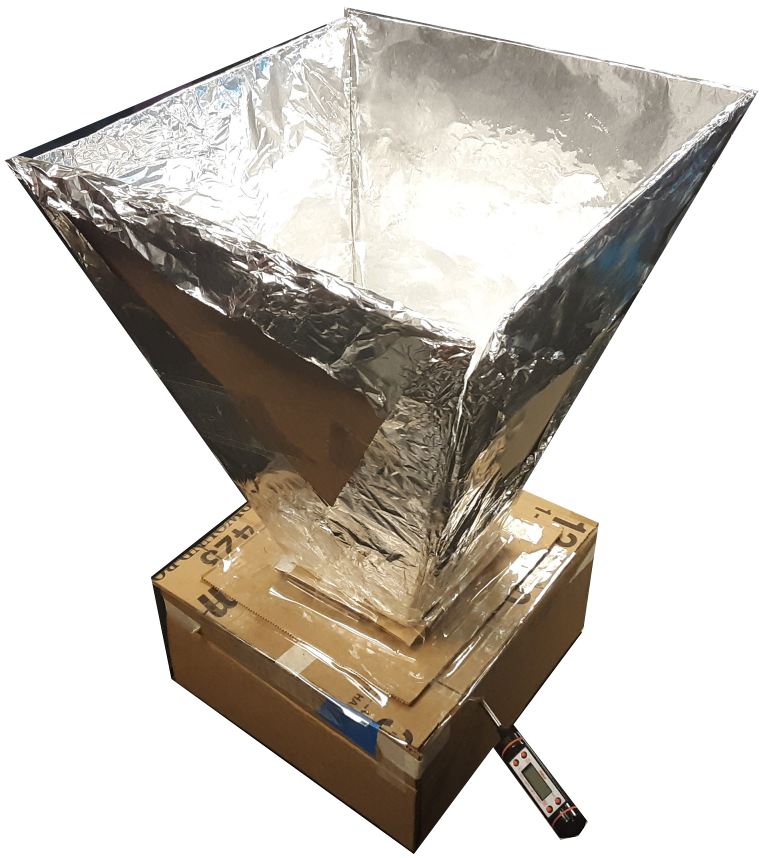


1 Abstract

This report covers the process of designing a solar oven to fulfill requirements given by the University of Arizona for an Engineering 102 class. The solar oven was constructed out of commonly available materials, such as cardboard, newspaper, aluminum foil, mylar, and various kinds of tape. A performance index was calculated from the amount of materials used and their weight. In this report, the mathematical model, design process, testing, and potential improvements that could be made on the solar oven built by our team are analyzed and explained.

Contents

2 Introduction

The purpose of this project was to investigate how Microsoft Excel could be used to assist in the design of a solar oven and to predict the performance index and maximum temperature of the solar oven using formulas provided in the "Solar Oven Project" packet. A goal seeking macro written in Excel was used to help calculate T_{io}, which was a process that no one in our team had experience with. Through the completion of this project, our team expected to learn a great deal about the principles of thermodynamics and how they can be applied in a practical setting (the solar oven), how to use Microsoft Excel more efficiently, and how to write a properly formatted scientific report. Data used in our calculations was usually experimentally found or found on the University of Arizona's OASIS data tool. The equations used for our calculations were obtained from the project packet. The project also enabled my team member and I to gain a better understanding of how proper planning, Microsoft Excel, and goalseeking macros could be better applied in a workplace scenario and used to maximize productivity.

3 Design Theory and Analytical Model

There are several equations vital to producing a mathematical model of the solar oven. First, we assume that energy is conserved within the oven/sun system and write the equation that shows this:

$$E_{out} = E_{in} \tag{1}$$

This equation shows that the total energy output of the oven, both that useful for cooking and the energy that is dissipated into the oven's surroundings, is equal to the total energy going into the oven. Most energy losses can be attributed to any one or a combination of three mechanisms for heat loss: conduction, convection, and radiation. Conduction losses are due to heated air within the solar oven being in contact with a material that has thermal conductivity, and therefore can conduct the heat away from the cooking chamber. Convection losses are due to the movement of surrounding air (i.e. wind) drawing cold air around the solar oven and drawing away heat within the solar oven. Radiation heat loss is due to infrared energy escaping the oven through the window or gaps in the container and heating the surrounding air or ground. A similar principle can be applied to the power absorbed by the cooking chamber's walls and the total power conducted out of the cooking chamber, since all the heat dissipated out of the cooking chamber in total must be equal to the amount of heat absorbed by the walls of the cooking chamber and the window.

$$P_{out} = P_{absorbed} \tag{2}$$

Next, we can need to calculate P_{out} and $P_{absorbed}$. These equations will serve as the backbone of our design. First, the solar power incident to the window must be calculated:

$$P_{incident} = I_o W \cdot L sin(\theta_s + \beta) \tag{3}$$

Where I_o is the incident solar power density in W/m^2, W is the width of the window in meters, and L is the length of the window in meters. θ_s is the angle the oven's window makes to the suns rays with respect to the ground, and β is the angle the oven's window makes with the ground (Figure 3).

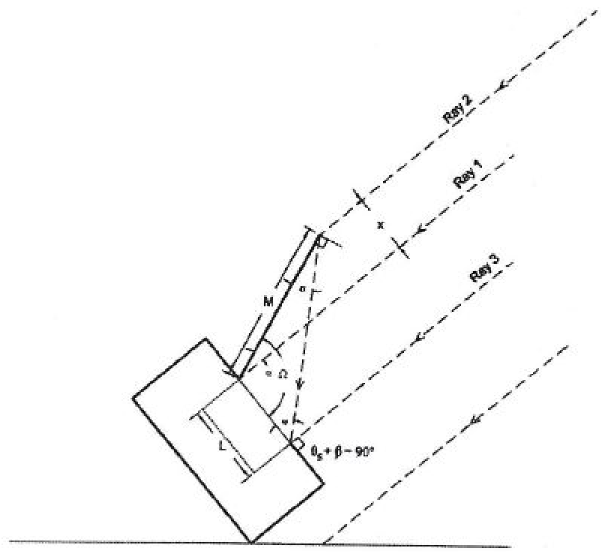


Figure 3. Solar Oven Geometry with Reflectors

In order to maximize $P_{incident}$, the sum of θ_s and β should be 90°so that the product of the expression $sin(\theta_s + \beta)$ is as close to one as possible. If $sin(\theta_s + \beta)$ is one or close to one, $P_{incident}$ is significantly closer to the raw value of $I_o W$, resulting in a higher $P_{incident}$ and therefore better performance. Then, we calculate the power transmitted through the window using the optical transmission coefficient τ:

$$P_{transmitted} = \tau \cdot P_{incident} \tag{4}$$

τ is given in Table 1 of the project packet:

T_{io}(°C)	$U_{w,single}(W/m^2$°C)	$U_{w,double}(W/m^2$°C)
66	10.10	4.88
93	13.90	6.69
121	18.66	8.96
149	24.34	11.74
177	31.6	15.2
204	40.11	19.35

And we can calculate the power absorbed by the walls of the cooking chamber using $P_{transmitted}$ and the absorption coefficient of the cavity walls, α, where ($0 \leq \alpha \geq 1$):

$$P_{absorbed} = \alpha \cdot P_{transmitted} \tag{5}$$

Finally, we can break down the equation for $P_{absorbed}$ into:

$$P_{absorbed} = I_o A_w \cdot \tau \cdot \alpha \cdot sin(\theta_s + \beta) \tag{6}$$

Before we wrap up the equation for P_{out}, we need the equation for the power of transferred heat:

$$P = U \cdot A \cdot (T_{surface} - T_{region\ INTO\ which\ power\ flows}) \tag{7}$$

And finally, to calculate P_{out}, we can combine equation (6) with equation (7) to get:

$$P_{out} = (U_{sb} \cdot A_{sb} + U_w \cdot A_w)(T_{io} - T_{ambient}) \tag{8}$$

Where U_{sb} is the overall heat transfer coefficient for the cooking chamber without the window, A_{sb} is the total area of the walls and floor of the cooking chamber, A_w is the area of the window, T_{io} is the temperature in the interior chamber of the oven, and $T_{ambient}$ is the temperature of the air surrounding the oven.

U_{sb} can be calculated by finding the overall heat transfer coefficient for the walls and floor of the chamber by using the formula

$$\frac{1}{\frac{x_1}{k_1} + \frac{x_2}{k_2} + \frac{x_3}{k_3} + \ldots \frac{x_n}{k_n}} \tag{9}$$

where x_n is the thickness of the material n, and k_n is the thermal conductivity in $W/(°Cm^2)$. The value of k for cardboard and wadded newspaper is given by Table 2:

Material	Thermal conductivity
Cardboard (Holman, 1981)	0.064
Wadded Newspaper (adapted from Johnson, 2003)	0.123

Next, from the previous equations we can find the equation for T_{io}:

$$T_{io} = T_{ambient} + \frac{I_o A_w \cdot \tau \cdot \alpha \cdot sin(\theta_s + \beta)}{(U_{sb} \cdot A_{sb} + U_w \cdot A_w)} \tag{10}$$

By graphing U_w versus T_{io} and creating a 2nd order polynomial best fit line, we get the equation of the best fit line (Fig 3):

$$U_w = \alpha T_{io}^2 + bT_{io} + c \tag{11}$$

After finding the 2nd order polynomial equation of the best fit line in Excel, we can solve for T_{io}, which gives the equation:

$$T_{io} = T_{ambient} + \frac{I_o A_w \cdot \tau \cdot \alpha \cdot sin(\theta_s + \beta)}{(U_{sb} \cdot A_{sb} + \{aT_{io}^2 + bT_{io} + c\} \cdot A_w)} \tag{12}$$

Next, we need the equation for power gain due to the presence of a reflector:

$$P_{absorbed\ with\ a\ reflector} = I_o A_w \cdot G \cdot \tau \cdot \alpha \cdot sin(\theta_s + \beta) \tag{13}$$

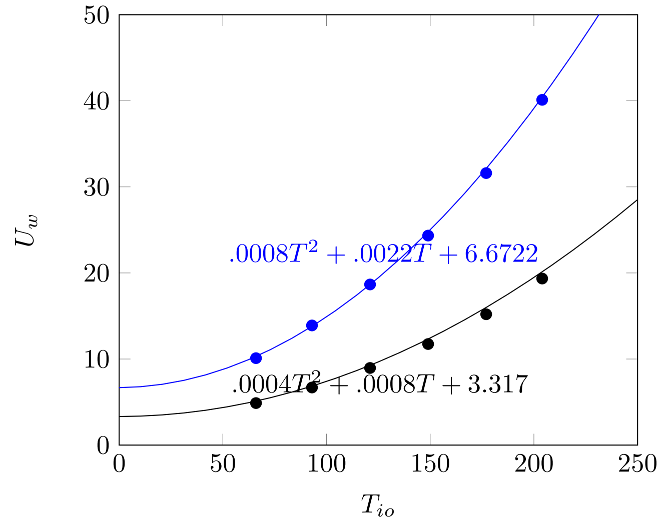


Figure 1: Equations of U_w single (blue) and double (black)

RAY DIAGRAM AND PAGE 15 EXPLANATION SHOULD GO HERE

The appropriate width and length of the reflectors (x and y, respectively) can be calculated via the following equations:

$$\begin{aligned} x &= M sin(\alpha) \\ y &= L sin(\theta_s + \beta) \end{aligned} \tag{14}$$

The power incident to the window due to the reflectors is given by the formula:

$$P_{incident\ to\ window} = I_o W(y + r \cdot x) \tag{15}$$

where r is the reflectivity of the mirror, given by Table 3:

Foil Condition	Reflectivity
Smooth	0.75
Almost Smooth	0.7
Crinkled	0.45-0.6

Then, the equation for the total power absorbed with a reflector can be found:

$$P_{absorbed\ with\ a\ reflector} = I_o A_w \cdot \alpha \cdot \tau \cdot [sin(\theta_s + \beta) + r \cdot (\frac{M}{L}) sin(\alpha)] \tag{16}$$

$(\frac{M}{L})$ can be solved for in a more elegant way by using only α :

$$(\frac{M}{L}) = \frac{sin(90 - 2\alpha)}{sin(\alpha)} \tag{17}$$

A better equation can be found by simplifying equation (16) using equation (13):

$$P_{absorbed\ with\ a\ reflector} = G \cdot P_{absorbed\ without\ a\ reflector} \tag{18}$$

$P_{absorbed\ without\ a\ reflector}$ is calculated through equation (6). G can be calculated using the equation

$$G = 1 + r \cdot (\frac{M}{L}) sin(\alpha) \tag{19}$$

which is simply equation 17 solved for G and simplified. G with N reflectors can be solved for using the equation:

$$G = 1 + N \cdot r \cdot (\frac{M}{L}) sin(\alpha) \tag{20}$$

Lastly, we must solve for T_{io} taking G into account:

$$T_{io} = T_{ambient} + \frac{I_o A_w \cdot G \cdot \tau \cdot \alpha}{(U_{sb} \cdot A_{sb} + U_w \cdot A_w)} \tag{21}$$

which is the final design equation for the oven.

4 Design Requirements

Design Requirements:

- The oven cooking chamber volume must be $1000 cm^3$. The length and height of the cooking chamber need to be at least 5cm each.
- The window opening should be square (W=L)
- The oven must be equipped with convenient access to the cooking chamber so that a small biscuit can be cooked during the demonstration. The oven should be equipped with a rack to support the biscuit off the walls of the oven chamber.
- The oven must accept a digital thermometer that will stick through an oven wall (but not the oven window) to measure the temperature inside the cooking chamber. The plastic housing for the thermometer gauge cannot be placed inside the oven.
- Teams MUST calculate the performance index of their ovens. This index is a ratio of the temperature increase in the oven to the delivered cost of the oven, as shown below:

$$PI = \frac{T_{io} - T_{ambient}}{C} \tag{22}$$

  T_{io} and $T_{ambient}$ are the air temperature inside and outside the oven. Compute the PI in terms of degrees Celsius. The value C is the cost of the oven which is a function of materials, labor and transportation. The labor cost is fixed at $5 and the transportation cost is $0.20 per pound.

- The maximum M/L ratio is 3.
- Minimum final oven temperature is 100°C (optimal is over 200°C)

– Focusing lenses or parabolic designs are not permitted.

These requirements were explicitly stated in the "Design Criteria/Constraints" section of the project packet. However, they were not the only requirements that existed. Other requirements included:

– Complete the project in 87 days

– Use the tools present in the school engineering space

– Use the material variants present in the school engineering space (brands of masking and duct tape, shades of dark paper, types of aluminum foil)

5 Design Description

Our design was a cardboard box cut out of one flat piece of cardboard and held together with masking and duct tape. This larger cardboard box was lined with wadded newspaper until a cavity just large enough to fit the 13x13x6cm (LxWxH) cooking chamber was left over. On top of the oven, a cardboard cover with a 10x10cm square hole in the center was placed to shield the newspaper and provide an extra layer of insulation while allowing a space for the mylar window so that sun could get into the cooking chamber. Our first mylar window was adhered to this cover. On top of the cover, four layers of cardboard were stacked on top of each other in such a way that the first mylar window would not be obstructed. On top of this 11.2mm high stack, another mylar window was placed and stuck down with duct tape. This air gap reduced the thermal conductivity of the window system and thus reduced heat loss through the mylar window in contact with the cooking chamber. The reflectors were designed in such a way that there were no gaps between adjacent reflectors, thus maximizing the area to which a reflective material could be applied and in turn increasing reflector gain.

6 Design Justification

Our design consists of a box within a box, insulated with wadded newspaper and topped with two mylar windows and a reflector assembly. The lid consists of the mylar windows and reflectors, which can simply be set on top of the box's cooking chamber and taped down to ready the solar oven for use. We chose this design because it was effective in that it fit the given and other (practical) requirements, but was the simplest solution that did so and fit our mathematical model. Cardboard was chosen as the box's main material due to easily workable nature and good insulation properties. In areas where additional insulation was vital to good performance, the cardboard was supplemented with thick layers of wadded newspaper. Wadded newspaper was used as the main insulation material due to its low effect on performance index (zero cost and low weight) and good insulation characteristics (low thermal conductivity). The thermal conductivity values for cardboard and wadded newspaper are given by Table 2 and are as follows:

Material	Thermal conductivity
Cardboard (Holman, 1981)	0.064
Wadded Newspaper (adapted from Johnson, 2003)	0.123

7 Test Procedure

To test the solar oven, we used a spare brick and propped our solar oven up on it such that the oven's centerline was directly normal to the sun for maximum efficiency and to ensure no shadows were throwing off our measurements and subsequently the temperature data. Additionally, care had to be taken to make sure that there were no trees, fences, or other structures casting a shadow over the oven. We recorded the temperature every five minutes until the inner mylar window failed, which would have invalidated future datapoints had we continued the test. Ambient temperature was taken with the same thermometer used to measure the temperature of the cooking chamber, and other environmental data (i.e. I_o) was taken from the University of Arizona's OASIS website. The data we recorded is as follows:

Test 1:

Datapoint	Temperature
1	87
2	101
3	119
4	125
5	97
6	125

Test 2:

Datapoint	Temperature
1	119.2
2	125.7
3	131.7
4	126.2
5	135.1

8 Test Results

The results of our tests can be seen in the last table of the previous section. Unfortunately, the data from the first test was lost due to a teammate's sibling accidentally deleting the spreadsheet that data was being recorded on. The data for test 1 is still accurate, but the decimals could not be remembered when the data was put back together and so they were left out. The results of test 1 were highly inaccurate in comparison to the predicted 2-window value. The temperature did not get up high enough to melt the mylar, so the problems encountered in test 2 were not an issue here and thus the 177.39 °C temperature prediction was used. Test 1 had poor results due to the lid not being sealed. We initially just thought hinging the lid/reflector assembly with a piece of tape would be sufficient, but between the first and second tests we decided to line the lid's edges more tape which helped hold heat in. The increase in performance between the two tests are apparent. The results of test 2 were inaccurate when compared to our initial 2-layer window prediction of 177.39 °C, but were very accurate when compared to our 1-layer window prediction of 137.59 °C. The reason our data must be compared to both of these predictions is because the inner mylar window melted due to the heat of the cooking chamber, then the temperature plateaued and maintained 135.1 °C for five minutes past our last datapoint. While we may have achieved the 2-layer window prediction had the inner layer of mylar not melted, we do not know for sure. In all likelihood, the second layer of mylar that was present did not contribute to the final temperature of the oven and instead just helped the oven reach the melting point of the inner layer of mylar more quickly, leaving only one window left, at which point the oven maintained 135.1 °C. The percent error for maximum temperature for the first test is as follows:

Test 1; 177.39°C Target

Datapoint	Percent Error (%)
1	50.9
2	43.1
3	32.9
4	29.5
5	45.3
6	29.5

And lastly, the 1-window and 2-window prediction percent error for maximum temperature for the second test, respectively, were as follows:

Test 2: 177.39°C Target

Datapoint	Percent Error (%)
1	32.8
2	29.1
3	25.7
4	28.8
5	23.8

Test 2: 137.58°C Target

Datapoint	Percent Error (%)
1	13.3
2	8.6
3	4.2
4	8.2
5	1.8

Temperature versus time data for test 1:

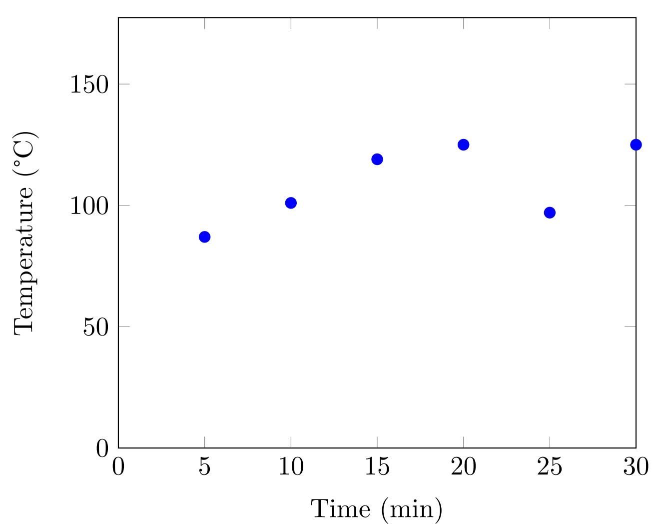


Figure 2: Test 1 Temperature (°C) versus Time (min)

Temperature versus time data for test 2:

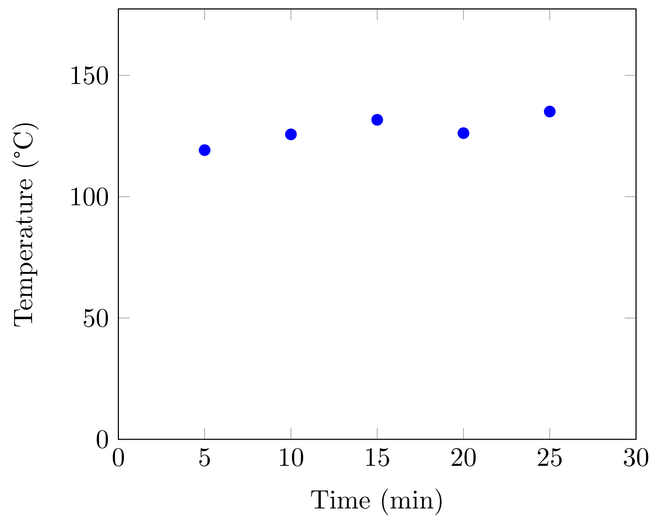
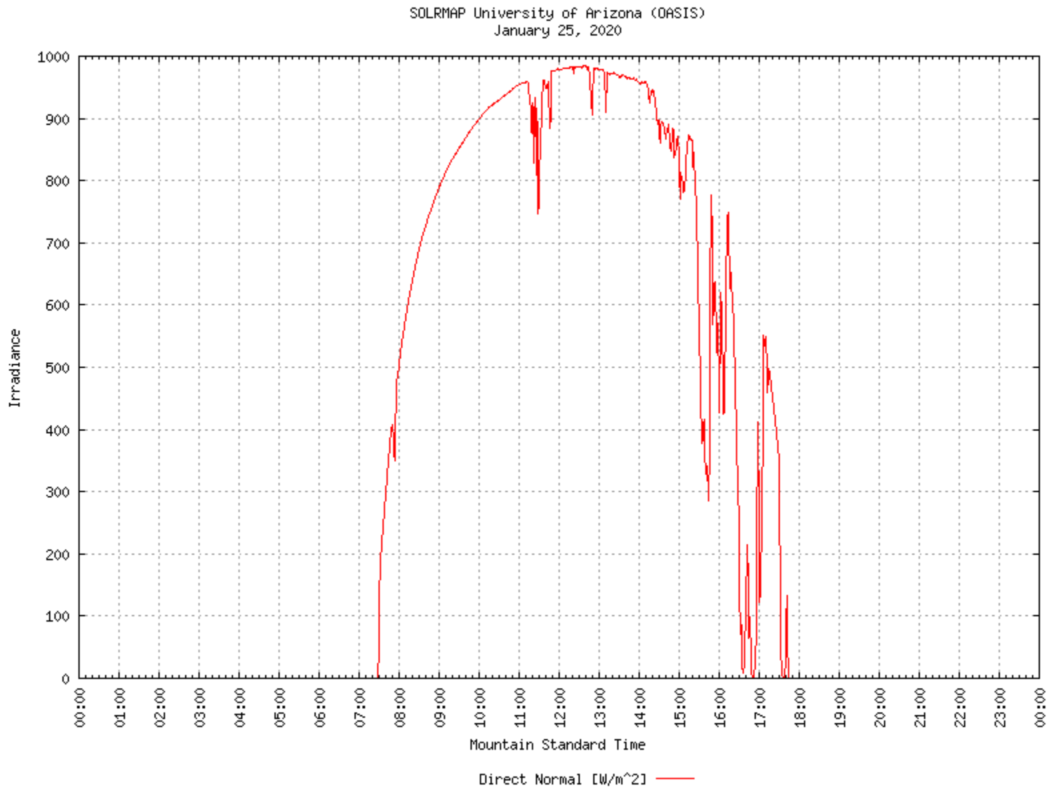


Figure 3: Test 2 Temperature (°C) versus Time (min)

Potential sources of error included small clouds moving in front of the sun for brief periods, variations in irradiance due to atmospheric disturbances, variations in how tightly packed the wadded newspaper was, variations in the density of the cardboard used, inconsistencies in the masking and duct tape used, and discrepancies in the oven's direct normal the sun as a result of the apparent motion of the sun across the sky over time. In the future, forecasts from a website like the National Weather Service could be used to find days with minimal clouds for testing, better ensuring optimal test conditions and therefore a test that would more accurately of the oven's performance. The performance index equation is as follows:

$$PI = \frac{T_{io} - T + ambient}{C} \tag{23}$$

According to this equation, the PI for our fully successful test (test 2) is 15.7. I_o plot for the testing day :



(Figure 5)

Due to the tests all being done in a single day, variation of I_o was kept to a minimum. Over the course of the testing period (12:55 to 14:00), I_o was fairly stable. The time range over which the solar oven was tested was slightly after noon, meaning that the irradiance throughout the day had peaked and was beginning to drop during the testing period. Based on the graph, it appears that 11:55 to 13:00 would have been a better time to test the oven, as the irradiance over this time range was more stable in comparison to the actual time range during which the oven was tested. The abrupt drops in irradiance shown on the graph are likely due to clouds passing over the sensor or very brief random variations in irradiance, and due to their short duration can be considered acceptable.

9 Team Dynamics

Our team dynamic was well balanced. Both group members participated sufficiently and got along very well. Team members were actively engaging in improving the design and giving constructive advice or other input to the group. The team members in our group and their respective functions throughout the project are:

Max Wolf:

- Construction of solar oven
- Fusion 360 model
- Oven Renderings

Emmett Hart:

- Construction of solar oven
- Excel sheet
- Sketches/initial plan

10 Design Critique and Summary

In hindsight, it is apparent that our group would have benefited from better constructed reflectors with smoother aluminum foil and better overall craftsmanship. While our build quality was adequate overall, any project can benefit from extra care and attention to detail. A better mechanism for holding the inner mylar window in place so that it would not warp and hurt the accuracy of the test when it melted could also be of benefit to the solar oven our team constructed. The main recommendation that would be of benefit to future teams doing this project would be to take extra care to make sure their working model was 100% in line with their design. Careful construction and attention to detail truly make all the difference in this project. Another recommendation is to use black paint for the cooking chamber instead of black paper. While a uniform, well done job with the black paper for making the cooking chamber more heat absorbent could be satisfactory, the use of black spray paint enables a team to get a more uniform, darker coating that is in direct, uniform contact with the cooking chamber. Our group had the advantage of having cardboard that was already black (it was a cut up presentation board), but the uniformity and shade of the coating was superior to that which could be achieved with black paper. Reading the forecast for two or three days into the future would also be beneficial to a group's test accuracy, as the group could choose one day out of several based on which one had the most optimal test conditions. This would reduce error provided that the oven was planned correctly, and consequently would make for a better report. Data taken should also be recorded on the cloud so that it cannot be easily lost due to a computer with data on it losing power suddenly and destroying the data. Alternatively, multiple people should have the data, or the data should be backed up onto a USB key. Additionally, making sure the workload on team members was balanced, hearing out other team members' ideas, and making sure that everyone felt good about the project made this project significantly more enjoyable for our team, so other teams should ensure they do the same. While our team dynamic was satisfactory for this project, having team members each come up with their own design right from the start, then combining ideas after (optionally) building individual prototypes and making a final oven would have been significantly more productive than just making one oven right from the start. However, this is only possible if groups have the time and time management skills to accomplish such a task.

Total Words: 3056

11 References

– All equations were obtained from ”Solar Oven Project” by the University of Arizona

Figure 3 was obtained from ”Solar Oven Project” by the University of Arizona

– Figure 5 is from SOLRMAP University of Arizona (OASIS) Daily Plots and Raw Data Files

12 Appendices

– All relevant files at https://github.com/KJ7ILJ/uofa-soven1